

ECON W3213 Spring 2014 Jón Steinsson

Midterm Review Session

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This is a set of questions to get you **started** on revision. They are very very basic. Don't expect the midterm to be this easy (just check out the sample midterm!) Don't be too happy if you can answer these. After all, these are the **bare basics**. If you can't answer them, however, you're in deep shit. In that case, as Gandalf says, you shall not pass.

1 Production

This is the Cobb-Douglas production function

$$Y = \bar{A}K^\alpha L^\beta$$

1. If $\beta = 1 - \alpha$ and $0 < \alpha < 1$
 - a) Show that the production function is increasing in both K and L
 - b) Show that it exhibits diminishing returns to K and diminishing returns to L
 - c) Show that it exhibits constant returns to scale
2. If $\alpha > 1$ and $\beta > 1$
 - a) Show that it is increasing in both K and L
 - b) Show that it does not exhibit diminishing returns to either K or L
 - c) Show that it exhibits increasing returns to scale
3. For a firm,
 - a) What is the cost of capital?
 - b) What is the cost of labor?

- c) What is the firm's profit?
 - d) How does a firm optimize capital input?
 - e) How does a firm optimize labor input?
 - f) What share of output goes to workers?
 - g) What share of output goes to capital?
4. What is the output $y = \frac{Y}{L}$ in per capita terms?

2 Household Behavior

Your overall utility is

$$U(C) - V(H)$$

1. Given that you love consuming and you hate working,
 - a) What does U represent?
 - b) What does V represent?
2. For $U(C)$
 - a) What is the sign of $U'(C) = \frac{\partial U(C)}{\partial C}$?
 - b) What is the sign of $U''(C) = \frac{\partial^2 U(C)}{\partial C^2}$?
 - c) Plot $U(C)$ against C
3. For $V(H)$
 - a) What is the sign of $V'(H)$?
 - b) What is the sign of $V''(H)$?
 - c) Plot $V(H)$ against H

You can only consume what you earn, hence

$$C = wH$$

1. Labor Supply using Constrained Optimization
 - a) Express $U(C)$ as a function of w and H
 - b) How does one maximize utility given the budget constraint?
 - c) What is the labor supply equation?

2. Labor Supply using Calculus of Variations
 - a) Derive the labor supply equation using calculus of variations
3. Labor Demand
 - a) What is the labor demand equation?
4. Substitution Effect
 - a) Without changing consumption C , what happens when wages w increase?
 - b) When wages w decrease?
5. Income Effect
 - a) Without changing wages w , what happens when consumption C increases?
 - b) When consumption C decreases?
6. Taxation
 - a) Does the utility function change due to taxation?
 - b) How does the budget constraint change when we consider income tax, consumption tax, and government transfers?
 - c) Find the equilibrium wage
 - d) Is this higher than or lower than the tax free wage?
7. Now let's use specific functional forms. $U(C) = \log C$ and $V(H) = \psi \frac{H^{1+\eta}}{1+\eta}$
 - a) Prove that these forms for $U(C)$ and $V(H)$ produces the signs that we want for $U'(C)$ $U''(C)$ $V'(H)$ and $V''(H)$
 - b) What is labor demand equation in this specific case?
 - c) What is the labor supply equation in this specific case?
 - d) Take the log of the equilibrium. What conclusions can we get regarding percentage change in hours of labor?
 - e) What is the Frisch elasticity of labor supply?

3 Market Efficiency

1. Pareto Efficiency
 - a) Define pareto efficiency in your own words
 - b) What are the 3 forms of efficiency that we need for pareto efficiency?
2. Exchange Efficiency

- a) Derive the condition that guarantees exchange efficiency
 - b) Express this on an edgeworth box and show the set of points that are exchange efficient
 - c) Convince yourself that this should also apply to all other pairs of outputs
- 3. Production Efficiency
 - a) Derive the condition that guarantees production efficiency
 - b) Convince yourself that this should also apply to all other pairs of inputs
- 4. Land Efficiency
 - a) Derive the condition that guarantees land efficiency
 - b) Convince yourself that this should also apply to all other pairs of input and land
- 5. Prices
 - a) How do prices ensure efficiency in each of these markets
- 6. Market Failure
 - a) In your own words, explain 5 types of market failure using examples

4 Consumption Savings

Your utility function is

$$U(C_1) + \beta U(C_2)$$

Your budget function is

$$C_1 + B = Y_1$$

$$C_2 = Y_2 + (1 + R)B$$

1. Using the utility function above
 - a) Explain what β is
2. Using the budget function
 - a) What are the lower and upper limits for B?
 - b) Combine them into a single equation without B

- c) Explain intuitively what that equation means using the concept of present value
3. To derive the Consumption Euler Equation,
 - a) Express C_2 in terms of C_1
 - b) Maximize utility subject to the budget constraint
 - c) Derive the Consumption Euler Equation
4. If $U(C) = \log C$
 - a) Show that this function is increasing and exhibits diminishing marginal utility
 - b) Derive the consumption euler equation for this specific functional form
5. Using the Consumption Euler Equation,
 - a) Show that level of consumption does not depend on level of period specific income, only total income
 - b) Level of consumption is a fraction of the present value of income. Find that fraction.
6. Extend what we have so far to 3 periods and derive the same two conclusions
7. Generalize this to n periods and show that the two conclusions still remain true

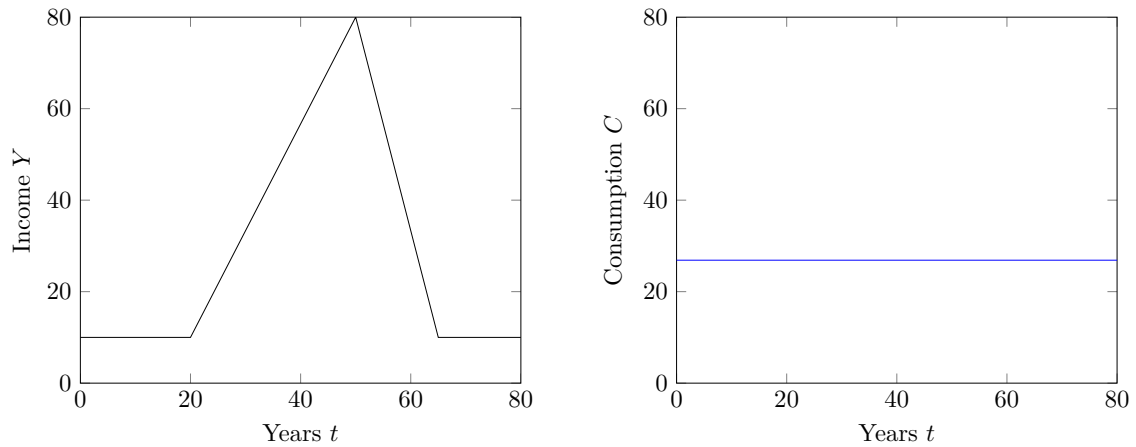


Figure 1: Income and Consumption Level throughout Lifetime

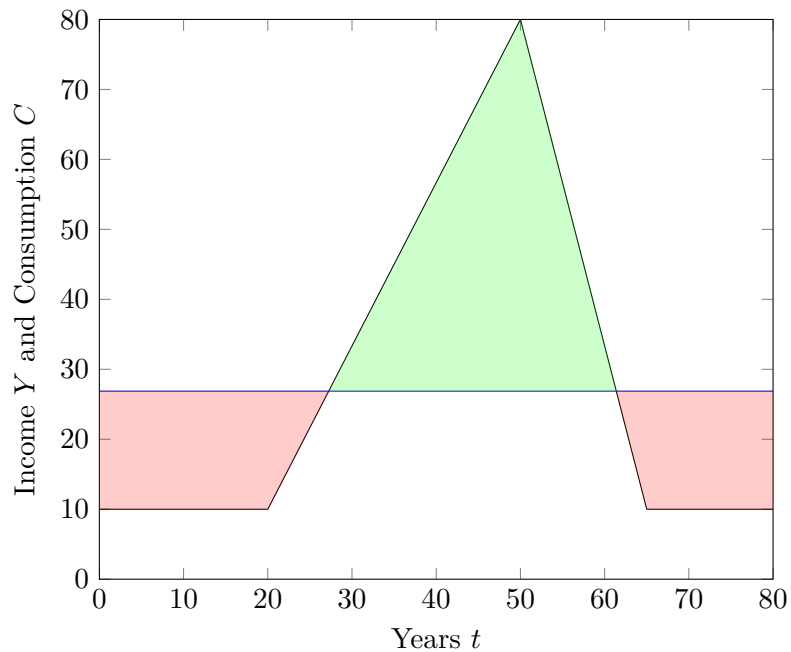


Figure 2: Borrowing, Saving and Dissavings

8. Referring to the graphs above
 - a) When is a person borrowing, saving, and dissaving?
 - b) Why is the consumption line a straight line?
 - c) Which of the shaded areas are equal to which others?

5 Malthus Growth Model

The production function is

$$Y_t = A_t D^\alpha L_t^{1-\alpha}$$

1. $0 < \alpha < 1$. Convince yourself that this function exhibits
 - a) Constant returns to scale
 - b) Diminishing returns to labor
2. Labor Demand and Supply
 - a) Derive the labor demand equation
 - b) State the labor supply equation

- c) Find the equilibrium wage

The Malthus Population Growth Equation is given by

$$\frac{N_{t+1}}{N_t} = \left(\frac{w_t}{w_s} \right)^\gamma \xi_t$$

3. What is
 - a) w_s
 - b) γ
 - c) ξ_t and what is the value that ξ_t should take under normal circumstances?
4. Express N_{t+1} in terms of N_t and graph that.
5. Explain how population changes if $N_t < N_{t+1}$ or $N_t > N_{t+1}$
6. Find the steady state population
7. What happens to wages at the steady state population? It is higher than or lower than the subsistence wage w_s ?
8. How will shocks affect both population and wages in the short run and long run if
 - a) The shock was a temporary disease that decreased population
 - b) The shock was a permanent increase in technology
 - c) For both of the above, illustrate what happens to population and wages in the short run and long run on two graphs – one for population and the other for wages

6 Solow-Swan Growth Model

The production function is

$$Y = \bar{A} K^{\frac{1}{3}} L^{\frac{2}{3}}$$

1. Convince yourself that this function exhibits
 - a) Constant returns to scale
 - b) Diminishing returns to labor
 - c) Diminishing returns to labor
2. Given that s of Y is saved, find investments
3. Find consumption

4. To find the steady state capital,
 - a) What is the inflow of capital for every period?
 - b) What is the outflow of capital for every period?
 - c) Derive the steady state capital
 - d) Graph dK , I , and Y on the same graph, showing the steady state and consumption at the steady state
 - e) What happens if the starting stock of capital is below the steady state? What about above?
5. How will shocks affect both capital and income in the short run and long run if
 - a) The shock is a temporary increase in capital (perhaps foreign aid?)
 - b) The shock is a permanent increase in technology \bar{A}
 - c) The shock is a permanent increase in d depreciation
6. Do 2, 3, 4, and 5 for the Solow-Swan model with Labor Changes where $L_{t+1} = L_t(1 + \bar{n})$
7. Is unconditional convergence empirically true? Is conditional convergence empirically true? Explain.