第六次作业

2020 年 5 月 10 日

1.

$f(x+y)=c_1(x+y)+c_2(x+y)=c_1x+c_2x+c_1y+c_2y=f(x)+f(y)$

$f(kx)=c_1(kx)+c_2(kx)=k(c_1x+c_2x)=kf(x)$

so $f(x)$ is a linear function

Because all the real functions make up a linear space,so I only need to

proof that X' is a subspace.

$f_1(x)+f_2(x)=c_{11}x_1+c_{21}x_2+c_{12}x_1+c_{22}x_2=(c_{11}+c_{12})x_1+(c_{21}+c_{22})x_2=(f_1+f_2)(x)$

$kf_1(x)=k(c_1x_1+c_2x_2)=(kc_1)x_1+(kc_2)x_2=(kf_1)(x)$

$x\in R^2, x=(x_1,x_2)$, denotes the basis as $g_1(x), g_2(x)$

$\begin{cases} g_1(x)=x_1 \\ g_2(x)=x_2 \end{cases}, proof: \forall f(x)\in X', f(x)=c_1x_1+c_2x_2=c_1g_1(x)+c_2g_2(x)$

$k_1(x_1,0)+k_2(0,x_2)=0$

$\Rightarrow (k_1x_1,k_2x_2)=0$

$\Rightarrow k_1=0, k_2=0$

so $g_1(x)$ and $g_2(x)$ is a basis of X'

2.

$$f(h_1+h_2)=(h_1+h_2)(s_1)=h_1(s_1)+h_2(s_1)$$
$$f(kh)=(kh)(s_1)=kh(s_1)=kf(h)$$

so $f(h)=h(s_1)$ is a linear function

3.

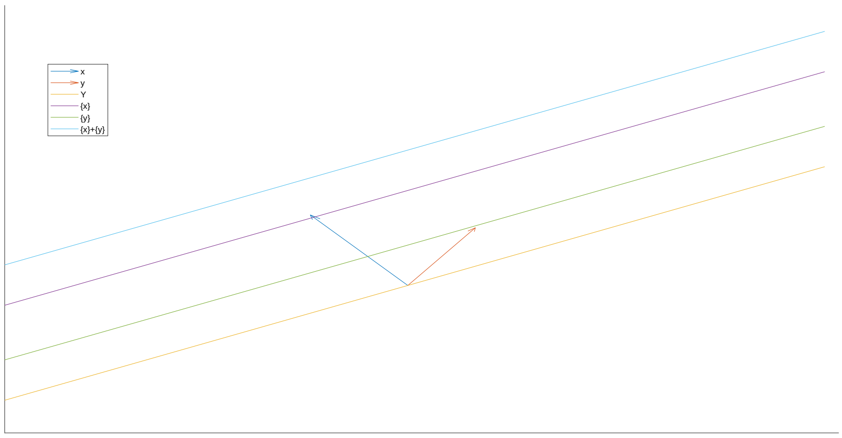

Figure 1: {x},{y},{x+y}