3170219111542 Ge Jintian

1.

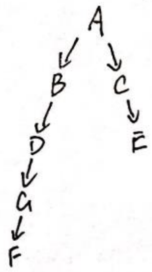
$$C_F = \frac{2\times 7}{5\times(5-1)} = \frac{14}{20} = 0.7$$

2.
(1) The diameter is measured by distance between A and F.

diameter $=4$

(2) True. Because in this case any other nodes will not be able to connect to F.

(3)
Break tie in alphabet order.

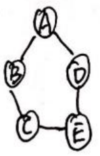


depth is 4

3.

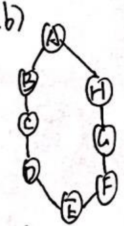
2.1

(a)



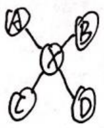
In this graph, B is pivotal of (A,C) since A→B→C is the only shortese path. And due to symmetry, every nodes are a pivotal of their adjacent nodes.

(b)



In this graph, B is pivotal of (A,C) and (A,D), since A→B→C and A→B→C→D are the only shortest path. Due to symmetry, every node is a pivotal of at least two paths.

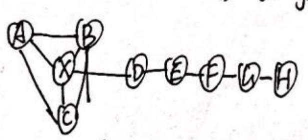
(c)



As in the graph, every path must go through X. So X must be a pivotal of any pair.

2.3

(a) The simplest case is a graph with some extensions, like:



Assume we have 5 extensions. Then we want to find n nodes in the complete part. We have.

$$A.D.(\text{average distance}) = \frac{C_n^2 + 2(n-1) + 3(n-1) + 4(n-1) + 5(n-1) + 6(n-1) + 1+2+3+4+5}{C_{n+5}^2}$$

Shortest distance $=b$

So, $b \geq 3A.D. \Rightarrow A.D. \leq 2$

Let's try $n=50$. In this case, $A.D.=1.49$

Therefore, a combination of 30 nodes in complete part and 5 nodes in extension is an example.

(b)

As described above, what we want is:

$$\frac{k+1}{c} \geq \frac{C_n^2 + (n-1)\cdot\frac{(k+3)k}{2} + \frac{k(k+1)}{2}}{C_{n+k}^2}$$

We can arbitrarily choose k and calculate n according to C.

3.2

It should be a weak tie.

If it is strong, then e and c should have a tie. otherwise it violates STC.

So, it is a weak tie.

5.3

(a) No such way

Since the edge of A, B, C are hostile, to form a balance triangle, the edges provided by D should be one friendly and one hostile. This means any arbitrary combination ~~a~~ of two edges among the three newly provided edges should be one friendly and one hostile, which is impossible.

(b) No such way

To make triangle (A,B,D) balance, Edge(A,D) and Edge(B,D) should have same property. It is the same for triangle (A,D,C). In this case, triangle (D,B,C) will be always unbalance.

(c) ~~因~~ It is impossible.

As we can see from (a) and (b), if one unbalance triangle exists, it is impossible to add a new node into it.

6.5

(M,M) is a N.E.

6.15

(1)

		player 1		
		A	B	N
player 2	A	-10, -10	10, 10	15, 0
	B	10, 10	5, 5	30, 0
	N	0, 15	0, 30	0, 0

(b)

As we can see, if we choose strategy B, at least we will earn 5 million. If we choose N, then at most we can earn 0. So this argument is correct.

(c)

No. If in (B, B) one player will move to A to earn a larger reward, so it is not a N.E.

(d)

(A,B) and (B,A)

(e)

Yes. Because as we can see, the N.E. will bring a sum of 20 million. And if ~~em~~merging, it can bring a sum of 30 million.

9.4

(a) Yes. Because B will only raise the price of the object. In this case, if A thinks it is value 1, then calling 1 can guarantee he will not suffer loss. If A thinks it is value 0, he doesnt need to compete with B.

(b)

$0.5\times0.75\times1+0.5\times0.75\times0+0.5\times0.25\times0+0.5\times0.75\times0$

$=0.375$

9.9.

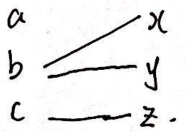
(a) $2\times\frac{1}{2}\times\frac{1}{2}+1\times\frac{1}{2}\times\frac{1}{2}+1\times\frac{1}{2}\times\frac{1}{2}+1\times\frac{1}{2}\times\frac{1}{2}=\frac{5}{4}$

(b) $2\times\frac{1}{2}\times\frac{1}{2}+R\times\frac{1}{2}\times\frac{1}{2}+R\times\frac{1}{2}\times\frac{1}{2}=0.5+0.5R$

(c) If $1\le R\le1.5$, $0.5+0.5R\le\frac{5}{4}$, which is the expectation of no R set.

So, R should be larger than 1.5.

10.9



		match
start	000	b b c
r1	010	b bc
r2	020	b,c b c
r3	031	a,b,c, b c

So, the final ~~et~~ market-clearing price is 0, 3, 1.

and x takes a, y takes b, z takes c