# Robotics
# Exercise 4

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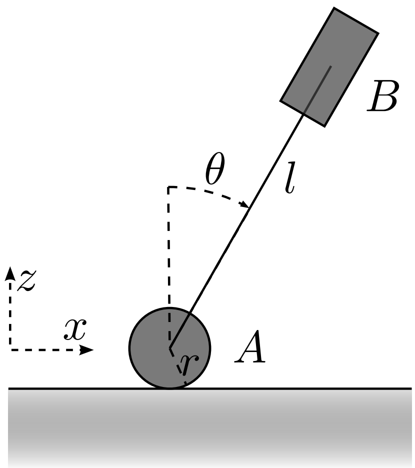
November 16, 2016

## 1 Fun with Euler-Lagrange




Consider an inverted pendulum mounted on a wheel in the 2D x-z-plane; similar to a Segway. The exercise is to derive the Euler-Lagrange equation for this system. Strictly follow the scheme we discussed on slide 03:23.

a) Describe the **pose** $p_i$ of every body (depending on $q$) in $(x, z, \phi)$ coordinates: its position in the x-z-plane, and its rotation $\phi$ relative to the world-vertical.

b) Describe the (linear and angular) velocity $v_i$ of every body.

c) Formulate the kinetic energy $T$

d) Formulate the potential energy $U$

e) Compute the Euler-Lagrange Equation:

$$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}$$

Tips:

- Use the generalized coordinates

$$q = (x, \theta) \tag{1}$$

  where $x$ is the position of the wheel and $\theta$ the angle of the pendulum relative to the world-vertical.

- The system can be parameterized by
  - $m_A, I_A, m_B, I_B$: masses and inertias of bodies $A$ (=wheel) and $B$ (=pendulum)
  - $r$: radius of the wheel
  - $l$: length of the pendulum (height of its COM)

- In this 3-dim space, the mass matrix of every body is

$$M_i = \begin{pmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & I_i \end{pmatrix} \tag{2}$$

(this allows to compute the kinetic energy of a body $i$ with $\frac{1}{2} v_i^\top M_i v_i$)