COMP 652: Assignment 3

Due on Tuesday, March 31 2015

Presented to Dr. Doina Precup

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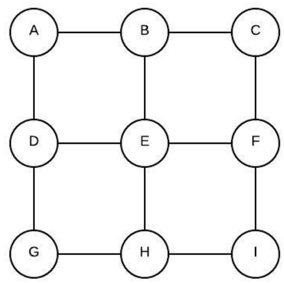
Question 1

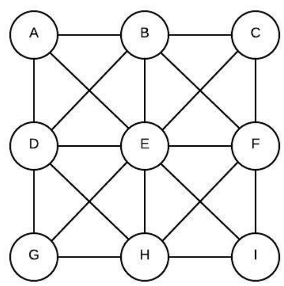
A)

In the 4-neighbor spin glass model the maximal cliques were the edges between each pixel. In an 8-neighbor spin glass model the maximal cliques become clusters of 4 pixels. As such, parametization for a 4 neighbor spin glass model will be as follows:



$$P(E) = \psi(B,E)\psi(D,E)\psi(E,F)\psi(E,H) \tag{1}$$

While the parameters for an 8 neighbor spin glass model will be as such:



$$P(E) = \psi(A,B,D,E)\psi(B,C,E,F)\psi(D,E,G,H)\psi(E,F,H,I) \tag{2}$$

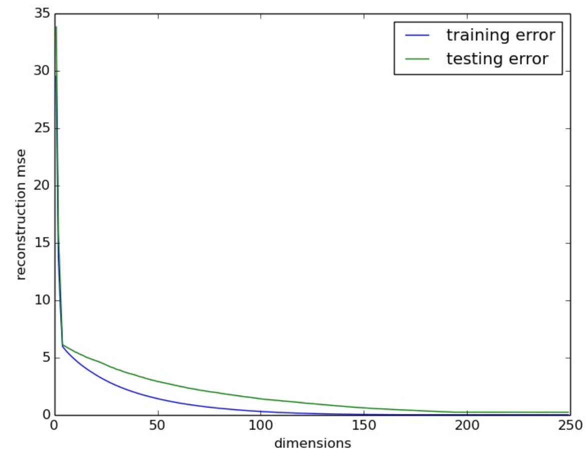
B)

The advantages and disadvantages would be related to a trade off between model precision and computation time.

With the 8 neighbor model more data will be used to infer the value a pixel, this could improve the models accuracy.This is at the expense of having to perform more calculations as well as increasing the potential of overfitting the model.

C)

Question 2



As dimensions are reduced from 250 the reconstruction error is initially quite small but becomes more important as dimensions approach 0. The shoulder of the reconstruction error line is at a dimension of approximately 25.

From this, we can conclude that given this input space we can use PCA to reduce dimensionality of the data to 25 in order to make computation more manageable without losing very much granularity in the feature data.

Question 3

A)

As with standard Hidden Markov Models, Coupled Hidden Markov models will have three categories of parameters. These are the initial probabilities, the transition probabilites and the emission probabilities. Given the system depicted in Figure 1 of assignment 3:

Initial Probabilities:

$$P(s0) \tag{3}$$

$$P(u0) \tag{4}$$

Transition Probabilities:

$$P(s_i|s_{i-1}, u_{i-1}) \tag{5}$$

$$P(u_i|u_{i-1}, s_{i-1}) \tag{6}$$

Emission Probabilities:

$$P(y_i|s_i) \tag{7}$$

$$P(z_i|u_i) \tag{8}$$

B)

In order to compute the joint probability of a sequence of observations a forward algorithm will need to be derived.

$$\begin{aligned}
\alpha_t(s_t, u_t) &= P(s_t, u_t, y_{0:T}, x_{0:T}) \\
&= \sum_{t-1=0}^{T} p(s_t, s_{t-1}, u_t, u_{t-1}, y_{1:T}, x_{1:T}) \\
&= \sum_{t-1=0}^{T} p(y_t|s_t)p(x_t|u_t)p(s_t|s_{t-1}, u_{t-1})p(u_t|s_{t-1}, u_{t-1})p(s_{t-1}, u_{t-1}, y_{1:T-1}, x_{1:T-1}) \\
&= \sum_{t-1=0}^{T} p(y_t|s_t)p(x_t|u_t)p(s_t|s_{t-1}, u_{t-1})p(u_t|s_{t-1}, u_{t-1})\alpha_{t-1}(y_{t-1}, x_{t-1})
\end{aligned} \tag{9}$$

$$\alpha_0(s_0, u_0) = p(y_0, z_0, s_0, u_0) = p(s_0)p(y_0|s_0)p(u_0)p(z_0|u_0) \tag{10}$$

C)

$$\begin{aligned}
\beta_k(s_k, u_k) &= p(y_{k+1:n}|s_k, u_k)p(z_{k+1:n}|s_k, u_k) \\
&= \sum_{z_{k+1=1}}^{m} p(y_{k+1:n}, z_{k+1:n}, s_{k+1}, u_{k+1}|s_k, u_k) \\
&= \sum_{z_{k+1=1}}^{m} p(y_{k+2:n}|s_{k+1}, u_{k+1})p(z_{k+2:n}|s_{k+1}, u_{k+1})p(y_{k+1}|s_{k+1})p(z_{k+1}|u_{k+1})p(s_{k+1}|s_k, u_k)p(u_{k+1}|s_k, u_k) \\
&= \sum_{z_{k+1=1}}^{m} \beta_{k+1}(s_{k+1}, u_{k+1})p(y_{k+1}|s_{k+1})p(z_{k+1}|u_{k+1})p(s_{k+1}|s_k, u_k)p(u_{k+1}|s_k, u_k)
\end{aligned} \tag{11}$$

$$\beta_n(s_n, u_n) = 1 \tag{12}$$

D)

The algorithm will remain largely the same except in the definition of the model parameters. More specifically, the initial and transition probabilities will need to be altered. K initial probabilities will be required and transition probabilities would be defined as follows in the current model context:

$$P(s_i|s_{i-k}, u_{i-k}) \tag{13}$$

$$P(u_i|u_{i-k}, s_{i-k}) \tag{14}$$

E)