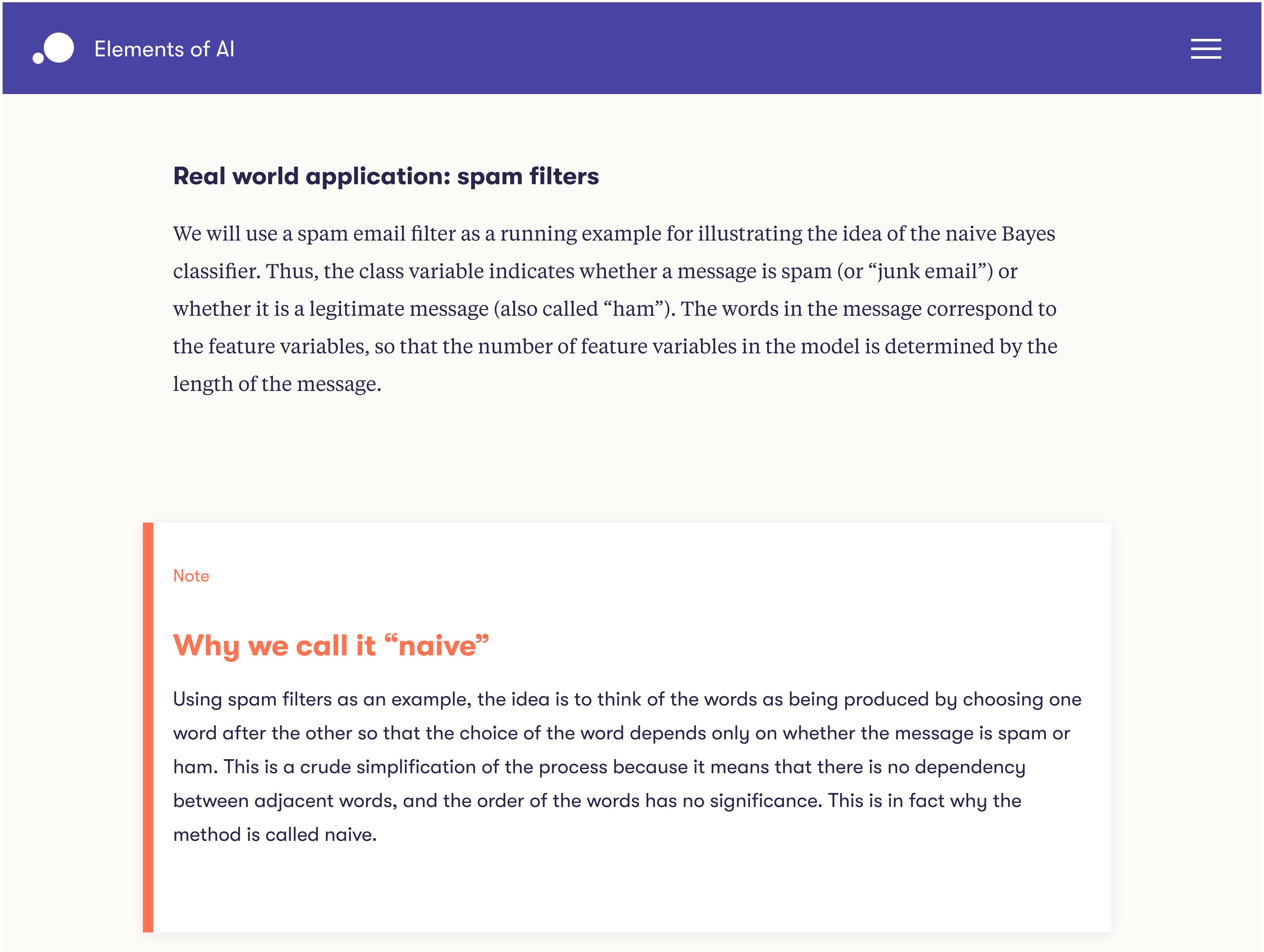
Elements of AI

III. Naive Bayes classification

One of the most useful applications of the Bayes rule is the so-called naive Bayes classifier.

The Bayes classifier is a machine learning technique that can be used to classify objects such as text documents into two or more classes. The classifier is trained by analyzing a set of training data, for which the correct classes are given.

The naive Bayes classifier can be used to determine the probabilities of the classes given a number of different observations. The assumption in the model is that the feature variables are conditionally independent given the class (we will not discuss the meaning of conditional



Real world application: spam filters

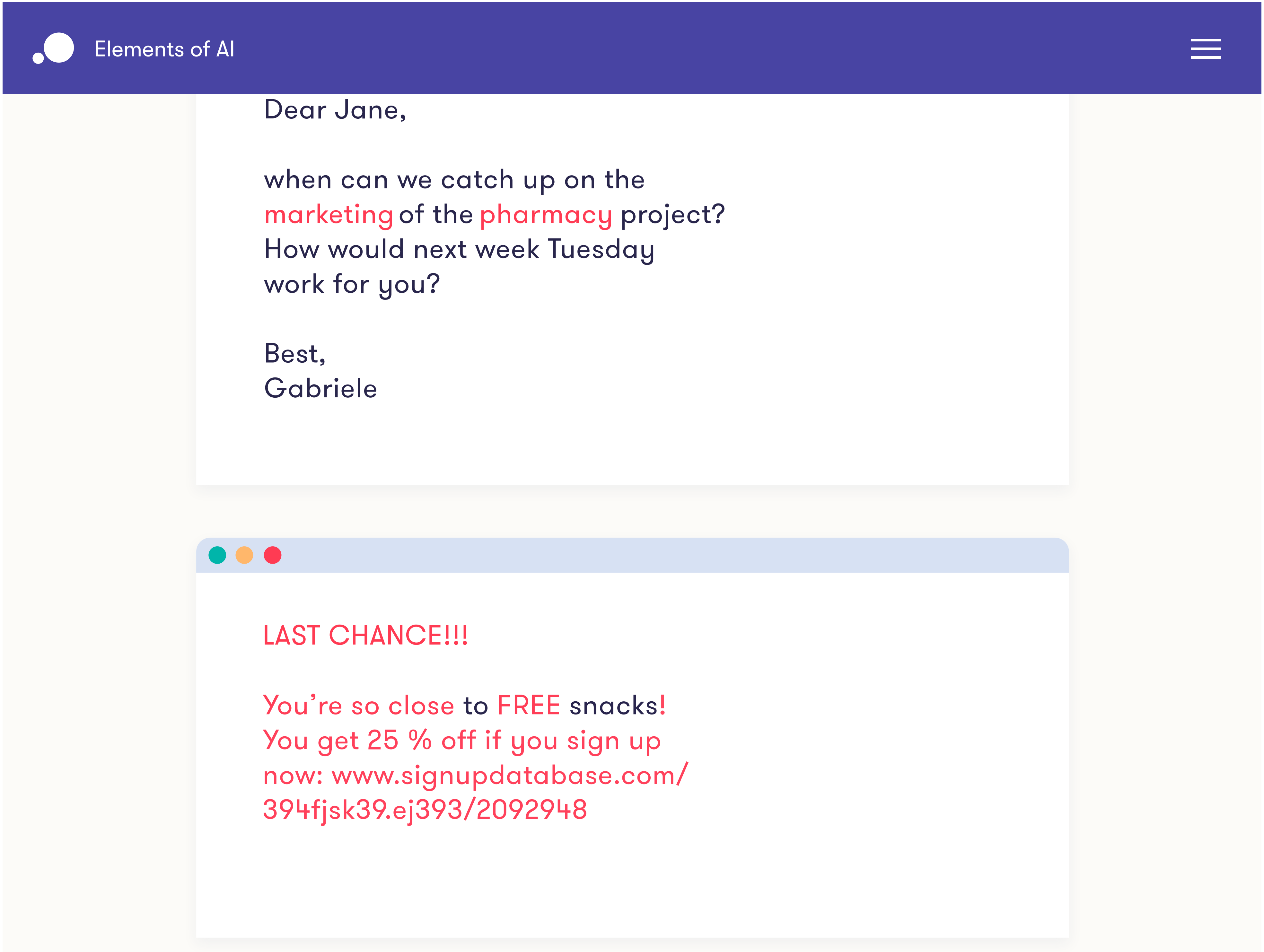
We will use a spam email filter as a running example for illustrating the idea of the naive Bayes classifier. Thus, the class variable indicates whether a message is spam (or “junk email”) or whether it is a legitimate message (also called “ham”). The words in the message correspond to the feature variables, so that the number of feature variables in the model is determined by the length of the message.

Note

Why we call it “naive”

Using spam filters as an example, the idea is to think of the words as being produced by choosing one word after the other so that the choice of the word depends only on whether the message is spam or ham. This is a crude simplification of the process because it means that there is no dependency between adjacent words, and the order of the words has no significance. This is in fact why the method is called naive.

"algorithm").



Dear Jane,

when can we catch up on the marketing of the pharmacy project? How would next week Tuesday work for you?

Best,
Gabriele

LAST CHANCE!!!

You're so close to FREE snacks! You get 25 % off if you sign up now: www.signupdatabase.com/394fjsk39.ej393/2092948

Despite its naivete, the naive Bayes method tends to work very well in practice. This is a good example of the common saying in statistics, “all models are wrong, but some are useful” means (the aphorism is generally attributed to statistician George E.P. Box).

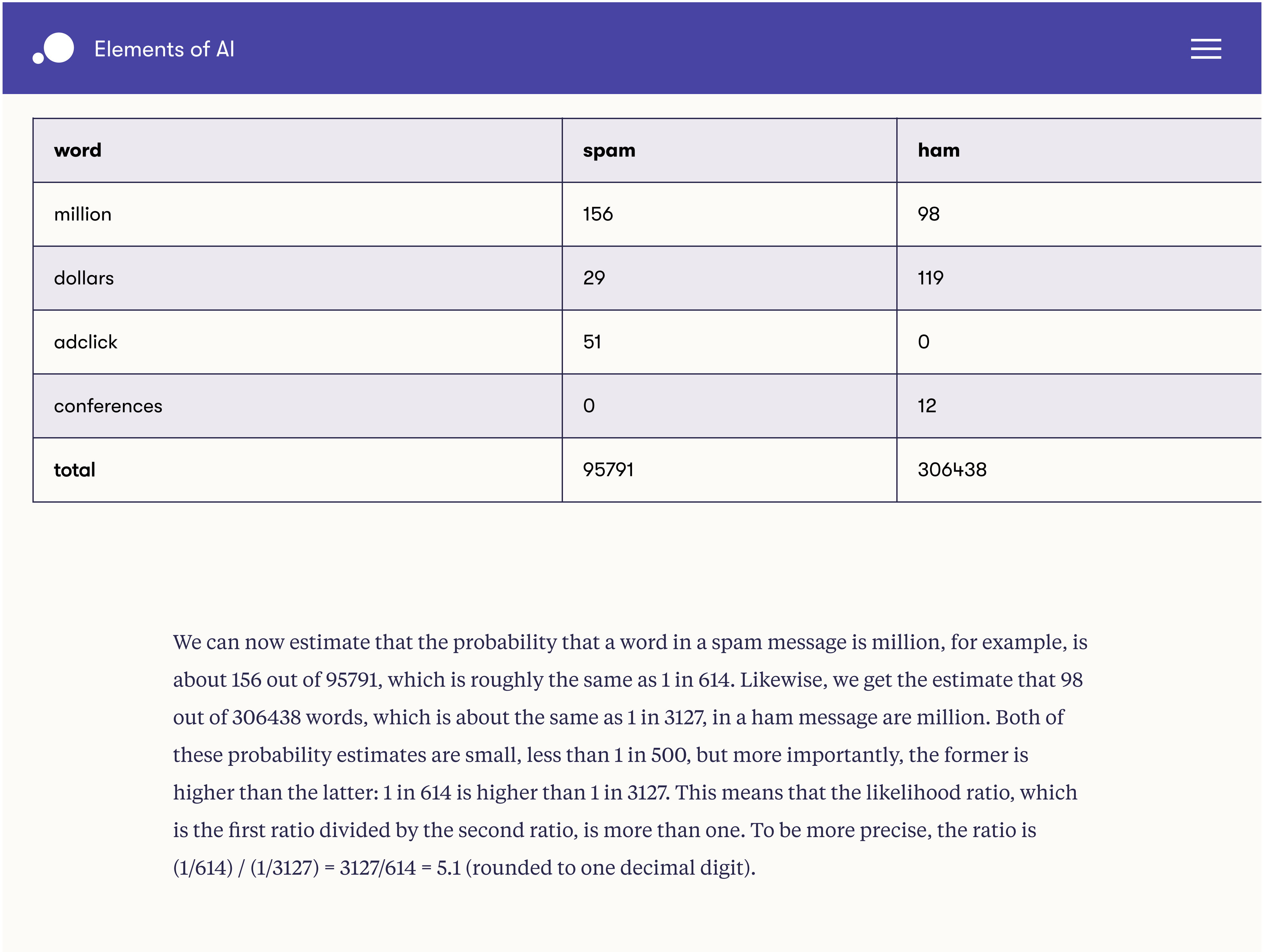
Estimating parameters

To get started, we need to specify the prior odds for spam (against ham). For simplicity assume this to be 1:1 which means that on the average half of the incoming messages are spam (in reality, the amount of spam is probably much higher).

To get our likelihood ratios, we need two different probabilities for any word occurring: one in spam messages and another one in ham messages.

The word distributions for the two classes are best estimated from actual training data that contains some spam messages as well as legitimate messages. The simplest way is to count how many times each word, abacus, acacia, ..., zurg, appears in the data and divide the number by the total word count.

To illustrate the idea, let’s assume that we have at our disposal some spam and some ham. You can easily obtain such data by saving a batch of your emails in two files.



word	spam	ham
million	156	98
dollars	29	119
adclick	51	0
conferences	0	12
total	95791	306438

We can now estimate that the probability that a word in a spam message is million, for example, is about 156 out of 95791, which is roughly the same as 1 in 614. Likewise, we get the estimate that 98 out of 306438 words, which is about the same as 1 in 3127, in a ham message are million. Both of these probability estimates are small, less than 1 in 500, but more importantly, the former is higher than the latter: 1 in 614 is higher than 1 in 3127. This means that the likelihood ratio, which is the first ratio divided by the second ratio, is more than one. To be more precise, the ratio is (1/614) / (1/3127) = 3127/614 = 5.1 (rounded to one decimal digit).

section *Odds and Probability*).

Note

Zero means trouble

One problem with estimating the probabilities directly from the counts is that zero counts lead to zero estimates. This can be quite harmful for the performance of the classifier – it easily leads to situations where the posterior odds are 0/0, which is nonsense. The simplest solution is to use a small lower bound for all probability estimates. The value 1/100000, for instance, does the job.

Using the above logic, we can determine the likelihood ratio for all possible words without having to use zero, giving us the following likelihood ratios:

word	likelihood ratio

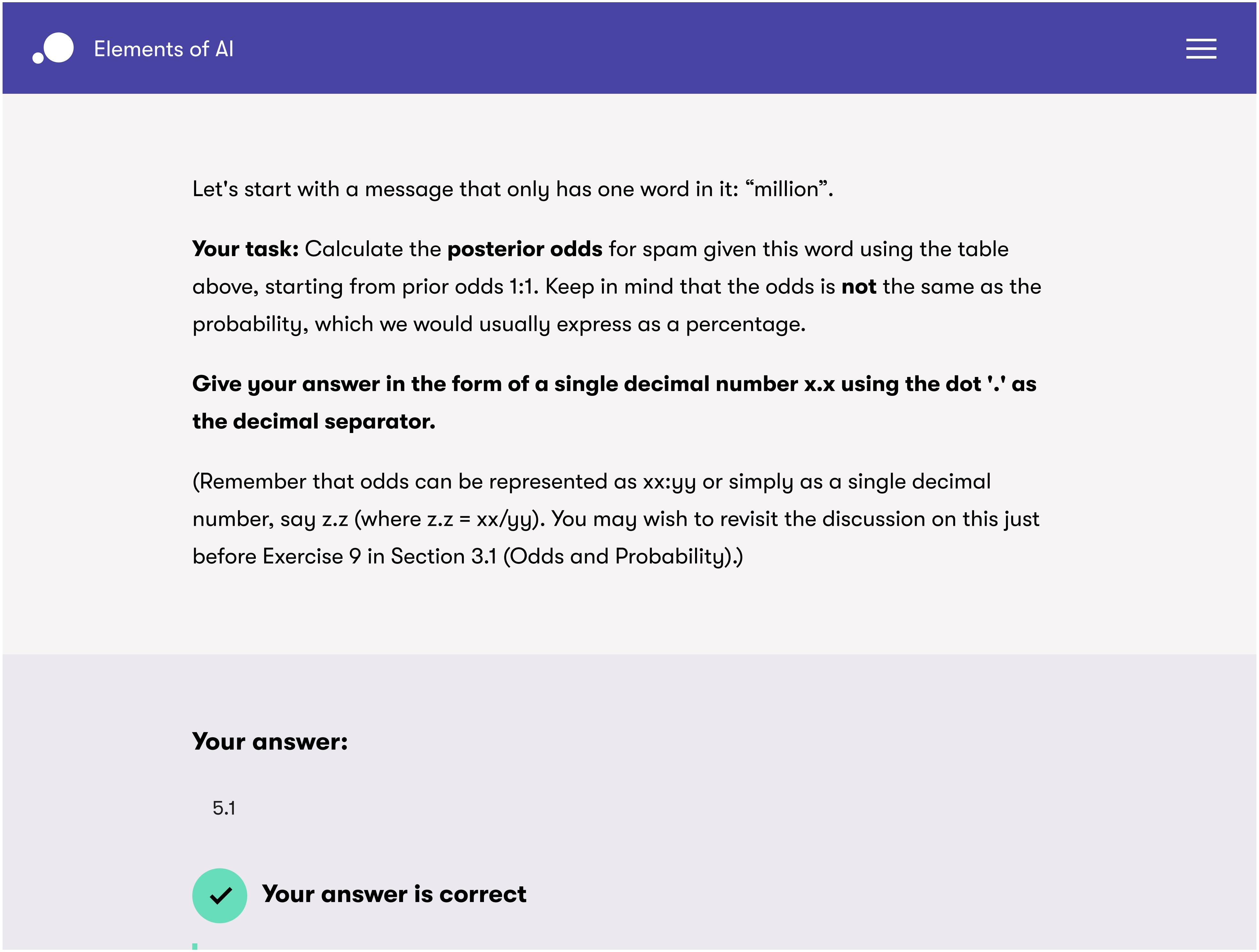
adclick	53.2
conferences	0.3

We are now ready to apply the method to classify new messages.

Example: is it spam or ham?

Once we have the prior odds and the likelihood ratios calculated, we are ready to apply the Bayes rule, which we already practiced in the medical diagnosis case as our example. The reasoning goes just like it did before: we update the odds of spam by multiplying it by the likelihood ratio. To remind ourselves of the procedure, let's try a message with a single word to begin with. For the prior odds, as agreed above, you should use odds 1:1.

✓ **Answered**



Let's start with a message that only has one word in it: “million”.

Your task: Calculate the **posterior odds** for spam given this word using the table above, starting from prior odds 1:1. Keep in mind that the odds is **not** the same as the probability, which we would usually express as a percentage.

Give your answer in the form of a single decimal number x.x using the dot '.' as the decimal separator.

(Remember that odds can be represented as xx:yy or simply as a single decimal number, say z.z (where z.z = xx/yy). You may wish to revisit the discussion on this just before Exercise 9 in Section 3.1 (Odds and Probability).)

Your answer:

5.1

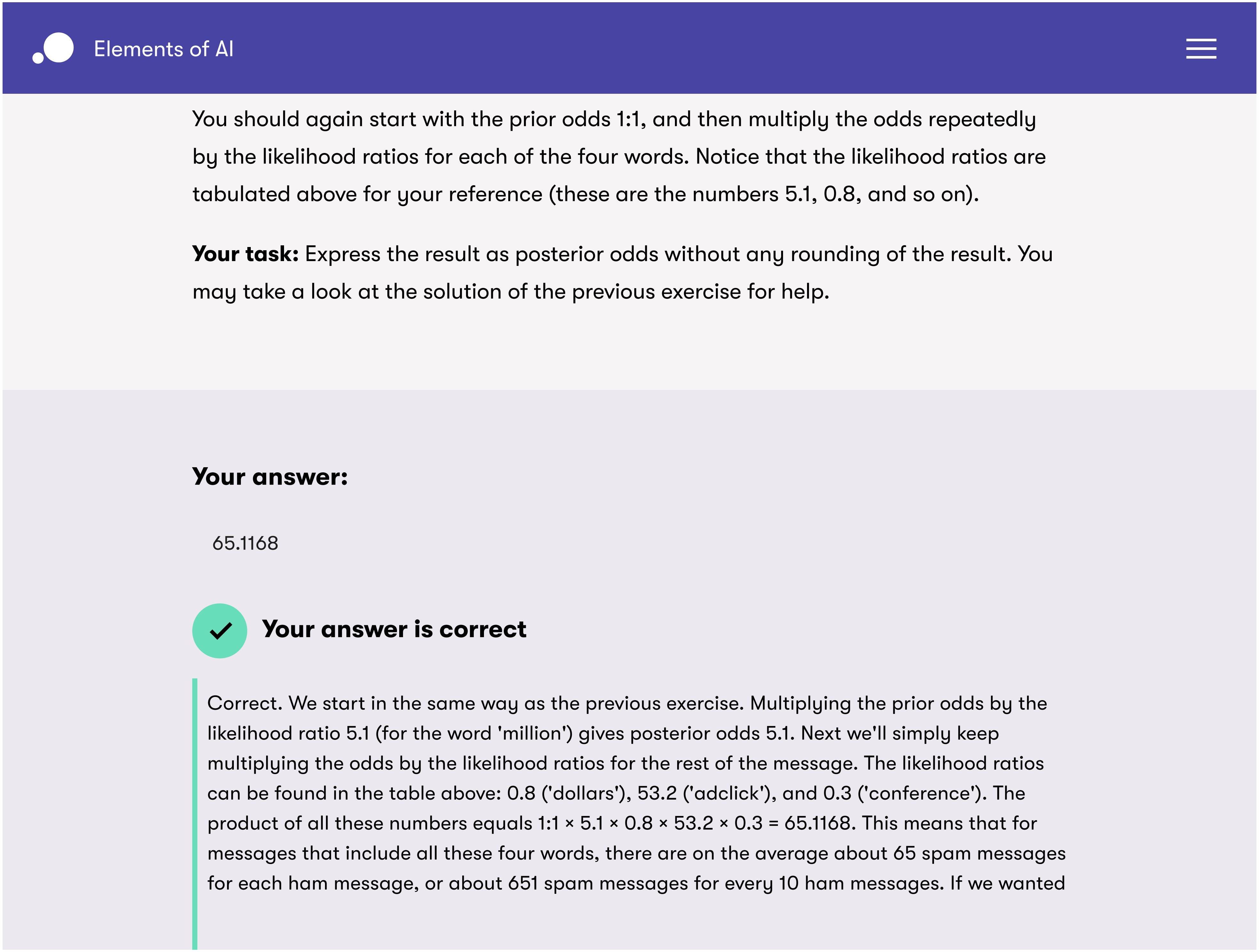
✓ **Your answer is correct**

> decided above that the prior odds are 1:1. The likelihood ratio is obtained by dividing the probability of the word 'million' in spam divided by the probability of the word 'million' in ham. This we already calculated above, and it can be found in the table of likelihood ratios: the value is 5.1. Now multiply the prior odds by the likelihood ratio to get 1:1 × 5.1 = 5.1. This is the posterior odds. Again, the posterior odds means that for messages that include the word 'million', there are on the average 5.1 spam messages for every ham message. Or to use whole numbers, there are 51 spam messages for every 10 ham messages. The probability value is therefore 51 / (51+10) = 51/61, or approximately 83.6 %.

To handle the rest of the words in a message, we can use exactly the same procedure. The posterior odds after one word, which you calculated in the previous exercise, will become the prior odds for the next word, and so on.

✓ **Answered**

Exercise 13: Full spam filter



You should again start with the prior odds 1:1, and then multiply the odds repeatedly by the likelihood ratios for each of the four words. Notice that the likelihood ratios are tabulated above for your reference (these are the numbers 5.1, 0.8, and so on).

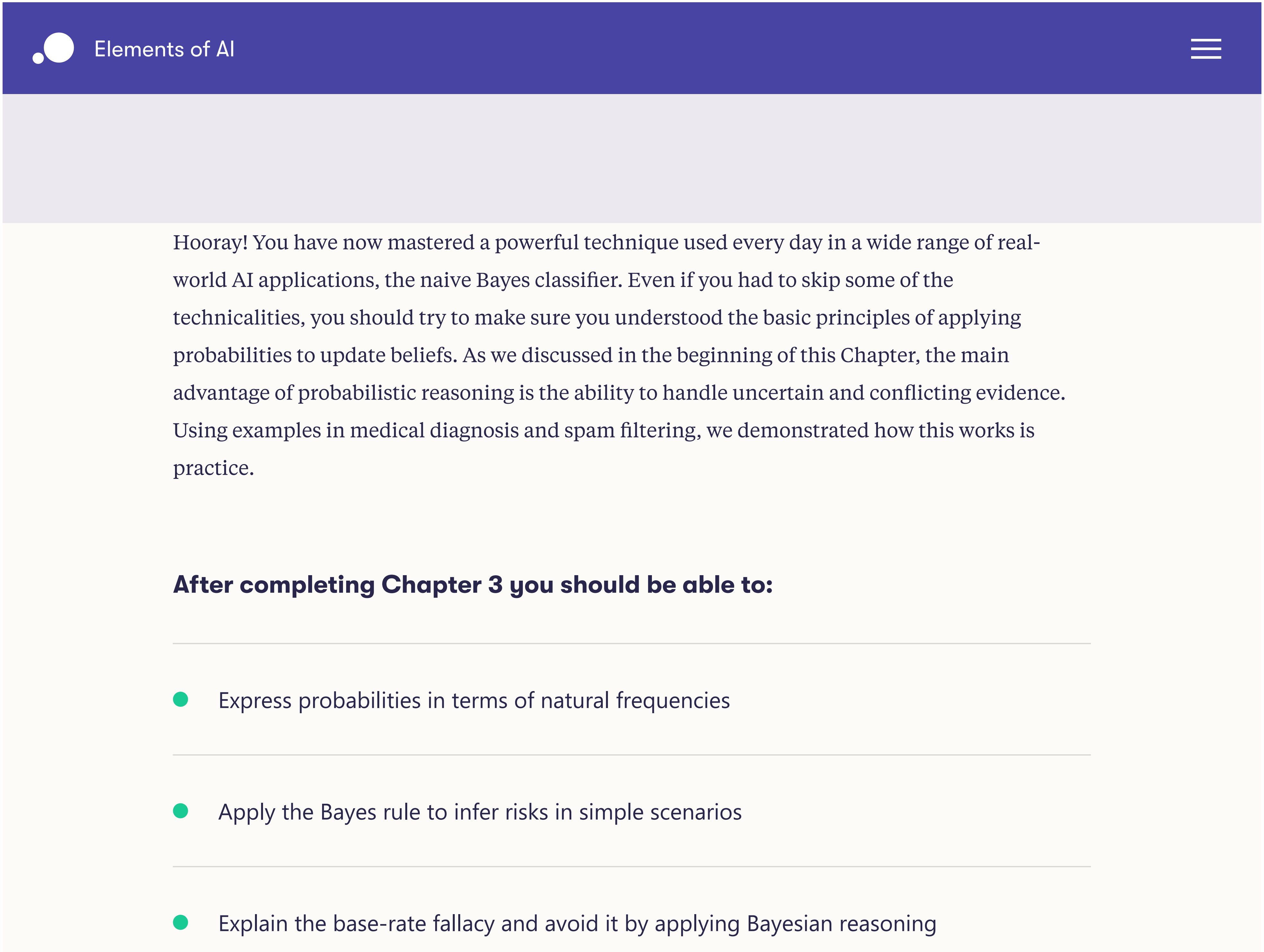
Your task: Express the result as posterior odds without any rounding of the result. You may take a look at the solution of the previous exercise for help.

Your answer:

65.1168

✓ **Your answer is correct**

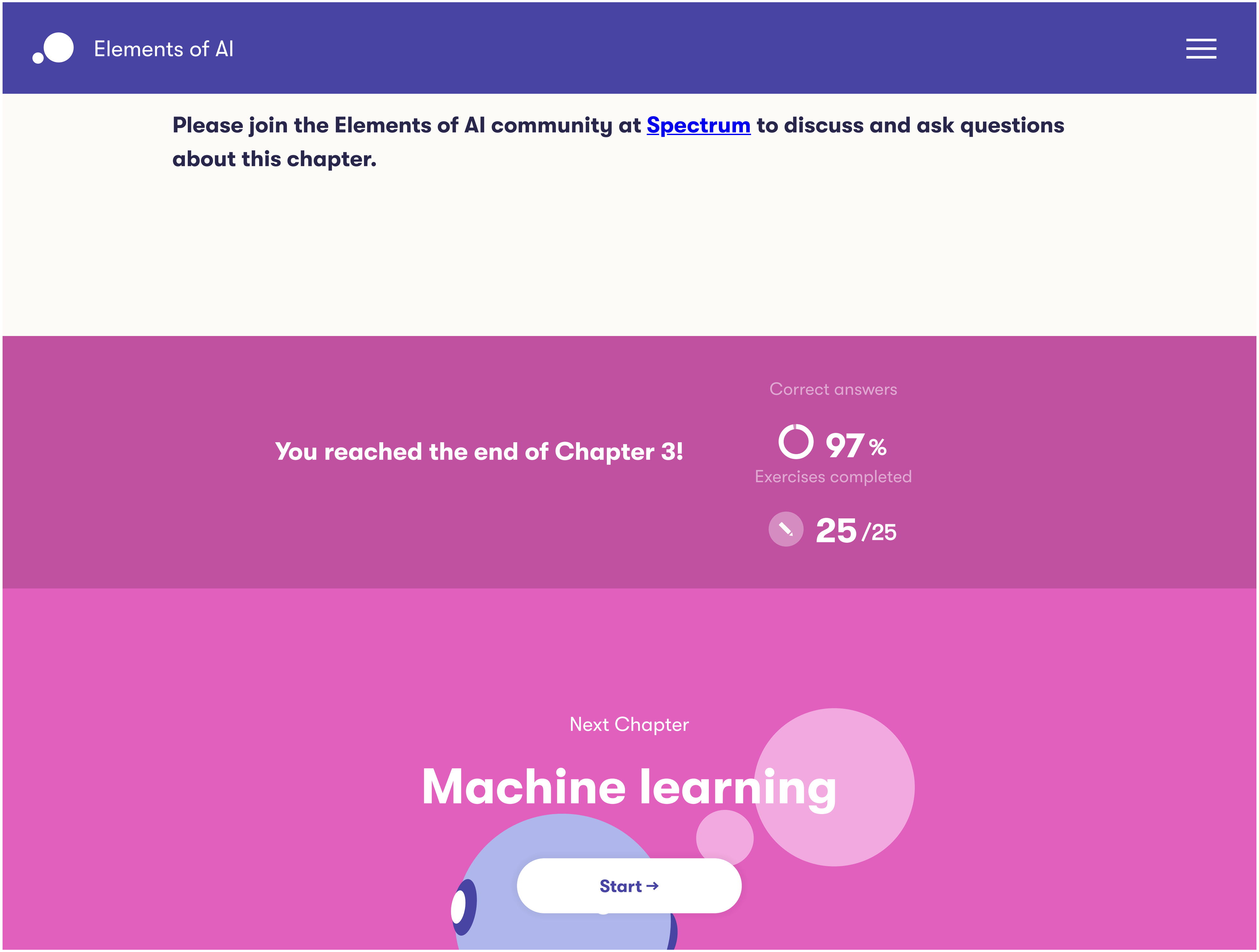
> Correct. We start in the same way as the previous exercise. Multiplying the prior odds by the likelihood ratio 5.1 (for the word 'million') gives posterior odds 5.1. Next we'll simply keep multiplying the odds by the likelihood ratios for the rest of the message. The likelihood ratios can be found in the table above: 0.8 ('dollars'), 53.2 ('adclick'), and 0.3 ('conference'). The product of all these numbers equals 1:1 × 5.1 × 0.8 × 53.2 × 0.3 = 65.1168. This means that for messages that include all these four words, there are on the average about 65 spam messages for each ham message, or about 651 spam messages for every 10 ham messages. If we wanted



Hooray! You have now mastered a powerful technique used every day in a wide range of real-world AI applications, the naive Bayes classifier. Even if you had to skip some of the technicalities, you should try to make sure you understood the basic principles of applying probabilities to update beliefs. As we discussed in the beginning of this Chapter, the main advantage of probabilistic reasoning is the ability to handle uncertain and conflicting evidence. Using examples in medical diagnosis and spam filtering, we demonstrated how this works is practice.

After completing Chapter 3 you should be able to:

- Express probabilities in terms of natural frequencies
- Apply the Bayes rule to infer risks in simple scenarios
- Explain the base-rate fallacy and avoid it by applying Bayesian reasoning



Please join the Elements of AI community at Spectrum to discuss and ask questions about this chapter.

You reached the end of Chapter 3!

Correct answers

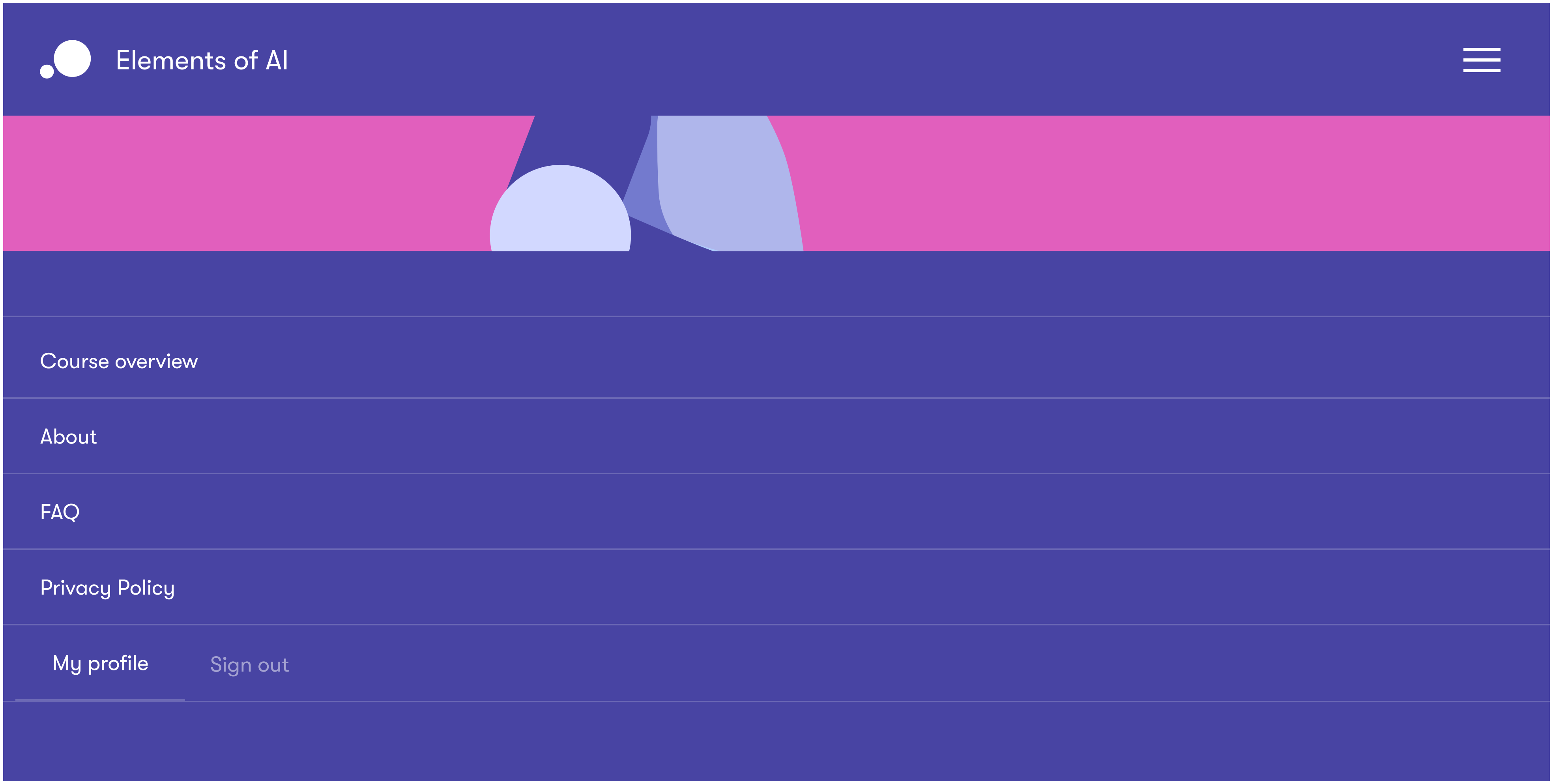
97 %

Exercises completed

25 /25

Next Chapter

Machine learning



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