

## 第六次作业

2020 年 5 月 10 日

1.

$$f(x+y) = c_1(x+y) + c_2(x+y) = c_1x + c_2x + c_1y + c_2y = f(x) + f(y)$$

$$f(kx) = c_1(kx) + c_2(kx) = k(c_1x + c_2x) = kf(x)$$

so  $f(x)$  is a linear function

Because all the real functions make up a linear space, so I only need to proof that  $X'$  is a subspace.

$$f_1(x) + f_2(x) = c_{11}x_1 + c_{21}x_2 + c_{12}x_1 + c_{22}x_2 = (c_{11} + c_{12})x_1 + (c_{21} + c_{22})x_2 = (f_1 + f_2)(x)$$

$$kf_1(x) = k(c_1x_1 + c_2x_2) = (kc_1)x_1 + (kc_2)x_2 = (kf_1)(x)$$

$x \in R^2, x = (x_1, x_2)$ , denotes the basis as  $g_1(x), g_2(x)$

$$\begin{cases} g_1(x) = x_1 \\ g_2(x) = x_2 \end{cases}, \text{proof: } \forall f(x) \in X', f(x) = c_1x_1 + c_2x_2 = c_1g_1(x) + c_2g_2(x)$$

$$k_1(x_1, 0) + k_2(0, x_2) = 0$$

$$\Rightarrow (k_1x_1, k_2x_2) = 0$$

$$\Rightarrow k_1 = 0, k_2 = 0$$

so  $g_1(x)$  and  $g_2(x)$  is a basis of  $X'$

2.

$$f(h_1 + h_2) = (h_1 + h_2)(s_1) = h_1(s_1) + h_2(s_1)$$

$$f(kh) = (kh)(s_1) = kh(s_1) = kf(h)$$

so  $f(h) = h(s_1)$  is a linear function

3.

因为用matlab画图，坐标轴不会调整成十字形，所以以o为原点

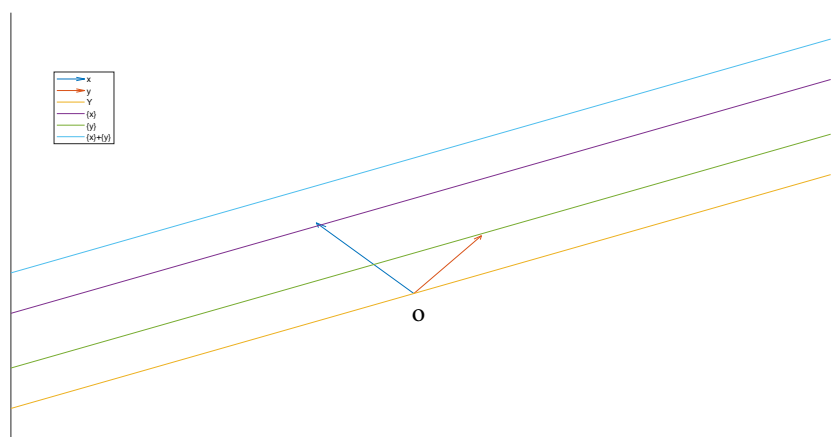


Figure 1:  $\{x\}, \{y\}, \{x+y\}$