第九次作业

2020 年 6 月 21 日

1.$bilinear$:

$$(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}) = (\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}) + (\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}) = |x_1y_1| + |x_2y_2| + |x_1z_1| + |x_2z_2|$$

$$(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}) = (\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 + z_1 \\ y_2 + z_2 \end{bmatrix}) = |x_1(y_1 + z_1)| + |x_2(y_2 + z_2)|$$

$$= |x_1y_1 + x_1z_1| + |x_2y_2 + x_2z_2| \leqslant |x_1y_1| + |x_2y_2| + |x_1z_1| + |x_2z_2|$$

so it does't satisfy bilinear,it's not an inner product.

2.(1)$(u+v, u-v) = (u,u) - (u,v) + (v,u) - (v,v)$

$= ||u||^2 - ||v||^2$

(2)if u,v have the same norm,then $||u|| = ||v||$

$(u+v, u-v) = ||u||^2 - ||v||^2 = 0$

so u + v is orthogonal to u - v

(3)let u and v be two side length of a rhombus,

then u + v and u - v are two digonals of the rhombus,and the length of u and v are same.

so $(u+v, u-v) = ||u||^2 - ||v||^2 = 0$, so the diagonals of the rhombus are perpendicular.

$3.||u|| \leqslant ||u+av|| \Rightarrow ||u||^2 \leqslant ||u+av||^2$

$0 \leqslant 2a(u,v) + a^2v^2$

$f(a) = a^2v^2 + 2a(u,v) = 0$

$then\ a_1 = 0, a_2 = \dfrac{-2(u,v)}{||v||^2}$

and between a_1 and $a_2, f(a) < 0$

so if $f(a) \geqslant 0$ to $\forall$a $\in$ R,then (u,v) = 0

4. $\displaystyle\int_{-\pi}^{\pi} \frac{\cos nx}{\sqrt{\pi}} \frac{\cos mx}{\sqrt{\pi}} dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \cos mx dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos((n+m)x) + \cos((n-m)x) dx$

$\displaystyle= \frac{1}{2\pi}[\frac{\sin((n+m)x)}{n+m} + \frac{\sin((n-m)x)}{n-m}]|_{-\pi}^{\pi} = 0$

$\displaystyle\int_{-\pi}^{\pi} \frac{\sin nx}{\sqrt{\pi}} \frac{\sin mx}{\sqrt{\pi}} dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx \sin mx dx = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \cos((n+m)x) - \cos((n-m)x) dx$

$\displaystyle= -\frac{1}{2\pi}[\frac{\sin((n+m)x)}{n+m} - \frac{\sin((n-m)x)}{n-m}]|_{-\pi}^{\pi} = 0$

$\displaystyle\int_{-\pi}^{\pi} \frac{\sin nx}{\sqrt{\pi}} \frac{\cos mx}{\sqrt{\pi}} dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx \cos mx dx = 0$

$\displaystyle\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}} \frac{\cos nx}{\sqrt{\pi}} dx = \frac{1}{\pi\sqrt{2}} \int_{-\pi}^{\pi} \cos nx = 0$

$\displaystyle\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}} \frac{\sin nx}{\sqrt{\pi}} dx = \frac{1}{\pi\sqrt{2}} \int_{-\pi}^{\pi} \sin nx = 0$

$\displaystyle\int_{-\pi}^{\pi} \frac{\sin nx}{\sqrt{\pi}} \frac{\sin nx}{\sqrt{\pi}} dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin^2 nx dx = 1$

$\displaystyle\int_{-\pi}^{\pi} \frac{\cos nx}{\sqrt{\pi}} \frac{\cos nx}{\sqrt{\pi}} dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos^2 nx dx = 1$

$\displaystyle\int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} dx = 1$

$5.v_1 = 1, v_2 = x, v_3 = x^2$

$u_1 = 1$

$$u_2 = v_2 - \frac{v_2 \cdot u_1}{u_1 \cdot u_1} u_1 = x - \frac{\int_0^1 x dx}{\int_0^1 1 dx} \cdot 1 = x - \frac{1}{2}$$

$$u_3 = v_3 - \frac{v_3 \cdot u_1}{u_1 \cdot u_1} u_1 - \frac{v_3 \cdot u_2}{u_2 \cdot u_2} u_2 = x^2 - \frac{\int_0^1 x^2 dx}{\int_0^1 1 dx} \cdot 1 - \frac{\int_0^1 x^2 (x - \frac{1}{2}) dx}{\int_0^1 (x - \frac{1}{2})^2 dx} (x - \frac{1}{2})$$

$$= x^2 - \frac{1}{3} - (x - \frac{1}{2}) = x^2 - x + \frac{1}{6}$$

$$\int_0^1 (x - \frac{1}{2})^2 = \frac{1}{12} \Rightarrow \int_0^1 (2\sqrt{3})^2 (x - \frac{1}{2})^2 = 1 \Rightarrow u_2 = 2\sqrt{3}(x - \frac{1}{2})$$

$$\int_0^1 (x^2 - x + \frac{1}{6})^2 = \frac{1}{180} \Rightarrow u_3 = \sqrt{5}(6x^2 - 6x + 1)$$

$$\begin{cases} u_1 = 1 \\ u_2 = 2\sqrt{3}(x - \frac{1}{2}) \\ u_3 = \sqrt{5}(6x^2 - 6x + 1) \end{cases}$$

6.the orthonormal basis of P^2[x] is

$$\begin{cases} u_1 = 1 \\ u_2 = 2\sqrt{3}(x - \frac{1}{2}) \\ u_3 = \sqrt{5}(6x^2 - 6x + 1) \end{cases}, q = k_1 u_1 + k_2 u_2 + k_3 u_3$$

$$k_1 = (q, u_1) = \int_0^1 q u_1 dx = u_1(\frac{1}{2}) = 1$$

$$k_2 = (q, u_2) = \int_0^1 q u_2 dx = u_2(\frac{1}{2}) = 0$$

$$k_3 = (q, u_3) = \int_0^1 q u_3 dx = u_3(\frac{1}{2}) = -\frac{\sqrt{5}}{2}$$

$$q = u_1 - \frac{\sqrt{5}}{2} u_3 = 1 - \frac{5}{2}(6x^2 - 6x + 1) = -15x^2 + 15x - \frac{3}{2}$$

$7.v_1 = (1, 2, 3, -4), v_2 = (-5, 4, 3, 2)$

$u_1 = v_1$

$u_2 = v_2 - \frac{v_2 \cdot u_1}{u_1 \cdot u_1} \cdot u_1 = (-\frac{77}{15}, \frac{56}{15}, \frac{13}{5}, \frac{38}{15})$

$x_1 = \frac{u_1}{||u_1||} = (\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{3}{\sqrt{30}}, \frac{-4}{\sqrt{30}})$

$x_2 = \frac{u_2}{||u_2||} = (-\frac{77}{15}\sqrt{\frac{15}{802}}, \frac{56}{15}\sqrt{\frac{15}{802}}, \frac{13}{5}\sqrt{\frac{15}{802}}, \frac{38}{15}\sqrt{\frac{15}{802}})$

suppose $v_1 \cdot x = 0, v_2 \cdot x = 0$

$$\begin{cases} x_1 + 2x_2 + 3x_3 - 4x_4 = 0 \\ -5x_1 + 4x_2 + 3x_3 + 2x_4 = 0 \end{cases}$$

$\Rightarrow (10, 9, 0, 7), (3, 9, -7, 0)$

$x_3 = (\frac{10}{\sqrt{230}}, \frac{9}{\sqrt{230}}, 0, \frac{7}{\sqrt{230}})$

$x_4 = (\frac{3}{\sqrt{139}}, \frac{9}{\sqrt{139}}, \frac{-7}{\sqrt{139}}, 0)$

x_3, x_4 are two orthonormal basis of U$^{\perp}$

$8.U = span((1, 1, 0, 0), (0, 0, 1, 2))$

$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$, we need to find x = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ so that Cx = b and b is near $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

$C^T Cx = C^T b, x = (C^T C)^{-1} C^T b$

so $u = Cx = C(C^T C)^{-1} C^T \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.5 \\ 2.2 \\ 4.4 \end{bmatrix}$