Robotics

Path Optimization – briefly

Marc Toussaint
University of Stuttgart
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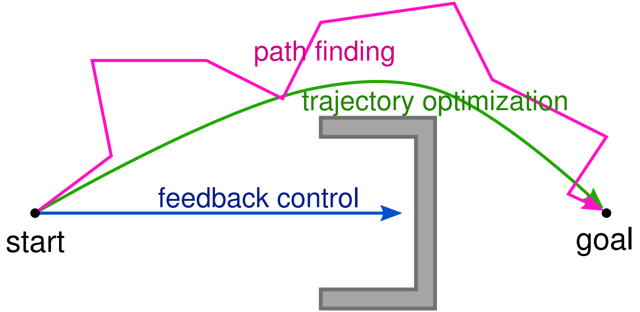
Lecturer: Peter Englert

Outline

- These are only some very brief notes on path optimization
- The aim is to explain how to *formulate* the optimization problem. Concerning the optimization algortihm itself, refer to the *Optimization* lecture.

Feedback control, path finding, trajectory optim.



- Feedback Control: E.g., $q_{t+1} = q_t + J^\sharp (y^* - \phi(q_t))$
- Trajectory Optimization: $\text{argmin}_{q_{0:T}} f(q_{0:T})$
- Path Finding: Find some $q_{0:T}$ with only valid configurations

From inverse kinematics to path costs

- Recall our optimality principle of inverse kinematics

$$\operatorname*{argmin}_q \|q - q_0\|^2_W + \|\Phi(q)\|^2$$

- A trajectory $q_{0:T}$ is a sequence of robot configurations $q_t \in \mathbb{R}^n$
- Consider the cost function

$$f(q_{0:T}) = \sum_{t=0}^{T} \|q_{t\text{-}1} - q_t\|^2_W + \sum_{t=0}^{T} \|\Phi_t(q_t)\|^2$$

(where $(q_{\text{-}1}$ is a given prefix)

- $\|q_{t\text{-}1} - q_t\|^2_W$ represents **control costs**
  $\Phi_t(q_t)$ represents **task costs**

General k-order cost terms

[Notation: x_t instead of q_t represents joint state]

$$\min_{x_{0:T}} \quad \sum_{t=0}^{T} f_t(x_{t-k:t})^\top f_t(x_{t-k:t})$$

$$\text{s.t.} \quad \forall_t : \; g_t(x_{t-k:t}) \le 0 \;, \quad h_t(x_{t-k:t}) = 0 \;.$$

Cost terms

- The $f_t(x_{t-k:t})$ terms can penalize various things:

$k=1$	$f_t(x_{t\text{-}1}, x_t) = x_t - x_{t\text{-}1}$	penalize velocity
$k=2$	$f_t(x_{t\text{-}2}, .., x_t) = x_t - 2x_{t\text{-}1} + x_{t\text{-}2}$	penalize acceleration
$k=3$	$f_t(x_{t\text{-}3}, .., x_t) = x_t - 3x_{t\text{-}1} + 3x_{t\text{-}2} - x_{t\text{-}3}$	penalize jerk

or in some arbitrary task spaces

$k=0$	$f_t(x_t) = \phi(x_t) - y^*$	penalize offset in some task space
$k=1$	$f_t(x_{t\text{-}1}, x_t) = \phi(x_t) - \phi(x_{t\text{-}1})$	
$k=2$	$f_t(x_{t\text{-}2}, .., x_t) = \phi(x_t) - 2\phi(x_{t\text{-}1}) + \phi(x_{t\text{-}2})$	
$k=3$	$f_t(x_{t\text{-}3}, .., x_t) = \phi(x_t) - 3\phi(x_{t\text{-}1} + 3\phi x_{t\text{-}2}) - \phi(x_{t\text{-}3})$	

- And terms f_t can be stacked arbitrarily

Choice of optimizer

$$\begin{aligned} \min_{x_{0:T}} \quad & \sum_{t=0}^{T} f_t(x_{t-k:t})^\top f_t(x_{t-k:t}) \\ \text{s.t.} \quad & \forall_t : \ g_t(x_{t-k:t}) \le 0 \ , \quad h_t(x_{t-k:t}) = 0 \ . \end{aligned}$$

- Constrained optimization methods:
  - Log-barrier, squared penalties
  - **Augmented Lagrangian**

- Note: also the Lagrangian is the form of the so-called **Gauss-Newton** form. The pseudo Hessian is a banded, symmetric, positive-definite matrix.