Ve460 Control Systems Analysis and Design
Chapter 1 An Overview and Brief History of Feedback Control

Jun Zhang

Shanghai Jiao Tong University

Contents

The main objectives of this chapter are:

1. A simple feedback system
2. A first analysis of feedback
3. A brief history of feedback control
4. Feedback system fundamentals

1. A simple feedback system

Example 1

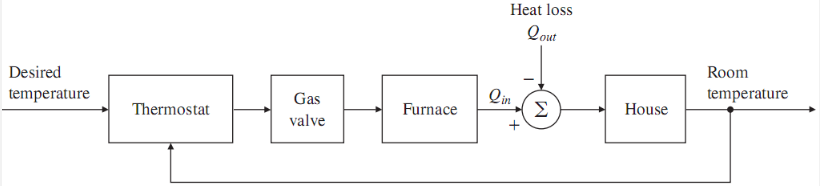


Figure 1.1 Component block diagram of a room temperature control system.

Consider the room temperature control:

Our objective is to maintain the room temperature at certain desired reference (also called the set point) by turning on and off the supply of furnace gas.

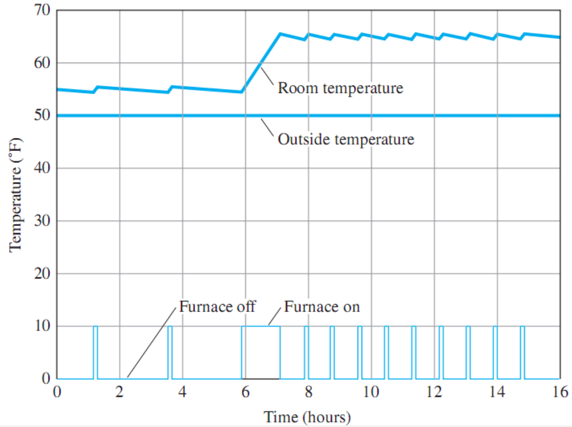
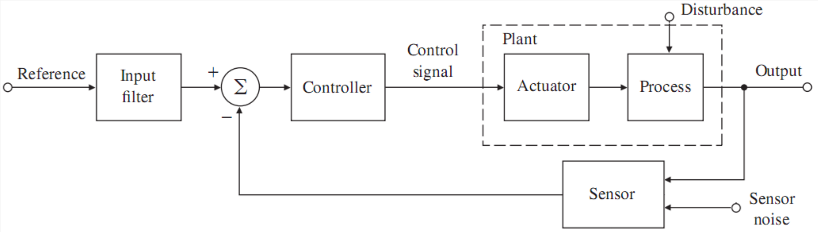
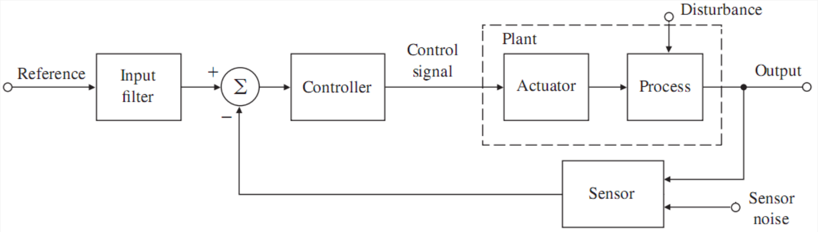


Figure 1.2 Plot of room temperature and furnace action.



We can now identify some generic components of an elementary feedback control system as above.

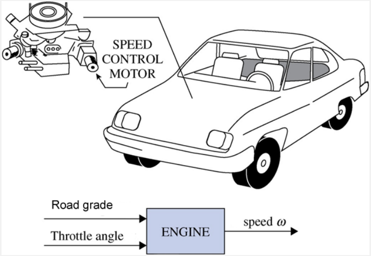
- Process: the component whose output is to be controlled (house);
- Actuator: the device that influences the controlled variable of the process (gas furnace);
- Plant: combination of process and actuator;
- Controller: the component that computes the desired control signal;



- Sensor: measures the output of the process (thermostat);
- Sensor noise: inevitable noise contained in the sensing devices;
- Reference: set point or trajectory that the output should follow;
- Input filter: converts or modifies the reference signal;
- Disturbance: the signal that influences the output but cannot be controlled (the flow of heat dissipated from the house)

2. A first analysis of feedback

Example 2



To demonstrate the value of feedback control, we analyze a simplified model of the cruise control of an automobile.

The objective is to control the speed, and

- Throttle angle is the **control variable** u;
- Vehicle speed is the **output variable** y.

Diagram of the plant

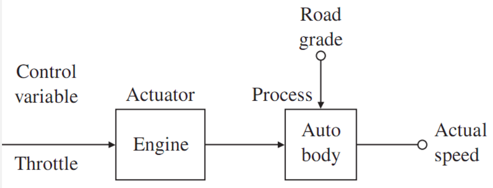
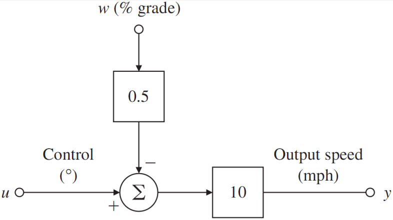


Figure 2.1 Block diagram of the cruise control plant.

- Measuring the speed of the vehicle on a level road at 65 mph, we find
  1° change in throttle angle (control variable u)
  $\Rightarrow$ 10 mph change in speed (output variable y)
- Driving up and down hills, we find
  grade changes by 1% (disturbance w)
  $\Rightarrow$ speed change of 5 mph (output variable y)

Mathematical model of the plant



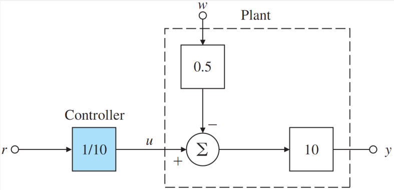
We can use a block diagram to describe the mathematical model of the cruise control plant, where

- The connecting lines carry signals;
- A block is like an ideal amplifier:

  Output = Gain × Input;
- A summer adds or subtracts the signals coming in, depending on the + or − beside each arrow.

The output speed is then determined by $y = 10(u - 0.5w)$.

Open-loop control



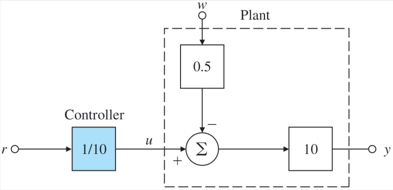
If the controller does not use the speedometer reading but sets

$$u = \frac{r}{10},$$

this is an open-loop control system. The term *open loop* means there is no closed path in the diagram, *i.e.*, u is independent of y. The speed is then given by

$$y_{ol} = 10(u - 0.5w) = 10\left(\frac{r}{10} - 0.5w\right) = r - 5w.$$

Open-loop control



The error in the output speed is

$$e_{ol} = r - y_{ol} = 5w,$$

and the percent error is

$$\%\ \text{error} = \frac{5w}{r} \times 100.$$

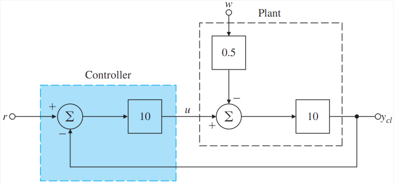
Consider the case when $r = 65$:

- If the road is level, $w = 0$, $\Rightarrow$ $y = 65$, no error;
- If 1% grade, $w = 1$, $\Rightarrow$ $y = 60$, 5-mph or 7.7% error;
- If 2% grade, $w = 2$, $\Rightarrow$ $y = 55$, 10-mph or 15.4% error;

No error only when

- There is no disturbance ($w = 0$); and
- The controller gain (1/10) is the exact inverse of the plant gain (10).

Closed-loop control



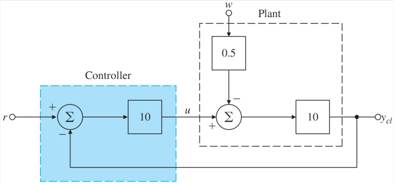
Now consider a closed-loop cruise control, where the sensor is assumed to be ideal. We then have

$$
\begin{aligned}
y_{cl} &= 10u - 5w, \\
u &= 10(r - y_{cl}).
\end{aligned}
$$

Then

$$
\begin{aligned}
y_{cl} &= 100r - 100y_{cl} - 5w, \\
101y_{cl} &= 100r - 5w, \\
y_{cl} &= \frac{100}{101}r - \frac{5}{101}w, \\
e_{cl} &= \frac{r}{101} + \frac{5w}{101}.
\end{aligned}
$$

Closed-loop control



Compare these two errors

$$e_{ol} = 5w,$$
$$e_{cl} = \frac{r}{101} + \frac{5w}{101},$$

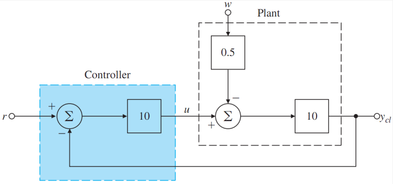
the feedback has reduced sensitivity of the speed error to the grade by a factor of 101. The percent error is

$$\%\text{ error} = \left(\frac{1}{101} + \frac{5w}{101r}\right) \times 100.$$

If again considering $r = 65$ and 1% grade, the percent error is 1.1% (cf. 7.7% in open-loop).

This reduction of the speed sensitivity to grade disturbances and plant gain is due to a *large* loop gain of 100.

Closed-loop control



However, there is now a small speed error on level ground even when $w = 0$:

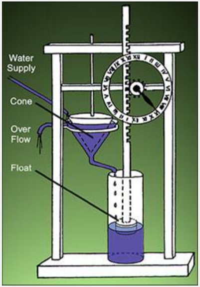
$$y_{cl} = \frac{100}{101} r = 0.99r \text{ mph}.$$

This error is small as long as the loop gain (product of plant and controller gains) is *large*.

Tricky part: we cannot increase the loop gain too large as it may cause stability issue (will discuss in Chapters 6, 8, and 9)!

3. Brief history of control

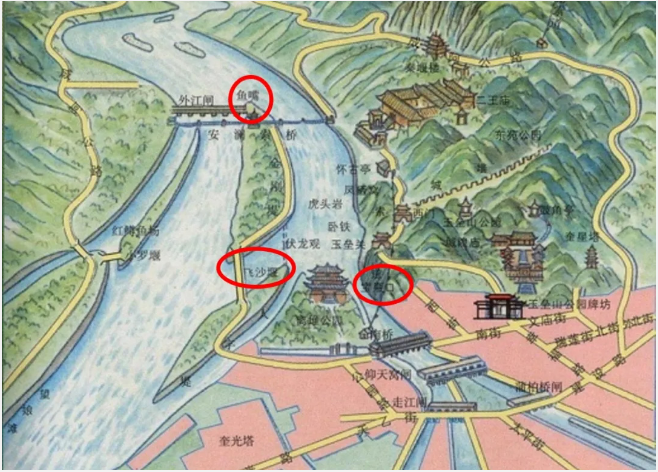
Early history of feedback control can be traced back to ancient times.



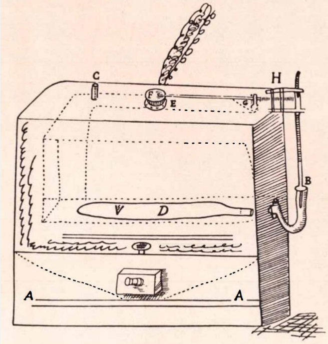
The first application of feedback control is believed to be the water clock appeared in 300 BC - 1 BC, invented by the Greek inventor and mathematician Ctesibius.

1. O. Mayr, The Origins of Feedback Control, Scientific American, October, 1970.
2. Stuart Bennett. A brief history of automatic control. IEEE Control Systems, 1996, 16(3):17-25.
3. Prof. ZhengXin Weng, Principle of Automatic Control (undergraduate course), Dept. of Automation, SJTU.
4. Prof. Yi Huang, Theory on system stability (graduate course), Institute of System Science, Chinese Academy of Sciences.

DuJiangYan irrigation system developed around 256 BC is also a great demonstration of feedback control system. Refer to documentary movies on the Internet for more details.

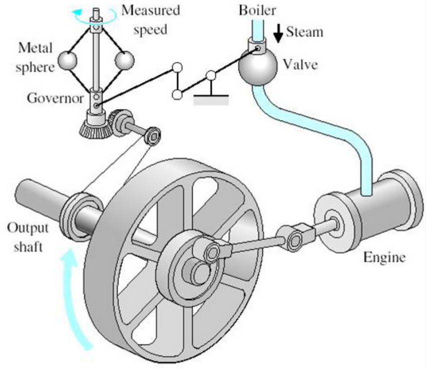


A more recent invention is a system designed by Dutch inventor Cornelis Drebbel in about 1620 to control the temperature of a furnace used to heat an incubator.



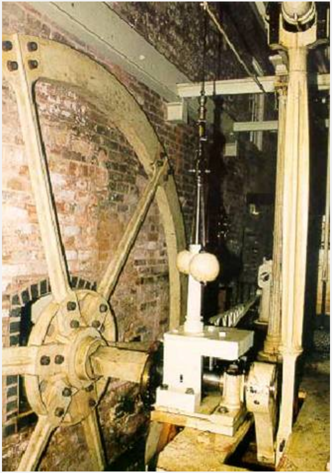
For more details, See Mayr's paper or Franklin's book.

Flyball governor



Flyball governor is possibly the most famous example in the early history of feedback control.

Starting from 1763, James Watt applied flyball governor to the steam engine to maintain the shaft rotating at a **constant** speed.

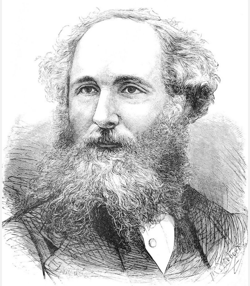


This photograph shows a fly-ball governor used on a steam engine in a cotton factory near Manchester in the UK.

Manchester was at the center of the industrial revolution.

As we all know, the unit of power is Watt, which was named after this great inventor.

Early design of the flyball governor was reported to sometimes have *Hunting* (*i.e.*, instability).



The following communications were read :—

I. "On Governors." By J. Clerk Maxwell, M.A., F.R.SS.L. & E. Received Feb. 20, 1868.

A Governor is a part of a machine by means of which the velocity of the machine is kept nearly uniform, notwithstanding variations in the driving-power or the resistance.

Most governors depend on the centrifugal force of a piece connected with a shaft of the machine. When the velocity increases, this force increases, and either increases the pressure of the piece against a surface or moves the piece, and so acts on a break or a valve.

In one class of regulators of machinery, which we may call *moderators* *, the resistance is increased by a quantity depending on the velocity. Thus in some pieces of clockwork the moderator consists of a conical pendulum revolving within a circular case. When the velocity increases, the ball of the pendulum presses against the inside of the case, and the friction checks the increase of velocity.

In Watt's governor for steam-engines the arms open outwards, and so contract the aperture of the steam-valve.

In a water-break invented by Professor J. Thomson, when the velocity

In 1868, James C. Maxwell published his paper *On Governor*, which is deemed as the first theoretical study of the stability of feedback control.

Yes, this is the same person who discovered **Maxwell's Equations**.

- In 1877, E. J. Routh developed a criteria to check stability, which is equivalent to determine whether all the roots of a polynomial equation have negative real parts (Chapter 6).

Example (Routh's array or Routh's tabulation)

Consider $a_6 s^6 + a_5 s^5 + \cdots + a_1 s + a_0 = 0$

$$
\begin{array}{c|c|cccc}
s^6 & a_6 & a_4 & a_2 & a_0 \\
s^5 & a_5 & a_3 & a_1 & 0 \\
s^4 & \dfrac{a_5 a_4 - a_3 a_6}{a_5} = A & \dfrac{a_5 a_2 - a_6 a_1}{a_5} = B & \dfrac{a_5 a_0}{a_5} = a_0 & 0 \\
s^3 & \dfrac{A a_3 - B a_5}{A} = C & \dfrac{A a_1 - a_0 a_5}{A} = D & 0 & 0 \\
s^2 & \dfrac{CB - AD}{C} = E & \dfrac{C a_0}{C} = a_0 & 0 & 0 \\
s^1 & \dfrac{ED - C a_0}{E} = F & 0 & 0 & 0 \\
s^0 & a_0 & & &
\end{array}
$$

- In 1892, A. M. Lyapunov defended his thesis *The General Problem of Stability of Motion* (discussed in Ve 562).

Classical control

- 1910, E. Sperry worked on ship autopilots with PID control (Chapter 10);
- 1927, H. S. Black proposed feedback electronic amplifier;
- 1932, H. Nyquist derived a frequency domain stability criterion (Chapter 9);
- 1938, H. Bode studied frequency response methods (Chapter 9);
- 1942, Ziegler-Nichols found a method for PID tuning;
- 1948, W. R. Evans developed Root Locus (Chapter 8).

Modern control

- 1948, N. Wiener published his seminal book *Cybernetics, or control and communication in the animal and the machine*;
- 1956, L. S. Pontryagin formulated the Maximum Principle (Ve 560);
- 1957, R. Bellman developed Dynamic Programming;
- 1960, R. E. Kalman established Optimal Estimation (Ve 560);

- 1970-1980, State variable methods and optimal control;
- 1980-1990, Robust and Nonlinear Control;
- 1990-2000, Switched control, hybrid control, and control using convex optimization, among others.

Underlying mathematics

- Classical control (frequency methods): complex analysis;
- Modern control theory (state-space approach): linear algebra;
- Nonlinear systems: differential geometry, Lie group;
- Optimal control: functional analysis;
- Stochastic control and System identification: probability theory, random processes;
- Optimization.

Control systems are abundant in modern civilization

- Automatic assembly lines;
- Space technology;
- Weapon systems;
- Transportation systems;
- Power systems;
- Robotics;
- Micro-Electro-Mechanical Systems (MEMS);
- Nanotechnology,

and many others.

Even the control of inventory, social, and economic systems may be approached from the theory of automatic control.

4. Feedback system fundamentals

The basic ingredients of a control system can be described by:

- Objectives of control;
- Control-system components;
- Results or outputs.

In more technical terms:

u is the **actuating signal**, or **control variable**;

y is the **output**, or **controlled variable**.

In general, the **objective** of the control system is:

Control the outputs in some prescribed manner by the inputs through the elements of the control system.

To achieve good control there are typical goals.

A. Stability

The system must be stable at all times. An absolute requirement! The requirement of stability is basic and instability may have two causes

- The system being controlled may be unstable (Segway, inverted pendulum, etc);
- The addition of feedback!

B. Tracking

The system output must track the command reference signal as closely as possible:

- Set point;
- Command tracking

C. Disturbance rejection

The system output must be as insensitive as possible to disturbance inputs.

One of the very oldest applications of feedback!

D. Robustness

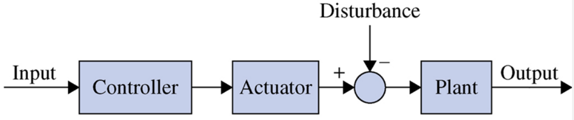
The preceding goals be met even if the model is not accurate or if the dynamics of the physical system change over time.

Model mismatch includes

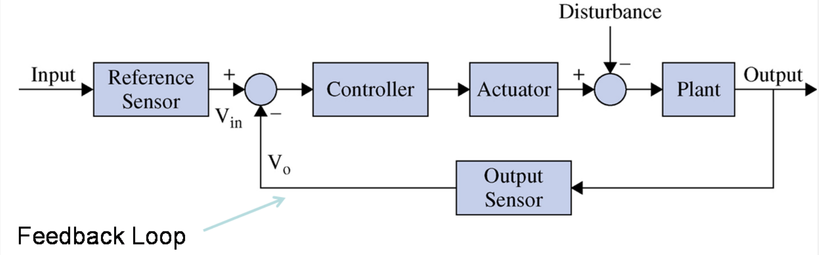
- parameter uncertainties
- nonlinearity
- time varying

Open-loop vs Closed-loop control

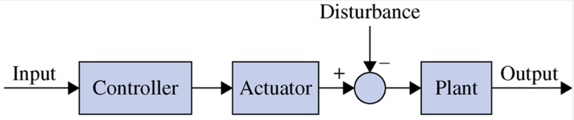
Open-loop control system



Closed-loop control system



Open-Loop Control Systems (Non-feedback Systems)



An open-loop control system usually contains only

- A controller; and
- A controlled process.

Advantages:

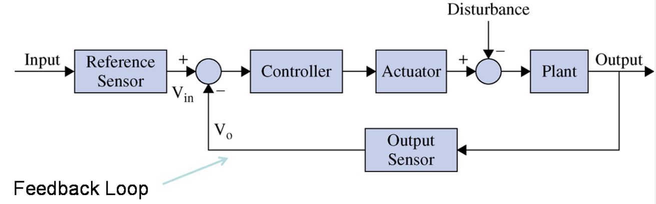
- Simple;
- Fast response.

Disadvantages:

- May not be able to eliminate static error (the error between the reference input and the system output after long time);
- Sensitive to disturbance and model mismatch.

Closed-Loop Control Systems (Feedback Control)

Closed-loop system: has one or more feedback paths



- Reduce static error (Chapter 7);
- Increase stability margin (Chapter 9);
- Increase robustness (Chapter 10);
- Other changes in bandwidth and gain (Chapter 9);

- May not be as fast as open-loop.