Quaternions (cont)

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$$q_1 \circ q_2 = \cancel{[s_1 s_2 - v_1 \cdot v_2 ; s_1 v_2 + s_2 v_1 + v_1 \times v_2]}$$

$$= [s_1 s_2 - v_1 \cdot v_2 ; s_1 * v_2 + s_2 * v_1 + v_1 \times v_2]$$

$$q_1 \circ q_2 = [1*5 - 65 ; 1*(6,7,8) + 5*(2,3,4) + (-4,8,-4)]$$

$$\cancel{[-60 ; (12, 30, 24)]}$$

$$[-60 ; (12, 30, 24)]$$

$^{-1}$:

$$q_1^{-1} = \frac{1}{\|q_1\|}(s_1 ; -v_1)$$

$$= \frac{1}{\sqrt{1^2+2^2+3^2+4^2}}(1 ; -(2,3,4)) = \frac{1}{\sqrt{30}}(1 ; -(2,3,4))$$

$$= \left(\frac{1}{\sqrt{30}} ; \left(-\frac{2}{\sqrt{30}}, -\frac{3}{\sqrt{30}}, -\frac{4}{\sqrt{30}}\right)\right)$$

Ex try to compute $q_1 \circ q_1^{-1}$ (at home)

$$\|q_1\| = \sqrt{s_1^2 + x_1^2 + y_1^2 + z_1^2}$$

rotate angle ϕ through the origin w/ normal n

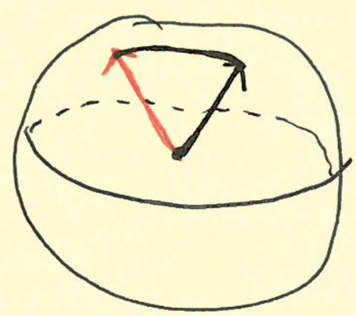
$$q = [\cos(\phi/2); \sin(\phi/2) * n]$$

rotate point $p \in \mathbb{R}^3$

1) convert to quaternion $[0; p]$
2) transform w/ quaternion product

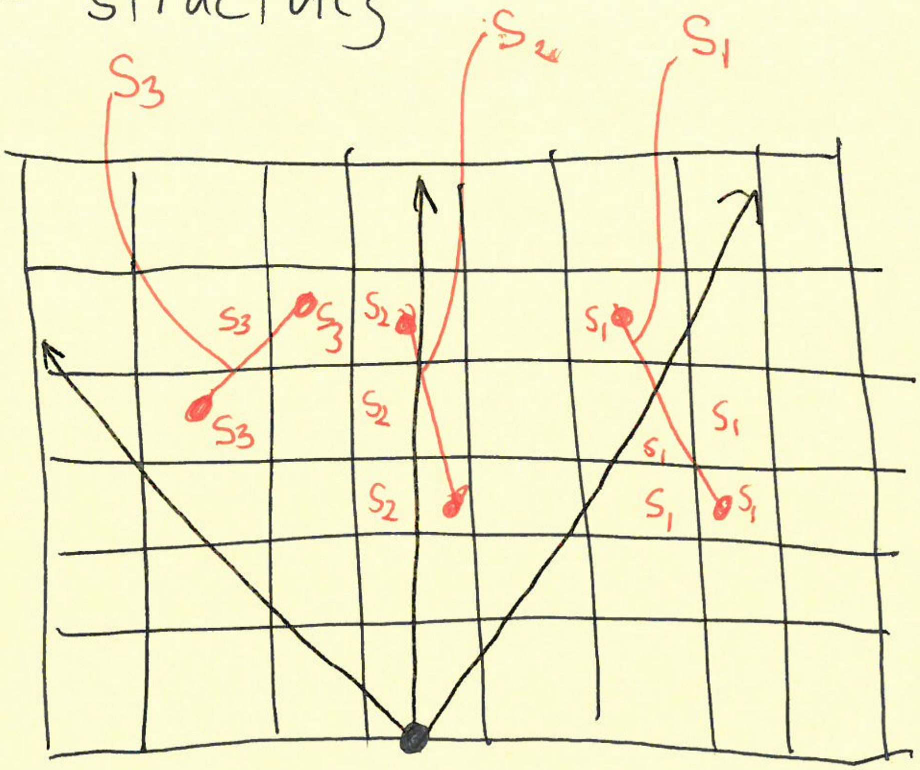
$$q_p = q p q^{-1}$$

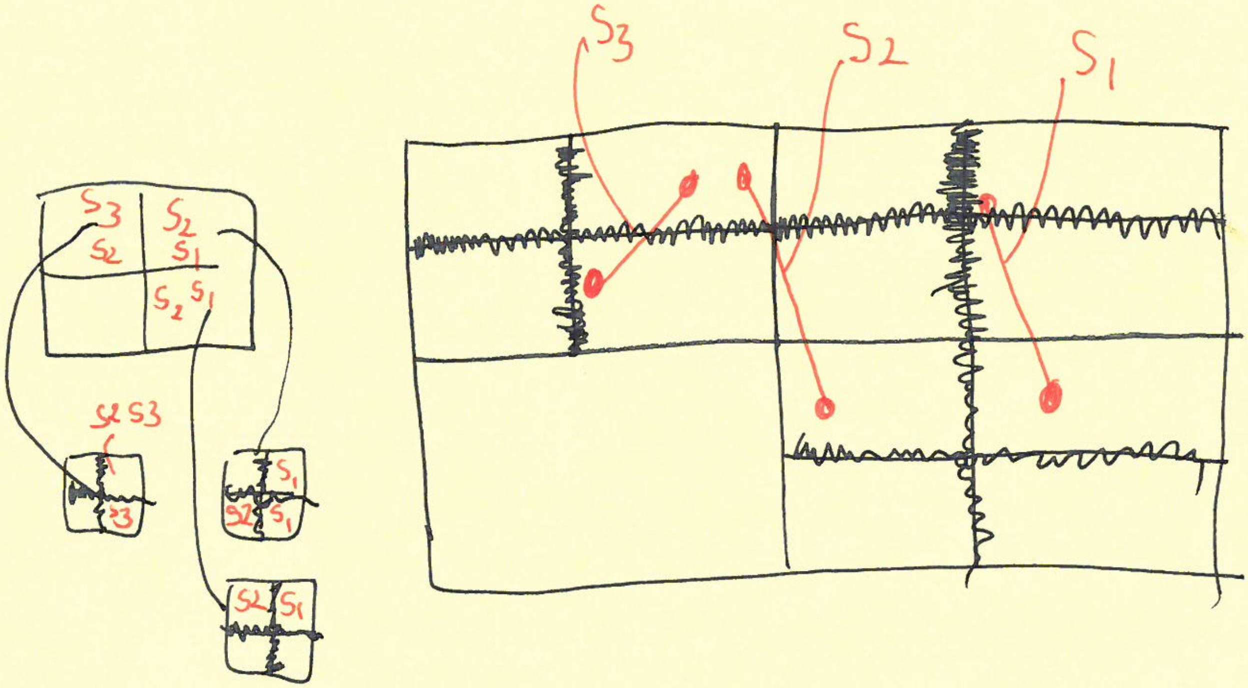
Spherical linear interpolation



$$\text{slerp}(q_1, q_2, t) = (q_2 q_1^{-1})^t q_1$$

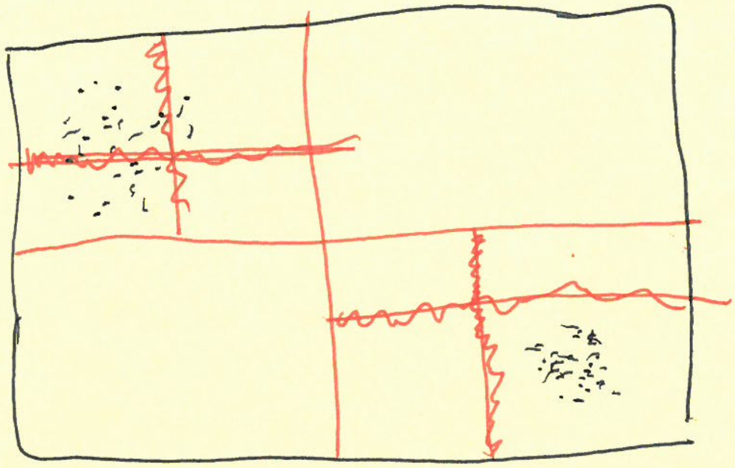
spatial data structures





Quadtree 2D (splits of 4 cells)

Octree 3D (splits of 8 cells)



Kd-tree