Robotics
Exercise 7

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1 Riccati equation in discrete time

Consider the time discrete linear quadratic system

$$f(x_t, u_t) = Ax_t + Bu_t$$

$$c(x_t, u_t) = x_t^\top Q x_t + u_t^\top R u_t$$

with the cost function

$$J^\pi = \sum_{t=0}^{\infty} c(x_t, u_t) \ .$$

The Bellman equation (slide 04.13) for this infinite-horizon discrete time system is

$$V(x) = \min_u \big[c(x,u) + V(f(x,u))\big] \ .$$

Start with the Bellman equation and derive the Riccati equation for the system. Similar to the continuous case, you can assume a value function of the form $V(x) = x^\top P x$ with a symmetric matrix P.

2 RRTs for peg in a hole task

Download the code framework here: https://ipvs.informatik.uni-stuttgart.de/mlr/16-Robotics/e07-code.tbz2
Unpack the code and copy the folder 'rrt' into 'robotics15/share/teaching/RoboticsCourse/'

The code demonstrates an RRT exploration and displays the explored endeffector positions.

a) First grow an RRT *backward* from the target configuration $q^* = (0.945499, 0.431195, -1.97155, 0.623969, 2.22355, -0.665206, -1.48356)$ to the initial configuration $q = 0$. Stop when there exists a node close (<`stepSize`) to the goal configuration. Read out the collision free path from the tree and display it. Why would it be more difficult to grow the tree *forward* from $q = 0$ to q^*?

b) Find a collision free path using bi-directional RRTs (that is, 2 RRTs growing together). Use q^* to root the backward tree and $q = 0$ to root the forward tree. Stop when a newly added node is close to the other tree. Read out the collision free path from the tree and display it.

c) Propose a simple method to make the found path smoother (while keeping it collision free). Implement the method and display the smooth trajectory.