

# Fuzzy Adaptive Control

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# Computer Discrete Control



- But, a computer can only offer discrete control. Every sampling period, the computer read the sensor data and then makes the decision to a new updated action.
- Sensing and decision at time steps  $kT$  ( $k=0, 1, 2, \dots$ ). The length of sampling period  $T=20, 30, 50\text{ms}, \dots$




$$e'(kT) = \frac{e(kT) - e(kT - T)}{T}$$



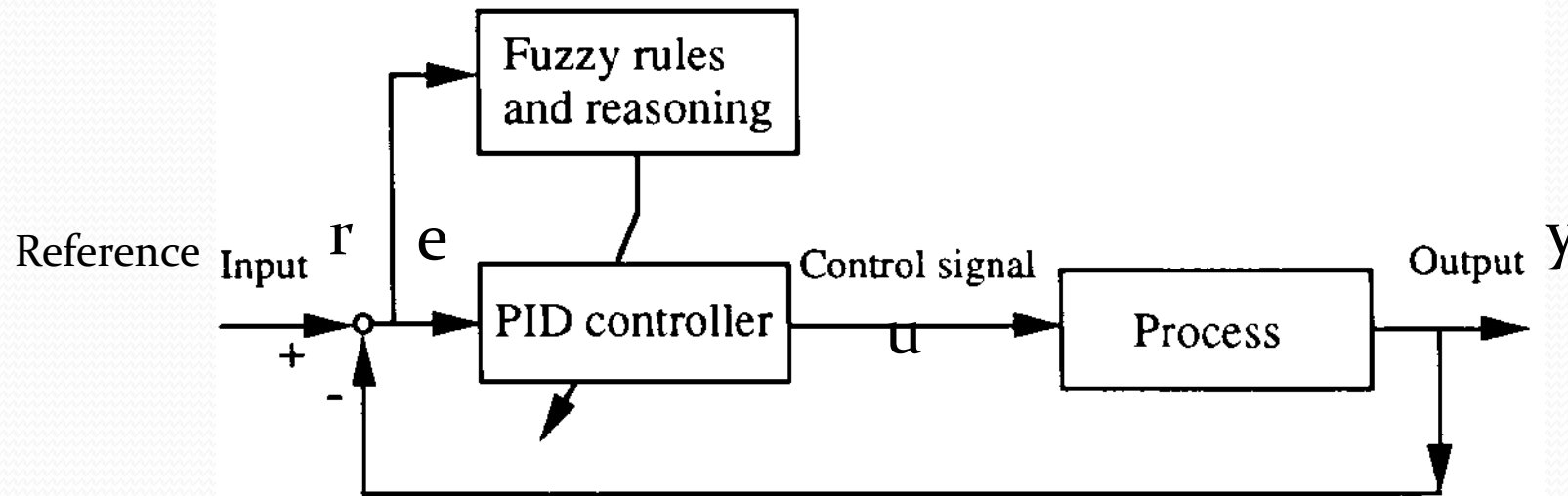
# Outline

- Fuzzy approach to adapting parameters of a conventional controller
  - Fuzzy gain tuning for PID controllers (fuzzy PID control)
- On-line adaptation of fuzzy controllers
  - Fuzzy reference model learning control
- Offline “adaptation” or learning of fuzzy controllers (offline learning and optimization)



# Fuzzy approach to controller parameter adaptation (Fuzzy PID)

# Fuzzy Tuning of PID Controller

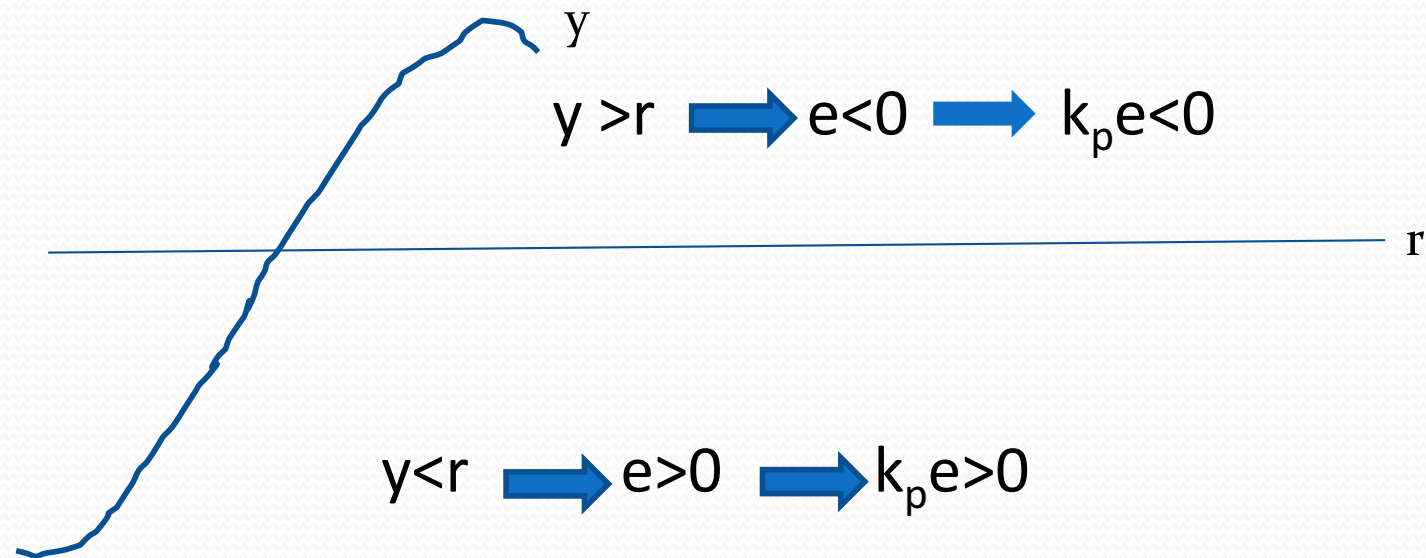


$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d e'(t)$$

$$u(k) = K_p e(k) + K_i T \sum_{j=0}^k e(j) + \frac{K_d}{T} [e(k) - e(k-1)]$$

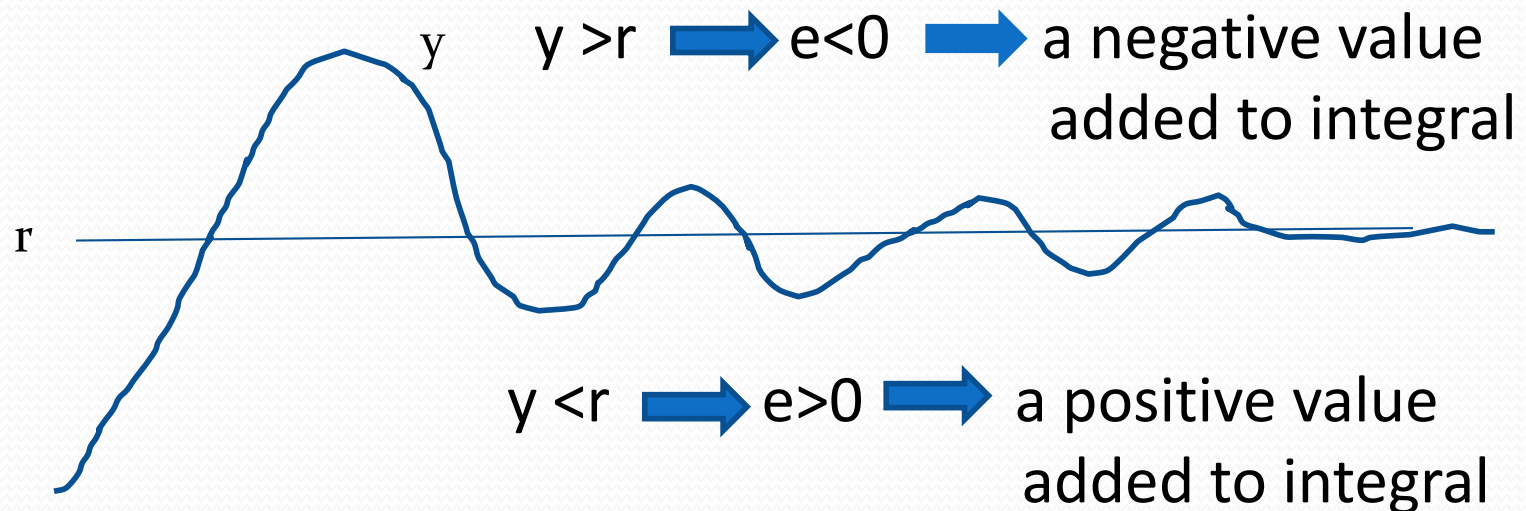
The **proportional gain**  $K_p$ , **integral gain**  $K_i$ , and **derivative gain**  $K_d$  are specified in advance. They remain constant in control process with conventional PID control

# The effect of proportional part



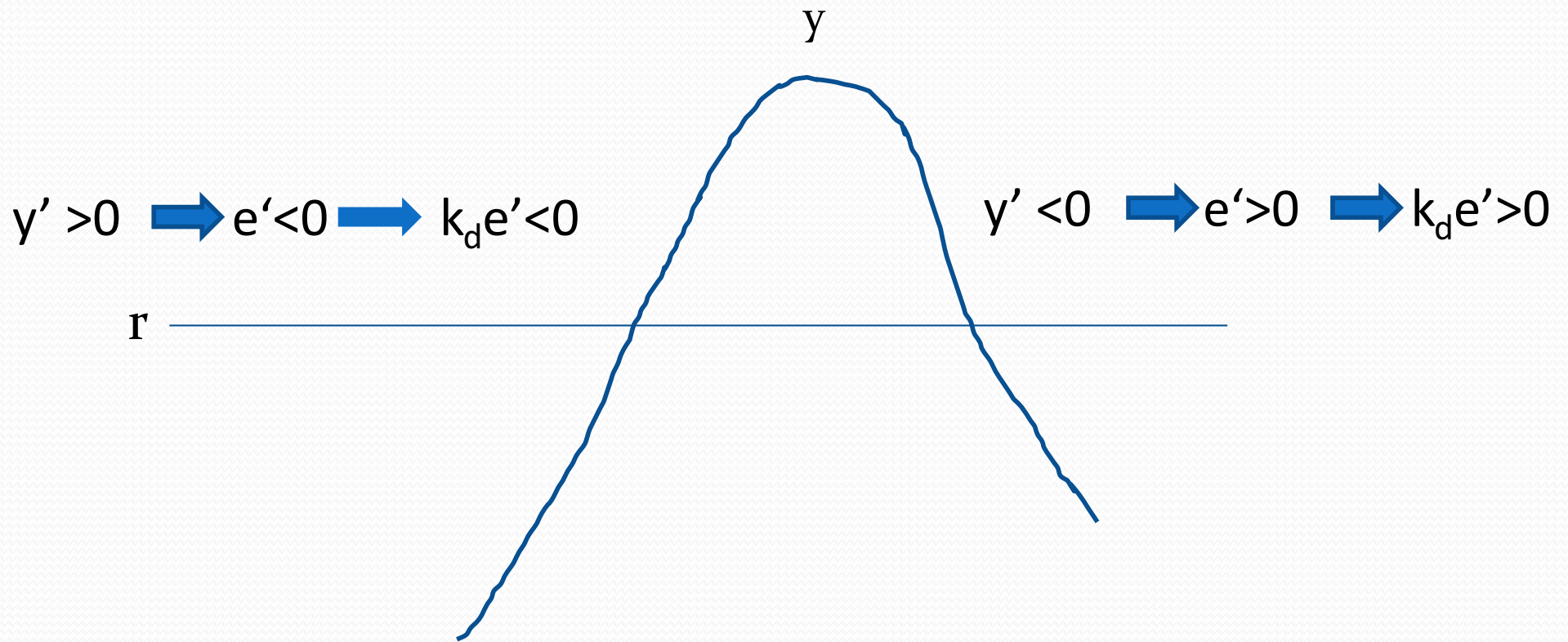
**Proportional part:** typically the main drive in a control loop, reducing a large part of the error.

# Effect of the integral part



**Integral part:** Reducing the final error to zero.  
Summing even a small error over time produces a drive signal to move the system toward a smaller error.

# Effect of the derivative part

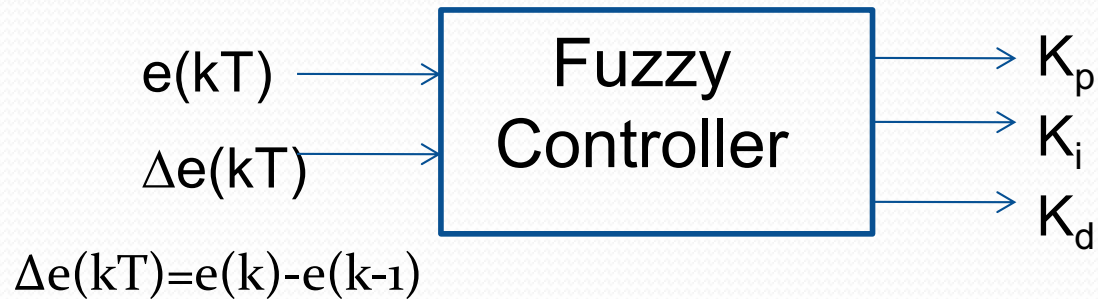


**Derivative part:** Curbing quick changes of the output. This helps reduce overshoot. It has no effect on final error.

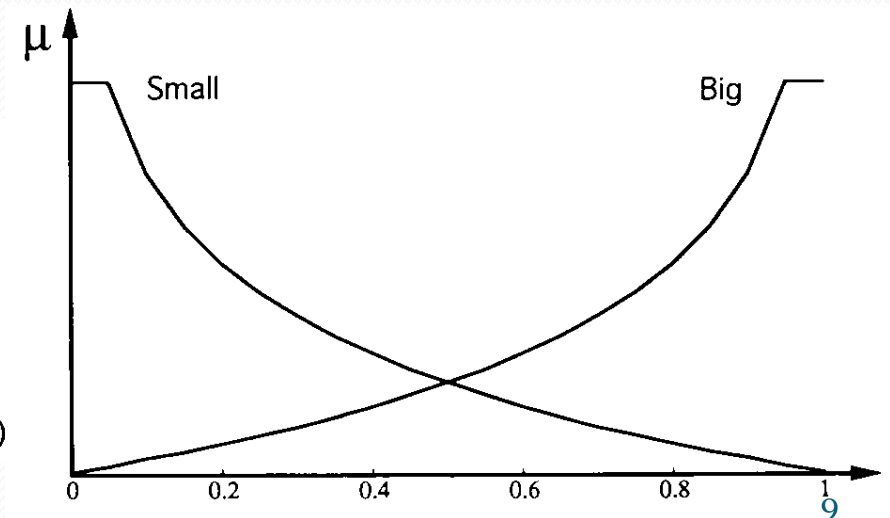
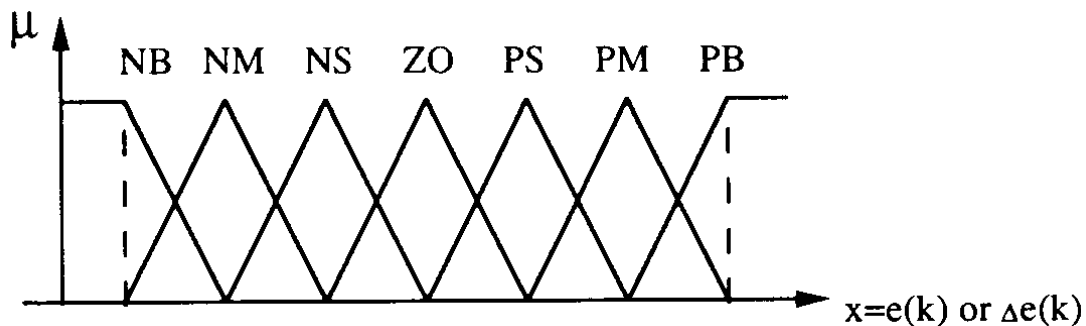


# Fuzzy Tuning of PID

Using fuzzy controller to enable varying PID gains in terms of the situation



Fuzzy membership functions for gains



# Fuzzy Gain Tuning Rules

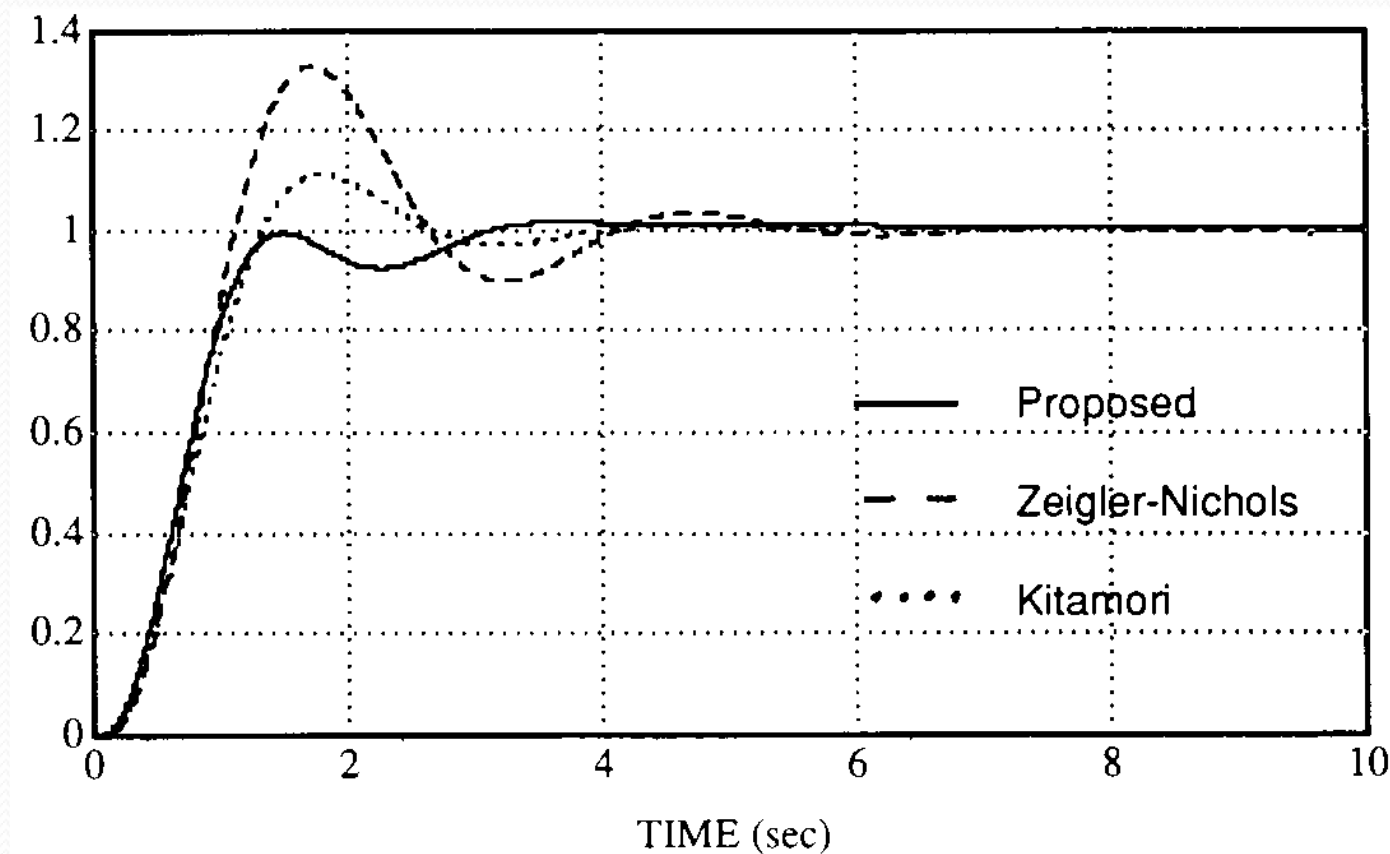
TABLE I  
FUZZY TUNING RULES FOR  $K'_p$

		$\Delta e(k)$						
		NB	NM	NS	ZO	PS	PM	PB
$e(k)$	NB	B	B	B	B	B	B	B
	NM	S	B	B	B	B	B	S
	NS	S	S	B	B	B	S	S
	ZO	S	S	S	B	S	S	S
	PS	S	S	B	B	B	S	S
	PM	S	B	B	B	B	B	S
	PB	B	B	B	B	B	B	B

Interested students (likely with Electrical Engineering background) can find further information from the paper:

“Fuzzy gain scheduling of PID controllers”, IEEE Trans. Systems, Man & Cybernetics, No. 5, 1993, pp. 1392-1398.

# Improved Performance with Fuzzy Tuning



Merits with fuzzy tuning: smaller overshoot, faster reaching stable status without error

# Inspiration from Fuzzy PID?

Can we use fuzzy to adjust parameters of a procedure/method during its operation?

## Examples:

- Adjust the mutation rate of GA with generations?
- Adjust the step size during a search process?



Fuzzy is like soy sauce for food, can make system more elegant. It is rewarding to use fuzzy techniques in many application areas



# Online adaptation of fuzzy controller

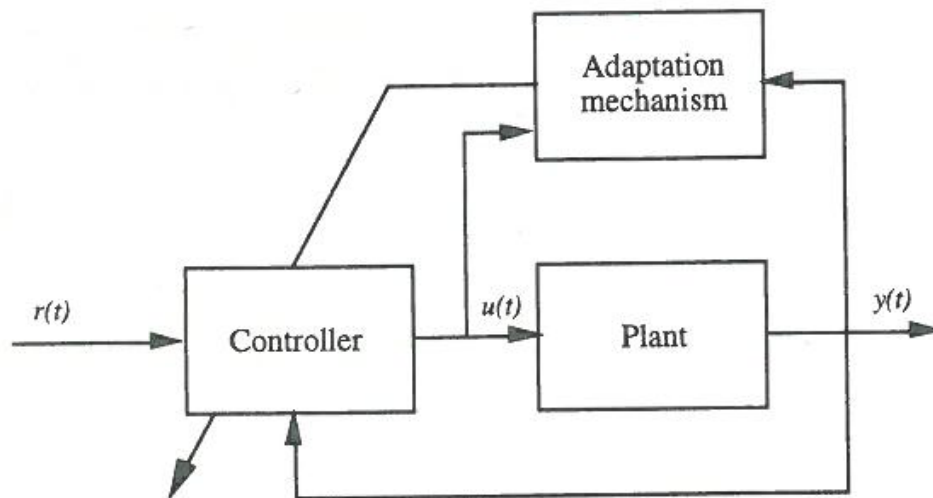


# Why Needing Online Adaptation for Fuzzy Controller ?

1. Human knowledge and experience is neither exact nor perfect
2. Sometimes humans can not precisely express their experience and knowledge
3. Environment /plant can change over time, fuzzy controller needs to adjust its strategy in response to such changes.

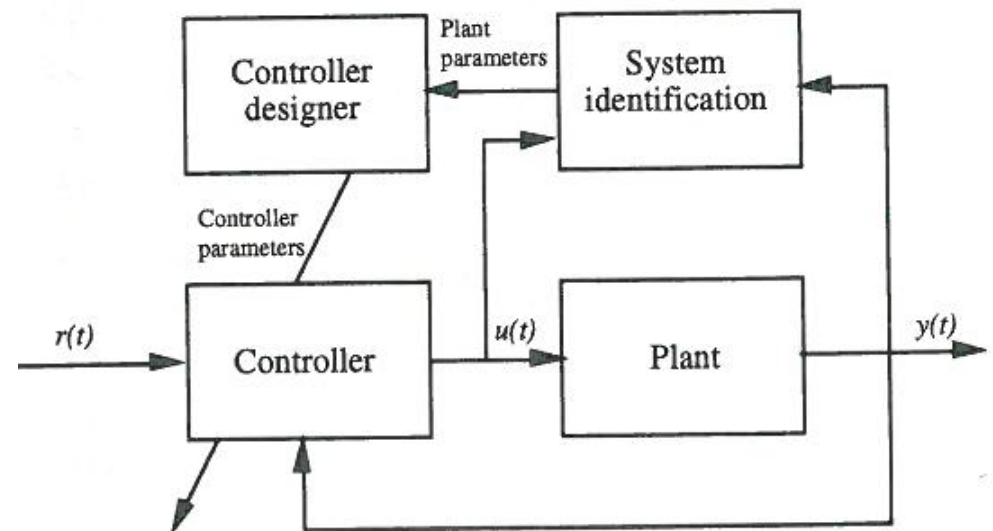
# Two Types of Adaptive Fuzzy Control

## Direct adaptive control



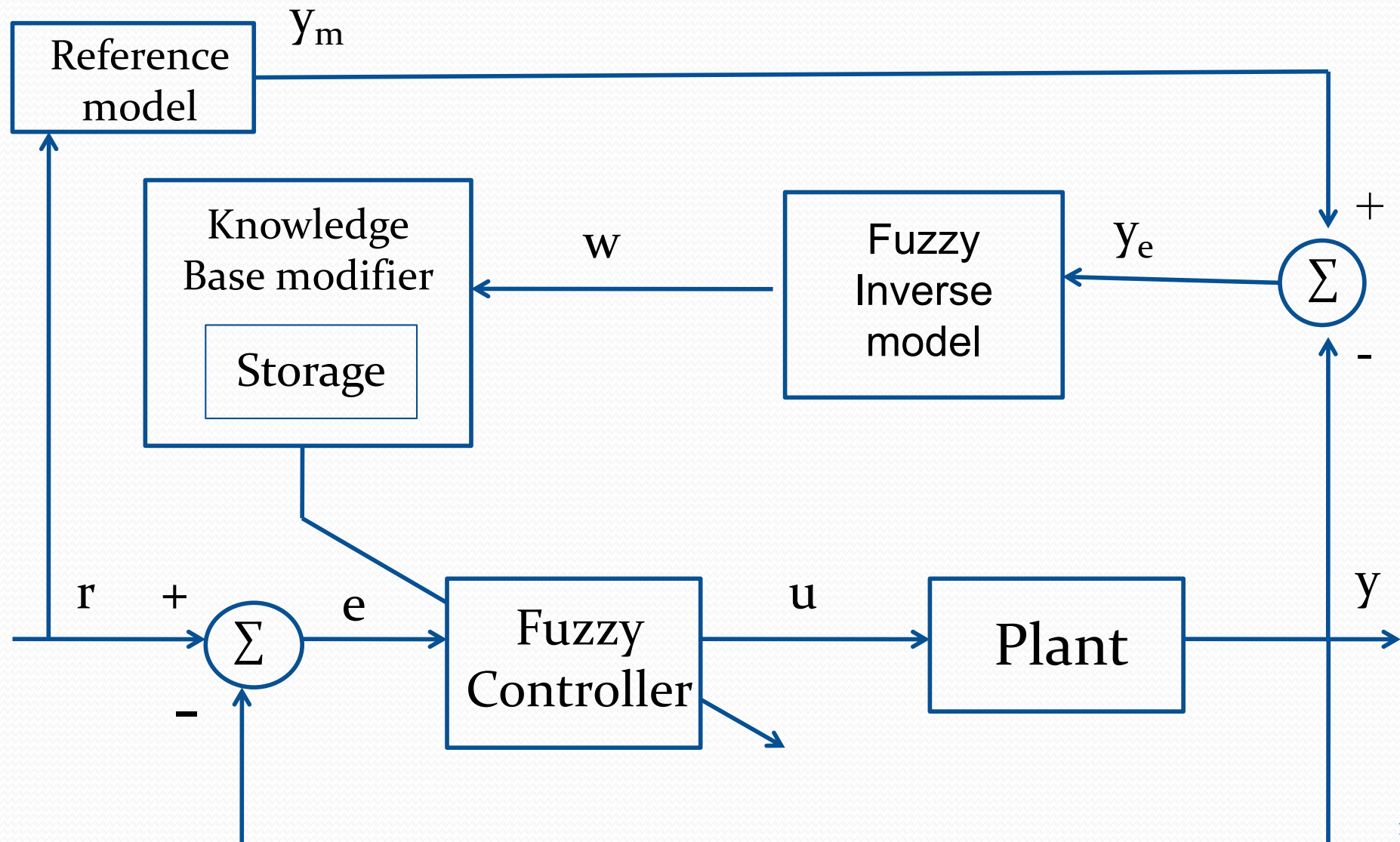
Utilize output signal to adapt

## Indirect adaptive control



Utilize parameters of the plant estimated via on-line system identification

# Fuzzy Reference Model Learning Control





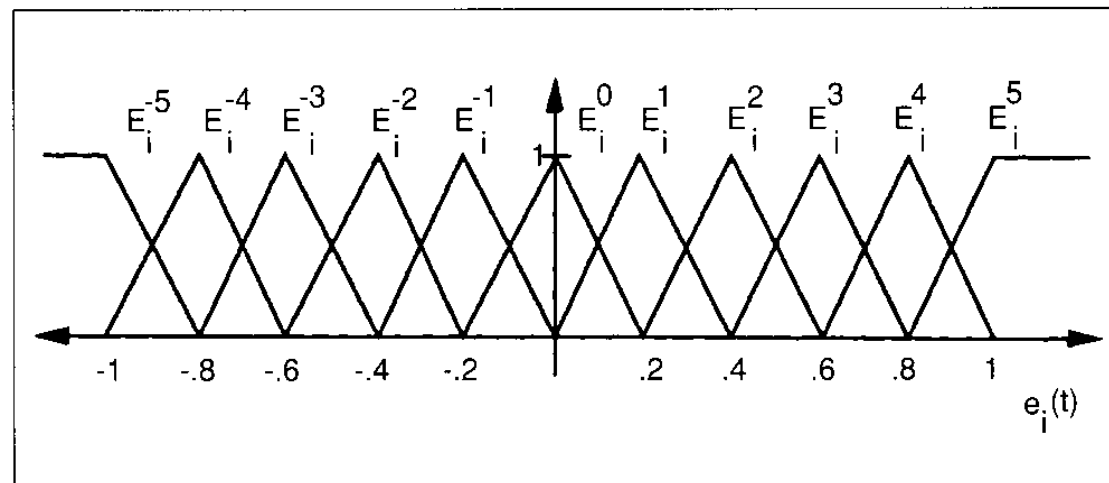
# Fuzzy Reference Model Learning Control

- Fuzzy controller: decides  $u(k)$  based on  $e(k)$  and  $e'(k)$ . Its goal is to control the plant to track the reference input
- Reference model: gives the requirement of how to track the reference input. It specifies the desired system output  $y_m$  at every time step given the reference signal
- Fuzzy inverse model: decides the value of change  $w$  for the controller output  $u$  such that the difference  $y_e$  between the desired output  $y_m$  from the reference model and the actual output  $y$  should have been reduced to zero. It has the same structure as the fuzzy controller
- Knowledge-base modifier: modifies the the fuzzy rules to realize the required change for output  $u$ .

# Fuzzy Controller



## Fuzzy membership function for inputs

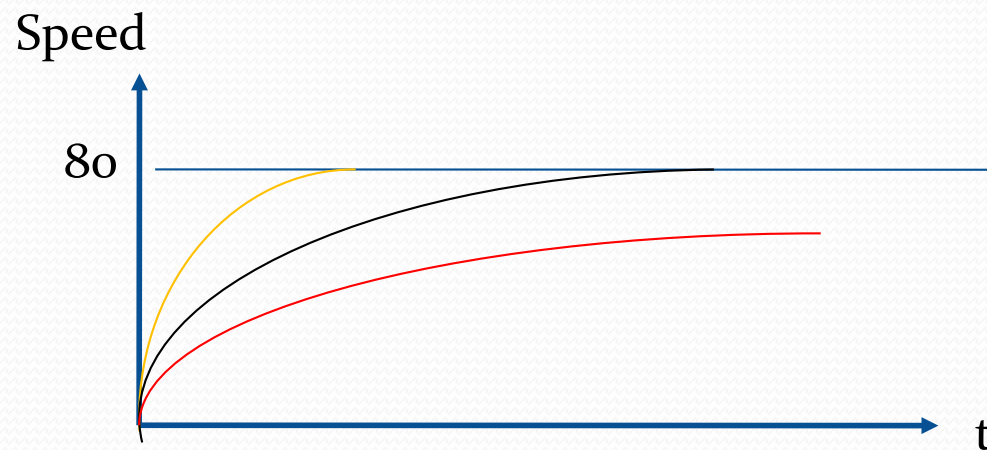


Initial conclusions of 121 rules:

- Best guess using available information
- Membership functions with centers at zero (no idea at all)

# Reference Model

A reference model gives a feasible specification of how to best track the input reference signal



# Reference Model Formulation

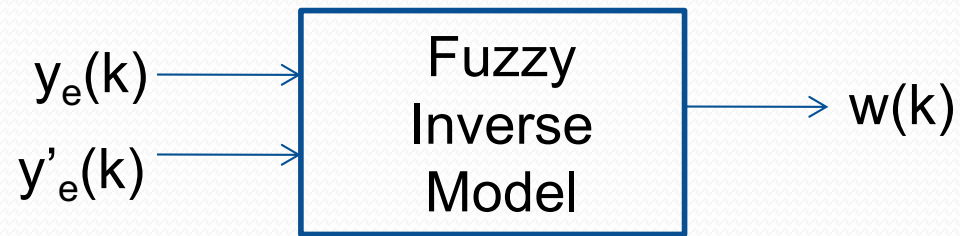
Reference model shows the desired relation between plant output  $y(t)$  and reference input  $r(t)$ , or how should  $y$  change in response to  $r$ .

Example:  $y'(t) + y(t) = r(t)$

Discrete form with  $T=0.1$  sec.:

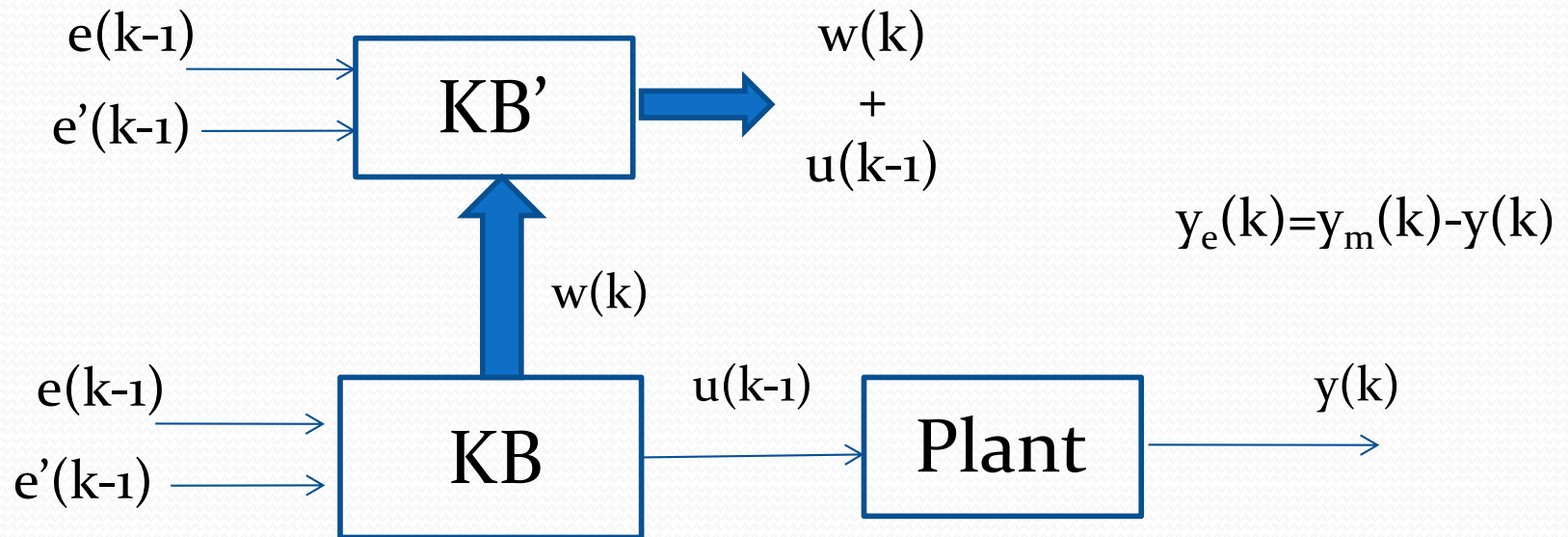
$$y_m(kT + T) = \frac{19}{21} y_m(kT) + \frac{1}{21} r(kT + T) + \frac{1}{21} r(kT)$$

# Fuzzy Inverse Model



- Fuzzy Inverse Model: similar to fuzzy controller
- Fuzzy Inverse model: fuzzy knowledge base, inference mechanism, fuzzification, defuzzification

# Knowledge Modifier



Focusing on those rules whose firing strength are non-zero, i.e.

$$t_i(e(k-1), e'(k-1)) > 0$$

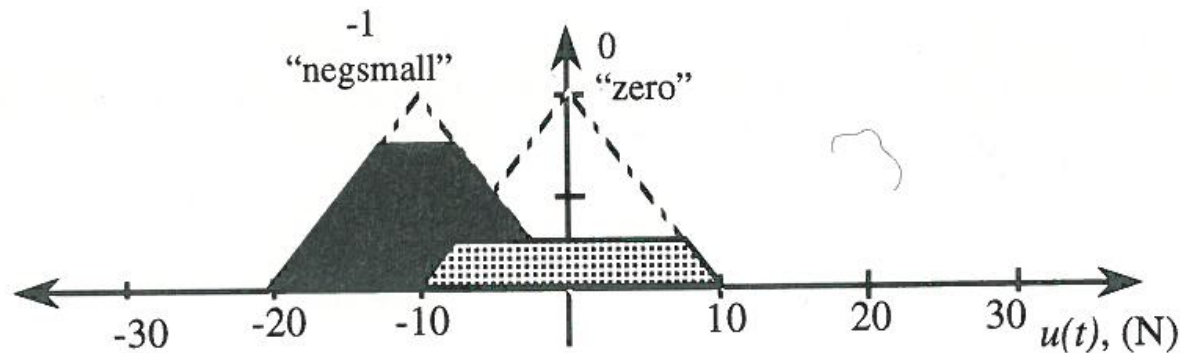
# Modifying Effective Rules

- Only consider and modify the effective rules
- Only move the consequent membership functions of the effective rules. Their shapes are not changed.
- Suppose  $b$  is the center of the output membership function of an effective rule, after time step  $kT$ , its value will be revised according to the following rule:

$$b(kT) = b(kT - T) + w(kT)$$

- If the centers of the membership functions of effective rules are revised in terms of the above rule, the controller output in the preceding step should have been  $u(kT - T) + w(kT)$

# Verifying the Modification



## Center of Gravity (COG)

$$u^{crisp} = \frac{\sum_i b_i \int \mu_{(i)}(u) du}{\sum_i \int \mu_{(i)}(u) du}$$

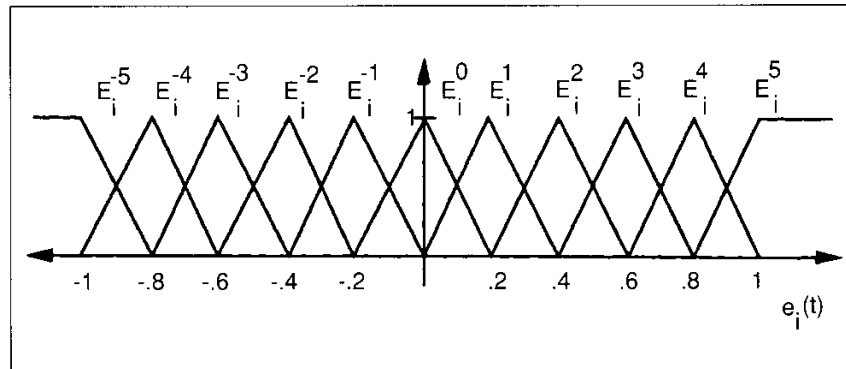
$$u = \frac{b_1 S_1 + b_2 S_2}{S_1 + S_2}$$

$$u' = \frac{(b_1 + w)S_1 + (b_2 + w)S_2}{S_1 + S_2} = u + w$$



# Example

Suppose  $w(k)=0.5$ ,  $e(k-1)=0.75$ ,  $e'(k-1)=-0.2$



Effective rules:

$R_1$ : if  $e$ ="3" and  $e'$ ="-1" then  $U_1$

$R_2$ : if  $e$ ="4" and  $e'$ ="-1" then  $U_2$

Modification of centers  $b_1$  and  $b_2$  for  $U_1$  and  $U_2$ :

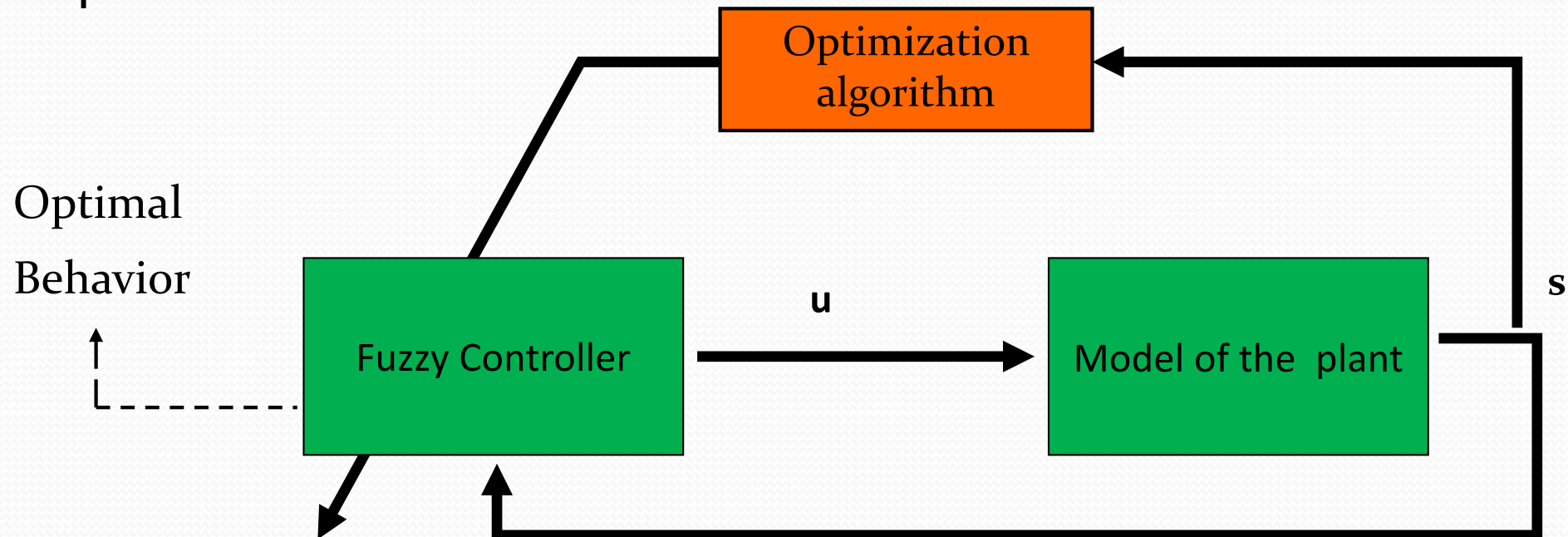
$$b_1(k)=b_1(k-1)+0.5, \quad b_2(k)=b_2(k-1)+0.5$$



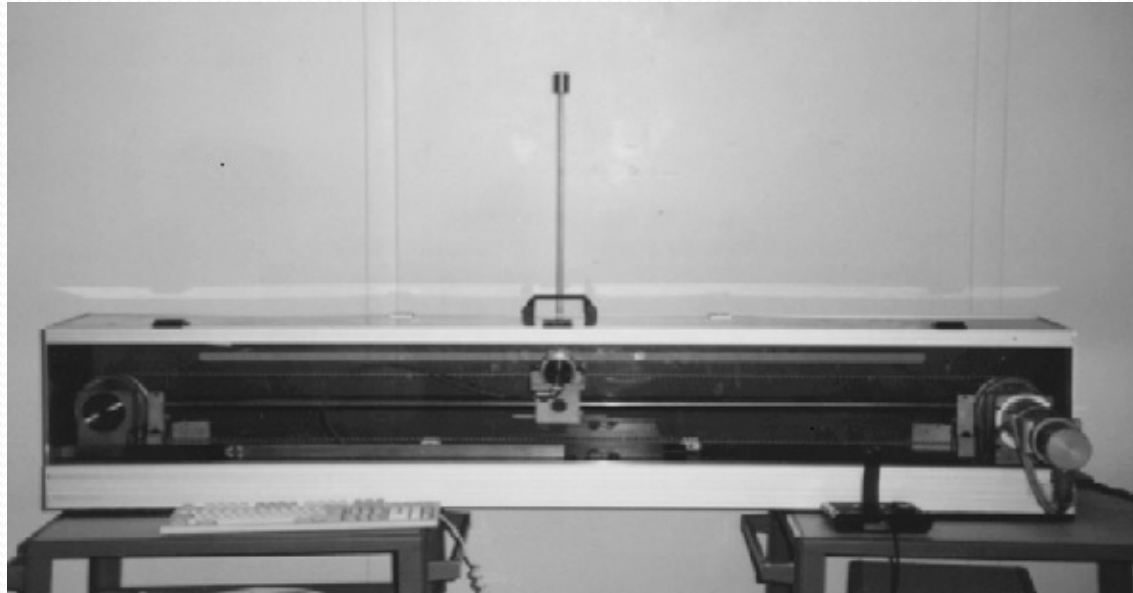
# Offline adaptation of fuzzy controller

# Model-Based Off-Line Fuzzy Learning

- If we know the model of the plant, we can predict the performance of a fuzzy controller by simulation.
- With the assessment of a controller by simulation, we can further apply optimization algorithm to find the best controller parameters



# Control Both Position and Angle

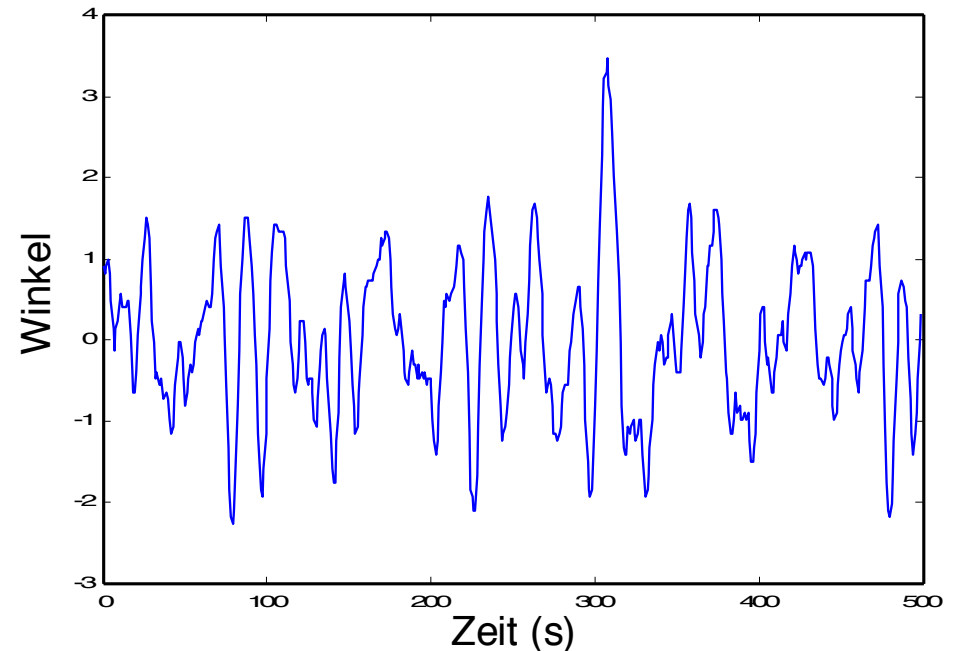
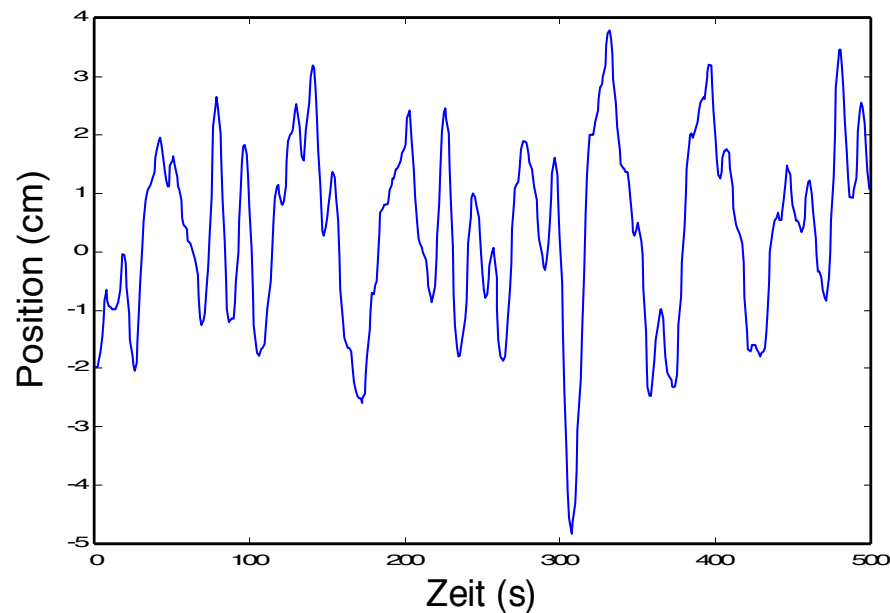


- Objectives:

- 1) Keep the cart at the center
- 2) Keep the pendulum upstraight

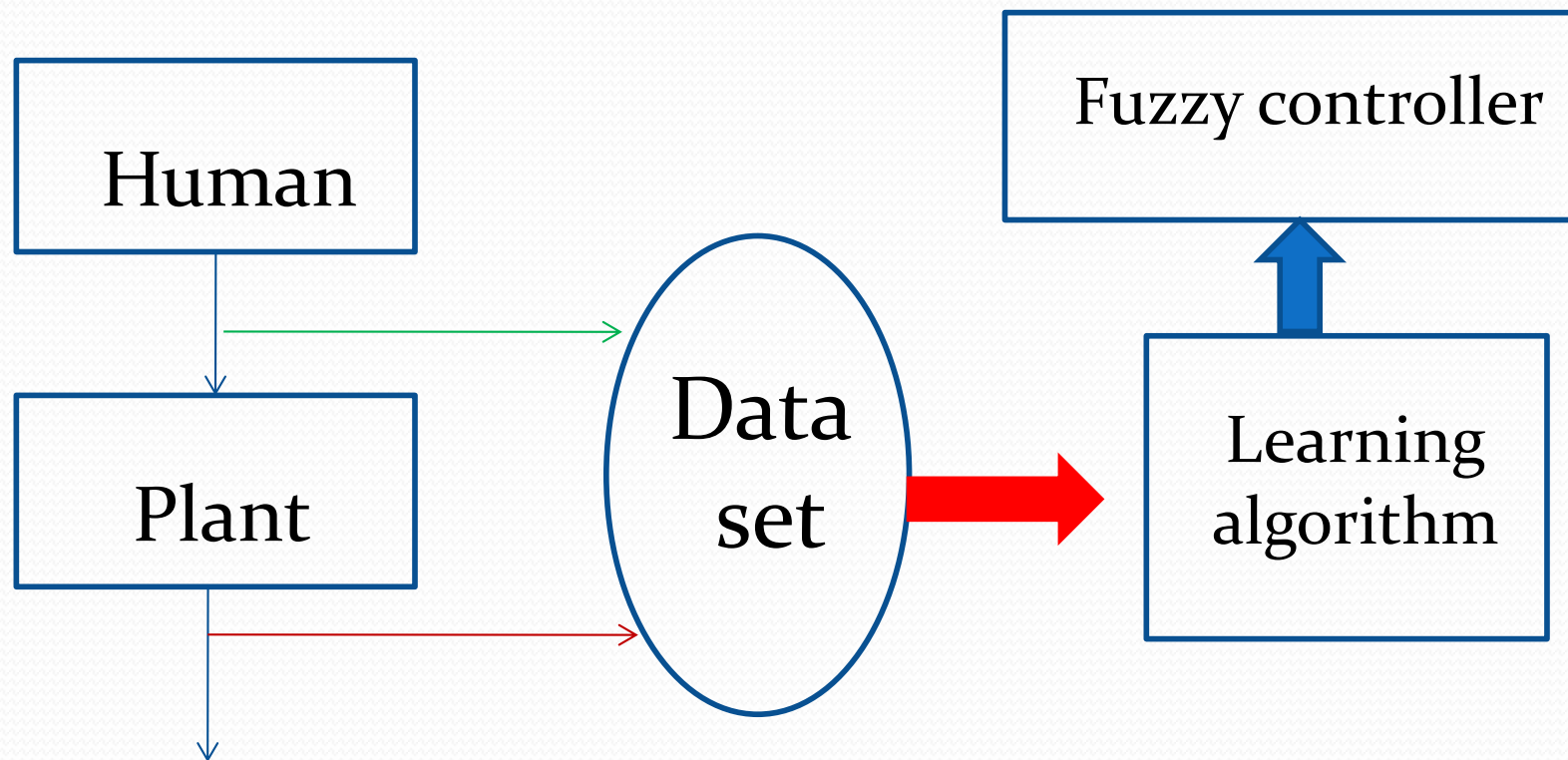
- Learning is based on the model of the pendulum equipment

# Performance in Real Balancing



Ning Xiong, Lothar Litz, “Reduction of fuzzy control rules by means of premise learning – Method and case study”, Fuzzy Sets and Systems, Vol. 132, 2002, pp. 217-231.

# Data-Based Fuzzy Controller Learning





# A Simple Method to Extract Rules from Data

L.X. Wang and J.M. Mendel, "Generating fuzzy rules by learning from examples", IEEE Trans. Systems, Man, & Cybernetics, Vol. 22, No. 6, 1992, pp. 1414-1427.

## Wang-Mendel Method:

1. Create a fuzzy rule from every training example
2. Assess every created rule with a truth value
3. Redundance and conflict removal

The rules created by Wang-Mendel method can be treated as a good starting point for further optimization of membership functions

# Recommendation for Reading

- It is important to carefully study and understand the content in the slides
- Optional reading:  
“Fuzzy gain scheduling of PID controllers”, IEEE Trans. Systems, Man & Cybernetics, No. 5, 1993, pp. 1392-1398. Available in the blackboard
- Compulsory reading:  
“Fuzzy model reference learning control for cargo ship steering”, IEEE Control Systems Magazine, 1993. Available in the blackboard.