第二次作业

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1.

define f:$R^n \to P^{n-1}$

$f(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$, and $x = (a_0, a_1, \cdots, a_{n-1})^T$

one - to - one:every column vector $x = (a_0, a_1, \cdots, a_{n-1})^T$ uniquely pins a polynomial $a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$

onto:$\forall$polynomial $a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$, there is a column vector $x = (a_0, a_1, \cdots, a_{n-1})^T$ in R^n, fullfilling $f(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$

Linear:(1)$f(x + y) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_{n-1} + b_{n-1})x^{n-1}$

$= a_0 + a_1x + \cdots + a_{n-1}x^{n-1} + b_0 + b_1x + \cdots + b_{n-1}x^{n-1} = f(x) + f(y)$

(2)$f(kx) = ka_0 + ka_1x + \cdots + ka_{n-1}x^{n-1} = kf(x)$

so R^n and P^{n-1} over the same field R (the field of real numbers) are isomorphic.

2.

suppose $\begin{bmatrix} a_{11} \cdots a_{1k} \\ . \\ . \\ . \\ a_{m1} \cdots a_{mk} \end{bmatrix}$ as a vector of X,mk = n

Let $f : X \to R^n$, $\begin{bmatrix} a_{11} \cdots a_{1k} \\ . \\ . \\ . \\ a_{m1} \cdots a_{mk} \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{12} \\ . \\ . \\ . \\ a_{mk} \end{bmatrix}$

$$\begin{bmatrix} a_{11} + \mathrm{b}_{11} \cdots a_{1k} + b_{1k} \\ . \\ . \\ . \\ a_{m1} + \mathrm{b}_{m1} \cdots a_{mk} + b_{mk} \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{12} \\ . \\ . \\ . \\ a_{mk} \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{12} \\ . \\ . \\ . \\ b_{mk} \end{bmatrix}$$

$$\begin{bmatrix} ca_{11} \cdots a_{1k} \\ . \\ . \\ . \\ ca_{m1} \cdots a_{mk} \end{bmatrix} = \begin{bmatrix} ca_{11} \\ ca_{12} \\ . \\ . \\ . \\ ca_{mk} \end{bmatrix} = c \begin{bmatrix} a_{11} \\ a_{12} \\ . \\ . \\ . \\ a_{mk} \end{bmatrix}$$

so f fulfils one - to - one and onto and linear

moreover we can define $g : X \to R^n$, $\begin{bmatrix} a_{11} \cdots a_{1k} \\ . \\ . \\ . \\ a_{m1} \cdots a_{mk} \end{bmatrix} = \begin{bmatrix} a_{mk} \\ a_{m(k-1)} \\ . \\ . \\ . \\ a_{11} \end{bmatrix}$

so the map is not unique

in a word,each finite - dimensional linear space X over field K is isomorphic to K^n, n = dimX and this isomorphism is not unique.