Robotics
Exercise 1

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1 Geometry

a) You have a book (coordinate frame B) lying on the table (world frame W). Initially B and W are identical. Now you move the book 1 unit to the right, then rotate it by 45° counter-clock-wise around its origin. Given a dot p marked on the book at position $p^B = (1, 1)$ in the book coordinate frame, what are the coordinates p^W of that dot with respect to the world frame?

b) Given a point x with coordinates $x^W = (0, 1)$ in world frame, what are its coordinates x^B in the book frame?

c) What is the *coordinate* transformation from world frame to book frame, and from book frame to world frame?

Please use homogeneous coordinates to derive these answers. (See http://ipvs.informatik.uni-stuttgart.de/mlr/marc/notes/3d-geometry.pdf for more details on 3D geometry.)

2 Matrix equations

a) Let X, A be arbitrary matrices, A invertible. Solve for X:

$$XA + A^\top = \mathbf{I}$$

b) Let X, A, B be arbitrary matrices, $(C - 2A^\top)$ invertible. Solve for X:

$$X^\top C = [2A(X + B)]^\top$$

c) Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, A \in \mathbb{R}^{d\times n}$. A obviously *not* invertible, but let $A^\top A$ be invertible. Solve for x:

$$(Ax - y)^\top A = \mathbf{0}_n^\top$$

d) As above, additionally $B \in \mathbb{R}^{n\times n}$, B positive-definite. Solve for x:

$$(Ax - y)^\top A + x^\top B = \mathbf{0}_n^\top$$

3 Vector derivatives

Let $x \in \mathbb{R}^n,\ y \in \mathbb{R}^d,\ f, g : \mathbb{R}^n \to \mathbb{R}^d,\ A \in \mathbb{R}^{d\times n},\ C \in \mathbb{R}^{d\times d}$. (Also provide the dimensionality of the results.)

a) What is $\frac{\partial}{\partial x} x$?

b) What is $\frac{\partial}{\partial x}[x^\top x]$?

c) What is $\frac{\partial}{\partial x}[f(x)^\top f(x)]$?

d) What is $\frac{\partial}{\partial x}[f(x)^\top C g(x)]$?

e) Let B and C be symmetric (and pos.def.). What is the minimum of $(Ax - y)^\top C(Ax - y) + x^\top Bx$?

4 Optimization

Given $x \in \mathbb{R}^n,\ f:\ \mathbb{R}^n \to \mathbb{R}$, we want to find $\text{argmin}_x\, f(x)$. (We assume f is uni-modal.)

a) What 1st-order optimization methods (querying $f(x), \nabla f(x)$ in each iteration) do you know?

b) What 2nd-order optimization methods (querying $f(x), \nabla f(x), \nabla^2 f(x)$ in each iteration) do you know?

c) What is backtracking line search?