General resolution

4. H_1: If the sky is clear, comets and stars can be seen.
   H_2: There was a full moon on last Sunday.
   H_3: The sky was clear last Sunday.
   C: Could comets and the moon be seen last Sunday?

Check the validity of the conclusion.
A propositional formula U is a theorem iff the empty clause can be derived from the CNF of $\neg U$ using the resolution algorithm.
Let $U_1, U_2, \dots, U_m, V$ be propositional formulas.
$U_1, U_2, \dots, U_m \vdash V$ iff $U_1, U_2, \dots, U_m \models$ iff $CNF(U_1 \wedge U_2 \wedge \dots \wedge U_m \wedge \neg V) \vdash_{Res} \square$

Propositional formulas:
$H_1: SC \to C \wedge S \equiv \neg SC \vee (C \wedge S) \equiv (\neg SC \vee C) \wedge (\neg SC \vee S) = C_1 \wedge C_2$
$H_2: sunday \to M \equiv \neg sunday \vee M : C_3$
$H_3: SC \wedge sunday = C_4 \wedge C_5$
$C: sunday \wedge C \wedge M$
$\neg C = \neg sunday \vee \neg C \vee \neg M : C_6$

$S = \{C_1, C_2, C_3, C_4, C_5, C_6\}$
$C_1: \neg SC \vee C$
$C_2: \neg SC \vee S$
$C_3: \neg sunday \vee M$
$C_4: SC$
$C_5: sunday$
$C_6: \neg sunday \vee \neg C \vee \neg M$

Variables:
SC – the sky is clear
C – comets can be seen
S – stars can be seen
sunday – it's Sunday
M – the moon can be seen

$Res_{sunday}(C_5, C_6) = \neg C \vee \neg M = C_7$
$Res_M(C_3, C_7) = \cancel{\neg sunday}\ \neg sunday \vee \neg C = C_8$
$Res_{sunday}(C_5, C_8) = \neg C = C_9$
$Res_C(C_1, C_9) = \neg SC = C_{10}$
$Res_{SC}(C_4, C_{10}) = \square$
$\Rightarrow CNF(H_1 \wedge H_2 \wedge H_3 \wedge \neg C) \vdash_{Res} \square$, so C is deducible from the hypotheses, therefore the conclusion C holds.

Liniar resolution

5. Consider the following hypotheses and check the validity of the conclusion.

H_1: In a zoo there are blue lizards, red lizards or white lizards.
H_2: There are no blue lizards.
H_3: In a zoo no red lizards were brought.
H_4: A small lizard that isn't red but may be white hatched.
Conclusion (C): There is a white lizard in a zoo.

$$H_1 \land H_2 \land H_3 \land H_4 \vdash C \text{ iff } H_1 \land H_2 \land H_3 \land H_4 \land \neg C \text{ is inconsistent}$$

Propositional formulas:

$H_1: B \lor R \lor W$
$H_2: \neg B$
$H_3: \neg R$
$H_4: \neg R \lor W$
$\neg C: \neg W$

The clausal forms are $S = \{C_1, C_2, C_3, C_4, C_5\}$
(in set of clauses)

$C_1: B \lor R \lor W$
$C_2: \neg B$
$C_3: \neg R$
$C_4: \neg R \lor W$
$C_5: \neg W$

Propositional variables
B -
R -
W -

$\boxed{C_5 = \neg W}$ $\boxed{C_1 = B \lor R \lor W}$
↓
$\boxed{C_6 = B \lor R}$ $\boxed{C_2 = \neg B}$
↓
$\boxed{C_7 = R}$ $\boxed{C_3 = \neg R}$
↓
$\boxed{C_8 = \square}$

$S \vdash^{lin}_{Res} \square$, therefore S is an inconsistent set of clauses, hence the conclusion C holds

Truth table

1. Consider the following hypotheses and check the validity of the conclusion.

H_1: A library is a place to concentrate.
H_2: Someone who reads a book needs to concentrate.
H_3. In order for someone to concentrate, there needs to be quiet.
H_4: If someone reads in a library, it has to be quiet.
C: ~~If in~~ In a library it is quiet.

Propositional variables:

$V(l, \cancel{b}, r, c, q) = (l \to \cancel{c}) \wedge (r \wedge b \to c) \wedge (c \to q) \wedge (r \wedge l \to q)$

$H_1: l \to c$
$H_2: r \to c$
$H_3: c \to q$
$H_4: r \wedge l \to q$
$C: \cancel{l \wedge q}\ l \to q$

$i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}, i_{13}, i_{14}, i_{15}, i_{16}$ are all models => the conclusion holds

l	r	c	q	$l \to c$	$r \to c$	$c \to q$	$r \wedge l \to q$	$l \to q$	V	$V \to l \to q$	
T	T	T	T	T	T	T	T	T	T	T	i_1
T	T	T	F	T	T	F	F	F	F	T	i_2
T	T	F	T	F	F	T	T	T	F	T	i_3
T	F	T	T	T	T	T	T	T	T	T	i_4
F	T	T	T	T	T	T	T	T	T	T	i_5
T	T	F	F	F	F	T	F	F	F	T	i_6
T	F	T	F	T	T	F	T	F	F	T	i_7
F	T	T	F	T	T	F	T	T	F	T	i_8
F	T	F	F	T	F	T	T	T	F	T	i_9
F	F	T	F	T	T	F	T	T	F	T	i_{10}
F	T	F	T	T	F	T	T	T	F	T	i_{11}
F	F	T	T	T	T	T	T	T	T	T	i_{12}
T	F	F	T	F	T	F	F	T	F	T	i_{13}
F	F	F	T	T	T	T	F	T	F	T	i_{14}
T	F	F	F	F	T	T	T	F	F	T	i_{15}
F	F	F	F	T	T	T	T	T	T	T	i_{16}

Definition of deduction

1. Consider the following hypotheses and check the validity of the conclusion.

H1: Ann will go to Tokyo if Bennet will go and Tokyo is a beautiful city.

H2: If Mary will go to Tokyo then Ann will go too.

H3: If Tokyo is a beautiful city Mary will go there.

H4: Tokyo is a beautiful city

Conclusion (C) Will Ann go to Tokyo?

Create the normal forms:

$H_1: \neg B \wedge Tk \rightarrow A$

$H_2: M \rightarrow A$

$H_3: Tk \rightarrow M$

$H_4: Tk$

$C: A$

Using $U_1, U_2, \dots U_n$, propositional formulas, called hypotheses an V is a formula, called conclusion. V is inferable from $U_1, \dots U_n$ and we denote by $U_1, \dots, U_n \vdash V$, if there exists a sequence $(f_1, f_2 \dots, f_m)$ of formulas such that $f_m = V$ and $(\forall i) \in \{1, \dots m\}$:

a) $f_i \in A_p$ (axioms)

b) $f_i \in \{U_1, \dots, U_n\}$

c) $f_{i_1}, f_{i_2} \vdash_{m.p} f_i$

The sequence $(f_1, f_2, \dots, f_m)$ is called a deduction of V from $U_1, U_2, \dots U_n$.

The deduction process:

$f_1 = H_1: \neg B \wedge Tk \rightarrow A$

$f_2 = H_2: M \rightarrow A$

$f_3: Tk \rightarrow M$

$f_4: Tk$

$f_3, f_4 \vdash_{m.p} M : f_5$

$f_2, f_5 \vdash_{m.p} A : f_6 = C$

The sequence of formulas $(f_1, f_2, f_3, f_4, f_5, f_6)$ is a deduction of C from the hypotheses, therefore, based on the hypotheses, Amir will go to Tokyo.

Semantic tableaux

3. Consider the following hypotheses and check the validity of the conclusion.

H1. If horses race today, there will be a winner.

H2. ~~Horses race to~~ If horses race and there is a winner then there will be a party.

H3. The preparations for the race are ready.

H4: If there is a tie, the party will still be held.

Conclusion (C): People attending the race will go to a party.

Propositional formulas:

H1: $R \to W$

H2: $R \land W \to P$

H3: R

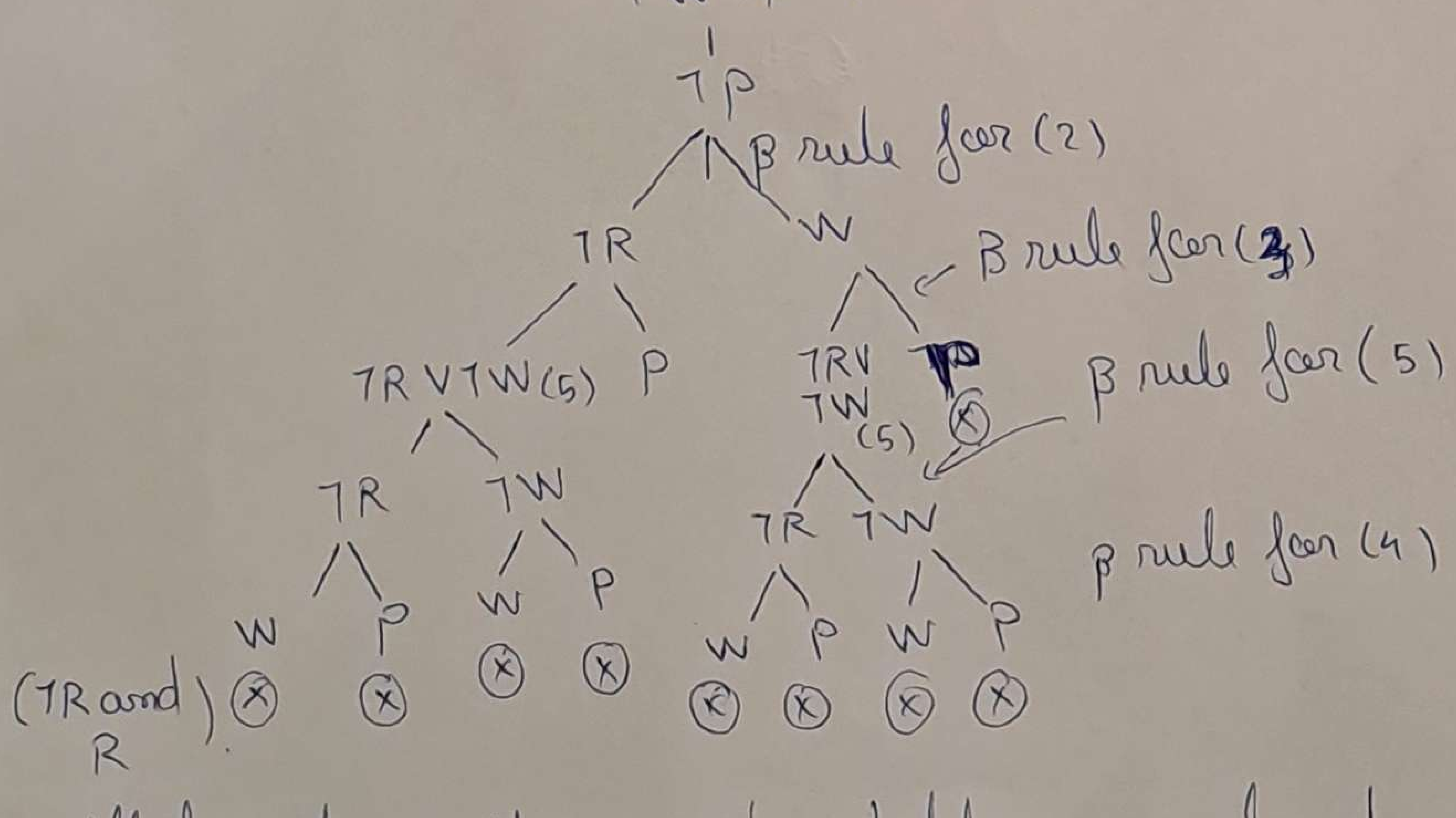
H4: $\neg W \to P$

C: P

The CNF form: $U((R \to W) \land (R \land W \to P) \land R \land (\neg W \to P) \land \neg P)$

$H_1, H_2, H_3, H_4 \vdash C$ iff $H_1 \land H_2 \land H_3 \land H_4 \land \neg C$ has a closed semantic tableaux

$(R \to W) \land (R \land W \to P) \land R \land (\neg W \to P) \land \neg P$ (1)



All branches of the semantic tableau are closed, containing pairs of opposite literals, therefore U is inconsistent and the conclusion C holds.

General predicate resolution

Consider the following hypotheses and check the validity of the conclusion

1. Hypotheses.

   H_1. All hounds howl at night.

   H_2. Anyone who has any cats will not have any mice.

   H_3. Light sleepers do not have anything which howls at night.

   H_4. John has either a cat or a hound.

   Conclusion

   C. If John is a light sleeper, then John does not have any mice.

$H_1, H_2, H_3, H_4 \vdash C$

Create the first-order (predicate) formulas:

$H_1: (\forall x)(h(x) \rightarrow hn(x))$

$H_2: (\forall x)((\forall y)(has(x,y) \wedge cat(y)) \rightarrow (\forall z)(\neg has(x,z) \wedge mice(z)))$

$H_3: (\forall x)(ls(x) \rightarrow (\forall y)(\neg has(x,y) \wedge hn(y)))$

$H_4:$ ~~has~~ $(\exists x)(has(John, x) \wedge cat(x)) \vee (\exists y)(has(John, y) \wedge h(y))$

$C: ls(John) \rightarrow (\forall x)(\neg has(John, x) \wedge mice(x))$

D - domain of the exercise

John - constant of the domain

Unary predicate symbols: h, hn, cat, mice, ls

Binary predicate symbols: has

$h: D \rightarrow \{T, F\}$, $h(x) = T$ if x is a hound

$hn: D \rightarrow \{T, F\}$, $hn(x) = T$ if h howls at night

$cat: D \rightarrow \{T, F\}$, $cat(x) = T$ if x is a cat

$mice: D \rightarrow \{T, F\}$, $mice(x) = T$ if x is a mouse

$ls: D \rightarrow \{T, F\}$, $ls(x) = T$ if x is a light sleeper

$has: D \rightarrow \{T, F\}$, $has(x,y) = T$ is x has y

Simplification of hypotheses:

$H_1: (\forall x)(h(x) \rightarrow hn(x))$

$H_2:$

We apply a refutation proof method: general predicate resolution.

The clausal normal forms corresponding to the hypotheses and the negation of the conclusion are as follows:

$H_1^c: \neg h(x) \vee hn(x) = C_1$

$H_2^c: \neg has(y,a) \vee cat(a) \vee \neg has(y,b) \wedge mice(b) = C_2 \wedge C_3$

$H_3^c: \neg ls(z) \vee \neg has(z,c) \wedge hn(c) = C_4 \wedge C_5$

H_4: $has(John, d) \wedge ((cat(d) \vee h(d))) = C_6 \wedge C_7$

$(\neg C)^C = \neg(\neg ls(John) \vee \neg has(John, e) \wedge mice(e)) = ls(John) \wedge has(John, e) \vee \neg mice(e)$
$= C_8 \wedge C_9$

≃ If John is a lightsleeper ~~he doesn~~ he has anything that isn't a mice.

To the set of clauses $S = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9\}$ general predicate resolution is applied.

$$C_{10} = Res^{Pr}_{[h \leftarrow e]}\ C_3, C_9 = has(John, e),\ e\text{ - universal} = C_{10}$$

$$C_{11} = Res^{Pr}_{\substack{[y \leftarrow John \\ a \leftarrow e \\ h \leftarrow e]}}\ C_2, C_{10} = cat(e) \cancel{\vee 7} = C_{11}$$

$$C_{12} = Res^{Pr}_{\substack{[z \leftarrow John \\ c \leftarrow e]}}\ C_4, C_{10} = \neg ls(John) = C_{12}$$

$$C_{13}, C_{12} = Res^{Pr}_{[John]} = \square$$

The most general unifier generated during the resolution process is the substitution:

$$[h \leftarrow e,\ y \leftarrow John,\ a \leftarrow e,\ z \leftarrow John,\ c \leftarrow e]$$

$S \vdash^{Pr}_{Res} \square$, therefore S is an inconsistent set and the deduction $H_1, H_2, H_3, H_4 \vdash C$ holds.

From the hypotheses we conclude that "If John is a light sleeper, then John does not have any mice."

Semantic tableaux

10. Consider the following hypotheses and check the validity of the conclusion.

H1: Every child loves every candy.

H2. Anyone who loves some candy is not a nutrition fanatic.

H3. Anyone who eats any pumpkin is a nutrition fanatic.

H4 Anyone who buys any pumpkin either carves it or eats it.

H5. John buys a pumpkin.

H6. Lifesavers is a candy.

Conclusion:

C: If John is a child, then John carves some pumpkin.

$H_1, H_2, H_3, H_4, H_5, H_6 \vdash C$

Create the first-order (predicate) formulas:

$H_1: (\forall x)(child(x) \to (\forall y)(loves(x,y) \wedge candy(y)))$

$H_2: (\forall x)((\exists y)(loves(x,y) \wedge candy(y)) \to \neg fanatic(x)))$

$H_3: (\forall x)((\forall y)(eat(x,y) \wedge pumpkin(y)) \to fanatic(x)))$

$H_4: (\forall x)(\forall y)(buy(x,y) \wedge pumpkin(y) \wedge (carve(x,y) \vee eat(x,y)))$

$H_5: (\forall y)(buy(John, y) \wedge pumkin(y))$

H_6: candy(Lifesavers)

C: $child(John) \to ((\exists x)(carve(John, x) \wedge pumpkin(x)))$

D-domain of the exercise

John - constant of the domain, Lifesavers - constant

Unary predicate symbols: child, candy, pumpkin, fanatic

Binary predicate symbols: loves, eat, carve

child: $D \to \{T, F\}$, child(x) = T if x is a child

candy: $D \to \{T, F\}$, candy(x) = T if x is a candy

pumpkin: $D \to \{T, F\}$, pumpkin(x) = T if x is a pumpkin

fanatic: $D \to \{T, F\}$, fanatic(x) = T if x is a fanatic

loves: $D \to \{T, F\}$, loves(x, y) = T if x loves y

eat: $D \to \{T, F\}$, eat(x, y) = T if x eats y

carve: $D \to \{T, F\}$, carve(x, y) = T if x carves y

The semantic tableaux method is a refutation method, thus we have to negate the conclusion and use the Theorem of soundness and completeness.

$H_1, H_2, H_3, H_4, H_5, H_6 \vdash C$ iff $H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge H_5 \wedge H_6 \wedge \neg C$ has a closed semantic tableaux

The semantic tableau corresponding to the conjunction of the hypotheses and the negation of the conclusion

Simplified hypotheses:

$H_1: (\forall x)(\forall y)(child(x) \to lc(x,y))$

$H_2: (\forall x)(\exists y)\, lc(x,y)$

$H_3: (\forall x)(\forall y)(eat(x,y) \to \neg lc(x,y))$

$H_4: (\forall x)(\forall y)(buy(x,y) \wedge eat(x,y))$ (factorized / absorbed $\neg eat(x,y) \vee eat(x,y)$)

$H_5: (\exists x)\, buy(John, x)$

$H_6:$ candy(Lifesavers)

$\neg C$: child(John) $\wedge (\forall x)($ eat(John, x)

$(\forall x)(\forall y)(child(x) \to lc(x,y)) \wedge (\forall x)(\exists y)\, lc(x,y) \wedge (\forall x)(\forall y)(eat(x,y) \to \neg lc(x,y))$
$\wedge (\forall x)(\forall y)(buy(x,y) \wedge eat(x,y)) \wedge (\exists x)\, buy(John,x) \wedge (\exists x)$ candy(Lifesavers)
$\wedge(child(John) \wedge (\forall x)\, eat(John,x))$ (1)

| α rule for (1)

$(\forall x)(\forall y)(child(x) \to lc(x,y))$ (2) $\equiv child(John) \to lc(John, c)$ univ. inst. $x \leftarrow John$, $y \leftarrow c$

| John, univ inst (x) c, univ inst (y) lc(John, c)

$(\forall x)(\forall y)\, lc(x,y)$ (3)

|

$(\forall x)(\forall y)(eat(x,y) \to \neg lc(x,y))$ (4) $\equiv (\forall x)(\forall y)(\neg eat(x,y) \vee \neg lc(x,y))$

|

$(\forall x)(\forall y)(buy(x,y) \wedge eat(x,y))$ (5)

|

$(\exists x)\, buy(John, x)$ (6)

|

candy(Lifesavers) (7)

|

$child(John) \wedge (\forall x)\, eat(John,x)$ (8)

| δ rule for (6)

buy(John, d) ~~(5)~~

| α rule for (5), d-instantiation

$(\forall x)(\forall y)\, eat(x,y)$ (9) $\equiv eat(John, c)$ $x \leftarrow John$, $y \leftarrow c$ univ. inst.

|

$(\forall x)(\forall y)\, buy(x,y)$ (10) $\equiv buy(John, c)$ $x \leftarrow John$, $y \leftarrow c$ univ. inst

| α rule for (8)

child(John)

|

$(\forall x)\, eat(John, x)$ (11) $\equiv eat(John, c)$ $x \leftarrow c$ univ. inst.

γ rule for (2)

$(\forall x)(\forall y)\, \neg child(x)$ (12) | $(\forall x)(\forall y)\, lc(x,y)$ (13 γ rule for (4)

$(\forall x)(\forall y)$ $\neg eat(x, y)$

univ. inst. $x \leftarrow$ John, $y \leftarrow C$

$\neg eat(John, C)$

$lc(John, C)$

$eat(John, C)$

$buy(John, C)$

⊗

$(\forall x)(\forall y)$ $\neg lc(x, y)$

univ. inst. $x \leftarrow$ John, $y \leftarrow C$

$\neg lc(John, C)$

$lc(John, C)$

$eat(John, C)$

$buy(John, C)$

⊗

γ rule for (4)

$(\forall x)(\forall y)$ $\neg eat(x, y)$

univ. inst. $x \leftarrow$ John, $y \leftarrow C$

$\neg eat(John, C)$

$lc(John, C)$

$eat(John, C)$

$buy(John, C)$

⊗

$(\forall x)(\forall y)$ $\neg lc(x, y)$

univ. inst. $x \leftarrow$ John, $y \leftarrow C$

$\neg lc(John, C)$

$lc(John, C)$

$eat(John, C)$

$buy(John, C)$

⊗

$\Rightarrow H_1, H_2, H_3, H_4, H_5, H_6 \vdash C$ is a valid theorem

---

$lc : D \to \{T, F\}$, $lc(x, y) = T$ if x loves candy y

$eat : D \to \{T, F\}$, $eat(x, y) = T$ if x eats ~~a~~ pumpkin y

$buy : D \to \{T, F\}$, $buy(x, y) = T$ if x buys pumpkin y

Linear resolution

22. Consider the following hypotheses and check the validity of the conclusion.

H_1. Everyone who feels warm either is drunk, or every costume they have is warm.

H_2. Every costume that is warm is furry.

H_3. Every AI student is a CS student

H_4. Every AI student has some robot costume.

H_5. No robot costume is furry.

Conclusion:

C: If every CS student feels warm, then every AI student is drunk.

$H_1, H_2, H_3, H_4, H_5 \vdash C$

Create the first-order (predicate) formulas:

H_1. $(\forall x)(warm(x) \rightarrow (drunk(x) \vee (\forall y)(costume(y) \wedge warm(y))))$

H_2: $(\forall x)(costume(x) \wedge warm(x) \rightarrow furry(x))$

H_3: $(\forall x)(AI(x) \rightarrow CS(x))$

H_4: $(\forall x)\ AI(x) \wedge \cancel{cost} (\exists y)(costume(y) \wedge robot(y))$

H_5: $\neg(\exists x)(costume(x) \wedge robot(x) \wedge furry(x))$

C: $(\forall x)(CS(x) \wedge warm(x)) \rightarrow (\forall y)(AI(y) \wedge drunk(y))$

D-domain of the exercise

Unary predicate symbols: warm, drunk, costume, furry, AI, CS, robot

warm: $D \rightarrow \{T, F\}$, $warm(x) = T$ if x is warm

drunk: $D \rightarrow \{T, F\}$, $drunk(x) = T$ if x is drunk

costume: $D \rightarrow \{T, F\}$, $costume(x) = T$ if x is a costume

furry: $D \rightarrow \{T, F\}$, $furry(x) = T$ if x is furry

AI: $D \rightarrow \{T, F\}$, $AI(x) = T$ if x is an AI student

CS: $D \rightarrow \{T, F\}$, $CS(x) = T$ if x is a CS student

robot: $D \rightarrow \{T, F\}$, $robot(x) = T$ if x is a robot

We apply the linear proof method (predicate resolution).

The pre-clausal normal forms corresponding to the hypotheses and the negation of the conclusion are as follows:

H_1: $(\forall x)(\neg warm(x) \vee drunk(x) \vee (\forall y)(costume(y) \wedge warm(y)))$

H_2: $(\forall x)(\neg costume(x) \vee \neg warm(x) \vee furry(x))$

H_3: $(\forall x)(\neg AI(x) \vee CS(x))$

H_4: $(\forall x)AI(x) \wedge (\exists y)(costume(y) \wedge robot(y))$

H_5: $(\forall x)(\neg costume(x) \vee \neg robot(x) \vee furry(x))$

~~C: $(\exists x)$~~ $\neg C^0$: $(\forall x)(CS(x) \wedge warm(x)) \wedge (\exists y)(\neg AI(y) \vee \neg drunk(y))$

= If every CS student feels warm, then there is a student that doesn't study Ai that could be drunk or not.

Clauses (in clausal normal forms).

$C_1: \neg warm(x) \vee drunk(x) \vee costume(y)$

$C_2: warm(z)$

$C_3: \neg Ai(m) \vee CS(m)$

$C_4: Ai(m)$

$C_5: costume(p)$

$C_6: robot(q)$

$C_7: \neg costume(r) \vee \neg robot(r) \vee furry(r)$

$C_8: CS(s)$

$C_9: warm(t) \equiv warm(z) = C_2$, so it is redundant

$C_9: \neg Ai(t) \vee \neg drunk(t)$

Constants of the domain: p, q

For linear resolution we choose the top clause:

$\boxed{C_3}$ $\boxed{C_4}$ → $\boxed{C_{10}}$ $CS(m)$

$\boxed{C_9}$ $\boxed{C_4}$ → $\boxed{C_{10}}$ $\neg drunk(t)$

$\boxed{C_7}$ $\boxed{C_5}$ → $\boxed{C_{10}}$ $\neg robot(p) \vee furry(p)$ - constants

$\boxed{C_7}$ $\boxed{C_6}$ → $\boxed{C_{10}}$ $\neg costume(q) \vee furry(q)$ - constants

$\boxed{C_1}$ $\boxed{C_2}$ → $\boxed{C_{10}}$ $drunk(x) \vee costume(y)$

We cannot derive the empty clause, so $S \nvdash^{P_r}_{Res} \square$, so the negated conclusion is consistent, hence C does not hold.

Definition of deduction

5. Consider the following hypotheses and check the validity of the conclusion:
   1. Anyone whom Mary loves is a football star.
   2. Any student who does not pass does not play.
   3. John is a student.
   4. Any student who does not study does not pass.
   5. Anyone who does not play is not a football star.
   6. (Conclusion): If John does not study, then Mary does not love John.

$H_1, H_2, H_3, H_4, H_5 \vdash C$

Create the first-order (predicate) formulas:

$H_1: (\forall x)(love(Mary, x) \to star(x))$

$H_2: (\forall x)(student(x) \wedge \neg pass(x) \to \neg play(x))$

$H_3:$ $student(John)$

$H_4: (\forall x)(student(x) \wedge \neg study(x) \to \neg pass(x))$

$H_5: (\forall x)(\neg play(x) \to \neg star(x))$

$C: \neg study(John) \to \neg love(Mary, John)$

D - domain of the exercise

John, Mary - constants of the domain

Unary predicate symbols: star, student, pass, play

Binary predicate symbols: love

$star: D \to \{T, F\}$, $star(x) = T$ if x is a football star

$student: D \to \{T, F\}$, $student(x) = T$ if x is a student

$pass: D \to \{T, F\}$, $pass(x) = T$ if x passes

$play: D \to \{T, F\}$, $play(x) = T$ if x plays

$love: D \to \{T, F\}$, $love(x, y) = T$ if x loves y

We apply the definition of deduction proof method.

Let $U_1, U_2, \dots U_n, V$ be first-order formulas, $U_1, U_2, \dots U_n$ are the hypotheses of the conclusion V. V is deducible (inferable, derivable) from $U_1, U_2 \dots U_n$, notation: $U_1, U_2, \dots, U_n \vdash V$, if there exists a sequence of formulas $(f_1, f_2, \dots f_m)$ such that $f_m = V$ and $\forall i \in \{1, \dots m\}$:

a) $f_i \in A_{Pr}$ (axiom of predicate logic)

b) $f_i \in \{U_1, U_2, \dots U_n\}$ (hypothesis formula)

c) $f_{i_1}, f_{i_2} \vdash_{mp} f_i$, $i_1 < i$ and $i_2 < i$

d) $f_j \vdash_{gen} f_i,\ j < i$

The sequence $(f_1, f_2, \dots f_m)$ is called the deduction of V from $U_1, U_2, \dots U_m$.

$(\forall x)(love(Mary, x) \to star(x)), (\forall y)(student(y) \land \neg pass(y) \to \neg play(y)),$
$, student(John), (\forall z)(student(z) \land \neg study(z) \to \neg pass(z)), (\forall s)(\neg play(s) \to$
$\to \neg star(s)) \vdash \neg study(John) \to \neg love(Mary, John)$

$f_1 : (\forall x)(love(Mary, x) \to star(x))$

$f_2 : (\forall y)(student(y) \land \neg pass(y) \to \neg play(y))$

$f_3 : student(John)$

$f_4 : (\forall z)(student(z) \land \neg study(z) \to \neg pass(z))$

$f_5 : (\forall s)(\neg play(s) \to \neg star(s))$

$f_2 \vdash_{univ-inst} f_6 = student(John) \land \neg pass(John) \to \neg play(John)$

$f_5 \vdash_{univ-inst} f_7 = \neg play(John) \to \neg star(John)$

$f_7, f_5 \vdash_{syllogism} f_8 = student(John) \land \neg pass(John) \to \neg star(John)$

$f_1 \vdash_{mt} f_9 = (\forall x)(\neg love(Mary, x) \to \neg star(x))$

$f_9 \vdash_{univ-inst} f_{10} = \neg love(Mary, John) \to \neg star(John)$

$f_8 \vdash_{mt} f_{11} = \neg student(John) \lor pass(John) \to star(John)$

$f_{11}, f_3 \vdash_{mp} f_{12} = pass(John) \to star(John)$

$f_{12} \vdash_{mt} f_{13} = \neg pass(John) \to \neg \cancel{star}(John)$

$f_{14}, f_{13} \vdash_{syllogism} f_{15} = student(John) \land \neg study(\cancel{John}) \to \neg \cancel{pass}\ star(John)$

$f_{\cancel{4}} \vdash_{univ-inst} f_{14} = student(John) \land \neg study(John) \to \neg pass(John)$

$f_{15} \vdash_{m.t} \neg student(John) \land study(John) \to pass(John) = f_{16}$

$f_{16} \vdash_{m.p.} study(John) \to pass(John) = f_{17}$

$f_{17} \vdash_{m.t} \neg study(John) \to \neg pass(John) = f_{18}$

$f_6 \vdash_{m.t} \neg student(John) \lor pass(John) \to play(John) = f_{19}$

$f_3, f_{19} \vdash_{m.p} pass(John) \to play(John) = f_{20}$ $f_{20} \vdash_{m.t} \neg pass(John) \to \neg play(John) = f_{21}$

$f_{16} \vdash_{m.t} \neg study(John) \to \neg pass(John) = f_{22}$ $f_{21}, f_{22} \vdash_{m.t,\ syllogism} \neg study(John) \to \neg play(John)$

Lock resolution

6. Consider the following hypotheses and check the validity of the conclusion

H1. Every coyote chases some roadrunner.

H2. Every roadrunner who says "beep-beep" is smart.

H3. No coyote chases any smart roadrunner.

H4. Any coyote who chases some roadrunner but does not catch it ~~are~~ is frustrated.

Conclusion:

C. If all roadrunners say "beep-beep", then all coyotes are frustrated.

$H_1, H_2, H_3, H_4 \vdash C$

Create the first-order (predicate) formulas:

$H_1: (\forall x)(coyote(x) \to (\exists y)(chases(x,y) \wedge roadrunner(y)))$

$H_2: (\forall x)(roadrunner(x) \wedge beep(x) \to smart(x))$

$H_3: \neg(\exists x)(coyote(x) \wedge (\forall y)(chases(x,y) \wedge roadrunner(y) \wedge smart(y))) =$

$= (\forall x)(\neg coyote(x) \vee (\exists y)(\neg chases(x,y) \vee \neg roadrunner(y) \wedge \neg smart(y)))$

$H_4: (\forall x)(coyote(x) \wedge (\exists y)(chases(x,y) \wedge roadrunner(y) \wedge \neg catches(x,y)) \to$
$\to frustrated(x))$

$C: (\forall x)(roadrunner(x) \wedge beep(x)) \to (\forall y)(frustrated(y) \wedge coyote(y))$

D - domain of the exercise

Unary predicate symbols: coyote, roadrunner, beep, smart, frustrated

Binary predicate symbols: chases, catches

$coyote: D \to \{T, F\}$, $coyote(x) = T$ if x is a coyote

$roadrunner: D \to \{T, F\}$, $roadrunner(x) = T$ if x is a roadrunner

$beep: D \to \{T, F\}$, $beep(x) = T$ if x says "beep-beep"

$smart: D \to \{T, F\}$, $smart(x) = T$ if x is smart

$frustrated: D \to \{T, F\}$, $frustrated(x) = T$ if x is frustrated

$chases: D \to \{T, F\}$, $chases(x,y) = T$ if x chases y

$catches: D \to \{T, F\}$, $catches(x,y) = T$ if x catches y

We apply a lock proof method: lock predicate resolution.

The pre-clausal normal forms corresponding to the hypotheses and the negation of the conclusion are as follows:

$H_1: (\forall x)(\neg coyote(x) \vee (\exists y)(chases(x,y) \wedge roadrunner(y)))$

$H_2: (\forall x)(\neg roadrunner(x) \vee \neg beep(x) \vee smart(x))$

$H_3: (\forall x)(\neg coyote(x) \vee (\exists y)(\neg chases(x,y) \vee \neg roadrunner(y) \wedge \neg smart(y))$

$H_4: (\forall x)(\neg coyote(x) \vee (\forall y)(\neg chases(x,y) \vee \neg roadrunner(y) \vee catches(x,y)) \vee \neg frustrated(x))$

$(\neg C)^c = \neg((\forall x)(roadrunner(x) \wedge beep(x)) \to (\forall y)(frustrated(y) \wedge coyote(y))) =$

$\cancel{= (\exists x)(\neg r} = \neg((\exists x)(\neg roadrunner(x) \vee \neg beep(x)) \vee (\forall y)(frustrated(y) \wedge coyote(y))) =$

$= (\forall x)(roadrunner(x) \wedge beep(x)) \wedge (\exists y)(\neg frustrated(y) \vee \neg coyote(y))$

= If all roadrunners say "beep-beep" there is something that is not frustrated and not a coyote.

Clauses:

$C_1: \underset{(1)}{\neg coyote(x)} \vee \underset{(2)}{chases(x,y)}$, $C_0 = \underset{(15)}{\neg roadrunner(w)} \vee \underset{(16)}{\neg beep(w)} \vee \underset{(17)}{smart(w)}$

$C_2: \underset{(3)}{roadrunner(y)}$

$C_3: \underset{(4)}{\neg coyote(z)} \vee \neg chases(z,a) \vee \neg roadrunner(a)$

$C_4: \underset{(6)}{\neg smart(b)}$

$C_5: \underset{(5)}{\neg coyote(t)} \vee \underset{(6)}{\neg chases(t,p)} \vee \underset{(7)}{\neg roadrunner(p)} \vee \underset{(8)}{catches(t,p)} \vee \underset{(9)}{\neg frustrated(t)}$

$C_6: \underset{(12)}{beep(m)}$

$C_7: \underset{(13)}{\neg frustrated(q)} \vee \underset{(14)}{\neg coyote(q)}$

Constants of the domain D: a, b, q

To the set of clauses $S = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_0\}$ lock predicate resolution is applied.

$C_8 = Res \underset{[y \leftarrow w]}{\overset{Pr}{\vdash}} C_0, C_2 = \neg beep(w) \vee smart(w)$

$C_9 = Res \underset{[w, b]}{\overset{Pr}{\vdash}} C_8, C_4 = \neg beep(b)$

$C_{10} = Res \underset{[m \leftarrow b]}{\overset{Pr}{\vdash}} C_9, C_6 = \square$

$S \underset{Res}{\overset{Pr}{\vdash}} \square$, therefore S is an inconsistent set and the deduction $H_1, H_2, H_3, H_4 \vdash C$ holds.