Robotics
Exercise 06

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1 Controllability

Consider the local linearization of the cart-pole,

$$\dot{x} = Ax + Bu \ , \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{c_2 g}{\frac{4}{3}l - c_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{\frac{4}{3}l - c_2} & 0 \end{pmatrix} , \quad B = \begin{pmatrix} 0 \\ c_1 + \frac{c_1 c_2}{\frac{4}{3}l - c_2} \\ 0 \\ \frac{-c_1}{\frac{4}{3}l - c_2} \end{pmatrix}$$

Use the parameters c_1, c_2, l, g from the last exercise sheet. Is the system controllable?

2 Stable control for the cart-pole

Consider a linear controller $u = w^\top x$ with 4 parameters $w \in \mathbb{R}^4$ for the cart-pole.

a) What is the closed-loop linear dynamics $\dot{x} = \widehat{A}x$ of the system?

b) Test if the controller with $w = (1.0000, 2.6088, 52.9484, 16.5952)$ (computed using ARE) is asymtotically stable. What are the eigenvalues?

c) Come up with a method that finds parameters w such that the closed-loop system is “maximally stable” around $x^* = (0, 0, 0, 0)$ (e.g., asymptotically stable with fastest convergence rate).

d) Output the optimal parameters and test them on the cart-pole simulation of the last exercise.