第八次作业

2020 年 6 月 2 日

1.$(2x)^2 + (3y)^2 = 1$

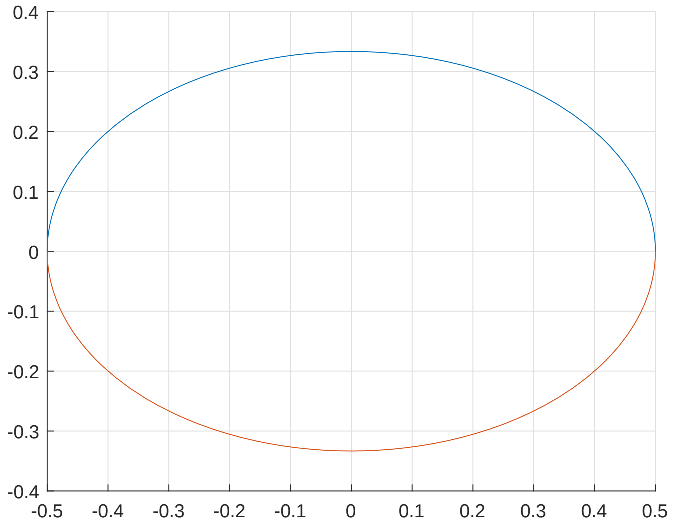
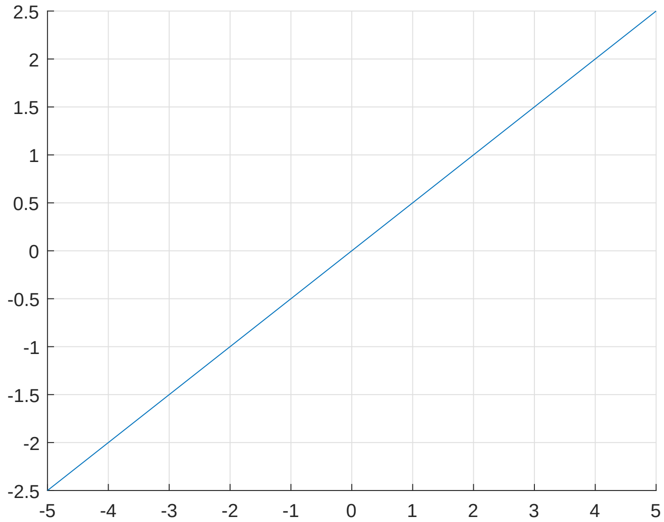


Figure 1: $(2x)^2 + (3y)^2 = 1$

2.

$$F\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ y \end{bmatrix} \Rightarrow F\begin{bmatrix} 2 \\ y \end{bmatrix} = \begin{bmatrix} 2y \\ y \end{bmatrix}$$



3.

$F(v) = F(a_1v_1 + a_2v_2 + \cdots a_nv_n) = a_1F(v_1) + a_2F(v_2) + \cdots a_nF(v_n)$

if $F(v_j)(1 \leqslant j \leqslant n)$ is already known

from one - to - one we can know that F is uniquely defined

if F is not a linear mapping,we can't conclude that F is uniquely defined

4.

$$T(x+y) = T(x) + T(y) = 0$$

$$T(kx) = kT(x) = 0$$

$$\Rightarrow x + y \in V, kx \in V$$

so $\{x \in V | T(x) = 0\}$ is a subspace of V

5.

$(1)T(A+B) = \dfrac{A+B+A^T+B^T}{2} = \dfrac{A+A^T}{2} + \dfrac{B+B^T}{2} = T(A)+T(B)$

$T(kA) = \dfrac{kA+kA^T}{2} = kT(A)$

so T is a linear mapping

$(2)T(A) = 0 \Rightarrow \dfrac{A+A^T}{2} = 0 \Rightarrow A+A^T = 0$

so the kernel of T consists in the linear space of all skew symmetric matrix

$(3)\forall A \in$ symmetric matrix,we can find countless matrices B

$A = \dfrac{B+B^T}{2} = T(B)$, so the range of T consists in the linear space of all symmetric matrices.

(4)the dimension of symmetric matrices is n

if n is an odd number,

then the dimension of skew symmetric matrices is n-1

if n is an even number,

then the dimension of skew symmetric matrices is n

6.

$$(1)\begin{cases} x=0 \\ x-y=0 \\ x-z=0 \\ x-y-z=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \\ z=0 \end{cases}$$

so the kernel of F is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$(2)F\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ x-y \\ x-z \\ x-y-z \end{bmatrix} = x\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - y\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} - z\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

so the range of F is R^3

7.

$$T\begin{bmatrix}1\ 0\ 0\\0\ 1\ 0\\0\ 0\ 1\end{bmatrix} = \begin{bmatrix}1\ 0\ 0\\0\ 1\ 0\\0\ 0\ 1\end{bmatrix}\begin{bmatrix}1\ 0\ 0\\0\ 2\ 0\\0\ 0\ 3\end{bmatrix}$$

$$T\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & -1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & -1\end{pmatrix}\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$$

$$\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix} = \begin{pmatrix}\frac{3}{2} & \frac{1}{2} & 0\\ \frac{1}{2} & \frac{3}{2} & 0\\ 0 & 0 & 3\end{pmatrix}$$

so the matrix associated with T under v_1, v_2, v_3 is $\begin{pmatrix}\frac{3}{2} & \frac{1}{2} & 0\\ \frac{1}{2} & \frac{3}{2} & 0\\ 0 & 0 & 3\end{pmatrix}$