第五次作业

葛旭

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1.

(1)$p^{n\times n}$ is a linear space

suppose $AB_1 = B_1A$

$AB_2 = B_2A$

$\forall B \in C(A), A(B_1 + B_2) = AB_1 + AB_2 = B_1A + B_2A = (B_1 + B_2)A$

$B_1 + B_2 \in C(A)$

$A(kB_1) = kAB_1 = kB_1A = (kB_1)A$

$kB_1 \in C(A)$

so C(A) is a subspace of P$^{n\times n}$

(2)suppose B = $\begin{pmatrix} b_{11} \cdots b_{n1} \\ \vdots \ddots \vdots \\ b_{n1} \cdots b_{nn} \end{pmatrix}$

$AB = \begin{pmatrix} b_{11} \cdots b_{n1} \\ \vdots \ddots \vdots \\ b_{n1} \cdots b_{nn} \end{pmatrix} = BA$

so C(A) = { B $\in P^{n\times n}$}

$$(3)\mathrm{A} = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & n \end{pmatrix}$$

$$AB = \begin{pmatrix} b_{11} \cdots\cdots b_{1n} \\ 2b_{21} \cdots 2b_{2n} \\ \vdots \qquad\quad \vdots \\ nb_{n1} \cdots nb_{nn} \end{pmatrix}$$

$$BA = \begin{pmatrix} b_{11} \;\; 2b_{12} \cdots nb_{1n} \\ \vdots \\ b_{n1} \;\; 2b_{n2} \cdots nb_{nn} \end{pmatrix}$$

$$AB = BA \Rightarrow \begin{cases} b_{12} = 0 \\ \vdots \\ b_{1n} = 0 \\ \vdots \\ b_{n(n-1)} = 0 \end{cases}$$

$$\Rightarrow \mathrm{B} = \begin{pmatrix} b_{11} & & \\ & \ddots & \\ & & b_{nn} \end{pmatrix}$$

the dimension of C(A) is n,a baisis is

$$\begin{pmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

2.

$$A = \begin{bmatrix} 3\ 2\ \text{-}\ 5\ 4 \\ 3\ \text{-}\ 1\ 3\ \text{-}\ 3 \\ 3\ 5\ \text{-}\ 13\ 11 \end{bmatrix}$$

$$Ax = 0 \Rightarrow \begin{bmatrix} 3\ 2\ \text{-}\ 5\ 4 \\ 3\ \text{-}\ 1\ 3\ \text{-}\ 3 \\ 3\ 5\ \text{-}\ 13\ 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3\ 2\ \text{-}\ 5\ 4 \\ 3\ \text{-}\ 1\ 3\ \text{-}\ 3 \\ 3\ 5\ \text{-}\ 13\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 3\ 2\ \text{-}\ 5\ 4 \\ 0\ \text{-}\ 3\ 8\ \text{-}\ 7 \\ 0\ 3\ \text{-}\ 8\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 3\ 2\ \text{-}\ 5\ 4 \\ 0\ \text{-}\ 3\ 8\ \text{-}\ 7 \\ 0\ 0\ 0\ 0 \end{bmatrix} \Rightarrow \begin{cases} 3x_1 + 2x_2 = 5x_3 - 4x_4 \\ -3x_2 = -8x_3 + 7x_4 \end{cases}$$

two basis: $\begin{pmatrix} -\frac{1}{9} \\ \frac{8}{3} \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} \frac{2}{9} \\ -\frac{7}{3} \\ 0 \\ 1 \end{pmatrix}$, the dimension of the solution space is 2