

# Modeling and Programming with Gecode

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The location of this document is:

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http://www.gecode.org/doc/3.7.3/MPG.pdf
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The location of this document corresponding to the latest Gecode version is:

http://www.gecode.org/doc-latest/MPG.pdf

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1

# Introduction

This document provides an introduction to modeling and programming with Gecode, an open, free, portable, accessible, and efficient environment for developing constraint-based systems and applications.

The hands-on, tutorial-style approach will get you started very quickly. The focus is on giving an overview of the key concepts and ideas required to model and program with Gecode. Each concept is introduced using concrete C++ code examples that are developed and explained step by step. This document is complemented by the complete Gecode reference documentation, as well as pointers to introductory and more advanced material throughout the text.

The first part of this document (Part M) is about *modeling* with Gecode. It explains modeling and solving constraint problems, and how to program, compile, link, and execute these models. This is complemented by a collection of interesting case studies of how to model with Gecode (Part C). The remaining, more advanced parts are about *programming* with Gecode: they explain how to use Gecode for implementing constraints (Part P), branchings (Part B), new variable types (Part V), and search engines (Part S).

#### 1.1 What is Gecode?

Gecode is an open, free, portable, accessible, and efficient environment for developing constraint-based systems and applications. Gecode is:

**open** Gecode is radically open for programming: it can be easily interfaced to other systems. It supports the programming of new propagators (as implementation of constraints), branching strategies, and search engines. New variables can be programmed at the same level of efficiency as integer and set variables that ship with Gecode.

**free** Gecode is distributed under the MIT license and is listed as free software by the FSF. All of its parts — including reference documentation, implementations of global constraints, and examples — are available as source code for download.

**portable** Gecode is implemented in C++ that rigidly follows the C++ standard. It can be compiled with modern C++ compilers and runs on a wide range of platforms.

**accessible** Gecode comes with complete tutorial and reference documentation that allows users to focus on different programming tasks with Gecode.

**efficient** Gecode offers excellent performance with respect to runtime, memory usage, and scalability. For example, Gecode won the MiniZinc Challenge in 2010, 2009, and 2008.

**parallel** Gecode complies with reality in that it exploits the multiple cores of today's commodity hardware for parallel search, giving an already efficient base system an additional edge.

alive Gecode has a sizeable user community and is being actively developed and maintained. In order to give you an idea: there has been a release every two to three month since the first release in December 2005.

### 1.2 What is this document?

We do not want to disappoint our readers, so let us get this out of the way as early as possible – here is *what this document is not*. This document is very definitely neither

- an introduction to constraint programming or modeling techniques, nor
- a collection of interesting implementations of constraints, nor
- an introduction to the architecture of constraint solvers, nor
- a reference documentation of the Gecode API.

The reader is therefore expected to have some background knowledge in constraint programming.

Furthermore, the document describes the C++ interface to Gecode, it is not about modeling with any of the available interfaces to Gecode.

**Keeping it simple.** Throughout this document, we will use simple examples to explain the concepts you have to understand in order to model and program with Gecode. However, these simple examples demonstrate the *complete array of techniques* that are sufficient to implement complex models, constraints, branchings, variables, and search engines. In fact, Gecode itself is based on the very same techniques – you will learn how to develop code that is just as good as (or maybe better than?) what Gecode itself provides.

**Gecode architecture.** This document follows the general architecture of Gecode, containing one part for each major component. Figure 1.1 gives an overview of the Gecode architecture. The kernel provides common functionality, upon which the modules for integer and set constraints as well as the search engines are built. The colored boxes refer to the topics covered in this document.

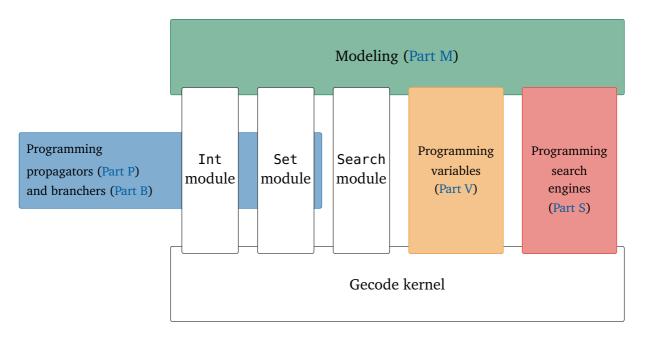


Figure 1.1: Gecode architecture

**Modeling.** The modeling part (Part M) of this document assumes some basic knowledge of modeling and solving constraint problems, as well as some basic C++ skills. The document restricts itself to simple and well known problems as examples. A constraint programming novice should have no difficulty to concentrate on the how-to-model with Gecode in particular, rather than the how-to-model with constraint programming in general.

The modeling part starts with a very simple constraint model that already touches on all the important concepts used in Gecode. There are detailed instructions how to compile and link the example code, so that you can get started right away. After that, the different variable types, the most important classes of constraints (including pointers to the Global Constraint Catalog [5], referred to by GCCAT), and the infrastructure provided by Gecode (such as search engines) are presented.

**Case studies.** This document includes a collection of case studies in Part C. The case studies mix modeling and programming with Gecode. Some case studies are just interesting constraint models. Other case studies are constraint models that include the programming of new constraints and/or branchings.

**Programming.** The programming parts of this document require the same knowledge as the modeling part, plus some additional basic knowledge of how constraint propagation is organized. Section 20.2 provides pointers to recommended background reading.

The programming parts of this document cover the following topics:

**programming propagators** Part P describes in detail how new propagators (as implementations of constraints) can be programmed using Gecode. The part gives a fairly com-

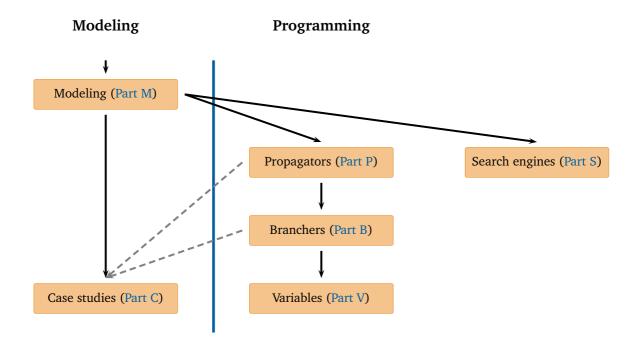


Figure 1.2: Dependencies among different parts of this document

plete account of concepts and techniques for programming propagators.

**programming branchers** Part B describes how new branchers (as implementations of branchings for search) can be programmed using Gecode. This part is short and straightforward.

**programming variables** Gecode supports the addition of new variables types: the modules for integer, Boolean and set variables use exactly the same programming interface as is available to any user. This interface is described in Part V.

**programming search** Part S describes how to program new search engines (Gecode comes with the most important search engines). The programming interface is simple yet very powerful as it is based on concurrency-enabled techniques such as recomputation and cloning.

#### 1.3 How to read this document?

The dependencies among the different parts of this document are sketched in Figure 1.2. Every part starts with a short overview section and sketches what constitutes the basic material of a part. A part only requires the basic material of the parts it depends on.

The dashed arrows from programming propagators (Part P) and programming branchers (Part B) capture that some but not all case studies require knowledge on how to program propagators and branchers. The individual case studies provide information on the required prerequisites.

**Downloading example programs.** All example program code used in this document is available for download, just click the download link in the upper right corner of an example.

Note that the code available for download is licensed under the same license as Gecode and not under the same license as this document. By this, you can use an example program as a starting point for your own programs.

If you prefer to download all example programs at once, you can do so here:

- example programs as gzipped tar archive
- example programs as 7z archive

#### 1.4 Do I need to be a C++ wizard?

You very definitely do not have to be a C++ wizard to program Gecode models, some basic C++ knowledge will do. Modeling constraint problems typically involves little programming and tends to follow a common and simple structure that is presented in this document. It should be sufficient to have a general idea of programming and object-oriented programming.

Even programming with Gecode requires only basic C++ knowledge. The implementation of propagators, branchings, variables, and search engines follows simple and predictable recipes. However, this estimate refers to the aspects of using the concepts and techniques provided by Gecode. Of course, implementing truly advanced propagation algorithms inside a propagator will be challenging!

If you want to brush up your C++ knowledge, then brushing up your knowledge about the following topics might be most rewarding when using Gecode:

- Classes and objects, inheritance, virtual member functions: models are typically implemented by inheritance from a Gecode-provided base class.
- Overloading and operator overloading: post functions for constraints and support for posting symbolic expressions and relations rely on overloading (several functions with different argument types share the same function name).
- Exceptions: Gecode throws exceptions if post functions are used erroneously, for example, when numerical overflow could occur or when parameters are used inconsistently.
- Templates: while the modeling layer uses only few generic classes implemented as templates, programming with Gecode requires some basic knowledge about how to program with templates in C++.

Any textbook covering these topics should be well suited, for example, [20] for a general introduction to C++, and [54] for an introduction to C++ for Java programmers.

## 1.5 Can you help me?

Gecode has a lively and sizeable user community that can be tapped for help. You can ask questions about Gecode on the mailing list users@gecode.org. But, please make sure to not waste your time and the time of others:

- Please check this document and the Gecode reference documentation before asking a question.
- Please check the archive of the Gecode users mailing list as to whether a similar question has been asked before (note that the archive can be searched) before asking a question.
- Please focus on questions specific to Gecode. For general questions about constraint programming more suitable forums exist.
- Please provide sufficient detail: describe your platform (operating system, compiler, Gecode version) and your problem (what does not work, what do you want to do, what is the problem you observe) as accurately as you can.
- Please do not contact the developers for general Gecode questions, we will not answer. First, we insist on the benefit to the entire user community to see questions and answers (and the contribution to the mailing list archive). Second, more importantly, our users are known to have very good answers indeed. Remember, they in contrast to the developers might be more familiar with your user perspective on Gecode.
- Never ask for solutions to homework. The only more offensive thing you could do is to provide a solution on the mailing list if someone has violated the no homework policy!

# 1.6 Does Gecode have bugs?

Yes, of course! But, Gecode is very thoroughly tested (tests cover almost 100%) and extensively used (several thousand users). If something does not work, we regret to inform you that this is most likely due to your program and not Gecode. Again, this does not mean that Gecode has no bugs. But it does mean that it might be worth searching for errors in *your* program first.

Likewise, all major program fragments in this document (those that can be downloaded) have been carefully tested as well.

And, yes. Please take our apologies in advance if that somewhat bold claim does turn out to be false... If you have accepted our apologies, you can submit your bug report here.

### 1.7 How to refer to this document?

We kindly ask you to refer to the individual parts of this document with their respective authors (each part has a dedicated set of authors). BIBTEX entries for the individual parts are available here.

If you refer to concepts introduced in Gecode, we kindly ask you to refer to the relevant academic publications.

# 1.8 Do you have comments?

If you have comments, suggestions, bug reports, wishes, or any other feedback for this document, please send a mail with your feedback to mpg@gecode.org.



# Modeling

Christian Schulte, Guido Tack, Mikael Z. Lagerkvist

This part explains modeling and solving constraint problems, and how to program, compile, link, and execute constraint models.

**Basic material.** The basic material needed for modeling with Gecode is as follows:

- Chapter 2 (Getting started) provides an overview of how to program, compile, link, and execute a constraint model in Gecode.
- Chapter 3 (Getting comfortable) discusses functionality in Gecode that makes modeling and execution of models more convenient.
- The three first sections of Chapter 4 (Integer and Boolean variables and constraints) explain integer and Boolean variables (Section 4.1), variable and argument arrays (Section 4.2), and how constraints are posted (Section 4.3).
- The first section of Chapter 5 (Set variables and constraints) gives an overview of set variables (Section 5.1).
- The first section of Chapter 7 (Search) explains branchings for modeling (Section 7.1). Even though not strictly necessary for modeling, it is recommended to also read Section 7.2 and Section 7.3 that explain how search (and in particular parallel search) works in Gecode.

Part C features a collection of example models for Gecode as further reading.

**Overview material.** The remaining chapters and sections provide an overview of the available functionality for modeling and solving:

- Constraints on integer and Boolean variables are summarized in Section 4.4 and Section 4.5 of Chapter 4 (Integer and Boolean variables and constraints).
- Section 5.2 and Section 5.3 of Chapter 5 (Set variables and constraints) summarize constraints on set variables.
- Chapter 6 (Modeling convenience: MiniModel) provides an overview of modeling convenience implemented by MiniModel.
- Section 7.4 of Chapter 7 (Search) summarizes how to use search engines.
- Chapter 8 (Gist) summarizes how to use Gist as a graphical and interactive search tool for developing constraint models.
- Chapter 9 (Script commandline driver) summarizes the commandline driver for Gecode models.

# 2

# **Getting started**

This chapter provides a basic overview of how to program, compile, link, and execute a constraint model in Gecode. The chapter restricts itself to the fundamental concepts available in Gecode, the following chapter presents functionality that makes programming models more comfortable.

**Overview.** Section 2.1 explains the basics of how a model is programmed in Gecode. This is followed in Section 2.2 by a discussion of how search is used to find solutions of a model. How a model is compiled, linked, and executed is explained for several different operating systems in Section 2.3. Section 2.4 shows how Gist as a graphical and interactive search tool can be used for developing constraint models. Search for a best solution of a model is explained in Section 2.5.

The chapter also includes an explanation of how to obtain Gecode in Section 2.6 which covers both installation of binary packages available for some platforms and compilation of source packages. There is no need to say, that this section is very important reading!

### 2.1 A first Gecode model

Models in Gecode are implemented using *spaces*. A space is *home* to the *variables*, *propagators* (implementations of constraints), *branchers* (implementations of branchings, describing the search tree's shape, also known as labelings), and – possibly – an *order* determining a best solution during search.

Not surprisingly in an object-oriented language such as C<sup>++</sup>, an elegant approach to programming a model is by inheritance: a model inherits from the class Space (implementing spaces) and the subclass constructor implements the model. In addition to the constructor, a model must implement a copy constructor and a copy function such that search for that model works (to be discussed later).

**Send More Money.** The model we choose as an example is Send More Money: find distinct digits for the letters S, E, N, D, M, O, R, and Y such that the well-formed equation (no leading zeros) SEND + MORE = MONEY holds.

The program (with some parts yet to be presented) is shown in Figure 2.1. Note that clicking a blue line starting with ▶ jumps to the corresponding code. Clicking [DOWNLOAD] in the upper right corner of the program provides access to the complete program text.

```
[DOWNLOAD]
SEND MORE MONEY ≡
 #include <gecode/int.hh>
 #include <gecode/search.hh>
 using namespace Gecode;
 class SendMoreMoney : public Space {
 protected:
    IntVarArray l;
 public:
    SendMoreMoney(void) : l(*this, 8, 0, 9) {
      IntVar s(l[0]), e(l[1]), n(l[2]), d(l[3]),
             m(l[4]), o(l[5]), r(l[6]), y(l[7]);
      ► NO LEADING ZEROS
      ► ALL LETTERS DISTINCT
      ► LINEAR EQUATION
      ► POST BRANCHING
    ► SEARCH SUPPORT
    ► PRINT SOLUTION
 };
  ► MAIN FUNCTION
```

Figure 2.1: A Gecode model for Send More Money

The program starts by including the relevant Gecode headers. To use integer variables and constraints, it includes <gecode/int.hh> and to access search engines it includes <gecode/search.hh>. All Gecode functionality is in the scope of the namespace Gecode, for convenience the program makes all functionality of the Gecode namespace visible by using namespace Gecode.

As discussed, the model is implemented as the class SendMoreMoney inheriting from the class Space. It declares an array l of integer variables and initializes this array to have 8 newly created integer variables as elements, where each variable in the array can take values from 0 to 9. Note that the constructor for the variable array l takes the current space (that is, \*this) as first argument. This is very common: any function that depends on a space takes the current space as argument (called *home space*) Examples are constructors for variables and variable arrays, functions that post constraints, and functions that post branchings.

To simplify the posting of constraints, the constructor defines a variable of type IntVar for each letter. Note the difference between creating a new integer variable (as done with creating the array of integer variables together with creating a new integer variable for each array element) and referring to the same integer variable through different C++ variables of type IntVar. This difference is discussed in more detail in Section 4.1.

**Posting constraints.** For each constraint there is a *constraint post function* that creates *propagators* implementing the constraint (in the home space that is passed as argument).

**Tip 2.1** (Space& versus Home). Actually, when you check the reference documentation, you will see that these functions do not take an argument of type Space& but of type Home instead. An object of type Home actually stores a reference to a space of type Space& (and a reference of type Space& is automatically coerced to an object of type Home). Additionally, a Home object might store other information that is useful for posting propagators and branchers. However, this is nothing you need to be concerned with when modeling with Gecode. Just think that Home reads as Space&. Using Home is important when programming propagators and branchers, see Section 20.5. ◀

The first constraints to be posted enforce that the equation is well formed in that it has no leading zeros:

```
NO LEADING ZEROS ≡
rel(*this, s, IRT_NQ, 0);
rel(*this, m, IRT_NQ, 0);
```

The family of rel post functions (functions with name rel overloaded with different argument types) implements simple relation constraints such as equality, inequalities, and disequality (see Section 4.4.3 and Simple relation constraints over integer variables). The constant IRT\_NQ requests a disequality constraint.

All letters are constrained to take pairwise distinct values by posting a distinct constraint (also known as alldifferent constraint):

```
ALL LETTERS DISTINCT ≡
distinct(*this, l);
```

See Section 4.4.7 and Distinct constraints for more information on the distinct constraint. The constraint that SEND + MORE = MONEY is posted as a linear equation where the individual letters are scaled to their appropriate decimal positions:

```
LINEAR EQUATION \equiv

IntArgs c(4+4+5); IntVarArgs x(4+4+5);

c[0]=1000; c[1]=100; c[2]=10; c[3]=1;

x[0]=s; x[1]=e; x[2]=n; x[3]=d;

c[4]=1000; c[5]=100; c[6]=10; c[7]=1;

x[4]=m; x[5]=o; x[6]=r; x[7]=e;

c[8]=-10000; c[9]=-1000; c[10]=-100; c[11]=-10; c[12]=-1;

x[8]=m; x[9]=o; x[10]=n; x[11]=e; x[12]=y;

linear(*this, c, x, IRT_EQ, 0);
```

The linear constraint (which, again, exists in many overloaded variants) posts the linear equation (as instructed by IRT\_EQ)

$$\sum_{i=0}^{|\mathsf{c}|-1} \mathsf{c}_i \cdot \mathsf{x}_i = 0$$

with coefficients c, integer variables x, and right-hand side constant 0 (see Section 4.4.6 and Linear constraints over integer variables). Here, |c| denotes the size (the number of elements) of the array c (which can be computed by c.size()). Post functions are designed to be as general as possible, hence the variant of linear that takes an array of coefficients and an array of integer variables as arguments. Other variants of linear exist that do not take coefficients (all coefficients are one) or accept an integer variable as the right-hand side instead of an integer constant.

Section 3.1 demonstrates additional support for posting linear expressions constructed from the usual arithmetic operators such as +, -, and \*.

**Posting branchings.** Branchings determine the shape of the search tree. Common branchings take a variable array of the variables to be assigned values during search, a variable selection strategy, and a value selection strategy.

Here, we select the variable with a smallest domain size first (INT\_VAR\_SIZE\_MIN) and assign the smallest value of the selected variable first (INT\_VAL\_MIN):

```
POST BRANCHING ≡
branch(*this, l, INT_VAR_SIZE_MIN, INT_VAL_MIN);
```

A *branching* is implemented by a *brancher* (like a constraint is implemented by a propagator). A brancher creates a number of *choices* where each choice is defined by a number of *alternatives*. For example, the brancher posted above will create as many choices as needed

to assign all variables in the integer variable array 1. Each of the choices is based on the variable selected by the brancher, say x, and the value selected by the brancher, say n. Then the alternatives of a choice are x = n and  $x \ne n$  and are tried by search in that order.

A space can have several branchers, where the brancher that is posted first is also used first for search. More information on branchings can be found in Section 7.1.

**Search support.** As mentioned before, a space must implement an additional copy() function that is capable of returning a fresh copy during search. Search in Gecode is based on a hybrid of *recomputation* and *cloning* (see Chapter 7). Cloning during search relies on the capability of a space to create a copy of itself.

To avoid confusion, by *cloning* we refer to the entire process of creating a clone of a space. By *copying*, we refer to the creation of a copy of a particular object during cloning, for example, a variable or a space.

```
SEARCH SUPPORT ≡
SendMoreMoney(bool share, SendMoreMoney& s) : Space(share, s) {
    l.update(*this, share, s.l);
}
virtual Space* copy(bool share) {
    return new SendMoreMoney(share,*this);
}
```

The actual copy() function is straightforward and uses an additional copy constructor. The copy() function is virtual such that cloning (used on behalf of a search engine) can create a copy of a space even though the space's exact subclass is not known to cloning. The Boolean argument share defines whether a shared copy is constructed and is of no further interest here.<sup>1</sup>

The obligation of the copy constructor is to invoke the copy constructor of the parent class, and to copy all data structures that contain variables. For SendMoreMoney this amounts to invoking Space(share,s) and updating the variable array. Please keep in mind that the copy constructor is run on the copy being created and is passed the space that needs to be copied as argument. Hence, updating the variable array l in the copy copies the array s.l from the space s being cloned (including all variables contained in the array). More on updating variables and variable arrays can be found in Section 4.1.8.

#### **Printing solutions.** Finally, the following prints the variable array 1:

<sup>&</sup>lt;sup>1</sup>For the curious reader: a shared (share is **true**) clone can only be used within the same thread. A clone without sharing (share is **false**) creates a clone where no data structures are shared between original and clone. Hence both original and cloned space can be used in different threads without problems. The appropriate value for share is passed to the clone() member function of a space by a search engine, see also Section 35.1.

```
PRINT SOLUTION ≡
  void print(void) const {
   std::cout << l << std::endl;
}</pre>
```

In a real application, one would use the solution in some other parts of the program. The point is that the space acts as a *closure* for the solution variables: the space maps member names to objects. The space for an actual solution is typically different from the space created initially. This is due to the fact that search for a solution returns a space that has been obtained by constraint propagation and cloning. The space members that refer to the solution variables (the member 1 in our example) provide the means to access a solution independent of a particular space.

### 2.2 Searching for solutions

Let us assume that we want to search for all solutions and that search is controlled by the main function of our program. Search consists of two parts:

- create a model and a search engine for that model; and
- use the search engine to find all solutions.

Hence, our main function looks as follows:

Creating a model is almost obvious: create an object of the subclass of Space that implements the model. Then, create a search engine (we will be using a search engine DFS for depth-first search) and initialize it with a model. Search engines are generic with respect to the type of model, implemented as a template in C++. Hence, we use a search engine of type DFS<SendMoreMoney> for the model SendMoreMoney.

When the engine is initialized, it takes a clone of the model passed to it (m in our example). As the engine takes a clone, several engines can be used without recreating the model. As we are interested in a single engine, we immediately delete the model m after the search engine has been initialized.

```
CREATE MODEL AND SEARCH ENGINE 
SendMoreMoney* m = new SendMoreMoney;

DFS<SendMoreMoney> e(m);
delete m;
```

**Tip 2.2** (Propagation is explicit). A common misconception is that constraint propagation is performed as soon as a space is created or as soon as a constraint is posted. Executing the following code

```
SendMoreMoney* m = new SendMoreMoney;
m->print();
```

prints

```
{[1..9], [0..9], [0..9], [0..9], [1..9], [0..9], [0..9]}
```

That is, only very simple and cheap propagation (nothing but modifying the domain of some variables) has been performed.

Constraint propagation is *explicit* and must be requested by the status() member function of a space (the function also returns information about the result of propagation but this is of no concern here). Requesting propagation by

```
(void) m->status();
m->print();
```

prints

```
{9, [4..7], [5..8], [2..8], 1, 0, [2..8], [2..8]}
```

A search engine first performs constraint propagation as only spaces that have been propagated can be cloned (so as to not duplicate propagation for the original and for the clone).

The DFS<SendMoreMoney> search engine has a simple interface: the engine features a next() function that returns the next solution or NULL if no more solutions exist. As we are interested in all solutions, a while loop iterates over all solutions that are found by the search engine:

```
SEARCH AND PRINT ALL SOLUTIONS 
while (SendMoreMoney* s = e.next()) {
   s->print(); delete s;
}
```

As you can see, a solution is nothing but a model again. A search engine ensures that constraint propagation is performed and that all variables are assigned as described by the branching(s) of the model passed to the search engine. When a search engine returns a model, the responsibility to delete the solution model is with the client of the search engine.

It is straightforward to see how one would search for a single solution instead: replace **while** by **if**. DFS is but one search engine and the behavior of a search engine can be configured (for example: how cloning or recomputation is used; how search can be interrupted) and it can be queried for statistical information. Search engines are discussed in more detail in Chapter 7.

**Tip 2.3** (Catching Gecode exceptions). Posting constraints, posting branchings, creating variables, and so on with Gecode might throw exceptions (for example, potential numerical overflow, illegal use of arguments). It is good practice to construct your programs right from the start to catch all these exceptions.

That is, you should wrap the entire body of the main function but the **return** statement into a **try** statement as follows:

```
try {
    ...
} catch (Exception e) {
    std::cerr << "Gecode exception: " << e.what() << std::endl;
    return 1;
}
return 0;</pre>
```

Even though this is good practice, the example programs in this document do not follow this advice, so as to keep the programs more readable.

## 2.3 Compiling, linking, and executing

This section assumes that you have Gecode with the right version (this document uses Gecode 3.7.3) already installed on your computer. If that is not the case, you should visit the download section on Gecode's website and read Section 2.6 on how to install and/or compile Gecode.

Naturally, the following sections are platform-specific.

#### 2.3.1 Microsoft Visual Studio

Gecode uses a technique called auto-linking with Visual Studio: by including Gecode header files, the compiler/linker knows which library and DLL must be linked against. Hence, it is sufficient to provide information to the compiler where the libraries reside.

The library and DLL names encode the Gecode version, the platform (x86, x64, or ia64), and whether Gecode has been built as release (optimized) or debug. That has the advantage that a single set of header files can be shared among different platforms and builds and the appropriate libraries and DLLs are selected automatically.

In the following we assume that you are using one of the pre-compiled Windows packages we provide (see Section 2.6.1). The packages define the environment variable %GECODEDIR% where Gecode has been installed, and update the Path environment variable so that Gecode DLLs are found for execution.

**Commandline.** In the following we assume that you use the Visual Studio Command Prompt. When compiling and linking with cl, you have to take the following into account:

- As Gecode uses exceptions, you have to add /EHsc as option on the commandline.
- You have to link dynamically against multithreaded libraries. That is, you have to add to the commandline either /MD (release build) or /MDd (debug build).
- If you want a release build, you need to switch off assertions by defining /DNDEBUG.
- You should instruct the compiler cl to search for the Gecode header files by adding /I"%GECODEDIR%\include" as an option.
- When using cl for linking, you should add at the very end of the commandline: /link /LIBPATH: "%GECODEDIR%\lib".
- By default, cl warns if **this** is used in an initializer list (Gecode uses this for the initialization of variables and variable arrays). You can suppress the warning by passing /wd4355.

The full command for compiling send-more-money.cpp as a release build (including optimization with /0x) is

```
cl /DNDEBUG /EHsc /MD /Ox /wd4355 -I"%GECODEDIR%\include" \
  -c -Fosend-more-money.obj -Tpsend-more-money.cpp
```

where the \ at the end of a line means that the line actually continues on the next line. The following command links the program:

```
cl /DNDEBUG /EHsc /MD /Ox /wd4355 -I"%GECODEDIR%\include" \
  -Fesend-more-money.exe send-more-money.obj \
  /link /LIBPATH:"%GECODEDIR%\lib"
```

**Integrated development environment.** When your Microsoft Visual Studio solution uses Gecode, all necessary settings can be configured in the properties dialog of your solution. We assume that Gecode is installed in the default location "<GECODEDIR>".²

- You must use dynamic linking against a multithreaded library. That is, either /MD (release build) or /MDd (debug build). Depending on whether /MD or /MDd is used, release or debug libraries and DLLs will be used automatically.
- As Gecode uses exceptions, you have to enable /EHsc as option (this is true by default).
- If you want a release build, you have to switch off assertions by defining /DNDEBUG (this is true by default).
- Configuration Properties, C<sup>++</sup>, General: set the "Additional Include Directories" to include "<GECODEDIR>\include" as the directory containing the Gecode header files.

<sup>&</sup>lt;sup>2</sup>Unfortunately, the development environment does not resolve %GECODEDIR% automatically. So you have to replace "<GECODEDIR>" in the following by the path where Gecode has been installed.

■ Configuration Properties, Linker, General: set the "Additional Library Directories" to "<GECODEDIR>\lib" as the path containing the libraries.

**Tip 2.4** (Cygwin with Microsoft Visual Studio). A setup that works well for us is to install Cygwin and Microsoft Visual Studio (Microsoft distributes a free Express Edition).

The easiest way to use the Microsoft Visual Studio C++ compiler (cl) from Cygwin is to add a line that loads the file vcvarsall.bat to the Cygwin.bat file that starts Cygwin. The file vcvarsall.bat comes with Microsoft Visual Studio (for example, it is used to start the Visual Studio Command Prompt). On my machine using Visual C++ 2008 Express Edition, the added line is

```
call "c:\Program Files\Microsoft Visual Studio 9.0\VC\vcvarsall.bat"
```

Or, you start the Visual Studio Command Prompt first, and then start the bash shell by hand with

```
chdir c:\Cygwin\bin
bash --login -i
```

provided that Cygwin is installed in c:\Cygwin.

### 2.3.2 Apple Mac OS

These compilation instructions assume that you use the Gecode binary package for Mac OS (see Section 2.6.1). If you compiled and installed Gecode from the source package, please read Section 2.3.3.

**Commandline.** When compiling your code using the gcc compiler (invoking it as g++), the Gecode header files are found automatically, they are on the default header search path. For linking against the Gecode framework, just add -framework gecode as option. Note that only versions 4.2 or better of gcc are supported.

The following command compiles and links send-more-money.cpp as a release build (including optimization):

```
g++ -03 -c send-more-money.cpp
g++ -framework gecode -o send-more-money send-more-money.cpp
```

**XCode.** You can easily use Gecode within the XCode development environment by choosing Add 
ightharpoonup Existing Frameworks... from the context menu on your target. Then pick the gecode framework from the list. You may have to edit your project settings to choose  $Mac\ OS\ 10.6$  as the base SDK.

#### 2.3.3 Linux and relatives

On Linux (and similar operating systems), Gecode is installed as a set of shared libraries. The default installation directory is "/usr/local", which means that the header files can be found in "/usr/local/include" and the libraries in "/usr/local/lib". Depending on your Linux distribution, a binary package may have been installed under the "/usr" path instead. If you installed Gecode from the source package, you may have chosen a different installation directory. For now, assume that Gecode is installed in "<dir>

**Commandline.** To compile your code using the gcc compiler, you have to add the option -I<dir>/include so that gcc can find the header files. Note that only versions 4.2 or better of gcc are supported.

For linking, the path has to be given as -L<dir>/lib, and in addition the individual Gecode libraries must be linked. You always have to link against the support and kernel libraries, using -lgecodesupport -lgecodekernel. For the remaining libraries, the rule of thumb is that if you include a header file <gecode/F00.hh>, then -lgecodeF00 must be given as a linker option. For instance, if you use integer variables and include gecode/int.hh, you have to link using -lgecodeint.

Some linkers require the list of libraries to be sorted such that libraries appear before all libraries they depend on. In this case, use the following order (and omit libraries you don't use): -lgecodeflatzinc, -lgecodedriver, -lgecodegist, -lgecodesearch, -lgecodeminimodel, -lgecodeset, -lgecodeint, -lgecodekernel, -lgecodesupport.

A complete example for compiling and linking the file send-more-money.cpp is as follows.

```
g++ -I<dir>/include -c send-more-money.cpp
g++ -o send-more-money -L<dir>/lib send-more-money.o \
    -lgecodesearch -lgecodeint -lgecodekernel -lgecodesupport
```

The \ at the end of a line means that the line actually continues on the next line.

In order to run programs that are linked against Gecode, the Gecode libraries must be found on the library path. They either have to be installed in one of the default locations (such as /usr/lib), or the environment variable LD\_LIBRARY\_PATH has to be set to include <dir>/lib.

**Eclipse development environment.** If you use the Eclipse IDE with the CDT ( $C/C^{++}$  development tools), you have to configure the paths to the Gecode header files and libraries.

In the *Project* menu, select the *Properties* dialog. Under *GCC C++ Compiler*, add <dir>/include to the *Directories*. Under *GCC C++ Linker*, add <dir>/lib to the *Library search path*, and the Gecode libraries you have to link against to the *Libraries* field.

In order to run programs that link against Gecode from within the Eclipse CDT, select *Open Run Dialog* from the *Run* menu. Either add a new launch configuration, or modify your existing launch configuration. In the *Environment* tab, add the environment variable LD\_LIBRARY\_PATH=<dir>/lib.

```
#include <gecode/int.hh>
#include <gecode/gist.hh>

using namespace Gecode;

class SendMoreMoney : public Space {
    ...
};

int main(int argc, char* argv[]) {
    SendMoreMoney* m = new SendMoreMoney;
    Gist::dfs(m);
    delete m;
    return 0;
}
```

Figure 2.2: Using Gist for Send More Money

**Tip 2.5** (Eclipse on Windows and Mac OS). If you use Eclipse on Windows or Mac OS, the procedure should be similar, except that you do not have to add the environment variable to the launch configuration, and on Windows you do not need to specify the libraries to link against. ◀

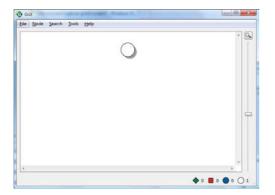
# 2.4 Using Gist

When developing a constraint model, the usual outcome of a first modeling attempt is that the model has no solutions or searching for a solution takes too much time to be feasible. What one really needs in these situations is additional insight as to: why does the model have no solutions, why is propagation not sufficient, or why is the branching not appropriate for the problem?

Gecode offers Gist as a graphical and interactive search tool with which you can explore any part of the search tree of a model step by step or automatically and inspect the nodes of the search tree.

Using Gist is absolutely straightforward. Figure 2.2 shows how Gist is used for the Send More Money problem. As before, a space m for the model is created. This space is passed to Gist, where Gist is instructed to work in dfs (depth-first search) mode. The call to Gecode::dfs terminates only after Gist's window is closed.

Figure 2.3 shows two screenshots of Gist. The left-hand side shows how Gist starts (with no node of the tree yet explored). The right-hand side shows the fully explored search tree of Send More Money.



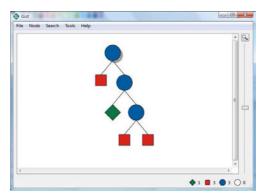


Figure 2.3: Gist screen shots

Gist is so intuitive that our recommendation is to just play a little with it. If you want to know more about Gist, consult Chapter 8.

One additional feature of Gist that comes in handy when developing constraint models is to inspect nodes of the search tree by double-clicking them. Figure 2.4 shows a modified program that instructs Gist to use the print() function of SendMoreMoney whenever a node is double-clicked. Note that the print function has been changed to take a standard outstream to print on as argument.

**Tip 2.6** (Gist scales). Do not be afraid to use Gist even on large problems. You can expect that per Gigabyte of main memory, Gist can maintain around eight to ten million nodes. And the runtime overhead is low (in our experiments, around 15% compared to the commandline search engine using one thread). Just be sure to increase the display refresh rate for larger trees (see Section 8.3.6). ◀

Section 3.3 explains how to use a commandline driver that supports to execute the same constraint model with different search engines (for example, DFS or Gist) by passing options on the commandline.

**Tip 2.7** (Linking against Gist). As discussed in Section 2.3, when you use Gist on a platform (Linux and relatives) that requires to state all libraries to link against, you also have to link against the library for Gist (that is, for Linux and relatives by adding -lgecodegist to the compiler options on the commandline). ◀

#### 2.5 Best solution search

The last aspect to be discussed in this chapter is how to search for a best solution. We are using a model for Send Most Money as an example: find distinct digits for the letters S, E, N, D, M, O, T, and Y such that the well-formed equation (no leading zeros) SEND + MOST = MONEY holds and that MONEY is maximal.

Searching for a best solution requires a best solution search engine and a function that constrains a space to yield a better solution. A Gecode model for Send Most Money is shown

```
[DOWNLOAD]
SEND MORE MONEY WITH GIST INSPECTION \equiv
 #include <gecode/int.hh>
 #include <gecode/gist.hh>
 class SendMoreMoney : public Space {
   void print(std::ostream& os) const {
     os << l << std::endl;
 };
 int main(int argc, char* argv[]) {
    SendMoreMoney* m = new SendMoreMoney;
   Gist::Print<SendMoreMoney> p("Print solution");
   Gist::Options o;
   o.inspect.click(&p);
   Gist::dfs(m,o);
   delete m;
   return 0;
 }
```

Figure 2.4: Using Gist for Send More Money with node inspection

```
SEND MOST MONEY 

...

class SendMostMoney : public Space {
    ...
    ▶ CONSTRAIN FUNCTION
};
MAIN FUNCTION
```

Figure 2.5: A Gecode model for Send Most Money finding a best solution

in Figure 2.5. The model differs from Send More Money only by using a different linear equation and the additional constrain() function.

Assume a new solution, say b, is found during best solution search: on the current search node s (a space) the member function constrain() is called and the so-far best solution b is passed as argument (that is, s.constrain(b) is executed). The constrain() member function must add a constraint to s such that s can only yield a better solution than b during search. For Send Most Money, the constrain() member function is as follows:

First, the integer value of money in the so-far best solution is computed from the values of the variables. Note that the search engine does not know what model it searches a solution for. The search engine passes a space \_b that the constrain member function must cast into a SendMostMoney space. Then the constraint is added that a better solution must yield more money.

**Using a best solution search engine.** The main function now uses a branch-and-bound search engine rather than a plain depth-first engine:

```
MAIN FUNCTION ≡
int main(int argc, char* argv[]) {
   SendMostMoney* m = new SendMostMoney;
   BAB<SendMostMoney> e(m);
   ...
}
```

The loop that iterates over all solutions found by the branch-and-bound search engine is exactly the same as before. That means that solutions are found and printed with an increasing value of *MONEY*. The best solution is printed last.

The branch-and-bound engine BAB (see also Section 7.4) calls the constrain() member function defined by the model. Note that every space defines a default constrain() member function (to keep the design of models simple). If a model does not redefine the constrain() member function (either directly or indirectly bu inheriting a constrain() function), the default function will be called which throws an exception of type SpaceConstrainUndefined (as this is a modeling error).

Using Gist for best solution search is straightforward. Instead of using Gist::dfs, one uses Gist::bab to put Gist into branch-and-bound mode.

In Section 3.2 it is discussed how a simple cost() function can be used for best solution search instead of a more general constrain() function.

# 2.6 Obtaining Gecode

This section explains how to obtain Gecode. There are two basic options: install Gecode as a binary package (explained in Section 2.6.1) or compile Gecode from its source (explained in Section 2.6.2 and Section 2.6.3).

We recommend to use the pre-compiled binaries we provide, unless you like to tinker. The advantage of the packages we provide is that required and additional software for those platforms not having automatic package management (for example, Microsoft Windows) and reference documentation in platform-specific format are already included.

#### 2.6.1 Installing Gecode

Naturally the following sections are platform specific.

#### **Installing Gecode on Windows**

The pre-compiled Gecode binaries available for download require Microsoft Visual Studio. Note that Visual Express Editions are available free of charge from Microsoft.

Pre-compiled binaries are available for Microsoft Visual C++ 2008 and 2010. The binary packages include everything needed (in particular, Qt for Gist).

**Tip 2.8** (Compiling on Windows for x86 versus x64). The download section on Gecode's website offers Windows packages for two different platforms: one for x86 (32 bit) and one for x64 (64 bit). When downloading and installing one of these packages you should make sure that the package's platform matches the compiler you are using (the Windows platform does not matter). The freely available Express Editions of Visual Studio only support x86. ◀

#### **Installing Gecode on Apple Mac OS**

The pre-compiled Gecode binaries for Mac OS require the XCode developer tools, version 3.1 or higher. XCode is available from the Mac OS install DVD, or from the Apple Mac Dev Center.

Furthermore, you will have to install a binary package of the Qt library. There are precompiled packages of the open source version of Qt on the Nokia Qt for Mac web page. You should download and install the Cocoa version (you do not need the debug libraries).

After installing these prerequisites, simply download and open the Gecode disk image for Mac OS, double-click the installer package, and follow the instructions.

#### **Installing Gecode on Linux and relatives**

The Debian and Ubuntu Linux distributions come with pre-compiled packages for Gecode. These packages (and all the packages they depend on) can be installed with the usual package management tools (for example, apt-get). Note that we do not maintain these packages, and that the repositories do not always provide the most up-to-date version.

If the Linux distribution of your choice does not provide a binary package for Gecode, or does not contain the latest version, please refer to the next section for instructions how to compile Gecode from the source.

# 2.6.2 Compiling Gecode

Gecode can be built on all recent versions of Windows, Linux, and MacOS X. Porting to other Unix flavors should be easy, if any change is necessary at all. The Gecode source code is available from the download section of the Gecode web site.

**Prerequisites.** In order to compile Gecode, you need a standard Unix toolchain including the following programs: a bash-compatible shell, GNU make, sed, cp, diff, tar, perl, grep.

These are available in all standard installations of Linux. On MacOS X, you need to install the XCode developer tools, version 3.1 or higher. XCode is available from the Mac OS install DVD, or from the Apple Mac Dev Center. For Windows, we require the Cygwin environment that provides all necessary tools (see also Tip 2.4).

We currently support:

■ Microsoft Visual C<sup>++</sup> compilers for Windows. The Microsoft Visual C<sup>++</sup> Express Edition is available free of charge from Microsoft.

■ GNU Compiler Collection (gcc) for Windows and Unix flavors such as Linux and MacOS X. The GNU gcc is open source software and available from the GCC home page. It is included in all Linux distributions and the Apple MacOS X developer tools.

**Important:** Gecode requires at least version 4.2 of gcc.

■ Intel C++ compiler, although we do not test the binaries produced by it.

**Configuring the sources.** After unpacking the sources, you need to run the configure script in the toplevel directory. This script (which uses GNU autoconf) acquires information about the system you try to compile Gecode on.

To setup Gecode for your particular system, you may need to add one or more of the following options to configure:

- When using the GNU compiler gcc on Windows instead of the Microsoft compiler, add
   CC=gcc CXX=g++ to your invocation of configure.
- To install Gecode somewhere else than the default /usr/local, please use the commandline --prefix=[···] switch.
- You can enable and disable the individual modules Gecode consists of by using --enable-[MODULE] and --disable-[MODULE].

You can get a list of all supported configuration options by calling configure with the --help switch. Section 2.6.3 explains the configuration options in more detail, if the defaults do not work for you.

**Compiling the sources.** After successful configuration, simply invoking

make

in the toplevel Gecode directory will compile the whole library and the examples.

**Installation.** After a successful compilation, you can install the Gecode library and all header files necessary for compiling against it by invoking

make install

in the build directory.

**Tip 2.9** (Do not forget the library path). In order to run programs that are linked against Gecode (such as the Gecode examples), the libraries must be found on the library path. See Section 2.3.3 for details. ◀

**Running the examples.** After compiling the examples, they can be run directly from the command line. For instance, try the Golomb Rulers Problem:

```
./examples/golomb
```

or (when running Windows):

```
./examples/golomb.exe
```

On some platforms, you may need to set environment variables like LD\_LIBRARY\_PATH (Linux) or DYLD\_LIBRARY\_PATH (Mac OS) to the toplevel compile directory or the installation directory (where the dynamic libraries are placed after compilation).

**Compilation with Gist.** The Gecode Interactive Search Tool (Gist) is a graphical search engine for Gecode, built on top of the Qt GUI toolkit.

In order to compile Gecode with Gist, you need an installation of the Qt library including the development header files. The Qt binary packages for Windows and Mac OS (available from the Nokia Qt web pages) as well as the Qt packages in the usual Linux distributions contain everything that is necessary. You can however also compile Qt from its sources, available from Nokia under both free and commercial licenses.

Please note that if you develop closed-source software with Gecode and Gist, you will have to either comply with the LGPL, or obtain a commercial license for Qt from Nokia!

If you are developing on Windows using the Microsoft Visual C++ compiler, make sure to compile the Qt library with the same compiler.

After installing Qt, make sure that the qmake tool is in your shell path. The Gecode configure script will then detect the presence of Qt and automatically compile with support for Gist.

**Tip 2.10** (Compatible compilers and installations for Gecode and Qt). Please make sure that the compiler with which Qt has been compiled is compatible with the compiler you intend to use for Gecode (most likely, the requirement is that both packages must be compiled with the very same compiler).

In particular, make sure that this is true when you install a Qt binary package. Watch out for pre-installed Qt packages! For example: the Qt packages on Windows available through Cygwin should be disabled or deinstalled when you want to use Qt and Gecode with the Microsoft Visual C++ compiler.

# 2.6.3 Advanced configuration and compilation

If the instructions from the previous section do not work for your system, please have a look at the following example configurations and advanced options to configure Gecode to your needs.

#### **Example configurations**

To compile Gecode on a Windows machine using the GNU compiler, use

```
./configure CC=gcc CXX=g++
```

To compile only the Gecode library without examples on a Unix machine, use

```
./configure --disable-examples
```

To compile on a **Unix machine** using a different than the default gcc compiler, and install under /opt/gecode, use

```
./configure --prefix=/opt/gecode CC=gcc-4.2 CXX=g++-4.2
```

To compile a **debug build** on Unix, turning on all assertions and not inlining anything, use

```
./configure --enable-debug
```

To compile on **Cygwin**, but linking against the **Windows libraries** instead of the Cygwin libraries, use

```
./configure CC="gcc-4 -mno-cygwin" CXX="g++-4 -mno-cygwin"
```

Note that we only support building static libraries on Cygwin and that you cannot use the standard gcc that comes with Cygwin.

To compile on a system using a different than the default compiler, and a /bin/sh that is not bash compatible (for example, on a **Solaris machine**), use

```
./configure --with-host-os=linux \
    CC="gcc-4.2" CXX="g++-4.2" \
    SHELL="/bin/bash"
make SHELL="/bin/bash"
```

You can compile as universal binary on a Mac OS machine. Configure with

```
./configure --with-architectures=i386,ppc
```

For building universal binaries on a PowerPC machine, you have to supply the path to the universal SDK (which is the default on Intel based Macs):

```
./configure --with-architectures=i386,ppc \
--with-sdk=/Developer/SDKs/MacOSX10.4u.sdk
```

Passing options for compilation. Additional options for compilation can be passed to the compiler from the make commandline via the variable CXXUSR. For example, to pass to gcc the additional option -mtune=i686 the following can be used:

```
make CXXUSR="-mtune=i686"
```

**Compiling in a separate directory.** The Gecode library can be built in a separate directory. This is useful if you do not want to clutter the source tree with all the object files and libraries.

Configuring Gecode in a separate directory is easy. Assume that the sources can be found in directory \$GSOURCEDIR, change to the directory where you want to compile Gecode and call

```
$GSOURCEDIR/configure [options]
```

This will generate all necessary files in the new build directory.

**Dependency management.** The dependencies between source files are not handled automatically. If you are using a Gecode version from our subversion repository or if you modified any of the source files, you will have to call make depend before compilation in order to determine the source dependencies.

Dependency management is only needed for recompiling Gecode after changing something. In an unmodified version (or after a make clean) all files are compiled anyway.

**Compiling for unsupported platforms.** If you want to try compiling Gecode on a platform that we do not mention, you can override the platform tests during configure. There are two options to specify the type of platform:

- --with-host-os=[linux|darwin|windows]
- --with-compiler-vendor=[gnu|microsoft]

Using the first option, you can state that your platform should behave like Linux, Darwin (which is actually BSD), or Windows. This affects mainly the filenames and the tools used to generate shared and static libraries.

The second option says that your compiler can be used very much like the gnu compiler gcc, or the Microsoft compiler cl. Please let us know of any successfull attempt at compiling Gecode on other platforms.

#### **Useful Makefile targets**

The main Gecode Makefile supports the following useful targets:

• all compiles all parts of the library that were enabled during configure, and the examples if enabled.

- install installs library, headers and examples (if enabled) into the prefix given at configure.
- clean removes object files.
- veryclean removes object files, libraries, and all files generated during make.
- distclean removes object files, libraries, and all generated files.
- depend generates dependencies between source files.
- test compiles the test suite.
- doc generates the reference documentation using doxygen.
- installdoc installs the documentation.
- dist creates a source distribution as a tgz archive, not including the contributions found in the contribs directory.
- distdir creates a source distribution in a directory, not including the contributions found in the contribs directory.
- distdoc creates tgz and zip archives of the documentation.

# 3

# Getting comfortable

This chapter provides an overview of some functionality in Gecode that makes modeling and execution of models more convenient.

**Overview.** Expressions constructed from standard arithmetic operators for posting linear constraints are discussed in Section 3.1, cost functions for best solution search are discussed in Section 3.2, and a script commandline driver that supports the most common options for running models from the commandline is discussed in Section 3.3.

# 3.1 Posting linear constraints de-mystified

As mentioned in the previous chapter, Gecode comes with simple modeling support for posting constraints defined by linear expressions and relations. The parts of the program for Send More Money from Figure 2.1 that change are shown in Figure 3.1. In order to use the modeling support, we have to include the MiniModel header.

The MiniModel module also supports Boolean expressions and relations, and much more, see Chapter 6 for more information. The module in itself does not implement any constraints. The function rel takes the description of the linear constraint, analyzes it, and posts a linear constraint by using the same linear function we have been using in Section 2.1.

# 3.2 Using a cost function

Figure 3.2 uses the class MaximizeSpace for cost-based optimization for Send Most Money. The class MaximizeSpace is also included in Gecode's MiniModel module (see Section 6.3 and Support for cost-based optimization).

The MaximizeSpace class is a sub-class of Space that defines a constrain() member function based on the cost of a space. Our model must implement a virtual cost() function that returns a variable defining the cost (the function must be **const**). In our example, we extend the model to maintain the cost (the amount of money) in a dedicated variable money (note that this variable must also be updated during cloning).

```
[DOWNLOAD]
SEND MORE MONEY DE-MYSTIFIED ≡
  #include <gecode/int.hh>
  #include <gecode/minimodel.hh>
  #include <gecode/search.hh>
  class SendMoreMoney : public Space {
  protected:
    IntVarArray l;
  public:
    SendMoreMoney(void) : l(*this, 8, 0, 9) {
      rel(*this,
                             1000*s + 100*e + 10*n + d
                           + 1000*m + 100*o + 10*r + e
                == 10000*m + 1000*o + 100*n + 10*e + y);
    }
  };
```

Figure 3.1: A Gecode model for Send More Money using modeling support

```
[DOWNLOAD]
SEND MOST MONEY WITH COST ≡
  class SendMostMoney : public MaximizeSpace {
  protected:
    IntVarArray l;
    IntVar money;
  public:
    SendMostMoney(void)
      : l(*this, 8, 0, 9), money(*this,0,100000) {
      rel(*this, money == 10000*m + 1000*o + 100*n + 10*e + y);
      . . .
    SendMostMoney(bool share, SendMostMoney& s)
      : MaximizeSpace(share, s) {
      l.update(*this, share, s.l);
      money.update(*this, share, s.money);
    }
    ► COST FUNCTION
  };
```

Figure 3.2: A Gecode model for Send Most Money using a cost function

The cost function then just returns the amount of money as follows:

```
COST FUNCTION ≡
  virtual IntVar cost(void) const {
   return money;
}
```

# 3.3 Using the script commandline driver

In order to experiment from the commandline with different model variants, different search engines, and so on, it is convenient to have support for passing different option values on the commandline that then can be used by a model. Gecode comes with a simple commandline driver that defines Script as a subclass of Space for modeling and support for commandline options.<sup>1</sup>

**Defining a script class.** Suppose that we want to experiment with two different variants of Send Most Money with a cost function: the first variant uses the model from Section 3.2 and the second variant models the equation SEND + MOST = MONEY by using carry variables.

Using carry variables with several linear equations instead of a single linear equation is straightforward. A carry variable is an integer variable with value 0 or 1 and each column in the equation SEND + MOST = MONEY is modeled by a linear equation involving the appropriate carry variable as follows:

Figure 3.3 shows a model for Send Most Money that uses the MaximizeScript class as base class (see Script classes) rather than MaximizeSpace (likewise, the driver module also offers a Script class to be used instead of Space). There are two main differences between MaximizeScript and MaximizeSpace (Script and Space). First, the constructor must accept a constant argument of type Options (actually, it must accept a constant argument of the type that is specified for the run member function to be explained below) that is used

<sup>&</sup>lt;sup>1</sup>One might wonder why a commandline driver should be part of Gecode but this is due to popular demand. The driver functionality has been there for the examples that ship with Gecode and everybody has been using it and got into trouble as it was not available as a module that could be easily reused. Here we are!

```
[DOWNLOAD]
SEND MOST MONEY WITH DRIVER ≡
 #include <gecode/driver.hh>
 #include <gecode/int.hh>
 #include <gecode/minimodel.hh>
 using namespace Gecode;
  class SendMostMoney : public MaximizeScript {
 public:
   enum {
     MODEL_SINGLE, MODEL_CARRY
   };
    SendMostMoney(const Options& opt)
      : l(*this, 8, 0, 9), money(*this,0,100000) {
      switch (opt.model()) {
      case MODEL_SINGLE:
        break;
      case MODEL_CARRY:
        ► USING CARRIES
        break;
     }
   }
   virtual void print(std::ostream& os) const {
      os << l << std::endl;
   }
 };
 int main(int argc, char* argv[]) {
    ► COMMANDLINE OPTIONS
    ► RUN SCRIPT
    return 0;
 }
```

Figure 3.3: A Gecode model for Send Most Money using the script commandline driver

to pass values computed from options passed on the commandline. Second, a subclass of MaximizeSpace must define a virtual print function that accepts a standard output stream as argument.

Note that one has to include <gecode/driver.hh> for a model that uses the script commandline driver. However, neither <gecode/search.hh> nor <gecode/gist.hh> need to be included as search is handled by the commandline driver.

**Tip 3.1** (Linking against the driver). As discussed in Section 2.3, when you use the commandline driver on a platform (Linux and relatives) that requires to state all libraries to link against, you also have to link against the library for the commandline driver (that is, for Linux and relatives by adding -lgecodedriver to the compiler options on the commandline).

The class SendMostMoney defines an enumeration type with values MODEL\_SINGLE (for a model where a single linear equation is posted for SEND + MOST = MONEY) and MODEL\_CARRY (for a model where several linear equations using carry variables are posted for SEND + MOST = MONEY). The options object opt provides a member function model that returns an enumeration value and posts the constraints accordingly.

**Defining commandline options.** The mapping between strings passed on the commandline and the values MODEL\_SINGLE and MODEL\_CARRY is established by configuring an object of class Options accordingly as follows:

This code creates a new Option object opt where the string "SEND + MOST = MONEY" serves as identification (such as when requesting to print help about the available commandline options).

The first call to opt.model() defines that single is a legal value for the option switch -model, that the string use single linear equation is the help text for the option value, and that the corresponding value (that is, the value returned by opt.model()) is SendMostMoney::MODEL\_SINGLE. The last call to opt.model defines the default value.

As we are performing best solution search, we want to compute all possible solutions. This is done by setting opt.solutions to 0 (the default value is 1 for searching for the first solution). Note that this default value can be changed on the commandline by passing an integer as value for the -solutions commandline option. Parsing the commandline by parse

now takes the configured values for the commandline options -model and -solutions into account.

The Options class supports most options that are useful for propagation, search, and so on similar to the model option. The full details are explained in Chapter 9.

The last piece of our model is calling the static run method of the script class by passing as template arguments our script class type SendMostMoney, BAB as the search engine we would like to use, and the type of options Options (as mentioned before, the constructor of the script must accept a single argument of this type).

```
RUN SCRIPT ≡
Script::run<SendMostMoney,BAB,Options>(opt);
```

Running the script from the commandline. Suppose that we have compiled our script example as the file send-most-money-with-driver.exe (for some platforms, just drop .exe). Then

```
send-most-money-with-driver.exe
```

prints something along the lines

```
SEND + MOST = MONEY
{9, 3, 4, 2, 1, 0, 5, 7}
{9, 3, 4, 2, 1, 0, 6, 8}
{9, 4, 5, 2, 1, 0, 6, 8}
{9, 5, 6, 3, 1, 0, 4, 7}
{9, 6, 7, 2, 1, 0, 3, 5}
{9, 6, 7, 3, 1, 0, 5, 8}
{9, 7, 8, 2, 1, 0, 3, 5}
{9, 7, 8, 2, 1, 0, 4, 6}
Initial
        propagators: 3
        branchers:
                     1
Summary
        runtime:
                      0.024 (24.000000 ms)
        solutions:
        propagations: 109
        nodes:
                      33
        failures:
                      9
        peak depth:
        peak memory:
                      14 KB
```

If we want to try the model with carry variables we can do that by running

#### send-most-money-with-driver.exe -model carry

We can also use Gist by giving a different mode of execution:

send-most-money-with-driver.exe -mode gist

As we call run with BAB as type of search, Gist will automatically start in branch-and-bound mode. Other supported modes are solution (the default mode shown above), time for printing average runtimes, and stat for just printing an execution statistics but no solutions.

Another important commandline option is -help which prints the options supported by the script together with some configuration information. The full details are explained in Chapter 9.

**Tip 3.2** (Aborting execution). In the solution and stat modes, the driver aborts the search gracefully if you send it a SIGINT signal, for example by pressing Ctrl-C on the command line. So if your model runs a long time without returning solutions, you can press Ctrl-C and still see the statistics that tell you how deep the search tree was up to that point, or how many nodes the search has explored.

When you press Ctrl-C twice, the process is interrupted immediately. This can be useful when debugging programs, e.g. if the process is stuck in an infinite loop. Alternatively, you can make the driver ignore Ctrl-C altogether, so that it immediately interrupts your program, using the command line option -interrupt false.

**Tip 3.3** (How Gecode has been configured). Depending on the hardware and software platform on which Gecode has been compiled, some features (such as Gist or thread support for parallel search) might not be available. For example, if Gist is not supported, the request for -mode gist will be silently ignored and the normal mode (-mode solution) is used instead.

To find out which features are available, just invoke a program using the commandline driver with the -help option. At the beginning of the printed text, you will find a short configuration summary.

Note that all features are enabled for the precompiled binaries that are available from the Gecode webpages.

**Tip 3.4** (Which version of Gecode are we using?). Some programs might have to deal with incompatible changes between different versions of Gecode. The macro GECODE\_VERSION\_NUMBER can be used to find out which version of Gecode is used during compilation. The macro's value is defined as  $100000 \times x + 100 \times y + z$  for Gecode version x.y.z.

# 4

# Integer and Boolean variables and constraints

This chapter gives an overview of integer and Boolean variables and the constraints available for them in Gecode. The chapter focuses on variables and constraints, a discussion of branching for integer and Boolean variables can be found in Section 7.1.

The chapter does not make an attempt to duplicate the reference documentation (see Using finite domain integers). It is concerned with the most important ideas and principles underlying integer and Boolean variables and constraints. In particular, the chapter provides entry points into the reference documentation and points to illustrating examples.

**Overview.** Section 4.1 details how integer and Boolean variables (and variables in general) can be used for modeling. Variable arrays and argument arrays are discussed in Section 4.2. Important aspects of how constraints are posted in Gecode are explained in Section 4.3. These sections belong to the basic reading material of Part M.

The remaining sections Section 4.4 and Section 4.5 provide an overview of the constraints that are available for integer and Boolean variables in Gecode.

**Important.** Do not forget to add

#include <gecode/int.hh>

to your program when you want to use integer or Boolean variables and constraints.

**Convention.** All program fragments and references to classes, namespaces, and other entities assume that declarations and definitions from the Gecode namespace are visible: for example, by adding

using namespace Gecode;

to your program.

The variable home refers to a space reference (of type Space&) and defines the home space in which new variables, propagators, and branchers are posted. Often (as in Chapter 2 and Chapter 3) home will be \*this, referring to the current space.

# 4.1 Integer and Boolean variables

Variables in Gecode are for modeling. They provide operations for creation, access, and update during cloning. By design, the only way to modify (constrain) a variable is by post functions for constraints and branchers.

Integer variables are instances of the class IntVar while Boolean variables are instances of the class BoolVar. Integer variables are *not* related to Boolean variables. A Boolean variable is *not* an integer variable with a domain that is included in {0,1}. The only way to get an integer variable that is equal to a Boolean variable is by posting a channeling constraint between them (see Section 4.4.11).

**Tip 4.1** (Do not use views for modeling). If you — after some browsing of the reference documentation — should come across integer views such as IntView, you might notice that views have a richer interface than integer variables. You might feel that this interface looks too powerful to be ignored. Now, you really should put some trust in this document: views are *not* for modeling.

The more powerful interface only works within propagators and branchers, see Part P and Part B.

#### 4.1.1 Creating integer variables

A variable provides a read-only interface to a *variable implementation* where the same variable implementation can be referred to by arbitrarily many variables.

New integer variables are created by using a constructor. A new integer variable x is created by

```
IntVar x(home, -4, 20);
```

This declares a variable x of type IntVar in the space home, creates a new integer *variable implementation* with domain  $\{-4, \ldots, 20\}$ , and points x to the newly created integer variable implementation.

The domain of a variable can also be specified by an integer set IntSet, for example by

```
IntVar x(home, IntSet(-4, 20));
```

which creates a new variable with domain  $\{-4,...,20\}$ . An attempt to create an integer variable with an empty domain throws an exception of type Int::VariableEmptyDomain.

**Tip 4.2** (Initializing variables). Variables do not have an initialization function. However, new variables can be also created by

```
IntVar x;
x = IntVar(home, -4, 20);
```

The default or copy constructor of a variable does not create a new variable (that is, a new variable implementation). Instead, the variable does not refer to any variable implementation (default constructor) or to the same variable implementation (copy constructor). For example, in

```
IntVar x(home, 1, 4);
IntVar y(x);
```

both x and y refer to the same integer variable implementation. Using a default constructor and an assignment operator is equivalent:

```
IntVar x(home, 1, 4);
IntVar y;
y=x;
```

#### 4.1.2 Limits for integer values

The set of values for an integer variable is a subset of the values of the type **int**. The set of values is symmetric: -Int::Limits::min = Int::Limits::max for the smallest possible integer variable value Int::Limits::min and the largest possible integer variable value Int::Limits::max. Moreover, Int::Limits::max is strictly smaller than the largest possible integer value INT\_MAX and Int::Limits::min is strictly larger than the smallest possible integer value INT\_MIN. These limits are defined in the namespace Int::Limits.

Any attempt to create a variable with values outside the defined limits throws an exception of type Int::OutOfLimits. The same holds true for any attempt to use an integer value outside the defined limits when posting a constraint or brancher.

# 4.1.3 Variable domains are never empty

An important invariant in Gecode is that the domain of a variable is never empty. When a variable domain should become empty during propagation, the space is failed but the variable's domain is kept. In fact, this is the very reason why an attempt to create a variable with an empty domain, for example by

```
IntVar x(home, 1, 0);
```

throws an exception of type Int::VariableEmptyDomain.

**Tip 4.3** (Small variable domains are beautiful). It is not an omission that an integer variable has no constructor that creates a variable with the largest possible domain. One could argue that a constructor like that would come in handy for creating temporary variables. After all, one would not have to worry about the exact domain!

Sorry, but one has to worry. The apparent omission is deliberate to make you worry indeed. For many propagators posted for a constraint a small domain is essential. For example, when posting a linear constraint (as in Section 2.1), variable domains that are

too large might result in an exception of type Int::OutOfLimits as during propagation numerical overflow might occur (even if Gecode resorts to a number type supporting larger numbers than **int** for propagating linear). Moreover, the runtime of other propagators (for example, many domain propagators such as domain consistent distinct) depend critically on the size of a domain. Again, Gecode tries to be clever in most of the cases. But, it is better to make it a habit to think about initial variable domains carefully (please remember: better safe than sorry).

For examples where small variable domains matter, see Tip 10.1 and Tip 13.2.

#### 4.1.4 Creating Boolean variables

The only difference between integer and Boolean variables is that Boolean variables can only take the values 0 or 1. Any attempt to create a Boolean variable with values different from 0 or 1 throws an exception of type Int::NotZeroOne.

**Convention.** If Boolean variables are not explicitly mentioned in the following, the same functionality for integer variables is also available for Boolean variables and has the same behavior.

#### 4.1.5 Variable access functions

Variables provide member functions for access, such as x.min() for the minimum value of the current domain for an integer or Boolean variable x. In particular, the member function x.val() accesses the integer value of an already assigned variable (if the variable is not yet assigned, an exception of type Int::ValOfUnassignedVar is thrown). In addition, variables can be printed by the standard output operator <<.

# 4.1.6 Iterating over integer variable domains

The entire domain of an integer variable can be accessed by a *value iterator* IntVarValues or a *range iterator* IntVarRanges. For example, the loop

```
for (IntVarValues i(x); i(); ++i)
  std::cout << i.val() << ' ';</pre>
```

uses the value iterator i to print all values of the domain of the integer variable x. The call operator i() tests whether there are more values to iterate for i, the prefix increment operator ++i moves the iterator i to the next value, and i.val() returns the current value of the iterator i. The values are iterated in strictly increasing order.

Similarly, the following loop

```
for (IntVarRanges i(x); i(); ++i)
  std::cout << i.min() << ".." << i.max() << ' ';</pre>
```

uses the range iterator i to print all *ranges* of the integer variable x. Given a finite set of integers d, the *range sequence* of d is the shortest (and unique) sequence of ranges (intervals)

$$\langle \lceil n_0 \dots m_0 \rceil \}, \dots, \lceil n_k \dots m_k \rceil \} \rangle$$

such that the sequence is ordered and non-adjacent  $(m_i + 1 < n_{i+1} \text{ for } 0 \le i < k)$ . A range iterator iterates over the ranges in the range sequence of a variable's domain. Like a value iterator, a range iterator implements the call operator i() to test whether there are more ranges to iterate for i and the prefix increment operator ++i to move i to the next range. As a range iterator i iterates over ranges, it implements the member functions i.min() and i.max() for the minimal, respectively maximal, value of the current range.

Iteration of values and ranges for Boolean variables is not available (as it is not needed).

#### 4.1.7 When to inspect a variable

Note that one must not change the domain of a variable (for example, by posting a constraint on that variable) while an iterator for that variable is still in use. This is the same as for most iterators, for example, for iterators in the C++ Standard Template Library (STL).

Otherwise, a variable can always be inspected: at any place (that is, not only in member functions of the variable's home) and at any time (regardless of the status of a space). If the variable's home is failed, the variable can still be inspected. However, it might be the case that the variable domain has more values than expected. For example, after creating a variable x with the singleton domain  $\{0\}$  and posting the constraint that x must be different from 0 by (read Tip 2.2 about status()):

```
IntVar x(home, 0, 0);
rel(home, x, IRT_NQ, 0);
(void) home.status();
```

the space home is failed but the variable x still contains the value 0 in its domain.

# 4.1.8 Updating variables

As discussed in Section 2.1, a variable must be updated during cloning in the copy constructor used by a space's copy() member function. For example, a variable x is updated by

```
x.update(home, share, y);
```

where share is the Boolean argument passed as argument to copy() and y is the variable from which x is to be updated. While x belongs to home, y belongs to the space being cloned.

A space only needs to update the variables that are part of the solution, so that their values can be accessed after a solution space has been found. Temporary variables do not need to be copied.

Assume that we want to constrain the integer variable p to be the product of x, y, and z. Gecode only offers multiplication of two variable, hence a temporary variable t is created (assume also that we know that the values for t are between 0 and 1000):

```
IntVar t(home, 0, 1000);
mult(home, x, y, t);
mult(home, t, z, p);
```

Here, t does not require updating. The multiplication propagators created by mult take care of updating the variable implementation of t.

# 4.2 Variable and argument arrays

Gecode has very few *proper* data structures. The proper data structures for integer and Boolean variables are the variables themselves, integer sets, and arrays of variables. Proper means that these data structures can be updated and hence be stored in a space. Other proper data structures are DFAs (deterministic finite automata) and tuple sets used for extensional constraints. These data structures are discussed in Section 4.4.13.

Of course, data structures that themselves do not contain proper data structures can be stored in a space, such as integers, pointers, and strings.

Gecode supports the programming of new proper data structures, this is discussed in Section 27.3.

### 4.2.1 Integer and Boolean variable arrays

Integer variable arrays of type IntVarArray can be used like variables. For example,

```
IntVarArray x(home, 4, -10, 10);
```

creates a new integer variable array with four variables containing newly created variables with domain  $\{-10, ..., 10\}$ . Boolean variable arrays of type BoolVarArray are analogous.

Creation of a variable array allocates memory from the home space. The memory is freed when the space is deleted (not when the destructor of the variable array is called). Variable arrays can be created without creating new variables by just passing the size. That is,

```
IntVarArray x(home, 4);
for (int i=0; i<4; i++)
x[i] = IntVar(home, -10, 10);</pre>
```

is equivalent to the previous example.

The other operations on variable arrays are as one would expect. For example, one can check whether all variables are assigned using the assigned() function. More importantly, variable arrays like variables have an update function and variable arrays must be updated during cloning. In the following, we will refer to the size of a variable array x by |x| (which can be computed by x.size()).

**Matrix interface.** Many models are naturally expressed by using matrices. Gecode offers support that superimposes a matrix interface for modeling on an array, see Section 6.2.

#### 4.2.2 Argument arrays

As mentioned above, the memory allocated for a variable array is freed only when its home space is deleted. That makes variable arrays *unsuited* for temporary variable arrays, in particular for arrays that are built dynamically or used as arguments for post functions.

For this reason, Gecode provides argument arrays: IntVarArgs for integer variables, BoolVarArgs for Boolean variables, IntArgs for integers, and IntSetArgs for integer sets (see Argument arrays). Internally, they allocate space from the heap<sup>1</sup> and the memory is freed when their destructor is executed. They are *not* proper data structures as they cannot be updated.

Argument arrays can be created empty:

```
IntVarArgs x;
```

with a certain size but without initializing the elements:

```
IntVarArgs x(5);
```

or fully initialized:

```
IntVarArgs x(home,5,0,10);
```

For a typical example, consider Section 2.1 where an integer argument array and an integer variable argument array are used to pass coefficients and variables to the linear post function.

**Dynamic argument arrays.** In contrast to variable arrays, argument arrays can grow dynamically by adding elements or whole arrays using **operator**<<:

```
IntVarArgs x;
x << IntVar(home,0,10);
IntVarArgs y;
y << IntVar(home,10,20);
y << x;
linear(home, IntVarArgs()<<x[0]<<x[1], IRT_EQ, 0);</pre>
```

Furthermore, argument arrays can be concatenated using **operator**+:

```
IntVarArgs z = x+y;
```

<sup>&</sup>lt;sup>1</sup>Actually, if an argument array has few fields it uses some space that is part of the object implementing the array rather than allocating memory from the heap. Hence, small argument arrays reside entirely on the stack.

**Slices.** It is sometimes necessary to post constraints on a subsequence of the variables in an array. This is made possible by the slice(start,inc,n) method of variable and argument arrays. The start parameter gives the starting index of the subsequence. The inc optional parameter gives the increment, i.e., how to get from one element to the next (its default is 1). The n parameter gives the maximal length of the resulting array (its default is -1, meaning as long as possible).

The following examples should make this clearer. Assume that the integer variable argument array x is initialized as follows:

```
IntVarArgs x(home, 10, 0, 10);
```

Then the following calls of slice() return:

- x.slice(5) returns an array with elements  $x[5], x[6], \dots, x[9]$ .
- x.slice(5,1,3) returns x[5],x[6],x[7].
- x.slice(5,-1) returns  $x[5], x[4], \dots, x[0]$ .
- x.slice(3,3) returns x[3],x[6],x[9].
- x.slice(8,-2) returns x[8],x[6],x[4],x[2],x[0].
- x.slice(8, -2, 3) returns x[8], x[6], x[4].

**Tip 4.4** (Reversing argument arrays). The slice() method can be used to compute an array with the elements of x in reverse order like this:

```
x.slice(x.size()-1,-1)
```

**Creating integer argument arrays.** Integer argument arrays with simple sequences of integers can be generated using the static method IntArgs::create(n,start,inc). The n parameter gives the length of the generated array. The start parameter is the starting value, and inc determines the increment from one value to the next.

Here are a few examples:

- IntArgs::create(5,0) creates an array with elements 0,1,2,3,4.
- IntArgs::create(5,4,-1) creates 4,3,2,1,0.
- IntArgs::create(3,2,0) creates 2,2,2.
- IntArgs::create(6,2,2) creates 2,4,6,8,10,12.

**Tip 4.5** (Dynamically constructing models). Sometimes the number of variables cannot be determined easily, for example when it depends on data read from a file.

Suppose the following script with a variable array x:

It is easy to use a variable argument array \_x for collecting variables as follows:

```
READ DATA ≡
   IntVarArgs _x;
   while (···) {
         ...
        _x << IntVar(*this,...);
   }</pre>
```

and then initialize the variable array x using the argument array:

```
INITIALIZE VARIABLE ARRAY ≡
x = IntVarArray(*this,_x);
```

In the following we do not distinguish between arrays and argument arrays unless operations require a certain type of array. In fact, all post functions for constraints and branchers only accept variable argument arrays. A variable array is automatically casted to a variable argument array if needed.

#### 4.2.3 STL-style iterators

All arrays in Gecode (including variable arrays and argument arrays) also support STL-style (Standard Template Library) iterators. For example, assume that a is an integer variable argument array. Then

```
for (IntVarArgs::iterator i = a.begin(); i != a.end(); ++i) {
    ...
}
```

creates an iterator i for the elements of a and iterates from the first to the last element in a.

More powerfully, iterators give you the ability to work with STL algorithms. Suppose that
f() is a function that takes an integer variable by reference such as in

```
void f(IntVar& x) { ··· }
```

and a is an integer variable argument array. Then

```
#include <algorithm>
std::for_each(a.begin(), a.end(), f);
```

applies the function f() to each integer variable in a.

# 4.3 Posting constraints

This section provides information about general principles for posting constraints over integer and Boolean variables.

**Post functions are clever.** A constraint post function carefully analyzes its arguments. Based on this analysis, the constraint post function chooses the best possible propagator for the constraint.

For example, when posting a distinct constraint (see Section 4.4.7) for the variable array x by

```
distinct(home, x);
```

where x has two elements, the much more efficient propagator for disequality  $x_0 \neq x_1$  is created.

**Everything is copied.** When passing arguments to a post function, all data structures that are needed for creating a propagator (or several propagators) implementing a constraint are copied. That is, none of the data structures that are passed as arguments are needed after a constraint has been posted.

**Reified constraints.** Many constraints also exist as a *reified* variant: the validity of a constraint is reflected to a Boolean control variable (reified constraints are also known as metaconstraints). Whether a reified version exists for a given constraint can be found in the reference documentation. If a reified version does exist, the Boolean control variable is passed as the last non-optional argument.

**Selecting the consistency level.** For many constraints, Gecode provides different propagators with different levels of consistency. All constraint post functions take an optional argument of type IntConLevel (see Using finite domain integers) controlling which propagator is chosen for a particular constraint.

The different values for IntConLevel have the following meaning:

- ICL\_VAL: perform value propagation. A typical example is naive distinct: wait until a variable becomes assigned to a value n, then prune n from all other variables.
- ICL\_BND: perform bounds propagation or achieve bounds consistency. This captures both bounds consistency over the integers (for example, for distinct, see Section 4.4.7) or bounds consistency over the real numbers (for example, for linear, see Section 4.4.6). For more information on bounds consistency over integers or real numbers, see [10].

Some propagators that are selected might not even achieve bounds consistency but the idea is that the propagator performs propagation by reasoning on the bounds of variable domains.

- ICL\_DOM: perform domain propagation or achieve domain consistency. Most propagators selected by ICL\_DOM achieve domain consistency but some just perform propagation by taking entire variable domains for propagation into account (for example, circuit, see Section 4.4.17).
- ICL\_DEF: choose default consistency level for this constraint.

Whether bounds or domain consistency is achieved and the default consistency level for a constraint are mentioned in the reference documentation for each post function.

**Tip 4.6** (Different consistency levels have different costs). Note that propagators of different consistency level for the very same constraint can have vastly different cost. In general, propagation for ICL\_VAL will be cheapest, while propagation for ICL\_DOM will be most expensive.

The reference documentation for a constraint lists whether propagation for a particular consistency level might have prohibitive cost for a large number of variables or a large number of values in the variables' domains. For example, for the linear constraint with n variables and at most d values for each variable, the complexity to perform bounds propagation (that is, ICL\_BND) is O(n) whereas the complexity for domain propagation (that is, ICL\_DOM) is  $O(d^n)$ .

**Exceptions.** Many post functions check their arguments for consistency before attempting to create a propagator. For each post function, the reference documentation lists which exceptions might be thrown.

**Unsharing arguments.** Some constraints can only deal with *non-shared* variable arrays: a variable is not allowed to appear more than once in the array (more precisely: no unassigned variable implementation appears more than once in the array). An attempt to post one of these constraints with shared variable arrays will throw an exception of type Int::ArgumentSame.

To be able to post one of these constraints on shared variable arrays, Gecode provides a function unshare (see Unsharing variables) that takes a variable argument array x as argument as in:

```
unshare(home, x);
```

It replaces each but the first occurrence of a variable y in x by a new variable z, and creates a propagator y = z for each new variable z.

Note that unshare requires a variable argument array and *not* a variable array. If x is a variable array, the following

```
IntVarArgs y(x);
```

creates a variable argument array y containing the same variables as x.

**Tip 4.7** (Unsharing is expensive). It is important to keep in mind that unshare creates new variables and propagators. This is also the reason why unsharing is not done implicitly by a post function for a constraint that does not accept shared variable arrays.

Consider the following example using extensional constraints for a variable argument array x possibly containing a variable more than once, where a and b are two different DFAs (see Section 4.4.13 for extensional constraints). By

```
unshare(home, x);
extensional(home, x, a);
extensional(home, x, b);
```

multiple occurrences of the same variable in x are unshared *once* and the propagators for extensional can work on the same non-shared array.

If unsharing were implicit, the following

```
extensional(home, x, a);
extensional(home, x, b);
```

would unshare x twice and create many more (useless) propagators and variables. Rather than implicitly unsharing the same array over and over again (and hence creating variables and propagators), unsharing is made explicit and should be done only once, if possible. ◀

#### 4.4 Constraint overview

This section provides an overview of the constraints and their post functions available for integer and Boolean variables.

```
IRT_EQ equality (=) IRT_NQ disequality (\neq)
IRT_LE strictly less inequality (<) IRT_LQ less or equal inequality (\leq)
IRT_GR strictly greater inequality (>) IRT_GQ greater or equal inequality (\geq)
```

Figure 4.1: Integer relation types

#### 4.4.1 Domain constraints

Domain constraints constrain integer variables and variable arrays to values from a given domain. For example, by

```
dom(home, x, 2, 12);
```

the values of the variable x (or of all variables in a variable array x) are constrained to be between 2 and 12. Domain constraints also take integers (assigning variables to the integer value) and integer sets of type IntSet. For example,

```
IntArgs a(4, 1,-3,5,-7)
IntSet d(a);
dom(home, x, d);
```

constrains the variable x (or, the variables in x) to take values from the set  $\{-7, -3, 1, 5\}$  (see also GCCAT: domain, in, in\_interval, in\_intervals, in\_set).

Note that there are no domain constraints for Boolean variables, please use relation constraints instead, see Section 4.4.4.

Domain constraints for a single variable also support reification. For examples using domain constraints, see n-Knight's tour (simple model) and Packing squares into a rectangle.

# 4.4.2 Membership constraints

Membership constraints constrain integer or Boolean variables to be included in an array of integer or Boolean variables. That is, for an integer variable array x and an integer variable y, the constraint

```
member(home, x, y);
```

forces that y is included in x:

$$y \in \left\{x_0, \dots, x_{|x|-1}\right\}$$

As mentioned, x and y can also be Boolean variables. Membership constraints also support reification.

#### 4.4.3 Simple relation constraints over integer variables

Simple relation constraints over integer variables enforce relations between integer variables and between integer variables and integer values. The relation depends on an integer relation type IntRelType (see Simple relation constraints over integer variables). Figure 4.1 lists the available integer relation types and their meaning.

**Binary relation constraints.** Assume that x and y are integer variables. Then

```
rel(home, x, IRT_LE, y);
```

constrains x to be strictly less than y. Similarly, by

x is constrained to be different from 4. Both variants of rel also support reification (see also GCCAT: eq, neq, lt, leq, gt, geq).

**Constraints between variable arrays and a single variable.** If x is an integer variable array and y is an integer variable, then

```
rel(home, x, IRT_LQ, y);
```

constrains all variables in x to be less than or equal to y. Likewise,

constrains all variables in x to be larger than 7 (see also GCCAT: arith).

**Constraints between array elements.** If x is an integer variable array, then

```
rel(home, x, IRT_LQ);
```

constrains the variables in x to be sorted in increasing order as follows:

$$x_0 \le x_1 \le \cdots \le x_{|x|-1}$$

The integer relation type values for inequalities (that is, IRT\_LE, IRT\_GQ, and IRT\_GR) are analogous. For an example, see Chapter 10 and Finding optimal Golomb rulers.

By

```
rel(home, x, IRT_EQ);
```

all variables in the integer variable array x are constrained to be equal:

$$\mathsf{x}_0 = \mathsf{x}_1 = \dots = \mathsf{x}_{|\mathsf{x}|-1}$$

By

```
BOT_AND conjunction (\land) BOT_OR disjunction (\lor) BOT_IMP implication (\rightarrow) BOT_EQV equivalence (\leftrightarrow)
```

Figure 4.2: Boolean operation types

```
rel(home, x, IRT_NQ);
```

the variables in x are constrained to be not all equal:

$$\neg \left( \mathsf{x}_0 = \mathsf{x}_1 = \dots = \mathsf{x}_{|\mathsf{x}|-1} \right)$$

For an example, see Schur's lemma (see also GCCAT: all\_equal, decreasing, increasing, not\_all\_equal, strictly\_decreasing, strictly\_increasing).

**Lexicographic constraints between variable arrays.** If x and y are integer variable arrays (where the sizes of x and y can be different),

```
rel(home, x, IRT_LE, y);
```

constrains x and y such that x is lexicographically strictly smaller than y (analogously for the other inequality relations). For IRT\_EQ and |x| = |y|, it is propagated that  $x_i = y_i$  for  $0 \le i < |x|$ . For IRT\_NQ and |x| = |y|, it is propagated that  $x_i \ne y_i$  for at least one i such that  $0 \le i < |x|$ . If  $|x| \ne |y|$ , for IRT\_EQ the space home is failed whereas for IRT\_NQ the constraint is ignored (see also GCCAT: lex\_greater, lex\_greatereq, lex\_less, lex\_lesseq).

See Balanced incomplete block design (BIBD) for an example (albeit over Boolean variables).

#### 4.4.4 Simple relation constraints over Boolean variables

Simple relation constraints over Boolean variables include the same post functions as simple relation constraints over integer variables. In addition, simple relation constraints over Boolean constraints provide support for the typical Boolean operations such as conjunction and disjunction. Boolean operations are defined by values of the type BoolOpType (see Simple relation constraints over integer variables). Figure 4.2 lists the available Boolean operation types.

For example, for Boolean variables x, y, and z,

```
rel(home, x, BOT_AND, y, z);
```

posts the constraint  $x \wedge y = z$ . Similarly,

```
rel(home, x, BOT_OR, y, 1);
```

posts that  $x \lor y$  must be true (see also GCCAT: and, equivalent, imply, or, xor).

Note that the integer value must be either zero or one, otherwise an exception of type Int::NotZeroOne is thrown.

For an example, see Balanced incomplete block design (BIBD).

**Tip 4.8** (Boolean negation). Boolean negation can be easily obtained by using IRT\_NQ as relation type. The constraint  $x = \neg y$  for Boolean variables x and y can be posted by

Additional constraints are available for Boolean variable arrays. For a Boolean variable array x and a Boolean variable y,

posts the constraint

$$\bigvee_{i=0}^{|\mathsf{x}|-1} \mathsf{x}_i = \mathsf{y}$$

Again, y can also be 0 or 1.

Note that Boolean implication is special in that it is not associative and Gecode follows normal notational convention. Hence for a Boolean variable array x and a Boolean variable y,

posts the constraint

$$x_0 \rightarrow (x_1 \rightarrow (... \rightarrow (x_{|x|-2} \rightarrow x_{|x|-1}))) = y$$

Again, y can also be 0 or 1.

In order to avoid many propagators for negation, the clause constraint accepts both positive and negative Boolean variables. For Boolean variable arrays x and y and a Boolean variable z (again, z can also be 0 or 1)

posts the constraint

$$\bigwedge_{i=0}^{|\mathsf{x}|-1} \mathsf{x}_i \wedge \bigwedge_{i=0}^{|\mathsf{y}|-1} \neg \mathsf{y}_i = \mathsf{z}$$

(see also GCCAT: clause\_and, clause\_or, nand, nor).

Note that only BOT\_AND and BOT\_OR as Boolean operation types are supported, other Boolean operations types throw an exception of type Int::IllegalOperation.

For an example, see CNF SAT solver.

post function	constraint posted	bnd	dom	GCCAT
min(home, x, y, z);	min(x, y) = z	✓	1	minimum
<pre>max(home, x, y, z);</pre>	max(x,y) = z	✓	✓	maximum
abs(home, x, y);	x  = y	✓	✓	abs_value
<pre>mult(home, x, y, z);</pre>	$x \cdot y = z$	✓	✓	
<pre>sqr(home, x, y);</pre>	$x^2 = y$	✓	✓	
<pre>sqrt(home, x, y);</pre>	$\left  \lfloor \sqrt{x} \right  = y$	✓	✓	
<pre>div(home, x, y, z);</pre>	$x \div y = z$	✓		
<pre>mod(home, x, y, z);</pre>	$x \mod y = z$	✓		
<pre>divmod(home, x, y, d, m);</pre>	$x \div y = d \wedge x \mod y = m$	✓		

Figure 4.3: Arithmetic constraints (x, y, z, d, and m are integer variables)

#### 4.4.5 Arithmetic constraints

Arithmetic constraints exist only over integer variables. In addition to the constraints summarized in Figure 4.3 (bnd abbreviates bounds consistency and dom abbreviates domain consistency), the minimum and maximum constraints are also available for integer variable arrays. That is, for an integer variable array x and an integer variable y

```
min(home, x, y);
```

constrains y to be the minimum of the variables in x (max is analogous).

#### 4.4.6 Linear constraints

Linear constraints over integer variables and Linear constraints over Boolean variables provide essentially the same post functions for integer and Boolean constraints (to be discussed below). The most general variant

```
linear(home, a, x, IRT_EQ, c);
```

posts the linear constraint

$$\sum_{i=0}^{|\mathsf{x}|-1} \mathsf{a}_i \cdot \mathsf{x}_i = \mathsf{c}$$

with integer coefficients a (of type IntArgs), integer variables x, and an integer constant c. Note that a and x must have the same size. Of course, all other integer relation types are supported, see Figure 4.1 for a table of integer relation types. Multiple occurrences of the same variable in x are explicitly allowed and common terms  $a \cdot y$  and  $b \cdot y$  for the same variable y are rewritten to  $(a + b) \cdot y$  to increase propagation. For an example, see Section 2.1.

The array of coefficients can be omitted if all coefficients are one. That is,

```
linear(home, x, IRT_GR, c);
```

posts the linear constraint

$$\sum_{i=0}^{|\mathsf{x}|-1} \mathsf{x}_i > \mathsf{c}$$

for a variable array x and an integer c.

Instead of an integer constant c as the right-hand side of the linear constraint, an integer variable can be used as well. This is true for linear constraints over both integer and Boolean variables: the right-hand side is always an integer value or an integer variable, even if the left-hand side involves Boolean variables. For example, when assuming that x is an array of Boolean variables,

```
linear(home, x, IRT_GQ, y);
```

imposes the constraint that there are at least y ones among the Boolean variables in x.

All variants of linear support reification and exist in variants that perform both bounds propagation (the default) and domain propagation (see also GCCAT: scalar\_product, sum\_ctr).

#### 4.4.7 Distinct constraints

The distinct constraint (see Distinct constraints) enforces that integer variables take pairwise distinct values (also known as alldifferent constraint). Obviously, distinct does not exist for Boolean variables.

**Posting** 

```
distinct(home, x);
```

constrains all variables in x to be pairwise different.

Posting

```
distinct(home, c, x);
```

for an array of integer values c (of type IntArgs) and an array of integer variables x of same size, constrains the variables in x such that

$$x_i + c_i \neq x_j + c_j$$
 for  $0 \le i, j < |x|$  and  $i \ne j$ 

Gecode offers value (the default), bounds (based on [21]), and domain propagation (based on [30]) for distinct (see also GCCAT: alldifferent, alldifferent\_cst).

For examples, see in particular Chapter 10, n-Queens puzzle, and Crowded chessboard.

#### 4.4.8 Counting constraints

**Counting single values.** Counting constraints count how often values are taken by an array of integer variables. The simplest case is

```
count(home, x, y, IRT_EQ, z);
```

which constrains z to be equal (controlled by IRT\_EQ, all integer relation types are supported, see Figure 4.1) to the number of integer variables in x that are equal to y. Here y and z can be integer variables as well as integer values (see also GCCAT: atleast, atmost, count, exactly).

The count constraints also support counting how many integer variables are included in an integer set. If y is an integer set, then

```
count(home, x, y, IRT_EQ, z);
```

constrains z to be equal to the number of integer variables in x that are included in y (see also GCCAT: among, among\_var, counts).

The following

```
count(home, x, c, IRT_EQ, z);
```

where x is an array of integer variables and c is an array of integers (of type IntArgs) with same size and z is an integer variable or value, constrains z to how often  $x_i = c_i$ , that is

$$z = \#\{i \in \{0, ..., |x| - 1\} \mid x_i = c_i\}$$

Here, #s denotes the cardinality (number of elements) of a set s.

**Counting multiple values.** The count constraint also supports counting multiple values (also known as gcc, or global cardinality constraint). Suppose that x and y (the *counting variables*) are two integer variable arrays (not necessarily of the same size). Then

```
count(home, x, y);
```

posts the constraints that the number of variables in x that are equal to a value j is  $y_j$  (for  $0 \le j < |y|$ ):

$$\#\{i \in \{0, \dots, |x| - 1\} \mid x_i = j\} = y_i \quad \text{for } 0 \le j < |y|$$

and that no other values are taken by x:

$$\bigcup_{i=0}^{|\mathsf{x}|-1} \{\mathsf{x}_i\} = \{0,\dots,|\mathsf{y}|-1\}$$

Rather than using counting variables, one can also use an array of integer sets (IntSetArgs). Then the number of values taken must be included in each individual set.

A more general variant also takes into account that the values under consideration are non contiguous but are defined by an additional array of integer values. Suppose that x and y (the counting variables) are two integer variable arrays (not necessarily of the same size) and c is an array of integers with the same size as y.

Then,

posts the constraints that the number of variables in x that are equal to the value  $c_j$  is  $y_j$  (for  $0 \le j < |y|$ ):

$$\#\{i \in \{0, \dots, |x| - 1\} \mid x_i = c_i\} = y_i \quad \text{for } 0 \le j < |y|$$

and that no other values but those in c are taken by x:

$$\bigcup_{i=0}^{|\mathbf{x}|-1}\{\mathbf{x}_i\} = \bigcup_{i=0}^{|\mathbf{c}|-1}\{\mathbf{c}_i\}$$

Again, y can also be an array of integer sets, where equality = is replaced by set inclusion  $\in$ .

A slightly simpler variant replaces the cardinality variables by a single integer set. That is, for an array of integer variables x, an integer set d, and an array of integer values c

posts the constraints that the number of variables in x that are equal to the value  $c_j$  is included in d (for  $0 \le j < |c|$ ):

$$\#\{i \in \{0, ..., |x| - 1\} \mid x_i = c_i\} \in d$$
 for  $0 \le j < |c|$ 

and that no other values but those in c are taken by x:

$$\bigcup_{i=0}^{|\mathsf{x}|-1} \{\mathsf{x}_i\} = \bigcup_{i=0}^{|\mathsf{c}|-1} \{\mathsf{c}_i\}$$

The last variant of count clarifies that count is a generalization of distinct (see Section 4.4.7): distinct constrains a value to occur at most once, whereas count offers more flexibility to constrain which values and how often these values can occur.

For example, if we know that the variables in the variable array x take values between 0 and n-1, then

```
count(home, x, IntSet(0,1), IntArgs::create(n,0,1));
```

is equivalent to

```
distinct(home, x);
```

Counting constraints only support integer variables, linear constraints can be used for Boolean variables, see Section 4.4.6. For examples, see the case studies in Chapter 11 and Chapter 13 or the examples Crowded chessboard and Magic sequence.

Note that Gecode implements the semantics of the original paper on the global cardinality constraint by Régin [28], where no other values except those specified may occur. This differs from the semantics in the Global Constraint Catalog [5], where values that are not mentioned can occur arbitrarily often.

Gecode offers value (the default), bounds (based on [27]), and domain propagation (based on [28]) for the global count constraint (see also GCCAT: global\_cardinality).

#### 4.4.9 Number of values constraints

Number of values constraints constrain how many values can be taken by an array of variables.

Assume that x is an array of integer variables and y is an integer variable. Then

constrains the number of distinct values in x to be equal to y, that is

$$\#\{x_0,\ldots,x_{|x|-1}\}=y$$

Instead of IRT\_EQ any other integer relation type can be used, see Figure 4.1 for an overview. For example,

```
nvalues(home, x, IRT_LQ, y);
```

constrains the number of distinct values in x to be less than or equal to y. The array x can also be an array of Boolean variables and y can be an integer value.

The constraint is implemented by the propagators introduced in [6] (see also GCCAT: nvalue, nvalues). For an example using the nvalues constraint, see Dominating Queens.

#### 4.4.10 Sequence constraints

Sequence constraints constrain how often values are taken by repeated subsequences of variables in an array of integer or Boolean variables. By

where x is an array of integer or Boolean variables, s is an integer set, and q, l, and u are integers, all subsequences of length q in the variable array x, that is, the sequences

$$\begin{aligned} \langle \mathsf{x}_0, \dots, \mathsf{x}_{\mathsf{q}+0-1} \rangle \\ \langle \mathsf{x}_1, \dots, \mathsf{x}_{\mathsf{q}+1-1} \rangle \\ & \cdots \\ \langle \mathsf{x}_{|\mathsf{x}|-\mathsf{q}}, \dots, \mathsf{x}_{|\mathsf{x}|-1} \rangle \end{aligned}$$

are constrained such that at least 1 and at most u variables in each subsequence are assigned to values from the integer set s.

In more mathematical notation, the constraint enforces

$$\bigwedge_{i=0}^{|\mathsf{x}|-\mathsf{q}} \mathrm{among}(\langle \mathsf{x}_i, \dots, \mathsf{x}_{i+\mathsf{q}-1} \rangle, \mathsf{s}, \mathsf{l}, \mathsf{u})$$

where the among constraint for the subsequence starting at position i is defined as

$$l \le \#\{j \in \{i, \dots, i + q - 1\} \mid x_j \in s\} \le u$$

The constraint is implemented by the domain consistent propagator introduced in [48] (see also GCCAT: among\_seq). For an example, see Car sequencing.

#### 4.4.11 Channel constraints

Channel constraints channel Boolean to integer variables and integer variables to integer variables.

**Channeling integer variables.** For two integer variable arrays x and y of same size,

posts the constraint

$$x_i = j \iff y_i = i \quad \text{for } 0 \le i, j < |x|$$

(see also GCCAT: inverse). The channel constraint between two integer variable arrays also supports integer offsets. For integers n and m,

posts the constraint

$$x_i - n = j \iff y_j - m = i$$
 for  $0 \le i, j < |x|$ 

(see also GCCAT: inverse\_offset). For examples, see n-Knight's tour (simple model) and Black hole patience.

**Channeling between integer and Boolean variables.** As integer and Boolean variables are unrelated (see Section 4.1), the only way to express that a Boolean variable x is equal to an integer variable y is by posting either

```
channel(home, x, y);
```

```
channel(home, y, x);
```

The channel constraint can also map an integer variable y to an array of Boolean variables x. The constraint

$$x_i = 1 \iff y = i \quad \text{for } 0 \le i < |x|$$

is posted by

```
channel(home, x, y);
```

(see also GCCAT: domain\_constraint). Note that an optional offset argument is supported. The constraint

$$x_i = 1 \iff y = i + n$$
 for  $0 \le i < |x|$ 

for an integer value n is posted by

```
channel(home, x, y, n);
```

For an example, see Pentominoes.

#### 4.4.12 Element constraints

Element constraints generalize array access to integer variables. For example,

```
IntArgs c(5, 1,4,9,16,25);
element(home, c, x, y);
```

constrains the integer variable y to be the element of the array c at index x (where the array starts at index 0 as is common in  $C^{++}$ ).

The index variable x is always an integer variable, but the array c can also be an array of integer variables, Boolean variables, or an array of integers between 0 and 1. The result variable y must be a Boolean variable or an integer between 0 and 1 if the array is an array of Boolean variables. It can be a Boolean variable if all integer values in the array are between 0 and 1.

Even if bounds propagation is requested for the element constraint, the propagators for element always perform domain reasoning on the index variable (see also GCCAT: elem, element). For examples, see the case study in Chapter 13 or the examples Steel-mill slab design problem and Travelling salesman problem (TSP).

**Tip 4.9** (Shared integer arrays). When checking the documentation for Element constraints it might come at a surprise that element constraints do not take integer argument arrays of type IntArgs but *shared integer arrays* of type IntSharedArray as argument. The reason is that the very same shared integer array can be used for several element constraints.

Consider the following example

```
IntArgs c(5, 1,4,9,16,25);
element(home, c, x, y);
element(home, c, a, b);
```

where x, y, a, and b are integer variables. Then, each time an element constraint is posted, a new shared integer array is created implicitly (that is, in the example above, two arrays are created). If the integer array is large or many element constraints are posted, it is beneficial to explicitly create a shared integer array, such as in:

```
IntArgs c(5, 1,4,9,16,25);
IntSharedArray cs(c);
element(home, cs, x, y);
element(home, cs, a, b);
```

Here only a single shared arrays is created and is used for both propagators created for posting the element constraints.

What is also obvious from the first example is that integer argument arrays of type IntArgs can automatically be coerced to integer shared arrays of type IntSharedArray. Hence, if performance is not that important, you even do not need to know that shared integer arrays exist.

For an example that uses shared integer arrays together with element constraints, see Chapter 19 and Crossword puzzle.

**Tip 4.10** (Shared arrays also provide STL-style iterators). Shared arrays also support STL-style (Standard Template Library) iterators, similar to other arrays provided by Gecode, see Section 4.2.3. ◀

#### 4.4.13 Extensional constraints

Extensional constraints (also known as user-defined or ad-hoc constraints) provide constraints that are specified in *extension*. The extension can be either defined by a DFA (deterministic finite automaton) or a tuple set TupleSet. DFAs can also be specified conveniently by regular expressions, see Section 6.4.

Note that both DFAs and tuple sets are proper Gecode data structures. When storing a DFA or a tuple set as a space member, the member must be updated with an update function during cloning exactly as described for variables in Section 4.1.8.<sup>2</sup>

**Deterministic finite automata.** Suppose we want to plan the activities of an evening that follows the Swedish drinking protocol: you may have as many drinks as you like, but now and then you sing a song after which you have to have a drink. We want to constrain an array of activities (Boolean or integer variables) such that the activities (drinking and singing) follow the protocol.

<sup>&</sup>lt;sup>2</sup>In most cases, the data structure is actually shared between spaces created by cloning. Only when a non-shared clone is requested (for parallel search), a copy of the data structure is created. This is the reason why updating is required.

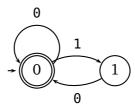


Figure 4.4: A DFA for the Swedish drinking protocol

The DFA in Figure 4.4 specifies legal sequences of activities according to the Swedish drinking protocol, where the state 0 is the start state and also the final state. The symbol 0 corresponds to drinking, whereas 1 corresponds to singing. That is, the sequence of activities must be a string of 0s and 1s accepted by the DFA.

The DFA d is initialized by

The array of transitions t is initialized by triples of integers (of type DFA::Transition). A triple  $\{a, s, b\}$  defines a transition from state a to state b with symbol s. States are denoted by non-negative integers and symbols are integer values (as always, restricted to integer values that can be taken on by an integer variable, see Section 4.1.2). A transition where a is -1 marks the last transition in a transition array. The array of final states f lists all final states of the DFA, where the array is terminated by -1. The first argument of the constructor of the DFA defines the start state.

Constraining an array of variables for four activities to the Swedish drinking protocol is done by

```
BoolVarArray x(home, 4, 0, 1); extensional(home, x, d);
```

Note that the same DFA would also work with an array of integer variables.

The propagator for the extensional constraint is domain consistent and is based on [26].

Examples that use regular expressions for defining DFAs can be found in Section 6.4.

**Tuple sets.** Constraints can also be defined by a list of tuples, where each tuple defines one solution of the extensional constraint. For example, the following defines the Swedish drinking protocol for three activities by a list of tuples:

```
TupleSet t;
t.add(IntArgs(3, 0,0,0));
t.add(IntArgs(3, 0,1,0));
t.add(IntArgs(3, 1,0,0));
t.finalize();
```

Note that before a tuple set can be used by a post function, it must be finalized as shown above. If a not-yet finalized tuple set is used for posting a constraint, an exception of type Int::NotYetFinalized is thrown.

The propagators for the extensional constraint are domain consistent and are based on [7] and [8] (see also GCCAT: in\_relation).

Constraining a sequence of variables to the Swedish drinking protocol using a tuple set is done by

```
BoolVarArray x(home, 3, 0, 1);
extensional(home, x, t);
```

For examples of extensional constraints using tuple sets, see Chapter 18, Black hole patience, and Kakuro.

#### 4.4.14 Sorted constraints

Sorted constraints relate an integer variable array to an array obtained by sorting the array. For example,

```
sorted(home, x, y);
```

constrains y to be x (of same size) sorted in increasing order. The more general variant features an additional integer variable array (again, of same size) z as in

```
sorted(home, x, y, z);
```

where z defines the sorting permutation, that is

$$x_i = y_{z_i}$$
 for  $0 \le i < |x|$ 

The propagator for sorted is bounds consistent and is based on [23] (see also GCCAT: sort, sort\_permutation).

#### 4.4.15 Bin-packing constraints

Bin packing constraints constrain how items can be packed into bins. The bin-packing constraint is posted as

```
binpacking(home, l, b, s);
```

where l is an array of integer variables (the *load* variables), b is an array of integer variables (the *bin* variables), and s is an array of integers (the item *sizes*).

The load variables 1 determine the load  $l_j$  of each bin j ( $0 \le j < |l|$ ) and the bin variables b determine for each item i ( $0 \le i < |b|$ ) into which bin  $b_i$  it is packed. The size of an item i ( $0 \le i < |b|$ ) is defined by its item size  $s_i$ . Naturally, the number of bin variables and item sizes must coincide (|b| = |s|).

The bin-packing constraint enforces that all items are packed into bins

$$b_i \in \{0, ..., |l| - 1\}$$
 for  $0 \le i < |b|$ 

and that the load of each bin corresponds to the items packed into it

$$l_j = \sum_{\{i \in \{0, \dots, |b|-1\} \mid b_i = i\}} s_i \quad \text{for } 0 \le j < |l|$$

The constraint is implemented by the propagator introduced in [41] (see also GCCAT: bin\_packing, bin\_packing\_capa). For an example using the bin-packing constraint and CDBF (complete decreasing best fit) [14] as a specialized branching for bin-packing, see Chapter 17 and Bin packing.

#### 4.4.16 Geometrical packing constraints

Geometrical packing constraints constrain how rectangles can be packed such that no two rectangles from a collection of rectangles overlap.

If x and y are integer variable arrays and w and h are integer arrays (where all arrays must be of the same size), then

```
nooverlap(home, x, w, y, h);
```

propagates that the rectangles defined by coordinates  $\langle x_i, y_i \rangle$ , widths  $w_i$ , and heights  $h_i$  for  $0 \le i < |x|$  do not overlap. That is, the following constraint is enforced (see also Section 6.1.2 for a picture):

$$(x_i + w_i \le x_j) \lor (x_j + w_j \le x_i) \lor (y_i + h_i \le y_j) \lor (y_j + h_j \le y_i)$$

Note that the width or the height of a rectangle can be zero. In this case, the rectangle does not occupy any space. However no other rectangle is allowed to be placed where the zero-sized rectangle is placed.

Rectangles can also be modeled as optional through a Boolean variable  $m_i$  for each rectangle i. If the Boolean variable  $m_i$  is 1 the rectangle is mandatory and considered by the packing constraint, if it is 0, the rectangle is ignored.

With an array of Boolean variables m the constraint taking optional rectangles into account is posted by

```
nooverlap(home, x, w, y, h, m);
```

The arrays defining the dimensions (that is, w and h for rectangles) can also be arrays of integer variables, where its values are constrained to be non-negative. In this case, the constraint post functions expects both a start and end coordinate. That is, by posting

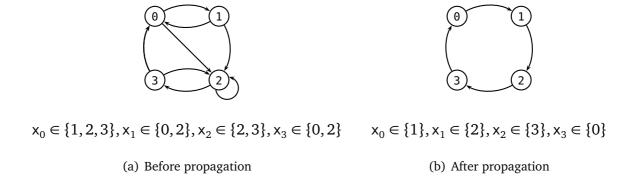


Figure 4.5: Representing edges and propagating circuit

it is enforced that the rectangles defined by the start coordinate  $\langle x0_i, y0_i \rangle$ , the dimension  $\langle w_i, h_i \rangle$ , and the end coordinate  $\langle x1_i, y1_i \rangle$  do not overlap. The end coordinates are *not* constrained to be the sum of the start coordinates and dimensions. That is, one has to explicitly post the linear constraints such that

$$(x0_i + w_i = x1_i) \land (y0_i + h_i = y1_i)$$

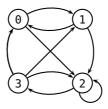
The constraints are implemented by a naive propagator (considering pairwise nooverlap between rectangles including constructive disjunction, see also GCCAT: diffn), this will change in the future. For an example using the no-overlap constraint, see Packing squares into a rectangle.

#### 4.4.17 Circuit and Hamiltonian path constraints

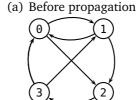
The circuit and path constraints (see Graph constraints) use values of variables in an integer variable array x as edges: if  $j \in x_i$ , the corresponding graph contains the edge  $i \to j$  for  $0 \le i, j < |x|$ . Obviously, the graph has the nodes 0 to |x| - 1.

**Circuit constraints.** Assume that x is an integer variable array (circuit does not support Boolean variables). Then,

constrains the values of x such that their corresponding edges form a Hamiltonian circuit (see also GCCAT: circuit). For an example before and after propagation of circuit see Figure 4.5. For an example, see Chapter 16 and n-Knights tour (model using circuit).



$$\begin{aligned} \mathbf{x}_0 \in \{1,2,3\}, \mathbf{x}_1 \in \{0,2\}, \mathbf{x}_2 \in \{2,3\}, \mathbf{x}_3 \in \{0,1,2\} \\ \mathbf{y}_i \in \{-100,\dots,100\} \quad (0 \leq i < 4) \\ \mathbf{z} \in \{-100,\dots,100\}, \end{aligned}$$



$$\begin{aligned} \mathbf{x}_0 &\in \{1,2\}, \mathbf{x}_1 \in \{0,2\}, \mathbf{x}_2 \in \{3\}, \mathbf{x}_3 \in \{0,1\} \\ \mathbf{y}_0 &\in \{3,5\}, \mathbf{y}_1 \in \{4,9\}, \mathbf{y}_2 \in \{5\}, \mathbf{y}_3 \in \{-7,8\} \\ \mathbf{z} &\in \{5,\dots,27\}, \\ \text{(b) After propagation} \end{aligned}$$

Figure 4.6: Representing edges and propagating circuit with cost

Common applications of circuit also require costs for the edges in the graph. Assume that the cost for an edge  $i \rightarrow j$  from node i to node j is defined by the following matrix:

$i \rightarrow j$	$\cdot \rightarrow 0$	$\cdot \rightarrow 1$	$\cdot \rightarrow 2$	$\cdot \rightarrow 3$
$0 \rightarrow \cdot$	0	3	5	7
$1 \rightarrow \cdot$	4	0	9	6
$2 \rightarrow \cdot$	2	1	0	5
3 → ·	-7	8	-2	0

Then, by

the integer variables x are constrained to the values forming the circuit as above, while the integer variables y define the cost of the edge for each node (these variables can be omitted), and the integer variable z defines the total cost of the edges in the circuit. Note that the matrix interface as described in Section 6.2 might come in handy for setting up the cost matrix.

Figure 4.6 shows a simple example for propagation circuit with cost where the cost matrix from above is used. For an example, see Travelling salesman problem (TSP).

**Hamiltonian path constraints.** The path constraint (see Graph constraints) is similar to the circuit constraint and enforces that nodes in a graph from a Hamiltonian path. Assume that x is an integer variable array (path does not support Boolean variables) and s (for start) and e (for end) are integer variables. Then,

```
path(home, x, s, e);
```

constrains the values of x, s, and e such that their corresponding edges form a Hamiltonian path that starts at node  $x_s$  and ends at node  $x_e$  (the value of the variable  $x_e$  is always |x|).

As an example, assume that the integer variable array x has three elements (that is, |x| = 3) with values between 0 and 3. Then all solutions to

```
path(home, x, s, e);
```

are as follows:

Х	S	е
$\langle 1, 2, 3 \rangle$	0	2
$\langle 1, 3, 0 \rangle$	2	1
⟨2,0,3⟩	1	2
$\langle 2, 3, 1 \rangle$	0	1
$\langle 3, 0, 1 \rangle$	2	0
$\langle 3, 2, 0 \rangle$	1	0

The path constraint provides, similar to circuit, variants for costs for edges in a Hamiltonian path, see Graph constraints.

#### 4.4.18 Scheduling constraints

This section provides an overview of scheduling constraints.

**Important.** The support for scheduling constraints is still experimental. It is not yet complete, as it still lacks modeling abstractions and specialized branchings. The existing code is tested and efficient, but the interfaces may change in future versions.

#### **Unary resource constraints**

A unary resource constraint models that a number of tasks to be executed on a single resource do not overlap, where each task is defined by its start time (an integer variable), its duration (an integer or integer variable), and possibly its end time (if the duration is a variable). Unary resource constraints are also known as disjunctive scheduling constraints.

For example, assume that four tasks with durations 2, 7, 4, and 11 are to be executed on the same resource where the start times are specified by an array of integer variables (of course, with four variables). Then, posting

```
IntArgs d(4, 2,7,4,11);
unary(home, s, d);
```

constrains the start times in s such that the execution of none of the tasks overlaps in time (see Scheduling constraints).

All propagators implementing unary use overload-checking, detectable precedences, not-first-not-last, and edge-finding, following [51] (see also GCCAT: disjunctive).

A common variant for unary resource constraints is where tasks can be optional: each task t has a Boolean variable b attached to it. If b=1 then the task is *mandatory* and is scheduled on the resource. If b=0 then the task is *excluded* and is not scheduled. Otherwise, the task is said to be *optional*. Assume that b refers to an array of Boolean variables also of size 4, then

```
IntArgs d(4, 2,7,4,11);
unary(home, s, d, b);
```

posts a propagator that constrains the start times s (of course, only if a task is mandatory) as well the Boolean variables in b (if a task becomes excluded as otherwise no feasible schedule would exist).

**Tasks with flexible duration.** The duration of a task can also be given as an integer variable instead of a constant integer. In this case, we say that the tasks are *flexible*. In addition to the flexible duration, the unary constraint also requires variables for the end time of each task.

Given variable arrays s, d, and e for the start times, durations, and end times, a unary resource constraint is posted as

```
unary(home, s, d, e);
```

However, the additional constraint for each task i that s[i] + d[i] = e[i] is not enforced automatically. Therefore, a model typically must contain additional constraints like

```
for (int i=0; i<s.size(); i++)
  post(home, s[i]+d[i]==e[i]);</pre>
```

The unary post function also exists in a variant with flexible, optional tasks.

#### **Cumulative scheduling constraints**

Gecode provides two generalizations of the unary resource scheduling constraint. The first one, called cumulative, models a resource where tasks can overlap. The resource has a limited *capacity*, and each task requires a certain *resource usage*. At each point in time, the sum of the resource usages of all tasks running at that point must not exceed the capacity. The second generalization, called cumulatives, deals with several cumulative resources at once.

Cumulative single-resource constraint. The single-resource constraint cumulative has nearly the same interface as unary. The only difference is a parameter c specifying the resource capacity, and an additional integer array u for the resource usage of each task. Assuming that s and d give the start times and durations as before, the following models a resource where two tasks can overlap, and the first three tasks require one unit of the resource, while the last task requires two:

```
IntArgs u(4, 1,1,1,2);
cumulative(home, 2, s, d, u);
```

The capacity can be an integer variable or a nonnegative integer. The resource usage must be stricly positive. As for unary, there exist versions with optional, flexible, and flexible and optional tasks.

The propagators implementing the cumulative constraint use time-tabling, overload checking [50], and edge-finding [49] (see also GCCAT: cumulative).

**Cumulative multi-resource constraint.** Given a *set* of resources that have some specified usage limit and a set of tasks that must be placed on these resources according to start-times, durations, resource usage, and resource compatibility, the cumulatives constraint (see Scheduling constraints) can be used for the placement of these tasks. The limit for the resources can be either a maximum or a minimum, and the resource usage of a task can be positive, negative, or zero. The limit is only valid over the intervals where there is at least one task assigned on that particular resource.

Consider the following code.

This code posts a constraint over a set of tasks T, where each task  $T_i$  is defined by  $\langle \mathsf{resource}_i, \mathsf{start}_i, \mathsf{duration}_i, \mathsf{end}_i, \mathsf{height}_i \rangle$ . The resource component indicates the potential resources that the task can use;  $\mathsf{start}$ ,  $\mathsf{duration}$ , and  $\mathsf{end}$  indicate when the task can occur; and finally the height component indicates the amount of resource the task uses (or "provides" in the case of a negative value). The resource  $R_i$  is defined by the limit  $\mathsf{limit}_i$  and the parameter  $\mathsf{atmost}$ . The latter is common for all resources, and indicates whether the limits are maximum limits ( $\mathsf{atmost}$  is true) or minimum limits ( $\mathsf{atmost}$  is false).

As for flexible tasks in Section 4.4.18, the cumulatives constraint does not enforce that  $start_i + duration_i = end_i$ . This additional constraint must be posted manually.

The parameters start and end are always integer variable arrays; resource, duration, and height can be either integer variable arrays or integer arrays; and limit is always an array of integers.

For an example using cumulatives see Packing squares into a rectangle, where the cumulatives constraints is used to model packing a set of squares. For an insightful discussion of how to use cumulatives for modeling, see [4] (the propagator for cumulative is implemented following this paper, see also GCCAT: cumulatives).

#### 4.4.19 Value precedence constraints

Value precedence constraints over integer variables enforce that a value precedes another value in an array of integer variables. By

```
precede(home, x, s, t);
```

where x is an array of integer variables and both s and t are integers, the following is enforced: if there exists j ( $0 \le j < |x|$ ) such that  $x_j = t$ , then there must exist i with i < j such that  $x_i = s$ . This is equivalent to:

- 1.  $x_0 \neq t$ , and
- 2. if  $x_j = t$  then  $\bigvee_{i=0}^{j-1} x_i = s$  for  $1 \le j < |x|$ .

A generalization is available for precedences between several integer values. By

```
precede(home, x, c);
```

where x is an array of integer variables and c is an array of integers, it is enforced that  $c_i$  precedes  $c_{i+1}$  in x for  $0 \le i < |c| - 1$ . That is

- 1.  $x_0 \neq c_{k+1}$  for  $0 \leq k < |c| 1$ , and
- 2. if  $x_j = c_{k+1}$  then  $\bigvee_{i=0}^{j-1} x_i = c_k$  for  $1 \le j < |x|$  and  $0 \le k < |c| 1$ .

The constraint is implemented by the domain consistent propagator introduced in [19] (see also GCCAT: int\_value\_precede, int\_value\_precede\_chain), the paper also explains how to use the precede constraint for breaking value symmetries. For an example, see Schur's lemma.

#### 4.5 Synchronized execution

Gecode offers support in Synchronized execution for executing a function (either a function or a static member function but not a member function) when integer or Boolean variables become assigned.

Assume that a function c (for continuation) is defined by

```
void c(Space& home) { ··· }
```

Then

```
wait(home, x, &c);
```

posts a propagator that waits until the integer or Boolean variable x (or, if x is an array of variables: all variables in x) is assigned. If x becomes assigned, the function c is executed with the current home space passed as argument.

**Tip 4.11** (Failing a space). If you want to fail a space home (for example when executing a continuation function as discussed above), you can do that by

```
home.fail();
```

Assume that two functions t (for then) and e (for else) are defined by

```
void t(Space& home) { ··· }
void e(Space& home) { ··· }
```

Then

```
when(home, x, &t, &e);
```

creates a propagator that will be run exactly once when the Boolean variable x becomes assigned. If x becomes assigned to 1, the function t is executed. If x becomes assigned to 0, the function t is executed. Both functions get the current home space of the propagator passed as argument. The else-function is optional and can be omitted.

5

## Set variables and constraints

This chapter gives an overview over set variables and set constraints in Gecode. Just like Chapter 4 does for integer and Boolean variables, this chapter serves as a starting point for using set variables. For the reference documentation, see Using finite integer sets.

**Overview.** Section 5.1 details how set variables can be used for modeling. The sections Section 5.2 and Section 5.3 provide an overview of the constraints that are available for set variables in Gecode.

**Important.** Do not forget to add

```
#include <gecode/set.hh>
```

to your program when you want to use set variables. Note that the same conventions hold as in Chapter 4.

#### 5.1 Set variables

Set variables in Gecode model sets of integers and are instances of the class SetVar.

**Tip 5.1** (Still do not use views for modeling). Just as for integer variables, you should not feel tempted to use views of set variables (such as SetView) for modeling. Views can only be used for implementing propagators and branchers, see Part P and Part B. ◀

**Representing set domains as intervals.** The domain of a set variable is a set of sets of integers (in contrast to a simple set of integers for an integer variable). For example, assume that the domain of the set variable x is the set of subsets of  $\{1, 2, 3\}$ :

Set variable domains can become very large – the set of subsets of  $\{1, ..., n\}$  has  $2^n$  elements. Gecode (like most constraint solvers) therefore approximates set variable domains by a set interval [l .. u] of a lower bound l and an upper bound u. The interval [l .. u] denotes the set of sets  $\{s \mid l \subseteq s \subseteq u\}$ . The lower bound l (commonly referred to as greatest lower bound or glb) contains all elements that are known to be included in the set, whereas the

upper bound u (commonly referred to as least upper bound or lub) contains the elements that may be included in the set. As only the two interval bounds are stored, this representation is space-efficient. The domain of x from the above example can be represented as  $[\{\}, \{1,2,3\}]$ .

Set intervals can only approximate set variable domains. For example, the domain

```
{ {1}, {2}, {3}, {1,2}, {1,3}, {2,3} }
```

cannot be captured exactly by an interval. The closest interval would be  $[\{\} ... \{1,2,3\}]$ . In order to get a closer approximation of set variable domains, Gecode additionally stores cardinality bounds. We write #[i..j] to express that the cardinality is at least i and at most j. The set interval bounds  $[\{\} ... \{1,2,3\}]$  together with cardinality bounds #[1...2] represent the above example domain exactly.

**Creating a set variable.** New set variables are created using a constructor. A new set variable x is created by

```
SetVar x(home, IntSet::empty, IntSet(1, 3), 1, 2);
```

This declares a variable x of type SetVar in the space home, creates a new set variable implementation with domain  $[\{\} ... \{1,2,3\}]$ , # [1...2], and makes x refer to the newly created set variable implementation.

There are several overloaded versions of the constructor, you can for example omit the cardinality bounds if you do not want to restrict the cardinality. You find the full interface in the reference documentation of the class SetVar. An attempt to create a set variable with an empty domain throws an exception of type Set::VariableEmptyDomain.

As for integer and Boolean variables, the default and copy constructors do not create new variable implementations. Instead, the variable does not refer to any variable implementation (default constructor) or to the same variable implementation (copy constructor). For example in

```
SetVar x(home, IntSet::empty, IntSet(1, 3), 1, 2);
SetVar y(x);
SetVar z;
z=y;
```

the variables x, y, and z all refer to the same set variable implementation.

**Limits for set elements.** All set variable bounds are subsets of the *universe*, defined as

```
[Set :: Limits :: min .. Set :: Limits :: max]
```

The universe is symmetric: -Set::Limits::min = Set::Limits::max. Furthermore, the cardinality of a set is limited to the unsigned integer interval

```
#[0.. Set :: Limits :: card]
```

The limits have been chosen such that an integer variable can hold the cardinality. This means that the maximal element of a set variable is Int :: Limits :: max/2 - 1. The limits are defined in the namespace Set::Limits.

Any attempt to create a set variable with values outside the defined limits throws an exception of type Set::OutOfLimits.

**Tip 5.2** (Small variable domains are still beautiful). Just like integer variables (see Tip 4.3), set variables do not have a constructor that creates a variable with the largest possible domain. And again, one has to worry and the omission is deliberate to make you worry. So think about the initial domains carefully when modeling. ◀

**Variable access functions.** You can access the current domain of a set variable x using member functions such as x.cardMax(), returning the upper bound of the cardinality, or x.glbMin(), returning the smallest element of the lower bound. Furthermore, you can print a set variable's domain using the standard output operator <<.

**Iterating variable domain interval bounds.** For access to the interval bounds of a set variable, Gecode provides three value iterators and corresponding range iterators. For example, the following loop

```
for (SetVarGlbValues i(x); i(); ++i)
  std::cout << i.val() << ' ';</pre>
```

uses the value iterator i to print all values of the greatest lower bound of the domain of x in increasing order. If x is assigned, this of course corresponds to the value of x. Similarly, the following loop

```
for (SetVarLubRanges i(x); i(); ++i)
  std::cout << i.min() << ".." << i.max() << ' ';</pre>
```

uses the range iterator i to print all ranges of the least upper bound of the domain of x. The third kind of iterator, SetVarUnknownValues or SetVarUnknownRanges, iterate the values resp. ranges that are still unknown to be part or not part of the set, that is  $u \setminus l$  for the domain  $[l \dots u]$ .

When to inspect a variable. The same restrictions hold as for integer variables (see Section 4.1.7). The important restriction is that one must not change the domain of a variable (for example, by posting a constraint on that variable) while an iterator for that variable is being used.

**Updating variables.** Set variables behave exactly like integer variables during cloning of a space. A set variable is updated by

```
x.update(home, share, y);
```

where share is the Boolean argument passed as argument to copy() and y is the variable from which x is to be updated. While home is the space x belongs to, y belongs to the space which is being cloned.

Variable and argument arrays. Set variable arrays can be allocated using the class SetVarArray. The constructors of this class take the same arguments as the set variable constructors, preceded by the size of the array. For example,

```
SetVarArray x(home, 4, IntSet::empty, IntSet(1, 3));
```

creates an array of four set variables, each with domain  $[\{\} .. \{1, 2, 3\}]$ .

To pass temporary data structures as arguments, you can use the SetVarArgs class (see Argument arrays). Some set constraints are defined in terms of arrays of sets of integers. These can be passed using IntSetArgs (see Argument arrays). Set variable argument arrays support the same operations introduced in Section 4.2.2.

#### 5.2 Constraint overview

This section introduces the different groups of constraints over set variables available in Gecode. The section serves only as an overview. For the details and the full list of available post functions, the section refers to the relevant reference documentation.

**Reified constraints.** Several set constraints also exist as a reified variant. Whether a reified version exists for a given constraint can be found in the reference documentation. If a reified version does exist, the Boolean control variable is passed as the last non-optional argument.

**Tip 5.3** (Reification by decomposition). If your model requires reification of a constraint for which no reified version exists in the library, you can often *decompose* the reification. For example, to reify the constraint  $x \cup y = z$  to a control variable b, you can introduce an auxiliary variable z0 and post the two constraints

```
rel(home, x, SOT_UNION, y, SRT_EQ, z0);
rel(home, z0, SRT_EQ, z, b);
```

#### 5.2.1 Domain constraints

Domain constraints restrict the domain of a set variable using a set constant (given as a single integer, an interval of two integers or an IntSet), depending on set relation types of type SetRelType (see Using finite integer sets). Figure 5.1 lists the available set relation types and their meaning. The relations SRT\_LQ, SRT\_LE, SRT\_GQ, and SRT\_GR establish a total order based on the lexicographic order of the characteristic functions of the two sets.

$SRT_{-}EQ$	equality (=)	$SRT_{-}NQ$	disequality (≠)
$SRT_{LQ}$	lex. less than or equal	SRT_LE	lex. less than
$SRT_{GQ}$	lex. greater than or equal	$SRT_{-}GR$	lex. greater than
SRT_SUB	subset (⊆)	SRT_SUP	superset (⊇)
SRT_DISJ	disjointness (  )	$SRT_{-}CMPL$	complement ( \cdot )

Figure 5.1: Set relation types

For example, the constraints

```
dom(home, x, SRT_SUB, 1, 10);
dom(home, x, SRT_SUP, 1, 3);
dom(home, y, SRT_DISJ, IntSet(4, 6));
```

result in the set variable x being a subset of  $\{1, ..., 10\}$  and a superset of  $\{1, 2, 3\}$ , while 4, 5, and 6 are not elements of the set y. The domain constraints for set variables support reification.

In addition to the above constraints,

```
cardinality(home, x, 3, 5);
```

restricts the cardinality of the set variable x to be between 3 and 5.

For examples using domain constraints, see Airline crew allocation, as well as the redundant constraints in Golf tournament.

#### 5.2.2 Relation constraints

Relation constraints enforce relations between set variables and between set and integer variables, depending on the set relation types introduced above.

For set variables x and y, the following constrains x to be a subset of y:

```
rel(home, x, SRT_SUB, y);

If x is a set variable and y is an integer variable, then

rel(home, x, SRT_SUP, y);
```

constrains x to be a superset of the singleton set  $\{y\}$ , which means that y must be an element of x

The last form of set relation constraint uses an integer relation type (see Figure 4.1) instead of a set relation type. This constraint restricts *all* elements of a set variable to be in the given relation to the value of an integer variable. For example,

```
rel(home, x, IRT_GR, y);
```

constrains all elements of the set variable x to be strictly greater than the value of the integer variable y (see also GCCAT: eq\_set, in, in\_set, not\_in).

Gecode provides reified versions of all set relation constraints. For an example, see Golf tournament and Chapter 15.

```
SOT_UNION union (\cup) SOT_INTER intersection (\cap) SOT_DUNION disjoint union (\uplus) SOT_MINUS set minus (\setminus)
```

Figure 5.2: Set operation types

#### 5.2.3 Set operations

Set operation/relation constraints perform set operations according to the type shown in Figure 5.2 and relate the result to a set variable. For example,

```
rel(home, x, SOT_UNION, y, SRT_EQ, z);
```

enforces the relation  $x \cup y = z$  for set variables x, y, and z. For an array of set variables x,

enforces the relation

$$\bigcup_{i=0}^{|\mathsf{x}|-1} \mathsf{x}_i = \mathsf{y}$$

Instead of set variables, the relation constraints also accept IntSet arguments as set constants. There are no reified versions of the set operation constraints (you can decompose using reified relation constraints on the result, see Tip 5.3).

Set operation constraints are used in most examples that contain set variables, such as Airline crew allocation or Generating Hamming codes.

#### 5.2.4 Element constraints

Element constraints generalize array access to set variables. The simplest version of element for set variables is stated as

```
element(home, x, y, z);
```

for an array of set variables or constants x, an integer variable y, and a set variable z. It constrains z to be the element of array x at index y (where the index starts at  $\theta$ ).

A further generalization uses a *set variable* as the index, thus selecting several sets at once. The result variable is constrained to be the union, disjoint union, or intersection of the selected set variables, depending on the set operation type argument. For example,

```
element(home, SOT_UNION, x, y, z);
```

for set variables y and z and an array of set variables x enforces the following relation:

$$z = \bigcup_{i \in y} x_i$$

Note that generalized element constraints follow the usual semantics of set operations if the index variable is the empty set: an empty union is the empty set, whereas an empty intersection is the full universe. Because of this semantics, the element constraint has an optional set constant argument so that you can specify the universe (i.e., usually the full set of elements your problem deals with) explicitly. For an example of a set element constraint, see Golf tournament and Chapter 15.

#### 5.2.5 Constraints connecting set and integer variables

Most models that involve set variables also involve integer variables. In addition to the set relation constraints that accept integer variables (interpreting them as singleton sets), Connection constraints to integer variables provide the necessary interface for models that use both set variables and integer or Boolean variables.

The most obvious constraint connecting integer and set variables is the cardinality constraint:

```
cardinality(home, x, y);
```

It states that the integer variable y is equal to the cardinality of the set variable x.

Gecode provides constraints for the minimal and maximal elements of a set. The following code

```
min(home, x, y);
```

constrains the integer variable y to be the minimum of the set x.

For an example of constraints connecting integer and set variables, see Steiner triples.

**Weighted sets.** The weights constraint assigns a weight to each possible element of a set variable x, and then constrains an integer variable y to be the sum of the weights of the elements of x. The mapping is given using two integer arrays, e and w. For example,

```
IntArgs e(6,    1, 3, 4, 5, 7, 9);
IntArgs w(6, -1, 4, 1, 1, 3, 3);
weights(home, e, w, x, y);
```

enforces that x is a subset of  $\{1,3,4,5,7,9\}$  (the set of elements), and that y is the sum of the weights of the elements in x, where the weight of the element 1 would be -1, the weight of 3 would be 4 and so on. Assigning x to the set  $\{3,7,9\}$  would therefore result in y being assigned to 4+3+3=10 (see also GCCAT: sum\_set).

**Channeling.** Finally, there are channeling constraints between set variables and integer and Boolean variables. For an array of integer variables x and an array of set variables y,

```
channel(home, x, y);
```

posts the constraint

$$x_i = j \iff i \in y_j$$
 for  $0 \le i, j < |x|$ 

The channel between a set variable y and an array of Boolean variables x,

```
channel(home, x, y);
```

enforces the constraint

$$x_i = 1 \iff i \in y$$
 for  $0 \le i < |x|$ 

An array of integer variables x can be channeled to a set variable y using

```
rel(home, SOT_UNION, x, y);
```

which constrains y to be the set  $\{x_0, \dots, x_{|x|-1}\}$ . An alias for this constraint is defined in the modeling convenience library, see Section 6.5 and Section 6.7.

A specialized version of the previous constraint is

```
channelSorted(home, x, y);
```

which constrains y to be the set  $\{x_0, \dots, x_{|x|-1}\}$ , and the integer variables in x are sorted in increasing order  $(x_i < x_{i+1} \text{ for } 0 \le i < |x|)$  (see also GCCAT: link\_set\_to\_booleans).

#### 5.2.6 Convexity constraints

Convexity constraints enforce that set variables are convex, which means that the elements form an integer interval. For example, the set  $\{1, 2, 3, 4, 5\}$  is convex, while  $\{1, 3, 4, 5\}$  is not, as it contains a hole. The *convex hull* of a set s is the smallest convex set containing s ( $\{1, 2, 3, 4, 5\}$ ) is the convex hull of  $\{1, 3, 4, 5\}$ ).

The constraint

```
convex(home, x);
```

states that the set variable x must be convex, and

```
convex(home, x, y);
```

enforces that the set variable y is the convex hull of the set variable x.

#### **5.2.7** Sequence constraints

Sequence constraints enforce an order among an array of set variables x. Posting the constraint

```
sequence(home, x);
```

results in the sets x being pairwise disjoint, and furthermore  $\max(x_i) < \min(x_{i+1})$  for all  $0 \le i < |x| - 1$ . Posting

```
sequence(home, x, y);
```

additionally constrains the set variable y to be the union of the x.

For an example of sequence constraints, see Steiner triples.

#### 5.2.8 Value precedence constraints

Value precedence constraints over set variables enforce that a value precedes another value in an array of set variables. By

```
precede(home, x, s, t);
```

where x is an array of set variables and both s and t are integers, the following is enforced: if there exists j ( $0 \le j < |x|$ ) such that  $s \notin x_j$  and  $t \in x_j$ , then there must exist i with i < j such that  $s \in x_i$  and  $t \notin x_j$ .

A generalization is available for precedences between several integer values. By

```
precede(home, x, c);
```

where x is an array of set variables and c is an array of integers, it is enforced that  $c_k$  precedes  $c_{k+1}$  in x for  $0 \le k < |c| - 1$ .

The constraint is implemented by the propagator introduced in [19] (see also GCCAT: set\_value\_precede), the paper also explains how to use the precede constraint for breaking value symmetries. For an example, see Golf tournament and Chapter 15.

#### 5.3 Synchronized execution

Gecode offers support in Synchronized execution for executing a function (either a function or a static member function but not a member function) when set variables become assigned. Assume that a function c (for continuation) is defined by

```
void c(Space& home) { ··· }
```

Then

```
wait(home, x, &c);
```

posts a propagator that waits until the set variable x (or, if x is an array of variables: all variables in x) is assigned. If x becomes assigned, the function c is executed with the current home space passed as argument.

# 6

### Modeling convenience: MiniModel

This chapter provides an overview of modeling convenience implemented by MiniModel. MiniModel (see Direct modeling support) provides some little helpers to the constraint modeler. However, it does not offer any new constraints or branchers.

**Overview.** Section 6.1 surveys how constraints represented by integer, Boolean, and set expressions and relations can be posted. How matrix interfaces for arrays can be defined and used is discussed in Section 6.2. Support for defining cost functions for cost-based optimization is presented in Section 6.3. Regular expressions for expressing extensional constraints are discussed in Section 6.4. Section 6.5 surveys channeling functions, whereas Section 6.6 and Section 6.7 discuss aliases for some commonly used constraints.

**Important.** Do not forget to add

```
#include <gecode/minimodel.hh>
```

to your program when you want to use MiniModel. Note that the same conventions hold as in Chapter 4.

#### 6.1 Expressions and relations

The main part of MiniModel consists of overloaded operators and functions that provide a more natural syntax for posting constraints. These operators can be used in two slightly different ways. You can post a relation, or create a new variable from an expression.

For example, the following code creates a fresh integer variable z that is costrained to be equal to the given *expression* 3\*x-4\*y+2, where both x and y are integer variables:

```
IntVar z=expr(home, 3*x-4*y+2);
```

An important aspect of posting an expression is that the returned variable is initialized with a reasonably small variable domain, see Tip 4.3.

A *relation* can be posted using the rel function, which posts the corresponding constraints and consequently returns **void**. Assume that z is an integer variable, then

```
rel(home, z == 3*x-4*y+2);
```

posts the same constraint as in the previous example.

MiniModel provides syntax for expressions and relations over integer, Boolean, and set variables, which can be freely mixed. For example, the following code snippet returns a Boolean variable that is true if and only if  $\{x\} \subseteq s$  and |s| = y, where x and y are integer variables, and s is a set variable:

```
BoolVar b = expr(home, (singleton(x) \leq s) && (cardinality(s) == y));
```

The rest of this section presents the different ways to construct expressions and relations, grouped by the type of the expressions.

#### **6.1.1** Integer expressions and relations

Integer expressions (i.e., expressions that evaluate to an integer) are constructed according to the structure sketched in Figure 6.1. We use the standard C++ operators (for an example, see Section 3.1), as well as several functions with intuitive names such as min or max. Integer expressions and relations can be constructed over integer, Boolean, and set variables. In Gecode, integer expressions are of type LinExpr, which are constructed using Linear expressions and relations, Arithmetic functions, and some Set expressions and relations.

Even arrays of variables (possibly with integer argument arrays as coefficients) can be used for posting some expressions and relations. For example, if x and y are integer variables and z is an array of integer variables, then

```
rel(home, x+2*sum(z) < 4*y);
```

posts a single linear constraint that involves all variables from the array z.

As long as an expression is *linear* (i.e., it can be represented as  $\sum a_i \cdot x_i$  where the  $a_i$  are integers and  $x_i$  are integer or Boolean variables), the constraint posted for the expression will be as few linear constraints as possible (see Section 4.4.6) to ensure maximal constraint propagation.<sup>1</sup>

*Non-linear* expressions, such as a multiplication of two variables, are handled by Mini-Model using *decomposition*. For example, posting the constraint

```
rel(home, a+b*(c+d) == 0);
```

for integer variables a, b, c, and d is equivalent to the decomposition

```
IntVar tmp0 = expr(home, c+d);
IntVar tmp1 = expr(home, b*tmp0);
rel(home, a+tmp1 == 0);
```

<sup>&</sup>lt;sup>1</sup>In case a linear expression has only integer variables or only Boolean variables, a single linear constraint is posted. If the expression contains both integer and Boolean variables, two linear constraints are posted.

```
\langle IntExpr \rangle
                        \langle n \rangle
                                                                        integer value
                                                                        integer or Boolean variable
                          \langle x \rangle
                          -⟨IntExpr⟩
                                                                         unary minus
                          \langle IntExpr \rangle + \langle IntExpr \rangle
                                                                         addition
                          \langle IntExpr \rangle - \langle IntExpr \rangle
                                                                        subtraction
                                                                        multiplication
                          \langle IntExpr \rangle * \langle IntExpr \rangle
                          ⟨IntExpr⟩ / ⟨IntExpr⟩
                                                                        integer division
                                                                        modulo
                          \langle IntExpr \rangle \% \langle IntExpr \rangle
                          sum(\langle \overline{x} \rangle)
                                                                        sum of integer or Boolean variables
                          sum(\langle \overline{n} \rangle, \langle \overline{x} \rangle)
                                                                         sum of integer or Boolean variables
                                                                        with integer coefficients
                          min(\langle IntExpr \rangle, \langle IntExpr \rangle)
                                                                         minimum
                          \min(\langle \overline{x} \rangle)
                                                                         minimum of integer variables
                          max(\langle IntExpr \rangle, \langle IntExpr \rangle)
                                                                        maximum
                          \max(\langle \overline{x} \rangle)
                                                                         maximum of integer variables
                                                                         absolute value
                          abs(\langle IntExpr \rangle)
                          sqr(\langle IntExpr \rangle)
                                                                         square
                          sqrt(\langle IntExpr\rangle)
                                                                        square root
                          element(\langle \overline{x} \rangle, \langle IntExpr \rangle)
                                                                         array element of integer variables
                          element(\langle \overline{n} \rangle, \langle IntExpr \rangle)
                                                                         array element of integers
                          min(\langle SetExpr \rangle)
                                                                         minimum of a set expression
                          max(\langle SetExpr \rangle)
                                                                         maximum of a set expression
                          cardinality(\langle SetExpr \rangle)
                                                                         cardinality of a set expression
  (IntRel)
                 ::=
                          \langle IntExpr \rangle \langle r \rangle \langle IntExpr \rangle
                                                                        relation
         \langle r \rangle
                ::= == | != | < | <= | > | >= relation symbol
        \langle \overline{x} \rangle ::= array of integer or Boolean variables
         \langle \overline{n} \rangle ::=
                          array of integers
```

Figure 6.1: Integer expressions and relations

```
\langle BoolExpr \rangle
                                                                        Boolean variable
                    ::=
                             \langle x \rangle
                             !\langle BoolExpr\rangle
                                                                        negation
                             ⟨BoolExpr⟩ && ⟨BoolExpr⟩
                                                                       conjunction
                             \langle BoolExpr \rangle \mid | \langle BoolExpr \rangle
                                                                       disjunction
                             \langle BoolExpr \rangle == \langle BoolExpr \rangle
                                                                       equivalence
                             \langle BoolExpr \rangle != \langle BoolExpr \rangle
                                                                       non-equivalence
                             \langle BoolExpr \rangle >> \langle BoolExpr \rangle
                                                                       implication
                                                                       reverse implication
                             \langle BoolExpr \rangle << \langle BoolExpr \rangle
                             \langle BoolExpr \rangle ^{\wedge} \langle BoolExpr \rangle
                                                                        exclusive or
                             element(\langle \overline{x} \rangle, \langle IntExpr \rangle)
                                                                        array element of Boolean variables
                                                                        reified integer relation
                             ⟨IntRel⟩
                             (SetRel)
                                                                        reified set relation
```

Figure 6.2: Boolean expressions

Like the post functions for integer and Boolean constraints presented in Section 4.4, posting integer expressions and relations supports an optional argument of type IntConLevel to select the consistency level. For more information, see Posting of expressions and relations and Section 4.3.

Using the expr function, you can enforce a particular decomposition, and you can specify the consistency level for each subexpression. For example,

```
rel(home, x+expr(home,y*z,ICL_DOM) == 0);
```

will propagate domain consistency for the multiplication, but bounds consistency (the default) for the sum.

An element expression such as element(x,e), where x is an array of integers or integer variables, and e is an integer expression, corresponds to an array access x[e], implemented using an element constraint (see Section 4.4.12).

MiniModel provides three integer expressions whose arguments are set expressions: the minimum of a set, the maximum of a set, and a set's cardinality. We will see later how set expressions are constructed.

For examples of integer expressions, see Alpha puzzle, SEND+MORE=MONEY puzzle, Grocery puzzle, Chapter 10, Chapter 13, and Section 3.1.

#### 6.1.2 Boolean expressions and relations

Boolean expressions are constructed using standard C++ operators according to the structure sketched in Figure 6.2.

Again, the purpose of a Boolean expression or relation is to post a corresponding constraint for it (see Posting of expressions and relations). Posting a Boolean expression returns a new Boolean variable that is constrained to the value of the expression. Several constraints might be posted for a single expression, however as few constraints as possible are posted.

For example, all negation constraints are eliminated by rewriting the Boolean expression into NNF (negation normal form) and conjunction and disjunction constraints are combined whenever possible.

For example, the Boolean expression x && (y >> z) (to be read as  $x \land (y \rightarrow z)$ ) for Boolean variables x, y, and z is posted by

```
BoolVar b=expr(home, x && (y >> z));
```

**Tip 6.1** (Boolean precedences). Note that the precedences of the Boolean connectives are different from the usual mathematical notation. In  $C^{++}$ , operator precedence cannot be changed, so the precedences are as follows (high to low): !, <<, >>, ==, !=, ^, &&, ||. For instance, this means that the expression  $b_0 == b_1 >> b_2$  will be interpreted as  $(b_0 \leftrightarrow b_1) \rightarrow b_2$  instead of the more canonical  $b_0 \leftrightarrow (b_1 \rightarrow b_2)$ . If in doubt, use parentheses!

Any Boolean expression e corresponds to the Boolean relation stating that e is true. Posting a Boolean relation posts the corresponding Boolean constraint. Using the Boolean expression from above,

```
rel(home, x && (y >> z));
```

posts that  $x \land (y \rightarrow z)$  must be true, whereas

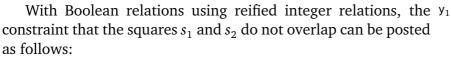
posts that  $x \land (y \rightarrow z)$  must be false.

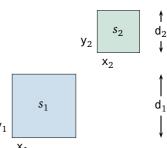
A Boolean element expression such as element(x,e), where x is an array of Boolean variables, and e is an integer expression, corresponds to an array access x[e], implemented using an element constraint (see Section 4.4.12).

Boolean expressions include reified integer relations. As an example consider the placement of two squares  $s_1$  and  $s_2$  such that the squares do not overlap. A well known model for this constraint is

$$x_1 + d_1 \le x_2 \quad \lor \quad x_2 + d_2 \le x_1 \quad \lor y_1 + d_1 \le y_2 \quad \lor \quad y_2 + d_2 \le y_1$$

The meaning of the integer variables  $x_i$  and  $y_i$ , and the integer values  $d_i$  is sketched to the right. The squares do not overlap, if the relative position of  $s_1$  with respect to  $s_2$  is either left, right, above, or below. As soon as one of the relationships is established, the squares do not overlap. Please also consult Section 4.4.16 for geometrical packing constraints.





Like the post functions for integer and Boolean variables presented above, posting Boolean expressions and relations supports an optional argument of type IntConLevel to select the consistency level. For more information, see Section 4.3.

Boolean expressions also include reified set relations, which will be covered below.

**Tip 6.2** (Reification of non-functional constraints). Reification of integer or set relations is mostly implemented through *decomposition*. For example, given integer variables x, y, and z, the reified division constraint

```
rel(home, (x / y == z) == b);
```

is actually equivalent to

```
IntVar tmp = expr(home, x / y);
rel(home, (tmp == z) == b);
```

Some constraints, such as division above, are not simple functions but impose side constraints. In the case of the division above, the side constraint is that y is not zero. It is important to understand the subtle semantics of decomposed reification here: If y happens to be zero, we get failure instead of b being constrained to false!

There are several expressions that have non-functional semantics: division, modulo, element, and disjoint set union (introduced below).

For more examples using Boolean expressions and Boolean relations including reification, see Chapter 12.

#### 6.1.3 Set expressions and relations

Set expressions and relations are constructed using the standard C++ operators and the functions listed in Figure 6.3. Just like for integer and Boolean expressions, posting of a set expression returns a new set variable that is constrained to the value of the expression.

For example, the set expression  $x \& (y \mid z)$  (to be read as  $x \cap (y \cup z)$ ) for set variables x, y, and z is posted by

```
SetVar s = expr(home, x \& (y | z));
```

Posting a set relation posts the corresponding constraint. Given an existing set variable s, the previous code fragment could therefore be written as

```
rel(home, s == (x \& (y | z)));
```

As noted above, set relations can be reified, turning them into Boolen expressions. The following code posts the constraint that b is true if and only if x is the complement of y:

```
BoolVar b = expr(home, (x == -y));
```

```
\langle SetExpr \rangle ::= \langle x \rangle
                                                            set variable
                                                            set constant (IntSet)
                         \langle s \rangle
                         - (SetExpr)
                                                            complement
                         ⟨SetExpr⟩ & ⟨SetExpr⟩
                                                            intersection
                         ⟨SetExpr⟩ | ⟨SetExpr⟩
                                                            union
                         \langle SetExpr \rangle + \langle SetExpr \rangle
                                                            disjoint union
                         \langle SetExpr \rangle - \langle SetExpr \rangle
                                                            set difference
                                                            intersection of variables
                         inter(\langle \overline{x} \rangle)
                         setunion(\langle \overline{x} \rangle)
                                                            union of variables
                         setdunion(\langle \overline{x} \rangle)
                                                            disjoint union of variables
                         singleton(\langle IntExpr \rangle)
                                                            singleton given by integer expression
  ⟨SetRel⟩
                ::= \langle SetExpr \rangle == \langle SetExpr \rangle
                                                            expressions are equal
                         \langle SetExpr \rangle != \langle SetExpr \rangle
                                                            expressions are not equal
                         \langle SetExpr \rangle \leftarrow \langle SetExpr \rangle first is subset of second expression
                         \langle SetExpr \rangle >= \langle SetExpr \rangle
                                                            first is superset of second expression
                         ⟨SetExpr⟩ | | ⟨SetExpr⟩ expressions are disjoint
        \langle \overline{x} \rangle ::= array of set variables
```

Figure 6.3: Set expressions and relations

Intead of a set variable, you can always use a constant IntSet, for example for reifying the fact that x is empty:

```
BoolVar b = expr(home, (x == IntSet::empty));
```

The subset relations can also be posted two-sided, such as

```
rel(home, IntSet(0,10) <= x <= IntSet(0,20));
```

# **6.2** Matrix interface for arrays

MiniModel provides a Matrix support class for accessing an array as a two dimensional matrix. The following

```
IntVarArgs x(n*m);
...
Matrix<IntVarArgs> mat(x, n, m);
```

declares an array of integer variables x and superimposes a matrix interface to x called mat with width n and height m. Note that the first argument specifies the number of columns, and the second argument specifies the number of rows.

The elements of the array can now be accessed at positions (i, j) in the matrix mat (that is, the element in column i and row j) using

```
IntVar mij = mat(i,j);
```

Furthermore, the rows and columns of the matrix can be accessed using mat.row(i) and mat.col(j). If a rectangular slice is required, the slice() member function can be used.

A matrix interface can be declared for any standard array or argument array used in Gecode, such as IntVarArray or IntSetArgs.

As an example of how the Matrix class can be used, consider the Sudoku problem (see Solving Sudoku puzzles using both set and integer). Given that there is a member IntVarArray x that contains  $9 \cdot 9$  integer variables with domain  $\{1, \ldots, 9\}$ , the following code posts constraints that implement the basic rules for a Sudoku.

```
Matrix<IntVarArray> m(x, 9, 9);

for (int i=0; i<9; i++)
   distinct(home, m.row(i));

for (int i=0; i<9; i++)
   distinct(home, m.col(i));

for (int i=0; i<9; i+=3)
   for (int j=0; j<9; j+=3)
    distinct(home, m.slice(i, i+3, j, j+3));</pre>
```

For more examples that use the Matrix class, see Chapter 19, Chapter 15, Chapter 18, Chapter 14, Magic squares, and Nonogram.

**Element constraints.** A matrix can also be used with an element constraint that propagates information about the row and column of matrix entries.

For example, the following code assumes that x is an integer array of type IntArgs with 12 elements.

```
Matrix<IntArgs> m(x, 3, 4);
IntVar r(home,0,1024), c(home,0,1024), v(home,0,1024);
element(home, m, r, c, v);
```

constrains the variable v to the value at position  $\langle r, c \rangle$  of the matrix m (see also GCCAT: element\_matrix).

**Tip 6.3** (Element for matrix can compromise propagation). Whenever it is possible one should use an array rather than a matrix for posting element constraints, as an element constraint for a matrix will provide rather weak propagation for the row and column variables.

Consider the following array of integers x together with its matrix interface m

```
IntArgs x(4, 0,2,2,1);
Matrix<IntArgs> m(x,2,2);
```

That is, m represents the matrix

$$\begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}$$

Consider the following example using an element constraint on an integer array:

```
IntVar i(home,0,8), v(home,0,1);
element(home, x, i, v);
```

After performing propagation, i will be constrained to the set  $\{0,3\}$  (as 2 is not included in the values of v).

Compare this to propagating an element constraint over the corresponding matrix as follows:

```
IntVar r(home,0,8), c(home,0,8), v(home,0,1);
element(home, m, r, c, v);
```

Propagation of element will determine that only the fields (0,0) and (1,1) are still possible. But propagating this information to the row and column variables, yields the values  $\{0,1\}$  for both r and c: each value for the coordinates is still possible even though some of their combinations are not.

# 6.3 Support for cost-based optimization

Support for cost-based optimization provides two subclasses of Space, MinimizeSpace and MaximizeSpace to search for a solution of minimal and maximal, respectively, cost. In order to use these abstract classes, a class inheriting from them must implement a virtual cost function of type

```
virtual IntVar cost(void) const { ··· }
```

The function must return an integer variable for the cost. For an example, see Section 3.2.

**Tip 6.4** (Cost must be assigned for solutions). In case the cost() function is called on a *solution*, the variable returned by cost() *must* be assigned. If the variable is unassigned for a solution, an exception of type Int::ValOfUnassignedVar is thrown. ◀

# 6.4 Regular expressions for extensional constraints

Regular expressions are implemented as instances of the class REG and provide an alternative, typically more convenient, interface for the specification of extensional constraints than DFAs do. The construction of regular expressions is summarized in Figure 6.4.

operation	meaning
REG r	initialize r as $\epsilon$ (empty)
REG r(4)	initialize r as single integer (symbol) 4
REG r(IntArgs(3, 0,2,4))	initialize r as alternative of integers 0 2 4
r + s	r followed by s
r   s	r or s
r += s	efficient shortcut for $r = r + s$
r  = s	efficient shortcut for r = r   s
*r	repeat r arbitrarily often (Kleene star)
+r	repeat r at least once
r(n)	repeat r at least n times
r(n,m)	repeat r at least n times, at most m times

Figure 6.4: Constructing regular expressions (r and s are regular expressions, n and m are unsigned integers)

Let us reconsider the Swedish drinking protocol from Section 4.4.13. The protocol can be described by a regular expression r constructed by

```
REG r = *REG(0) + *(REG(1) + +REG(0));
```

A sequence of activities x (an integer or Boolean variable array) can be constrained by

```
DFA d(r);
extensional(home, x, d);
```

after a DFA for the regular expression has been computed.

**Tip 6.5** (Creating a DFA only once). Please make it a habit to create a DFA explicitly from a regular expression r rather than implicitly by

```
extensional(home, x, r);
```

Both variants work, however the implicit variant disguises the fact that each time the code fragment is executed, a new DFA for the regular expression r is computed (think about the code fragment being executed inside a loop and your C++ compiler being not too smart about it)!<sup>2</sup>

For examples on using regular expressions for extensional constraints, see the nonogram case study in Chapter 14 or the examples Solitaire domino, Nonogram, and Pentominoes. The models are based on ideas described in [17], where regular expressions for extensional constraints nicely demonstrate their usefulness.

<sup>&</sup>lt;sup>2</sup>The integer module cannot know anything about regular expressions. Hence, it is impossible in C<sup>++</sup> to avoid the implicit conversion. This is due to the fact that the conversion is controlled by a type operator (that must reside in the MiniModel module) and not by a constructor that could be made **explicit**.

alias	constraint posted	GCCAT
atmost(home, x, u, v);	<pre>count(home, x, u, IRT_LQ, v);</pre>	atmost
atleast(home, x, u, v);	<pre>count(home, x, u, IRT_GQ, v);</pre>	atleast
exactly(home, x, u, v);	count(home, x, u, IRT_EQ, v);	exactly
lex(home, x, r, y);	rel(home, x, r, y);	lex
<pre>values(home, x, s);</pre>	<pre>dom(home, x, s);</pre>	
	<pre>nvalues(home, x, IRT_EQ, s.size());</pre>	

Figure 6.5: Aliases for integer constraints (x and y are integer variable arrays, u and v are integers or integer variables, r is an integer relation type, s is an integer set)

# 6.5 Channeling functions

Channel functions are functions to channel a Boolean variable to an integer variable and vice versa, and to channel between integer variables and a set variable. For an integer variable x,

```
channel(home, x);
```

returns a new Boolean variable that is equal to x. Likewise, for a Boolean variable x an equal integer variable is returned.

For an array of integer variables x, channel (home, x) returns a set variable equal to all the integers in x.

# 6.6 Aliases for integer constraints

Aliases for integer constraints provide some popular aliases. Figure 6.5 lists the aliases and their corresponding definitions.

#### 6.7 Aliases for set constraints

Aliases for set constraints provide aliases and convenience post functions for useful set constraints.

channel (home, x, y) is an alias for rel (home, SOT\_UNION, x, y), posting the constraint that y is exactly the set of integers  $\{x_0, \ldots, x_{|x|-1}\}$ . In addition to the union constraint, it posts an nvalues constraint for stronger propagation (see Section 4.4.9).

range(home, x, y, z), where x is an array of integer variables and y and z are set variables, is an alias for element(home, SOT\_UNION, x, y, z). This constraints treats x as defining a function, and constrains z to be the range of the function restricted to y:

$$z = \bigcup_{i \in y} \{x_i\}$$

Conversely, roots(home, x, y, z) constrains y to be the roots of the elements in z, i.e., those indices mapping to elements in z:

$$y = \bigcup_{i \in z} \{j \mid x_j = i\}$$

(see also GCCAT: roots).

# 7

# Search

This chapter discusses how search is used for solving Gecode models. Search involves two techniques: *branching* and *exploration*. Branching defines the shape of the search tree. Exploration defines a strategy how to explore parts of the search tree and how to possibly modify the tree's shape during exploration (for example, during branch-and-bound best solution search by adding new constraints). This chapter restricts itself to simple search engines to find solutions, Gist as an interactive and graphical search engine is discussed in Chapter 8.

**Overview.** Section 7.1 explains important details of how branching in Gecode can be used for modeling and solving problems. This section belongs to the basic reading material of Part M.

Section 7.2 explains how search in Gecode makes use of hybrid recomputation and why it is efficient. Even though this section does not belong to the basic reading material, you are highly encouraged to read it.

Section 7.3 explains how parallel search can be used in Gecode and what can be expected of parallel search in principle. How search engines can be used is explained in Section 7.4.

**Convention.** Note that the same conventions hold as in Chapter 4.

# 7.1 Branching

A space in Gecode can have *several* branchers posted on behalf of a *branching* that are executed in order of creation. Assume that in

```
branch(home, x, INT_VAR_SIZE_MIN, INT_VAL_MIN);
...
branch(home, y, INT_VAR_SIZE_MIN, INT_VAL_MIN);
```

both calls to branch create a brancher. Search branches first on the variables x and then on the variables y. Here, it does not matter whether propagators are created in between the creation of branchers.

#### 7.1.1 Branching on integer and set variables

**Important.** Do not forget to add

```
#include <gecode/int.hh>
```

to your program when you want to branch on integer and Boolean variables. Likewise, do not forget to add

```
#include <gecode/set.hh>
```

for branching on set variables.

Gecode offers predefined *variable-value branching*: when calling branch to post a branching, the third argument defines which variable is selected for branching, whereas the fourth argument defines which values are selected for branching. For example, for an array of integer or Boolean variables x the following call to branch

```
branch(home, x, INT_VAR_MIN_MIN, INT_VAL_SPLIT_MIN);
```

selects a variable y with the smallest minimum value (in case of ties, the first such variable in x is selected) and creates a choice with two alternatives  $y \le n$  and y > n where

$$n = \left| \frac{\min(y) + \max(y)}{2} \right|$$

The posted brancher assigns all variables and then ceases to exist. If more branchers exist, search continues with the next brancher.

For integer and Boolean variables, variable selection is defined by a value of type IntVarBranch and value selection is defined by a value of type IntValBranch (see Branching). For an overview of the available value and variable selection strategies, see Figure 7.1.

The degree of a variable is the number of propagators depending on the variable (useful as an approximate measure of how constrained a variables is). The accumulated failure count (AFC) of a variable is the sum of the AFCs of all propagators depending on the variable plus its degree (to give a good initial value if the AFCs of all propagators are still zero). The AFC of a propagator counts how often the propagator has failed during search. The AFC is also known as the weighted degree of a variable [9].

The minimum-regret is the difference between the smallest and second smallest value in the domain of a variable (maximum-regret is analogous).

**Tip 7.1** (Variables are re-selected during branching). A variable-value branching selects a variable for each choice it creates. Consider as an example a script using an integer variable array x with three variables and domains [1 .. 4] created by

```
IntVarArray x(home, 3, 1, 4);
```

INT_VAR_NONE INT_VAR_RND INT_VAR_DEGREE_MIN INT_VAR_DEGREE_MAX INT_VAR_AFC_MIN INT_VAR_AFC_MAX INT_VAR_MIN_MIN INT_VAR_MIN_MAX INT_VAR_MAX_MIN INT_VAR_SIZE_MIN INT_VAR_SIZE_MAX INT_VAR_SIZE_DEGREE_MIN INT_VAR_SIZE_DEGREE_MAX INT_VAR_SIZE_AFC_MIN INT_VAR_SIZE_AFC_MIN INT_VAR_SIZE_AFC_MIN INT_VAR_SIZE_AFC_MIN INT_VAR_SIZE_AFC_MIN INT_VAR_SIZE_AFC_MAX INT_VAR_REGRET_MIN_MIN INT_VAR_REGRET_MIN_MAX INT_VAR_REGRET_MAX_MIN INT_VAR_REGRET_MAX_MIN	first unassigned randomly smallest degree largest degree smallest accumulated failure count (AFC) largest accumulated failure count (AFC) smallest minimum value largest minimum value smallest maximum value largest maximum value smallest domain size largest domain size smallest domain size divided by degree largest domain size divided by AFC largest domain size divided by AFC smallest minimum-regret largest minimum-regret
INT_VAR_REGRET_MAX_MAX	largest maximum-regret

INT_VAL_MIN	smallest value
THE VAL MED	greatest value not greater th

INT\_VAL\_MED greatest value not greater than the median

INT\_VAL\_MAX largest value INT\_VAL\_RND random value

INT\_VAL\_SPLIT\_MIN values not greater than mean of smallest and largest value INT\_VAL\_SPLIT\_MAX values greater than mean of smallest and largest value INT\_VAL\_RANGE\_MIN values from smallest range, if domain has several ranges;

otherwise, values not greater than mean of smallest

and largest value

INT\_VAL\_RANGE\_MAX values from largest range, if domain has several ranges;

otherwise, values greater than mean of smallest and largest value

INT\_VALUES\_MIN all values starting from smallest INT\_VALUES\_MAX all values starting from largest

Figure 7.1: Integer and Boolean variable and value selection

Let us assume that no constraints are posted on the variables in x and that a branching is posted by

```
branch(home, x, INT_VAR_SIZE_MAX, INT_VAL_SPLIT_MIN);
```

The branching starts by selecting x[0] as the first variable with the largest domain in the array x and creates the choice

$$(x[0] \le 2) \lor (x[0] > 2)$$

Now assume that search explores the first alternative which results in the domain  $\{1,2\}$  for x[0]. When search continues, the branching again selects the first variable with a largest domain: hence x[1] is selected and *not* x[0].

In other words, a variable-value branching does not stick to a selected variable until the variable becomes assigned. Instead, a variable-value branching re-selects a variable for each choice it creates.

**Tip 7.2** (Do not try all values). Note that for INT\_VALUES\_MIN and INT\_VALUES\_MAX, a variable-value branching creates a choice for each selected variable with one alternative per value of the variable. This is typically a poor choice, as none of the alternatives can benefit from propagation that arises when other values of the same variable are tried. These branchings exist for instructional purposes (well, they do create beautiful trees in Gist). ◀

Likewise, for set variables, variable selection is defined by a value of type SetVarBranch and value selection is defined by a value of type SetValBranch (see Branching). For an overview of the available value and variable selection strategies, see Figure 7.2.

#### 7.1.2 Random variable and value selection

One particular strategy for variable and value selection is by random. For integer and Boolean variables, INT\_VAR\_RND selects a random variable and INT\_VAL\_RND selects a random value. For set variables, SET\_VAR\_RND selects a random variable and SET\_VAL\_RND\_INC and SET\_VAL\_RND\_EXC include and exclude a random value from a set variable.

The random number generators used for random variable and value selection follow a uniform distribution and can be initialized by a seed value. In order to pass seed values, variable-value branching accepts optional arguments of type VarBranchOptions and ValBranchOptions (see Generic branching support). These classes feature a member seed that defines the seed value used for the random number generator.

Both classes support the generation of seed values based on the current time. The following selects both variables and values randomly and uses the current time as seed for both variable and value selection.

```
first unassigned
SET_VAR_NONE
SET_VAR_RND
                           randomly
                           smallest degree
SET_VAR_DEGREE_MIN
                           largest degree
SET_VAR_DEGREE_MAX
                           smallest accumulated failure count (AFC)
SET_VAR_AFC_MIN
                           largest accumulated failure count (AFC)
SET_VAR_AFC_MAX
                           smallest minimum unknown element
SET_VAR_MIN_MIN
                           largest minimum unknown element
SET_VAR_MIN_MAX
                           smallest maximum unknown element
SET_VAR_MAX_MIN
                           largest maximum unknown element
SET_VAR_MAX_MAX
                           smallest unknown set
SET_VAR_SIZE_MIN
                           largest unknown set
SET_VAR_SIZE_MAX
                           smallest domain size divided by degree
SET_VAR_SIZE_DEGREE_MIN
                           largest domain size divided by degree
SET_VAR_SIZE_DEGREE_MAX
                           smallest domain size divided by AFC
SET_VAR_SIZE_AFC_MIN
SET_VAR_SIZE_AFC_MAX
                           largest domain size divided by AFC
                   include smallest element
 SET_VAL_MIN_INC
                   exclude smallest element
 SET_VAL_MIN_EXC
 SET_VAL_MED_INC
                   include median element (rounding downwards)
                   exclude median element (rounding downwards)
 SET_VAL_MED_EXC
                   include largest element
 SET_VAL_MAX_INC
 SET_VAL_MAX_EXC
                   exclude largest element
 SET_VAL_RND_INC
                   include random element
                   exclude random element
 SET_VAL_RND_EXC
```

Figure 7.2: Set variable and value selection

#### 7.1.3 Tie-breaking

The default behavior for tie-breaking during variable selection is that the first variable (that is the variable with the lowest index in the array) satisfying the selection criteria is selected. For many applications that is not sufficient.

A typical example for integer variables is to select a most constrained variable first (the variable most propagators depend on, that is, with largest degree). Then, among the most constrained variables select the variable with the smallest domain. This can be achieved by using a tiebreak function:

The overloaded function tiebreak (see Tie-breaking for variable selection) takes up to four variable selection values.

Random selection is particularly interesting for tie-breaking. For example, breaking ties by first selecting a variable with smallest domain and then selecting a random variable among those with smallest domain:

To be able to pass seed values to a particular variable selection in tie-breaking, the tiebreak function is also overloaded for the class VarBranchOptions. For example,

passes a seed value based on the current time to the random variable selection used for tie-breaking. The same can also be achieved by

## 7.1.4 Using branch filter functions

By default, a variable-value branching continues to branch until all variables passed to the branching are assigned. This behavior can be changed by using a *branch filter function*.

A branch filter function is called during branching for each variable to be branched on. If the filter function returns **true**, the variable is considered for branching. Otherwise, the variable is simply ignored.

```
BRANCH FILTER FUNCTION SKETCH ≡
  class Model : public Space {
  protected:
    IntVarArray x;
  public:
    Model(void) : ··· {
        ...
        ▶ POST BRANCHING
    }
    ...
    ▶ DEFINE FILTER FUNCTION
};
```

Figure 7.3: Model sketch for branch filter function

A branch filter function can be specified when a branching is posted. The filter function to be used by a branching can be passed by creating an object of class VarBranchOptions (see Generic branching support) and passing that object to the branch() function.

A branch filter function is of type BranchFilter (see Generic branching support) defined as

```
typedef bool (*BranchFilter)(const Space& home, int i, const Var& y);
```

That is, a branch filter function takes the home space and the position i of the variable y as argument. The position i refers to the position of the variable y in the array of variables used for posting the branching.

Assume, for example, that we want to branch only on variables from a variable array x for branching with a domain size of at least 4. Consider the sketch of a model shown in Figure 7.3.

The branch filter function can be defined as a static member function of the class Model as follows:

```
DEFINE FILTER FUNCTION ≡
  static bool filter(const Space& home, int i, const Var& y) {
   return static_cast<const IntVar&>(y).size() >= 4;
}
```

Note that the variable y must first be cast to the variable type used for branching (IntVar in our case).

Specifying that the branching should use the filter function is done as follows:

```
POST BRANCHING ≡

VarBranchOptions o(&filter);
branch(home, x, ···, ···, o);
```

One could also use the member bf of a variable branch option to set a branch filter function:

```
VarBranchOptions o;
o.bf = &filter;
branch(home, x, ..., o);
```

In many cases it can be more convenient to perform the actual filtering in a **const** member function, say filter(), that has access to all members of the script Model. Then, the filter() member function can be called by some other static member function:

where the function trampoline() now serves as branch filter function.

Branch filter functions and tie-breaking. Tie-breaking requires that several objects of class VarBranchOptions are combined via the tiebreak() function (see Section 7.1.3). However, the branch filter function (if defined) is always taken from the first object. That is, when defining option objects by

the branch filter function f1() is used.

#### 7.1.5 Assigning integer and set variables

A special variant of branching is *assigning* variables: for a not yet assigned variable the branching creates a single alternative which assigns the variable a value. The effect of assigning is that assignment is interleaved with constraint propagation. That is, after an assignment has been done, the next assignment will be done only after the effect of the previous assignment has been propagated.

For example, the next code fragment assigns all integer variables in x their smallest possible value:

```
assign(home, x, INT_ASSIGN_MIN);
```

```
INT_ASSIGN_MIN smallest value
                       median value (rounding downwards)
      INT_ASSIGN_MED
      INT_ASSIGN_MAX maximum value
                       random value
      INT_ASSIGN_RND
SET_ASSIGN_MIN_INC include smallest element
                    exclude smallest element
SET_ASSIGN_MIN_EXC
                    include median element (rounding downwards)
SET_ASSIGN_MED_INC
                    exclude median element (rounding downwards)
SET_ASSIGN_MED_EXC
                    include largest element
SET_ASSIGN_MAX_INC
SET_ASSIGN_MAX_EXC
                    exclude largest element
SET_ASSIGN_RND_INC
                    include random element
SET_ASSIGN_RND_EXC exclude random element
```

Figure 7.4: Value selection for assigning variables

The strategy to select the value to be assigned is defined by a value of type IntAssign (see Assigning) for integer and Boolean variables and by a value of type SetAssign (see Assigning). Figure 7.4 summarizes the value selection strategies for assigning integer, Boolean, and set variables.

Assignments also support random value selection and hence also accept an optional argument of type ValBranchOptions.

#### 7.1.6 Branching on and assigning single variables

In addition to branching on an array of variables or assigning an array of variables, Gecode also supports branching on and assigning a single variable.

For example, if x is an integer variable of type IntVar, then

```
branch(home, x, INT_VAL_MIN);
```

branches on the single variable x by first trying the smallest value of x.

Assume that x is an array of integer variables. Then the following code

```
for (int i=0; i<x.size(); i++)
branch(home, x[i], INT_VAL_MIN);</pre>
```

is equivalent, albeit considerably less efficient, to

```
branch(home, x, INT_VAR_NONE, INT_VAL_MIN);
```

#### 7.1.7 Executing code between branchers

A common scenario is to post some constraints only after part of the branching has been executed. This is supported in Gecode by a brancher (see Branch with a function) that executes a function (either a function or a static member function but not a member function). Suppose the following code fragment defining a model Model:

where the constructor posts two branchers

```
POST BRANCHINGS ≡
branch(home, x, INT_VAR_NONE, INT_VAL_MIN);
branch(home, &Model::post);
```

The second branching takes a function pointer to the static member function Model::post which is defined as

```
void more(void) {
    ...
}
static void post(Space& home) {
    static_cast<Model&>(home).more();
}
```

As soon as the first branching is finished, the second branching is executed. This branching provides just a single alternative that calls the function Model::post with the current space as its argument. Then, the function casts the home to Model& and calls more on home. While one could post the additional constraints and/or branchings in Model::post directly, the member function Model::more is more convenient to use.

**Tip 7.3** (Propagation is still explicit). It is tempting to believe that the variables in x in the above example are all assigned when more() is executed. This is not necessarily true.

It will be true for the first time more() is executed. But more() will be executed possibly quite often during recomputation (see the following Section). And then, the only guarantee one can rely on is that the brancher has created enough alternatives to guarantee that the variables in x are assigned *but only after constraint propagation has been performed* (see Tip 2.2).

# 7.2 Hybrid recomputation

A central requirement for search is that it can return to previous states: as spaces constitute nodes of the search tree, a previous state is nothing but a space. Returning to a previous space might be necessary because an alternative suggested by a branching did not lead to a solution, or even if a solution has been found more solutions might be requested. As propagation and branching change spaces, provisions must be taken that search can actually return to a previous space, or an equivalent version of that space.

Two space are *equivalent* if propagation and branching and hence search behave exactly the same on both spaces. Equivalent spaces can be different, for example, they contain different yet equivalent propagators, or are allocated at a different memory area.

Gecode employs a hybrid of two techniques for restoring spaces: *recomputation* and *cloning*.

If you want to know how search engines can be programmed in Gecode, please consult Part S.

#### **7.2.1** Cloning

Cloning creates a clone of a space (this is supported by the virtual copy member function as discussed in Chapter 2). A clone and the original space are of course equivalent. Restoration with cloning is straightforward: before following a particular alternative during search, a clone of the space is made and used later if necessary.

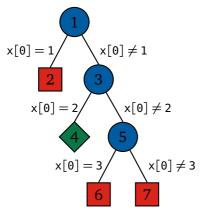
# 7.2.2 Recomputation

Recomputation remembers what has happened during branching: rather than storing an entire clone of a space just enough information to redo the effect of a brancher is stored. The information stored is called a *choice* in Gecode. Redoing the effect is called to *commit* a space: given a space and a choice committing reexecutes the brancher as described by the choice and the alternative to be explored (for example, left or right).

Consider the following part of a model, which constrains both the sum and the product of x[0] and x[1] to be equal to x[2]:

```
IntVarArray x(home, 3, 1, 6);
rel(home, x[0] + x[1] == x[2]);
mult(home, x[0], x[1], x[2]);
branch(home, x, INT_VAR_NONE, INT_VAL_MIN);
```

The corresponding search tree is shown in Figure 7.5. A red box corresponds to a failed node, a green diamond to a solution, and a blue circle to a choice node (a node that has a not-yet finished brancher left). An example choice for node 3 is  $(x[0] = 2) \lor (x[0] \neq 2)$  where the left alternative (or the 0-th alternative) is x[0] = 2 and the right alternative (1-st alternative) is  $x[0] \neq 2$ . Committing a space for node 3 to the 1-st alternative posts the constraint  $x[0] \neq 2$ .



node	x[0]	x[1]	x[2]
1	{1,,5}	{1,,5}	{2,,6}
3	{2,,5}	{1,,3}	{3,,6}
4	2	2	4
5	{3,,5}	{1,,2}	{4,,6}

Figure 7.5: Example search tree

More precisely, a choice does not store the actual variables but the position among the variables of the brancher (storing 0 rather than x[0]). By that, a choice can be used with an equivalent yet different space. This is essential as the space used during recomputation will be different from the space for which the choice has been created.

**Tip 7.4** (Search is indeterministic). Gecode has been carefully designed to support non-monotonic propagators: they are essential for example for randomized or approximation propagation algorithms. A propagator in Gecode must be *weakly* monotonic: essentially, a propagator must be correct but it does not need to always prune exactly the same way. A consequence of this is that search is indeterministic: it might be that two different searches find solutions in a different order (possibly returning a different first solution) or that the number of explored nodes is different. However, search is always sound and complete: it never misses any solution, it does not duplicate solutions, nor does it report non-solutions as solutions.

If you want to know more about weakly monotonic propagators and their interaction with search, we recommend to consult [40].

#### 7.2.3 Hybrid recomputation

The hybrid of recomputation and cloning works as follows. For each new choice node, a choice is stored. Then, every now and then search also stores a clone of a space (say, every eight steps). Now, restoring a space at a certain position in the search tree traverses the path in the tree upwards until a clone c is found on the path. Then recomputation creates a clone c' of c (in certain cases, recomputation might use c directly as an optimization). Then all choices on the path are committed on c' yielding an equivalent space.

To recompute the node? for the example shown in Figure 7.6, the following operations are executed:

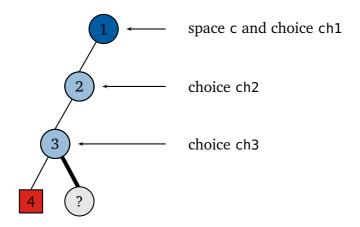


Figure 7.6: Hybrid recomputation

```
Space* s = c->clone();
s->commit(ch1, 0);
s->commit(ch2, 0);
s->commit(ch3, 1);
```

#### 7.2.4 Why recomputation is almost for free

An absolutely fundamental property of the above hybrid is that an equivalent space is computed without performing any constraint propagation! Remember: committing just reposts constraints but does not perform constraint propagation.

Reconsider the example from Figure 7.6. Search has just failed at node 4 and must compute a space for node ?.

Suppose that only cloning but no recomputation is used. Then, a clone of the space for node 3 is created (from the clone that is stored in node 3) and that clone is committed to the first alternative of d3 (this corresponds to the slightly thicker edge in Figure 7.6). After that, constraint propagation is performed (by executing the status() function of a space, see also Tip 2.2) to find out if and how search must continue. That is: there is one clone operation, one commit operation, and one status operation to perform constraint propagation.

With hybrid recomputation, one clone operation, three commit operations, and one status operation to perform constraint propagation are needed (as shown above). The good news is that commit operations are very cheap (most often, just modifying a single variable or posting a constraint). What is essential is that in both cases only a *single* status operation is executed. Hence, the cost for constraint propagation during hybrid recomputation turns out to be not much higher than the cost without recomputation.

For hybrid recomputation, some additional propagation might have to be done compared to cloning. As it turns out, the additional cost is rather small. This is due to the fact that constraint propagation executes all propagators that might be able to remove values for vari-

ables until no more propagation is possible (a fixpoint for the propagators is computed). Due to the approximative nature of "might be able to remove values" the additional propagation tends to show only a very small increase in runtime.

#### 7.2.5 Adaptive recomputation

Consider the case that a search engine finds a failed node. That means that some brancher has made an erroneous decision and now search has to recover from that decision. It is quite likely that not only the last decision is wrong but that the decision that lead to failure is somewhere higher up in the search tree. With other words, it is quite likely that search following a depth-first left-most strategy must explore an entire failed subtree to recover from the erroneous decision. In that case it would be better for hybrid recomputation if there was a clone close to the failed node rather than far away.

To optimize recomputation in this example scenario, Gecode uses *adaptive recomputation*: if a node must be recomputed, adaptive recomputation creates an additional clone in the middle of the recomputation path. A clone created during adaptive recomputation is likely to be a good investment. Most likely, an entire failed subtree will be explored. Hence, the clone will be reused several times for reducing the amount of constraint propagation during recomputation.

More information about search based on recomputation (although not using choices) can be found in [33]. Search using choices has been inspired by batch recomputation [11] and decomposition-based search [24]. For an empirical evaluation of different techniques for search, see [29].

## 7.2.6 Controlling recomputation

Hybrid and adaptive recomputation can be easily controlled by two integers  $c_d$  (commit distance) and  $a_d$  (adaptive distance). The value for  $c_d$  controls how many clones are created during exploration: a search engine creates clones during exploration to ensure that recomputation executes at most  $c_d$  commit operations. The value for  $a_d$  controls adaptive recomputation: only if the clone for recomputation is more than  $a_d$  commit operations away from the node to be recomputed, adaptive recomputation is used.

Values for  $c_d$  and  $a_d$  are used to configure the behavior of search engines using hybrid and adaptive recomputation, see more in the next Section.

The number of commit operations as distance measure is approximately the same as the length of a path in the search tree. It is only an approximation as search engines use additional techniques to avoid some unused clone and commit operations.

**Tip 7.5** (Values for  $c_d$  and  $a_d$ ). If  $c_d = 1$ , recomputation is never used (you might not want to try that for any other reason but curiosity; it takes too much memory to be useful). Likewise, to switch off cloning, you can use a value for  $c_d$  that is larger than the expected depth of the search tree. If  $a_d \ge c_d$ , adaptive recomputation is never used.

#### 7.3 Parallel search

Parallel search has but one motivation: try to make search more efficient by employing several threads (or workers) to explore different parts of the search tree in parallel.

Gecode uses a standard work-stealing architecture for parallel search: initially, all work (the entire search tree to be explored) is given to a single worker for exploration, making the worker busy. All other workers are initially idle, and try to steal work from a busy worker. Stealing work means that part of the search tree is given from a busy worker to an idle worker such that the idle worker can become busy itself. If a busy worker becomes idle, it tries to steal new work from a busy worker.

As work-stealing is indeterministic (depending on how threads are scheduled, machine load, and other factors), the work that is stolen varies over different runs for the very same problem: an idle worker could potentially steal different subtrees from different busy workers. As different subtrees contain different solutions, it is indeterministic which solution is found first.

When using parallel search one needs to take the following facts into account (note that some facts are not particular to parallel search, check Tip 7.4: they are just more likely to occur):

- The order in which solutions are found might be different compared to the order in which sequential search finds solutions. Likewise, the order in which solutions are found might differ from one parallel search to the next. This is just a direct consequence of the indeterministic nature of parallel search.
- Naturally, the amount of search needed to find a first solution might differ both from sequential search and among different parallel searches. Note that this might actually lead to super-linear speedup (for n workers, the time to find a first solution is less than 1/n the time of sequential search) or also to real slowdown.
- For best solution search, the number of solutions until a best solution is found as well as the solutions found are indeterministic. First, any better solution is legal (it does not matter which one) and different runs will sometimes be lucky (or not so lucky) to find a good solution rather quickly. Second, as a better solution prunes the remaining search space the size of the search space depends crucially on how quickly good solutions are found.
- As a corollary to the above items, the deviation in runtime and number of nodes explored for parallel search can be quite high for different runs of the same problem.
- Parallel search needs more memory. As a rule of thumb, the amount of memory needed scales linearly with the number of workers used.
- For parallel search to deliver some speedup, the search tree must be sufficiently large. Otherwise, not all threads might be able to find work and idle threads might slow down busy threads by the overhead of unsuccesful work-stealing.

```
GolombRuler
        m[12] = \{0, 1, 3, 7, 12, 20, 30, 44, 65, 80, 96, 122\}
        ··· (additional solutions omitted)
        m[12] = \{0, 2, 6, 24, 29, 40, 43, 55, 68, 75, 76, 85\}
Initial
        propagators: 67
        branchers:
                      1
Summary
        runtime:
                       15.349 (15349.000000 ms)
        solutions:
                       17
        propagations: 584967706
        nodes:
                       3708507
        failures:
                       1854221
        peak depth:
                       52
        peak memory:
                       1636 KB
```

Figure 7.7: Output for golomb rulers with eight workers

■ From all the facts listed, it should be clear that for depth-first left-most search for just a single solution it is notoriously difficult to obtain consistent speedup. If the heuristic is very good (there are almost no failures), sequential left-most depth-first search is optimal in exploring the single path to the first solution. Hence, all additional work will be wasted and the work-stealing overhead might slow down the otherwise optimal search.

**Tip 7.6** (Be optimistic about parallel search). After reading the above list of facts you might have come to the conclusion that parallel search is not worth it as it does not exploit the parallelism of your computer very well. Well, why not turn the argument upside down: your machine will almost for sure have more than a single processing unit and maybe quite some. With sequential search, all units but one will be idle anyway.

The point of parallel search is to make search go faster. It is not to perfectly utilize your parallel hardware. Parallel search makes good use (and very often excellent use for large problems with large search trees) of the additional processing power your computer has anyway.

For example, on my machine with eight cores and using Gecode 3.4.0, running Finding optimal Golomb rulers for size 12 as follows

```
golomb-ruler.exe -threads 8 12
```

prints something like shown in Figure 7.7.

Compared to sequential search where one gets something like shown in Figure 7.8 one gets a speedup of 7.3.

```
GolombRuler
        m[12] = \{0, 1, 3, 7, 12, 20, 30, 44, 65, 80, 96, 122\}
        ··· (additional solutions omitted)
        m[12] = \{0, 2, 6, 24, 29, 40, 43, 55, 68, 75, 76, 85\}
Initial
        propagators: 67
        branchers:
Summary
                      1:52.336 (112336.000000 ms)
        runtime:
        solutions:
                      16
        propagations: 813022663
        nodes:
                      5313357
        failures:
                      2656663
        peak depth:
                      57
        peak memory:
                      262 KB
```

Figure 7.8: Output for golomb rulers with one worker

Parallel search is controlled by the number of threads (or workers) used for search. If a single worker is requested, sequential search is used. The number of threads to be used for search is controlled by the search options passed to a search engine, see the following section for details.

**Tip 7.7** (Do not optimize by branching alone). A common modeling technique for optimization problems that does not work for parallel search is the following. Suppose, one has a variable c for the cost of a problem and one wants to minimize the cost. Then, one could use the following code fragment

```
branch(home, c, INT_VAL_MIN);
```

which will try the values for c in increasing order.

With sequential search, searching for the first solution with a standard depth-first left-most search engine will deliver a best solution, that is, a solution with least cost for c.

With parallel search, the first solution found might of course not be a best one. Hence, instead of using plain left-most depth-first search, one should use best solution search with a proper constrain function that guarantees that c will be minimized. This will as always guarantee that the last solution found is the best.

For an example, see the naive model for the bin packing case study in Section 17.2 where a branching first branches on the number of required bins.

member	type	meaning
propagate	unsigned long int	propagators executed
fail	unsigned long int	failed nodes explored
node	unsigned long int	nodes explored
depth	unsigned long int	maximal depth of explored tree
memory	size_t	peak memory allocated

Figure 7.9: Search statistics (partial)

# 7.4 Search engines

**Important.** Do not forget to add

#include <gecode/search.hh>

to your program when you want to use search engines.

All search engines in Gecode are parametric (are templates) with respect to a subclass T of Space (for example, SendMoreMoney in Section 2.1). Moreover, all search engines share the same interface:

■ The search engine is initialized by a constructor taking a pointer to an instance of the space subclass T as argument. By default, the search engine takes a clone of the space passed.

This behavior can be changed, as can be other aspects of a search engine, see Section 7.4.1.

- A next solution can be requested by a next() member function. If no more solutions exist, next() returns NULL. Otherwise, the engine returns a solution which again is an instance of T. The client of the search engine is responsible for deleting solutions.
- A search engine can be asked for statistics information by the statistics() member function. The function returns an object of type Search::Statistics. The statistics information provided is partially summarized in Figure 7.9.
- A search engine can be queried by stopped() whether the search engine has been stopped by a *stop object*. Stop objects are discussed in Section 7.4.2.
- The destructor deletes all resources used by the search engine.

Note that search engines use pointers to objects rather than references to objects. The reason is that some pointers might be NULL-pointers (for example, if next() fails to find a solution) and that users of search engines have to think about deleting solutions computed by search engines.

engine	shortcut	exploration	best solution	parallel
DFS	dfs	depth-first left-most		✓
BAB	bab	branch-and-bound	✓	✓
Restart	restart	depth-first left-most restart	✓	✓

Figure 7.10: Available search engines

member	type	meaning
threads	double	number of parallel threads to use
c_d	unsigned int	commit recomputation distance
a_d	unsigned int	adaptive recomputation distance
clone	bool	whether engine uses a clone when created
stop	Search::Stop*	stop object (NULL if none)

Figure 7.11: Search options

For each search engine there also exists a convenient shortcut function (of the same name but entirely in lowercase letters) that returns either the first solution or, in the case of best solution search, the last (and hence best) solution. The available search engines are summarized in Figure 7.10.

BAB continues search when a solution is found by adding a constraint (through the constrain() function as discussed in Section 2.5) to search for a better solution to all remaining nodes of the search tree. Restart restarts the search after the constraint has been added to the root node of the search tree (again, by using the constrain() function).

# 7.4.1 Search options

All search engines can take a default option value of type Search::Options when being created. The options are summarized in Figure 7.11. The default values for the options are defined in the namespace Search::Config.

The meaning of the values for the search options are straightforward but for threads. Assume that your computer has m processing units<sup>1</sup> and that the value for threads is n.

- If n = 0, then m threads are used (as many as available processing units).
- If  $n \ge 1$ , then n threads are used (absolute number of threads to be used).
- If  $n \le -1$ , then m + n threads are used (absolute number of processing units not to be used). For example, when n = -6 and m = 8, then 2 threads are used.

<sup>&</sup>lt;sup>1</sup>This is a very rough characterization: a processing unit could be a CPU, a processor core, or a multithreading unit. If you want to find out how many processing units Gecode believes your machine has, invoke the configuration summary as described in Tip 3.3.

class	description
Search::MemoryStop	memory limit exceeded
Search::NodeStop	node limit exceeded
Search::FailStop	failure limit exceeded
Search::TimeStop	time limit exceeded

Figure 7.12: Predefined stop objects

- If 0 < n < 1, then  $n \cdot m$  threads are used (relative number of processing units to be used). For example, when n = 0.5 and m = 8, then 4 threads are used.
- If -1 < n < 0, then  $(1+n) \cdot m$  threads are used (relative number of processing units not to be used). For example, when n = -0.25 and m = 8, then 6 threads are used.

Note that all values are of course rounded and that at least one thread will be used.

#### 7.4.2 Stop objects

A stop object (a subclass of Search::Stop) implements a single virtual member function stop() that takes two arguments, the first of type Search::Statistics and the second of type Search::Options, and returns either **true** or **false**. If a stop object is passed to a search engine (by passing it as stop member of a search option), the search engine calls the stop() function of the stop object before every exploration step and passes the current statistics as argument. If the stop() function returns true, the search engine stops its execution.

When a search engine is stopped its next() function returns NULL as solution. To find out whether a search engine has been stopped or whether there are no more solutions, the stopped() member function of a search engine can be used. Search can be resumed by calling next() again after the stop object has been modified (for example, by increasing the node or time limit).

Note that when using several threads for parallel search, each thread checks whether it is stopped independently using the very same stop object. If one thread is stopped, then the entire search engine is stopped.

Gecode provides several predefined stop objects, see Stop-objects for stopping search. For an overview see Figure 7.12.

**Tip 7.8** (Number of threads for stop objects). As mentioned above, each thread in parallel search uses the very same stop object. For example, when using the predefined Search::NodeStop stop object with a node limit of n, then each thread can explore up to n nodes.

If you want to have finer control (say, only allow each thread to explore up to n/m nodes where m is the number of threads) you can use the search option argument that is passed as

the second argument to the stop member function to scale the node limit according to the number of available threads. ◀

8

# **Gist**

The Graphical Interactive Search Tool, Gist, provides user-controlled search, search tree visualization, and inspection of arbitrary nodes in the search tree. Gist can be helpful when experimenting with different branching strategies, with different models for the same problem (for instance adding redundant constraints), or with propagation strength (for instance bounds versus domain propagation). It gives you direct feedback how the search tree looks like, if your branching heuristic works, or where propagation is weaker than you expected.

**Overview.** How the search tree of a problem is used as the central metaphor in Gist is sketched in Section 8.1. Section 8.2 explains how to invoke Gist, whereas Section 8.3 explains how to use Gist.

**Important.** Do not forget to add

#include <gecode/gist.hh>

to your program when you want to use Gist.

# 8.1 The search tree

The central metaphor in Gist is the *search tree*. Each node in the tree represents a fixpoint of propagatation (that is, an invocation of status(), see Tip 2.2). Inner nodes stand for

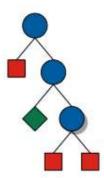


Figure 8.1: A search tree

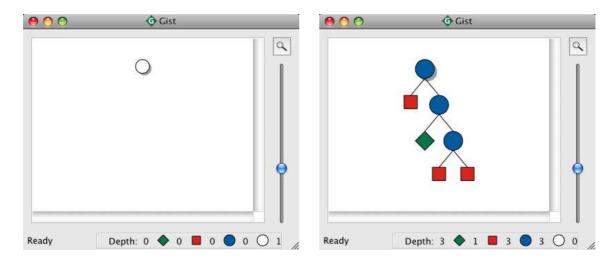


Figure 8.2: Gist, solving the Send More Money problem

choices (fixpoints with a subsequent brancher), while leaf nodes correspond to failure (dead ends in the search) or solutions of the problem. Figure 8.1 shows a search tree as drawn by Gist. The inner nodes (blue circles) are choices, the red square leaf nodes are failures, and the green diamond leaf node is a solution.

Conceptually, every node in the tree contains a corresponding space, which the user can access in order to inspect the node. Internally, Gist does not store all these spaces, but recomputes them on demand. That way, Gist can handle very large search trees (with millions of nodes) efficiently.

# 8.2 Invoking Gist

Gist is implemented using the Qt application framework. It can be invoked either as a standalone component, or as part of a bigger Qt-based application.

The screenshots in this chapter show Gist running on Mac OS X, but the functionality, the menus and the keyboard shortcuts are the same on all platforms (with small exceptions that will be mentioned later).

#### 8.2.1 Standalone use

When used as a standalone component, invoking Gist merely amounts to calling

```
Gist::dfs(m);
```

where m is a pointer to the space that contains the model to be solved. This call opens a new instance of Gist, with the root node initialized with m. Figure 8.2 (left) shows Gist initialized with the Send More Money puzzle from Chapter 2.

If you want to solve an optimization problem using branch-and-bound search, you can invoke Gist with optimization turned on:

Gist::bab(m);

#### 8.2.2 Use as a Qt widget

If you are developing an application with a graphical user interface using the Qt toolkit, you can embed Gist as a widget. Either use Gist::GistMainWindow, which gives you an independent widget that inherits from QMainWindow, or directly embed the Gist::Gist widget into your own widgets. You have to include the files gecode/gist/mainwindow.hh for the independent widget, or gecode/gist/qtgist.hh for the widget you can embed.

Apart from the integration into your own application, the advantage over the standalone approach is that you get access to Gist's *signals* and *slots*. For example, you can use more than one inspector, or you can control the search programatically instead of by user input. The details of this are beyond the scope of this document, please refer to the reference documentation of the corresponding classes for more information, and have a look at the directory gecode/gist/standalone-example in the Gecode source distribution.

# 8.3 Using Gist

This section gives an overview of the functionality available in Gist. Most of Gist is intuitive to use, and the easiest way of learning how to use it is to start one of the examples that come with Gecode using the -mode gist commandline option, and then play around.

#### 8.3.1 Automatic search

When you invoke Gist, it initializes the root node of the search tree, as seen in Figure 8.2. Any node that has not yet been explored by the search is drawn as a white circle, indicating that it is not yet known whether this node is in fact a choice, failure, or solution. White nodes are called *open*.

Obviously, the first thing a user will then want to do is to search for a solution of the given problem. Gist provides an automatic first-solution and all-solution depth-first search engine. It can be activated from the *Search* menu:



If you click *All solutions* (or alternatively press the *A* key), Gist will explore the entire tree. The result appears in Figure 8.2 (right). Depending on the preferences, Gist may collapse subtrees that contain only failed nodes (such as the rightmost subtree in the figure) into big red triangles.

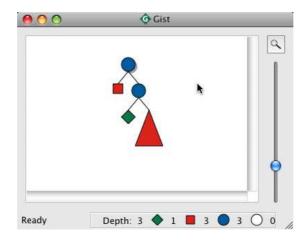


Figure 8.3: A hidden subtree in Gist

The search engines always only explore the subtree under the *currently selected node*, which is marked by a shadow. For example, the root node is selected after initialization, and the rightmost choice node is selected in Figure 8.2 (right). A single click selects a node, and there is always exactly one selected node.

**Stopping search after exhausting a branching.** If you want to learn more about how your branchings affect the shape of the search tree, you can add *stop branchings* to your problem using the Gist::stopBranch() function. This will install a brancher that does not modify the search tree (in fact, it simply inserts a single unary choice), but Gist will recognize the brancher and halt exploration. A special node (shaped like a stop-sign) marks the position in the tree where the stop brancher was active. You can toggle whether Gist continues exploration beyond the brancher using the options in the *Node* menu. Obviously, this is most useful if the call to stopBranch is placed *between* calls to other branchings.

#### 8.3.2 Interactive search

As an alternative to automatic search, you can also explore the search tree interactively. Just select an arbitrary open (white) node and choose *Inspect* from the *Node-Inspect* menu (or press the *Return* key).

You can navigate in the tree using the arrow keys (or the corresponding options from the *Node* menu). Automatic and interactive search can be freely mixed. In order to focus on interesting parts of the search tree, you can hide subtrees using the *Hide/unhide* option, or hide all subtrees below the selected node that are completely failed. The option *Unhide all* expands all hidden subtrees below the current node again. The red triangle in Figure 8.3 is a hidden subtree.

If you want to start over, the Search menu provides an option to reset Gist.

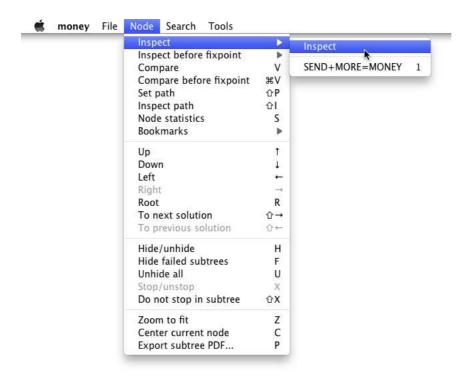


Figure 8.4: The Node menu

**Bookmarks.** The *Node* menu has a submenu *Bookmarks*, where you can collect nodes for quick access. You can set a bookmark for the currently selected node by chosing *Add/remove bookmark* from the submenue, or by pressing *Shift+B*. You can then enter a name for the bookmark (or leave it empty, then the bookmark will get a number). Chosing *Add/remove bookmark* for an already bookmarked node removes the bookmark. Bookmarked nodes are drawn with a small black circle. Selecting a bookmark moves you directly to the corresponding node.

#### 8.3.3 Branch-and-bound search

If Gist has been invoked in branch-and-bound mode, it prunes the search tree with each new solution that is found. Branch-and-bound works with any order of exploration, so you can for instance explore interactively, or start automatic search for just a subtree.

The last solution that was found, which in branch-and-bound is always the best solution known so far, is displayed in orange instead of green.

## 8.3.4 Inspecting and comparing nodes

Of course just looking at the shape of the tree is most of the time not very enlightening by itself. We want to see the content of the nodes. For this, you can add *inspectors* and



Figure 8.5: The Tools menu

*comparators* to Gist. Inspectors and comparators can be called on solution, choice, or failure nodes (but obviously not on open nodes).

**Invoking inspectors and comparators.** The *Node* menu (Figure 8.4) provides several options for inspecting and comparing nodes. If you choose *Inspect* from the *Inspect* submenu (or simply press *Return*), all double-click inspectors that are active in the *Tools* menu (see below) will be invoked for the currently selected node. You can also invoke a particular inspector by choosing it from th *Inspect* menu or typing its shortcut (the first nine inspectors get the shortcuts 0–9).

If you choose an inspector from the *Inspect before fixpoint* menu instead, the state of the node after branching but before fixpoint propagation is shown.

When choosing the *Compare* option, the mouse cursor turns into a crosshair, and clicking on another node will invoke the comparators that have been activated in the *Tools* menu. Again, *Compare before fixpoint* considers the state of the second node after branching but before fixpoint propagation. This is especially useful to find out what the branching does at a particular node: Select a node, choose *Compare before fixpoint*, and click on the child node you are interested in. The comparison will show you exactly how the branching has changed the variable domains.

**Choosing the active inspectors and comparators.** Gist distinguishes between three groups of inspectors, and the group of comparators, which can be chosen from the *Tools* menu (Figure 8.5):

- *Move inspectors* are called whenever the user moves to a different node.
- *Solution inspectors* are called for each new solution that is found.
- *Double-click inspectors* are called when the user explicitly inspects a node, for instance by choosing *Inspect* from the *Node* menu or by double-clicking.
- *Comparators* are called when the user chooses *Compare* or *Compare before fixpoint*, and then clicks on another node to compare the currently selected one to.

**The printing inspector.** The simplest way to add an inspector to your model is to use the Gist::Print inspector, as demonstrated in Section 2.4. Figure 8.6 shows the Gist::Print inspector after double-clicking the solution of the Send More Money problem.

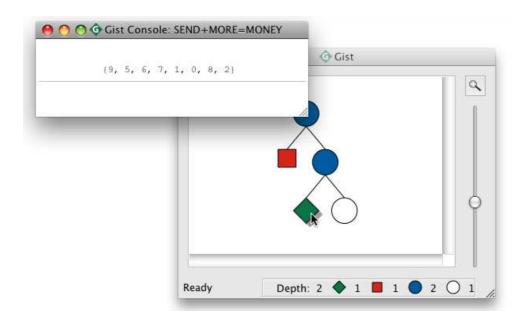


Figure 8.6: Inspecting a solution in Gist

Implementing inspectors. An inspector is an object that inherits from the abstract base class Gist::Inspector. The abstract base class declares a virtual member function, inspect(const Space& s), which is called when one of the events described above happens. The space that is passed as an argument corresponds to the inspected node in the tree. The inspector is free to perform any const operation on the space.

The variable comparator. Similar to the printing inspector, there is a predefined comparator Gist::VarComparator that can be easily added to scripts. It requires the script to implement a member function compare() in which it outputs the result of comparing itself to another space. Figure 8.7 shows how to add comparison to the example from Section 2.4. It uses a convenience member function from the class Gist::Comparator that produces a string representation of the differences between two variable arrays.

Implementing comparators. A comparator inherits from Gist::Comparator and implements at least its compare(const Space& s0, const Space& s1) member function. This function is called when the currently selected node with space s0 is compared to the node with space s1. As for inspectors, a comparator can perform any const operation on these two spaces.

**Subtree statistics.** The *Node* menu provides an option *Node statistics*, which when clicked opens a small window that displays statistics of the subtree rooted at the currently selected node, as shown in Figure 8.8. The statistics include the depth of the current node (the upper right number in Figure 8.8), the maximum depth of the subtree (the lower right number),

```
[DOWNLOAD]
SEND MORE MONEY WITH GIST COMPARISON \equiv
 #include <gecode/int.hh>
 #include <gecode/gist.hh>
 class SendMoreMoney : public Space {
   void compare(const Space& s, std::ostream& os) const {
      os << Gist::Comparator::compare<IntVar>("l",l,
        static_cast<const SendMoreMoney&>(s).l);
   }
 };
 int main(int argc, char* argv[]) {
    SendMoreMoney* m = new SendMoreMoney;
    Gist::Print<SendMoreMoney> p("Print solution");
   Gist::VarComparator<SendMoreMoney> c("Compare nodes");
   Gist::Options o;
   o.inspect.click(&p);
   o.inspect.compare(&c);
   Gist::dfs(m,o);
   delete m;
    return 0;
```

Figure 8.7: Using Gist for Send More Money with node comparison

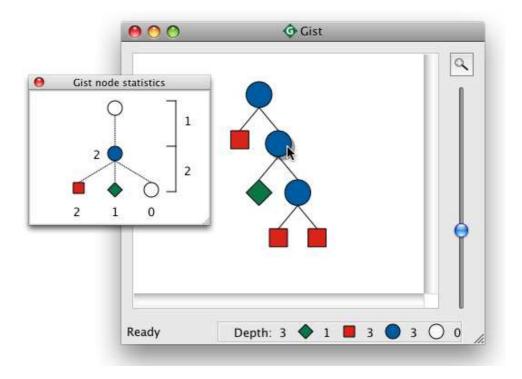


Figure 8.8: Node statistics

and how many of the different node types the subtree contains (the numbers at the small nodes). The information is automatically updated when you select a different node.

## 8.3.5 Zooming, centering, exporting, printing

Here is some functionality you may have already found during your experiments with Gist.

**Mouse wheel zoom.** You probably noticed that the slider right of the search tree lets you zoom in and out. Another way of zooming is to hold *Shift* while using the mouse wheel. It will zoom in and out keeping the area under the mouse cursor visible.

**Zoom and center.** In addition to manual zooming, Gist provides automatic options. The button with the magnifying glass icon above the zoom slider toggles the auto-zoom feature, which always zooms the tree such that as much of it as possible is visible in the window. During auto-zoom, the manual zoom is disabled. Instead of auto-zoom, you can also select *Zoom to fit* from the *Node* menu (or press *Z*) in order to adjust the zoom so that the current tree fits. When working with large trees, it is sometimes useful to scroll back to the currently selected node by choosing *Center current node* from the *Node* menu or pressing *C*.

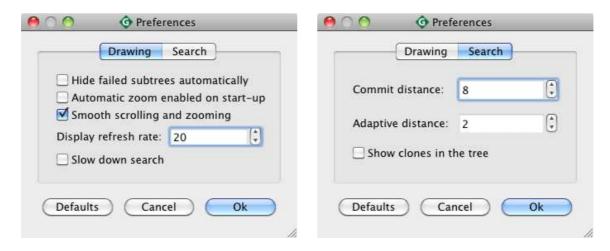


Figure 8.9: Gist preferences

**Exporting and printing.** The *File* menu provides options for exporting the current search tree as a PDF file or printing it. If you want to export a single subtree, select *Export subtree PDF* from the *Node* menu. The tree is exported or printed as seen, including hidden nodes.

## 8.3.6 Options and preferences

When invoking Gist, you can pass it an optional argument of type Gist::Options. This options class inherits from the standard search options discussed in Section 7.4.1, but adds a class with two member functions, inspect.click() and inspect.solution(), that you can use to pass inspectors to Gist.

The two options for recomputation, c\_d and a\_d, as well as the clone option of Search::Options are honored by Gist; the remaining options are ignored.

During execution, Gist can be configured using the *Preferences* dialog, available from the program menu on Mac OS or the *File* menu on Windows and Linux.

The drawing preferences (Figure 8.9, left) let you specify whether failed subtrees are hidden automatically during search, and whether the auto-zoom and smooth scrolling features are enabled at start-up. Furthermore, you can set the refresh interval – this is the number of nodes that are explored before the tree is redrawn. If you set this to a large number, search will be faster, but you get less visual feedback. Finally, you can enable slow search, which will explore only around three nodes per second. Drawing preferences are remembered between sessions and across different invocations of Gist.

The search preferences are exactly the parameters you can pass using the Gist::Options class. In addition, you can switch on the display of where Gist actually stores spaces in the tree, as shown in Figure 8.10. A small red circle indicates a clone used for recomputation, while a small yellow circle shows that the node has an active space used for exploration (a so-called *working clone*).

The recomputation parameters are not remembered between sessions.

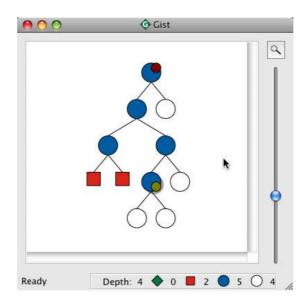


Figure 8.10: Displaying where Gist stores spaces in the tree

## Script commandline driver

The commandline driver (see Script commandline driver) provides support for passing common commandline options to programs and a sub-class for spaces called Script that can take advantage of the options.

**Overview.** Section 9.1 summarizes the commandline options supported by the commandline driver. The base classes for scripts that work together with the commandline driver are sketched in Section 9.2.

**Important.** Do not forget to add

#include <gecode/driver.hh>

to your program when you want to use the script commandline driver.

## 9.1 Commandline options

The commandline driver provides classes Options, SizeOptions, and InstanceOptions that support parsing commandline options. All classes support the options as summarized in Figure 9.1. Here, for a commandline option with name -name, the option classes provide two functions with name name(): one that takes no argument and returns the current value of the option, and one that takes an argument of the listed type and sets the option to that value. The only exceptions are -c-d, -a-d, and -print-last where the member functions are named  $c_d()$ ,  $a_d()$ , and  $print_last()$  respectively.

The values for -threads are interpreted as described in Section 7.4.1.

The only option for which no value exists is -help: it prints some configuration information and a help text for the options and stops program execution.

The class SizeOptions accepts an unsigned integer as the last value on the commandline (of course, without an option). The value can be retrieved or set by member functions size().

The class InstanceOptions accepts a string as the last value on the commandline (of course, without an option). The value can be retrieved or set by member functions instance().

option	type	explanation						
	propagation options							
-icl {def, val, bnd, dom} integer consistency level								

search options							
-solutions	unsigned int	how many solutions (0 for all)					
-threads	double	how many threads					
-c-d (c_d)	unsigned int	commit recomputation distance					
-a-d (a_d)	unsigned int	adaptive recomputation distance					
-node	unsigned int	cutoff for number of nodes					
-fail	unsigned int	cutoff for number of failures					
-time	unsigned int	cutoff for time in milliseconds					
-interrupt	{false,true}	whether driver catches Ctrl-C					

execution options								
-mode	{solution,time,stat,gist}	script mode to run						
-samples	unsigned int	how many samples						
-iterations	unsigned int	how many iterations per sample						
-print-last	{false,true}	whether to only print the last solution						

	user-definable options							
-branching	string	branching options						
-model	string	general model options						
-propagation	string	propagation options						
-symmetry	string	symmetry breaking options						
-search	string	search options						

Figure 9.1: Predefined commandline options

The different modes passed as argument for the option -mode have the following meaning:

- solution prints solutions together with some runtime statistics.
- time can be used for benchmarking: average runtime and coefficient of deviation is printed, where the example is run -samples times. For examples with short runtime, -iterations can be used to repeat the example several times before measuring runtime. Note that the runtime includes also setup time (the creation of the space for the model).
- stat prints short execution statistics.
- gist runs Gist rather than search engines that print information. Gist is put into depth-first mode, when a non best solution search engine is used (that is, DFS), and into branch-and-bound mode otherwise (that is, BAB and Restart).

If Gecode has not been compiled with support for Gist (see Tip 3.3 for how to find out about Gecode's configuration), the mode gist will be ignored and the mode solution is used instead.

For an example, in particular, how to use the user-defined options, see Section 3.3. As all examples that come with Gecode use the script commandline driver, a plethora of examples is available (see Example scripts (models)).

**Gist inspectors and comparators.** The driver options can pass inspectors and comparators (see Section 8.3.4) to Gist. To register an inspector i, use the inspect.click(&i), inspect.solution(&i), or inspect.move(&i) methods of the option object, for a comparator c, use inspect.compare(&c).

## 9.2 Scripts

Scripts (see Script classes) are subclasses of Space that are designed to work together with option objects of class Options and SizeOptions.

In particular, the driver module defines scripts MinimizeScript and MaximizeScript that can be used for finding best solutions based on a virtual cost() function, see also Section 3.2 and Section 6.3.

As scripts are absolutely straightforward, all can be understood by following some examples. For an example see Section 3.3 or all examples that come with Gecode, see Example scripts (models).

C

## Case studies

Christian Schulte, Guido Tack, Mikael Z. Lagerkvist

This part presents a collection of modeling case studies. The case studies are ordered (roughly) according to their complexity.

**Basic models.** The basic models use classic constraint programming problems in order to demonstrate how typical modeling tasks are done with Gecode. The basic models are:

- Chapter 10 (Golomb rulers) shows a simple problem with a single distinct constraint and a few rel constraints. As the problem is so well known, it might serve as an initial case study of how to model with Gecode.
- Chapter 11 (Magic sequence) shows how to use counting constraints (count).
- Chapter 12 (Photo alignment) shows how to use reified constraints for solving an overconstrained problem.
- Chapter 13 (Locating warehouses) shows how to use element, linear, and global counting (count) constraints.
- Chapter 14 (Nonogram) shows how to use regular expressions and extensional constraints.
- Chapter 15 (Social golfers) presents a case study on modeling problems using set variables and constraints.

**Advanced models.** The following models are slightly more advanced:

- Chapter 16 (Knight's tour) demonstrates a problem-specific brancher inspired by a classic heuristic for a classic problem.
- Chapter 17 (Bin packing) also demonstrates a problem-specific brancher including techniques for breaking symmetries during branching.
- Chapter 18 (Kakuro) presents a model that employs user-defined constraints implemented as extensional constraints using tuple set specifications. Interestingly, the tuple set specifications are computed by solving a simple constraint problem.
- Chapter 19 (Crossword puzzle) presents a simple model using nothing but distinct and element constraints. The simple model is shown to work quite well compared to a dedicated problem-specific constraint solver. This underlines that an efficient general-purpose constraint programming system actually can go a long way.

Note that the first two case studies require knowledge on programming branchers, see Part B.

**Acknowledgments.** We thank Pierre Flener and Håkan Kjellerstrand for numerous detailed and helpful comments on the case studies. Their comments have considerably improved the presentation of the case studies.

## Golomb rulers

This chapter studies a simple problem that is commonly used as an example for constraint programming. The model uses nothing but a single distinct constraint, a few rel constraints, and posts linear expressions. As the problem is so well known, it might serve as an initial case study of how to model with Gecode.

## 10.1 Problem

The problem is to find an optimal *Golomb ruler* (see Problem 6 in CSPLib) of size n. A Golomb ruler has n marks  $0 = \mathsf{m}_0 < \mathsf{m}_1 < \dots < \mathsf{m}_{n-1}$  such that the distances  $\mathsf{d}_{i,j} = \mathsf{m}_j - \mathsf{m}_i$  for  $0 \le i < j < n$  are pairwise distinct. An optimal Golomb ruler is of minimal length (that is,  $\mathsf{m}_{n-1}$  is minimal). Figure 10.1 shows an optimal Golomb ruler with 6 marks.



Figure 10.1: An optimal Golomb ruler with 6 marks

In the model for Golomb rulers, we are going to use the following construction for a Golomb ruler (a non-optimal ruler, though) as it provides upper bounds on the values for the marks of a ruler. The upper bounds improve the efficiency of our model, see below for more details.

Assume that the distance between marks i and i+1 is  $m_{i+1}-m_i=2^{i+1}$  (that is, for example,  $m_1-m_0=1$ ,  $m_2-m_1=2$ ,  $m_3-m_2=4$ , and so on). Then the marks are

$$m_i = \sum_{k=1}^i 2^{k-1} = 2^i - 1.$$

Figure 10.2 shows a Golomb ruler with 6 marks following this construction.

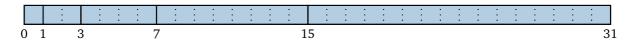


Figure 10.2: A constructed Golomb ruler with 6 marks

```
[DOWNLOAD]
GOLOMB ≡
  class GolombRuler : public MinimizeScript {
  protected:
    IntVarArray m;
  public:
    GolombRuler(const SizeOptions& opt)
      : m(*this,opt.size(),0,
          (opt.size() < 31)
            ? (1 << (opt.size()-1)) - 1
            : Int::Limits::max) {
      CONSTRAINING MARKS
      NUMBER OF MARKS AND DISTANCES
      ► POSTING DISTANCE CONSTRAINTS
      ► IMPLIED CONSTRAINTS
      ► DISTANCES MUST BE DISTINCT
      ► SYMMETRY BREAKING
      ▶ BRANCHING
    virtual IntVar cost(void) const {
      return m[m.size()-1];
  };
```

Figure 10.3: A script for computing Golomb rulers

Now consider the bit representation of  $m_i$ : exactly the least i bits are one. For the distances

$$d_{i,j} = m_j - m_i = \sum_{k=1}^{j} 2^{k-1} - \sum_{k=1}^{i} 2^{k-1}$$

we can easily see that in their bit representation the least i bits are zero, followed by j-i ones. That means for  $0 \le i < j < n$  the bit representations of the  $d_{i,j}$  are pairwise distinct. In other words, we can always construct a Golomb ruler with n marks of length  $m_{n-1} = 2^{n-1} - 1$ .

### **10.2** Model

Figure 10.3 shows the script for implementing the Golomb ruler model. The script stores a variable array m for the marks. The largest possible value of a mark is set to  $2^{n-1} - 1$ 

according to the construction of a Golomb ruler in the previous section, provided that this value does not exceed the possible size limit of an integer (integers in Gecode are at least 32 bits, this is checked when Gecode is configured for compilation). If  $n \ge 31$  we just choose the largest possible integer value for an integer variable (see Int::Limits).

**Tip 10.1** (Small variable domains are still beautiful). As mentioned in Tip 4.3, initializing variable domains to be small makes sense. For example, if we always chose Int::Limits::max rather than the smaller upper bounds for Golomb rulers with n < 31, the propagators for linear constraining the distances would have to resort to extended precision as the internal computations during propagation exceed the integer precision. That would mean that scripts for n < 31 would run approximately 15% slower!

The script does not store a variable array for the distances (unlike the array for the marks), they are stored in an integer variable argument array. As the distances are only needed for posting constraints but not for printing the solution, it is more efficient to store them in an variable argument array but not in a variable array. More details on argument arrays and their relation to variable arrays can be found in Section 4.2.

The cost() function as required by the class MinimizeScript (see Section 9.2) just returns the largest mark on the ruler.

**Marks.** Assigning the first mark to zero and ordering the marks in increasing order is done by posting rel constraints (see Section 4.4.3):

```
CONSTRAINING MARKS ≡
rel(*this, m[0], IRT_EQ, 0);
rel(*this, m, IRT_LE);
```

**Distances.** The number of marks n and number of distances  $n_d$  are initialized so that they can be used for posting constraints:

```
NUMBER OF MARKS AND DISTANCES ≡
const int n = m.size();
const int n_d = (n*n-n)/2;
```

As mentioned, the distances are stored in an integer variable argument array d. The fields of the array d are initialized by the variable returned by the expr() function for linear expressions (see Section 6.1):

```
POSTING DISTANCE CONSTRAINTS 
IntVarArgs d(n_d);
for (int k=0, i=0; i<n-1; i++)
  for (int j=i+1; j<n; j++, k++)
   d[k] = expr(*this, m[j] - m[i]);</pre>
```

One might be tempted to optimize the posting of distance constraints for  $d_{0,j}$  for 0 < j < n as  $m_0 = 0$  and hence  $d_{0,j} = m_j$  for 0 < j < n. Optimizing avoids to create new variables (that is, the variables  $m_j$  are stored as  $d_{0,j}$  for 0 < j < n) and posting propagators to implement the equality constraints  $d_{0,j} = m_j$  for 0 < j < n.

However, the expr() function does this automatically. As  $m_0$  is already assigned by posting a rel constraint, the expr() function simplifies the posted expressions accordingly.

Finally, all distances must be pairwise distinct (see Section 4.4.7) where bounds propagation is requested (see Section 4.3):

```
DISTANCES MUST BE DISTINCT ≡
distinct(*this, d, ICL_BND);
```

Intuitively, bounds propagation is sufficient as also the propagation for the distances is using bounds propagation.

**Implied constraints.** The following implied constraints are due to [43]. A distance  $d_{i,j}$  for  $0 \le i < j < n$  satisfies the property that it is equal to the sum of all distances between marks  $m_i$  and  $m_j$ . That is

$$d_{i,i} = d_{i,i+1} + d_{i+1,i+2} + \cdots + d_{i-1,i}$$

This can be verified as follows:

$$\begin{split} \mathbf{d}_{i,j} &= \mathbf{m}_{j} - \mathbf{m}_{i} \\ &= (\mathbf{m}_{j} - \mathbf{m}_{j-1}) + (\mathbf{m}_{j-1} - \mathbf{m}_{j-2}) + \dots + (\mathbf{m}_{i+1} - \mathbf{m}_{i}) \\ &= \mathbf{d}_{j-1,j} + \mathbf{d}_{j-2,j-1} + \dots + \mathbf{d}_{i,i+1} \\ &= \mathbf{d}_{i,i+1} + \mathbf{d}_{i+1,i+2} + \dots + \mathbf{d}_{j-1,j} \end{split}$$

As all distances  $d_{i,j}$  for  $0 \le i < j < n$  must be pairwise distinct, also the j-i distances

$$\mathsf{d}_{i,i+1},\mathsf{d}_{i+1,i+2},\ldots,\mathsf{d}_{j-1,j}$$

must be pairwise distinct and hence must be j-i distinct integers. That means that

$$d_{i,i} = d_{i,i+1} + d_{i+1,i+2} + \cdots + d_{i-1,i}$$

must be at least the sum of the first j - i integers:

$$d_{i,j} \ge \sum_{l=1}^{j-i} l = (j-i)(j-i+1)/2$$

The implied constraints can be posted as a lower bound with a rel constraint (see Section 4.4.3) for the distances as follows:

```
IMPLIED CONSTRAINTS ≡
  for (int k=0, i=0; i<n-1; i++)
   for (int j=i+1; j<n; j++, k++)
    rel(*this, d[k], IRT_GQ, (j-i)*(j-i+1)/2);</pre>
```

Note that one could also combine the posting of the distance constraints with constraining the lower bounds of the distances for efficiency. However, we separate both for clarity. Anyway, the time spent on posting constraints is insignificant to the time spent on solving the model!

**Symmetry breaking.** Provided that the ruler has a sufficient number of marks (that is, n > 2) we can break (a few) symmetries by constraining the distance  $d_{0,1}$  (stored at the first position in the array d) between the first and second mark to be smaller than the distance  $d_{n-2,n-1}$  (stored at the last position in the array d) between the next to last and last mark as follows:

```
SYMMETRY BREAKING ≡
if (n > 2)
rel(*this, d[0], IRT_LE, d[n_d-1]);
```

**Branching.** The branching chooses the marks from left to right on the ruler and assigns the smallest possible value for a mark first:

```
BRANCHING ≡
branch(*this, m, INT_VAR_NONE, INT_VAL_MIN);
```

## 10.3 More information

This case study is also available as an example, see Finding optimal Golomb rulers. For a detailed discussion of how to model the Golomb ruler problem, see [43].

## Magic sequence

This chapter shows how to use counting constraints for solving magic sequence puzzles.

### 11.1 Problem

The Magic Sequence puzzle (Problem 19 in CSPLib, first introduced as a constraint problem in [46]) requires finding a sequence of integers  $x_0, ..., x_{n-1}$  such that for all  $0 \le i < n$ , the number i occurs exactly  $x_i$  times in the sequence. For example, a magic sequence of length n = 8 is

$$\langle 4, 2, 1, 0, 1, 0, 0, 0 \rangle$$

### **11.2** Model

The outline of the script for solving magic sequence puzzles is shown in Figure 11.1. It contains the integer variable array x (see Section 4.2.1), which is initialized according to the problem specification.

**Counting constraints.** The problem can be modeled directly using one counting constraint (see Section 4.4.8) per variable. We will see later that there is a global constraint that combines all these individual counting constraints.

```
COUNTING CONSTRAINTS ≡
for (int i=0; i<x.size(); i++)
count(*this, x, i, IRT_EQ, x[i]);</pre>
```

**Implied linear constraints.** The model as described so far completely captures the problem. Therefore, given enough time, Gecode will return all its solutions and only the solutions. However, it is sometimes useful to post additional, *implied constraints*, which do not change the meaning of the model (they do not change the set of solutions), but which provide additional constraint propagation that results in a smaller search tree.

Figure 11.1: A script for solving magic sequence puzzles

The two implied constraints that we will use for the magic sequence problem result from the fact that any magic sequence of length *n* satisfies the following two equations:

$$\sum_{i=0}^{n-1} x_i = n \qquad \sum_{i=0}^{n-1} (i-1) \cdot x_i = 0$$

The first equation is true because the sum of all occurrences, i.e. the overall number of items in the sequence, must be equal to the length of the sequence.

The second equation can be rewritten as

$$\sum_{i=0}^{n-1} i \cdot \mathsf{x}_i = \sum_{i=0}^{n-1} \mathsf{x}_i \iff \sum_{i=0}^{n-1} i \cdot \mathsf{x}_i = n$$

So it remains to be shown that  $\sum_{i=0}^{n-1} i \cdot x_i$  is also equal to the length of the sequence. This follows from the fact that  $x_i$  is the number of times i occurs in the sequence, so  $i \cdot x_i$  is the number of positions occupied by the sequence elements that are equal to i, and the sum over those must be equal to the length of the sequence.

The two equations translate easily into linear constraints (see Section 4.4.6), using integer argument arrays of type IntArgs (see Section 4.2.2) to supply coefficients.

```
IMPLIED CONSTRAINTS ≡
linear(*this, x, IRT_EQ, x.size());
linear(*this, IntArgs::create(x.size(),-1,1), x, IRT_EQ, 0);
```

You can do your own experiments, comparing runtime and search tree size of the model with and without implied constraints.

```
[DOWNLOAD]
MAGIC SEQUENCE GCC ≡
  class MagicSequence : public Script {
    IntVarArray x;
  public:
    MagicSequence(const SizeOptions& opt)
      : x(*this,opt.size(),0,opt.size()-1) {
      // Global counting constraint
      count(*this, x, x, opt.icl());
      // Implied constraint
      linear(*this, IntArgs::create(x.size(),-1,1), x, IRT_EQ, 0);
      // Branching
      branch(*this, x, INT_VAR_NONE, INT_VAL_MAX);
   }
    . . .
  };
```

Figure 11.2: Magic sequence puzzles with a global counting constraint

**Branching.** For large sequences, many variables in the sequence will be 0 because the overall sum is only n (see previous paragraph on implied constraints). Therefore,  $x_0$  should take a large value. We simply branch in the given order of the variables, starting with the largest values.

```
BRANCHING =
branch(*this, x, INT_VAR_NONE, INT_VAL_MAX);
```

**Global counting constraints.** The global counting constraint (also known as global cardinality constraint, see Section 4.4.8) can express the combination of all the individual counting constraints, and yields stronger propagation. It also includes the propagation of the first of the two implied linear constraints. So, as an alternative to the n counting constraints above, we can use the code in Figure 11.2.

#### 11.3 More information

The magic sequence puzzle is also included as a Gecode example, see Magic sequence. The example contains both the model using individual counting constraints and the one using a single global counting constraint.

## Photo alignment

This chapter shows how to use reified constraints for solving an overconstrained problem.

## 12.1 Problem

Betty, Chris, Donald, Fred, Gary, Mary, Paul, Peter, and Susan want to align in a row for taking a photo. They have the following preferences:

- 1. Betty wants to stand next to Donald, Gary, and Peter.
- 2. Chris wants to stand next to Gary and Susan.
- 3. Donald wants to stand next to Fred and Gary.
- 4. Fred wants to stand next to Betty and Gary.
- 5. Gary wants to stand next to Mary and Betty.
- 6. Mary wants to stand next to Betty and Susan.
- 7. Paul wants to stand next to Donald and Peter.
- 8. Peter wants to stand next to Susan and Paul.

These preferences are obviously not satisfiable all at once (e.g., Betty cannot possibly stand next to three people at once). The problem is *overconstrained*. To solve an overconstrained problem, we turn it into an optimization problem: The task is to find an alignment that violates as few preferences as possible.

## **12.2** Model

We model the photo alignment as an array of integer variables pos such that pos[p] represents the position of person p in the final left-to-right order. The outline of a script for this problem is shown in Figure 12.1.

The cost() function as required by the class MinimizeScript (see Section 9.2) just returns the number of violations.

```
[DOWNLOAD]
рното ≡
  . . .
 enum {
    Betty, Chris, Donald, Fred, Gary,
    Mary, Paul, Peter, Susan
 };
 const int n = 9;
  const int n_prefs = 17;
 int spec[n_prefs][2] = {
    {Betty,Donald}, {Betty,Gary}, {Betty,Peter},
    {Chris,Gary}, {Chris,Susan},
    {Donald, Fred}, {Donald, Gary},
    {Fred, Betty}, {Fred, Gary},
    {Gary, Mary}, {Gary, Betty},
    {Mary,Betty}, {Mary,Susan},
    {Paul, Donald}, {Paul, Peter},
    {Peter, Susan}, {Peter, Paul}
 };
  class Photo : public MinimizeScript {
    IntVarArray pos;
    IntVar
                violations;
 public:
    Photo(const Options& opt)
      : pos(*this,n,0,n-1), violations(*this,0,n_prefs) {
      ► CONSTRAIN POSITIONS
      ► COMPUTE VIOLATIONS
      ► SYMMETRY BREAKING
    virtual IntVar cost(void) const {
      return violations;
    . . .
 };
```

Figure 12.1: A script for the photo alignment problem

There are only two hard constraints for this model: no person can be in more than one place, and no two persons can stand in the same place. The first constraint is enforced automatically by the choice of variables, as each pos variable represents the unique position of a person (see also Tip 13.1). For the second constraint, the variables in the pos array must be pairwise distinct (see Section 4.4.7):

```
CONSTRAIN POSITIONS ≡
distinct(*this, pos, ICL_BND);
```

We choose the bounds consistent variant of distinct (by giving the extra argument ICL\_BND, see Section 4.3) as also the other propagators perform only bounds reasoning.

The remaining constraints implement the preferences and turn them into a measure of *violation*, which expresses how many preferences are not fulfilled in a solution. A preference (i, j) is not fulfilled if the distance between the positions of person i and person j is greater than one. This can be implemented using a linear constraint, an absolute value constraint, and a reified constraint for each preference, as well as one linear constraint that constrains the sum of the violations:

Using the MiniModel library (see Figure 6.2 and Section 6.1) yields more compact and readable code:

```
COMPUTE VIOLATIONS =
BoolVarArgs viol(n_prefs);
for (int i=0; i<n_prefs; i++) {
  viol[i] = expr(*this, abs(pos[spec[i][0]]-pos[spec[i][1]]) > 1);
}
rel(*this, violations == sum(viol));
```

We can observe that this problem has a symmetry, as reversing a solution yields again a solution. Symmetric solutions like this can be ruled out by arbitrarily picking two persons, and always placing one somewhere to the left of the other. For example, let us always place Betty somewhere to the left of Chris:

```
SYMMETRY BREAKING ≡
rel(*this, pos[Betty] < pos[Chris]);</pre>
```

## 12.3 More information

This case study is also available as a Gecode example, see Placing people on a photo.

## Locating warehouses

This chapter demonstrates the warehouse location problem. It shows how to use element, global counting (count), and linear constraints.

## 13.1 Problem

The problem is taken from [47, Chapter 10], see also Problem 34 in CSPLib. A company needs to construct warehouses to supply stores with goods. Each candidate warehouse has a certain capacity defining how many stores it can supply. Each store shall be supplied by exactly one warehouse. Maintaining a warehouse incurs a fixed cost. Costs for transportation from warehouses to stores depend on the locations of warehouses and stores.

We want to determine which warehouses should be opened (that is, supply to at least one store) and which warehouse should supply which store such that the overall cost (transportation costs plus fixed maintenance costs) is smallest.

In the following problem instance, the fixed maintenance cost c\_fixed for a warehouse is 30. There are five candidate warehouses  $w_0, \ldots, w_4$  and ten stores  $s_0, \ldots, s_9$ . The candidate warehouses have the following capacity:

$w_0$	$w_1$	$w_2$	$w_3$	$w_4$
1	4	2	1	3

The costs to supply a store by a candidate warehouse are defined by a matrix c\_supply<sub>i,j</sub> ( $0 \le i < 10, 0 \le j < 5$ ) as follows:

	$w_0$	$w_1$	$w_2$	$w_3$	$w_4$
$s_0$	20	24	11	25	30
$s_1$	28	27	82	83	74
$s_2$	74	97	71	96	70
$s_3$	2	55	73	69	61
s <sub>4</sub>	46	96	59	83	4
<i>s</i> <sub>5</sub>	42	22	29	67	59
<i>s</i> <sub>6</sub>	1	5	73	59	56
s <sub>7</sub>	10	73	13	43	96
<i>s</i> <sub>8</sub>	93	35	63	85	46
<i>s</i> <sub>9</sub>	47	65	55	71	95

```
[DOWNLOAD]
WAREHOUSES ≡
 const int n_warehouses = 5;
 const int n_stores = 10;
 const int capacity[n_warehouses] = {
    1, 4, 2, 1, 3
 };
 const int c_fixed = 30;
 const int c_supply[n_stores][n_warehouses] = {
 };
 class Warehouses : public MinimizeScript {
 protected:
    ► VARIABLES
 public:
    Warehouses(const Options&)
      ► VARIABLE INITIALIZATION
    {
        ▶ DO NOT EXCEED CAPACITY
      ► OPEN WAREHOUSES
      ► COST FOR EACH WAREHOUSE
      ► TOTAL COST
      ▶ BRANCHING
    ► COST FUNCTION
 };
```

Figure 13.1: A script for locating warehouses

### **13.2** Model

The outline for the script implementing our model is shown in Figure 13.1. The data definitions are as described in the previous section.

As we need to minimize the total cost, the script inherits from the class MinimizeScript which is a driver-defined subclass (similar to Script and Space) of MinimizeSpace (see Section 6.3), see Section 9.2. This in particular means that the class must define a virtual cost() function, see below.

**Variables.** The script declares the following variables:

```
VARIABLES 
IntVarArray supplier;
BoolVarArray open;
IntVarArray c_store;
IntVar c_total;
```

#### where:

- for each store s, there is a variable supplier<sub>s</sub> such that supplier<sub>s</sub> = w if warehouse w supplies store s;
- for each warehouse w, there is a Boolean variable open $_w$  which equals one, if the warehouse w supplies at least one store;
- for each store s, there is a variable c\_store<sub>s</sub> which defines the cost for s to be supplied by warehouse supplier<sub>s</sub>;
- a variable c\_total which defines the total cost.

**Tip 13.1** (Choose variables to avoid constraints). Just by the choice of supplier variables where supplier<sub>s</sub> = w if warehouse w supplies store s, the problem constraint that each store shall be supplied by exactly one warehouse is enforced. Hence, no explicit constraints must be posted in our model. After all, no supplier variable can take on two different values!

This is good modeling practice: choosing variables such that some of the problem constraints are automatically enforced.

The variables are initialized as follows:

```
VARIABLE INITIALIZATION ≡
  : supplier(*this, n_stores, 0, n_warehouses-1),
    open(*this, n_warehouses, 0, 1),
    c_store(*this, n_stores)
```

We only declare but do not initialize the cost variables c\_store and c\_total, as we will assign them by results obtained by posting expressions, see Section 6.1. The difference between declaring variables and initializing them such that new variables are created is explained in detail in Section 4.1.1.

**Constraints.** For a given warehouse w the following must hold: the number of stores s supplied by w is not allowed to exceed the capacity of w. This can be expressed by a counting constraint count (see Section 4.4.8) as follows:

```
DO NOT EXCEED CAPACITY =
IntSetArgs c(n_warehouses);
for (int w=0; w<n_warehouses; w++)
  c[w] = IntSet(0,capacity[w]);
count(*this, supplier, c, ICL_DOM);</pre>
```

Here, the array of integer sets c defines how many stores can be supplied by a warehouse, where  $c_w$  contains every legal number of occurrences of w in supplier. To achieve strong propagation for the count constraint, we choose domain propagation by providing the additional argument ICL\_DOM (see Section 4.3).

For a given warehouse w the following must hold: if the number of stores s supplied by w is at least one, then  $\mathsf{open}_w$  equals one. That is,  $\mathsf{open}_{\mathsf{supplier}_s}$  must be 1 for all stores s. This is expressed by using element constraints (see Section 4.4.12) as follows:

```
OPEN WAREHOUSES ≡
for (int s=0; s<n_stores; s++)
  element(*this, open, supplier[s], 1);</pre>
```

**Cost computation.** The cost c\_store<sub>s</sub> for each store s is computed by an element constraint (see Section 4.4.12), mapping the warehouse supplying s to the appropriate cost:

```
COST FOR EACH WAREHOUSE 
for (int s=0; s<n_stores; s++) {
   IntArgs c(n_warehouses, c_supply[s]);
   c_store[s] = expr(*this, element(c, supplier[s]));
}</pre>
```

The total cost  $c_{total}$  is defined by the cost of open warehouses (that is, the sum of  $open_{w}$  for all warehouses w multiplied with the fixed maintenance cost for a warehouse) and the cost of stores (that is, the sum of the  $c_{store}$  for all stores s):

```
TOTAL COST ≡
c_total = expr(*this, c_fixed*sum(open) + sum(c_store));
```

Note that the linear expression posted for defining the total cost mixes integer and Boolean variables and is automatically decomposed into the appropriate linear constraints, see Section 6.1.

**Tip 13.2** (Small variable domains are still beautiful). As mentioned in Tip 4.3, initializing variable domains to be small makes sense.

Here we choose to post expressions (see Section 6.1) via the expr() function that returns an integer variable. The integer variable returned by expr() has automatically computed and reasonable bounds.

A different choice would be to initialize the variables c\_store and c\_total in the constructor of the script and constrain them explicitly (for example, via the rel() function where the expression for expr() is turned into an equality relation). But that would mean that we either have to compute some estimates for the bounds used for initializing the variables or resort — without real necessity — to the largest possible integer value.

**Branching.** The branching proceeds in two steps, implemented by two different branchings. The first branching assigns values to the variables c\_store<sub>s</sub> and follows the strategy of maximal regret: select the variable for which the difference between the smallest value and the next larger value is maximal (see Section 7.1.1). The second branching makes sure that all stores are being assigned a warehouse. This branching is necessary as supplying a store could have the same cost for several different warehouses (depending on the data used for the model). Hence, even though all variables in c\_store are assigned, not all variables in supplier must be assigned and hence the total cost is also not assigned. The branchings are posted in the appropriate order as follows:

```
BRANCHING ≡
branch(*this, c_store, INT_VAR_REGRET_MIN_MAX, INT_VAL_MIN);
branch(*this, supplier, INT_VAR_NONE, INT_VAL_MIN);
```

**Cost function.** The cost function cost() to be used by the search engine is defined to return the total cost  $c_{-}$ total:

```
COST FUNCTION ≡
  virtual IntVar cost(void) const {
   return c_total;
}
```

#### 13.3 More information

This problem is also available as a Gecode example, see Locating warehouses. The model presented in [47, Chapter 10] proposes a better branching than the branching shown in the previous section.

## Nonogram

This chapter shows how to use regular expressions and extensional constraints for solving nonogram puzzles.

### 14.1 Problem

Nonograms (Problem 12 in CSPLib) are popular puzzles in which the puzzler shades squares in a matrix. Each instance of the puzzle has constraints on the rows and columns of the matrix, specifying the number and length of the groups of consecutive marks in that row or column. For example, a row that in the solution has the marks

has the *hint* 2 3 1, indicating that there are three separate groups of marks, with lengths 2, 3, and 1. Given two groups of marks, there must be at least one empty square in between. An example nonogram is given in Figure 14.1 and its solution is shown in Figure 14.2. The general nonogram problem is NP-complete, as shown in [45].

#### **14.2** Model

The model follows naturally from the constraints of the problem. The variables needed are a matrix  $x_{ij}$  of 0-1 variables, representing the squares to shade. For the hint 2 3 1 on row i, we post the following extensional constraint (see Section 4.4.13):

extensional(
$$x_{i\bullet}$$
,  $0^*1^20^+1^30^+10^*$ )

The regular expression starts and ends with zero or more zeroes. Each group is represented by as many ones as the group length. In between the groups, one or more zeroes are placed. Using this construction, we get one constraint per row and column of the matrix. The outline of the script is shown in Figure 14.3.

**Puzzle specification.** The puzzle is specified by an array of integers. The first two integers specify the width and the height of the grid. These are followed first by the column and then the row hints. Each hint specifies the number of groups and the length of each group. For example, the hint used as an example above is specified as 3, 2, 3, 1. The puzzle from Figure 14.1 is written as follows.

				2	2	2	2	2	2	2	
			3	3	2	2	2	2	2	3	3
-	2	2									
	4	4									
1	3	1									
2	1	2									
	1	1									
	2	2									
	2	2									
		3									
		1									

Figure 14.1: Example nonogram puzzle

				2	2	2	2	2	2	2	
			3	3	2	2	2	2	2	3	3
	2	2									
	4	4									
1	3	1									
2	1	2									
	1	1									
	2	2									
	2	2									
		3									
		1									

Figure 14.2: Solution to the example puzzle

```
[DOWNLOAD]
NONOGRAM ≡
 . . .
  ► PUZZLE
 class Nonogram : public Script {
   int width, height;
   BoolVarArray b;
   DFA line(const int*& p) {
      ► LINE FUNCTION
   }
 public:
   Nonogram(const Options& opt)
      : width(spec[0]), height(spec[1]),
        b(*this,width*height,0,1) {
      Matrix<BoolVarArray> m(b, width, height);
      ► INTIALIZE HINT POINTER
      ► COLUMN CONSTRAINTS
      ► ROW CONSTRAINTS
      ▶ BRANCHING
   }
 };
 int main(int argc, char* argv[]) {
 }
```

Figure 14.3: A script for solving nonogram puzzles

**Line function.** For the hint that starts at p (that is, p points to a position in the array of integers spec[] where a hint starts), the following code constructs a regular expression (see Section 6.4) that matches that hint.

```
int nhints = *p++;
REG r0(0), r1(1);
REG border = *r0;
REG separator = +r0;
REG result = border;
if (nhints > 0) {
    result += r1(*p,*p);
    p++;
    for (int i=nhints-1; i--; p++)
        result += separator + r1(*p,*p);
}
return result + border;
```

The variables r0 and r1 represent the constants 0 and 1 (this is a slight optimization to construct a regular expression for the constants 0 and 1 just once). The variables border and separator represent sequences of zeroes at the borders and between marks. The loop adds all hints (as repeated r1s) to the result expression with separators in between. Note that if the hint is just 0 (representing an empty line with no mark), then the result will be just the same as a border+border, which is  $0^*0^* = 0^*$ .

**Constraints.** Given the line() function, posting the appropriate constraints is as follows. The pointer p is initialized to point to the first hint:

```
INTIALIZE HINT POINTER ≡
const int* p = spec+2;
```

Two loops go through all the hints, get the regular expression for the line, and post the constraints for the appropriate variables. First, the column constraints are posted:

```
COLUMN CONSTRAINTS ≡
for (int w=0; w<width; w++)
  extensional(*this, m.col(w), line(p));</pre>
```

followed by the row constraints:

```
ROW CONSTRAINTS ≡
for (int h=0; h<height; h++)
  extensional(*this, m.row(h), line(p));</pre>
```

**Branching.** Choosing a branching for nonograms is not obvious. For many nonogram puzzles, using an AFC-based branching (see Section 7.1.1) is a good idea:

```
BRANCHING ≡
branch(*this, b, INT_VAR_AFC_MAX, INT_VAL_MAX);
```

The choice to use INT\_VAL\_MAX is because most puzzles will have fewer marks than empty spaces. For the example puzzle from Figure 14.1, propagation alone solves the puzzle. To solve really hard puzzles, a custom branching may be needed.

### 14.3 More information

The nonogram puzzle is also included as a Gecode example, see Nonogram. The example in particular features several grids to try the model on.

Despite its simplicity, the program for solving nonograms works amazingly well. Extensive information on nonogram puzzles and a comparison of different nonogram solvers (including the model described in this chapter) is Survey of Paint-by-Number Puzzle Solvers.

# **15**

# Social golfers

This chapter presents a case study on modeling problems using set variables and constraints.

#### 15.1 Problem

The social golfers' problem (Problem 10 in CSPLib) requires finding a schedule for a golf tournament. There are  $g \cdot s$  golfers who want to play a tournament in g groups of s golfers each over w weeks, such that no two golfers play against each other more than once during the tournament.

Here is a solution for the instance w = 4, g = 3, and s = 3, where the players are numbered from 0 to 8:

	Group 0			Group 1			Group 2		
Week 0	0	1	2	3	4	5	6	7	8
Week 1	0	3	6	1	4	7	2	5	8
Week 2	0	4	8	1	5	6	2	3	7
Week 3	0	5	7	1	3	8	2	4	6

### **15.2** Model

The model for the social golfers' problem closely follows the above problem description. Its outline is shown in Figure 15.1. The script defines an array of set variables groups of size  $g \cdot w$ , where each group can contain the players  $0 \dots g \cdot s - 1$  and has cardinality s (see Section 5.1).

The script also defines a matrix schedule with g columns and w rows on top of the variable array, such that schedule(i,j) is the set of members of group i in week j.

The constraints are straightforward. For each week, the union of all groups must be disjoint and contain all players. This can be expressed directly using a disjoint union constraint (see Section 6.1) on the rows of the schedule:

```
GROUPS IN A WEEK ≡
SetVar allPlayers(*this, 0,g*s-1, 0,g*s-1);
for (int i=0; i<w; i++)
rel(*this, setdunion(schedule.row(i)) == allPlayers);</pre>
```

```
[DOWNLOAD]
GOLF ≡
  class GolfOptions : public Options {
 };
  class Golf : public Script {
    int g, s, w;
    SetVarArray groups;
  public:
    Golf(const GolfOptions& opt)
    : g(opt.g()), s(opt.s()), w(opt.w()),
      groups(*this,g*w,IntSet::empty,0,g*s-1,s,s) {
      Matrix<SetVarArray> schedule(groups,g,w);
      ► GROUPS IN A WEEK
      ► OVERLAP BETWEEN GROUPS
      ► BREAK GROUP SYMMETRY
      ▶ BREAK WEEK SYMMETRY
      ▶ BREAK PLAYER SYMMETRY
      branch(*this, groups, SET_VAR_MIN_MIN, SET_VAL_MIN_INC);
    }
    . . .
  };
```

Figure 15.1: A script for the social golfers' problem

Each group can have at most one player in common with any other group. This can be expressed by a constraint that states that the cardinality of the intersection between any two groups must be at most 1:

```
OVERLAP BETWEEN GROUPS ≡
for (int i=0; i<groups.size()-1; i++)
for (int j=i+1; j<groups.size(); j++)
rel(*this, cardinality(groups[i] & groups[j]) <= 1);</pre>
```

**Symmetry breaking.** Using set variables to model the groups already avoids introducing symmetry among the players in a group. For example, if we had modeled each group as *s* integer variables, any permutation of these variables would produce an equivalent solution.

But there are more symmetries in this problem, and some of them can be avoided easily by introducing additional *symmetry breaking constraints*.

Within a week, the order of the groups is irrelevant. Therefore, we can impose a static order requiring that all minimal elements of each group are ordered increasingly (see Section 5.2.5 for the minimal element constraint, Section 6.1 for the MiniModel support, and Section 4.4.3 for ordering integer variables):

```
BREAK GROUP SYMMETRY =
  for (int j=0; j<w; j++) {
    IntVarArgs m(g);
    for (int i=0; i<g; i++)
        m[i] = expr(*this, min(schedule(i,j)));
    rel(*this, m, IRT_LE);
}</pre>
```

Similarly to the group symmetry, the order of the weeks is irrelevant. Again, the symmetry can be broken by imposing an order on the group elements. The previous constraint made sure that player 0 will always be in schedule(0,j) for any week j. So imposing an order on the second smallest element of schedule(0,j) will do the trick:

```
BREAK WEEK SYMMETRY =
IntVarArgs m(w);
for (int j=0; j<w; j++)
   m[j] = expr(*this, min(schedule(0,j)-IntSet(0,0)));
rel(*this, m, IRT_LE);</pre>
```

Finally, the players can be permuted arbitrarily. For example, swapping the numbers 2 and 6 in the initial example produces a symmetric solution:

	Group 0			Group 1			Group 2		
Week 0	0	1	6	3	4	5	2	7	8
Week 1	0	3	2	1	4	7	6	5	8
Week 2	0	4	8	1	5	2	6	3	7
Week 3	0	5	7	1	3	8	6	4	2

This symmetry can be broken using the precede constraint (see Section 5.2.8):

```
BREAK PLAYER SYMMETRY ≡
precede(*this, groups, IntArgs::create(groups.size(),0));
```

It enforces for any pair of players *s* and *t* that *t* can only appear in a group without *s* if there is an earlier group where *s* appears without *t*. This establishes an order that breaks the value symmetry between the players. In the example above, the constraint rules out that 6 appears in group 0, week 0, because that would require 2 to appear in an earlier group. The only solution that remains after symmetry breaking is the one in the initial table in Section 15.1.

Note that these symmetry breaking constraints do not necessarily break all symmetries of the problem completely. We mainly discussed them as additional examples of modeling with set variables and constraints.

## 15.3 More information

The case study is also available as a Gecode example, see Golf tournament. You can find a discussion of the symmetry breaking constraints presented here and a number of additional implied constraints in [2].

# 16

# Knight's tour

This chapter demonstrates a problem-specific brancher inspired by a classic heuristic for a classic problem.

**Important.** This case study requires knowledge on programming branchers, see Part B.

### 16.1 Problem

The problem of a knight's tour is to find a series of knight's moves (a knight can move two fields vertically and simultaneously one field horizontally, or vice versa) on an empty  $n \times n$  board starting from some initial field such that:

- each field is visited exactly once, and
- there is a further knight's move from the last field of the tour to the initial field.

Figure 16.1 shows a knight's tour for n = 8, where the tour starts at the lower left corner.

#### **16.2** Model

The model for the knight's tour uses a successor representation for the knight's moves to make posting the constraints straightforward. To further simplify posting the constraints, the variables in the model use *fields* on the board as values. The field 0 has  $\langle x, y \rangle$ -coordinates  $\langle 0, 0 \rangle$  on the board, the field 1 has coordinates  $\langle 1, 0 \rangle$ , the field n on an  $n \times n$  board has coordinates  $\langle 0, 1 \rangle$ , and so on. That is, fields on the board are counted first in x-direction and then in y-direction.

The successor representation means that a variable in the model has the field numbers as possible values that can be reached by a knight's move. The model uses the circuit constraint (see Section 4.4.17) to enforce that the tour is in fact a Hamiltonian circuit. The only additional constraints needed are that fields must be reachable only by knight's moves.

Figure 16.2 outlines the program to implement the knight's tour model. An object of class Knights stores the size of the board as its member n. The variables for the knight moves are stored in the integer variable array (see Section 4.2.1) succ. The array succ has  $n^2$  elements where the variable at position f stores the successor of the field f. The function

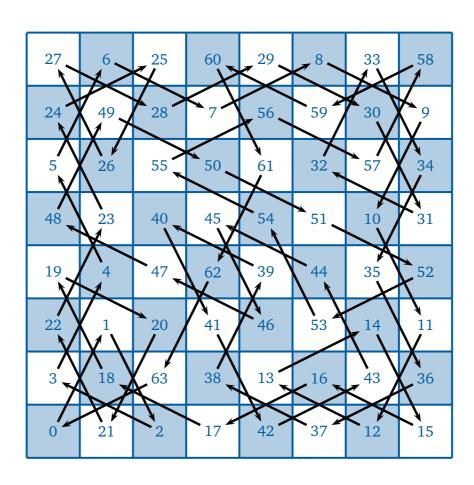


Figure 16.1:  $8 \times 8$ -knight's tour

```
KNIGHTS ≡
                                                               [DOWNLOAD]
 #include <climits>
  ▶ BRANCHER
 class Knights : public Script {
 protected:
   const int n;
    IntVarArray succ;
 public:
   int f(int x, int y) const { return x + y*n; }
   IntSet neighbors(int i) {
   Knights(const SizeOptions& opt)
      : n(opt.size()), succ(*this,n*n,0,n*n-1) {
      ► KNIGHT'S MOVES
      ► FIX FIRST MOVE
      ► HAMILTONIAN CIRCUIT
     warnsdorff(*this, succ);
   }
 };
```

Figure 16.2: A script for the knight's tour problem

f(x,y) computes the field number for coordinates (x,y). For example, f(0,0)=0, f(1,0)=1, and f(0,1)=n.

**Enforcing knight's moves.** Domain constraints dom (see Section 4.4.1) are used to constrain moves to knight's moves:

```
KNIGHT'S MOVES ≡
for (int i=0; i<n*n; i++)
dom(*this, succ[i], neighbors(i));</pre>
```

The function neighbors(i) returns an integer set which contains the fields that are reachable from field i by a knight's move. For example, for n = 8, neighbors(0) (the field 0 has coordinates (0,0)) returns the integer set  $\{17,10\}$  (that is, the fields with coordinates (2,1) and (1,2)) and neighbors(27) (the field 27 has coordinates (3,3)) returns the integer set  $\{17,33,10,42,12,44,21,37\}$  (that is, the fields with coordinates (2,1), (4,1), (1,2), (5,2), (1,4), (5,4), (2,5), and (4,5)).

**Fixing the first move.** Without loss of generality we fix that the knight's first move goes to field f(1,2):

```
FIX FIRST MOVE \equiv rel(*this, succ[0], IRT_EQ, f(1,2));
```

Fixing the first move can be seen as breaking a symmetry in the model and hence reduces the amount of search needed for finding a knight's tour.

**Enforcing a Hamiltonian circuit.** The circuit constraint (see Section 4.4.17) enforces that the tour of knight's moves forms a Hamiltonian circuit:

```
HAMILTONIAN CIRCUIT ≡
  circuit(*this, succ, ICL_DOM);
```

We request domain propagation for the circuit constraint by providing ICL\_DOM as argument (see Section 4.3). Intuitively, we want to have the strongest possible propagation available for circuit as it is the only constraint.

## 16.3 Branching

The really interesting aspect of solving the knight's tour puzzle is to find a branching that works well. A classic approach is Warnsdorff's heuristic [52]: move the knight to a field that has the least number of further possible moves.

<sup>&</sup>lt;sup>1</sup>Of course, domain propagation for circuit does not achieve domain consistency, as the problem of finding Hamiltonian circuits is NP-complete [12, p. 199].

In terms of our model, Warnsdorff's heuristic has of course no procedural notion of *moving* the knight! Instead, our branching finds a yet unassigned field i on the board (a field whose successor is not known yet). Then, it first tries a value for  $succ_i$  that moves the knight to a field with the least number of further possible moves. That is, the branching first tries a value n for  $succ_i$  such that the domain size of  $succ_n$  is smallest. If there are several such values, it just tries the smallest first (as it is most natural to implement).

The brancher. Figure 16.3 shows an outline of a branching and a brancher implementing Warnsdorff's heuristic. The members of Warnsdorff are exactly the same as for the example brancher in Section 28.2.2: the view array x stores the knight's moves, the mutable integer start points to the current unassigned view in x, and the PosVal choice stores the position of the view and the value to be used for branching.

**Status computation.** The status() function tries to find a yet unassigned view for branching in the view array x. It starts inspecting the views from position start. If it finds an assigned view, it moves start to the position in the view array as defined by the assigned view's value. In other words, status() follows partially constructed knight's tours. If status() finds an unassigned view, it returns **true**:

```
status function \( \)
  virtual bool status(const Space&) const {
    for (int n=0; n<x.size(); n++) {
        if (!x[start].assigned())
            return true;
        start = x[start].val();
    }
    return false;
}</pre>
```

The number of attempts to find an unassigned view is limited by the number of views in the view array x. If the limit is exceeded, all views are assigned and hence the status() function returns **false**.

**Choice computation.** The choice() function implements the actual heuristic. It chooses the value of x[start] for branching that has the smallest domain size as follows:

```
choice function =
  virtual Choice* choice(Space&) {
   int n=-1; unsigned int size=UINT_MAX;
  for (Int::ViewValues<Int::IntView> i(x[start]); i(); ++i)
   if (x[i.val()].size() < size) {
      n=i.val(); size=x[n].size();
   }
  return new PosVal(*this,start,n);
}</pre>
```

As mentioned above, to keep the implementation simple, if there are several values that have smallest domain size, the first is chosen (and hence the smallest).

Note that the value UINT\_MAX is larger than the size of any view domain.<sup>2</sup> Hence, it is guaranteed that the **for**-loop will always choose a value from the domain of x[start]. The choice returned is the typical implementation to store position and value, similar to the examples in Section 28.2.2.

## 16.4 More information

The model is also available as a Gecode example, see n-Knights tour (model using circuit). The Gecode example also features a simple standard branching that can be compared to Warnsdorff's branching and a naive model using reification instead of circuit (see n-Knight's tour (simple model)).

 $<sup>^2</sup>$ UINT\_MAX (for the maximal value of an unsigned integer) is available as the program includes the <climits> header file.

Figure 16.3: A brancher for Warnsdorff's heuristic

# **17**

# Bin packing

This chapter studies the classic bin packing problem. Three models are presented: a first and naive model (presented in Section 17.2) that suffers from poor propagation to be feasible. This is followed by a model (Section 17.3) that uses the special binpacking constraint to drastically improve constraint propagation. A final model improves the second model by a problem-specific branching (Section 17.4) that also breaks many symmetries during search.

**Important.** This case study requires knowledge on programming branchers, see Part B.

### 17.1 Problem

The bin packing problem consists of packing n items of sizes  $size_i$  ( $0 \le i < n$ ) into the smallest number of bins such that the capacity c of each bin is not exceeded.

For example, the 11 items of sizes

require at least four bins of capacity 10 as shown in Figure 17.1 where the items are numbered starting from zero.

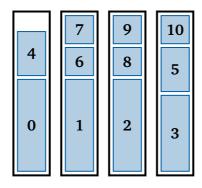


Figure 17.1: An example optimal bin packing

```
INSTANCE DATA =
  const int c = 100;
  const int n = 50;
  const int size[n] = {
    99,98,95,95,95,94,94,91,88,87,86,85,76,74,73,71,68,60,55,54,51,
    45,42,40,39,39,36,34,33,32,32,31,31,30,29,26,26,23,21,21,21,19,
    18,18,16,15,5,5,4,1
  };
```

Figure 17.2: Instance data for a bin packing problem

```
COMPUTE LOWER BOUND ≡
  int lower(void) {
   int s=0;
   for (int i=0; i<n; i++)
       s += size[i];
   return (s + c - 1) / c;
}</pre>
```

Figure 17.3: Computing a lower bound for the number of bins

### 17.2 A naive model

Before turning our attention to a naive model for the bin packing problem, this section discusses instance data for the bin packing problem and how to compute lower and upper bounds for the number of required bins.

**Instance data.** Figure 17.2 shows an example instance of a bin packing problem, where n defines the number of items, c defines the capacity of each bin, and the array size defines the size of each item. For simplicity, we assume that the item size are ordered in decreasing order.

The data corresponds to the instance N1C1W1\_N taken from [31]. More information on other data instances can be found in Section 17.5.

**Computing a lower bound.** A simple lower bound  $L_1$  (following Martello and Toth [22]) for the number of bins required for a bin packing problem just considers the size of all items and the bin capacity as follows:

$$L_1 = \left\lceil \frac{1}{c} \sum_{i=0}^{\mathsf{n}-1} \mathsf{size}_i \right\rceil$$

```
int upper(void) {
  int* free = new int[n];
  for (int i=0; i<n; i++)
    free[i] = c;
  int u=0;
    PACK ITEMS INTO FREE BINS
  delete [] free;
  return u+1;
}</pre>
```

Figure 17.4: Computing an upper bound for the number of bins

The computation of the lower bound  $L_1$  is as to be expected and is shown in Figure 17.3. The ceiling operation is replaced by adding c-1 followed by truncating integer division with c.

Note that more accurate lower bounds are known, see Section 17.5 for more information.

**Computing an upper bound.** An obvious upper bound for the number of bins required is the number of items: each item is packed into a separate bin (provided that no item size exceeds the bin capacity c). A better upper bound can be computed by constructing a solution by packing items into bins following a first-fit strategy: pack all items into the first bin of sufficient free capacity.

Figure 17.4 shows the function upper() that returns an upper bound for the number of bins. It initializes an array free of n integers with the bin capacity c. The integer u refers to the index of the last used bin (that is, a bin into which an item has been packed). The function returns the number of used bins (that is, the index u plus one).

Each item is packed into a bin with sufficient free capacity where j refers to the next free bin:

```
PACK ITEMS INTO FREE BINS ≡

for (int i=0; i<n; i++) {
   int j=0;
   ▶ FIND FREE BIN
   u = std::max(u,j);
}</pre>
```

The next free bin j is searched for as follows:

```
FIND FREE BIN 
  while (free[j] < size[i])
    j++;
  free[j] -= size[i];</pre>
```

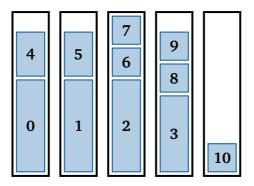


Figure 17.5: An non-optimal bin packing found during upper bound computation

The loop always terminates as there is one bin for each item.

Note that upper() has  $O(n^2)$  complexity in the worst case but could be made more efficient by speeding up finding a fitting bin.

The solution constructed during computation of upper() is not necessarily optimal. As an example, consider the packing computed by upper() for the example from Section 17.1 shown in Figure 17.5: it takes five rather than four bins because one of the items 4 and 5 should be packed together with item 3 rather than with one of the items 0, 1, and 2.

If both lower and upper bound coincide the solution constructed is of course optimal and we are done solving the bin packing problem. For reasons of simplicity, our model just forsakes this opportunity and re-computes an optimal solution by constraint programming.

**Model proper.** Figure 17.6 shows a script for the bin packing model. The script defines integers l and u that store the lower and upper bound as discussed above. A *load variable* l and i (taking values from [0..c]) defines the total size of all items packed into bin i. The script uses u load variables as the upper bound guarantees that u bins are sufficient to find an optimal solution. A *bin variable*  $bin_i$  (taking values from [0..u-1] defines for each item i into which bin it is packed. The variable bins defines the number of *used bins*. A bin is *used* if at least one item is packed into it, otherwise it is an *excess* bin.

The integer s is initialized as the size of all items and sizes is initialized as an integer argument array of all sizes.

The cost() function as required by the class MinimizeScript (see Section 9.2) returns the number of bins.

**Excess bins.** If the script finds a solution that uses less than u bins, say k (the value of the bins variable), then u - k of the load variables corresponding to excess bins are zero. To remove many symmetrical solutions that only differ in which bins are excess bins, the script constrains the excess bins to be the bins k, ..., u - 1:

```
EXCESS BINS ≡
for (int i=1; i<=u; i++)
rel(*this, (bins < i) == (load[i-1] == 0));</pre>
```

```
[DOWNLOAD]
BIN PACKING NAIVE ≡
  ► INSTANCE DATA
  ► COMPUTE LOWER BOUND
  ► COMPUTE UPPER BOUND
 class BinPacking : public MinimizeScript {
 protected:
    const int l;
    const int u;
    IntVarArray load;
    IntVarArray bin;
    IntVar bins;
 public:
    BinPacking(const Options& opt)
      : l(lower()), u(upper()),
        load(*this, u, 0, c),
        bin(*this, n, 0, u-1), bins(*this, l, u) {
      EXCESS BINS
      int s=0;
      for (int i=0; i<n; i++)
        s += size[i];
      IntArgs sizes(n, size);
      ► LOADS ADD UP TO ITEM SIZES
      ► LOADS ARE EQUAL TO PACKED ITEMS
      ► SYMMETRY BREAKING
      ► PACK ITEMS THAT REQUIRE A BIN
      ▶ BRANCHING
    virtual IntVar cost(void) const {
      return bins;
    }
   . . .
 };
```

Figure 17.6: A naive script for solving a bin packing problem

**Constraining load and bin variables.** The sum of all load variables must be equal to the size of all items:

```
LOADS ADD UP TO ITEM SIZES ≡
linear(*this, load, IRT_EQ, s);
```

The load variable for a bin must be constrained according to which items are packed into the bin. A standard formulation of this constraint uses Boolean variables  $x_{i,j}$  which determine whether item i has been packed into bin j. That is, for each item  $0 \le i < n$  the following constraint must hold:

$$\mathbf{x}_{i,j} = 1 \iff \mathrm{bin}_i = j \qquad (0 \leq j < \mathbf{u})$$

A more efficient propagator for the very same constraint is available as a channel constraint between an array of Boolean variables and a single integer variable, see Section 4.4.11. That is, for each item  $0 \le i < n$  the following constraint must hold:

$$channel(\langle x_{i,0}, x_{i,1}, \dots, x_{i,u-1} \rangle, bin_i)$$

Note that  $\langle x_{i,0}, x_{i,1}, \dots, x_{i,u-1} \rangle$  corresponds to x.col(i).

Furthermore, the size of all items packed into a bin must equal the corresponding load variable. Both constraints are expressed as follows, using a matrix x (see Section 6.2) of Boolean variables  $_x$ :

```
LOADS ARE EQUAL TO PACKED ITEMS ≡
BoolVarArgs _x(*this, n*u, 0, 1);
Matrix<BoolVarArgs> x(_x, n, u);
for (int i=0; i<n; i++)
  channel(*this, x.col(i), bin[i]);
for (int j=0; j<u; j++)
  linear(*this, sizes, x.row(j), IRT_EQ, load[j]);</pre>
```

**Symmetry breaking.** Items of the same size are equivalent as far as the model is concerned. To break symmetries, the bins for items of the same size are ordered:

```
SYMMETRY BREAKING ≡
  for (int i=1; i<n; i++)
   if (size[i-1] == size[i])
    rel(*this, bin[i-1] <= bin[i]);</pre>
```

The loop exploits that items are ordered according to size and hence items of the same size are adjacent.

**Pack items that require a bin.** If the size s of an item exceeds  $\lceil \frac{c}{2} \rceil$  (or, equivalently, 2s > c), the item cannot share a bin with any other item also exceeding half of the capacity. That is, items exceeding half of the capacity can be directly assigned to different bins:

```
PACK ITEMS THAT REQUIRE A BIN ≡
for (int i=0; (i < n) && (i < u) && (size[i] * 2 > c); i++)
rel(*this, bin[i] == i);
```

The assignment of items to bins is compatible with the symmetry breaking constraints discussed previously.

**Branching.** We choose a naive branching strategy that first branches on the number of required bins, followed by trying to assign items to bins.

```
BRANCHING ≡
branch(*this, bins, INT_VAL_MIN);
branch(*this, bin, INT_VAR_NONE, INT_VAL_MIN);
```

Note that by choosing the bin variables with order INT\_VAR\_NONE assigns the largest item to a bin first as items are sorted by decreasing size.

The script in Figure 17.6 does not show that the script uses branch-and-bound search to find a best solution. Why depth-first search is not sufficient with parallel search is discussed in Tip 7.7.

**Running the model.** When running the naive model<sup>1</sup>, it becomes apparent that the model is indeed naive. Finding the best solution takes 29.5 seconds and 2451 018 failures. Clearly, that leaves ample room for improvement in the following sections!

# 17.3 Improving propagation

This section improves (and simplifies) the naive model from the previous section by using a dedicated binpacking constraint.

**Model.** The improved model is shown in Figure 17.7. Instead of using Boolean variables x, linear constraints, and channel constraints it uses the binpacking constraint (see also Section 4.4.15). The constraint enforces that the packing of items as defined by the bin variables corresponds to the load variables.

**Running the model.** Finding a best solution using the model with improved propagation takes 1.5 seconds and 64477 failures. That is, this model runs almost 20 times faster than the naive model and reduces the number of failures by a factor of 38.

<sup>&</sup>lt;sup>1</sup>All measurements in this chapter have been made on a laptop with an Intel i5 M430 processor (2.27 GHz, two cores, hyper-threading), 4 GB of main memory, running Windows 7 x64, and using Gecode 3.4.3.

```
BIN PACKING PROPAGATION 

class BinPacking: public MinimizeScript {
...

public:
BinPacking(const Options& opt)

: ··· {
...

IntArgs sizes(n, size);
binpacking(*this, load, bin, sizes);
...
}
...
};
...
```

Figure 17.7: A script with improved propagation for solving a bin packing problem

## 17.4 Improving branching

This section describes a problem specific branching to improve the model of the previous section even further.

Complete decreasing best fit branching. The improved branching for bin packing is called complete decreasing best-fit (CDBF) and is due to Gent and Walsh [14]. The branching uses some additional improvements suggested by Shaw in [41].

The branching tries to assign items to bins during search where the items are tried in order of decreasing size. The bin is selected according to a best fit strategy: try to put the item into a bin with sufficient but least free space. The space of the bin after packing an item is called the bin's *slack*. If there is no bin with sufficient free space left, CDBF fails.

Suppose that CDBF selects item i and bin b. Then the following actions are taken during branching:

- If there is a perfect fit (that is, the slack is zero), branching assigns item i to bin b. This corresponds to a branching with a single alternative.
- If all possible bins have the same slack, branching assigns item i to bin b. Again, this corresponds to a branching with a single alternative.
- Otherwise, CDBF tries two alternatives in the following order:
  - Assign item i to bin b.

```
BIN PACKING BRANCHING 

...

▶ CDBF

class BinPacking : public MinimizeScript {
    ...

public:
    BinPacking(const Options& opt)
        : ... {
        ...
        branch(*this, bins, INT_VAL_MIN);
        cdbf(*this, load, bin, sizes);
    }
    ...
};
....
```

Figure 17.8: A script with improved branching for solving a bin packing problem

 Not only prune bin b from the potential bins for item i but also prune all bins with the same slack as b from the potential bins for all items with the same size as i.

Note that the second alternative of CDBF performs symmetry breaking during search as it prunes also with respect to equivalent items and bins.

**Model.** The only change to the model compared to Section 17.3 is that it uses the branching cdbf for assigning items to bins during search.

**Brancher creation.** The cdbf branching takes load variables (for computing the free space of a bin), bin variables (to pack items into bins), and the item sizes (to compute how much space an item requires) as input and posts the CDBF brancher as shown in Figure 17.9.

The branching post function cdbf() creates view arrays for the respective variables, creates a shared integer array of type IntSharedArray (see Tip 4.9) and posts a CDBF brancher. The advantage of using a shared array is that the sizes are stored only once in memory and branchers in different spaces have shared access to the same memory area (see also Section 27.3).

The brancher CDBF stores the load variables, bin variables, and item sizes together with an integer item. The integer item is used to find the next unassigned item. It is declared **mutable** so that the **const** status() function (see below) can modify it. The brancher exploits that the items are sorted by decreasing size: by initializing item to zero the brancher is trying to pack the largest item first.

```
CDBF \equiv
 class CDBF : public Brancher {
  protected:
    ViewArray<Int::IntView> load;
   ViewArray<Int::IntView> bin;
    IntSharedArray size;
    mutable int item;
    ► CDBF CHOICE
 public:
    CDBF(Home home, ViewArray<Int::IntView>& l,
                    ViewArray<Int::IntView>& b,
                    IntSharedArray& s)
      : Brancher(home), load(l), bin(b), size(s), item(0) {
      home.notice(*this,AP_DISPOSE);
    static void post(Home home, ViewArray<Int::IntView>& l,
                                ViewArray<Int::IntView>& b,
                                 IntSharedArray& s) {
      (void) new (home) CDBF(home, l, b, s);
    ► STATUS FUNCTION
    ► CHOICE FUNCTION
    ► COMMIT FUNCTION
    virtual size_t dispose(Space& home) {
      home.ignore(*this,AP_DISPOSE);
      size.~IntSharedArray();
      return sizeof(*this);
   }
 };
  void cdbf(Home home, const IntVarArgs& l, const IntVarArgs& b,
                       const IntArgs& s) {
    if (b.size() != s.size())
      throw Int::ArgumentSizeMismatch("cdbf");
    ViewArray<Int::IntView> load(home, l);
   ViewArray<Int::IntView> bin(home, b);
    IntSharedArray size(s);
    CDBF::post(home, load, bin, size);
```

Figure 17.9: CDBF brancher and branching

By default, the dispose() member function of a brancher is not called when the brancher's home space is deleted. However, the dispose() function of the CDBF brancher must call the destructor of the shared integer array size. Hence, the constructor of CDBF calls the notice() function of the home space so that the brancher's dispose() function is called when home is deleted (see also Section 20.9). Likewise, the dispose() function calls the ignore() function before the brancher is disposed.

**Status computation.** The status() function tries to find a yet unassigned view for branching in the view array bin. It starts inspecting the views at position item and skips all already assigned views. If there is a not yet assigned view left, item is updated to that unassigned view and **true** is returned (that is, more branching is needed). Otherwise, the brancher returns **false** as no more branching is needed:

```
virtual bool status(const Space&) const {
  for (int i = item; i < bin.size(); i++)
    if (!bin[i].assigned()) {
     item = i; return true;
    }
  return false;
}</pre>
```

As the items are sorted by decreasing size, the integer item refers to the largest not-yet packed item.

**Choice computation:** initialization. The choice() function implements the actual heuristic. The function uses n for the number of items, m for the number of bins, and initializes a region for managing temporary memory (see Section 27.1) as follows:

```
CHOICE FUNCTION =
  virtual Gecode::Choice* choice(Space& home) {
    int n = bin.size(), m = load.size();
    Region region(home);
    INITIALIZE FREE SPACE IN BINS
    INITIALIZE BINS WITH SAME SLACK
    FIND BEST FIT
    CREATE CHOICE
}
```

The choice() function can rely on the fact that it is immediately executed after the status() function has been executed. That entails that item refers to the largest not-yet packed item.

The function computes in free the free space of each bin. From the maximal load the size of items that have already been packed (that is, the item's bin variable is already assigned) is subtracted:

```
INITIALIZE FREE SPACE IN BINS =
  int* free = region.alloc<int>(m);
  for (int j=0; j<m; j++)
    free[j] = load[j].max();
  for (int i=0; i<n; i++)
    if (bin[i].assigned())
     free[bin[i].val()] -= size[i];</pre>
```

The choice() function uses the integer slack to track the slack of the so-far best fit (initialized with INT\_MAX such that any fit will be better). The integer n\_possible counts the number of possible bins for the item whereas n\_same counts the number of best fits. The array same stores all bins with the same so-far smallest slack.

```
INITIALIZE BINS WITH SAME SLACK ≡
int slack = INT_MAX;
unsigned int n_possible = 0;
unsigned int n_same = 0;
int* same = region.alloc<int>(m+1);
same[n_same++] = -1;
```

The array same is initialized to contain the bin -1: if no bin has sufficient space for the current item this will guarantee that the commit() function (see below) leads to failure.

**Choice computation: create choice.** In order to find all best fits, all bins are examined. If the current item fits into a bin, the number of possible bins n\_possible is incremented and all best fits are remembered in the array same as follows:

```
find BEST FIT =
  for (Int::ViewValues<Int::IntView> j(bin[item]); j(); ++j)
  if (size[item] <= free[j.val()]) {
    n_possible++;
    if (free[j.val()] - size[item] < slack) {
        slack = free[j.val()] - size[item];
        n_same = 0;
        same[n_same++] = j.val();
    } else if (free[j.val()] - size[item] == slack) {
        same[n_same++] = j.val();
    }
}</pre>
```

Note that finding a better fit updates slack and resets the bins stored in same.

Now, the choice() function determines whether a special case needs to be dealt with:

- Is the best fit a perfect fit: that is, slack is zero?
- Are all fits a best fit: that is, n\_same is equal to n\_possible?
- Is there no fitting bin: that is, n\_possible is zero?

In these cases a choice with a single alternative and otherwise a choice with two alternatives is created:

```
CREATE CHOICE =
if ((slack == 0) ||
    (n_same == n_possible) ||
    (n_possible == 0))
  return new Choice(*this, 1, item, same, 1);
else
  return new Choice(*this, 2, item, same, n_same);
```

The definition of the choice class is shown in Figure 17.10. The choice stores the current item and in the array same all bins with the same slack. The choice does not need to store any information regarding items of same size as this information is available from the brancher.

**Commit function.** The commit() function takes a choice of type CDBF::Choice and the alternative a (either 0 or 1) as input:

Committing to the first alternative tries to pack item into the first bin stored in same as follows:

```
COMMIT TO FIRST ALTERNATIVE ≡
GECODE_ME_CHECK(bin[c.item].eq(home, c.same[0]));
```

Committing to the second alternative removes all  $n_s$  ame bins stored in same from all items that have the same size as item as follows:

```
CDBF CHOICE ≡
 class Choice : public Gecode::Choice {
 public:
   int item;
   int* same;
   int n_same;
   Choice(const Brancher& b, unsigned int a, int i, int* s, int n_s)
      : Gecode::Choice(b,a), item(i),
        same(heap.alloc<int>(n_s)), n_same(n_s) {
      for (int k=0; k<n_same; k++)</pre>
        same[k] = s[k];
   virtual size_t size(void) const {
      return sizeof(Choice) + sizeof(int) * n_same;
   virtual ~Choice(void) {
      heap.free<int>(same, n_same);
   virtual void archive(Archive& e) const {
      Gecode::Choice::archive(e);
      e << alternatives() << item << n_same;
     for (int i=n_same; i--;) e << same[i];</pre>
   }
 };
```

Figure 17.10: CDBF choice

The iterator Iter::Values::Array iterates over all values stored in an array (they must be in sorted order) and the operation minus\_v() prunes all values as defined by an iterator from a view, see Section 23.2.

**Running the model.** Finding a best solution using the model with improved propagation and improved branching takes 84 milliseconds and 3 098 failures. That is, this model runs 352 times faster than the naive model and reduces the number of failures by a factor of 791.

### 17.5 More information

Bin packing featuring all models presented in this chapter is also available as a Gecode example, see Bin packing. The example also makes use of a more accurate lower bound known as  $L_2$  [22].

# 18

# **Kakuro**

This chapter studies Kakuro puzzles, a variant of the well-known Sudoku puzzles. Two models are presented: a first and obvious model that suffers from too little propagation to be feasible. This is followed by a model that employs user-defined constraints implemented as extensional constraints using tuple set specifications. Interestingly, the tuple set specifications are computed by solving a simple constraint problem.

### 18.1 Problem

Solving a Kakuro puzzle (see Figure 18.1 for an example) amounts to finding digits between 1 and 9 for the non-hint fields on a board. A hint field can specify a *vertical hint* and/or a *horizontal hint*:

- A vertical hint contains a number *s* above the diagonal in the hint field. It requires that all digits on the fields extending from the field left of the hint up to the next hint field or to the end of the row are pairwise distinct and sum up to the value *s*.
- A horizontal hint contains a number *s* below the diagonal in the hint field. The requirements are analogous.

The number contained in a hint is called its *value* and the number of fields constrained by a hint is called its *length*.

The solution for the Kakuro puzzle from Figure 18.1 is shown in Figure 18.2. Kakuro puzzles are always designed (at least meant to be) to have a unique solution.

#### 18.2 A naive model

A script for the Kakuro model is shown in Figure 18.3. The script stores the width (w) and height (h) of the board. The fields are stored in an integer variable array f which is initialized to have  $w \cdot h$  elements. Note that none of the fields is initialized in the constructor of Kakuro; their initialization is discussed below.

**Board specification.** The specification of the Kakuro board, as shown in Figure 18.3, stores width and height of the board, followed by a specification of the hints. The hints are provided in two groups: first vertical hints, then horizontal hints, separated by the integer -1. A hint

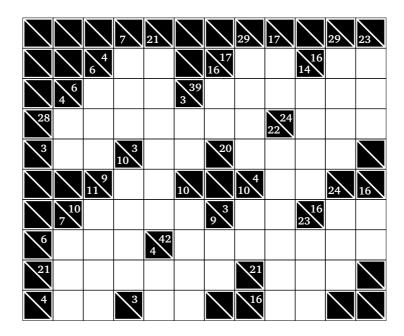


Figure 18.1: A Kakuro puzzle

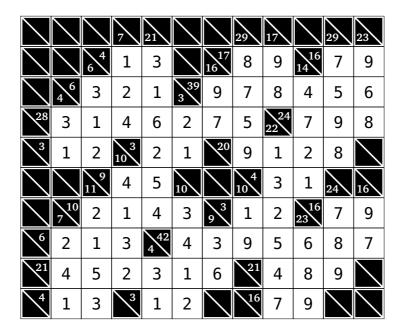


Figure 18.2: Solution for Kakuro puzzle from Figure 18.1

```
[DOWNLOAD]
KAKURO NAIVE ≡
  ▶ BOARD SPECIFICATION
 class Kakuro : public Script {
 protected:
    const int w, h;
    IntVarArray f;
 public:
    ► INIT FUNCTION
    ► POSTING HINT CONSTRAINTS
    Kakuro(const Options& opt)
      : w(board[0]), h(board[1]), f(*this,w*h) {
      ► FIELD INITIALIZATION
      ► SETUP
      ► PROCESS VERTICAL HINTS
      ▶ BRANCHING
    }
 };
```

```
board specification =
const int board[] = {
    // Dimension w x h
    12, 10,
    // Vertical hints
    3, 0, 3, 7,    4, 0, 6,21,    7, 0, 4,29,    8, 0, 2,17,
    ...
    -1,
    // Horizontal hints
    2, 1, 2, 4,    6, 1, 2,17,    9, 1, 2,16,    1, 2, 3, 6,
    ...
};
```

Figure 18.3: A naive script and board specification for solving Kakuro puzzles

is described by its coordinates on the board, followed by its length and value. Note that the specification assumes that the field with coordinate (0,0) is in the left upper corner.

**Initializing fields.** All fields are initialized to a single shared integer variable black that is assigned to zero:

```
FIELD INITIALIZATION ≡
IntVar black(*this,0,0);
for (int i=0; i<w*h; i++)
f[i] = black;</pre>
```

Only if a field is actually used by a hint, the field will be initialized to an integer variable taking digit values by the following init() function:

```
INIT FUNCTION =
  IntVar init(IntVar& x) {
   if (x.min() == 0)
      x = IntVar(*this,1,9);
   return x;
}
```

The test whether the minimum of variable x equals zero is true, if and only if x still refers to the variable black. In this case, a new variable is created with non-zero digits as variable domain. As x is passed by reference, assigning x to the newly created variable also assigns the corresponding field on the board to the newly created variable. This guarantees that a new variable is created at most once for each field.

**Posting hint constraints.** Posting hint constraints is done best by using a matrix interface b (see Section 6.2) to the fields in f. The specification of the hints will be accessed by the variable k, where the dimension of the board has already been skipped:

```
SETUP =
Matrix<IntVarArray> b(f,w,h);
const int* k = &board[2];
```

Processing the vertical hints is straightforward. After retrieving the coordinates x and y, the length n, and the value s for a hint from the board specification, the variables covered by the hint are collected in the integer variable argument array (see Section 4.2.2) col. The constraint for the hint on the collected variables is posted by the member function hint():

```
PROCESS VERTICAL HINTS =
while (*k >= 0) {
   int x=*k++; int y=*k++; int n=*k++; int s=*k++;
   IntVarArgs col(n);
   for (int i=0; i<n; i++)
      col[i]=init(b(x,y+i+1));
   hint(col,s);
}</pre>
```

The hint() function must constrain that all variables are distinct (using a distinct constraint) and that they sum up to the value of the hint (using a linear constraint). To achieve strong propagation, we want to use domain propagation for both distinct and linear. However, the complexity of domain propagation for linear is exponential, hence it is a good idea to avoid posting linear constraints as much a possible.

Consider a hint of length 9. Then obviously, the single possible value of the hint is  $\sum_{i=1}^{9} i = 9(9+1)/2$  and hence no linear constraint needs to be posted. Now consider a hint of length 8 with value s. Then, the fields covered by the hint take on all but one digit. That is, all fields must be different from  $\sum_{i=1}^{9} i - s = 9(9+1)/2 - s$ . Taking these two observations into account, the constraints for a hint can be posted as follows, where the value ICL\_DOM requests domain propagation (see Section 4.3):

```
POSTING HINT CONSTRAINTS =
  void hint(const IntVarArgs& x, int s) {
    if (x.size() < 8)
        linear(*this, x, IRT_EQ, s, ICL_DOM);
    else if (x.size() == 8)
        rel(*this, x, IRT_NQ, 9*(9+1)/2 - s);
    distinct(*this, x, ICL_DOM);
}</pre>
```

Note that there are other special cases where no linear constraint needs to be posted, for example if for a hint of length n and value s it holds that  $\sum_{i=1}^{n} i = s$  (that is, only digits from 1 to n are possible). See Section 18.4 for more information.

Vertical hints are of course analogous and are hence omitted.

**Branching.** We choose a branching that selects the variable where the quotient of domain size and AFC is smallest (see Section 7.1.1). Values are tried by interval bisection:

```
BRANCHING ≡
branch(*this, f, INT_VAR_SIZE_AFC_MIN, INT_VAL_SPLIT_MIN);
```

Why the model is poor. When running the script, solving even the tiny board of Figure 18.1 requires 19 search nodes. There exist commercially available boards with thousands of hints, which are of course completely out of reach with the naive model. Figure 18.4 shows the possible digits for each field after performing propagation for the Kakuro script but before any search. Consider the two green fields for the hint of length 2 and value 4. The only possible combination for the two fields is (3,1). However, propagation does not prune the value 2 for both fields. The reason is that a hint constraint is decomposed into a distinct constraint and into a linear constraint and neither constraint by itself warrants more pruning than shown.

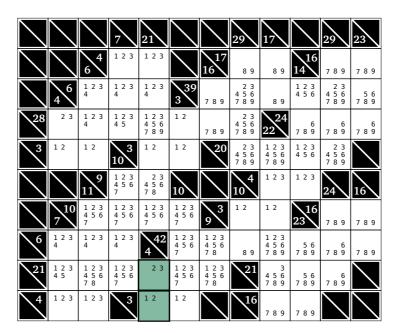


Figure 18.4: Propagation for the Kakuro puzzle

## 18.3 A working model

The naive model from the previous section suffers from the fact that hint constraints are decomposed into a distinct constraint and a linear constraint. One remedy would be to implement a dedicated propagator for a distinctlinear constraint. This is impractical: too complicated and too much effort for such a specialized constraint.

**Model idea.** This section implements distinctlinear constraints as extensional constraints using tuple sets as specification of the possible solutions of distinctlinear constraints. For example, for a hint of length 3 and value 8, the possible solutions for the corresponding distinctlinear constraint are:

$$\langle 1,2,5 \rangle$$
  $\langle 1,3,4 \rangle$   $\langle 1,4,3 \rangle$   $\langle 1,5,2 \rangle$   $\langle 2,1,5 \rangle$   $\langle 2,5,1 \rangle$   $\langle 3,1,4 \rangle$   $\langle 3,4,1 \rangle$   $\langle 4,1,3 \rangle$   $\langle 4,3,1 \rangle$   $\langle 5,1,2 \rangle$   $\langle 5,2,1 \rangle$ 

The model needs a method to compute all solutions of a distinctlinear constraint. To simplify matters, we are going to compute all solutions of a distinctlinear constraint by computing all solutions of a trivial constraint problem: the decomposition of a distinctlinear constraint into a distinct and linear constraint.

Figure 18.5 shows the outline for a working script for solving Kakuro puzzles. The class DistinctLinear defines the script used for computing all solutions of a distinctlinear constraint and the function distinctlinear() serves as constraint post function. Apart from how hint constraints are posted, the Kakuro script is the same as in the previous section.

```
[DOWNLOAD]
KAKURO ≡
 class DistinctLinear : public Space {
 protected:
    IntVarArray x;
 public:
    ► DISTINCT LINEAR SCRIPT
    ► RETURNING A SOLUTION
 };
 void distinctlinear(Home home, const IntVarArgs& x, int c) {
    ► SET UP SEARCH ENGINE
    ► COMPUTE TUPLE SET
    ► POST EXTENSIONAL CONSTRAINT
 }
 class Kakuro : public Script {
    ► POSTING HINT CONSTRAINTS
 };
```

Figure 18.5: A working script for solving Kakuro puzzles

**Computing distinct linear solutions.** As mentioned, the script for DistinctLinear just posts a linear and distinct constraint for n variables and value s. As the search space of the problem is small anyway, we neither need strong propagation for distinct and linear nor do we need a clever branching:

```
DISTINCT LINEAR SCRIPT ≡

DistinctLinear(int n, int s) : x(*this,n,1,9) {
    distinct(*this, x);
    linear(*this, x, IRT_EQ, s);
    branch(*this, x, INT_VAR_NONE, INT_VAL_SPLIT_MIN);
}
```

When solving the DistinctLinear script, we need its solutions as integer argument arrays for computing a tuple set. The solution() member function of DistinctLinear returns an integer argument array for a solution as follows:

```
RETURNING A SOLUTION ≡
IntArgs solution(void) const {
   IntArgs s(x.size());
   for (int i=0; i<x.size(); i++)
      s[i]=x[i].val();
   return s;
}</pre>
```

**Posting distinctlinear constraints.** The search engine (see Section 7.4) for computing all solutions of a DistinctLinear script is initialized as follows:

```
SET UP SEARCH ENGINE ≡
DistinctLinear* e = new DistinctLinear(x.size(),c);
DFS<DistinctLinear> d(e);
delete e;
```

Computing a tuple set (see Section 4.4.13) for all solutions of a distinctlinear constraints is straightforward:

```
COMPUTE TUPLE SET =
TupleSet ts;
while (DistinctLinear* s = d.next()) {
  ts.add(s->solution()); delete s;
}
ts.finalize();
```

Note that after all solutions have been added to the tuple set ts, it must be finalized before it can be used by an extensional constraint (see Section 4.4.13).

Finally, posting the extensional constraint using the tuple set ts is as to be expected:

```
POST EXTENSIONAL CONSTRAINT ≡
extensional(home, x, ts);
```

**Posting hint constraints.** Posting a hint constraint follows a similar line of reasoning as in the previous section. If the length of a hint is 0, no constraint needs to be posted (hints of length 0 are black fields without hints). If the length is 1, the single variable is constrained to s directly. For lengths 8 and 9, distinct is used as it achieves the same propagation as distinctlinear. Note that the case for length 8 continues (as it does not have a **break** statement) with the case for length 9 and hence also posts a distinct constraint. In all other cases, distinctlinear is used:

```
POSTING HINT CONSTRAINTS =
  void hint(const IntVarArgs& x, int s) {
    switch (x.size()) {
    case 0:
        break;
    case 1:
        rel(*this, x[0], IRT_EQ, s); break;
    case 8:
        rel(*this, x, IRT_NQ, 9*(9+1)/2 - s);
    case 9:
        distinct(*this, x, ICL_DOM); break;
    default:
        distinctlinear(*this, x, s); break;
    }
}
```

There is a further important optimization which we will not show (but see Section 18.4). Each time distinctlinear is called, it computes a new tuple set, even though the tuple set is exactly the same for all hints of equal length and value. To guarantee that the same tuple set is computed at most once, one could cache tuple sets: if a tuple set for a certain length and value has already been computed earlier, it is not computed again but taken from a cache (where it had been stored when it was computed for the first time).

**This model works.** For the example puzzle, propagation alone is sufficient to solve the puzzle. Even puzzles with thousands of hints are solved without search in a fraction of a second (including computing the tuple sets, provided they are cached as sketched above).

#### 18.4 More information

Kakuro puzzles with some more examples are available as a Gecode example, see Kakuro. In particular, the model caches tuple sets such that for each type of hint its corresponding tuple set is computed at most once as discussed in Section 18.3. Furthermore, the example exploits further special cases where posting a distinct constraint rather than a complete hint constraint is sufficient as discussed in Section 18.2.

More constraint-based techniques for solving Kakuro puzzles are discussed in [42].

# 19

# Crossword puzzle

This chapter studies solving crossword puzzles and presents a simple model using nothing but distinct and element constraints.

The simple model for this classical problem is shown to work quite well compared to a constraint-based approach to solving crossword puzzles using a dedicated problem-specific constraint solver [1]. This underlines that an efficient general-purpose constraint programming system actually can go a long way.

#### 19.1 Problem

To solve a crossword puzzle problem, a crossword grid (see Figure 19.1 for an example) must be filled with words (from a predefined dictionary) extending both in horizontal and vertical directions such that:

- If words cross at a field of the grid, the words' letters at the crossing field are the same.
- No word is used twice.

Words use lowercase letters only and extend as far as they can. That is, the beginning (and the end) of a word must either be adjacent to a black field on the grid or must be a field on the grid's border.

An example solution for the grid from Figure 19.1 is shown in Figure 19.2.

#### **19.2** Model

The model uses two sets of variables:

■ The model uses for each word on the grid a *word variable*. A value for a word variable is a *dictionary index* defining the index of the word chosen from the dictionary of all words.

The script for the model uses the word variables only as temporary variables for posting constraints. To simplify posting constraints, words are processed in groups of words of the same length. In particular, dictionary indices are also defined with respect to words of the same length in the dictionary.

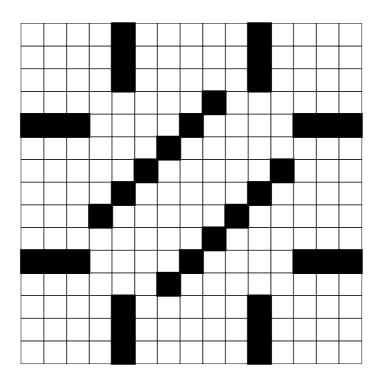


Figure 19.1: A crossword puzzle grid

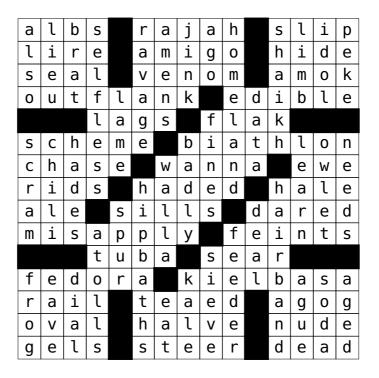


Figure 19.2: Solution for crossword puzzle grid from Figure 19.1

```
[DOWNLOAD]
CROSSWORD ≡
  ► GRID SPECIFICATION
  ► WORDS SPECIFICATION
  class Crossword : public Script {
  protected:
    const int w, h;
    IntVarArray letters;
  public:
    Crossword(const Options& opt)
      : w(grid[0]), h(grid[1]), letters(*this,w*h,'a','z') {
      ► SET UP
      ► INITIALIZE BLACK FIELDS
      ► PROCESS WORDS BY LENGTH
      ▶ BRANCHING
    }
  };
```

Figure 19.3: Crossword script

■ For each field on the grid, the model uses a *letter variable*. The values for a letter variable are either 0 (for a black field on the grid) or a character code between 'a' and 'z' for lowercase letters.

Given the sets of variables, the constraints for the model are straightforward:

- All word variables for words of the same length must be distinct. Only word variables for words of the same length need to be constrained to be distinct, as words of different length are distinct by definition.
- Assume that w is a word variable for a word of length n on the grid and  $x_0, \ldots, x_{n-1}$  are the letter variables that correspond to the word on the grid. Assume further that  $0 \le p < n$  and that an array w2l (for word to letter) maps the dictionary indices of all words of length n to their p-th letter. Then, the letter variable  $x_p$  can be linked to the word variable w by posting the constraint that w2l $_w = x_p$  (this is an element constraint).

An outline for the script implementing the crossword puzzle is shown in Figure 19.3. The script stores the width w and the height h of the grid and an integer variable array letters for the letter variables (including the black fields). The values for letters range from 'a' to 'z' (black fields are discussed below).

```
GRID SPECIFICATION ≡
    const int grid[] = {
        // Width and height of crossword grid
        15, 15,
        // Number of black fields
        36,
        // Black field coordinates
        0,4, 0,10, 1,4, 1,10, 2,4, 2,10, 3,8, 4,0, 4,1,
        ...
        // Length and number of words of that length
        8, 8,
        // Coordinates where words start and direction (0 = horizontal)
        0,3,0, 0,9,0, 3,0,1, 5,7,1, 7,5,0, 7,11,0, 9,0,1, 11,7,1,
        ...
        // End marker
        0
};
```

```
words specification \( \exists \)
  const int n_words[] = {
    1, 26, 66, 633, 2443, 4763, 7585, 10380, 10974
  };
  const char** words[] = {
    ...
};
```

Figure 19.4: Grid and words specification

**Grid and words specification.** The specification for the grid used in this case study (see Section 19.4 for more information) and the word dictionary are shown in Figure 19.4. The grid specification contains information about the dimension of the grid (as used in Figure 19.3), the number and coordinates of black fields on the grid, and the start coordinates of words and their direction on the grid for each word length permitted by the grid.

For each word length l, the array  $n\_words_l$  defines how many words of length l exist in the dictionary of words. That is, for a word length l, the set of dictionary indices is  $\{0,\ldots,n\_words_l-1\}$ . The array words provides access to the letters of a word of some given length with a given dictionary index. That is, for a given word length l and for a position in the word p with  $0 \le p < l$ ,  $words_{l,i,p}$  (or words[l][i][p] in  $C^{++}$ ) is the p-th letter of the word with dictionary index i among all words of length l (where  $0 \le i < n\_words_l$ ) in the dictionary. The dictionary of words just contains words of length at most eight as this is sufficient for the example grid used in this case study.

The word list is based on SCOWL-55 (Spell Checking Oriented Word Lists) truncated to

words of length at most eight, see wordlist.sourceforge.net. Please check the source file available from Figure 19.3 for copyright information.

**Grid initialization.** The grid specification is accessed by the pointer g (with the width and height part already skipped). The matrix ml (see Section 6.2) supports access to the letters as a matrix:

```
SET UP =
  const int* g = &grid[2];
  Matrix<IntVarArray> ml(letters, w, h);
```

The black fields of the grid are initialized by storing a variable black at the respective coordinates:

```
INITIALIZE BLACK FIELDS ≡
  IntVar black(*this,0,0);
  for (int n = *g++; n--; ) {
    int x=*g++, y=*g++; ml(x,y)=black;
}
```

At first sight, the treatment of black fields in the grid appears to be inefficient. First, each element in the integer variable array letters is initialized (in the initialization list of the constructor Crossword()) to a new integer variable with values ranging from 'a' to 'z'. Then, some variables become redundant as their fields are overwritten by black. However, this only matters initially when a space of class Crossword is created. As soon as a clone of that space is created, the redundant variables are not copied and hence do not matter any longer. Moreover, all black fields on the grid share a single variable black which saves memory compared to a variable for each black field on the grid.

**Processing words by length.** As suggested by the grid specification, words are processed in groups of the same length. The loop that processes all words of the same length \(\partial\) has the following structure:

```
PROCESS WORDS BY LENGTH ≡
while (int l = *g++) {
   int n = *g++;
   ▶ INITIALIZE ARRAY OF WORDS
   ▶ PROCESS WORD ON GRID
}
```

Here, n is initialized to the number of words with length l in the grid.

To enforce that all n words of the same length l are distinct, an integer argument array wosl (for words of same length) is created (see Section 4.2.1), where each variable takes the possible dictionary indices for words of length l as values. The word variables in wosl are constrained to be distinct as follows:

```
INITIALIZE ARRAY OF WORDS ≡
IntVarArgs wosl(*this,n,0,n_words[l]-1);
distinct(*this, wosl);
```

**Constraining letters by words.** The remaining constraints link a word variable to the variables for its letters. All words of length 1 are processed as follows:

```
PROCESS WORD ON GRID ≡
IntArgs w2l(n_words[l]);
for (int i=0; i<n; i++) {
  int x = *g++, y = *g++; bool h = (*g++ == 0);
  ▶PROCESS EACH LETTER POSITION
}</pre>
```

The integer argument array w2l is used to map the dictionary indices of all words of length l in the dictionary to their letters. The x-coordinate and the y-coordinate and whether the word extends horizontally (h is **true**) or vertically (h is **false**) is retrieved from the grid specification.

Linking a word variable to a single letter is done for all 1 letters in a word, where the integer p refers to the position of a letter in a word:

```
PROCESS EACH LETTER POSITION ≡

for (int p=0; p<l; p++) {

► CONSTRAIN LETTERS
}
```

The integer argument array w2l is used to map all words in the dictionary of length l to their p-th letters. Then, for each letter position an element constraint (see Section 4.4.12) is posted that links the word variables to the respective letter variable:

```
CONSTRAIN LETTERS ≡
  for (int j=0; j<n_words[l]; j++)
  w2l[j] = words[l][j][p];
  element(*this, w2l, wosl[i], h ? ml(x+p,y) : ml(x,y+p));</pre>
```

**Branching.** We choose a simple branching that selects a variable where the quotient of domain size and AFC is smallest (see Section 7.1.1). The first value for the selected variable to be tried is the smallest:

```
BRANCHING ≡
branch(*this, letters, INT_VAR_SIZE_AFC_MIN, INT_VAL_MIN);
```

```
CROSSWORD OPTIMIZED 

...

class Crossword : public Script {
...

Crossword(const Options& opt)

: w(grid[0]), h(grid[1]), letters(*this,w*h,'a','z') {
...

while (int l = *g++) {
 int n = *g++;
 ...

for (int p=0; p<l; p++) {
    ▶ INITIALIZE WORD TO LETTER ARRAY
    ▶ CONSTRAIN LETTERS
    }
    g += 3*n;
    }
    ...
}
...

};
...

};
...
```

Figure 19.5: An optimized crossword script

## 19.3 An optimized model

The model in the previous section wastes some memory: for all word variables for words of length l, the array w2l is the same for a given position p. However, the very same array is computed n times: for each word variable for words of length l.

The first optimization is to swap the loops that iterate over the dictionary index i and the letter position p, as shown in Figure 19.5. However, one can go even further. By default, each time an element constraint is posted, a new shared array for the integer argument array is created (these will be still shared among all spaces of the same thread). To just have a single copy of the array, we can create a shared integer array of type IntSharedArray instead. Then, for each word length l and each position p there will be a single shared array only. See Tip 4.9 for more on shared arrays.

The shared integer array is initialized as follows:

```
INITIALIZE WORD TO LETTER ARRAY ≡
IntSharedArray w2l(n_words[l]);
for (int j=0; j<n_words[l]; j++)
w2l[j] = words[l][j][p];</pre>
```

The very same shared integer array is used for all words of the same length from the dictionary as follows:

```
CONSTRAIN LETTERS ≡
for (int i=0; i<n; i++) {
  int x = g[3*i+0], y = g[3*i+1];
  bool h = (g[3*i+2] == 0);
  element(*this, w2l, wosl[i], h ? ml(x+p,y) : ml(x,y+p));
}</pre>
```

In summary, the optimization does not offer a better model but a more memory-efficient implementation of the same model.

#### 19.4 More information

The script that is shown in this case study is also available as a Gecode example called Crossword puzzle. The example features a number of different crossword grids and supports branching on the word variables or on the letter variables. In addition, arbitrary dictionaries can be used provided they are available as a list of words in a file.

**Related work.** Solving crossword puzzles is a classic example for search methods (see for example [16]) and also for constraint programming (see for example [3] and [1]).

Anbulagan and Botea introduce Combus in [1]. Combus is a constraint-based solver specialized at solving crossword puzzles. Its distinctive feature is that it uses no good-learning to speed up search.

In the following, we are going to compare Combus to the model presented in this chapter. The purpose of the comparison is to shed light on the respective advantages of a problem-specific solver such as Combus and a simple model using a modern off-the-shelf constraint programming system such as Gecode.

**Used hardware and software platform.** All experiments have been run on a laptop with an Intel i5 M430 processor (2.27 GHz, two cores, hyper-threading), 4 GB of main memory, running Windows 7 x64 and using Gecode 3.4.0. The model has been run with a single thread only (unless mentioned otherwise). The runtimes (unless mentioned otherwise) are measured as wall-time and are the average of five runs. The coefficient of deviation is less than 3% and typically less than 1%.

The hardware platform used in [1] is an Intel Core Duo 2.4 GHz.

**Comparison with Combus.** The purpose of the comparison is to understand better the relative merits of the two different approaches. An exact comparison of runtimes is therefore not really meaningful, in particular as different hardware platforms are used.

The comparison makes only approximate statements for runtime and number of nodes explored during search. If the runtime for the Gecode model and Combus differ by at most a factor of two in either direction, the two approaches are roughly the same, denoted by  $\approx$ . If the runtime for the Gecode model is two to ten times faster, we use +; if it is ten to 100 times faster, we use +++. Analogously, we use -, --, and --- if the Gecode model is slower than Combus. We also use the same symbols for comparing the number of nodes explored during search, where fewer nodes are of course better.

Figure 19.6 shows the results for the Gecode model and their comparison to Combus for the dictionaries words (containing 45 371 words) and uk (containing 225 349 words), where both dictionaries are the same as in [1]. For each grid size  $15 \times 15$ ,  $19 \times 19$ ,  $21 \times 21$ , and  $23 \times 23$  ten different grids 01 to 10 are used. The runtime is in seconds.

For the Gecode model as well as Combus a timeout of 20 minutes is used. Orange fields are instances where neither the Gecode model nor Combus finds a solution (or proves that there is none) within 20 minutes. Red fields are instances where Combus finds a solution but the Gecode model fails to find a solution.

Just by judging how many instances can be solved by either approach (74 for the Gecode model, 77 for Combus), it becomes clear that Combus is, as to be expected, the more robust approach. Likewise, considering the number of nodes explored during search, Combus shows the clear advantage of the approach taken.

On the other hand, in most cases the Gecode model can explore more than two orders of magnitude more nodes and always at least one order of magnitude more nodes per second than Combus. This difference in efficiency explains why the simple Gecode model can solve that many instances at all.

Moreover, one needs to consider the modeling and programming effort. The Gecode model is straightforward, does have considerably less than 100 lines of code (excluding grid

							words dict	ionary						$\neg$
	II II					words dictionary					22	22		
	15 × 15			19 × 19			21 × 21			23 × 23				
	time nodes		time nodes		time nodes		time		nodes					
01	2.0	++	789	-	0.6	++	145 ≈	174.3	~	26 567	0.0	+++	0	~
02	5.9	~	5 5 2 1		4.5	+	2546	2.7	++	601 -	4.3	++	1384	-
03	0.5	++	143	-	8.2	+	2538 -	1.5	++	427 -	100.0	+	84375	-
04	51.9	-	34519		0.8	++	436 -	40.7	++	9932 ≈	35.4	+	5 2 5 5	-
05	1.3	++	382	-	0.3	++	141 ≈	8.3	+	3 938	3.0	++	1358	-
06	532.3	-	122870		0.4	+++	240 ≈	1.9	++	305 -	_	~	_	
07	37.8	+	5 535		0.8	++	254 -	3.0	++	1010 -	3.5	++	1088	-
08	0.3	+++	130	~	0.7	++	221 ≈	1.3	++	172 ≈	_		_	
09	0.3	++	112	~	0.8	++	300 -	1.7	++	185 ≈	49.5	++	9716	~
10	_		-		0.3	++	138 ≈	_	~	-	_	~	_	

uk dictionary									
	15	× 15	19 >	< 19	21 ×	21	23 × 23		
	time nodes		time	nodes	time	nodes	time	nodes	
01	0.7 +++	108 ≈	1.8 +++	200 ≈	3.2 +++	163 ≈	3.3 ++	241 ≈	
02	0.8 +++	96 ≈	1.7 +++	355 -	2.7 +++	196 ≈	4.1 +++	380 -	
03	0.8 +++	104 ≈	2.0 +++	175 ≈	2.8 +++	194 ≈	6.8 +++	853 -	
04	0.5 +++	95 ≈	0.9 +++	183 ≈	6.4 ++	390 -	15.6 ++	1734	
05	0.5 +++	85 ≈	0.7 +++	145 ≈	4.0 ++	266 -	3.7 +++	255 ≈	
06	2.3 +++	110 ≈	0.9 +++	171 ≈	2.0 +++	168 ≈	59.7 +	2804	
07	1.4 +++	114 ≈	0.9 +++	166 ≈	2.2 +++	183 ≈	3.8 +++	218 ≈	
08	0.6 +++	122 ≈	1.1 +++	154 ≈	1.9 +++	188 ≈	6.3 +++	379 -	
09	0.6 +++	117 ≈	_	-	2.7 +++	193 ≈	4.1 ++	212 ≈	
10	0.9 +++	93 ≈	1.0 +++	173 ≈	8.8 ++	323 -	27.5 ++	1311 -	

Figure 19.6: Comparison of Gecode model with Combus

instance	time	nodes		
words-15×15-10	319.2 -	228 017		
words-23×23-08	24.1 ++	10193 ≈		

Figure 19.7: Results for some hard words dictionary instances

specifications and dictionary support), and can be programmed in a few hours. One can expect that designing and programming a powerful problem-specific solver such as Combus requires considerably more time and expertise.

Last, but very definitely not least, one can use parallel search with Gecode, see Section 7.3. The two word instances that Combus could solve within 20 minutes, but that Gecode could not, were re-tried with parallel search. The instances are run with four threads instead of a single thread (that is, exploiting two cores and hyper-threading on the used hardware platform) and the results are shown in Figure 19.7. The two instances could now be solved within the timeout. None of the instances that neither Combus nor Gecode could solve were helped by parallel search though, even when the timeout was increased to two hours. Note that the runtime is the measure of a single run only (several runs show a quite high coefficient of deviation due to parallel search).

**Acknowledgments.** We are grateful to Peter Van Beek for access to example grids and to Adi Botea for providing us with the dictionaries words and uk from [1].

P

# **Programming propagators**

Christian Schulte, Guido Tack

This part explains how to program propagators as implementations of constraints.

**Basic material.** Chapter 20 (Getting started) shows how to implement simple propagators for simple constraints over integer variables. It introduces the basic concepts and techniques that are necessary for any propagator.

**Programming techniques.** The bulk of this part describes a wide range of techniques for programming efficient propagators:

- Chapter 21 (Avoiding execution) discusses techniques for avoiding propagator execution. The examples used in this chapter introduce view arrays for propagators and Boolean views.
- Chapter 22 (Reification and rewriting) discusses how to implement propagators for reified constraints and how to optimize constraint propagation by propagator rewriting.
- Chapter 23 (Domain propagation) explains various programming techniques for propagators that perform domain propagation. The chapter also describes modification event deltas as information available to propagators and staging as a technique for speeding up domain propagation.
- Chapter 24 (Advisors) is concerned with advisors for efficient incremental propagation. Advisors can be used to provide information to a propagator which of its views have changed and how they have changed.
- Chapter 25 (Views) shows how to straightforwardly and efficiently reuse propagators for implementing several different constraints by using views. As it comes to importance, this chapter ranks second after Chapter 20. However, it comes rather late to be able to draw on the example propagators presented in the previous chapters.

**Overview material.** Chapter 26 (Propagators for set constraints) summarizes how to implement propagators for constraints over set variables. An overview of memory management for propagators (and branchers, see Part B) is provided by Chapter 27 (Managing memory).

# 20

# **Getting started**

This chapter shows how to implement simple propagators for simple constraints over integer variables. It introduces the basic concepts and techniques that are necessary for any propagator.

Here, and in the following chapters, the focus is on propagators over integer and Boolean variables. Part of the concepts introduced are specific to integer and Boolean propagators, however the techniques how to program efficient propagators are largely orthogonal to the type of variables. In Chapter 26, the corresponding concepts for set variables are presented.

**Important.** This chapter introduces concepts and techniques step-by-step, starting with a naive and inefficient first version of a propagator that then is stepwise refined. Even if you feel compelled to start programming right after you have seen the first, naive variant, you should very definitely read on until having read the entire chapter.

Overview. The first three sections set the stage for programming propagators. Section 20.1 sketches how propagators perform constraint propagation and is followed by an overview of some useful background reading material (Section 20.2). Section 20.3 provides an overview of what needs to be implemented for a constraint followed by the first naive implementation of a simple constraint (Section 20.4). The naive implementation is improved in Section 20.5 by both taking advantage of some predefined abstractions in Gecode and straightforward optimizations. This is followed by a discussion of propagation conditions as a further optimization to avoid redundant propagator executions (Section 20.6). The next section (Section 20.7) presents a first reasonable propagator that takes advantage of predefined patterns to cut down on programming effort. The last two sections discuss the obligations a propagator must meet (Section 20.8) and how some of these obligations can be waived by a propagator (Section 20.9).

# 20.1 Constraint propagation in a nutshell

Constraints and variables are used for modeling constraint problems. However, the only reason for actually modeling a problem is to be able to solve it with constraint propagation by removing values from variables that are in conflict with a constraint. In order to implement constraint propagation, a constraint (typically) requires a *propagator* (or several propagators) as its implementation.

**Views versus variables.** The essence of a propagator is to remove values from variables that are in conflict with the constraint the propagator implements. However, a propagator does not use variables directly as they only offer operations for accessing but not removing values. Instead, a propagator uses *variable views* (or just views) as they offer operations for both value access and removal.

Views and variables have a simple relationship in that they both offer interfaces to variable implementations. When a variable is created for modeling, also a *variable implementation* is created. The variable serves as a read-only interface to the variable implementation. A view can be initialized from a variable: the view becomes just another interface to the variable's variable implementation. For more information on the relationship between variables, views, and variable implementations see Part V.

In the following we often refer to the variables of a propagator as the variable implementations that the propagator refers to through views. That is, we will often not distinguish between variables, views, and variable implementations. There is little risk of confusion as propagators always compute with views and variable implementations are never exposed for programming propagators. Much more on the relationship between variables, views, and variable implementations can be found in Part V.

By the *domain* of a variable (or a view, or a variable implementation) we refer to the set of values the variable still can take. We will often use notation such as  $x \in \{1, 2, 5\}$  which means that the domain of x is  $\{1, 2, 5\}$ .

**Executing propagators.** A propagator is implemented in Gecode as a subclass of the class Propagator where the different tasks a propagator must be able to perform are implemented as virtual member functions. Before we describe these functions and their purpose, we sketch how a propagator actually performs constraint propagation.

As mentioned above, a propagator has several views on which it performs constraint propagation according to the constraint it implements. Like variables in modeling, propagators and views (more precisely, their variable implementations) belong to a *home space* (or just *home*). When a propagator is created, it *subscribes* to some of its views: subscriptions control the execution of a propagator. As soon as a view changes (the only way how a view can change is that some of its values are removed), all propagators that are subscribed to the view are *scheduled* for execution. Actually, subscriptions offer additional control in that only certain types of value removals schedule a propagator, this is discussed in Section 20.6. A propagator that is not scheduled, is called *idle*. Scheduling and execution are sketched in Figure 20.1.

Eventually, the space chooses a scheduled propagator for execution and executes it by running the propagate() member function of the propagator. This function possibly removes values and by this might schedule more propagators for eventual execution (possibly re-scheduling the currently executing propagator itself). Besides removing values and scheduling propagators, the propagate() member function reports about propagation. The details of what can be reported by a propagator are detailed in the following sections, but a particularly important report is: the propagator reports failure as it found out that the

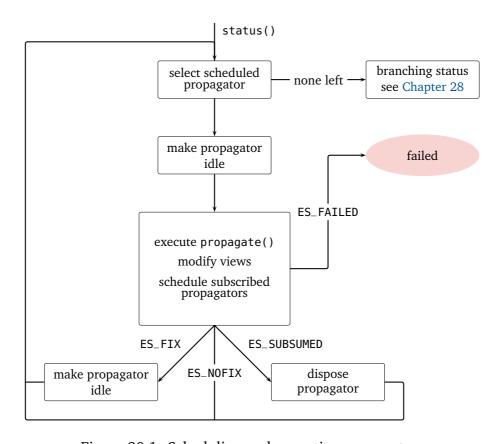


Figure 20.1: Scheduling and executing propagators

constraint it implements is unsatisfiable with the values left for the propagator's views (this is described by returning the value ES\_FAILED). The remaining return values are discussed in Section 20.4 and Section 20.5.

To get the entire propagation process started, some but not necessary all propagators are scheduled as soon as they are created. Which propagators are scheduled initially is discussed in detail in Section 20.6.

Propagation is interleaved in that a space executes only one propagator at a time. The process of choosing a scheduled propagator and executing it is repeated by the space until no more propagators are available for execution. If no more propagation is possible, a space is called *stable*. This process implements constraint propagation and must be explicitly triggered by executing the status() member function of a space (please consult Tip 2.2). The status() function is typically invoked by a search engine (see Part S for details).

**Space and propagator fixpoints.** We often refer to the fact that no more propagation is possible for a space by saying that the space is at *fixpoint*. Likewise, we say that a propagator that cannot remove any more values is at *fixpoint*. The term fixpoint is intuitive when one looks at typical models for constraint propagation: propagators are modeled as functions that take variables and their values as input and output, often referred to as stores or domains. A domain that is a fixpoint means that input and output for a propagator are the same and hence the propagator did not perform any propagation.

## 20.2 Background reading

This document does not present any formal or mathematical model of how propagation is organized and which properties propagation has. There are numerous publications that do that in more detail than is possible in this document:

- A general overview of how a constraint programming system works can be found in [34].
- Realistic models that present many ideas that have been developed in the context of Gecode can be found in [37], [35], and [44].
- A truly general model for propagation that is used in Gecode is introduced in [40]. This publication is rather specialized and the generality of the model will be only briefly discussed later (see Section 20.8 and Section 36.2).

It is highly recommended to read about the general setup of constraint propagation in one of these papers. For more advanced ideas, we often refer to certain sections in the above publications or to further publications. Remember, one of the advantages of Gecode is that many of its key ideas have been introduced by Gecode and are backed by academic publications.

## 20.3 What to implement?

Our first propagator implements the less constraint x < y for two integer variables x and y.

**Constraint post functions.** Before discussing what a propagator must do in detail, we need to discuss how to implement the function for our less constraint that can be used for modeling. As known from Part M, the function should have a declaration such as

```
void less(Space& home, IntVar x0, IntVar x1);
```

We call a function implementing a constraint a *constraint post function*. The constraint post function less() takes a space home where to post the constraint (actually, where to post the propagator implementing the constraint) and variables x0 and x1. The responsibilities of a constraint post function are straightforward: it checks whether its arguments are valid (and throws an exception otherwise), creates variable views for the variables passed to it, and then posts an appropriate propagator. This is detailed below.

**Tip 20.1** (Variables and views are passed by value). As discussed before, variables and views are nothing but different interfaces to variable implementations. Their implementations are hence strikingly simple: they just carry a pointer to a variable implementation and their member functions just call the appropriate member functions of the variable implementation (some views might store an additional integer as well). For that reason, it is sufficient to pass variables and views simply by value rather than by reference. ◀

What a propagator must do. The previous section focused on the execution of a propagator that then prunes variables. While performing propagation is typically the most involved part of a propagator, a propagator must also handle several other tasks: how to post and initialize it, how to dispose it, how to copy it during cloning for search, and when to execute it. How propagation is organized in Gecode is sketched in Figure 20.2, this paragraph will fill in the missing details.

**posting** A constraint post function typically initiates the posting of one or several propagators. Posting a propagator is organized into two steps. The first step is implemented by a *propagator post function*, while the second is implemented by the propagator's constructor.

Typical tasks in a propagator post function are as follows:

- Decide whether the propagator really needs to be posted.
- Decide whether a related, simpler, and hence more efficient propagator should be posted instead.

 $<sup>^{1}</sup>$ As we will discuss later (and as you might have noticed when modeling with Gecode, see Tip 2.1), a constraint post function takes a value of class Home instead of Space&. A value of type Home includes a reference to a space together with potentially additional information useful for posting. This aspect is discussed in Section 20.5 in more detail.

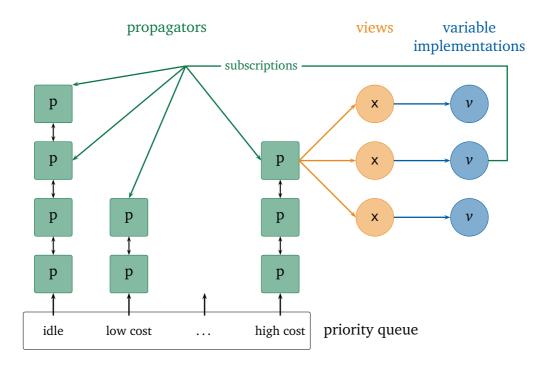


Figure 20.2: Propagators, views, and variable implementations

- Enforce certain invariants the propagator might require (for example, restricting the values of variables so that the propagator becomes simpler).
- Perform some initial propagation such that variable domains become reasonably small (for a discussion why small variable domains are useful, see Tip 4.3).
- Last but definitely not least, create an instance of the propagator.

The reason why posting requires a constraint post function as well as a propagator post function is to separate concerns. We will see later that the propagator post function is typically being reused by several constraints and also for propagator rewriting, an important technique which is discussed in Chapter 22. The post function of a propagator is conveniently implemented as a static member function post() of the propagator's class.

The propagator's constructor initializes the propagator and performs another essential task: it creates *subscriptions* to views for the propagator. Only if a propagator *subscribes* to a view (together with a propagation condition which is ignored for the moment and discussed later), the propagator is scheduled for execution whenever the domain of the view changes. Subscribing also automatically schedules the propagator for execution if needed (detailed in Section 20.6).

disposal Gecode does not automatically garbage collect propagators.<sup>2</sup> Propagators must be

<sup>&</sup>lt;sup>2</sup>One of the key design principles of Gecode is: *no magic!* Whatever needs to be done must be done explicitly.

explicitly disposed. When a propagator is disposed it must also explicitly *cancel* its subscriptions (the subscriptions the propagator created in the constructor).

The only exception to this rule is that subscriptions on assigned variables do not need to be canceled. In fact, as an optimization, assigned variables do not maintain subscriptions: subscribing to an assigned variable schedules the propagator and canceling a subscription on an assigned variable does nothing. Disposal must also free other resources currently used by the propagator.

Propagator disposal is implemented by a virtual member function dispose(). A propagator has no destructor, the dispose function assumes this role instead. The reason to have a dispose() function rather than a destructor is that disposal requires the home space of a propagator which is passed as an argument to the dispose() function (destructors cannot take arguments in C++).

Disposal might be triggered by the propagator itself (in fact, this is the most common case) as we will see later. It is important to understand that when the space of a propagator is deleted, its propagators are *not* being disposed by default. A propagator can explicitly request to be disposed when its home is deleted (see Section 20.8). A typical case where this is needed is when the propagator has allocated memory that is not managed by the space being deleted. In Chapter 27, memory management is discussed in detail.

**copying** Copying for propagators works exactly as does copying for spaces: a virtual copy () member function returns a copy of a propagator during cloning and a copy constructor is in charge of copying and updating the propagator's data structures (in particular, its views).

**cost computation** Scheduling a propagator guarantees that it is executed eventually but it does not specify when. When a propagator is executed, is defined by its *cost*. The cheaper it is to execute the propagator, the earlier the propagator should be executed. This is based on the intuition that a cheaper propagator might either already fail a space and hence no expensive propagators must be executed, or that a cheaper propagator might perform propagation from which a more expensive propagator can take advantage.

As to be expected, every propagator must implement a virtual cost() member function. This member function is called when the propagator is scheduled. In fact, the cost() function might be called several times when certain information (so-called modification event deltas) for a propagator changes. We postpone a discussion of the details to Section 23.5.

The cost of executing a propagator is just an approximation that helps propagation in Gecode to be fast and also to prevent pathological behavior. Propagators of same cost are executed according to a first scheduled, first executed policy (basically, scheduling is organized fairly into queues of propagators with similar cost). However, Gecode

does not give hard guarantees on the order of propagator execution: the cost of a propagator should be understood as a suggestion and not a requirement.

For the interested reader, the design of cost-based scheduling for Gecode together with its evaluation can be found in [37, Section 6].

**propagation** The core of a propagator is how it performs propagation by removing values from its views that are in conflict with the constraint it implements. This is implemented by a virtual member function propagate() that returns an *execution status*.

The execution status returned by the propagate() function must capture several important aspects:

- As mentioned before, a propagator must report whether failure occurred by returning an appropriate value for the execution status. The propagator can either find out by some propagation rules that failure must be reported. Or, when it attempts to modify view domains, the *modification operation* reports that it failed (a so-called *domain wipe-out* occurred, as all values would have been removed). Modification operations are also called *tell operations*.
- A propagator must report when the propagator has become *subsumed* (also known as *entailed*): that is, when the propagator can never ever again perform any propagation and should be disposed.
  - The requirement to report subsumption is rather weak in that a propagator must at the very latest report subsumption if all of its views are assigned (of course, it might report subsumption earlier, as will be discussed in Section 20.5). With other words, a propagator must *always* report subsumption but can wait until all its views are assigned (unless it reports failure, of course).
- A propagator can characterize what it actually has computed: either a fixpoint for itself or not. We ignore this aspect for the time being and return to it in Section 20.5 and continue this discussion in Section 21.1.

**Obligations of a propagator.** What becomes quite clear is that a propagator has to meet certain obligations (disposing subscriptions, detecting failure, detecting subsumption, and so on). Some obligations must be met in order to comply with Gecode's requirements of a well-behaved propagator, other obligations must be met so that a propagator becomes a faithful implementation of a constraint. Section 20.8 provides an overview of all obligations a propagator must meet.

# 20.4 Implementing the less constraint

Figure 20.3 shows the class definition Less for our less propagator and the definition of the constraint post function. Unsurprisingly, the propagator Less uses two views for integer

```
[DOWNLOAD]
LESS ≡
 #include <gecode/int.hh>
  using namespace Gecode;
  class Less : public Propagator {
  protected:
    Int::IntView x0, x1;
  public:
    ► POSTING
    ▶ DISPOSAL
    ► COPYING
    ► COST COMPUTATION
    ▶ PROPAGATION
  };
  void less(Space& home, IntVar x0, IntVar x1) {
    ► CONSTRAINT POST FUNCTION
  }
```

Figure 20.3: A constraint and propagator for less

variables of type Int::IntView and propagates that the values for x0 must be less than the values for x1.

The Less propagator inherits from the class Propagator defined by the Gecode kernel (as any propagator must do) and stores two integer views which are defined by Gecode's integer module. Hence, we need to include <gecode/int.hh>. Note that only the constraint post functions of the integer module are available in the Gecode namespace. All other functionality, including Int::IntView, is defined in the namespace Gecode::Int.

**Constraint post function.** The constraint post function is implemented as follows:

```
CONSTRAINT POST FUNCTION ≡
Int::IntView y0(x0), y1(x1);
if (Less::post(home,y0,y1) != ES_OK)
home.fail();
```

The constraint post function creates two integer variable views y0 and y1 for its integer variable arguments and calls the static propagator post function as defined by the Less class. A propagator post function also returns an execution status of type ExecStatus (see Status of constraint propagation and branching commit) where the only two values that can be returned by a propagator post function are ES\_OK (posting was successful) and ES\_FAILED (the post function determined even without actually posting the propagator that the constraint Less is unsatisfiable). In case ES\_FAILED is returned, the constraint post function must mark the current space home as failed (by using home.fail()).

**Propagator posting.** The posting of the Less propagator is defined by a constructor designed for initialization and a static post function returning an execution status as follows:

The constructor initializes its integer views and creates subscriptions to both x0 and x1. Subscribing to an integer view takes the home space, the propagator as subscriber (as a reference), and a propagation condition of type PropCond. We do not look any further into propagation conditions right here (Section 20.6 does this in detail) but give two hints. First, the values for propagation conditions depend on the variable view as witnessed by the

fact that the value Int::PC\_INT\_DOM is declared in the namespace Gecode::Int. Second, Int::PC\_INT\_DOM creates a subscription such that the propagator is executed whenever the domain of x0 (respectively x1) changes.

The propagator post function is entirely naive in that it always creates a Less propagator and always succeeds (and hence returns ES\_0K). Note that propagators can only be created in a space. Hence, the **new** operator is used in a placement version with placement argument (home) which allocates memory for the Less propagator from home.

**Disposal.** The virtual dispose() function for the Less propagator takes a home space as argument and returns the size of the just disposed propagator (as type size\_t).<sup>3</sup> Otherwise, the dispose() function does exactly what has been described above: it cancels the subscriptions created by the constructor used for posting:

```
DISPOSAL =
  virtual size_t dispose(Space& home) {
    x0.cancel(home,*this,Int::PC_INT_DOM);
    x1.cancel(home,*this,Int::PC_INT_DOM);
    (void) Propagator::dispose(home);
    return sizeof(*this);
}
```

Note that the arguments for canceling a subscription are (and must be) exactly the same as the arguments for creating a subscription.

**Copying.** The virtual copy() function and the corresponding copy constructor are unsurprising in that they follow exactly the same structure as the corresponding function and constructor for spaces used for modeling:

```
COPYING \( \)
Less(Space& home, bool share, Less& p)
: Propagator(home, share, p) {
    x0.update(home, share, p.x0);
    x1.update(home, share, p.x1);
}
virtual Propagator* copy(Space& home, bool share) {
    return new (home) Less(home, share, *this);
}
```

The only aspect that deserves some attention is that a propagator must be created in a home space and hence a placement **new** operator is used (analogous to propagator posting).

<sup>&</sup>lt;sup>3</sup>Following the discussion from above, the size of a disposed propagator must be returned explicitly, as propagators do not use destructors that implicitly handle size.

#### static cost functions

unary	propagator with single variable view
binary	propagator with two variable views
ternary	propagator with three variable view

#### dynamic cost functions

dynamic cost functions					
linear	propagator with $\approx$ linear complexity (or $O(n \log n)$ )				
quadratic	propagator with $\approx$ quadratic complexity				
cubic	propagator with $\approx$ cubic complexity				
crazy	propagator with $\approx$ exponential (or large polynomial) complexity				

Figure 20.4: Summary of propagation cost functions

**Cost computation.** Cost values for propagators are defined by the class PropCost. The class PropCost defines several static member functions with which cost values can be created. For Less, the cost() function returns a cost value for a binary propagator with low cost (as we will see below, the propagate() function is really cheap to execute):

```
COST COMPUTATION ≡
  virtual PropCost cost(const Space&, const ModEventDelta&) const {
   return PropCost::binary(PropCost::L0);
}
```

Please ignore the additional argument of type ModEventDelta to cost() for now, see Section 23.4.

The static member functions provided by PropCost are summarized in Figure 20.4. Each function takes either the value PropCost::L0 (for low cost) or PropCost::HI (for high cost). The dynamic cost functions take an additional integer (or unsigned integer) value defining how many views the propagator is computing with.

For example, a propagator with n variables and complexity  $O(n \log n)$  might return a cost value constructed by

```
PropCost::linear(PropCost::HI,n);
```

As mentioned before, propagation cost is nothing but an approximation of the real cost of the next execution of the propagate() function of the propagator. The only hard fact you can rely on is that a propagator with a cost value using PropCost::HI is never given preference over a propagator with a cost value using PropCost::LO.

**Propagation proper.** Before starting with the code for propagation, we have to work out how the propagator should prune. This can be rather involved, leading to specialized *pruning* or *filtering* algorithms. For our Less propagator, the filtering rules are simple:

■ All values for x0 must be less than the largest possible value of x1.

```
eq(home,n) assign to value n
nq(home,n) remove value n
le(home,n) restrict values to be less than n
lq(home,n) restrict values to be less or equal than n
gr(home,n) restrict values to be greater than n
restrict values to be greater or equal than n
```

Figure 20.5: Value-based modification functions for integer variable views

• All values for x1 must be larger than the smallest possible value of x0.

These two rules can be directly implemented as follows (again, please ignore the additional argument of type ModEventDelta to propagate() for now):

```
PROPAGATION =
  virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
    if (x0.le(home,x1.max()) == Int::ME_INT_FAILED)
        return ES_FAILED;
    if (x1.gr(home,x0.min()) == Int::ME_INT_FAILED)
        return ES_FAILED;
    if (x0.assigned() && x1.assigned())
        return home.ES_SUBSUMED(*this);
    else
        return ES_NOFIX;
}
```

The le() modification operation applied to an integer view x takes a home space and an integer value n and keeps only those values from the domain of x that are smaller than n (gr() is analogous). A view modification operation returns a modification event of type ModEvent (see Generic modification events and propagation conditions). A modification event describes how the domain of a view has changed, in particular the value Int::ME\_INT\_FAILED is returned if a domain wipe-out has occurred. In that case, the propagator has found out (just by attempting to perform a view modification operation) that the constraint it implements is unsatisfiable. In this case, a propagator immediately returns with the execution status ES\_FAILED. Naturally, the member functions min() and max() of an integer view x just return the smallest and largest possible value of the domain of x.

If the propagator had not reported failure by returning ES\_FAILED it would be faulty: it would incorrectly claim that values for the views are solutions of the constraint it implements. Being correct with respect to the constraint a propagator implements is one of the obligations of a propagator we will discuss in Section 20.8.

The modification operations for integer variable views taking a single integer value n as argument are listed in Figure 20.5. Integer variable views also support modification opera-

```
LESS BETTER 

...

class Less: public Propagator {
...

public:
    Less(Home home, Int::IntView y0, Int::IntView y1)
...
    }
    ▶ POSTING
...

▶ PROPAGATION
};
```

Figure 20.6: A better constraint and propagator for less

tions that simultaneously can operate on sets of values. These operations are discussed in Chapter 23.

The second part of the propagator is concerned with deciding subsumption: if both x0 and x1 are assigned, the propagator executes the function ES\_SUBSUMED(). The function ES\_SUBSUMED() disposes the propagator by calling its dispose() member function and returns a value for ExecStatus that signals that the propagator is subsumed. After returning, the memory for the propagator will be reused. It is important to understand that subsumption is not an optimization but a requirement: at the very latest when all of its views are assigned, a propagator must report subsumption! The propagator is of course free to report subsumption earlier as is exploited in Section 20.5.

In case the propagator is neither failed nor subsumed, it reports an execution status ES\_NOFIX. Returning the value ES\_NOFIX means that the propagator will be scheduled if one of its views (x0 and x1 in our example) have been modified. If none of its views have been modified, the propagator is not scheduled. This rather naive statement of what the propagator has computed is improved in Section 20.5.

**Tip 20.2** (Immediately return after subsumption). As mentioned above, the function ES\_SUBSUMED() disposes a propagator. To not crash Gecode, you must immediately return after calling ES\_SUBSUMED(). In particular, executing view modification operations or creating subscriptions by a disposed propagator will surely crash Gecode! ◀

### 20.5 Improving the Less propagator

Figure 20.6 shows an improved implementation of the Less propagator together with the corresponding constraint post function less(). The propagator features improved posting,

improved propagation, and improved readability.

A recurring theme in improving propagators in this and in the next section is not about sophisticated propagation rules (admittedly, Less does not have much scope for cleverness). In contrast, all improvements will try to achieve the best of all optimizations: avoid executing the propagator in the first place!

**Improving posting.** As mentioned above, all functions related to posting (constructor, propagator post function, and constraint post function) should take a value of type Home rather than Space&. The improved propagator honors this without any further changes (a Space is automatically casted to a Home if needed, and vice-versa). An example of how passing a Home value is actually useful can be found in Section 22.2.

The improved constraint post function is as follows:

```
CONSTRAINT POST FUNCTION ≡
  void less(Home home, IntVar x0, IntVar x1) {
   if (home.failed()) return;
   GECODE_ES_FAIL(Less::post(home,x0,x1));
}
```

The constraint post function features three improvements:

- The most obvious improvement: if the home space is already failed, no propagator is posted.
- The variable views are initialized implicitly. Note that the propagator post function Less::post() is called with integer variables x0 and x1 which are automatically coerced to integer views (as Int::IntView has a non-explicit constructor with argument type IntVar).
- Instead of testing whether Less::post() returns ES\_FAILED, the constraint post function uses the macro GECODE\_ES\_FAIL for convenience (doing exactly the same as shown before). Macros for checking and failing are summarized below.

The improved post function is a little bit more sophisticated:

It includes the following improvements:

- The propagator is not posted if x0 and x1 happen to refer to the very same variable implementation. In that case, of course, x0 can never be less than x1 and post() can immediately report failure.
- The propagator performs one initial round of propagation already during posting. This yields small variable domains for other propagators to be posted (see Tip 4.3).
- If all values of x0 are already less than all values of x1 (that is, x0.max() < x1.min()), then the constraint that x0 is less than x1 already holds (it is subsumed). Hence, no propagator needs to be posted.

**Improving propagation.** The propagate() function is improved as follows:

```
PROPAGATION =
virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
   GECODE_ME_CHECK(x0.le(home,x1.max()));
   GECODE_ME_CHECK(x1.gr(home,x0.min()));
   if (x0.max() < x1.min())
      return home.ES_SUBSUMED(*this);
   else
      return ES_FIX;
}</pre>
```

with the following improvements:

- The checking macro GECODE\_ME\_CHECK checks whether a modification operation returns failure and then returns ES\_FAILED.
- The propagator tries to detect subsumption early, it does not wait until both x0 and x1 are assigned. It uses exactly the same criterion that is used for avoiding posting of the propagator in the first place.
  - This is entirely legal: a propagator can report subsumption as soon as the propagator will never propagate again (or, with other words, the propagator will always be at fixpoint). The obligation is: it must report subsumption (or failure) at the very latest when all of its views are assigned. Early subsumption means fewer propagator executions.
- If the propagator is not yet subsumed it returns ES\_FIX instead of ES\_NOFIX. This means that the propagator tells the Gecode kernel that what it has computed happens to be a fixpoint for itself.
  - This is easy enough to see for Less: executing the propagate() function twice does not perform any propagation in the second call to propagate(). After all, only after x0.min() or x1.max() change, the propagator can prune again. As neither x0.min()

#### check macros

GECODE\_ME\_CHECK(me) Checks whether me signals failure and returns ES\_FAILED.

GECODE\_ES\_CHECK(es) Checks whether es signals failure or subsumption and returns es.

#### fail macros

GECODE\_ME\_FAIL(me) Checks whether me signals failure, fails home, and returns. GECODE\_ES\_FAIL(es) Checks whether es signals failure, fails home, and returns.

Figure 20.7: Check and fail macros

nor x1.max() change (the propagator might change x1.min() or x0.max() instead), the propagator should report that it has computed a fixpoint.

The situation where returning ES\_FIX instead of ES\_NOFIX differs, is when the propagator actually prunes the values for x0 or x1. If the propagator returns ES\_NOFIX it will be scheduled in this situation. If it returns ES\_FIX it will not be scheduled. The difference between ES\_FIX and ES\_NOFIX is sketched in Figure 20.1. Again, not scheduling a propagator means fewer propagator executions. Chapter 21 takes a second look at fixpoint reasoning for propagators.

Check and fail macros. Check and fail macros available in Gecode are summarized in Figure 20.7. Note that a check macro can be used in a propagator post function or in a propagate() function, whereas a fail macro can only be used in a constraint post function. Note also that both fail macros assume that the identifier home refers to the current home space. For an example of how to use GECODE\_ES\_CHECK, see Section 21.3. For an example of how to use GECODE\_ME\_FAIL, see Section 21.2.

In fact, explicitly testing whether a modification event returned by a view modification operation is equal to Int::ME\_INT\_FAILED is highly discouraged. If not using GECODE\_ME\_CHECK, the proper way to test for failure is as in:

```
if (me_failed(x0.le(home,x1.max())))
  return ES_FAILED;
```

Note that both GECODE\_ME\_CHECK as well as me\_failed work for all variable views and not only for integer variable views.

## 20.6 Propagation conditions

This section discusses modification events and propagation conditions in more detail. A modification event describes how a modification operation has changed a view. A propaga-

tion condition describes when a propagator is scheduled depending on how the views it is subscribed to are modified.

**Modification events.** Modification operations on integer views might return the following values for ModEvent (we assume that the view x has the domain  $\{0, 2, 3\}$ ):

- Int::ME\_INT\_NONE: the view has not been changed. For example, both x.le(home,5) and x.nq(home,1) return Int::ME\_INT\_NONE.
- Int::ME\_INT\_FAILED: the values of a view have been wiped out. For example, both x.le(home,0) and x.eq(home,1) return Int::ME\_INT\_FAILED.
  - Note that when a modification operation signals failure, the values of a view might change unpredictably, see also Section 4.1.7. For example, the values might entirely remain or only part of the values are removed. This also clarifies that a view (or its variable implementation) does not maintain the information that a modification operation on it failed. This also stresses that it is essential to check the modification event returned by a modification operation for failure.
- Int::ME\_INT\_DOM: an inner value (that is neither the smallest nor the largest value) has been removed. For example, x.nq(home,2) ( $x \in \{0,3\}$ ) returns Int::ME\_INT\_DOM.
- Int::ME\_INT\_BND: the smallest or largest value of a view has changed but the view has not been assigned. For example, both x.nq(home,0) ( $x \in \{2,3\}$ ) and x.lq(home,2) ( $x \in \{0,2\}$ ) return Int::ME\_INT\_BND.
- Int::ME\_INT\_VAL: the view has been assigned to a single value. For example, both x.le(home,2) (x = 0) and x.gq(home,3) (x = 3) return Int::ME\_INT\_VAL.

**Propagation conditions.** The propagation condition used in subscriptions determine, based on modification events, when a propagator is scheduled. Assume that a propagator p subscribes to the integer view x with one of the following propagation conditions:

- Int::PC\_INT\_DOM: whenever the view x is modified (that is, for modification events Int::ME\_INT\_DOM, Int::ME\_INT\_BND, and ME\_INT\_VAL on x), schedule p.
- Int::PC\_INT\_BND: whenever the bounds of x are modified (that is, for modification events Int::ME\_INT\_BND and Int::ME\_INT\_VAL on x), schedule p.
- Int::PC\_INT\_VAL: whenever x becomes assigned (that is, for modification event Int::ME\_INT\_VAL on x), schedule p.

For our Less propagator, the right propagation condition for both views x0 and x1 is of course Int::PC\_INT\_BND: the propagator can only propagate if either the lower bound of x0 or the upper bound of x1 changes. Otherwise, the propagator is at fixpoint. Again, the

```
[DOWNLOAD]
Less even better \equiv
 class Less : public Propagator {
 public:
    Less(Home home, Int::IntView y0, Int::IntView y1)
      : Propagator(home), x0(y0), x1(y1) {
      x0.subscribe(home,*this,Int::PC_INT_BND);
      x1.subscribe(home,*this,Int::PC_INT_BND);
   }
   virtual size_t dispose(Space& home) {
      x0.cancel(home,*this,Int::PC_INT_BND);
      x1.cancel(home,*this,Int::PC_INT_BND);
      (void) Propagator::dispose(home);
      return sizeof(*this);
   }
   ...
 };
```

Figure 20.8: An even better constraint and propagator for less

```
[DOWNLOAD]
DISEQUALITY ≡
  class Disequal : public Propagator {
  public:
    Disequal(Home home, Int::IntView y0, Int::IntView y1)
      : Propagator(home), x0(y0), x1(y1) {
      x0.subscribe(home, *this, Int::PC_INT_VAL);
      x1.subscribe(home,*this,Int::PC_INT_VAL);
    }
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      if (x0.assigned())
        GECODE_ME_CHECK(x1.ng(home,x0.val()));
      else
        GECODE_ME_CHECK(x0.nq(home,x1.val()));
      return home.ES_SUBSUMED(*this);
    }
  };
```

Figure 20.9: A propagator for disequality

very point of propagation conditions is to avoid executing a propagator that is known to be at fixpoint. An even better propagator for less using the proper propagation conditions is shown in Figure 20.8.

The idea to avoid execution depending on how variables change is well known and typically realized through so-called *events*. For an evaluation, see [37, Section 5]. Distinguishing between modification events and propagation conditions has been introduced by Gecode, for a discussion see [44, Chapter 5].

The Less propagator subscribes to both of its views (that is, x0 and x1). For some propagators it is actually sufficient to only subscribe to some but not all of its views. Section 21.3 discusses partial and dynamically changing subscriptions for propagators.

**Scheduling when posting.** As mentioned earlier, a propagator also needs to be scheduled when it is created to get the process of constraint propagation started. More precisely, a propagator p might be scheduled when it subscribes to a view x. If x is assigned, p is always scheduled regardless of the propagation condition used for subscribing. If x is not assigned, p is only scheduled if the propagation condition is different from Int::PC\_INT\_VAL.

With other words: propagation conditions provide a guarantee a propagator can rely on. In particular, for the propagator condition Int::PC\_INT\_VAL, the guarantee is that the propagator is only executed if a view is assigned.

### single view patterns

UnaryPropagator BinaryPropagator TernaryPropagator NaryPropagator NaryOnePropagator unary propagator with view x0 binary propagator with views x0 and x1 ternary propagator with views x0, x1, and x2 *n*-ary propagator with view array x *n*-ary propagator with view array x and view y

#### mixed view patterns

MixBinaryPropagator MixTernaryPropagator MixNaryOnePropagator binary propagator with views x0 and x1 ternary propagator with views x0, x1, and x2 *n*-ary propagator with view array x and view y

Figure 20.10: Propagator patterns

Consider, for example, the implementation of the propagate() method for a disequality propagator shown in Figure 20.9. The propagator waits until at least x0 or x1 are assigned (due to the propagation condition PC\_INT\_VAL). When its propagate() method is executed, the propagator can exploit that at least one of the views x0 and x1 is assigned by testing only x0 for assignment.

Scheduling when creating subscriptions can be avoided by giving **false** as an optional last argument to subscribe(). This is typically used when subscriptions are created during propagation, and the propagator does not need to be scheduled.

### 20.7 Using propagator patterns

Gecode's kernel defines common propagator patterns (see Propagator patterns) which are summarized in Figure 20.10. The single view patterns are templates that require a view as first argument and a propagation condition as second argument. The mixed view patterns require for each view or view array a view argument and a propagation condition. The mixed view patterns are useful when different types of views and/or different propagation conditions for the views are needed (see Section 25.1.2 for an example).

The propagator patterns accept also a value Int::PC\_INT\_NONE for the propagation conditions that avoid creating subscriptions at all. This comes in handy when the propagator patterns are used in situations where no subscriptions are needed, see for example Section 24.3.

The patterns define a constructor for creation (that also creates subscriptions to their views with the defined propagation conditions), a constructor for cloning, a dispose() member function, and a cost() member function where the cost value is always the PropCost::LO variant of the corresponding cost function (that is, PropCost::unary() for UnaryPropagator, PropCost::linear() for NaryPropagator and NaryOnePropagator, and so on). The integer module additionally defines patterns for reified propagators which

are discussed in Section 22.2.4

Figure 20.11 shows how to use the BinaryPropagator pattern for the less constraint. To give an impression of what needs to be implemented, the code for implementing the less constraint is shown in full. Note that one must define a propagator post function and the virtual member functions copy() and propagate(). Of course, one could also choose to overwrite the virtual member functions dispose() and cost() if needed.

### 20.8 Propagator obligations

A propagator has to meet three different kinds of obligations: obligations towards the constraint it implements, obligations towards the amount of propagation it performs, and obligations that are Gecode specific. Some obligations can be waived by notifying the Gecode kernel that a propagator does not comply (see Section 20.9).

### **Constraint implementation.** A propagator must be

**correct** A propagator must be correct in that it never prunes values that can appear in a solution of the constraint it implements.

**checking** A propagator must be checking: at the very latest when all views are assigned, the propagator must decide whether the assignment is a solution of the constraint or not (in which case the propagator must report failure).

### **Amount of propagation.** A propagator must be

**contracting** A propagator is only allowed to remove values. The modification operations we have been discussing so-far naturally satisfy this property. For some operations that are discussed in Section 23.2 extra carefulness is required.

**monotonic** By default, a propagator must be monotonic: a propagator is not allowed to perform more pruning when executed for views with more values than when executed for views with less values.

Propagators are typically monotonic unless they use randomization, approximation, or something similar. For more details see below. This obligation can be waived.

**subscription complete** A propagator must create sufficient subscriptions such that it is scheduled for execution when it is not at fixpoint.

**fixpoint and subsumption honest** A propagator is not allowed to claim that it has computed a fixpoint or is subsumed if it is not (that is, it could still propagate).

<sup>&</sup>lt;sup>4</sup>As reified propagators require Boolean views, reified patterns are provided by the integer module and not by the kernel. The kernel knows nothing about the different views programmed on top of it.

```
[DOWNLOAD]
LESS CONCISE \equiv
 class Less : public BinaryPropagator<Int::IntView,Int::PC_INT_BND> {
    Less(Home home, Int::IntView x0, Int::IntView x1)
      : BinaryPropagator<Int::IntView,Int::PC_INT_BND>(home,x0,x1) {}
    static ExecStatus post(Home home,
                           Int::IntView x0, Int::IntView x1) {
      if (same(x0,x1))
        return ES_FAILED;
      GECODE_ME_CHECK(x0.le(home,x1.max()));
      GECODE_ME_CHECK(x1.gr(home, x0.min()));
      if (x0.max() >= x1.min())
        (void) new (home) Less(home, x0, x1);
      return ES_0K;
    Less(Space& home, bool share, Less& p)
      : BinaryPropagator<Int::IntView,Int::PC_INT_BND>(home,share,p) {}
    virtual Propagator* copy(Space& home, bool share) {
      return new (home) Less(home, share, *this);
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      GECODE_ME_CHECK(x0.le(home,x1.max()));
      GECODE_ME_CHECK(x1.gr(home,x0.min()));
      if (x0.max() < x1.min())
        return home.ES_SUBSUMED(*this);
      else
        return ES_FIX;
   }
 };
 void less(Home home, IntVar x0, IntVar x1) {
    if (home.failed()) return;
    GECODE_ES_FAIL(Less::post(home,x0,x1));
  }
```

Figure 20.11: A concise constraint and propagator for less

Implementation specific obligations. A propagator must be

**subsumption complete** At the very latest, a propagator must report subsumption if all views it has subscribed to are assigned.

external resource free By default, a propagator cannot allocate memory from any other source (or any other resource) but its home space as the dispose() member function is not automatically called when the propagator's home space is deleted. This obligation can be waived.

**update complete** All views a propagator subscribes to must be updated during cloning (that is, a propagator can only subscribe to views it actually stores).

The reason for this obligation is that a space does not know its views (and variable implementations). It only gets to know them during cloning when they are explicitly updated. Having subscription information maintained by Gecode's kernel without updating the corresponding variable implementations will leave a space after cloning in an inconsistent state (crashes are guaranteed).

**cloning conservative** When a propagator is copied during cloning, it is not allowed to perform any variable modification operations nor is it allowed to change its subscriptions.

**subscription correct** Any subscription to a view must eventually be canceled (typically in the dispose() member function), unless the view is assigned.

### 20.9 Waiving obligations

A propagator can notify its home space about some of its properties (as defined by ActorProperty, see Programming actors) that relate to some of the obligations mentioned in the previous section.

**Weakly monotonic propagators.** If a propagator p intends to be non-monotonic, it can notify its home space home by

```
home.notice(p,AP_WEAKLY);
```

It can revoke this notice later (it must revoke this at latest in its dispose() function) by

```
home.ignore(p,AP_WEAKLY);
```

This is typically done in the constructor of the propagator and means that the propagator intends to be only *weakly monotonic*: it is sufficient for the propagator to be checking and correct. For a discussion of weak monotonicity together with an example, see [40].

**Calling dispose() during space deletion.** If a propagator p needs to use external resources or non-space allocated memory, it must inform its home space during posting about this fact by:

```
home.notice(p,AP_DISPOSE);
```

This will ensure that the dispose() function of the propagator p will be called when its home space is deleted.

In its dispose function the propagator p must revoke this notice by

```
home.ignore(p,AP_DISPOSE);
```

For examples, see Section 24.2 and Section 17.4. In Chapter 27, memory management is discussed in detail.

# 21

## **Avoiding execution**

This chapter serves two purposes. First, it discusses techniques for avoiding propagator execution. Second, the examples used in this chapter introduce view arrays for propagators and Boolean views.

**Overview.** Fixpoint reasoning as an important technique for avoiding propagator execution is discussed in detail in Section 21.1 (we already briefly touched on this subject in Section 20.5). The following section (Section 21.2) presents an example propagator for Boolean disjunction introducing Boolean variable views and view arrays for propagators with arbitrarily many views. The Boolean disjunction propagator is used in Section 21.3 as an example for dynamic subscriptions (a propagator only subscribes to a subset of its views and the subscriptions change while the propagator is being executed) as another technique to avoid propagator execution.

### 21.1 Fixpoint reasoning reconsidered

In this section, we develop a propagator for equality x = y that performs bounds reasoning to further discuss how a propagator can do fixpoint reasoning. A stronger domain propagator for equality is discussed in Chapter 23. Background information on fixpoint reasoning can be found in [37, Section 4].

A naive propagator. The propagate() member function of an equality propagator Equal is shown in Figure 21.1. The remaining functions are as to be expected.

The propagation rules implemented by Equal is that the values of both x0 and x1 must be greater or equal than std::max(x0.min(),x1.min()) and less or equal than std::min(x0.max(),x1.max()). The above implementation follows these rules even though it avoids the computation of std::min and std::max. Let us look to the adjustment of the lower bounds of x0 and x1 (the upper bounds are analogous):

- If x0.min()==x1.min(), nothing is pruned.
- If x0.min()<x1.min(), then x0 is pruned.
- If x0.min()>x1.min(), then x1 is pruned.

```
EQUAL NAIVE 
...

class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_BND> {
  public:
    ...

    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
        GECODE_ME_CHECK(x0.gq(home,x1.min()));
        GECODE_ME_CHECK(x1.gq(home,x0.min()));
        GECODE_ME_CHECK(x0.lq(home,x1.max()));
        GECODE_ME_CHECK(x1.lq(home,x0.max()));
        if (x0.assigned() && x1.assigned())
            return home.ES_SUBSUMED(*this);
        else
            return ES_NOFIX;
        }
    };
    ...
}
```

Figure 21.1: A naive equality bounds propagator

As can be seen, the propagator does not perform any fixpoint reasoning, it always returns ES\_NOFIX (unless it is subsumed or failed).

The propagator might actually sometimes compute a fixpoint and sometimes not. Consider  $x0 \in \{1,2,3,4\}$  and  $x1 \in \{0,1,2,5\}$ . Then Equal propagates that  $x0 \in \{1,2,3,4\}$  and  $x1 \in \{1,2\}$  which happens to be not a fixpoint. The reason (which is common when performing bounds propagation) is that a bounds modification (here x1.lq(home,4)) has resulted in an even smaller upper bound (or an even larger lower bound when the lower bound is modified). In a way, the modification operation updating the bound fell into a hole in the variable domain.

**Reporting fixpoints.** The above example shows that the equality propagator computes a fixpoint if and only if, after propagation, x0.min()==x1.min() and x0.max()==x1.max(). Figure 21.2 shows the propagate() member function of an improved Equal propagator that takes advantage of fixpoint reasoning.

An idempotent propagator. The previous propagator reports when it computes a fixpoint. However, we can change any propagator so that it always computes a fixpoint. Propagators which always compute a fixpoint (unless they are subsumed) are known as *idempotent* propagators. The idea to turn an arbitrary propagator into an idempotent propagator is simple: repeat propagation until it has computed a fixpoint. Figure 21.3 shows the propagate() member function of an idempotent equality propagator.

```
virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
    GECODE_ME_CHECK(x0.gq(home,x1.min()));
    GECODE_ME_CHECK(x1.gq(home,x0.min()));
    GECODE_ME_CHECK(x0.lq(home,x1.max()));
    GECODE_ME_CHECK(x1.lq(home,x0.max()));
    if (x0.assigned() && x1.assigned())
        return home.ES_SUBSUMED(*this);
    else if ((x0.min() == x1.min()) && (x0.max() == x1.max()))
        return ES_FIX;
    else
        return ES_NOFIX;
    }
...
```

Figure 21.2: An equality bounds propagator with fixpoint reasoning

```
[DOWNLOAD]
EQUAL IDEMPOTENT ≡
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      do {
        GECODE_ME_CHECK(x0.gq(home,x1.min()));
        GECODE_ME_CHECK(x1.gq(home,x0.min()));
      } while (x0.min() != x1.min());
      do {
        GECODE_ME_CHECK(x0.lq(home,x1.max()));
        GECODE_ME_CHECK(x1.lq(home,x0.max()));
      } while (x0.max() != x1.max());
      if (x0.assigned() && x1.assigned())
        return home.ES_SUBSUMED(*this);
      else
        return ES_FIX;
    }
```

Figure 21.3: An idempotent equality bounds propagator

Figure 21.4: An idempotent equality bounds propagator using modification events

The idempotent propagator always computes a fixpoint. That means that it does not need to use the generic mechanisms provided by the Gecode kernel for scheduling and executing propagators in order to compute a fixpoint. Instead, a tight inner loop inside the propagate() function computes the fixpoint. If the propagator is cheap (which it is in our example) it might be better to make the propagator idempotent and hence avoid the overhead of the Gecode kernel (even though the Gecode kernel is very efficient as it comes to scheduling and executing propagators).

An idempotent propagator using modification events. The idempotent propagator shown above tests a propagator-specific criterion to determine whether a fixpoint has been computed. With the help of modification events there is a generic approach to computing a fixpoint within the propagate() member function of a propagator. Figure 21.4 shows the propagate() member function where the Boolean variable nafp (for: not at fixpoint) tracks whether the propagator has computed a fixpoint. The function me\_modified(me) checks whether the modification event me does not signal failure or that the view did change (that is, for integer views, me is different from Int::ME\_INT\_FAILED and Int::ME\_INT\_NONE). Whenever a view is modified, nafp is accordingly set to true. The remaining modification operations are omitted as they are analogous.

**Tip 21.1** (Understanding ES\_NOFIX). The above technique for making a propagator idempotent is based on the idea of repeating propagation until no more views are modified.

Unfortunately, we have seen (more than once) a similar but not very good idea in propagators for finding out whether a propagator is at fixpoint. The idea can be sketched as

follows: record in a Boolean flag modified whether a modification operation actually modified a view during propagation. For our bounds equality propagator the idea can be sketched as follows:

```
virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
   bool modified = false;

ModEvent me = x0.gq(home,x1.min());
   if (me_failed(me))
      return ES_FAILED;
   else if (me_modified(me))
      modified = true;
   ...
   if (x0.assigned() && x1.assigned())
      return home.ES_SUBSUMED(*this);
   else
      return modified ? ES_NOFIX : ES_FIX;
}
```

That means, if modified is **true** the propagator might not be at fixpoint and hence ES\_NOFIX is returned. This makes not much sense: ES\_NOFIX means *that a propagator is not considered to be at fixpoint if it has modified a view!*. If no view has been modified the propagator must be at fixpoint and just returning ES\_NOFIX does the trick.

This not-so-hot idea can be summarized as: achieving nothing with quite some effort! ◀

For the equality constraint, checking the propagator-specific fixpoint condition is simple enough. For more involved propagators, the generic approach using modification events always works and is to be preferred. As the generic approach is so useful, a macro <code>GECODE\_ME\_CHECK\_MODIFIED</code> is available. With this macro, the loop insided the <code>propagate()</code> function can be expressed as:

```
while (nafp) {
  nafp = false;
  GECODE_ME_CHECK_MODIFIED(nafp,x0.gq(home,x1.min()));
  GECODE_ME_CHECK_MODIFIED(nafp,x1.gq(home,x0.min()));
  GECODE_ME_CHECK_MODIFIED(nafp,x0.lq(home,x1.max()));
  GECODE_ME_CHECK_MODIFIED(nafp,x1.lq(home,x0.max()));
}
```

### 21.2 A Boolean disjunction propagator

Before we demonstrate dynamic subscriptions in the next section, we discuss a propagator for Boolean disjunction. The propagator is rather simple, but we use it as an example for Boolean views, arrays of views, and several other aspects.

```
[DOWNLOAD]
OR TRUE ≡
 class OrTrue : public Propagator {
 protected:
   ViewArray<Int::BoolView> x;
 public:
   OrTrue(Home home, ViewArray<Int::BoolView>& y)
      : Propagator(home), x(y) {
     x.subscribe(home,*this,Int::PC_BOOL_VAL);
   }
    ▶ PROPAGATION
 };
 void dis(Home home, const BoolVarArgs& x, int n) {
   if ((n != 0) \&\& (n != 1))
      throw Int::NotZeroOne("dis");
   if (home.failed()) return;
   if (n == 0) {
      for (int i=x.size(); i--; )
        GECODE_ME_FAIL(Int::BoolView(x[i]).zero(home));
   } else {
      ViewArray<Int::BoolView> y(home,x);
      GECODE_ES_FAIL(OrTrue::post(home,y));
   }
 }
```

Figure 21.5: Naive Boolean disjunction

Figure 21.5 shows the OrTrue propagator that propagates that an array of Boolean views x is 1 (that is, the Boolean disjunction on x is true). That is, at least one of the views in x must be 1. The propagator uses an array ViewArray of Boolean views (a ViewArray is generic with respect to the views it stores). Similar to views, view arrays have subscribe() and cancel() functions for subscriptions, where the operations are applied to all views in the view array.

**Tip 21.2** (View arrays also provide STL-style iterators). View arrays also support STL-style (Standard Template Library) iterators, similar to other arrays provided by Gecode, see Section 4.2.3. ◀

**Constraint post function.** The constraint post function dis() constrains the disjunction of the Boolean variables in x to be equal to the integer n:

$$\bigvee_{i=0}^{|\mathbf{x}|-1} \mathbf{x}_i = \mathbf{n}$$

The constraint post function checks that the value for n is legal. If n is neither 0 nor 1, the constraint post function throws an exception. Here, we use an appropriate Gecode-defined exception, but any exception of course works.

If n is 0, all variables in x must be 0 as well: rather than posting a propagator to do that, we assign the views immediately in the constraint post function. Note that we have to get an integer view to be able to assign x[i] to zero as only views provide modification operations.

Otherwise, a view array y is created (newly allocated in the space home) and its fields are initialized to views that correspond to the respective integer variables in x. Then, posting the OrTrue propagator is as expected.

A more general case of Boolean disjunction, where n is an integer variable instead of an integer value is discussed in Section 24.3.

**Propagation.** Propagation for Boolean disjunction is straightforward: first, all views are inspected whether they are assigned to 1 (in which case the propagator is subsumed) or to 0 (in which case the assigned view is eliminated from the view array). If no views remain, the propagator is failed. If a single (by elimination, an unassigned view) remains, it is assigned to 1. This can be implemented as shown in Figure 21.6.

The operation  $x.move_lst(i)$  of a view array x moves the last element of x to position i and shrinks the array by one element. Shrinking from the end is in particular simple if the array elements are iterated backwards as in our example. The class ViewArray provides several operations to shrink view arrays. Note that  $x.move_lst(i)$  requires that the view at position i is actually assigned or has no subscription, otherwise the subscription for x[i] needs to be canceled before x[i] is overwritten. View arrays also provide operations for simultaneously moving elements and canceling subscriptions.

```
PROPAGATION \(\begin{align*} \text{virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
    for (int i=x.size(); i--; )
        if (x[i].one())
            return home.ES_SUBSUMED(*this);
    else if (x[i].zero())
            x.move_lst(i);
    if (x.size() == 0)
        return ES_FAILED;
    if (x.size() == 1) {
        GECODE_ME_CHECK(x[0].one(home));
        return home.ES_SUBSUMED(*this);
    }
    return ES_FIX;
}
```

Figure 21.6: Propagation for naive Boolean disjunction

**Tip 21.3** (View arrays have non-copying copy constructors). The constructor for posting in Figure 21.5 uses the copy constructor of ViewArray to initialize the member x. The copy constructor does *not* copy a view array. Instead, after its execution both original and copy have shared access to the same views. ◀

**Using propagator patterns.** How to use a propagator pattern with an array of views is shown in Figure 21.7 for Boolean disjunction.

**Boolean views.** A Boolean view Int::BoolView provides operations for testing whether it is assigned to 1 (one()) or 0 (zero()), or whether it is not assigned yet (none()). As modification operations Boolean views offer one(home) and zero(home). Also, Boolean views only support PC\_BOOL\_NONE and PC\_BOOL\_VAL as propagation conditions and ME\_BOOL\_NONE, ME\_BOOL\_FAILED, and ME\_BOOL\_VAL as modification events.

Boolean views (variables and variable implementations likewise) are not related to integer views by design: the very point is that Boolean variable implementations have a specially optimized implementation that is in fact not related to the implementation of integer variables.

Boolean views also implement all operations that integer views implement and integer propagation conditions can also be used with Boolean views. PC\_INT\_DOM, PC\_INT\_BND, and PC\_INT\_VAL are mapped to the single Boolean propagation condition PC\_BOOL\_VAL. Having the same interface for integer and Boolean views is essential: Section 25.3 shows how integer propagators can be reused as Boolean propagators without requiring any modification.

Figure 21.7: Naive Boolean disjunction using a propagator pattern

### 21.3 Dynamic subscriptions

The previous section has been nothing but a warm-up to the presentation of dynamic subscriptions as another technique to avoid propagator execution.

Watched literals. The naive propagator presented above is disastrous: every time the propagator is executed, it checks all of its views to determine whether a view has been assigned to 0 or 1. Worse yet, pretty much all of the propagator executions are entirely pointless for propagation (but not for determining subsumption as is discussed below).

One idea would be to only check until two unassigned views have been encountered. In this case, it is clear that the propagator cannot perform any propagation. Of course, this might prevent the propagator from detecting subsumption as early as possible: as it does not scan all views but stops scanning after two unassigned views, it might miss out on a view already assigned to 1.

The idea to stop scanning after two unassigned views have been encountered can be taken even further. A well-known technique for the efficient implementation of Boolean SAT (satisfiability) solvers are *watched literals* [25]: it is sufficient to subscribe to two Boolean views for propagating a Boolean disjunction to be satisfied. Subscribing to a single Boolean view is not enough: if all views but the subscription view are assigned to 0 the subscription view must be assigned to 1 to perform propagation, however the propagator will not be scheduled as the single subscription view is not assigned. More than two subscriptions are not needed for propagation, as the propagator can only propagate if a single unassigned view remains. It might be the case that a view to which the propagator has not subscribed is assigned to 1. That means that the propagator is not subsumed as early as possible but that does not affect propagation.

```
[DOWNLOAD]
OR TRUE WITH DYNAMIC SUBSCRIPTIONS ≡
 class OrTrue :
   public BinaryPropagator<Int::BoolView,Int::PC_BOOL_VAL> {
 protected:
    ViewArray<Int::BoolView> x;
 public:
   OrTrue(Home home, ViewArray<Int::BoolView>& y)
      : BinaryPropagator<Int::BoolView,Int::PC_BOOL_VAL>
        (home,y[0],y[1]), x(y) {
     x.drop_fst(2);
   }
   virtual size_t dispose(Space& home) {
      (void) BinaryPropagator<Int::BoolView,Int::PC_BOOL_VAL>
        ::dispose(home);
      return sizeof(*this);
   }
    ► COPY
    ► RESUBSCRIBE
    ▶ PROPAGATION
 };
```

Figure 21.8: Boolean disjunction with dynamic subscriptions

The propagator. The propagator using dynamic subscriptions maintains exactly two subscriptions to its Boolean views. Figure 21.8 shows the OrTrue propagator with dynamic subscriptions. It inherits from the BinaryPropagator pattern and the views x0 and x1 of the pattern are exactly the two views with subscriptions. All remaining views without subscriptions are stored in the view array x. The operation drop\_fst(n) for an integer n drops the first n elements of a view array and shortens the array by n elements accordingly (that is, drop\_fst() is dual to drop\_lst() as used in the previous section). Note that the propagator must define a dispose() member function: this is needed not because dispose() must cancel additional subscriptions (the very point is that x has no subscriptions) but that it must return the correct size of OrTrue.

**Tip 21.4** (drop\_fst() and drop\_lst() are efficient). Due to the very special memory allocation policy used for view arrays (their memory is allocated in a space and is never freed as they only can shrink, see Chapter 27), both drop\_fst() and drop\_lst() have constant time complexity. ◀

**Propagation.** The idea of how to perform propagation is fairly simple: if one of the views with subscriptions is assigned to 1, the propagator is subsumed. If one of the subscription views is assigned to 0, say x0, a function resubscribe() tries to find a yet unassigned view to subscribe to it and store it as x0. If there is no such view but there is a view assigned to 1, the propagator is subsumed. If there is no such view, then the propagator tries to assign 1 to x1, and, if successful, the propagator is also subsumed. The implementation is as follows:

```
PROPAGATION \(\overline{\text{virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
    if (x0.one() || x1.one())
        return home.ES_SUBSUMED(*this);
    if (x0.zero())
        GECODE_ES_CHECK(resubscribe(home, x0, x1));
    if (x1.zero())
        GECODE_ES_CHECK(resubscribe(home, x1, x0));
    return ES_FIX;
}
```

**Resubscribing.** The function resubscribe() implements the search for a yet unassigned view for subscription and is shown in Figure 21.9.

**Copying.** The assigned views in x do not really matter much: all views assigned to 0 can be discarded. If there is a view assigned to 1 all other views can be discarded (of course, a single view assigned to 1 must be kept for correctness). Hence a good idea for copying is: copy only those views that still matter. This leads to a smaller view array requiring less memory. We decide to discard assigned views as much as we can in the copy() function rather than in the constructor used for copying. By this, also the original and not only the copy profits

Figure 21.9: Resubscribing for Boolean disjunction with dynamic subscriptions

from fewer views. While the copy benefits because there are less views to be stored, the original propagator benefits because resubscribe() does not have to scan assigned views as they already have been eliminated. Following this discussion, the copy() function can be implemented as:

```
copy =
virtual Propagator* copy(Space& home, bool share) {
  for (int i=x.size(); i--; )
    if (x[i].one()) {
       x[0]=x[i]; x.size(1); break;
    } else if (x[i].zero()) {
       x.move_lst(i);
    }
  return new (home) OrTrue(home, share, *this);
}
```

There is another optimization during copying that looks promising. If all views in x are eliminated, a propagator without the view array x is sufficient as a copy. If there is a view assigned to 1, then a propagator should be created that is subsumed. How this can be achieved is discussed in Section 22.4.

# **22**

# Reification and rewriting

This chapter discusses how to implement propagators for reified constraints and how to optimize constraint propagation by propagator rewriting. Gecode does not offer any special support for reification but uses propagator rewriting as a more general technique.

**Overview.** The definition of a reified constraint and how it can be propagated is reviewed in Section 22.1. The following section (Section 22.2) presents a refied less or equal propagator as an example. General propagator rewriting during constraint propagation is discussed in Section 22.3, whereas Section 22.4 presents how to rewrite propagators during cloning.

### 22.1 Reification

A propagator for the reified constraint  $c \Leftrightarrow b = 1$  for a Boolean *control variable* b propagates as follows:

- 1. If b is 1, propagate c.
- 2. If b is 0, propagate  $\neg c$  (it is not always easy to propagate the negation of a constraint c effectively and efficiently).
- 3. If *c* is subsumed, propagate that b is 1.
- 4. If  $\neg c$  is subsumed, propagate that b is 0.

The idea how to implement reification in Gecode is quite simple: if the first (or second) case is true, the reified propagator implementing the reified constraint rewrites itself into a propagator for c (or  $\neg c$ ). The remaining two cases are nothing but testing criteria for subsumption.

The advantage of rewriting a reified propagator for  $c \Leftrightarrow b = 1$  into a non-reified propagator for c or  $\neg c$  is that the propagators created by rewriting are typically available anyway, and that after rewriting no more overhead for reification is incurred.

### 22.2 A reified less or equal propagator

We will discuss a reified less or equal propagator  $x \le y \Leftrightarrow b = 1$  for integer variables x and y as an example. Taking the above propagation rules for a reified constraint, we obtain:

- 1. If b is 1, propagate  $x \le y$ .
- 2. If b is 0, propagate x > y (actually, we choose to propagate y < x instead).
- 3. If  $x \le y$  is subsumed, propagate that b is 1.
- 4. If x > y is subsumed, propagate that b is 0.
- 5. If none of the above rules apply, the propagator is at fixpoint.

The propagator ReLeEq for reified less or equal relies on propagators Less for less and LeEq for less or equal (the propagators are not shown, see Section 20.7 instead). The propagate() function as shown in Figure 22.1 follows the propagation rules sketched above.

**Propagator rewriting.** The GECODE\_REWRITE macro takes the propagator (here, \*this) to be rewritten and an expression that posts the new propagator as arguments. It relies on the fact that the identifier home refers to the current home space (like the fail macros in Section 20.5). The macro expands to something along the following lines:

```
size_t s = this->dispose(home);
GECODE_ES_CHECK(LeEq::post(home(*this),x0,x1));
return home.ES_SUBSUMED_DISPOSED(*this,s);
```

That is, the ReLeEq propagator is disposed, the new LeEq propagator is posted, and subsumption is reported. The order is essential for performance: by first disposing ReLeEq, the data structures that store the subscriptions for x0 and x1 will have at least one free slot for the subscriptions that are created by posting LeEq and hence avoid (rather inefficient and memory consuming) resizing. Note that it is important to understand that the old propagator is disposed before the new propagator is posted. In case that some data structures that are deleted during disposal are needed for posting, one either has to make sure that the data structures outlive the call to dispose() or that one does not use the GECODE\_REWRITE macro but first posts the new propagator and only then reports subsumption.

Another essential part is that after calling ES\_SUBSUMED(), a propagator is not allowed to do anything that can schedule propagators (such as performing modification operations or creating subscriptions). That is the reason that first dispose() is called and later the special variant ES\_SUBSUMED\_DISPOSED() is used for actually reporting subsumption. Do not use ES\_SUBSUMED\_DISPOSED() unless you really, really have to!

```
[DOWNLOAD]
LESS OR EQUAL REIFIED \equiv
 class Less
 };
 class LeEq
  . . .
 };
 class ReLeEq
    : public Int::ReBinaryPropagator<Int::IntView,Int::PC_INT_BND,</pre>
                                       Int::BoolView> {
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      if (b.one())
        GECODE_REWRITE(*this, LeEq::post(home(*this), x0, x1));
      if (b.zero())
        GECODE_REWRITE(*this, Less::post(home(*this), x1, x0));
      switch (Int::rtest_lq(x0,x1)) {
      case Int::RT_TRUE:
        GECODE_ME_CHECK(b.one(home)); break;
      case Int::RT_FALSE:
        GECODE_ME_CHECK(b.zero(home)); break;
      case Int::RT_MAYBE:
         return ES_FIX;
      }
      return home ES_SUBSUMED(*this);
    }
 };
```

Figure 22.1: A constraint and propagator for reified less or equal

Adding information to Home. As has been discussed in Tip 2.1 and Section 20.5, posting uses a value of class Home instead of a reference to a Space. In the expansion of GECODE\_REWRITE as shown above, the call operator () as in

### home(\*this)

returns a new value of type Home with the additional information added that the propagator to be posted is in fact a rewrite of the propagator \*this. This information is important, for example, for inheriting the AFC (accumulated failure count, see Section 7.1.1): the newly created propagator will inherit the number of accumulated failures from the propagator being rewritten. There will be other applications of Home in future versions of Gecode.

**Testing relations between views.** Rather than testing whether  $x0 \le x1$  or x0 > x1 hold individually, we use the function Int::rtest\_lq() that tests whether two views are less or equal and returns whether the relation holds (Int::RT\_TRUE), does not hold (Int::RT\_FALSE), or might hold or not (Int::RT\_MAYBE).

Relation tests are available for two views (integer or Boolean) or for a view (again, integer or Boolean) and an integer. Relation tests exist for all inequality relations: Int::rtest\_le() (for <), Int::rtest\_lq() (for  $\leq$ ), Int::rtest\_gr() (for >), and Int::rtest\_gq() (for  $\geq$ ). For equality, a bounds test (Int::rtest\_eq\_bnd()) as well as a domain test exists (Int::rtest\_eq\_dom()). While Int::rtest\_eq\_bnd() only uses the bounds for testing the relation (with constant time complexity), Int::rtest\_eq\_dom() uses the full set of values in the domain of the views to determine the relation (having linear time complexity in the length of the range representation of the views' domains, see Section 23.2 for more information on range representations of view domains). The same holds true for disequality: Int::rtest\_nq\_bnd() performs the bounds-only test, whereas Int::rtest\_nq\_dom() performs the full domain test. For example, Int::rtest\_eq\_bnd(x,y) for  $x \in \{0,2\}$  and  $y \in \{1,3\}$  returns Int::RT\_MAYBE, whereas Int::rtest\_eq\_dom() returns Int::RT\_FALSE.

Reified propagator patterns. The integer module (as these patterns require Boolean views they are part of the integer module) provides reified propagator patterns for unary propagators (Int::ReUnaryPropagator) and binary propagators (Int::ReBinaryPropagator and Int::ReMixBinaryPropagator). In addition to views x0 (and x1 for the binary variants), they define a Boolean control variable b. Please note that in Figure 22.1 a reified propagator pattern requires an additional template argument for the Boolean control view used (the mystery why this is useful is lifted in Chapter 25).

### 22.3 Rewriting during propagation

Rewriting during propagation is a useful technique that is not limited to implementing reification. We consider a Max propagator that propagates  $\max(x_0, x_1) = x_2$ . The propagation

rules (we are giving bounds propagation rules that achieve bounds( $\mathbb{Z}$ ) consistency, see [10]) for Max are easy to understand when looking at an equivalent formulation of max [36]:

$$\max(\mathsf{x}_0,\mathsf{x}_1) = \mathsf{x}_2 \iff (\mathsf{x}_0 \le \mathsf{x}_2) \land (\mathsf{x}_1 \le \mathsf{x}_2) \land \big( (\mathsf{x}_0 = \mathsf{x}_2) \lor (\mathsf{x}_1 = \mathsf{x}_2) \big)$$
$$\iff (\mathsf{x}_0 \le \mathsf{x}_2) \land \big( \mathsf{x}_1 \le \mathsf{x}_2 \big) \land \big( (\mathsf{x}_0 \ge \mathsf{x}_2) \lor (\mathsf{x}_1 \ge \mathsf{x}_2) \big)$$

Then, the propagation rules can be turned into the following C++-code to be executed by the propagate() member function of Max:

```
GECODE_ME_CHECK(x2.lq(home,std::max(x0.max(),x1.max())));
GECODE_ME_CHECK(x2.gq(home,std::max(x0.min(),x1.min())));
GECODE_ME_CHECK(x0.lq(home,x2.max()));
GECODE_ME_CHECK(x1.lq(home,x2.max()));
if ((x1.max() <= x0.min()) ||
    (x1.max() < x2.min()))
GECODE_ME_CHECK(x0.gq(home,x2.min()));
if ((x0.max() <= x1.min()) ||
    (x0.max() < x2.min()))
GECODE_ME_CHECK(x1.gq(home,x2.min()));</pre>
```

Please take a second look: the condition of the first **if**-statement tests whether  $x_1$  is less or equal than  $x_0$ , or whether  $x_1$  is less than  $x_2$ . In both cases,  $max(x_0, x_1) = x_2$  simplifies to  $x_0 = x_2$ . Exactly this simplification can be implemented by propagator rewriting, please check Figure 22.2. Note that the propagator is naive in that it does not implement the propagator to be idempotent (however, this can be done exactly as demonstrated in Section 21.1).

Another idea would be to perform rewriting during cloning: the copy() function could return an Equal propagator rather than a Max propagator in the cases where rewriting is possible. However, this is illegal: it would create a propagator with only two views, hence one view would not be updated even though there is a subscription for it (violating obligation "update complete" from Section 20.8). Also, canceling a subscription is illegal during cloning (violating obligation "cloning conservative" from Section 20.8). The next section shows an example with opposite properties: rewriting during propagation makes no sense (even though it would be legal) whereas rewriting during cloning is legal and useful.

### 22.4 Rewriting during cloning

In Section 21.3, the propagator 0rTrue is simplified during copying by eliminating assigned views. Here we show that the propagator created as a copy during cloning can be optimized

```
[DOWNLOAD]
MAX USING REWRITING ≡
  class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_BND> {
 };
  class Max : public TernaryPropagator<Int::IntView,Int::PC_INT_BND> {
  public:
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      GECODE_ME_CHECK(x2.lq(home,std::max(x0.max(),x1.max())));
      GECODE_ME_CHECK(x2.gq(home,std::max(x0.min(),x1.min())));
      GECODE_ME_CHECK(x0.lq(home,x2.max()));
      GECODE_ME_CHECK(x1.lq(home,x2.max()));
      if ((x1.max() \le x0.min()) || (x1.max() < x2.min()))
        GECODE_REWRITE(*this, Equal::post(home(*this), x0, x2));
      if ((x0.max() \le x1.min()) || (x0.max() < x2.min()))
        GECODE_REWRITE(*this, Equal::post(home(*this),x1,x2));
      if (x0.assigned() && x1.assigned() && x2.assigned())
        return home.ES_SUBSUMED(*this);
      else
        return ES_NOFIX;
   }
  };
```

Figure 22.2: A maximum propagator using rewriting

further by rewriting it during copying.

**Copying.** The modified copy() function is as follows:

```
virtual Propagator* copy(Space& home, bool share) {
  for (int i=x.size(); i--; )
    if (x[i].one()) {
        x[0]=x[i]; x.size(1);
        return new (home) SubsumedOrTrue(home, share, *this);
    } else if (x[i].zero()) {
        x.move_lst(i);
    }
  if (x.size() == 0)
    return new (home) BinaryOrTrue(home, share, *this);
  else
    return new (home) OrTrue(home, share, *this);
}
```

The copy() function takes advantage of two cases:

1. If there is a view that is assigned to 1, the propagator is subsumed. However, during cloning a propagator cannot be deleted by subsumption. Reporting subsumption is only possible during propagation. The copy() function does the next best thing: it creates a propagator SubsumedOrTrue that next time it is executed will in fact report subsumption.

Note that in this situation rewriting during cloning is preferable to rewriting during propagation. An important aspect of the OrTrue propagator is that it does not inspect all of its views during finding a view for resubscribing (as implemented by the resubscribe() member function). Instead, inspection stops as soon as the first unassigned view is found. That entails that a view that is assigned to 1 might be missed during propagation. In other words, rewriting during cloning also optimizes subsumption detection.

2. If all views in the view array x are assigned to 0, the view array is actually not longer needed (it has no elements). In this case, the copy() function creates a propagator BinaryOrTrue that is a special variant of OrTrue limited to just two views. Note that the two views x0 and x1 are known to be not yet assigned: otherwise, the propagator would not be at fixpoint. This is impossible as only spaces that are at fixpoint (and hence all of its propagators must be at fixpoint) can be cloned (this invariant is discussed in Section 35.1).

Rewriting during propagation would be entirely pointless in this situation: the propagator (be it 0rTrue or Binary0rTrue) will be executed at most one more time and

```
OR TRUE USING REWRITING \equiv
                                                               [DOWNLOAD]
 class BinaryOrTrue :
   BinaryOrTrue(Space& home, bool share,
                 BinaryPropagator<Int::BoolView,Int::PC_BOOL_VAL>& p)
   virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      if (x0.zero())
        GECODE_ME_CHECK(x1.one(home));
      if (x1.zero())
        GECODE_ME_CHECK(x0.one(home));
      return home.ES_SUBSUMED(*this);
   }
 };
 class SubsumedOrTrue :
   SubsumedOrTrue(Space& home, bool share,
                   BinaryPropagator<Int::BoolView,Int::PC_BOOL_VAL>& p)
   virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      return home.ES_SUBSUMED(*this);
 };
  class OrTrue :
    ► COPY
 };
```

Figure 22.3: A Boolean disjunction propagator using rewriting

will become subsumed then (or, due to failure, it is not executed at all). As rewriting incurs the cost of creating and disposing propagators and subscriptions, rewriting in this case would actually slow down execution.

Constructors for copying. The relevant parts of the two special propagators for rewriting SubsumedOrTrue and BinaryOrTrue are shown in Figure 22.3. Their propagate() functions are exactly as sketched above. The only other non-obvious aspect are the constructors used for copying during cloning: they are now called for a propagator of class OrTrue. In this example, it is sufficient to have a single constructor for copying as all the propagators OrTrue, SubsumedOrTrue, and BinaryOrTrue inherit from BinaryPropagator. In other cases, it might be necessary to have more than a single constructor for copying defined by a propagator class C: one for copying a propagator of class C and one for creating propagators as rewrites of other propagators.

# 23

### Domain propagation

The propagators in previous chapters use simple modification operations on variable views in that only a single integer defines how a view's domain is changed. When programming propagators that perform more elaborate domain reasoning, these modification operations are typically insufficient as they easily lead to incorrect and inefficient propagators.

To conveniently program efficient propagators that perform domain reasoning, Gecode offers modification operations that take entire sets of integers as arguments. To be efficient, these sets are not implemented as a typical set data structure but as *iterators*: the integers (or entire ranges of integers) can be iterated in increasing order (smallest first).

**Overview.** This chapters motivates why special domain operations on variable views are needed (Section 23.1) and demonstrates how iterators are used for domain propagation (Section 23.2 and Section 23.3). Section 23.4 and Section 23.5 describe modification event deltas and staging for efficiently combining bounds with domain propagation.

### 23.1 Why domain operations are needed

Let us consider an attempt to implement domain propagation for an equality constraint on views x0 and x1. The idea for pruning is simple: only keep those values of x0 and x1 that are common to the domains of both x0 and x1. This leads to a first version of a domain equality propagator as shown in Figure 23.1. Beware, the "propagator" is not only naive but also incorrect: it will crash Gecode!

The propagator uses iterators of type Int::ViewValues to iterate over the values of an integer view in increasing order (the template argument defines the type of the view to iterate on). The increment operator ++ moves the iterator to the next value, the application operator () tests whether the iterator has more values to iterate, and the function val() returns the iterator's current value. The reason why the propagator will crash is simple: it uses the iterator i for iterating over the values of  $x\theta$  and modifies the domain of  $x\theta$  while iterating! This is illegal: when using an iterator for a view, the view cannot be modified (or, in  $C^{++}$  lingua: modifying the variable invalidates the iterator).

One idea how to fix this problem is to use a temporary data structure in which the newly computed intersection domain is stored. That is not very tempting: allocating and initializing an intermediate data structure is costly.

```
INCORRECT DOMAIN EQUAL ≡
 class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
 public:
    Equal(Home home, Int::IntView x0, Int::IntView x1)
      : BinaryPropagator<Int::IntView,Int::PC_INT_DOM>(home,x0,x1) {}
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      Int::ViewValues<Int::IntView> i(x0), j(x1);
      while (i() && j())
        if (i.val() < j.val()) {
          GECODE_ME_CHECK(x0.nq(home,i.val())); ++i;
        } else if (j.val() < i.val()) {</pre>
          GECODE_ME_CHECK(x1.nq(home,j.val())); ++j;
        } else {
          ++i; ++j;
      while (i()) {
        GECODE_ME_CHECK(x0.nq(home,i.val())); ++i;
      while (j()) {
        GECODE_ME_CHECK(x1.nq(home,j.val())); ++j;
      }
      if (x0.assigned() && x1.assigned())
        return home.ES_SUBSUMED(*this);
      else
        return ES_FIX;
   }
 };
```

Figure 23.1: An incorrect propagator for domain equal

```
[DOWNLOAD]
NAIVE DOMAIN EQUAL ≡
  class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
    virtual PropCost cost(const Space&, const ModEventDelta&) const {
      return PropCost::binary(PropCost::HI);
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      Int::ViewRanges<Int::IntView> r0(x0);
      GECODE_ME_CHECK(x1.inter_r(home, r0));
      Int::ViewRanges<Int::IntView> r1(x1);
      GECODE_ME_CHECK(x0.narrow_r(home,r1));
      if (x0.assigned() && x1.assigned())
        return home.ES_SUBSUMED(*this);
      else
        return ES_FIX;
    }
  };
```

Figure 23.2: A naive propagator for domain equal

But even then, the approach is flawed from the beginning: a single nq() operation on a view has linear runtime in the size of the view's domain (actually, in the length of its range sequence, see below). As potentially a linear number of nq() operations are executed, the propagator will have quadratic complexity even though it should have linear (as the values of the domains are available in strictly increasing order).

### 23.2 Iterator-based modification operations

Figure 23.2 shows a working, yet still naive, implementation of an equality propagator performing domain reasoning. Note that the cost() function is overridden: even though the propagator is binary, it now incurs a higher cost due to the domain operations to be performed. The propagate() function uses two *range iterators* for iterating over the range sequence of the domains of x0 and x1. For the definition of a range sequence, see Section 4.1.6.

A range iterator for a range sequence  $s = \langle [n_i ... m_i] \rangle_{i=0}^k$  is an object that provides iteration over s: each of the  $[m_i ... n_i]$  can be obtained in sequential order but only one at a time. A range iterator provides the following operations: the application operator () tests whether there are more ranges to iterate, the increment operator ++ moves to the next range, the function  $\min()$  ( $\max()$ ) returns the smallest (largest) value of the current range, and the function width() returns the width of the current range (that is, its number of elements) as

an unsigned integer.

The motivation to iterate over range sequences rather than individual values is efficiency: as there are typically less ranges than indvidual values, iteration over ranges can be more efficient. Moreover, many operations are sufficiently easy to express in terms of ranges rather than values (see Section 23.3 for an example).

**Iterator-based modification operations.** The propagator uses two modification operations for range iterators:  $x1.inter_r()$  intersects the current domain of x1 with the set defined by the range iterator r0 for x0. After this operation (provided no failure occurred), the view x1 has the correct domain: the intersection of the domains of x0 and x1. The operation  $x0.narrow_r()$  replaces the current domain of x0 by the set defined by the range iterator r1 (which iterates the intersection of the domains of x0 and x1).

A third operation available for range iterators is minus\_r() which removes the values as described by the range iterator passed as argument.

Instead of using range iterators for modification operations, one can also use value iterators instead. Similarly, a view provides operations inter\_v(), narrow\_v(), and minus\_v() for intersecting, replacing, and removing values.

**Tip 23.1** (Narrow is dangerous). The narrow\_r() operation used in the above example is dangerous (as is the narrow\_v() operation). As discussed in Section 20.8, a propagator must be contracting: the domain of a view is only allowed to get smaller. If narrow\_r() is used incorrectly, then one could actually replace the view's domain by a larger domain.

In the above example, the propagator is contracting: r1 refers to the intersection of x0 and x1, which of course has no more values than x0.

**Tip 23.2** (Iterators must be increasing). The range sequence iterated by a range iterator *must* be sorted in increasing order and *must* not overlap as described above. Otherwise, domain operations using range iterators become incorrect.

For value iterators, the values must be increasing in value with iteration. But it is okay if the same value is iterated over multiply. That is, the values must be increasing but not necessarily strictly increasing.

**Avoiding shared iterators.** The problem that made our attempt to implement propagation for equality in Section 23.1 incorrect is to use iterators for iterating over views that are simultaneously modified.

Iterator-based modification operations automatically take care of potential sharing between the iterator they use and the view domain they update. By default, an iterator-based modification operation assumes that iterator and domain are shared. The operation first constructs a new domain and iterates to the end of the iterator. Only then the old domain is replaced by the newly constructed domain. In many cases, however, there is no sharing between iterator and domain and the operations could be performed more efficiently by in-place update operations on the domain.

```
[DOWNLOAD]
NON-SHARED DOMAIN EQUAL ≡
 class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
    static ExecStatus post(Home home,
                           Int::IntView x0, Int::IntView x1) {
      if (!same(x0,x1))
        (void) new (home) Equal(home, x0, x1);
      return ES_0K;
   }
   virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      Int::ViewRanges<Int::IntView> r0(x0);
      GECODE_ME_CHECK(x1.inter_r(home,r0,false));
      Int::ViewRanges<Int::IntView> r1(x1);
      GECODE_ME_CHECK(x0.narrow_r(home,r1,false));
   }
 };
```

Figure 23.3: A propagator for domain equal without sharing

In our example, there is no sharing if the views x0 and x1 do not refer to the very same variable implementation. If they do, the propagator should not even be posted as it is subsumed anyway (views of the same type referring to the same variable implementation are trivially equal). Figure 23.3 shows the propagator post function of an improved propagator for equality: the propagator is posted only if x0 and x1 do not refer to the same variable implementation. The propagate() function is improved by giving an additional **false** argument to both inter\_r() and narrow\_r(). Hence, the two operations use more efficient operations performing in-place updates on domains.

### 23.3 Taking advantage of iterators

This section shows how the combination of simple iterators can help in implementing domain propagation.

Suppose that we want to implement a close variant of the equality constraint, namely x = y + c for integer variables x and y and some integer value c. It is easy to see that the new domain for x must be all values of x intersected with the values  $\{n+c \mid n \in y\}$ . Likewise, the new domain for y must be all values of y intersected with the values  $\{n-c \mid n \in x\}$ . But how can we implement these simple propagation rules?

**Mapping range sequences.** Assume a range sequence  $\langle [m_i ... n_i] \rangle_{i=0}^k$  for the values in the domain of y. Then, what we want to compute is a range sequence

$$\langle [m_i + c ... n_i + c] \rangle_{i=0}^k$$
.

With other words, we want to map the first into the second range sequence. Luckily, this is easy. Suppose that r is a range iterator for the view y. A range iterator for our desired range sequence can be constructed using the Iter::Range::Map iterator and an object that describes how to map the ranges of r to the desired ranges.

For this, we define the following class:

```
OFFSET MAP \( \)
    class OffsetMap {
    protected:
        int o;
    public:
        OffsetMap(int o0) : o(o0) {}
        int min(int m) const {
            return m+o;
        }
        int max(int m) const {
            return m+o;
        }
    }
}
```

```
[DOWNLOAD]
DOMAIN EQUAL WITH OFFSET ≡
  ▶ OFFSET MAP
  class EqualOffset :
    public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
  protected:
    int c;
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      Int::ViewRanges<Int::IntView> r0(x0);
      OffsetMap om0(-c);
      Iter::Ranges::Map<Int::ViewRanges<Int::IntView>,OffsetMap>
        mr0(r0,om0);
      GECODE_ME_CHECK(x1.inter_r(home, mr0, false));
      Int::ViewRanges<Int::IntView> r1(x1);
      OffsetMap om1(c);
      Iter::Ranges::Map<Int::ViewRanges<Int::IntView>,OffsetMap>
        mr1(r1,om1);
      GECODE_ME_CHECK(x0.narrow_r(home,mr1,false));
   }
  };
  . . .
```

Figure 23.4: A propagator for domain equal with offset

OffsetMap defines to which values the minimum (min()) and the maximum (max()) of each input range must be mapped. Using OffsetMap, we can construct a range iterator m for our desired sequence by

With the help of the map range iterator, propagation for OffsetEqual as shown in Figure 23.4 is straightforward. Note that several functions need to be modified to deal with the additional integer constant c used by the propagator (the details can be inspected by downloading the full program code). Further note that the constraint post function is slightly to liberal in that it does not check whether the values with the integer constant c added exceed the limits for legal integer values.

While the propagator is reasonably easy to construct using map iterators, Section 25.1.2 shows how to obtain exactly the same propagator without any programming effort but the implementation of an additional constraint post function.

**Using and defining iterators.** Gecode comes with a multitude of range and value iterators to transform range and value sequences of other iterators. These iterators are defined in the namespace Iter::Ranges and value iterators in the namespace Iter::Values. Example iterators include: iterators to convert value into range iterators and vice versa, iterators to compute the union and intersection, iterators to iterate over arrays and bitsets, just to name a few.

But even if the predefined iterators are not sufficient, it is straightforward to implement new iterators: the only requirement is that they implement the appropriate interface mentioned above for range or value iterators. The namespace Iter contains a multitude of simple and advanced examples.

**Benefits of iterators.** The true benefit of using iterators for performing value or range transformations is that the iterator-based domain modification operation with which an iterator is used is automatically specialized at compile time. Typically, no intermediate data structures are created and the modification operations are optimized at compile time for each individual iterator.<sup>1</sup>

A scenario where this in particular matters is when iterators are used as *adaptors* of a propagator-specific data structure. Assume that a propagator uses a specific (typically, quite involved) data structure to perform propagation. Most often this data structure encodes in some form which views should take which values. Then, one can implement simple iterators that inspect the propagator-specific data structure and provide the information about values for views so that they can be used directly with iterator-based modification operations. Again, intermediate data structures are avoided by this approach.

### 23.4 Modification event deltas

There is a rather obvious approach to improving the efficiency of domain operations performed by propagators: make the domains as small as possible! One typical approach to reduce the size of the domains is to perform bounds propagation first. After bounds propagation, the domains are likely to be smaller and hence the domain operations are likely to be more efficient.

For propagating equality, the simplest idea is to first perform bounds propagation for equality as discussed in Section 21.1, directly followed by domain propagation. However, we can improve further by exploiting an additional token of information about the views of a propagator that is supplied to both the cost() and propagate() function of a propagator.

The cost() and propagate() functions of a propagator take an additional *modification* event delta value of type ModEventDelta (see Programming actors) as argument. Every propagator maintains a modification event delta that describes how its views have changed since

<sup>&</sup>lt;sup>1</sup>Some predefined iterators actually have to resort to intermediate data structures. For example, iterators that need to revert the order of its values or ranges (think of a value iterator for values that are the negation of values of some other value iterator).

```
[DOWNLOAD]
DOMAIN EQUAL USING BOUNDS PROPAGATION ≡
 class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
   virtual ExecStatus propagate(Space& home,
                                 const ModEventDelta& med) {
     if (Int::IntView::me(med) != Int::ME_INT_DOM) {
          GECODE_ME_CHECK(x0.gq(home,x1.min()));
          GECODE_ME_CHECK(x1.gq(home,x0.min()));
       } while (x0.min() != x1.min());
       do {
          GECODE_ME_CHECK(x0.lq(home,x1.max()));
          GECODE_ME_CHECK(x1.lq(home,x0.max()));
       } while (x0.max() != x1.max());
       if (x0.assigned() && x1.assigned())
          return home.ES_SUBSUMED(*this);
       if (x0.range() && x1.range())
          return ES_FIX;
     }
   }
 };
```

Figure 23.5: A propagator for domain equal using bounds propagation

the last time the propagator has been executed. For each view type (that is, integer, Boolean, set, ...) a modification event delta stores a modification event that can be extracted from the modification event delta: If med is a modification event delta, then Int::IntView::me(med) returns the modification event for integer views.

The extracted modification event describes how all views of a certain view type have changed. For example, for integer views, the modification event Int::ME\_INT\_VAL signals that there is at least one integer view that has been assigned since the propagator has been executed last (analogous for Int::ME\_INT\_BND and Int::ME\_INT\_DOM). Even the modification event Int::ME\_INT\_NONE carries some information: none of the propagator's integer views have been modified (which, of course, can only happen if the propagator also uses views of some other type).

Figure 23.5 shows a propagator that combines both bounds and domain propagation for propagating equality. It first extracts the modification event for integer views from the modification event delta med. Only if the bounds (that is, modification events Int::ME\_INT\_VAL or Int::ME\_INT\_BND, hence different from Int::ME\_INT\_DOM) have changed for at least one

of the views x0 and x1, the propagator performs bounds propagation.

After performing bounds propagation, the propagator checks whether it is subsumed. Then it does some more fixpoint reasoning: if the domains of x0 and x1 are ranges (that is, they do not have holes), the propagator is at fixpoint. Otherwise, domain propagation is done as shown before.

### 23.5 Staging

Taking the perspective of a single propagator, first performing bounds propagation directly followed by domain propagation seems to be appropriate. However, when taking into account that some other cheap propagators (at least cheaper than performing domain propagation by this propagator) might already be able to take advantage of the results of bounds propagation, it might actually be better to postpone domain propagation and give cheaper propagators the opportunity to execute. This idea is known as *staging* and has been introduced in Gecode [37, Section 7]. Note that staging is not limited to bounds and domain propagation but captures any stages in propagation that differ in cost.

Here, we focus on staging for first performing bounds propagation (stage "bounds") and then domain propagation (stage "domain") for equality. Additionally, a propagator can be idle (stage "idle"). The stage of a propagator is controlled by how its modification event delta changes:

- Initially, the propagator is idle, its modification event delta is empty, and the propagator is in stage "idle".
- When the modification event delta for integer views changes to Int::ME\_INT\_DOM and the propagator is in stage "idle", the propagator is put into stage "domain" with high propagation cost.
- When the modification event delta for integer views changes to Int::ME\_INT\_VAL or Int::ME\_INT\_BND, the propagator is put into stage "bounds" with low propagation cost.

Note that this in particular includes the case where the modification event delta for integer views has been Int::ME\_INT\_DOM and where the propagator had already been in stage "domain". As soon as the modification event delta changes to Int::ME\_INT\_BND or Int::ME\_INT\_VAL the propagator is put into stage "bounds".

By the very construction of modification event deltas, the modification event delta for integer views can neither change from Int::ME\_INT\_VAL to Int::ME\_INT\_BND nor from Int::ME\_INT\_BND (or Int::ME\_INT\_VAL) to Int::ME\_INT\_DOM. That is, if the equality propagator using staging is in stage "bounds" it stays in that stage until it is executed.

When the propagator is executed, it can be either in stage "bounds" or stage "domain". If it is executed in stage "bounds", it performs bounds propagation and then returns that it wants to be put into stage "domain". The essential point is that the propagator does not

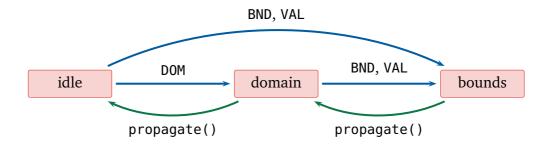


Figure 23.6: Stage transitions for the equality propagator

continue with domain propagation but gives other propagators the opportunity to execute first. If the propagator is executed in stage "domain", it performs domain propagation and returns that it is at fixpoint (and hence the propagator is put into stage "idle"). Figure 23.6 summarizes the stage transitions, where a blue transition is triggered by a change in the modification event delta (Int::ME\_INT\_DOM is abbreviated by DOM and so on) and a green transition is performed be executing the propagator.

**Rescheduling propagators.** The cost of a propagator depends on its modification event delta. This connection goes even further: only if the modification event delta of a propagator changes, a propagator is rescheduled according to its cost by recomputing the cost() function.

Not recomputing cost each time a propagator might be scheduled is done for two reasons. First, the number of possibly expensive cost computations is reduced. Second, always rescheduling would also violate the fairness among all propagators already scheduled with same cost. If a propagator is scheduled often, it would be penalized as its execution would be deferred.

Section 24.4 discusses a technique to force rescheduling of a propagator, irrespective of its modification event delta.

**Controlling staging by modification event deltas.** The cost() function and the essential parts of the propagate() function of a propagator that uses staging to combine bounds and domain propagation for equality are shown in Figure 23.7.

The cost() function returns the cost based on the modification event delta med: if med only includes Int::ME\_INT\_DOM, then the propagator returns that next time it executes, it executes at high-binary cost (according to stage "domain"). Otherwise, the propagator returns that next time it executes, it executes at low-binary cost (according to stage "bounds").

The propagate() function is almost identical to the function shown in the previous section. The only difference is that propagate() returns after having performed bounds propagation. The call to ES\_FIX\_PARTIAL() specifies that the current propagator \*this has computed a partial fixpoint for all modification events but Int::ME\_INT\_DOM. The function Int::IntView::med creates a modification event delta from a modification event for integer views. As an effect, the modification event delta of the current propagator is set to

```
[DOWNLOAD]
DOMAIN EQUAL USING STAGING \equiv
  class Equal : public BinaryPropagator<Int::IntView,Int::PC_INT_DOM> {
    virtual PropCost cost(const Space&,
                          const ModEventDelta& med) const {
      if (Int::IntView::me(med) != Int::ME_INT_DOM)
        return PropCost::binary(PropCost::L0);
      else
        return PropCost::binary(PropCost::HI);
    virtual ExecStatus propagate(Space& home,
                                 const ModEventDelta& med) {
      if (Int::IntView::me(med) != Int::ME_INT_DOM) {
        return home.ES_FIX_PARTIAL
          (*this,Int::IntView::med(Int::ME_INT_DOM));
      }
    }
  };
```

Figure 23.7: A propagator for domain equal using staging

include nothing but Int::ME\_INT\_DOM and the propagator is scheduled: as defined by the cost() function, the propagator is scheduled for high binary cost. That means that other propagators of lower cost might be executed first.

Constructing modification event deltas. Every view type provides a static med() function that translates a modification event of that view type into a modification event delta. Modification event deltas for different view types can be combined with the | operator. For example, the following expression combines the modification event ME\_INT\_BND for integer views with the modification event ME\_BOOL\_VAL for Boolean views:

```
Int::IntView::med(ME_INT_BND) | Int::BoolView::med(ME_B00L_VAL)
```

Note that only modification event deltas for different view types can be combined using |.

# 24

# **Advisors**

This chapter is concerned with advisors for efficient incremental propagation. Advisors can be used to provide information to a propagator which of its views have changed and how they have changed.

**Overview.** In Section 24.1, a motivation and a model for advisors is presented. The following two sections demonstrate advisors. Section 24.2 shows an example propagator that exploits the information provided by an advisor about which view has changed. Section 24.3 shows an example propagator that exploits information on how the domain of its views have changed. Section 24.4 sketches how advisors can be used for forcing propagators to be rescheduled.

### 24.1 Advisors for incremental propagation

Consider the following, rather simple, example constraint. The constraint samedom for an array of integer variables x and an integer set d holds, if and only if:

- either all variables in x take values from d (that is,  $x_i \in d$  for  $0 \le i < |x|$ ),
- or none of the x take values in d (that is,  $x_i \notin d$  for  $0 \le i < |x|$ ).

**More knowledge is needed.** Obviously, there are two different approaches to realize samedom:

**decomposition** We decompose the samedom constraint as follows. We create a Boolean variable b and post reified dom constraints (see Domain constraints) such that  $x_i \in d \Leftrightarrow b = 1$  (for  $0 \le i < |x|$ ). As the single Boolean variable b is the same for all reified dom constraints, samedom is automatically enforced.

**implementation** A different approach is to implement a dedicated propagator for samedom. Propagation is quite simple: whenever the propagator is executed, try to find a view among the  $x_i$  such that either  $x_i \in d$  or  $x_i \notin d$ . If there is no such  $x_i$ , the propagator is at fixpoint. Otherwise, the propagator constrains all  $x_i$  accordingly.

Let us compare the individual merits of the two approaches (note that both achieve domain consistency). Decomposition requires O(|x|) propagators consuming quite some

memory. Implementation requires a single propagator only and hence has a lower overhead both for runtime (only a single propagator needs to be scheduled) and memory consumption.

However, the implementation approach is dreadful and is in fact less efficient than decomposition! The problem is the "try to find a view" approach: each time the propagator is executed, it only knows that some of its views have changed since last time the propagator has been executed. It just does not know which views changed! That is, each time the propagator is executed, it needs to scan all its views. For samedom, the propagator takes  $O(|x| \cdot |d|)$  runtime as scanning needs to inspect the entire domain of each view. This is considerably worse than decomposition: when the domain of a  $x_i$  changes, only a single reified propagator for dom is executed with cost O(|d|).

The problem is the little amount of information available to a propagator when it is executed. The propagator only knows that some of its views changed but not which views. Hence, just finding out which views have changed always incurs linear cost in the number of views.

For a simple constraint such as samedom, the linear overhead is prohibitive. For more involved constraints, decomposition might be infeasible as it would compromise propagation (think of domain consistent distinct as an example).

**Advisors.** Gecode provides *advisors* to inform propagators about view changes. An advisor belongs to a propagator and can be defined (by inheriting from Advisor) to store information as needed by its propagator. The sole purpose of an advisor is to subscribe to a view of its propagator: each time the view changes, an advise() function of the advisor's propagator is executed with the advisor as argument (sometimes we will be a little sloppy by saying that the advisor is executed itself).

In more detail:

- An advisor must inherit from the class Advisor.
- When an advisor is created, it is created with respect to its propagator and a council of advisors. Each advisor belongs to a council and a propagator can have at most one council. The sole purpose of a council is to manage its advisors for cloning and disposal. In particular, when the propagator is disposed, the council must be disposed as well.

A council also provides access to all of its advisors (we will exploit this in the following sections).

• An advisor can subscribe to views (and, hence, an advisor subscription like any other subscription must eventually be canceled). Unlike subscriptions of propagators to views, subscriptions of advisors do not use propagation conditions: an advisor is always executed when its subscription view changes.

Also, an advisor is never executed when the subscription is created, only when the subscription view changes.

■ An advisor is executed as follows: the advise() function of its propagator is executed. The function takes the advisor as argument and an additional argument of type Delta. The *delta* describes how the domain of the view has changed. Clearly, which kind of information a delta provides depends on the type of the view. Deltas, in particular, provide access to the modification event of the operation that triggered the advisor's execution.

For integer and Boolean views, deltas provide some approximate information about which values have been removed. For an example, see Section 24.3.

■ The advise() function is *not* allowed to perform propagation by executing modification operations on views. It can change the state of the propagator and must return an execution status: ES\_FAILED means that the propagator is failed; ES\_FIX means that the propagator is at fixpoint (hence, it is not scheduled); ES\_NOFIX means that the propagator is not any longer at fixpoint (hence, it is scheduled). That is, an advisor does exactly what its name suggests as it provides advice to its propagator: when should the propagator be executed and the advisor can also provide information for the next propagator execution.

The advise() function can also return after calling the functions ES\_FIX\_DISPOSE() or ES\_NOFIX\_DISPOSE() which are analogous to ES\_FIX and ES\_NOFIX but also dispose the advisor.

There are two more functions that force the advisor's propagator to be rescheduled. They are discussed in Section 24.4.

Note that a propagator using advisors must make sure that its advisors schedule the propagator when it is not at fixpoint. Otherwise, it would not meet its "subscription complete" obligation (make sure to read Tip 24.2). A propagator is free to mix subscriptions using advisors and subscriptions using propagation conditions, see Section 24.3.

For more information on advisors including design and evaluation, see [18].

#### 24.2 The samedom constraint

The idea how the SameDom propagator implements the samedom constraint is straightforward. The propagator creates an advisor for each of its views and as soon as the advise() function decides for one of the views x that either  $x \subseteq d$  or  $x \cap d = \emptyset$  holds, the propagator is informed what it has to do and is scheduled. Then, the propagator performs the required propagation and becomes subsumed. That is, a SameDom propagator is executed at most once.

**Todo information.** A SameDom propagator stores in todo what it has to do when it is executed:

```
TODO INFORMATION ≡
enum ToDo { INCLUDE, EXCLUDE, NOTHING };
ToDo todo;
```

Initially, todo is NOTHING. When the propagator is scheduled on behalf of an advisor todo is either INCLUDE (that is, the propagator must propagate that  $x_i \subseteq d$  for  $0 \le i < |x|$ ) or EXCLUDE (that is, the propagator must propagate that  $x_i \cap d = \emptyset$  for  $0 \le i < |x|$ ).

**View advisors.** Each advisor used by the SameDom propagator stores the view it is subscribed to. By this, the advise() function can use the view stored with an advisor to decide what the propagator needs to do. A view advisor is defined as follows:

```
ADVISOR ≡
  class ViewAdvisor : public Advisor {
  public:
    Int::IntView x;
    ViewAdvisor(Space& home, Propagator& p,
                Council<ViewAdvisor>& c, Int::IntView x0)
      : Advisor(home,p,c), x(x0) {
      x.subscribe(home,*this);
    ViewAdvisor(Space& home, bool share, ViewAdvisor& a)
      : Advisor(home, share, a) {
      x.update(home, share, a.x);
    void dispose(Space& home, Council<ViewAdvisor>& c) {
      x.cancel(home,*this);
      Advisor::dispose(home,c);
    }
  };
  Council<ViewAdvisor> c;
```

An advisor must implement a constructor for creation which takes the home space, the advisor's propagator, and the council of advisors as arguments. Additionally, a view advisor also takes the view as input and subscribes to the view.

An advisor does neither have an update() nor a copy() function, a constructor for cloning with the typical arguments is sufficient. The dispose() function of an advisor does not have to report the advisor's size (in contrast to a propagator's dispose() function).

The propagator maintains a council of view advisors c. A council controls disposal and copying during cloning. Moreover, a council provides access to all advisors in the council (where already disposed advisors are excluded). The SameDom propagator does not store its

```
[DOWNLOAD]
SAMEDOM ≡
  class SameDom : public Propagator {
  protected:
    ► TODO INFORMATION
    ► ADVISOR
    IntSet d;
    static ModEvent include(Space& home, Int::IntView x,
                             const IntSet& d) {
    static ModEvent exclude(Space& home, Int::IntView x,
                             const IntSet& d) {
    static ToDo dom(Int::IntView x, const IntSet& d) {
  public:
    ► CONSTRUCTOR FOR POSTING
    ▶ DISPOSAL
    ► ADVISE FUNCTION
    ▶ PROPAGATION
  };
```

Figure 24.1: A samedom propagator using advisors

views explicitly in a view array. As the council provides access to its advisors, all views can be accessed from the council's advisors.

**Tip 24.1** (Different types of advisors for the same propagator). Any propagator that uses advisors must have exactly one council. As the council depends on the advisor type, only one advisor type per propagator is possible.

This is not a real restriction. If several different types of advisors are needed, one can either use advisors with virtual member functions or encode the advisor's type by some member stored by the advisor.

The propagator proper. The SameDom propagator is shown in Figure 24.1. The function include(home,x,d) constrains the view x to only take values from d, whereas exclude(home,x,d) constrains the view x to not take any values from d. The function dom(x,d) returns whether the values for x are included in d (INCLUDE is returned) or whether values for x are excluded from d (EXCLUDE is returned). All these functions are

implemented with range iterators as discussed in Chapter 23.

**Posting the propagator.** The propagator post function (not shown) makes sure that the propagator is only posted if for all views  $x_i$ , it cannot be decided whether  $x_i \in d$  or  $x_i \notin d$ . If this is not the case, the post function already performs the necessary propagation. Note that the propagator post function by performing some propagation ensures the central invariant of the SameDom propagator: the value of todo (which is NOTHING initially) corresponds to the current domains of the propagator's views.

The constructor for posting creates the required advisors as follows:

```
CONSTRUCTOR FOR POSTING ≡
SameDom(Home home, const IntVarArgs& x, const IntSet& d0)
: Propagator(home), todo(NOTHING), c(home), d(d0) {
  for (int i=x.size(); i--; )
     (void) new (home) ViewAdvisor(home,*this,c,x[i]);
  home.notice(*this,AP_DISPOSE);
}
```

**Tip 24.2** (Getting a propagator started with advisors). The SameDom propagator has the property that when it is posted, it is known to be at fixpoint (the post() function ensures this by checking for each view  $x_i$  whether  $x_i \in d$  or  $x_i \notin d$ ).

In general, it might not be true that a propagator using advisors is at fixpoint when it is posted. In that case, the constructor of the propagator must not only create advisors but also make sure that the propagator is scheduled for execution.

A propagator can be scheduled by using the static schedule() function of a view. For example, assume that the propagator to be scheduled should be scheduled because one of its integer views of type IntView is assigned. This can be achieved by:

```
IntView::schedule(home, *this, ME_INT_VAL);
```

where \*this refers to the current propagator.

```
Likewise,
```

```
IntView::schedule(home, *this, ME_INT_BND);
```

schedules the propagator with the information that the bounds of some of its integer views have changed.

Mandatory propagator disposal. The constructor also puts a notice on the propagator that it must always be disposed, even if the home space is deleted (as discussed in Section 20.8). Putting a notice is required because the integer set d of type IntSet is a proper data structure and must hence be deleted when a SameDom propagator is disposed.

Accordingly, the dispose() function deletes the integer set d as follows:

```
DISPOSAL =
  virtual size_t dispose(Space& home) {
    home.ignore(*this,AP_DISPOSE);
    c.dispose(home);
    d.~IntSet();
    (void) Propagator::dispose(home);
    return sizeof(*this);
}
```

Disposing the council c also disposes all view advisors. Hence, also all subscriptions are canceled (as the dispose() function of a ViewAdvisor cancels its subscription).

It is essential to ignore the notice in dispose() by calling home.notice(): otherwise Gecode might attempt to dispose a now already disposed propagator just over and over again!

**Propagation with advice.** The advise() function is straightforward:

```
ADVISE FUNCTION =
  virtual ExecStatus advise(Space&, Advisor& a, const Delta&) {
   if (todo != NOTHING)
     return ES_FIX;
   todo = dom(static_cast<ViewAdvisor&>(a).x,d);
   return (todo == NOTHING) ? ES_FIX : ES_NOFIX;
}
```

If todo is already different from NOTHING, the propagator has already been scheduled, and the advise() function returns ES\_FIX to signal that the propagator does not need to be scheduled again.<sup>1</sup> Otherwise, depending on the result of dom(), the advise() function updates the todo value of the propagator and returns ES\_NOFIX if the propagator needs scheduling (as todo is different from NOTHING).

Note that the advise() function of SameDom ignores its Delta argument. In the next section we will see a complementary design: the advisor does not carry any information, but only uses the information provided by the Delta.

The propagate() member function is exactly as to be expected:

```
PROPAGATION =
  virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
   if (todo == INCLUDE)
     for (Advisors<ViewAdvisor> a(c); a(); ++a)
        GECODE_ME_CHECK(include(home,a.advisor().x,d));
   ...
}
```

 $<sup>^1</sup>$ Actually, it would also be okay to return ES $\_$ N0FIX. Scheduling an already scheduled propagator is okay.

The Advisors class provides an iterator over all advisors in the council c. As mentioned earlier, the iterator (and hence the council) provides sufficient information to retrieve all views of interest for propagation.

**Tip 24.3** (Advisors and propagator obligations). The astute reader might have wondered whether a SameDom propagator is actually "update complete" in the sense of Section 20.8. The answer is yes, because the council copies all of its view advisors and each view advisor updates its view in turn.

Likewise, the obligations "subscription complete" and "subscription correct" need to be satisfied regardless of whether a propagator uses propagator condition-based or advisor-based subscriptions to views.

**Advisor disposal.** The SameDom propagator leaves the disposal of its advisors to its own dispose() function. However, it could also request disposal of an advisor in the advise() function itself. A different implementation of the advise function would be:

```
if (todo != NOTHING)
   return home.ES_FIX_DISPOSE(c,static_cast<ViewAdvisor&>(a));
todo = dom(static_cast<ViewAdvisor&>(a).x,d);
return (todo == NOTHING) ? ES_FIX :
   home.ES_NOFIX_DISPOSE(c,static_cast<ViewAdvisor&>(a));
```

With this design, all advisors would be disposed on behalf of the advise() function and not by the propagator's dispose() function. This is due to the following two facts:

- A single advisor finds out that todo must be either EXCLUDE or INCLUDE. This advisor returns ES\_NOFIX\_DISPOSE() and hence is disposed.
- All other advisors will be executed at the very latest when the propagator performs propagation and are disposed as well.

**Using predefined view advisors.** Unsurprisingly, view advisors are a common abstraction for propagators using advisors. Hence, Gecode provides view advisors as predefined abstractions that are parametric with respect to their view type. Figure 24.2 shows how SameDom can be implemented using predefined view advisors.

## 24.3 General Boolean disjunction

Let us consider an efficient propagator 0r for implementing the Boolean disjunction

$$\bigvee_{i=0}^{|\mathbf{x}|-1} \mathbf{x}_i = \mathbf{y}$$

```
SAMEDOM USING PREDEFINED VIEW ADVISORS = 
...

class SameDom : public Propagator {
  protected:
    ...
    Council<ViewAdvisor<Int::IntView> > c;
    IntSet d;
    ...

public:
    ...

virtual ExecStatus advise(Space&, Advisor& a, const Delta&) {
    if (todo != NOTHING)
        return ES_FIX;
    todo = dom(static_cast<ViewAdvisor<Int::IntView>&>(a).view(),d);
    return (todo == NOTHING) ? ES_FIX : ES_NOFIX;
    }
    ...
};
...
```

Figure 24.2: A samedom propagator using predefined view advisors

```
or ≡
                                                                [DOWNLOAD]
  typedef MixNaryOnePropagator<Int::BoolView,Int::PC_BOOL_NONE,</pre>
                                Int::BoolView,Int::PC_B00L_VAL>
          OrBase:
  class Or : public OrBase {
  protected:
    int n_zero;
    Council<Advisor> c;
  public:
    Or(Home home, ViewArray<Int::BoolView>& x, Int::BoolView y)
      : OrBase(home,x,y), n_zero(0), c(home) {
      x.subscribe(home,*new (home) Advisor(home,*this,c));
    }
    ► ADVISE
    ▶ PROPAGATION
  };
```

Figure 24.3: A Boolean disjunction propagator using advisors

where all  $x_i$  and y are Boolean views. When y becomes assigned, propagation is straightforward. If y is assigned to 0, all  $x_i$  must be assigned to 0 as well. If y is assigned to 1, 0r is rewritten into the propagator 0rTrue from Section 21.3.

As it comes to the  $x_i$ , the 0r propagator uses a similar technique to the SameDom propagator. It can use advisors to find out which view has been assigned instead of inspecting all  $x_i$ . However, the propagator requires very little information: it does not need to know which view has changed, it only needs to know whether a view among the  $x_i$  has been assigned to 0 or 1. Our 0r propagator uses the delta information passed to the advise() function to determine the value to which a view has been assigned. The advantage is that the propagator only needs a single advisor instead of one advisor per view.

**Tip 24.4** (Advisor space requirements). A subscription requires one pointer, regardless of whether a propagator or an advisor is subscribed to a view. Without any additional information stored by an advisor, an advisor requires two pointers. Hence, it pays off to use as few advisors as possible. ◀

The Or propagator. The Or propagator inherits from the MixNaryOnePropagator template (to increase readability, a base class OrBase is defined as a type) and uses PC\_BOOL\_NONE as propagation condition for the views in the view array x. That actually means that no subscriptions are created for the views in x. A subscription with propagation condition

PC\_B00L\_VAL is created for the single Boolean view y. The constructor for posting creates a single advisor which subscribes to all views in x. In fact, the 0r propagator mixes advisors with normal subscriptions.

The advise() function uses the delta information to decide whether one of the views the advisor has subscribed to is assigned to 0 (then Int::BoolView::zero() returns true):

```
ADVISE =
  virtual ExecStatus advise(Space&, Advisor&, const Delta& d) {
   if (Int::BoolView::zero(d) && (++n_zero < x.size()))
     return ES_FIX;
   else
     return ES_NOFIX;
}</pre>
```

The advise function counts the number of views assigned to zero in  $n_zero$ . It reports that the propagator must be scheduled, if all views have been assigned to zero (that is,  $n_zero$  equals the number of views in x) or if a view has been assigned to one. Note that the propagator post function makes sure that the propagator is only posted when none of the views in x are assigned.

The propagate() function is straightforward:

```
PROPAGATION \(\equiv \)
  virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
    if (y.one())
        GECODE_REWRITE(*this,OrTrue::post(home(*this),x));
    if (y.zero()) {
        for (int i = x.size(); i--; )
            GECODE_ME_CHECK(x[i].zero(home));
    } else if (n_zero == x.size()) {
        GECODE_ME_CHECK(y.zero(home));
    } else {
        GECODE_ME_CHECK(y.one(home));
    }
    return home.ES_SUBSUMED(*this);
}
```

It first checks whether the propagator has been executed because y has been assigned and propagates as sketched before. Then it uses the value of  $n_z$  to determine how to propagate.

**Delta information for integer views.** The delta information provided to the advise() function can be interpreted through the view the advisor has subscribed to. Boolean views provide static member functions zero() and one() to find out whether the view has been assigned to 0 or 1.

For integer views (and set views, see Section 26.2), one must use the view to which the advisor has subscribed to for accessing the delta.

For example, suppose that the advisor a has subscribed to the view x by

#### x.subscribe(home,a);

When the advise() function is executed for the advisor a with delta d, the view x provides access to the delta information (typically, this will mean that a in fact stores the view x).

The modification event of the operation that modified the view x is available by x.modevent(d). If x.any(d) returns true, no further information about how the domain of x has changed is available. Only if x.any(d) is false, x.min(d) returns the previously (that is, the value before the domain modification) smallest value of x and x.max(d) returns the previously largest value of x. With other words, the delta x0 only provides approximate information on how the old view domain has been changed.

For Boolean views, one can rely on the fact that the delta information is always accurate. For integer views, it might be the case that x.any(d) returns true even though only the bounds of x have changed.

## 24.4 Forced propagator rescheduling

An advisor can force its propagator to be rescheduled even though the propagator's modification event delta has not changed. As discussed in Section 23.5, the cost() function of a propagator is only recomputed when its modification event delta changes.

When the advise() of a propagator returns ES\_NOFIX\_FORCE (or, the advise() function calls ES\_NOFIX\_DISPOSE\_FORCE()), the propagator is rescheduled regardless of its current modification event delta. See also Status of constraint propagation and branching commit.

# **25**

# **Views**

This chapter should come as a welcome diversion from the previous chapters in this part. Instead of introducing more concepts and techniques for programming propagators, it shows how to straightforwardly and efficiently reuse propagators for implementing several different constraints. In a way, the chapter tells you how to cache in on all the effort that goes into developing a propagator.

The idea is to make a propagator generic with respect to the views the propagator computes with. As we are talking C++, generic propagators will be nothing but templates where the template arguments happen to be view types. Then, by instantiating the template propagator, one can obtain implementations for several constraints from a single propagator. More on views (a concept introduced by Gecode) can be found in [39] and [38].

As it comes to importance, this chapter should be the second in this part. However, the chapter comes rather late to be able to draw on the example propagators presented in the previous chapters.

**Overview.** Integer variable views are discussed in Section 25.1 and Boolean variable views are discussed in Section 25.2. How integer propagators can be reused for Boolean views is presented in Section 25.3.

### 25.1 Integer views

Assume that we need an implementation for the min constraint. Of course, we could implement a Min propagator analogous to the Max propagator from Section 22.3. But let us assume that we need to be lazy in that we do not have the time to implement Min (after all, there are more interesting constraints out there that we want to implement).

What we could do to implement min(x, y) = z is to introduce three new variables x', y', and z', post three constraints such that x = -x', y = -y', and z = -z', and finally post a max constraint instead: max(x', y') = z'. While the strength of propagation is uncompromised, efficiency is poor: three additional variables and three additional propagators are needed.

#### 25.1.1 Minus views

Minus views can do exactly what we discussed above but without creating additional variables or propagators. Assume that we have an integer view x that serves as an interface to

```
[DOWNLOAD]
MIN AND MAX \equiv
  template<class View>
  class Max : public TernaryPropagator<View,Int::PC_INT_BND> {
  protected:
      using TernaryPropagator<View,Int::PC_INT_BND>::x0;
      using TernaryPropagator<View,Int::PC_INT_BND>::x1;
      using TernaryPropagator<View,Int::PC_INT_BND>::x2;
  };
  void min(Home home, IntVar x0, IntVar x1, IntVar x2) {
    if (home.failed()) return;
    Int::MinusView y0(x0), y1(x1), y2(x2);
    GECODE_ES_FAIL(Max<Int::MinusView>::post(home,y0,y1,y2));
  void max(Home home, IntVar x0, IntVar x1, IntVar x2) {
    if (home.failed()) return;
    GECODE_ES_FAIL(Max<Int::IntView>::post(home,x0,x1,x2));
  }
```

Figure 25.1: Minimum and maximum constraints implemented by a Max propagator

a variable implementation  $\nu$ . Then, a *minus integer view* m for  $\nu$  is also an interface to  $\nu$ , however the interface implements operations such that m is an interface to  $-\nu$ .

For example, assume that the domain of x is  $\{-1,1,3,4,6\}$  (which also means that  $v \in \{-1,1,3,4,6\}$ ). Then, the domain for m is  $\{-6,-4,-3,-1,1\}$ . For example, m.min() returns -6 (which, of course, is nothing but -x.max()) and the modification operation m.gq(home, -3) results in domains  $m \in \{-3,-1,1\}$  and  $x \in \{-1,1,3\}$  (which, of course, is the same as x.lq(home, -(-3)) and hence as x.lq(home, 3)).

The very point of this exercise is: a minus view is just a different interface to an *existing* variable implementation and does not require a *new* variable implementation. Moreover, the operations performed by the minus view interface are optimized away at compile time.

Figure 25.1 shows how to obtain both min and max constraints from the very same Max propagator using Int::IntView and Int::MinusView views. The only change needed compared to the Max propagator from Section 22.3 is that the propagator does not hardwire its view type. Instead, the propagator is generic by being implemented as a template over the view type View it uses. The constraint post functions then just instantiate the Max propagator with the appropriate view types.

**Tip 25.1** (Using **using** clauses). Note the **using** clauses in Figure 25.1. They make x0, x1, and x2 visible for the Max propagator. This is necessary in  $C^{++}$  as Max inherits from a base

```
[DOWNLOAD]
DOMAIN EQUAL WITH AND WITHOUT OFFSET ≡
 template<class View0, class View1>
 class Equal
    : public MixBinaryPropagator<View0,Int::PC_INT_DOM,
                                 View1,Int::PC_INT_DOM> {
 };
 void equal(Home home, IntVar x0, IntVar x1) {
   if (home.failed()) return;
   GECODE_ES_FAIL((Equal<Int::IntView,Int::IntView>
                    ::post(home,x0,x1)));
 }
 void equal(Home home, IntVar x0, IntVar x1, int c) {
   if (home.failed()) return;
   GECODE_ES_FAIL((Equal<Int::IntView,Int::OffsetView>
                    ::post(home,x0,Int::OffsetView(x1,c))));
 }
```

Figure 25.2: Domain equality with and without offset

class that itself depends on the template argument View of Max. There are other possibilities to refer to members such as x0 that also work, for example by writing **this**->x0 instead of just x0. We choose the variant with **using** clauses to keep the code of the propagator uncluttered.

•

#### 25.1.2 Offset views

An *offset view* o with offset c (an integer value) for a variable implementation  $\nu$  provides operations such that o behaves as  $\nu + c$ .

Figure 25.2 shows how a domain equality constraint (see Section 23.2) and a domain equality constraint with offset (see Section 23.3) can be obtained from the same domain equality propagator Equal. Equal has two template arguments View0 and View1 for its views x0 and x1 respectively. With two view template arguments, the propagator can be instantiated with different view types for x0 and x1. Therefore, the propagator uses MixBinaryPropagator as base class as it supports different view types as well.

**shared versus same.** The domain modification operations inter\_r and narrow\_r used in the Equal propagator from Section 23.2 are used such that the operations perform a more

efficient in-place update of the view domain (with an additional Boolean value **false** as last and optional argument). This is only legal because the range iterator passed as argument to the modification operations does not depend on the view being modified. The post function of Equal ensures this by only posting the propagator if the two views x0 and x1 are not referring to the very same variable implementation (that is, same(x0,x1) is false).

With arbitrary views, the situation becomes a little bit more involved. Assume that x0 is an integer view referring to the variable implementation v and that x1 is an offset integer view for the same variable implementation v and an integer value  $c \neq 0$ . In this case, the views x0 and x1 share the same variable implementation v but are *not* the same.

The function shared() tests whether two views share the same variable implementation. Hence, the use of domain modification operations in Equal have to be modified as follows:

```
DOMAIN PROPAGATION ≡
  Int::ViewRanges<View0> r0(x0);
  GECODE_ME_CHECK(x1.inter_r(home,r0,shared(x0,x1)));
  Int::ViewRanges<View1> r1(x1);
  GECODE_ME_CHECK(x0.narrow_r(home,r1,shared(x0,x1)));
```

Now, the more efficient in-place operations are used only if x0 and x1 do not share the same variable implementation.

#### 25.1.3 Constant and scale views

In addition to minus and offset views, Gecode offers *scale views* and *constant views* for integer variable implementations.

A scale view for a variable implementation  $\nu$  with an integer scale factor a where a>0 implements operations for  $a\cdot\nu$ . Scale views exist in two variants differing in the precision of multiplication: IntScaleView performs multiplication over integers, whereas DoubleScaleView performs multiplication over doubles (see Integer views and Int::ScaleView).

An integer constant view Int::ConstIntView provides an integer view interface to an integer constant c. With other words, an integer constant view for the integer c behaves as an integer view assigned to the value c.

#### 25.2 Boolean views

For Boolean views, the view resembling a minus view over integers is a view for negation. For example, with Boolean negation views Int::NegBoolView both disjunction and conjunction constraints can be obtained from a propagator for disjunction (see Figure 25.3).

```
[DOWNLOAD]
or and and from or \equiv
  template<class View>
  class OrTrue :
    public BinaryPropagator<View,Int::PC_B00L_VAL> {
  };
  void dis(Home home, const BoolVarArgs& x, int n) {
  }
  void con(Home home, const BoolVarArgs& x, int n) {
    } else {
      ViewArray<Int::NegBoolView> y(home,x.size());
      for (int i=x.size(); i--; )
        y[i]=Int::NegBoolView(x[i]);
      GECODE_ES_FAIL(OrTrue<Int::NegBoolView>::post(home,y));
    }
  }
```

Figure 25.3: Disjunction and conjunction from same propagator

Figure 25.4: Less constraints for both integer and Boolean variables

### 25.3 Integer propagators on Boolean views

As has been discussed in Section 21.2, Boolean views feature all operations available on integer views (such as lq() or gr()) in addition to the dedicated Boolean operations (such as one() or zero()). Due to the availability of integer operations on Boolean views, integer propagators can be used to implement Boolean constraints.

Figure 25.4 shows how the propagator Less can be used to implement the less constraint for both integer and Boolean variables.

**Tip 25.2** (Boolean variables are not integer variables). The above example uses a template to obtain an implementation of a constraint on both integer and Boolean variables. This is necessary as Boolean variables are not integer variables (in the sense that BoolVar is not a subclass of IntVar). The same holds true for their views and variable implementations.

This is by design: Boolean variables are not integer variables as they have a specially optimized implementation (taking advantage of the limited possible variable domains and that only PC\_B00L\_VAL as propagation condition is needed).

# **26**

# Propagators for set constraints

This chapter shows how to implement propagators for constraints over set variables. We assume that you have worked through the chapters on implementing integer propagators, as most of the techniques readily carry over and are not explained again here.

We also assume a basic knowledge of propagation for set constraints. To read more about this topic, please refer to [15, 44].

**Overview.** Section 26.1 demonstrates a propagator that implements set interesection. Set views and their related concepts are summarized in Section 26.2.

### 26.1 A simple example

Figure 26.1 shows a propagator for the ternary intersection constraint  $x_0 \cap x_1 = x_2$  for three set variables  $x_0$ ,  $x_1$ , and  $x_2$ .

As you can see, propagators for set constraints follow exactly the same structure as propagators for integer or Boolean constraints. The same propagator patterns can be used (see Section 20.7). The appropriate views and propagation conditions are defined in the namespace Gecode::Set.

In order to understand the propagate() function, we have to look at how set variable domains are represented.

The set bounds approximation. We already saw in Chapter 5 that set variable domains are represented as intervals in order to avoid an exponential representation. For example, recall that

cannot be captured exactly by an interval, but is instead approximated by the smallest enclosing interval  $[\{\} .. \{1,2,3\}]$ .

Set propagators therefore access and modify the interval bounds. Naturally, set-valued domain operations similar to the ones for integer variables (see Chapter 23) play an important role for set propagators.

For each set view, Set::GlbRanges provides a range iterator for its lower bound, and Set::LubRanges iterates the upper bound. The main iterator-based modification operations

```
[DOWNLOAD]
INTERSECTION ≡
 #include <gecode/set.hh>
 using namespace Gecode;
  class Intersection
    : public TernaryPropagator<Set::SetView,Set::PC_SET_ANY> {
 public:
    Intersection(Home home, Set::SetView x0, Set::SetView x1,
                 Set::SetView x2)
      : TernaryPropagator<Set::SetView,Set::PC_SET_ANY>(home,
                                                         x0, x1, x2) {}
    virtual ExecStatus propagate(Space& home, const ModEventDelta&) {
      using namespace Iter::Ranges; using namespace Set;
      bool assigned = x0.assigned() && x1.assigned() && x2.assigned();
      ► RULE 1
      ► RULE 2
      ► RULE 3
      ► RULE 4
      ► RULE 5
      ► RULE 6
      ► CARDINALITY
      return assigned ? home.ES_SUBSUMED(*this) : ES_NOFIX;
 };
 void intersection(Home home, SetVar x0, SetVar x1, SetVar x2) {
   if (home.failed()) return;
   GECODE_ES_FAIL(Intersection::post(home,x0,x1,x2));
 }
```

Figure 26.1: A constraint and propagator for set intersection

on set views are includeI (adding a set to the lower bound), excludeI (removing a set from the upper bound), and intersectI (intersecting the upper bound with a set).

**Filtering rules.** Coming back to the example propagator for ternary intersection, we have to devise filtering rules that express the constraint in terms of the interval bounds. In the following, we write  $\underline{x}$  and  $\overline{x}$  for the lower bound resp. upper bound of a view x. Then, ternary intersection can be propagated with the following rules and implemented with set domain operations:

1.  $x_0 \cap x_1 \subseteq x_2$ 

```
RULE 1 ≡
{
    GlbRanges<SetView> x0lb(x0), x1lb(x1);
    Inter<GlbRanges<SetView>, GlbRanges<SetView> > i(x0lb,x1lb);
    GECODE_ME_CHECK(x2.includeI(home,i));
}
```

2.  $\overline{x_0} \cap \overline{x_1} \supseteq x_2$ 

```
RULE 2 =
{
    LubRanges<SetView> x0ub(x0), x1ub(x1);
    Inter<LubRanges<SetView>, LubRanges<SetView> > i1(x0ub,x1ub);
    GECODE_ME_CHECK(x2.intersectI(home,i1));
}
```

 $3. \ \underline{\mathsf{x}_2} \subseteq \mathsf{x}_0$ 

```
RULE 3 =
{
    GlbRanges<SetView> x2lb(x2);
    GECODE_ME_CHECK(x0.includeI(home,x2lb));
}
```

4.  $x_2 \subseteq x_1$ 

```
RULE 4 =
{
    GlbRanges<SetView> x2lb(x2);
    GECODE_ME_CHECK(x1.includeI(home,x2lb));
}
```

#### integer-valued bounds operations

```
Return minimum/maximum of lower bound
glbMin() / glbMax()
lubMin() / lubMax()
                        Return minimum/maximum of upper bound
                        Return size of lower/upper bound
glbSize() / lubSize()
                        Return number of elements in upper but not in lower bound
unknownSize()
                        test whether lower bound contains element
contains()
                        test whether upper bound does not contain element
notContains()
                        add element (or range) to lower bound
include()
                        remove element (or range) from upper bound
exclude()
intersect()
                        intersect upper bound with element or range
                        set-valued bounds modifications
includeI()
                        add elements to lower bound
                        remove elements from upper bound
excludeI()
intersectI()
                        intersect upper bound with given set
                             cardinality operations
                        Return/modify minimum cardinality
cardMin()
                        Return/modify maximum cardinality
cardMax()
```

Figure 26.2: Set view operations

```
5. \underline{\mathsf{x}_0} \setminus \overline{\mathsf{x}_2} \not\subseteq \mathsf{x}_1
```

```
RULE 5 =
{
   GlbRanges<SetView> x0lb(x0); LubRanges<SetView> x2ub(x2);
   Diff<GlbRanges<SetView>, LubRanges<SetView> > diff(x0lb, x2ub);
   GECODE_ME_CHECK(x1.excludeI(home,diff));
}
```

```
6. x_1 \setminus \overline{x_2} \not\subseteq x_0
```

```
RULE 6 ≡
  {
    GlbRanges<SetView> x1lb(x1); LubRanges<SetView> x2ub(x2);
    Diff<GlbRanges<SetView>, LubRanges<SetView> > diff(x1lb, x2ub);
    GECODE_ME_CHECK(x0.excludeI(home,diff));
}
```

The first four rules should be self-explanatory. The last two rules state that anything that is in  $x_0$  but not in  $x_2$  cannot be in  $x_1$  (and the same for  $x_0$  and  $x_1$  swapped). The full list of operations on set views appears in Figure 26.2.

**Fixpoint.** Note how the propagator determines which execution status to return. Before applying any of the filtering rules, it checks whether all of the variables are already assigned. If they are, then propagation will compute a fixpoint and the propagator can return that it is subsumed after applying the filtering rules. Otherwise, it has not necessarily computed a fixpoint (e.g. rule 6 may modify the upper bound of  $x_0$ , making it necessary to apply rule 2 again).

**Cardinality.** In addition to the interval bounds, set variables store *cardinality bounds*, that is, the minimum and maximum cardinality of the set variable. These bounds are stored and modified independently of the interval bounds, but of course modifications to these different bounds affect each other.

For example, consider a set variable with a domain represented by the interval  $[\{\} ... \{1,2\}]$  and the cardinality #[1...2]. Adding 1 to the lower bound would result in the cardinality lower bound being increased to 1. Removing 1 from the upper bound would result in 2 being added to the lower bound to satisfy the minimum cardinality of 1.

Using cardinality information, propagation for some set constraints can be strengthened. For the ternary intersection example, we can for instance add the following filtering rules:

```
LubRanges<SetView> x0ub(x0), x1ub(x1);
Union<LubRanges<SetView>, LubRanges<SetView> > u_lub(x0ub,x1ub);
unsigned int s_lub = size(u_lub);
if (x0.cardMin() + x1.cardMin() > s_lub)
   GECODE_ME_CHECK(x2.cardMin(home, x0.cardMin()+x1.cardMin()-s_lub));
GlbRanges<SetView> x0lb(x0), x1lb(x1);
Union<GlbRanges<SetView>, GlbRanges<SetView> > u_glb(x0lb,x1lb);
unsigned int s_glb = size(u_glb);
GECODE_ME_CHECK(x2.cardMax(home,x0.cardMax()+x1.cardMax()-s_glb));
GECODE_ME_CHECK(x0.cardMin(home,x2.cardMin()));
GECODE_ME_CHECK(x1.cardMin(home,x2.cardMin()));
```

When dealing with cardinality, it is important to handle overflow or signedness issues. In the above example, we have to check whether x0.cardMin()+x1.cardMin()>s, because otherwise the expression x0.cardMin()+x1.cardMin()-s may underflow (as we are dealing with unsigned integers here). This is not the case for the second cardinality rule. Here, we can be sure that the size of the union of the lower bounds is always greater than the sum of the maximum cardinalities.

# 26.2 Modification events, propagation conditions, views, and advisors

This section summarizes how these concepts are specialized for set variables and propagators.

**Modification events and propagation conditions.** The modification events and propagation conditions for set propagators (see Figure 26.3) capture the parts of a set variable domain that can change.

One could imagine a richer set, for example distinguishing between lower and upper bound changes of the cardinality, or separating the cardinality changes from the interval bound changes. However, the number of propagation conditions has a direct influence on the size of a variable, see Section 31.1. Just like for integer views, this set of modification events and propagation conditions has been chosen as a compromise between expressiveness on the one hand, and keeping the set small on the other.

**Set variable views.** In addition to the basic Set::SetView class, there are five other set views: Set::ConstSetView, Set::EmptyView, Set::UniverseView, Set::SingletonView, and Set::ComplementView.

The first three are constant views. A SingletonView wraps an integer view x in the interface of a set view, so that it acts like the singleton set  $\{x\}$ . A ComplementView is like Boolean negation, it provides the set complement with respect to the global Gecode universe for set variables (defined as [Set::Limits::min.. Set::Limits::max], see Set::Limits).

Advisors for set propagators. Advisors for set constraints get informed about the domain modifications using a Set::SetDelta. The set delta provides only information about the minimum and maximum values that were added to the lower bound and/or removed from the upper bound.

#### set modification events

Set::ME_SET_NONE	the view has not been changed	
Set::ME_SET_FAILED	the domain has become empty	
Set::ME_SET_VAL	the view has been assigned to a single set	
Set::ME_SET_CARD	the view has been assigned to a single set	
Set::ME_SET_LUB	the upper bound has been changed	
Set::ME_SET_GLB	the lower bound has been changed	
Set::ME_SET_BB	both bounds have been changed	
Set::ME_SET_CLUB	cardinality and upper bound have changed	
Set::ME_SET_CGLB	cardinality and lower bound have changed	
Set::ME_SET_CBB	cardinality and both bounds have changed	
set propagation conditions		
Set::PC_SET_VAL	schedule when the view is assigned	
Set::PC_SET_CARD	schedule when the cardinality changes	
Set::PC_SET_CLUB	schedule when the cardinality or the upper bound changes	
C-+ DC CET CCLD		

Set::PC_SEI_CARD	schedule when the cardinality changes
Set::PC_SET_CLUB	schedule when the cardinality or the upper bound changes
Set::PC SET CGLB	schedule when the cardinality or the lower bound changes

Set::PC\_SET\_ANY schedule at any change

Set::PC\_SET\_NONE do not schedule

Figure 26.3: Set modification events and propagation conditions

# **27**

# Managing memory

This chapter provides an overview of memory management for propagators. In fact, the memory management aspects discussed here are also valid for branchers (Part B) and to some extent even for variables (Part V).

**Overview.** Section 27.1 describes the different memory areas available in Gecode together with their allocation policies. The following section (Section 27.2) discusses how a propagator can efficiently manage its own state. Section 27.3 discusses an abstraction for sharing data structures among all spaces for one thread, whereas Section 27.4 discusses an abstraction for sharing data structures among several propagators that belong to the same space.

### 27.1 Memory areas

Gecode manages several different memory areas with different allocation policies: *spaces* provide access to space-allocated memory, *regions* provide access to temporary memory with implicit deallocation, and *space-allocated freelists* provide efficient access to small chunks of memory which require frequent allocation and deallocation.

All memory areas but freelists provide operations alloc(), realloc(), and free() for allocation, reallocation, and deallocation of memory from the respective area. To provide a uniform interface, Gecode also defines a heap object (see Heap memory management) implementing the very same allocation functions. All memory-management related operations are documented in Memory management.

**Memory management functions.** Let us consider allocation from the heap as an example. By

```
int* i = heap.alloc<int>(n);
```

a memory chunk for n integers is allocated. Likewise, by

```
heap.free<int>(i,n);
```

the memory is freed (for heap, the memory is returned to the operating system). By

```
int* j = heap.realloc<int>(i,n,m);
```

a memory chunk is allocated for m integers. If possible, j will refer to the same memory chunk as i (but there is no guarantee, of course).

The memory management functions implement C++ semantics: alloc() calls the default constructor for each element allocated; realloc() calls the destructor, default constructor, or copy constructor (depending on whether the memory area must shrink or grow); free() calls the destructor for each element freed.

**Space.** Space-allocated memory (see Space-memory management) is managed per each individual space. All space-allocated memory is returned to the operating system if a space is deleted. Freeing memory (via the free() operation) enables the memory to be reused by later alloc() operations.

Spaces manage several blocks of memory allocated from the operating system, the block sizes are automatically chosen based on the recent memory allocations of a space (and, if a space has been created by cloning, the memory allocation history of the original space). The exact policies can be configured, see the namespace MemoryConfig.

Memory chunks allocated from the operating system for spaces are cached among all spaces for a single thread. This cache for the current thread can be flushed by invoking the flush() member function of Space.

Variable implementations, propagators, branchers, view arrays, for example, are allocated from their home space. An important characteristic is that all these data structures have fixed size throughout their lifetimes or even shrink. As space-allocated memory is managed for later reusal if it is freed, frequent allocation/reallocation/deallocation leads to highly fragmented memory with little chance of reusal. Hence, space-allocated memory is absolutely unsuited for data structures that require frequent allocation/deallocation and/or resizing. For these data structures it is better to use the heap or freelists, if possible. See Section 27.2 for more information.

Note that space-allocated memory is available with allocators compatible with the C++ STL, see Using allocators with Gecode.

**Region.** All spaces for one thread share a chunk of memory for temporary data structures with very efficient allocation and deallocation (again, its exact size is defined in the namespace MemoryConfig). Assume that home refers to a space (say, during the execution of the propagate() function of a propagator), then

#### Region r(home);

creates a new region r for memory management (see Region memory management). A region does not impose any size limit on the memory blocks allocated from it. If necessary, a region transparently falls back to heap-allocated memory.

Several regions for the same space can exist simultaneously. For example,

```
Region r1(home), r2(home);
int* i = r1.alloc<int>(n);
int* j = r2.alloc<int>(m);
```

The specialty of a region is that it does not require free() operations. If a region is destructed, all memory allocated from it is freed (unless several regions for the same space exist). If several regions for the same space are in use, the memory is entirely freed when all regions are destructed.

Even though a region does not require free() operations, it can profit from it: if the memory allocated last is freed first, the freed memory becomes immediately available for future allocation.

**Heap.** The heap (a global variable, see Heap memory management) is nothing but a C++-wrapper around malloc() and free() as provided by the underlying operating system. In case memory is exhausted, an exception of type MemoryExhausted is thrown.

**Space-allocated freelists.** Freelists are allocated from space memory. Any object to be managed by a freelist must inherit from the class FreeList which already defines a pointer to a next freelist element. The sizes for freelist objects are quite constrained, check the values fl\_size\_min and fl\_size\_max as defined in the namespace MemoryConfig.

Allocation and deallocation is available through the member functions fl\_alloc() and fl\_dispose() of the class Space.

### 27.2 Managing propagator state

Many propagators require sophisticated data structures to perform propagation. The data structures are typically kept between successive executions of a propagator. There are two main issues for these data structures: *where* to allocate them and *when* to allocate them.

Where to allocate. Typically, the data structures used by a propagator are of dynamic size and hence cannot be stored in a simple member of the propagator. This means that the propagator is free to allocate the memory from either its own space or from the heap. Allocation from the heap could also mean to use other operations to allocate and free memory, such as malloc() and free() provided by the operating system or **new** and **delete** provided by C++.

In case the data structure does not change its size often, it is best to allocate from a space: allocation is more efficient and deallocation is automatic when the space is deleted.

In case the data structure requires frequent reallocation operations, it is better to allocate from the heap. Then, the memory will not be automatically freed when a space is deleted. The memory must be freed by the propagator's dispose() function. Furthermore, the Gecode kernel must be informed that the dispose() function must be called when the propagator's home space is deleted, see Section 20.9.

Reporting allocated memory. A propagator that has allocated non-space memory must implement the virtual member function allocated() that returns the amount (as a value of type size\_t) of non-space memory allocated by the propagator. The function allocated() is used by the function allocated() for a space to compute the total amount of memory allocated by a space. Note that only propagators are considered that have registered for disposal with a space (only these propagators can have non-space memory allocated anyway).

When to allocate. An important fact on which search engines in Gecode rely (see Part S) is that they always store a space created by cloning for backtracking and never a space that has already been used for propagation. The reason is that in order to perform propagation, the propagators, variables, and branchers might require some additional memory. Hence, a space that has performed propagation is likely to require more memory than a pristine clone. As the spaces stored by a search engine define the total amount of memory allocated for solving a problem with Gecode, it pays to save memory by storing pristine clones.

The most obvious policy is to allocate *eagerly*: the data structures are allocated when the propagator is created and they are copied exactly when the propagator is copied.

However, very often it is better to *lazily* recompute the data structures as follows. The data structures are initialized and allocated the first time a propagator is executed. Likewise, the data structures are not copied and construction is again postponed until the propagator is executed again. Creating and allocating data structures lazily is often as simple as creating them eagerly. The benefit is that clones of spaces on which no propagation has been performed yet require considerably less memory.

Most propagators in Gecode that require involved data structures construct their data structures lazily (for example, the propagator for domain consistent distinct using the algorithm from [30], see Int::Distinct::Dom). Some use a hybrid approach where some data structures are created eagerly and others lazily (for example the propagator for extensional for finite automata using the algorithm from [26], see Int::Extensional::LayeredGraph).

### 27.3 Shared objects and handles

A common request for managing a data structure (we will refer to data structure here just as object) is that it is used and shared by several propagators from different spaces with the restriction that all spaces are used by a single thread. Gecode provides *shared objects* and *shared handles* for this form of sharing. Shared handles and objects are used to implement proper data structures as introduced in Section 4.2.

A shared object is heap-allocated and is referred to by several shared handles. If the last shared handle is deleted, also the shared object is deleted (implemented by reference counting). In addition, shared handles provide an update() function that creates a new copy of the shared object in case the copy is to be used by a different thread (that is, the Boolean share argument is true).

```
[DOWNLOAD]
Shared object and handle \equiv
 class SI : public SharedHandle {
 protected:
    class SIO : public SharedHandle::Object {
   public:
      int data;
      SIO(int d) : data(d) {}
      SIO(const SIO& sio) : data(sio.data) {}
      virtual Object* copy(void) const {
        return new SIO(*this);
      }
      virtual ~SIO(void) {}
   };
 public:
   SI(int d) : SharedHandle(new SIO(d)) {}
    int get(void) const {
      return static_cast<SIO*>(object())->data;
   void set(int d) {
      static_cast<SI0*>(object())->data = d;
    ► SOME INHERITED MEMBERS
 };
```

Figure 27.1: A simple shared object and handle

Figure 27.1 shows a simple example of a shared object and the shared handle using the object. A shared object SIO (for Shared Integer Object) stores a single integer data to be shared (normally, this will be an interesting data structure). A shared object must inherit from SharedHandle::Object and must implement a virtual copy() function that returns a new shared object. If needed, a virtual destructor can be defined.

The shared handle SI (for Shared Integer) is the only class that has access to a shared object of type SIO. A shared handle must inherit from SharedHandle and can use the object() member function to access and update its shared object.

The most interesting members of SI that are inherited are the following:

```
SOME INHERITED MEMBERS ≡
  void update(Space& home, bool share, SharedHandle& sh) {
    ...
}
~SI(void) {}
```

Other inherited members include a copy constructor and an assignment operator. Shared integer arrays are an example for shared objects, see Tip 4.9.

#### 27.4 Local objects and handles

For some applications, it is necessary to share data structures between different propagators (and/or branchers, see Part B) that belong to the same space. For example, several scheduling propagators might record precedences between tasks in a shared data structure. *Local objects* and *local handles* implement this form of sharing.

A local object is space-allocated and is referred to by several local handles. A local object is deleted when its home space is deleted. Local handles provide an update() function that creates a new copy of the local object when the space is cloned (while maintaining the sharing of local objects within a space).

Figure 27.2 shows a simple local object and handle. Similar to the shared integer objects from Figure 27.1, the local integer objects here provide shared access to a single integer. However, this integer object is copied whenever the space is copied, so changes to the object are kept within the same space.

If a local object additionally allocates external resources or non-space allocated memory, it must inform its home space about this fact, very much like propagators (see Section 20.9). Figure 27.3 shows a local object and handle that implement an array of integers. In the constructor and the dispose function, the local object registers and de-registers itself for disposal using AP\_DISPOSE.

```
[DOWNLOAD]
LOCAL OBJECT AND HANDLE \equiv
  class LI : public LocalHandle {
 protected:
    class LIO : public LocalObject {
    public:
      int data;
      LIO(Space& home, int d) : LocalObject(home), data(d) {}
      LIO(Space& home, bool share, LIO& l)
        : LocalObject(home, share, l), data(l.data) {}
      virtual LocalObject* copy(Space& home, bool share) {
        return new (home) LIO(home, share, *this);
      }
     virtual size_t dispose(Space&) { return sizeof(*this); }
   };
 public:
    LI(Space& home, int d) : LocalHandle(new (home) LIO(home,d)) {}
    int get(void) const {
      return static_cast<LIO*>(object())->data;
   void set(int d) {
      static_cast<LIO*>(object())->data = d;
 };
```

Figure 27.2: A simple local object and handle

```
[DOWNLOAD]
LOCAL OBJECT WITH EXTERNAL RESOURCES \equiv
 class LI : public LocalHandle {
 protected:
    class LIO : public LocalObject {
    public:
      int* data;
      int n;
      LIO(Space& home, int n0)
       : LocalObject(home), data(heap.alloc<int>(n0)), n(n0) {
        home.notice(*this,AP_DISPOSE);
      LIO(Space& home, bool share, LIO& l)
        : LocalObject(home, share, l), data(l.data) {}
      virtual LocalObject* copy(Space& home, bool share) {
        return new (home) LIO(home, share, *this);
      }
      virtual size_t dispose(Space& home) {
        home.ignore(*this,AP_DISPOSE);
        heap.free<int>(data,n);
        return sizeof(*this);
      }
   };
  public:
    LI(Space& home, int n) : LocalHandle(new (home) LIO(home, n)) {}
    int operator [](int i) const {
      return static_cast<const LIO*>(object())->data[i];
   int& operator [](int i) {
      return static_cast<LIO*>(object())->data[i];
    }
 };
```

Figure 27.3: A local object and handle with external resources

B

### **Programming branchers**

**Christian Schulte** 

This part presents how to program branchers as implementations of branchings.

Chapter 28 (Getting started) shows how to implement branchers. An overview of the predefined view-value brancher pattern is given in Chapter 29 (View-value branchers).

## 28

### **Getting started**

This chapters presents how to program branchers as implementations of branchings.

**Important.** You need to read about search and programming propagators before reading this chapter. For search, you need to read Chapter 7 first, in particular Section 7.2. For programming propagators, the initial Chapter 20 is sufficient.

**Overview.** Section 28.1 sets the stage for programming branchers by explaining how search engines, spaces, and branchers together implement the branching process during search. A simple brancher is used as an initial example in Section 28.2. A brancher that chooses its views for branching according to some criterion is demonstrated in Section 28.3. Section 28.4 presents a specialized brancher for assigning views rather than branching on them. Variable views for branchers are discussed in Section 28.5.

#### 28.1 What to implement?

A *branching* is used in modeling for describing the shape of the search tree. A *brancher* implements a branching. That is, a branching is similar to a constraint, whereas a brancher is similar to a propagator. Branchings take variables as arguments and branchers compute with variable views.

**Brancher order.** Creating a brancher registers it with its home space. A space maintains a queue of its branchers in that the brancher that is registered first is also used first for branching. The first brancher in the queue of branchers is referred to as the *current brancher*.

**Executing branchers.** A brancher in Gecode is implemented as a subclass of the class Brancher. Similar to a propagator, a brancher must implement several virtual member functions defining the brancher's behavior.

The most important functions of a brancher can be sketched as follows:

- The status() function tests whether the current brancher has anything left to do.
- The choice() function creates a *choice* that describes the next alternatives for search. The choice must be independent of the brancher's home space, and must contain all necessary information for the alternatives.

■ The commit() function takes a previously created choice and commits to one of the choice's alternatives. Note that commit() must use only the information in the choice, the domains of the brancher's views might be weaker, stronger, or incomparable to the domains when the choice was created (due to recomputation, see Section 36.2.1).

Branchers and propagators are both actors, they both inherit from the class Actor. That entails that copying and disposal of branchers is the same as it is for propagators and hence does not require any further discussion (see Section 20.3). Also memory management is identical, see Chapter 27.

How branchers execute is best understood when studying the operations that are performed by a search engine on a space.

**Status computation.** A search engine calls the status() function of a space to determine whether a space is failed, solved, or must be branched on. When the status() function of a space is executed, constraint propagation is performed as described in Section 20.1. If the space is failed, status() returns the value SS\_FAILED (a value of type SpaceStatus, see Programming search engines).

Assume that constraint propagation has finished (hence, the space is stable) and the space is not failed. Then, status() computes the status of the current brancher. The status of a brancher is computed by the virtual status() member function a brancher implements. If the brancher's status() function returns **false**, the space's status() function tries to select the next brancher in the queue of branchers as the current brancher. If there are no branchers left in the queue of branchers, the space's status() function terminates and reports that the space is solved (by returning SS\_SOLVED). Otherwise the process is repeated until the space has a current brancher such that its status() function has returned **true**. In this case, the space's status() function returns SS\_BRANCH to signal to the search engine that the space requires branching.

The status() function of a brancher does not do anything to actually perform branching, it just checks whether there is something left to do for the brancher (for example, whether there are unassigned views left in an array of views to be branched on).

**Choice creation.** In case the space's status() function has returned SS\_BRANCH, the search engine typically branches. In order to actually branch, the search engine must know how to branch. To keep search engines orthogonal to the space type they compute with (that is, the problem a search engine tries to find solutions for), a search engine can request a *choice* as a description of how to branch. With a choice, a search engine can *commit* a space to a particular alternative as defined by the choice. Alternatives are numbered from zero to the number of alternatives minus one, where the number of alternatives is defined by the choice. A choice is specific to a brancher and must provide sufficient information such that the brancher together with the choice can commit to all possible alternatives.

A search engine requests the computation of a choice by executing the choice() function of a space. The space's choice() function does nothing but returning the choice created by the choice() function of the current brancher. Let us postpone the discussion of how

a choice can store the information needed for branching. A choice always provides two important pieces of information:

- The number of its alternatives (an unsigned integer greater than zero) is available through the function alternatives().
- A unique identity inherited from the brancher that has created the choice. This information is not available to a programmer but makes it possible for a space to find the corresponding brancher for a given choice (to be detailed below).

A search engine must execute the choice() function of a space *immediately* after executing the space's status() function. Any other action on the space (such as adding propagators or modifying views) might change the current brancher of the space and hence choice() might fail to compute the correct choice.

**Committing to alternatives.** Assume that the engine has a choice ch with two alternatives and that the engine has created a clone c of the current space s  $by^1$ 

```
Space* c = s->clone();
```

The search engine typically commits s to the first alternative (the alternative with number 0) as defined by the space and later it might commit c to the second alternative (the alternative with number 1). Let us first consider committing the space s to the first alternative by:

```
s->commit(*ch,0);
```

The space's commit() function attempts to find a brancher that corresponds to the choice ch (based on the identity that is stored by the choice). Assume, the space's commit() function finds a brancher b. Then it invokes the commit() function of the brancher b as follows:

```
b.commit(*s,*ch,0);
```

Now the brancher b executes its commit() function. The function typically modifies a view as defined by the choice (\*ch in our example) and the number of the alternative (0 in our example) passed as arguments. The brancher's commit() function must return an executions status that captures whether the operations performed by commit() have failed (ES\_FAILED is returned) or not (ES\_OK is returned).

If the space's commit() function does not find a brancher that corresponds to the choice ch, an exception of type SpaceNoBrancher is thrown. The reason for being unable to find a corresponding brancher is that the search engine is implemented incorrectly (see Part S).

<sup>&</sup>lt;sup>1</sup>Note that search engines compute with pointers to spaces and choices rather than references, hence s, c, and ch are pointers.

At some later time, the search engine might decide to explore the second alternative of the choice ch by using the clone c:

```
c->commit(*ch,1);
```

Then, the commit() function of the space c tries to find the brancher corresponding to ch. This of course will be a different brancher in a different space: however, the brancher found will be a copy of the brancher b.

**More on choices.** A consequence of the discussion of commit() is that choices cannot store information that belongs to a particular space: the very idea of a choice is that it can be used with different spaces (above: original and clone)! That also entails that a choice is allocated independently from any space and must be deleted explicitly by a search engine.

Consider as an example a brancher that wants to create the choice  $(x_i = n) \lor (x_i \ne n)$  where x is an array of integer views (that is,  $x_i = n$  is alternative 0 and  $x_i \ne n$  is alternative 1). The choice is not allowed to store  $x_i$  directly (as  $x_i$  belongs to a space), instead it stores the position i in the array x and the integer value n. When the brancher's commit() function is executed, the brancher provides the view array x (the part that is specific to a space) and the choice provides the position i and the value n (the parts that are space-independent).

A choice must inherit from the class Choice and needs to implement a single virtual member function size() which returns the size (of type size\_t) of the choice. This function can be used by a search engine to compute the amount of memory allocated by a choice (this can be useful for statistics).

**Even more on (archives of) choices.** Branchers and choices must support one more operation, the *(un-)archiving* of choices. Archiving enables choices to be used not only in the same search engine with a different space, but transferred to a search engine in a different process, possibly on a different computer. The main application for this are distributed search engines.

An Archive is simply an array of unsigned integers, which is easy to transmit e.g. over a network. Every choice must implement the archive() method, which in turn calls archive() on the super class and then writes the contents of the choice into the archive. Conversely, branchers must implement a choice method that gets an archive as the argument and returns a new choice.

Brancher invariants and life cycle. The very idea of a choice is that it is a space-independent description of how to perform commits on a space. Consider the following example scenario: a search engine has a space s and a clone c of s. Then, the search engine explores part of the search tree starting from s and stores a sequence of choices while exploring. Then, the engine wants to recompute a node in the search tree and uses the clone c for it: it can recompute by performing commits with the choices it has stored.

In this scenario, the brancher to which the choices correspond *must* be able to perform the commits as described by the choices. That in particular entails that a brancher cannot

modify its state arbitrarily: it must always guarantee that choices it created earlier (actually, that a copy of it created earlier) can be interpreted correctly.

The status() and choice() functions of a brancher can be seen as being orthogonal to its commit() function. The former functions compute new choices. The latter function must be capable to commit according to other choices that are unrelated to the choice that might be computed by the choice() function. Even if the status() function of a brancher has already returned **false**, the brancher must still be capable of performing commits. Hence, a brancher cannot be disposed immediately when its status() returns **false**.

Spaces impose an important invariant on the use of its commit() and choice() function. After having executed the choice() function, no calls to commit() are allowed with choices created earlier (that is, choice() invalidates all previously created choices for commit()). That also clarifies when branchers are disposed: the choice() function of a space disposes all branchers for which their status() functions have returned **false** before. This invariant is essential for recomputation, see Section 36.2.1.

Garbage collection of branchers. As the choice() function of a space performs garbage collection of branchers, it can also be called in case the space's status() function returned SS\_SOLVED (signaling that no more branchers are left). In this case, the branchers are garbage collected but no choice is returned (instead, NULL is returned). See Section 35.2 for an example and Section 35.1 for a discussion.

#### 28.2 Implementing a nonemin branching

This section presents an example for a branching and its implementing brancher.

The branching

```
void nonemin(Home home, const IntVarArgs& x);
```

branches by selecting the first unassigned variable  $x_i$  in x and tries to assign  $x_i$  to the smallest possible value of  $x_i$ .

That is, nonemin is equivalent to the predefined branching (see Section 7.1.1) used as

```
branch(home, x, INT_VAR_NONE, INT_VAL_MIN);
```

For examples of problem-specific branchers see Chapter 16 and Chapter 17.

#### 28.2.1 A naive brancher

Figure 28.1 shows the class NoneMin implementing the brancher for the branching nonemin. NoneMin inherits from the class Brancher. The branching post function creates a view array and posts a brancher of class NoneMin. The brancher post function does not return an execution status as posting a brancher does not fail (in contrast to a propagator post function, see Section 20.4). The constructor as well as the brancher post function of NoneMin are exactly as to be expected from our knowledge on implementing propagators (of course, branchers do not use subscriptions).

```
[DOWNLOAD]
NONE MIN \equiv
  class NoneMin : public Brancher {
  protected:
    ViewArray<Int::IntView> x;
    ► CHOICE DEFINITION
  public:
    NoneMin(Home home, ViewArray<Int::IntView>& x0)
      : Brancher(home), x(x0) {}
    static void post(Home home, ViewArray<Int::IntView>& x) {
      (void) new (home) NoneMin(home,x);
    }
    ► STATUS
    ► CHOICE
    ► COMMIT
  };
  void nonemin(Home home, const IntVarArgs& x) {
    if (home.failed()) return;
    ViewArray<Int::IntView> y(home,x);
    NoneMin::post(home,y);
```

Figure 28.1: A branching and brancher for nonemin

**Status computation.** The status computation of a NoneMin brancher is straightforward. The status() function scans all views in the view array x and returns **true** if there is an unassigned view left:

```
virtual bool status(const Space& home) const {
  for (int i=0; i<x.size(); i++)
    if (!x[i].assigned())
     return true;
  return false;
}</pre>
```

**Choice computation.** Computing the choice for a brancher involves two aspects: the definition of the class for the choice object and the choice() member function of a brancher that creates choice objects.

The choice object PosVal inherits from the class Choice. The information that is required for committing is which view and which value should be used. As discussed above, the PosVal choice does not store the view directly, but the position pos of the view in the view array x. A PosVal choice stores both position and value as integers as follows:

```
choice Definition =
  class PosVal : public Choice {
  public:
    int pos; int val;
    PosVal(const NoneMin& b, int p, int v)
        : Choice(b,2), pos(p), val(v) {}
    virtual size_t size(void) const {
        return sizeof(*this);
    }
    virtual void archive(Archive& e) const {
        Choice::archive(e);
        e << pos << val;
    }
};</pre>
```

The constructor of PosVal uses the constructor of Choice for initialization, where both the brancher b and the number of alternatives of the choice (2 for PosVal) are passed as arguments. The size() function just returns the size of a PosVal object. The archive() function calls Choice::archive() and then writes the position and value to the archive.

The choice(Space& home) member function of the brancher is called directly (by the choice(Space& home) function of a space) after the status() function of the brancher in case the status() of the brancher has returned **true**. That is, the brancher is the current brancher of the space and still needs to create choices for branching. For a NoneMin brancher that means that there is definitely a not yet assigned view in the view array x, and that the brancher must choose the first unassigned view:

```
choice \( \)
  virtual Choice* choice(Space& home) {
    for (int i=0; true; i++)
        if (!x[i].assigned())
            return new PosVal(*this,i,x[i].min());
    GECODE_NEVER;
    return NULL;
}
  virtual Choice* choice(const Space&, Archive& e) {
    int pos, val;
    e >> pos >> val;
    return new PosVal(*this, pos, val);
}
```

The choice(Space& home) function returns a new PosVal object for position i and the smallest value of x[i]. The choice(const Space&, Archive& e) function creates a choice from the information contained in the archive.

**Tip 28.1** (Never execute). The macro GECODE\_NEVER states that it will never be executed. If compiled in debug mode, GECODE\_NEVER corresponds to the assertion assert(**false**). If compiled in release mode, the macro informs the compiler (depending on the platform) that GECODE\_NEVER is never executed so that the compiler can take advantage of this information for optimization. ◀

**Committing to alternatives.** The commit() function of NoneMin can safely assume that the choice c passed as argument is in fact an object pv of class PosVal. This is due to the fact that the space commit() function uses the identity stored in a choice object to find the corresponding brancher.

From the PosVal object the position of the view pos and the value val are extracted. Then, if the commit is for the first alternative (a is 0), the brancher assigns the view x[pos] to val using the view modification operation eq (see Figure 20.5). If the modification operation

returns a modification event signaling failure, me\_failed is true and hence the execution status ES\_FAILED is returned. Otherwise, ES\_0K is returned as execution status. If the commit is for the second alternative (a is 1), the value val is excluded from the view x[pos].

#### 28.2.2 Improving status and choice

The status() and choice() functions of NoneMin as defined in Figure 28.1 are inefficient as they always inspect the entire view array for finding the first unassigned view.

The brancher shown in Figure 28.2 stores an integer start to remember which of the views in x are already assigned. The brancher maintains the invariant that all  $x_i$  are assigned for  $0 \le i < \text{start}$ . The value of start will be updated by the status() function which must be declared as **const**. As start is updated by a **const**-function, it is declared as **mutable**.

The choice() function of an improved NoneMin brancher can rely on the fact that x[start] is the first unassigned view in the view array x.

#### 28.3 Implementing a sizemin branching

This section presents an example for a brancher that implements a more interesting selection of the view to branch on. The branching

```
void sizemin(Home home, const IntVarArgs& x);
```

branches by selecting the variable in x with smallest domain size first and tries to assign the selected variable to its smallest possible value. That is, sizemin is equivalent to the predefined branching (see Section 7.1.1) if used as

```
branch(home, x, INT_VAR_SIZE_MIN, INT_VAL_MIN);
```

Figure 28.3 shows the brancher SizeMin implementing the sizemin branching. The status() function is not shown as it is exactly the same function as for NoneMin from the previous section: after all, the brancher is done only after all of its views are assigned. Likewise, the PosVal choice and the commit() function are unchanged.

The choice() function uses the integer p for the position of the unassigned view with the so-far smallest domain size and the unsigned integer s for the so-far smallest size. Then, all views that are not known to be assigned are scanned to find the view with smallest domain size. Keep in mind that due to the invariant enforced by the brancher's status() function, x[start] is not assigned.

It is absolutely essential to skip already assigned views. If assigned views were not skipped, then the same choice would be created over and over again leading to an infinite search tree.

```
NONE MIN IMPROVED =
...
class NoneMin : public Brancher {
  protected:
    ViewArray<Int::IntView> x;
    mutable int start;
...
    virtual bool status(const Space& home) const {
        for (int i=start; i<x.size(); i++)
            if (!x[i].assigned()) {
            start = i; return true;
            }
        return false;
    }
    virtual Choice* choice(Space& home) {
        return new PosVal(*this,start,x[start].min());
    }
    ...
};
...</pre>
```

Figure 28.2: An improved brancher for nonemin

```
[DOWNLOAD]
SIZE MIN ≡
  . . .
  class SizeMin : public Brancher {
    virtual Choice* choice(Space& home) {
      int p = start;
      unsigned int s = x[p].size();
      for (int i=start+1; i<x.size(); i++)</pre>
        if (!x[i].assigned() \&\& (x[i].size() < s)) {
          p = i; s = x[p].size();
        }
      return new PosVal(*this,p,x[p].min());
    }
    . . .
  };
  • • •
```

Figure 28.3: A brancher for sizemin

```
[DOWNLOAD]
ASSIGN MIN \equiv
 class AssignMin : public Brancher {
   class PosVal : public Choice {
   public:
      int pos; int val;
      PosVal(const AssignMin& b, int p, int v)
        : Choice(b,1), pos(p), val(v) {}
   };
   virtual ExecStatus commit(Space& home,
                               const Choice& c,
                               unsigned int a) {
      const PosVal& pv = static_cast<const PosVal&>(c);
      int pos=pv.pos, val=pv.val;
      return me_failed(x[pos].eq(home,val)) ? ES_FAILED : ES_OK;
   }
 };
```

Figure 28.4: A brancher for assignmin

#### 28.4 Assignment branchers

This section presents an example for a brancher that assigns all of its views rather than branches on its views. The branching

```
void assignmin(Home home, const IntVarArgs& x);
```

assigns all variables in x to their smallest possible value. That is, assignmin is equivalent to the predefined branching (see Section 7.1.5) used as

```
assign(home, x, INT_ASSIGN_MIN);
```

Figure 28.4 shows the relevant parts of the brancher AssignMin. Unsurprisingly, both the status() and choice() function are identical to those shown in Figure 28.2 (and hence are omitted).

The changes concern the created choice of type PosVal: the constructor now initializes a choice with a single alternative only (the second argument to the call of the constructor Choice). The commit() function is a specialized version of the commit() function defined in Figure 28.2. It only needs to be capable of handling a single alternative.

```
NONE MIN AND NONE MAX =
...

template < class View>
    class NoneMin : public Brancher {
        ...
};

void nonemin(Home home, const IntVarArgs& x) {
        ...
}

void nonemax(Home home, const IntVarArgs& x) {
        if (home.failed()) return;
        ViewArray<Int::MinusView> y(home,x.size());
        for (int i=x.size(); i--; )
            y[i]=Int::MinusView(x[i]);
        NoneMin<Int::MinusView>::post(home,y);
}
```

Figure 28.5: Branchings for nonemin and nonemax

#### 28.5 Using variable views

Variable views can also be used for reusing branchers to obtain several branchings, similar to reusing propagators for several constraints, see Chapter 25.

While in principle all different variable views introduced in Chapter 25 can be used for branchers, the only meaningful variable view for branchers is the minus integer view (see Section 25.1.1).

As an example consider the branchings nonemin (see Section 28.2) and nonemax where the latter tries to assign the maximal value of a view first. The corresponding program fragment is shown in Figure 28.5. The class NoneMin is made generic with respect to the type of view it uses. Then, both nonemin and nonemax can be obtained by instantiating NoneMin with integer views or integer minus views.

# 29

### View-value branchers

This chapter gives an overview of the predefined view-value brancher pattern that is generic with respect to how views and values for branching are selected.

**Overview.** Section 29.1 demonstrates how the view-value pattern for branchers can be used to straightforwardly implement common branchings. Additional support for view-value branchers is summarized in Section 29.2.

#### 29.1 Using view-value branchers

A view-value brancher follows a pattern for conveniently programming branchers that selects a view from an array of views for branching and then creates a choice that involves a single value from the selected view's domain. The view-value pattern is implemented by the class ViewValBrancher which is a template expecting two types: the first type must be a view selector that describes how views are selected, whereas the second type must be a value selector that describes how values are selected for branching.

A view selector must inherit from the class ViewSelBase which itself is a template expecting a variable view type as its argument type. A value selector must inherit from the class ValSelBase which, again, itself is a template expecting a variable view type and a value type as its argument types.

Even though a single view-value brancher uses a view selector as well as a value selector, the view types for view selector and value selector can actually differ. However, a view-value brancher for integer views can only combine integer views and minus views (these are the only views that are actually meaningful for branchers on integer views, see Section 28.5).

#### 29.1.1 Implementing sizemin

Figure 29.1 shows how the sizemin branching (see Section 28.3) can be implemented by a view-value brancher. The sizemin() function creates a view selector sm of type SizeMin and a value selector vm of type ValueMin and then posts a view-value brancher with the view and value selectors.

**View selectors.** The view selector SizeMin is shown in Figure 29.2. Every view selector must inherit from the class ViewSelBase. It must implement the following constructor and

Figure 29.1: Using view-value brancher for sizemin

```
VIEW SELECTION ≡
  class SizeMin : public ViewSelBase<Int::IntView> {
  protected:
    unsigned int size;
  public:
    SizeMin(void) : size(0U) {}
    SizeMin(Space& home, const VarBranchOptions& vbo)
      : ViewSelBase<Int::IntView>(home,vbo) {}
    ViewSelStatus init(Space& home, Int::IntView x) {
      size = x.size();
      return (size == 2) ? VSS_BEST : VSS_BETTER;
   ViewSelStatus select(Space& home, Int::IntView x) {
      if (x.size() < size) {</pre>
        size = x.size();
        return (size == 2) ? VSS_BEST : VSS_BETTER;
      } else if (x.size() > size) {
        return VSS_WORSE;
      } else {
        return VSS_TIE;
      }
    }
 };
```

Figure 29.2: View selection for sizemin

#### functions:

- A constructor that initializes the view selector. The constructor takes a home space and an object of class VarBranchOptions as arguments (to be discussed in Section 29.2.4).
  - This constructor is used for initializing data that is used throughout the entire lifetime of the view-value brancher. As an example, think of a random view selector. Then the constructor would initialize the random number generator.
- A function init() that returns a view selection status of type ViewSelStatus (see Generic brancher based on view and value selection). The initialization function is called for a view x (the first unassigned view of the view-value brancher) and should initialize the data structures that are required to select the appropriate view. In our case, the size is initialized to the so-far smallest domain size of a view (which, of course, is x.size()).

The init function must return either VSS\_BETTER or VSS\_BEST. In case the size of x is two, it is clear that x is a best view to be selected (as no view can have domain size 0 and as views for selection are not assigned).

■ A selection function select(). The purpose of the selection function becomes clear when looking at the example.

If the view x passed as argument has a domain of smaller size, the size member is updated, and VSS\_BETTER or VSS\_BEST is returned (the view x is better than the previously selected view or even a best view). If x has a domain of larger size, VSS\_WORSE is returned. If the domain size of x agrees with the so-far smallest domain size size, the selection function returns VSS\_TIE.

In the case of ties, tie-breaking can be used for combining several view selectors, see Section 29.2.1 for details. Tie-breaking also interprets VSS\_BEST as VSS\_BETTER such that even several best views are considered for tie-breaking.

The function select() is called for all unassigned views but the first one (for which init() is called) of a view-value brancher and hence implements the actual view selection.

If needed, a view selector can also implement a destructor.

The default constructor is not really needed but is implemented to avoid compiler warnings.

Note that the view type used for init() and select() must coincide with the view type used for instantiating ViewSelBase.

**Value selectors.** The value selector ValMin is shown in Figure 29.3. Every value selector must inherit from the class ValSelBase. It must implement the following constructor and functions:

```
value selection \( \)
    class ValMin : public ValSelBase<Int::IntView,int> {
    public:
        ValMin(void) {}
        ValMin(Space& home, const ValBranchOptions& vbo)
            : ValSelBase<Int::IntView,int>(home,vbo) {}
        int val(Space& home, Int::IntView x) const {
            return x.min();
        }
        ModEvent tell(Space& home, unsigned int a, Int::IntView x, int n) {
            return (a == 0) ? x.eq(home,n) : x.gr(home,n);
        }
    };
}
```

Figure 29.3: Value selection for sizemin

- A constructor that initializes the value selector. The constructor takes a home space and an object of class ValBranchOptions as argument (to be discussed in Section 29.2.4).
   The constructor serves the same purpose as the constructor for a view selector.
- A value function val() that returns the value to be used for branching.
- A tell function tell() that performs a modification operation on the view x with respect to the selected value n for the alternative a. The tell() function must return the modification event of the performed modification operation.

A view-value brancher is only interested in whether the modification operation failed or not. Hence, if several modification operations are executed in the tell() function it is sufficient to report failure by an appropriate modification event (for example, ME\_INT\_FAILED). The details of non-failed modification events are of no concern to a view-value brancher.

Again, the default constructor is defined to avoid compiler warnings.

The view type and value type used for val() and tell() must coincide with the view type and value type used for instantiating ValSelBase.

For an example that implements a custom branching using a value selector, please see The balanced academic curriculum problem.

#### 29.1.2 Implementing nonemin

Implementing the nonemin branching (see Section 28.2) by a view-value brancher is straightforward. Figure 29.4 shows the view selector None, the value selector is the same

(ValMin) as in Section 29.1.1. Both the init() and select() functions of None immediately report that the view x is a best one. Note that select() will actually never be called unless None is used in tie-breaking (which would render None useless in the first place).

#### 29.1.3 Implementing assignmin

Even assignment branchings can be obtained from view-value branchers. Figure 29.5 shows how to define a value selector with a single alternative only. The definition of ValMin is the same as in Section 29.1.1 and Section 29.1.2 but with a redefinition of the number of alternatives.

#### 29.2 Support for view-value branchers

This section provides a brief overview of additional functionality that supports the construction of view-value branchers.

#### 29.2.1 Tie-breaking

A common need is to combine several view selectors for tie-breaking. For example, assume that you want to a select a view first by largest degree (see Section 7.1.1) and then by smallest domain size. That is, among the views with largest degree, a view with smallest domain size is selected.

Gecode provides two approaches for combining view selectors for tie-breaking. The class ViewSelTieBreakStatic statically combines two view selectors into a new view selector using tie-breaking. The static combination is resolved at compile time and hence does not incur any runtime overhead. The class ViewSelTieBreakDynamic provides dynamic tie-breaking that defines a view selector that implements tie-breaking over an array of view selectors.

#### 29.2.2 Predefined view selectors

Gecode offers a plethora of predefined view selectors, a summary is given in Figure 29.6. For more information on the different view selectors, please consult Section 7.1.1.

The view selectors that are generic (that is, not specific to a particular view type) are templates expecting a variable view type as its template argument and can be used with integer, Boolean, and set views.

The integer and set view selectors are specific to integer and set views and hence are not available as templates.

```
VIEW VAL NONE MIN =
...
class None : public ViewSelBase<Int::IntView> {
  public:
    None(void) {}
    None(Space& home, const VarBranchOptions& vbo)
    : ViewSelBase<Int::IntView>(home, vbo) {}
    ViewSelStatus init(Space& home, Int::IntView x) {
      return VSS_BEST;
    }
    ViewSelStatus select(Space& home, Int::IntView x) {
      return VSS_BEST;
    }
};
...
```

Figure 29.4: Using view-value brancher for nonemin

Figure 29.5: Using view-value brancher for assignmin

#### generic (template with view type)

ViewSelNonefirstViewSelDegreeMinsmallest degreeViewSelDegreeMaxlargest degreeViewSelAfcMinsmallest AFCViewSelAfcMaxlargest AFCViewSelRndrandom

#### integer views

Int::Branch::BySizeDegreeMin smallest domain size divided by degree Int::Branch::BySizeDegreeMax largest domain size divided by degree smallest domain size divided by AFC Int::Branch::BySizeAfcMax largest domain size divided by AFC

#### set views

Set::Branch::ByMinMin smallest minimum element
Set::Branch::ByMinMax largest minimum element
Set::Branch::ByMaxMin smallest maximum element
Set::Branch::ByMaxMax largest maximum element
Set::Branch::BySizeMin smallest domain size
Set::Branch::BySizeMax largest domain size

Set::Branch::BySizeDegreeMin smallest domain size divided by degree largest domain size divided by degree set::Branch::BySizeAfcMin smallest domain size divided by AFC Set::Branch::BySizeAfcMax largest domain size divided by AFC

Figure 29.6: Predefined view selectors

#### integer views (template with view type)

Int::Branch::ValMin minimum
Int::Branch::ValMed median

Int::Branch::ValSplitMin split according to median
Int::Branch::ValRangeMin split according to ranges

Int::Branch::ValRnd random

#### set views (template with Boolean value)

Set::Branch::ValMin minimum
Set::Branch::ValMed median
Set::Branch::ValMax maximum
Set::Branch::ValRnd random

Figure 29.7: Predefined value selectors

#### 29.2.3 Predefined value selectors

Gecode offers a plethora of predefined value selectors, a summary is given in Figure 29.7. For more information on the different value selectors, please consult Section 7.1.1.

The value selectors for integer views are templates where the argument can either be an integer view Int::IntView or a minus integer view Int::MinusView.

The value selectors for set views are templates requiring a Boolean value as template argument. The Boolean argument defines whether the integer value chosen for branching is first included (the argument must be **true**) or excluded (the argument must be **false**).

#### 29.2.4 Variable and value branch options

Branch options for both view selectors (see VarBranchOptions) and value selectors (see ValBranchOptions) support passing seed values for random view and value selectors. For details, see the individual classes.

V

### **Programming variables**

**Christian Schulte** 

This part explains how new variable types can be programmed with Gecode.

In order to be able to program variables, all chapters in this part should be read. The chapters capture the following:

- Chapter 30 (Getting started) outlines how a new variable type can be programmed for Gecode.
- Chapter 31 (Variable implementations) shows how the kernel-specific and variable domain-specific aspects of a variable implementation are specified and programmed.
- Chapter 32 (Variables and variable arrays) shows how variables and variable arrays for modeling are programmed for a given variable implementation.
- Chapter 33 (Views) shows how views for programming propagators and branchers are programmed for a given variable implementation.
- Chapter 34 (Putting everything together) finally explains how a new variable type can be used with Gecode.

**Important.** Programming variables requires to configure and recompile Gecode from its source code. Using one of the pre-compiled packages is not sufficient. More details can be found in Chapter 34.

# 30

### **Getting started**

This chapter outlines how a new variable type can be programmed with Gecode. The chapter (and the entire part on programming variables) chooses integer interval variables as its running example.

**Overview.** An overview of what needs to be designed and programmed is presented in Section 30.1. The structure of how the implementation of a variable type is organized is presented in Section 30.2.

**Important.** Programming variables requires to configure and recompile Gecode from its source code. Using one of the pre-compiled packages is not sufficient. More details can be found in Chapter 34.

#### 30.1 Overview

We are going to use *integer interval variables* as the running example for the entire part on programming variables. Integer interval variables take integer values (like the integer variables that come pre-defined with Gecode do) but their domain is defined by a lower and an upper bound only. That is, the domain is always an interval. This is in contrast to the integer variables that come with Gecode, where their domain can be any finite set of integer values.

The focus of this part is on understanding what needs to be done for implementing new variables, so we deliberately choose very simple variables together with very few operations on them as an example.

Even though integer interval variables seem not very interesting, variants of them could in fact be interesting. For example, the integer values used for the lower and upper bound could be integers of arbitrary precision, or instead of integer values one could choose floating point values.

**What must be programmed.** Programming variables includes the following programming tasks:

■ *Variable implementations* (Chapter 31): Programming a variable implementation consists of two tasks:

- 1. The first task is to specify domain-independent aspects of a variable implementation. This includes specifying a name for the variable implementation type, scope information, modification events, and propagation conditions. From a simple specification file containing this information a domain-independent base class for a variable implementation and the corresponding C++ definitions of modification events and propagation conditions is generated.
  - The generated base class together with the definition of modification events and propagation conditions actually become part of Gecode's kernel. The kernel needs these definitions to schedule propagators that have subscribed to a variable implementation and to maintain these subscriptions during cloning.
- 2. The second task consists of programming the domain-dependent operations of a variable implementation. This is achieved by defining a class for a variable implementation that inherits from the generated, domain-independent base class and defines the respective domain operations.
- *Variables* and *variable arrays* (Chapter 32): As a variable is nothing but a simple and read-only interface to a variable implementation, a variable is obtained by inheriting from a base class for variables that depends on the variable implementation type. The actual programming amounts to defining read-only variable operations that invoke the corresponding operations on the variable's variable implementation.
  - Variable arrays and variable argument arrays are, as variables, needed for modeling. Their programming requires the definition of several traits classes so that the defined arrays can be used with Gecode-provided functionality (for example, with the matrix interface for arrays, see Section 6.2).
- *Views* (Chapter 33): Programming a view depends on the type of the view: whether the view is a direct interface to a variable implementation (a *variable implementation view*), whether it is a *constant view*, or whether it is derived (a *derived view*) from some other view. As examples, we are going to implement an integer view as a variable implementation view, minus and offset views as derived views, and an integer constant view as a constant view.

In addition to the classes for views, some additional functions on views must be defined for testing in which order views are and whether two views are shared or the same (see Section 25.1.2).

**Putting everything together.** Even though we are presenting the implementation of integer interval variables only as an example, Chapter 34 shows how everything is put together. This includes examples of propagators, branchers, post functions using various views, and a simple script (Golomb rulers, see Chapter 10) using integer interval variables.

More importantly, it also shows how Gecode must be configured and compiled such that integer interval variables are supported by Gecode's kernel.

#### 30.2 Structure

The implementation of integer interval variables is contained in a single header file int.hh, which is shown in Figure 30.1.

**Namespaces.** The implementation is contained in the namespace MPG (for Modeling and Programming with Gecode) to avoid name-clashes with functionality provided by Gecode. To keep the implementation of integer interval variables concise, some important definitions in the Gecode namespace are made available by **using** declarations (see Figure 30.1).

Similar to the organization of namespaces in Gecode, definitions that are used for modeling (variables and variable arrays) are contained in the namespace MPG, while definitions that are used for programming (variable implementations, views, and additional support) are in the namespace MPG::Int. As the structure of namespaces matters (part of the support for variable arrays must be defined inside the Gecode namespace), each program fragment is shown in its appropriate namespace.

As an example, consider the definition of two exceptions that are thrown by the constructor creating a new integer interval variable, in case the variable domain is ill-specified:

```
namespace MPG { namespace Int {

  class OutOfLimits : public Exception {
   public:
     OutOfLimits(const char* l)
          : Exception(l, "Number out of limits") {}
     };

  class VariableEmptyDomain : public Exception {
     ...
  };
}
```

As discussed above, Exception is Gecode::Exception (see Exception) and has been introduced by a **using** declaration.

**Naming scheme.** The naming scheme follows the same naming scheme for integer variables as defined by Gecode (albeit defined in the namespace MPG instead of Gecode):

■ Variable implementations: The base class is named IntVarImpBase whereas the variable implementation class is named IntVarImp. The names of modification events start with ME\_INT\_ whereas the names of propagation conditions start with PC\_INT\_. As mentioned above, these classes and identifiers are defined within the namespace MPG::Int.

```
[DOWNLOAD]
INT.HH ≡
 #ifndef __MPG_INT_HH__
 #define __MPG_INT_HH__
 #include <gecode/kernel.hh>
  using Gecode::Advisor;
  EXCEPTIONS
  ► VARIABLE IMPLEMENTATION
  ► VARIABLE
  ► ARRAY TRAITS
  ► VARIABLE ARRAYS
  ► INTEGER VIEW
  ► CONSTANT INTEGER VIEW
  ► MINUS VIEW
  ► OFFSET VIEW
  #endif
```

Figure 30.1: The header file for integer interval variables

- Variables and variable arrays: Integer interval variables are implemented by the class IntVar. Variable arrays of integer interval variables are implemented by the class IntVarArray, whereas the corresponding variable argument array is implemented by the class IntVarArgs. These classes are defined in the namespace MPG.
- Views: the respective views are implemented by classes IntView, ConstIntView, MinusView, and OffsetView. They are all defined within the namespace MPG::Int.

## 31

# Variable implementations

This chapter describes how variable implementations can be programmed with Gecode. The chapter uses integer interval variables as introduced in Chapter 30 as its running example.

**Overview.** The design of integer interval variables is detailed in Section 31.1. After having finalized the design, Section 31.2 explains how the domain-independent base class for the variable implementation together with definitions of modification events and propagation conditions can be generated from a simple specification. Section 31.3 shows how the actual variable implementation is programmed from the generated base class for a variable implementation. Section 31.4 provides an overview of additional options for generating a variable implementation base class from its specification.

#### 31.1 Design decisions

Before starting with the description of the implementation of integer interval variables, let us detail their design. This includes the design of the variable domain including access and modification operations, deltas for advisors (see Chapter 24), modification events, and propagation conditions.

**Variable domain and operations.** Unsurprisingly, the variable domain of an integer variable implementation is represented by two integers l (lower bound) and u (upper bound). The variable implementation provides access operations min() and max() that return these integers.

To modify an integer interval variable implementation, the operation gq(home,n) modifies the domain such that its values must be greater or equal to n, whereas the operation lq(home,n) modifies the domain such that its values must be less or equal to n.

The values 1 and u can only be initialized (when creating a new variable, see Section 32.1) and modified such that they obey the following invariants:

- 1. The domain is never empty, that is,  $l \le u$ .
- 2. The domain values never exceed the limits defined by (INT\_MAX is the largest possible value for an **int**):

```
LIMITS ≡
  namespace Limits {
   const int max = (INT_MAX / 2) - 1;
  const int min = -max;
}
```

That is, Int :: Limits ::  $min \le l \le u \le Int :: Limits :: max$ .

The choice of values for Limits::min and Limits::max are motivated by simplicity only. To keep the example propagators used in Chapter 34 simple, the limits are chosen such that the addition and subtraction of two integer values within the limits do not lead to numerical overflow. A real-life variable implementation would try to make as many values as possible available for a variable domain, see for example Section 4.1.2.

**Tip 31.1** (Correctness matters). While the decision to restrict the possible values of a variable implementation is motivated by simplicity, the decision for a real-life variable implementation is absolutely essential.

Being unclear about which values can correctly be maintained by a variable implementation, not ensuring that no numerical overflow occurs, or not checking for the necessary invariants when a new variable is created, renders the very idea of constraint programming obsolete: that whenever a solution is found by Gecode, it *actually happens to be a solution*. Hence correctness does not only matter for implementing propagators and branchers but also for getting the basic design of variables right.

**Assigned variables.** An integer interval variable is assigned iff l = u.

**Deltas for advisors.** We design the delta information for an advisor computed by a modification operation on the variable implementation to be an interval as well. The interval defines the values that are removed by a modification operation. Due to the nature of the modification operations lq() and qq(), the removed values always form an interval.

The design of deltas to be used by advisors for a variable implementation depends directly on the design of the modification operations provided by a variable implementation. For example, if our integer interval variable implementation also featured an operation eq() to assign a variable implementation to a value, then one also would have to choose a different design for the delta information. Assume a variable implementation with domain [l .. u] and that the modification operation eq(home,n) is executed where l < n < u. Then one could design the delta information to either accurately represent the set of removed values  $[l .. n - 1] \cup [n + 1 .. u]$  or to provide support for signaling that the domain has changed arbitrarily (this is the design chosen for integer variables in Gecode, see Section 24.3).

**Modification events.** Any variable implementation must support the mandatory events for no modification (to be implemented as ME\_INT\_NONE), for failure (to be implemented as ME\_INT\_FAILED), and for assignment to a value (to be implemented as ME\_INT\_VAL).

The additional events must be chosen such that they take the following two aspects into account:

- The modification operations should return meaningful values that describe how the domain of a variable implementation has changed. They must return ME\_INT\_VAL if the variable implementation becomes assigned. Otherwise, we choose to return ME\_INT\_MIN if the lower bound changes and to return ME\_INT\_MAX if the upper bound changes.
- When a new propagator is posted and the propagator subscribes to some views (and hence to some variable implementations), the propagator must be scheduled with respect to some modification event. This modification event should capture that "somehow the variable has changed for the propagator". In case an integer interval variable implementation is not yet assigned (otherwise the propagator will be scheduled with the modification event ME\_INT\_VAL anyway), we use an additional modification event ME\_INT\_BND capturing that one or both of the bounds have changed.

Again, there is quite some degree of freedom in the choice of modification events. Another design would be to only provide the modification event ME\_INT\_BND instead (apart from the mandatory modification events). An important aspect in which design to choose is the relation between modification events and propagation conditions to be discussed below.

**Propagation conditions.** To make our example variable implementations sufficiently interesting, we design the propagation conditions such that they can take full advantage of the modification events.

That is, apart from the mandatory propagation condition for not creating any subscription (to be implemented as PC\_INT\_NONE) and the mandatory propagation condition for an assigned variable implementation (to be implemented as PC\_INT\_VAL), we have three propagation conditions as follows:

- PC\_INT\_MIN: schedule a propagator if the lower bound of a variable implementation changes.
- PC\_INT\_MAX: schedule a propagator if the upper bound changes.
- PC\_INT\_BND: schedule a propagator if lower or upper bound changes.

This design can also be reformulated in terms of modification events that are generated by a modification operation:

- PC\_INT\_MIN: schedule a propagator for ME\_INT\_VAL, ME\_INT\_MIN, and ME\_INT\_BND.
- PC\_INT\_MAX: schedule a propagator for ME\_INT\_VAL, ME\_INT\_MAX, and ME\_INT\_BND.
- PC\_INT\_BND: schedule a propagator for ME\_INT\_VAL, ME\_INT\_MIN, ME\_INT\_MAX, and ME\_INT\_BND.

A simpler design would be to have the single non-mandatory propagation condition PC\_INT\_BND. The decision which design is best is not straightforward, as the tradeoff between the cost for additional propagation conditions (see below) and the gain from avoiding propagator executions depends on many different aspects. For a discussion and an evaluation in the context of Gecode's integer variables, see [37, Section 5].

**Costs and limits for modification events and propagation conditions.** The cost per each individual modification event and propagation condition is as follows:

- Assume that a variable implementation uses n different modification events (including the mandatory ones). The size of n does not affect efficiency. To represent these modification events, the Gecode kernel reserves  $\lceil \log_2(n-1) \rceil$  bits in each propagator for maintaining modification event deltas, see Section 23.4.
  - The totally available number of bits for all variable implementation types used by Gecode is 32 (independent of whether Gecode is run on a 32 bit or 64 bit platform). That is, if we assume less than ten modification events per variable implementation type, the Gecode kernel can support at least ten different variable implementation types.<sup>1</sup>
- The number of different propagation conditions *m* per variable implementation type is only limited by the largest value of an unsigned integer in C<sup>++</sup>.
  - For each propagation condition, every variable implementation needs a 32-bit word, that is a variable implementation requires at least O(m) space (which is typically dwarfed by the space consumed for actually storing the subscriptions of a propagator or an advisor to a variable implementation).
  - Subscribing to a variable implementation requires O(m) time. Canceling a subscription with propagation condition p requires O(m + k) time, where k is the number of subscriptions with propagation condition p.

#### 31.2 Base definitions

The variable implementation base class together with definitions of modification events and propagation conditions are not programmed but are generated from a simple specification file. The specification contains three sections: a general section for naming, a section for modification events, and a section for propagation conditions.

In the following we describe how to turn the parts of the design from the previous section that is concerned with modification events and propagation conditions into the specification. The member functions of the generated base class are used and explained in the next section.

<sup>&</sup>lt;sup>1</sup>If you ever exceed this limit, please let us know. Adding more bits is easy, even though we do not expect that to happen anytime soon.



Figure 31.1: Variable implementation specification

General section. The specification file (named int.vis, where vis stands for variable implementation specification; however the file extension does not matter) is shown in Figure 31.1. The specification file must start with [General] defining the start of the general section and must end with a line [End]. The Name option defines the names of the entities to be generated. In our example, a variable implementation base class IntVarImpBase (that is, the specified name is prepended to VarImpBase) generated, the identifiers for modification events start with ME\_INT\_ (that is, the specified name is put after the ME\_ in capital letters), and the identifiers for propagation conditions start with PC\_INT\_. All these definitions are contained within the namespace as defined by the Namespace option.

The general section (and also the other sections discussed below) supports additional specification options, see Section 31.4 for a summary and a specification file template for download.

**Modification event section.** Every modification event requires a definition that is preceded by a line containing [ModEvent] as shown in Figure 31.2. The option Name defines the name of the modification event (in fact, just the part after ME\_INT\_ for our example). The values on the right-hand side of = specify that some modification events are special:

- The modification events named FAILED (that is, ME\_INT\_FAILED) and NONE (that is, ME\_INT\_NONE) are defined to be the events for failure (=FAILED) and no change (=NONE).
- The modification event named VAL is defined to be the event when a variable implementation becomes assigned (=ASSIGNED) to a value.
- The modification event named BND is defined to be used for scheduling a propagator when the propagator subscribes to a non-assigned variable implementation (=SUBSCRIBE).

Any variable implementation must define special events with =NONE, =FAILED, and =ASSIGNED.

The section for modification events also defines how modification events are combined with a Combine option. The combination of modification events is needed for the correctness

MODIFICATION EVENTS ≡ [ModEvent] Name: FAILED=FAILED [ModEvent] Name: NONE=NONE [ModEvent] Name: VAL=ASSIGNED Combine: VAL=VAL, MIN=VAL, MAX=VAL, BND=VAL [ModEvent] Name: BND=SUBSCRIBE Combine: VAL=VAL, MIN=BND, MAX=BND, BND=BND [ModEvent] Name: MIN Combine: VAL=VAL, MIN=MIN, MAX=BND, BND=BND [ModEvent] Name: MAX Combine: VAL=VAL, MIN=BND, MAX=MAX, BND=BND

Figure 31.2: Modification event section

of scheduling propagators and also for modification event deltas, see Section 23.4. An entry l = r for the modification event m defines that m combined with l is r.

The definition of the combination of modification events can be expressed as a table:

	VAL	MIN	MAX	BND
VAL	VAL	VAL	VAL	VAL
MIN	VAL	MIN	BND	BND
MAX	VAL	BND	MAX	BND
BND	VAL	BND	BND	BND

This table is exactly what is specified by the Combine options. The special modification events NONE and FAILED do not have a Combine option.

We will not present the full mathematical detail of the properties that must hold for the combination of modification events, the theory is presented in [44, Section 5.5].

**Propagation condition section.** Every propagation condition requires a definition that is preceded by a line containing [**PropCond**] as shown in Figure 31.3. The option Name defines the name of the propagation condition (in fact, just the part after PC\_INT\_ for our example). The values on the right-hand side of = specify that some propagation conditions are special:

■ The propagation condition named NONE (that is, PC\_INT\_NONE) is defined to be the propagation condition for not creating any subscription (=NONE).

```
PROPAGATION CONDITIONS ≡
  [PropCond]
  Name:
               NONE=NONE
  [PropCond]
  Name:
               VAL=ASSIGNED
  ScheduledBy: VAL
  [PropCond]
               BND
  Name:
  ScheduledBy: VAL, BND, MIN, MAX
  [PropCond]
  Name:
               MIN
  ScheduledBy: VAL, BND, MIN
  [PropCond]
  Name:
               MAX
  ScheduledBy: VAL, BND, MAX
```

Figure 31.3: Propagation condition section

■ The propagation condition named VAL (that is, PC\_INT\_VAL) is defined to be the condition when a propagator wants to subscribe to the event that a variable implementation becomes assigned (=ASSIGNED).

Any variable implementation must define the special propagation conditions =NONE and =ASSIGNED.

For each propagation condition (but for =NONE), it must be defined by a ScheduledBy option which modification events schedule a propagator for execution. That is, when defining a propagation condition p, an entry m in the list of modification events defines the following: a propagator subscribed to a variable implementation x with propagation condition p is scheduled for execution when a modification operation on x returns the modification event x. The modification events in our example correspond to the design presented in Section 31.1.

#### 31.3 Variable implementation

The variable implementation for integer interval variables is shown in Figure 31.4. As discussed in the previous sections, the variable implementation inherits from the generated base class IntVarImpBase and implements a lower bound l and an upper bound u.

**Access operations.** Every variable implementation must implement a member function assigned() that tests whether the variable is assigned to a value:

```
VARIABLE IMPLEMENTATION ≡
  namespace MPG { namespace Int {
    ► LIMITS
    ► DELTA FOR ADVISORS
    class IntVarImp : public IntVarImpBase {
    protected:
      int l, u;
    public:
      IntVarImp(Space& home, int min, int max)
        : IntVarImpBase(home), l(min), u(max) {}
      ► ACCESS OPERATIONS
      ► ASSIGNMENT TEST
      ► MODIFICATION OPERATIONS
      ► SUBSCRIPTIONS
      ► COPYING
      ▶ DELTA INFORMATION
    };
  }}
```

Figure 31.4: Variable implementation

```
ASSIGNMENT TEST ≡
bool assigned(void) const {
  return l == u;
}
```

The test for assignment is used in the implementation of other member functions of the variable implementation. Furthermore, variables and views automatically provide implementations of a member function assigned() that calls the assigned() function of their variable implementation.

The access operations for the lower and upper bound are straightforward. Here, and in the following, we only show one of the operations, the operation for the other bound is analogous:

```
ACCESS OPERATIONS ≡

int min(void) const {

return l;
}
...
```

**Modification operations.** The modification operations must notify the Gecode kernel if a variable implementation is modified. As a description how a variable implementation changes, they must pass a modification event and delta information for advisors to a member function notify(). The notify() function executes subscribed advisors and schedules subscribed propagators (depending on the passed modification event and the propagators' propagation conditions). The notify() function is inherited from the generated variable implementation base class and depends on the specified modification events and propagation conditions.

The delta information is implemented as discussed in Section 31.1 as an interval with lower and upper bound:

```
class IntDelta : public Delta {
  private:
    int l, u;
  public:
    IntDelta(int min, int max) : l(min), u(max) {}
    int min(void) const {
      return l;
    }
    ...
};
```

The actual modification operations first test whether the variable implementation does not require modification or whether the operation fails and only then perform the actual modification. Before updating the upper bound u to n, the lq() operation creates the delta information d that describes that values between n+1 and u are being removed.

The notify() function is given the home space, a modification event, and the variable delta d as argument. The modification event passed to notify() must capture how the domain has changed. In particular, it must reflect whether the variable implementation has been assigned. The notify() function executes the advisors subscribed to this variable implementation and schedules all subscribed propagators with appropriate propagation conditions. Note that the notify() function returns a modification event. In case an advisor reports failure after its execution, notify() returns ME\_INT\_FAILED. Otherwise it returns the modification event that has been passed as argument:

```
MODIFICATION OPERATIONS =
  ModEvent lq(Space& home, int n) {
   if (n >= u) return ME_INT_NONE;
   if (n < l) return ME_INT_FAILED;
   IntDelta d(n+1,u); u = n;
   return notify(home, assigned() ? ME_INT_VAL : ME_INT_MAX, d);
}
...</pre>
```

**Tip 31.2** (Variable implementations must always be consistent). Even if a modification operation fails, the data structures for the variable implementation must be still consistent. That is, all operations must still work. See also Section 4.1.3. ◀

**Delta information access.** The variable implementation must also implement functions that provide access to the delta information:

```
DELTA INFORMATION ≡
  static int min(const Delta& d) {
   return static_cast<const IntDelta&>(d).min();
  }
  ...
```

This construction appears nonsensical at first sight, however there are two good reasons why a variable implementation interprets the information stored in the delta information (of course, in that case one would have to declare the operation as **const** but not **static**). First, the variable implementation can change the information based on its own state. Second, the very same idea is needed for views (see Section 33.4.2 for an example) and hence this design keeps the interfaces of views and variable implementations as similar as possible.

**Subscriptions.** A variable implementation must implement subscribe() and cancel() operations for both propagators and advisors. The implementation of these operations always follow the same structure as shown below.

The reason why these functions have to be implemented in the variable implementation class even though they are (in slightly different form) already defined in the variable implementation base class is that they require information about whether a variable implementation is assigned. The definitions are as follows:

**Copying during cloning.** Copying a variable implementation during cloning is implemented by a constructor and a copy() function. The constructor is straightforward and the copy() function only creates a new variable implementation if the variable implementation has not been copied before. If it has been copied before (that is, copied() returns **true**), the copy() function must return the forwarding pointer to the previously created copy as follows:

```
COPYING =
  IntVarImp(Space& home, bool share, IntVarImp& y)
    : IntVarImpBase(home, share, y), l(y.l), u(y.u) {}
  IntVarImp* copy(Space& home, bool share) {
    if (copied())
      return static_cast<IntVarImp*>(forward());
    else
      return new (home) IntVarImp(home, share, *this);
}
```

Additional inherited member functions. In addition to the constructor and the member functions defined and used by our variable implementation, several other member functions are typically just inherited and are defined by the class VarImp. The most important inherited member functions are summarized in Figure 31.5. For an explanation of degree and accumulated failure count, see Section 7.1.1.

# access operations degree() returns degree (number of subscriptions) afc() returns accumulated failure count scheduling support schedule() schedule propagator modification event deltas me() extract modification event med() construct modification event delta delta information access modevent() return modification event from delta

Figure 31.5: Summary of member functions predefined by variable implementations

#### 31.4 Additional specification options

This section provides an overview of additional specification options not discussed in Section 31.2.

**Comments.** Any line starting with # is discarded and hence can serve as a comment in the specification file.

**Generating headers, footers, and comments.** Any text after the options for a **[ModEvent]** and **[PropCond]** definition until the next definition is added to the generated C++-code before the generated identifier definition. This can be used for defining comments to be added to the generated C++-code. For example, by

```
[PropCond]
Name: NONE=NONE
// Propagation condition to be ignored
[PropCond]
...
```

the comment

```
// Propagation condition to be ignored
```

is put before the definition of the generated propagation condition.

Related support exists for putting a header before (or a footer after) all generated definitions for modification events and propagation conditions: The text following [ModEventHeader], [ModEventFooter], [PropCondHeader], and [PropCondFooter] is inserted at the respective places in the generated code.

**Conditional compilation.** Giving an option Ifdef in the general section followed by some C++-preprocessor identifier IDENT wraps the entire generated code in preprocessor directives as follows:

```
#ifdef IDENT
...
#endif
```

By this, the Gecode kernel can be compiled with or without a particular variable type without being forced to reconfigure the Gecode kernel, see also Section 34.2.

**Explicitly disposing variable implementations.** Our example variables are entirely spaceallocated and do not require external memory or other resources. However, for some variable types, the variable implementation might use external resources or memory that is not spaceallocated and must explicitly be freed.

For an example, suppose the integer interval variables had been implemented by using arbitrary precision integers for the lower and upper bound and that these bounds must explicitly be freed.

By specifying in the general section

```
Dispose: true
```

and implementing in the variable implementation class a dispose(Space& home) member function, all variable implementations are disposed by calling their dispose() functions. The variable implementations are disposed when their home space is deleted.

Additionally, an object must be created that controls the disposal of variable implementations. Assume that our example integer variables used external memory and that its specification file contains

```
Dispose: true
```

and that MPG::IntVarImp implements a dispose() function. Then, your program must create a variable implementation disposer as follows:

```
Gecode::VarImpDisposer<MPG::IntVarImp> disposer;
```

The disposer object must be initialized before the first variable using MPG::IntVarImp is created.

**Reserving bits.** A limited number of bits *b* can be reserved within each variable implementation by specifying in the general section

```
Bits: b
```

Then, the variable implementation can get a reference to a value of type **unsigned int** by calling the member function bits() where the least b bits can be used freely. However, the maximal number of subscriptions (both propagators and advisors) for that variable implementation type is reduced from  $2^{31} - 1$  to  $2^{31-b} - 1$ . Furthermore, any attempt to use more than the specified number of bits will crash Gecode in a truly spectacular fashion!

**Specification file template.** The specification template contains all possible specification options to assist in defining your own variable types.

### **32**

## Variables and variable arrays

This chapter describes how variables can be programmed from variable implementations and how variable arrays and variable argument arrays can be programmed. The chapter uses integer interval variables as introduced in Chapter 30 together with their implementations as defined in Chapter 31 as its running example.

**Overview.** How integer interval variables are implemented is detailed in Section 32.1. Variable arrays and variable argument arrays are discussed in Section 32.2.

#### 32.1 Variables

As a variable is just a read-only interface to a variable implementation, its implementation is straightforward. The definition of integer variables is shown in Figure 32.1. The copy constructor uses the member function varimp() that returns the pointer to the variable's variable implementation.

Note that within the class IntVar, a pointer to the corresponding variable implementation is available as protected member x (see Tip 25.1 for information on **using**).

One also must define an output operator << for a variable as shown in Figure 32.1.

It is important to remember that variables are defined in the namespace MPG. This is in contrast to variable implementations, which are defined in the namespace MPG::Int.

**Variable creation.** Creating a new variable is done with the following constructor that creates a new variable implementation as follows:

```
VARIABLE ≡
 namespace MPG {
   class IntVar : public VarImpVar<Int::IntVarImp> {
   protected:
      using VarImpVar<Int::IntVarImp>::x;
   public:
      IntVar(void) {}
      IntVar(const IntVar& y)
        : VarImpVar<Int::IntVarImp>(y.varimp()) {}
      ► VARIABLE CREATION
      ► ACCESS OPERATIONS
   };
   template<class Char, class Traits>
    std::basic_ostream<Char,Traits>&
   operator <<(std::basic_ostream<Char,Traits>& os, const IntVar& x) {
   }
 }
```

Figure 32.1: Variable programmed from a variable implementation

## access operations varimp() returns pointer to variable implementation assigned() whether variable is assigned degree() returns degree (number of subscriptions) afc() returns accumulated failure count update during cloning update() updates variable during cloning

Figure 32.2: Summary of member functions predefined by variables

Note that the constructor ensures the invariants for the lower and upper bound of a variable as discussed in Section 31.1 by possibly throwing exceptions.

**Access operations.** In addition to constructors, variables typically implement the same access operations as their corresponding variable implementation:

```
ACCESS OPERATIONS ≡
int min(void) const {
   return x->min();
}
...
```

Additional inherited member functions. In addition to the constructor and member functions defined by our variables, several other member functions are typically just inherited and are defined by the class VarImpVar. The most important inherited member functions are summarized in Figure 32.2. For an explanation of degree and accumulated failure count, see Section 7.1.1.

#### 32.2 Variable arrays and variable argument arrays

Defining variable arrays and variable argument arrays (see also Section 4.2) requires the implementation of the arrays proper together with some traits. The traits classes for variable arrays and variable argument arrays ensure that Gecode-provided functionality for arrays can be used with the newly defined arrays.

**Array traits.** The definition of the array traits classes is shown in Figure 32.3. The definition is done in two steps. The first step provides forward declarations of the array types IntVarArgs and IntVarArray in the namespace MPG (because that is where these arrays will be defined).

```
ARRAY TRAITS ≡
 namespace MPG {
   class IntVarArgs; class IntVarArray;
 }
 namespace Gecode {
   template<>
   class ArrayTraits<Gecode::VarArray<MPG::IntVar> > {
   public:
     typedef MPG::IntVarArray StorageType;
     typedef MPG::IntVar ValueType;
     typedef MPG::IntVarArgs ArgsType;
   };
   template<>
   class ArrayTraits<MPG::IntVarArray> {
   };
   template<>
   class ArrayTraits<Gecode::VarArgArray<MPG::IntVar> > {
   public:
     typedef MPG::IntVarArgs StorageType;
     typedef MPG::IntVar ValueType;
     typedef MPG::IntVarArgs ArgsType;
   template<>
   class ArrayTraits<MPG::IntVarArgs> {
   };
 }
```

Figure 32.3: Array traits for variable arrays

```
VARIABLE ARRAYS ≡
  namespace MPG {
    class IntVarArgs : public VarArgArray<IntVar> {
    public:
      IntVarArgs(void) {}
      explicit IntVarArgs(int n) : VarArgArray<IntVar>(n) {}
      IntVarArgs(const IntVarArgs& a) : VarArgArray<IntVar>(a) {}
      IntVarArgs(const VarArray<IntVar>& a) : VarArgArray<IntVar>(a) {}
      IntVarArgs(Space& home, int n, int min, int max)
        : VarArgArray<IntVar>(n) {
        for (int i=0; i<n; i++)
          (*this)[i] = IntVar(home,min,max);
     }
    };
    class IntVarArray : public VarArray<IntVar> {
    public:
      IntVarArray(void) {}
      IntVarArray(const IntVarArray& a)
        : VarArray<IntVar>(a) {}
      IntVarArray(Space& home, int n, int min, int max)
      }
   };
  }
```

Figure 32.4: Variable arrays

The second step requires to define traits for these two array types. The trait classes must be defined in the namespace Gecode. For each array type, two traits classes are needed: one for the base class (for example, Gecode::VarArray<MPG::IntVar>) and one for the class to be implemented (for example, MPG::IntVarArray). The definitions for the array type and its base class must be identical and follow the examples shown in Figure 32.3.

**Variable arrays.** The implementation of variable arrays and variable argument arrays typically only require the implementation of various constructors when inheriting from the base classes VarArray and VarArgArray. The minimal set of constructors such that the arrays are compatible to arrays as used by Gecode is shown in Figure 32.4.

## 33

#### Views

This chapter describes how views as needed for programming propagators and branchers can be programmed. The chapter uses integer interval variables as introduced in Chapter 30 together with their implementations as defined in Chapter 31 as its running example.

**Overview.** Section 33.1 provides an overview of the different types of views available in Gecode. The remaining sections provide examples for each different view type: Section 33.2 shows how an integer view IntView is constructed as a variable implementation view; Section 33.3 shows how a constant integer view ConstIntView is programmed as a constant view; Section 33.4 shows how a minus view MinusView and an offset view OffsetView are programmed as derived views.

#### 33.1 View types

Gecode provides three different types of views:

- Variable implementation views: a variable implementation view is nothing but a direct interface to a variable implementation. A variable implementation view must inherit from VarImpView. The class VarImpView is parametric with respect to a variable and not a variable implementation (as one might expect). This is due to the fact that the type of the variable implementation can be obtained automatically from the type of a variable. Making a variable implementation view parametric with respect to a variable type has the advantage that information on both the variable type and variable implementation type become available.
- Constant views: a constant view must implement the same interface and must perform the same operations as some assigned variable implementation view. This particular variable implementation view is called the *corresponding* variable implementation view. A constant view must inherit from ConstView which is parametric with respect to the corresponding variable implementation view.
- *Derived views*: a derived view is a view that is implemented in terms of some other view (all view types are possible: variable implementation, constant, and derived). The view from which the derived view is derived, is called the *base* view. A derived view must inherit from <code>DerivedView</code> which is parametric with respect to the base view.

#### access operations returns pointer to variable implementation varimp() assigned() whether variable is assigned returns degree (number of subscriptions) degree() afc() returns accumulated failure count subscriptions subscribe() subscribe propagator/advisor cancel propagator/advisor cancel() scheduling support schedule propagator schedule() modification event deltas extract modification event me() construct modification event delta med() delta information access return modification event from delta modevent() update during cloning updates view during cloning update()

Figure 33.1: Summary of member functions predefined by views

**Predefined member functions.** The classes VarImpView, ConstView, and DerivedView define already many member functions that simplify the implementation of new views. The most important predefined member functions are summarized in Figure 33.1.

Note that the varimp() function for a constant view or for a view derived from a constant view returns NULL, as no variable implementation exists.

**View test functions.** There are three different functions predefined for views:

- The function shared(x,y) returns **true**, if both views x and y share a common variable implementation (see Section 25.1.2). Typically, the definition of shared() does not need to be overloaded for newly defined views.
- The function same(x,y) returns **true**, if both views x and y are identical (see Section 25.1.2). In some cases, the definition of same() must be overloaded for newly defined views (see Section 33.3 and Section 33.4.2 for examples).
- The function before(x,y) returns **true**, if x comes before y in some arbitrary total and strict order for ordering views. The function is mainly used for sorting arrays of

```
INTEGER VIEW ≡
  namespace MPG { namespace Int {
    class IntView : public VarImpView<IntVar> {
    protected:
      using VarImpView<IntVar>::x;
    public:
      IntView(void) {}
      IntView(const IntVar& y)
        : VarImpView<IntVar>(y.varimp()) {}
      IntView(IntVarImp* y)
        : VarImpView<IntVar>(y) {}
      ► ACCESS OPERATIONS
      ► MODIFICATION OPERATIONS
      ▶ DELTA INFORMATION
    };
    template<class Char, class Traits>
    std::basic_ostream<Char,Traits>&
    operator<<(std::basic_ostream<Char,Traits>& os, const IntView& x) {
    }
  }}
```

Figure 33.2: Integer view

views into some order (in particular for detecting duplicate views). In some cases, the definition of before() must be overloaded for newly defined views (see Section 33.3 and Section 33.4.2 for examples).

**Output operator.** For every view also an output operator << must be defined. We sketch this only for integer views in Section 33.2, for all other views the definition is analogous.

#### 33.2 Variable implementation views: integer view

Figure 33.2 shows the definition of the class IntView for integer views from the class VarImpView for variable implementation views. Please remember that a variable implementation view is parametric with respect to a variable type (IntVar in our example, such that IntView uses the same variable implementation type IntVarImp as IntVar does).

Similar to variables obtained from variable implementations, a variable implementation

view has a protected member x that is a pointer to its variable implementation (see Tip 25.1 for information on **using**). A variable implementation view must implement at least the shown constructors such that it can be initialized both from the corresponding variable type and from the corresponding variable implementation type.

The remaining implementation tasks for variable implementation views are straightforward: all operations that are specific to a variable type (in our case, specific to integer interval variables) must be implemented. The implementation is straightforward as only the corresponding operations of the variable implementation are invoked:

■ The access operations must be implemented:

```
ACCESS OPERATIONS ≡

int min(void) const {

return x->min();
}
...
```

• The modification operations must be implemented:

```
MODIFICATION OPERATIONS ≡

ModEvent lq(Space& home, int n) {
   return x->lq(home,n);
}
...
```

• Finally, the operations for accessing delta information must be implemented:

```
DELTA INFORMATION ≡
  int min(const Delta& d) const {
   return IntVarImp::min(d);
}
...
```

#### 33.3 Constant views: constant integer view

Figure 33.3 shows the implementation of a constant integer view with IntView as the corresponding variable implementation view. A constant integer view ConstIntView stores an integer value x and must implement all variable-specific operations that are implemented by the corresponding IntView class (as shown in Figure 33.3).

Slightly less obvious is the implementation of operations that access delta information. While these operations must be implemented such that constant integer views can be used instead of integer views, they will never be executed (by definition, a constant view can never change). Hence we use the macro GECODE\_NEVER (see Tip 28.1) to clarify that the delta information operations are never executed:

```
CONSTANT INTEGER VIEW ≡
  namespace MPG { namespace Int {
    class ConstIntView : public ConstView<IntView> {
    protected:
      int x;
    public:
      ConstIntView(void) : x(0) {}
      ConstIntView(int n) : x(n) {}
      int min(void) const {
        return x;
      }
      ModEvent lq(Space& home, int n) {
        return (x <= n) ? ME_INT_NONE : ME_INT_FAILED;</pre>
      }
      ▶ DELTA INFORMATION
      ▶ UPDATE DURING CLONING
    };
    ► VIEW TESTS
  }}
```

Figure 33.3: Constant integer view

```
DELTA INFORMATION ≡
  int min(const Delta& d) const {
   GECODE_NEVER; return 0;
}
...
```

**Update during cloning.** The definition of the update() member function of ConstView does not take care of the integer value x. Hence we need to provide a new update() function that updates the value of x as follows:

```
UPDATE DURING CLONING =
  void update(Space& home, bool share, ConstIntView& y) {
   ConstView<IntView>::update(home, share, y);
   x = y.x;
}
```

**View tests.** Also the default definitions of the view test functions same() and before() for constant views do not take the integer value x of the view into account. Overloaded versions for constant integer views are as follows:

```
VIEW TESTS ≡
  inline bool same(const ConstIntView& x, const ConstIntView& y) {
    return x.min() == y.min();
}
inline bool before(const ConstIntView& x, const ConstIntView& y) {
    return x.min() < y.min();
}</pre>
```

#### 33.4 Derived views

This section exemplifies two different derived views: minus views and offset views. Why these views are useful and what their semantics is can be seen in Section 25.1.1 for minus views and in Section 25.1.2 for offset views.

#### 33.4.1 Minus view

Figure 33.4 shows that a minus view is derived from an integer view IntView. The protected member x refers to the base view, that is the integer view from which the minus view is

```
MINUS VIEW ≡
  namespace MPG { namespace Int {
    class MinusView : public DerivedView<IntView> {
    protected:
      using DerivedView<IntView>::x;
      ► MODIFICATION EVENTS AND PROPAGATION CONDITIONS
    public:
      MinusView(void) {}
      explicit MinusView(const IntView& y)
        : DerivedView<IntView>(y) {}
      ► ACCESS OPERATIONS
      ► MODIFICATION OPERATIONS
      ► SUPPORT OPERATIONS
      ► SUBSCRIPTIONS
      ▶ DELTA INFORMATION
    };
  }}
```

Figure 33.4: Minus view

```
MODIFICATION EVENTS AND PROPAGATION CONDITIONS ≡

static ModEvent minusme(ModEvent me) {
    switch (me) {
    case ME_INT_MIN: return ME_INT_MAX;
    case ME_INT_MAX: return ME_INT_MIN;
    default: return me;
    }
}
static PropCond minuspc(PropCond pc) {
    ...
}
```

Figure 33.5: Negation of modification events and propagation conditions

derived (see Tip 25.1 for information on **using**).

**Access operations.** The access operations are as to be expected for a minus view. That is, the lower bound of the derived view is the negation of the upper bound of the base view:

```
ACCESS OPERATIONS ≡
  int min(void) const {
   return -x.max();
}
...
```

**Modification operations.** The modification operations are slightly more involved than the access operations as they return a modification event. If the modification event of the base view is ME\_INT\_MAX (the upper bound of the base view has changed), then the modification event for the derived view must be ME\_INT\_MIN (the lower bound of the derived view has changed).

Figure 33.5 shows functions minusme() and minuspc() that return the negation of modification events and propagation conditions (to be discussed later).

Using the function minusme, the modification operations can be defined as follows:

```
MODIFICATION OPERATIONS ≡

ModEvent lq(Space& home, int n) {
   return minusme(x.gq(home,-n));
}
...
```

**Accessing delta information.** Accessing delta information must also take into account that the modification event stored in a delta must be converted with minusme(). Also the other operations for accessing delta information must be adopted accordingly:

```
static ModEvent modevent(const Delta& d) {
   return minusme(IntView::modevent(d));
}
int min(const Delta& d) const {
   return -x.max(d);
}
...
```

Additional operations. Any operation that is concerned with either modification events or propagation conditions must be implemented to take the switch between lower bound and upper bound into account. These operations include the operations for handling subscriptions of propagators (the function minuspc() is defined analogously to minusme() in Figure 33.5):

Note that the operations that subscribe and cancel advisors must be re-implemented even though they are unchanged. This is due to inheritance in C++: as the overloaded functions for propagators are redefined, also the functions for advisors are considered to be redefined. The remaining operations to be implemented are support operations:

```
support Operations =
  static void schedule(Space& home, Propagator& p, ModEvent me) {
    return IntView::schedule(home,p,minusme(me));
  }
  static ModEvent me(const ModEventDelta& med) {
    return minusme(IntView::me(med));
  }
  static ModEventDelta med(ModEvent me) {
    return IntView::med(minusme(me));
  }
```

#### 33.4.2 Offset view

Figure 33.6 shows that an offset view is derived from an integer view IntView and stores an additional integer value c for the offset. The protected member x refers to the base view,

```
OFFSET VIEW ≡
  namespace MPG { namespace Int {
    class OffsetView : public DerivedView<IntView> {
    protected:
      using DerivedView<IntView>::x;
      int c;
    public:
      OffsetView(void) {}
      OffsetView(const IntView& y, int d)
        : DerivedView<IntView>(y), c(d) {}
      int offset(void) const {
        return c;
      int min(void) const {
        return x.min()+c;
      }
      ModEvent lq(Space& home, int n) {
        return x.lq(home,n-c);
      }
      int min(const Delta& d) const {
        return x.min(d)+c;
      }
      ► UPDATE DURING CLONING
    };
    ► VIEW TESTS
  }}
```

Figure 33.6: Offset view

that is the integer view from which the offset view is derived (see Tip 25.1 for information on **using**). The access, modification, and delta information access operations of an offset view are as to be expected.

The update() function must also update the integer offset c as follows:

```
UPDATE DURING CLONING ≡
  void update(Space& home, bool share, OffsetView& y) {
    x.update(home, share, y.x);
    c=y.c;
}
```

Likewise, the view test functions same() and before() must take into account the integer offset c:

```
VIEW TESTS ≡
inline bool same(const OffsetView& x, const OffsetView& y) {
   return same(x.base(),y.base()) && (x.offset() == y.offset());
}
inline bool before(const OffsetView& x, const OffsetView& y) {
   return before(x.base(),y.base())
   || (same(x.base(),y.base()) && (x.offset() < y.offset()));
}</pre>
```

## 34

## Putting everything together

This chapter finally explains how integer interval variables can be used with Gecode.

**Overview.** Section 34.1 sketches an example script together with implementations of constraints and branchings using integer interval variables. The following section, Section 34.2, shows how Gecode can be configured to use integer interval variables and how to compile and run the example script.

**Important.** Please make sure to carefully read Section 2.6.2, before reading any further in this chapter!

#### 34.1 Golomb rulers à la integer interval variables

Figure 34.1 shows the top-level structure of a single C++-file containing a script together with all required implementations of post functions, propagators, and branchers. The example script implements a naive version of the Golomb ruler model presented in Chapter 10. The reason to package everything into a single C++-file is to simplify compiling the example.

The implementations of the constraints in the C++-file are carefully constructed to exercise most of the functionality described in the previous chapters in this part. In particular, some constraints have a slightly non-standard implementation to exercise all views presented in Chapter 33.

#### 34.2 Configuring and compiling Gecode

The following steps configure and compile Gecode with integer interval variables:

- 1. Start a shell.
- 2. Create a new directory, say MPG, and make it the current directory:

mkdir MPG; cd MPG

3. Download a Gecode 3.7.3 source package or check-out Gecode 3.7.3 from svn. We assume that the Gecode source code is contained in a directory named gecode-3.7.3.

Figure 34.1: Golomb rulers à la integer interval variables

- 4. If you have not yet done so, download and copy all files required for integer interval variables and the example into the current directory:
  - The header file int.hh containing the implementation of integer interval variables.
  - The variable implementation specification file int.vis.
  - The file putting-everything-together.cpp from the previous section.
- 5. Configure Gecode to incorporate integer interval variables:

```
cd gecode-3.7.3
./configure --with-vis=../int.vis
```

After this step, Gecode has been configured to incorporate the generated definitions as described by the specification file int.vis. If you need to pass other options to configure to successfully build Gecode, please do so.

6. Compile Gecode and leave the directory

```
make;cd ..
```

7. Set the PATH environment variable to point to the just compiled Gecode installation:

export PATH="gecode-3.7.3:\$PATH"

Depending on the platform you use, you might also have to set the environment variable LD\_LIBRARY\_PATH accordingly.

Finally: compile, link, and run the example script putting-everything-together.cpp as described in Section 2.3, where you need to make sure that the directory for include files and library files is gecode-3.7.3.

S

#### Programming search engines

**Christian Schulte** 

This part shows how to program search engines in Gecode.

Chapter 35 (Getting started) presents how to implement simple search engines, where the focus is on understanding the basic operations available on spaces to implement search engines. Chapter 36 (Recomputation) explains recomputation as the most essential technique for efficient search in Gecode. Chapter 37 (An example engine) puts all techniques for search and recomputation from Chapter 35 and Chapter 36 together and presents a realistic search engine.

# 35

# **Getting started**

This chapters presents how to implement simple search engines. The focus is on understanding the basic operations available on spaces to implement search engines. None of the engines presented here is realistic as they do not use recomputation. The full picture is developed in Chapter 36 and Chapter 37.

**Overview.** Section 35.1 sets the stage by explaining space operations for programming search engines. A depth-first search engine that makes the simplifying assumption that all choices explored during search are binary is shown in Section 35.2. The next section, Section 35.3, shows depth-first search for choices with an arbitrary number of alternatives. How best solution search can be programmed from spaces is exemplified by a simple branch-and-bound search engine in Section 35.4.

## 35.1 Space-based search

Search engines compute with spaces: a space implements a constraint model and exploration of its search space is implemented by operation on spaces. The operations on spaces include: computing the status of a space by the status() function, creating a clone of a space by the clone() function, and committing to an alternative of a choice by the commit() function. To commit to an alternative, a space provides the function choice() that returns a choice defining how the space can be committed to one of its alternatives. Another operation required to program exploration is the function alternatives() defined by a choice that returns the number of alternatives of a choice.

Spaces implement also a constrain() function for best solution search. Its discussion is postponed to Section 35.4.

This section reviews the above operations from the perspective of a search engine, the perspective how branchers are controlled by these operations is detailed in Section 28.1. Gecode's architecture for search is designed such that a search engine does not need to know *which* problem is being solved by a search engine: any problem implemented with spaces can be solved by a search engine, and different search engines can be used for solving the same problem. The basic idea of this factorization is due to [33].

Note that here and in the following, spaces and choices are always assumed to be *pointers* to the respective objects. Pointers are necessary as search engines dynamically create and delete spaces and choices.

**Status computation.** A search engine needs to decide how to proceed during search by computing the *status* of a space by invoking its status() function. The status() function performs constraint propagation (see Section 20.1) followed by determining the next brancher for branching, if possible (see Section 28.1). Depending on the result of constraint propagation and brancher selection, the status() function returns one of the following values of the type SpaceStatus (see Programming search engines):

■ SS\_FAILED: the space is *failed*. The search engine needs to backtrack and revisit other spaces encountered during exploration.

An important responsibility of a search engine is to perform resource management for spaces. In the case of failure, the typical action is to delete the failed space.

■ SS\_SOLVED: the space is *solved*. Hence the search engine has found a solution and typically returns the solution.

For most engines, the responsibility for deleting a solution lies with the user of a search engine.

Following the discussion in Section 28.1, calling the choice() function of a solved space performs garbage collection for branchers that are not any longer needed. Section 35.2 shows an example search engine that performs garbage collection on solved spaces.

• SS\_BRANCH: the space requires branching for search to proceed.

The first step in branching is to compute a choice by calling the choice() function of a space. The returned choice can be used for committing to alternatives of a space. In particular, a choice returned by the choice() function provides a function alternatives() that returns how many alternatives the choice has.

The pointer to the choice that is returned by the choice() function of a space s is **const**. That is, the following code:

```
const Choice* ch = s->choice();
```

gets a **const** pointer to a choice (the choice cannot be modified). Note that it is the obligation of the search engine to eventually delete the choice by

```
delete ch;
```

**Cloning spaces.** A central requirement for a search engine is that it can return to a previous state: as spaces constitute the nodes of the search tree, a previous state is nothing but a space again. Returning to a previous space might be necessary because an alternative suggested by a branching did not lead to a solution, or, even if a solution has been found, more solutions might be requested.

As propagation and branching modify spaces, provisions must be taken that search can actually return to the clone of a previous space. This is provided by the clone() function of a space: it returns a clone of a space. This clone can be stored by a search engine such that the engine can return to a previous state. Spaces that are clones of each other are *equivalent*: space operations will have exactly the same effect on equivalent spaces.

The clone() function of a space can only be called on a space that is stable and not failed (that is, the status() function on a space must return SS\_SOLVED or SS\_BRANCH). Otherwise, Gecode throws an exception of type SpaceNotStable if the space is not stable and of type SpaceFailed if the space is failed.

**Committing to alternatives.** Given a space s and a choice ch (assumed to be a **const** pointer), the space s can be committed to the i-th alternative by calling the commit() function of a space as follows:

```
s->commit(*ch,i);
```

The choice ch must be *compatible* with the space s. Before defining when a choice is compatible with a space, let us look at two examples.

Suppose a search engine has invoked status() on a space s which returned SS\_BRANCH. The next step is to obtain a choice ch for s and a clone c of s by:

```
const Choice* ch = s->choice();
Space* c = s->clone();
```

Further assume that the choice is binary (that is, ch->alternatives() returns 2). A search engine can explore both alternatives (typically, the search engine performs the commit() for the second<sup>1</sup> alternative much later) by:

```
s->commit(*ch,0);
c->commit(*ch,1);
```

That is, a choice ch is compatible with the space s from which it has been computed and with the clone c of s.

A search engine for best solution search performs slightly different operations. Let us follow an example scenario. First, the search engine starts exploring the first alternative by:

```
s->commit(*ch,0);
```

Then search continues with s. Let us assume that the search engine finds a better solution when continuing search from s. Hence, the search engine adds additional constraints to the clone c to make sure that exploration from c yields a better solution (the constraints are added by calling the constrain() function of a space, see Section 35.4). And only then the search engine commits the clone c to the second alternative by:

<sup>&</sup>lt;sup>1</sup>Even though the alternatives are numbered starting from 0 we refer to the alternative with number 0 as the first alternative and the alternative with number 1 as the second alternative.

```
c->commit(*ch,1);
```

That is, a choice ch is also compatible with the clone c of s, even though additional constraints have been added to c after it had been created by cloning.

In fact, the relation that a choice is compatible with a space is quite liberal. The full notion of compatibility is needed for recomputation and is discussed in Section 36.2.1.

**Parallel search.** Gecode's kernel is constructed such it does not need to know anything about parallelism: it performs no synchronization<sup>2</sup> in order to keep propagation and search simple and efficient.

By default, a space or clones of a space can only be used by a single thread. However, one can request clones that can be used by different threads as they are created independently. This is achieved by providing **false** as an additional argument to the clone() member function as follows:

```
Space* c = s->clone(false);
```

The additional argument requests that a non-shared clone is created (in fact, it requests that non-shared copies of data structures used by the space s are created, see Section 27.3 for details). The spaces s and c can now be used by different threads in parallel.

**Statistics support.** The three main space operations (status(), clone(), and commit()) provide support for execution statistics. For example, statistics from the execution of status() on a space s can be collected in the object stat by:

```
StatusStatistics stat;
s->status(stat);
```

The classes for the statistics correspond to the space operations:

```
status() StatusStatistics
clone() CloneStatistics
commit() CommitStatistics
```

Statistics information is collected by accumulation. That is, for spaces s1 and s2, the following:

```
StatusStatistics stat;
s1->status(stat);
s2->status(stat);
```

<sup>&</sup>lt;sup>2</sup>That is a slight over-simplification. Gecode's kernel performs just enough synchronization to maintain global information such as AFC (see Section 7.1.1).

collects the combined statistics of performing status() on s1 and s2.

The statistics classes also implement addition operators. The following is equivalent to the previous example:

```
StatusStatistics stat;
{
   StatusStatistics a, b;
   s1->status(a);
   s2->status(b);
   stat = a + b;
}
```

which is also equivalent to:

```
StatusStatistics stat;
{
   StatusStatistics a, b;
   s1->status(a); stat += a;
   s2->status(b); stat += b;
}
```

# 35.2 Binary depth-first search

This section shows a simple search engine that performs left-most depth-first search. It makes the additional simplification that all choices are binary, the general case is discussed in Section 35.3.

Figure 35.1 shows the definition of the function dfs() that implements the search engine. It takes a space as input and returns a space as a solution or NULL if no solution exists. The resource policy it implements is that it takes responsibility for deleting the space s with which dfs() is called initially. The solution it returns must eventually be deleted by the caller of dfs() (if the initial space happens to be a solution, the engine does not delete it). The search engine starts by executing the status() function on s and hence triggers propagation and possibly brancher selection.

In this chapter and in Chapter 36 we use recursive functions to implement exploration during search. This is rather inefficient with respect to both runtime and space in C++. A more realistic implementation uses an explicit stack, for an example see Chapter 37.

**Failure and solutions.** In case the space s is failed, the search engine deletes the space and returns NULL as specified:

```
FAILED ≡
  delete s; return NULL;
```

Figure 35.1: Depth-first search for binary choices

If the space s is solved, the search engine triggers garbage collection of remaining branchers as mentioned in Section 35.1 and returns the solution:

```
SOLVED =
  (void) s->choice(); return s;
```

**Branching.** Following the discussion in Section 35.1, before the search engine can start committing to alternatives and perform recursive search, it needs to compute a choice for committing and a clone for backtracking:

```
PREPARE FOR BRANCHING ≡

const Choice* ch = s->choice();

Space* c = s->clone();
```

The search engine tries the first alternative by committing the space s to it and continues search recursively:

```
FIRST ALTERNATIVE \( s \cdots \cdot \cdots \cdot \cdots \cdot \cdot
```

If the recursive call to dfs() returns a solution (that is, t is different from NULL and hence the condition of the if statement is true) the engine deletes both choice and clone and returns the solution t.

**Saving memory.** It is *absolutely essential* that the search engine uses the original space s for further exploration and stores the clone c for backtracking. Exchanging the roles of s and c by:

```
c->commit(*ch,0);
if (Space* t = dfs(c)) {
   delete ch; delete s;
   return t;
}
```

would also find the same solution. However, this search engine would most likely need more memory. Spaces that already have been used for propagation (such as s) typically require more memory than a pristine clone (see also Section 27.2). Hence, any search engine should maintain the invariant that it stores pristine clones for backtracking, but never spaces that have been used for propagation.

If the first alternative did not lead to a solution, search commits the clone c to the second alternative, deletes the now unneeded choice, and recursively continues search:

```
SECOND ALTERNATIVE 

    c->commit(*ch,1);
    delete ch;
    return dfs(c);
```

# 35.3 Depth-first search

This section demonstrates how left-most depth-first search with choices having an arbitrary number of alternatives can be implemented. By this, the section presents the general version of the search engine from Section 35.2.

Figure 35.2 outlines the depth-first search engine, where computing the space status and handling failed and solved spaces is the same as in Section 35.2. If the search engine needs to branch, it computes the choice ch for branching and the number of alternatives n.

Choices can actually have a single alternative, for example for assigning variables (see Section 7.1.5). This special case should be optimized as in fact no clone needs to be stored for backtracking. Hence:

```
if (n == 1) {
   s->commit(*ch,0);
   delete ch;
   return dfs(s);
}
```

Figure 35.2: Depth-first search

If the choice has more than a single alternative, a clone c is created and a loop iterates over all alternatives:

If the loop terminates, no solution has been found and hence NULL is returned.

When trying the a-th alternative, the search engine determines which space e to choose to continue exploration:

```
SPACE TO EXPLORE =
   Space* e;
   if (a == 0)
      e = s;
   else if (a == n-1)
      e = c;
   else
      e = c->clone();
```

The choice of e avoids the creation of an unnecessary clone for the last alternative.

After committing the space to explore the a-th alternative, search continues recursively. If a solution t has been found, it is returned after the search engine deletes the clone (unless it has already been used for the last alternative) and the choice:

```
RECURSIVE SEARCH \( \ext{ e->commit(*ch,a);} \)
  if (Space* t = dfs(e)) {
    if (a != n-1) delete c;
    delete ch;
    return t;
  }
}
```

#### 35.4 Branch-and-bound search

This section shows how to program a best solution search engine. It chooses branch-and-bound search as an example where choices are again assumed to binary for simplicity. The non-binary case can be programmed similar to Section 35.3.

**Constraining spaces.** A space to be used for best solution search must implement a constrain() function as discussed in Section 2.5. The key aspect of a best solution search engine is that it must be able to add constraints to a space such that the space can only lead to solutions that are better than a previously found solution.

Assume that a best solution search engine has found a so-far best solution b (a space). Then, by

```
s->constrain(*b);
```

the engine can add constraints to the space s that guarantee that only solutions that are better than b are found by search starting from s.

The Space class actually already implements a constrain() function which throws the exception SpaceConstrainUndefined. That is, a space to be used with a best solution search engine must redefine the default constrain() function by inheritance.

**Search engine.** The basic structure of the branch-and-bound search engine is shown in Figure 35.3. A user of the search engine calls the function bab() taking a single space as argument. The function either returns the best solution or NULL if no solution exists.

The function bab() that takes three arguments implements the actual exploration. The space s is the space that is currently being explored, the unsigned integer n counts the number of solutions found so far, and the space b is the so-far best solution. Note that both n and b are passed by reference and hence the variables are shared between all recursive invocations of the search engine. The number of solutions n is used for deciding when a space must be constrained to yield better solutions.

```
[DOWNLOAD]
BAB \equiv
  void bab(Space* s, unsigned int& n, Space*& b) {
    switch (s->status()) {
    case SS_SOLVED:
      ► SOLVED
      break;
    case SS_BRANCH:
        const Choice* ch = s->choice();
        Space* c = s->clone();
        ► REMEMBER NUMBER OF SOLUTIONS
        ► EXPLORE FIRST ALTERNATIVE
        ► CONSTRAIN CLONE
        ► EXPLORE SECOND ALTERNATIVE
        delete ch;
      }
      break;
  }
  Space* bab(Space* s) {
    unsigned int n = 0; Space* b = NULL;
    bab(s,n,b);
    return b;
  }
```

Figure 35.3: Branch-and-bound search

The single argument bab() function initializes n and b to capture that no solution has been found yet. After executing the bab() search engine, b refers to the best solution (or is NULL) and is returned after garbage collecting remaining branchers.

**Finding a solution.** The search engine is constructed such that every solution found is better than the previous. Hence, when a solution is found, the previous so-far best solution is deleted<sup>3</sup> and is updated to the newly found solution. As a new solution is found also the number of solutions n is incremented:

```
solved =
n++;
delete b;
(void) s->choice(); b = s->clone(); delete s;
```

The search engine first garbage collects branchers (by calling choice()) and remembers a pristine clone of the solution found.

**Branching.** Exploring the first alternative differs considerably from exploring the second alternative of a choice. When exploring the first alternative, it is guaranteed that the current space s can only lead to better solutions. If a solution is found by exploring the first alternative (or if several solutions are found), then a constraint must be added to the clone c such that only better solutions can be found when continuing exploration with c for the second alternative. To detect whether a solution has been found when exploring the first alternative, the search engine remembers the number of solutions m before starting to explore the first alternative as follows:

```
REMEMBER NUMBER OF SOLUTIONS ≡
unsigned int m=n;
```

Exploring the first alternative is as to be expected:

```
EXPLORE FIRST ALTERNATIVE ≡
s->commit(*ch,0);
bab(s,n,b);
```

Before exploring the second alternative, the engine checks whether new solutions have been found during the exploration of the first alternative. If new solutions have been found, the clone c is constrained to yield better solutions:

```
CONSTRAIN CLONE ≡
if (n > m)
    c->constrain(*b);
```

<sup>&</sup>lt;sup>3</sup>Actually, b is NULL for the first solution found. However, it is legal in C<sup>++</sup> to invoke the **delete** operator on a NULL-pointer.

The second alternative is explored as follows:

```
EXPLORE SECOND ALTERNATIVE ≡

c->commit(*ch,1);

bab(c,n,b);
```

Note that execution of the constrain() function might constrain some variables and possibly add new propagators (even new variables). Even though c might not be any longer an identical clone of s, the choice ch is still compatible with the space c (see Section 35.1).

# 36

# Recomputation

This chapter demonstrates recomputation as the most essential technique for efficient search in Gecode. It is highly recommended to read Section 7.2 before reading this chapter.

**Overview.** The simplest possible search engine based on recomputation, where all spaces needed for search are recomputed from the root of the search tree, is discussed in Section 36.1. Important invariants for recomputation and how they must be taken into account by search engines using recomputation are discussed in Section 36.2. How best solution search is combined with recomputation is discussed in Section 36.3. The following three sections present important optimizations for search engines using recomputation: Section 36.4 shows how last alternative optimization can avoid commit() operations during recomputation; Section 36.5 shows how hybrid recomputation that stores additional spaces can be used to speed up search; Section 36.6 shows adaptive recomputation that helps speeding up search in case of failures.

**Important.** All sections in this chapter make the simplifying assumption that choices are binary, dealing with non binary choices is similar to Section 35.3. The search engine in Chapter 37 combines all recomputation techniques of this chapter for the general case.

## 36.1 Full recomputation

This section demonstrates search based on full recomputation. While full recomputation is unrealistic, the search engine presented here helps in understanding the ideas behind recomputation. In particular, the search engine serves as an example for important invariants for recomputation that are discussed in Section 36.2.

**Search engine.** Figure 36.1 shows the outline for the search engine that implements depth-first search with full recomputation. The function dfs() that takes a single space s as its argument is called by the user, the function dfs() that takes three arguments implements exploration with full recomputation.

The basic idea of the search engine is to always keep a single space r as the *root* space of the search tree to be explored. Exploration maintains a current space and a path consisting of edges that defines which node in the search tree is currently being explored. Exploration

```
[DOWNLOAD]
DFS USING FULL RECOMPUTATION \equiv
  ...
  ► EDGE CLASS
  Space* dfs(Space* s, Space* r, Edge* p) {
    switch (s->status()) {
    case SS_BRANCH:
        ► EXPLORE FIRST ALTERNATIVE
        ► EXPLORE SECOND ALTERNATIVE
      }
    }
  }
  Space* dfs(Space* s) {
    if (s->status() == SS_FAILED) {
      delete s; return NULL;
    Space* r = s -> clone();
    Space* t = dfs(s,r,NULL);
    delete r;
    return t;
```

Figure 36.1: Depth-first search using full recomputation

```
EDGE CLASS \equiv
  class Edge {
  protected:
    Edge* p;
    const Choice* ch;
    unsigned int a;
  public:
    Edge(Space* s, Edge* e)
      : p(e), ch(s->choice()), a(0) {}
    ► NEXT ALTERNATIVE
    ► COMMITTING A SPACE
    ► RECOMPUTING A SPACE
    ~Edge(void) {
      delete ch;
    }
  };
```

Figure 36.2: Edge class for depth-first search using full recomputation

is governed by the invariant that the current space can always be recomputed from the path of edges maintained by the search engine.

Initially, when the user calls the dfs() function taking a single argument, the root space r is computed as a clone of the space s passed as argument. As only stable and non-failed spaces can be cloned (see Section 35.1), the status() function is used to find out whether propagation on s results in failure. If not, the root space r is created as a clone of s and the search engine dfs() is called with s as the current space, r as the root space, and NULL as the current path from the current space to the root space.

**Edges for recomputation.** Figure 36.2 shows the class Edge implementing an edge of a path to be used for recomputation. The class stores a pointer p to the predecessor edge, a choice ch, and the number of the alternative a that corresponds to the edge. Edges are organized from the current node (space) of the search tree upwards to the root: the last edge of a path connects to the root of the search tree and has NULL as its predecessor p. Note that we restrict our attention in this chapter to binary choices only.

Initialization by the Edge's constructor takes the current space of the search engine s and the predecessor edge e and initializes the edge with the choice for s and the first alternative. Deleting an edge also deletes the stored choice ch.

An edge provides a next() function that redirects the edge to the next alternative:

```
NEXT ALTERNATIVE 
void next(void) {
   a++;
}
```

The commit() function of an edge commits a space s to the alternative that corresponds to the edge:

```
COMMITTING A SPACE ≡
Space* commit(Space* s) const {
   s->commit(*ch,a); return s;
}
```

The function returns the space just for convenience as can be seen below.

Finally, recomputing a space corresponding to an entire path of edges is implemented by the recompute() function. The function takes the root space r as argument and is implemented as follows:

```
RECOMPUTING A SPACE =
Space* recompute(Space* r) const {
   return commit((p == NULL) ? r->clone() : p->recompute(r));
}
```

First, the function traverses the path of edges upwards until the root of the path is reached. Then it creates a clone of the root space r and performs the commit() operations for each edge on the path.

**Exploring alternatives.** The central invariant that the current path of edges p must always correspond to the current space is essential for how the search engine implementing full recomputation explores the search tree.

Before exploring the first alternative recursively, a new edge is created for the first alternative, the current space s is committed to the first alternative, and exploration continues recursively:

```
EXPLORE FIRST ALTERNATIVE ≡

Edge e(s,p);
if (Space* t = dfs(e.commit(s),r,&e))
return t;
```

Note that all resource management for handling edges is done automatically: as soon as the edge e goes out of scope, it is automatically deleted (and hence also the choice for the edge is deleted).

Again, exploration of the second alternative maintains the central invariant: the edge is redirected to the next alternative and then a space corresponding to the path of edges is recomputed:

```
e.next();
return dfs(e.recompute(r),r,&e);
```

**Cost of recomputation.** It is important to notice the following facts about the cost of recomputation, where we compare the search engine using full recomputation to the search engine without recomputation in Section 35.2:

- While the number of commit() operations on spaces drastically increases for full recomputation, the number of status() operations executed remains exactly the same. However, execution of status() during recomputation is more expensive as more constraint propagation needs to be done: full recomputation always starts from the root space where only little constraint propagation has been performed.
- The number of clone() operations executed by full recomputation is never larger than the number of clone() operations without recomputation.

Typically, the number of clone() operations is much smaller: recomputation is *optimistic* in the sense that a clone is created and recomputation is performed only if a node is required for exploration. A search engine without recomputation is *pessimistic* in that it always creates a clone *before* continuing exploration to be able to backtrack to a space that might be required for further exploration.

Section 36.5 presents hybrid recomputation as a technique that makes recomputation less optimistic in that it creates more clones before continuing exploration. Section 36.6 presents adaptive recomputation that makes recomputation even less optimistic in cases where it is likely that exploration can benefit from additional clones.

A detailed evaluation of recomputation in Gecode and a comparison to other techniques for implementing search can be found in [29]. An evaluation of different recomputation techniques (although in a different setup) can be found in [32] and [33, Chapter 7].

## 36.2 Recomputation invariants

This section discusses two important invariants that govern recomputation. The first invariant is that recomputation can only use compatible choices for commit operations: the notion of choice compatibility has been sketched in Section 35.1 and is detailed in Section 36.2.1. The second invariant is concerned with the fact that recomputation is not always deterministic. However, Section 36.2.2 explains why recomputation still works even though it is not deterministic.

## 36.2.1 Choice compatibility

Section 35.1 introduced the notion that a space s is compatible with a choice ch (that is, ch can be used for a commit() operation on s). For recomputation, a more general notion of compatibility is needed: during recomputation, a search engine performs commit() operations using choices that have been computed earlier on a path in the search tree.

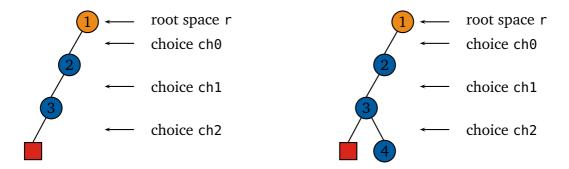


Figure 36.3: Example situations during recomputation

Consider the situation sketched in the left part of Figure 36.3 with a root space r and choices ch0, ch1, and ch2 as stored on the path of edges for recomputation. That is, all choices have been computed by the choice() function from a clone of the root space r where in between the calls to choice() other operations on the space have been performed. For example, the search engine implementing full recomputation from Section 36.1 has executed the status() and commit() functions several times in order to compute the choices.

Suppose that s is a clone of r (computed by s=r->clone()). Then all choices are compatible with s. Now suppose that t is a clone of s. Then all choices are still compatible with t. Hence, if a choice ch has been created for a space r all spaces that are related by cloning to r are compatible with ch. Compatibility continues to hold even if other operations (such as commit() operations for other choices, computing the status of a space, or the creation of new variables, propagators, and branchers) are performed on a space that is clone-related. The only exception is the choice() function of a space.

Suppose that recomputation proceeds by recomputing the space s for node 4 as shown in the right part of Figure 36.3. Assume that s requires branching. Hence, a search engine is executing s->choice(). After executing the choice() function, all previous choices ch0, ch1, and ch2 are not any longer compatible with s.

**Order of commit() operations.** The choices ch0, ch1, and ch2 do not have to be used for commit() in the same order in which they have been created. However, it is more efficient to use them in the order ch0, ch1, and ch2. The difference is that if a space has n branchers and the choices correspond to different branchers, then a commit() operation uses O(1) time to find the corresponding brancher for a choice if the choices are used in order. If they are not used in order, a commit() operation uses O(n) time to find the corresponding brancher. Having said all that, n is typically just one or two.

#### 36.2.2 Recomputation is not deterministic

Assume that a search engine has recorded a path from a node in the search tree and that the space s corresponds to that node in the tree. Then, when a space t for that node

is recomputed, the two spaces s and t might actually differ: the amount of propagation performed by s and t can be different.

The difference in propagation is due to the fact that Gecode supports propagators that are weakly monotonic but not necessarily monotonic. In summary, a weakly monotonic propagator can perform more or less propagation (that is, prune more values from variables domains or prune fewer values from variable domains) but is still correct and complete, see also Section 20.9.

As weakly monotonic constraint propagation is still correct and complete, also recomputation is correct in the following sense: All solutions that are found by search starting from space s are also found by search starting from space t. However, search might find the solutions in different order when starting from either s or t.

Consider the special case that s is failed. The recomputed space t might not necessarily be failed. That is, s performed more constraint propagation than t. However, additional search from t will never find any solution. The same is also true with the roles of s and t exchanged: even though s is not failed, the recomputed space t can be failed.

For search engines using recomputation this typically does not pose any problems. In most cases, a search engine does not attempt to recompute a space that it assumes to be non-failed. Consider the depth-first search engine using full recomputation from Section 36.1. There, the spaces that are recomputed correspond to nodes in the search tree not yet explored. In other words, when recomputing spaces that have been explored previously, a search engine cannot make the assumption that the space is not failed just because the space explored previously has not been failed.

An exception is adaptive recomputation to be discussed in Section 36.6 as an optimization for recomputation. Adaptive recomputation recomputes previously explored spaces to speed up further search.

For a detailed discussion of weakly monotonic propagation together with the consequences for search including recomputation, please consult [40].

#### 36.3 Branch-and-bound search

The first idea to combine recomputation with best solution search is to take the engine for best solution search without recomputation (see Section 35.4) and add recomputation to it along the lines of Section 36.1. A search engine implementing this approach would work roughly as follows: recompute a space and, if needed, add the constraints to the space such that the space can only lead to a better solution.

A tighter integration of recomputation and best solution search is in fact much better: instead of adding the constraints for a better solution to the space that is recomputed, add it to the space from which recomputation starts (with full recomputation: the root space). The advantage is that if adding the constraints to the space from which recomputation starts already leads to failure, the entire subtree starting from that space can be discarded (with full recomputation: search is done).

Figure 36.4 sketches branch-and-bound best solution search using full recomputation.

```
[DOWNLOAD]
BAB USING FULL RECOMPUTATION \equiv
  void bab(Space* s, Space*& r, Space*& b, Edge* p) {
    switch (s->status()) {
    case SS_SOLVED:
      ► SOLVED
    case SS_BRANCH:
      {
        EXPLORE SECOND ALTERNATIVE
      }
    }
  }
  Space* bab(Space* s) {
    Space* r = s -> clone();
    Space* b = NULL;
    bab(s,r,b,NULL);
  }
```

Figure 36.4: Branch-and-bound search using full recomputation

The search engine takes the current space s, the root space r, the so-far best solution b, and the predecessor edge p as arguments. Note that both the root space r and the so-far best solution b are passed by reference as they change during exploration.

When the search engine finds a new solution s, it replaces the so-far best solution b by s and updates the root space by adding the constraints that r must yield better solutions than b as follows:

```
delete b;
(void) s->choice(); b = s->clone(); delete s;
r->constrain(*b);
if (r->status() == SS_FAILED) {
   delete r; r = NULL;
} else {
   Space* c = r->clone(); delete r; r = c;
}
break;
```

The root space then must be checked for failure. If the root space is failed, it is deleted and the pointer to the root space becomes NULL. Otherwise, the root space becomes a clone of the newly computed space to save memory (as mentioned earlier, it is better to store a pristine clone of a space than a space on which propagation has been performed).

Exploring the second alternative of a choice checks whether the root space is already failed:

```
EXPLORE SECOND ALTERNATIVE 

if (r != NULL)
bab(e.recompute(r),r,b,&e);
```

# 36.4 Last alternative optimization

This section presents an important optimization for recomputation that helps to avoid many commit operations during recomputation. Even though the optimization is discussed in the context of full recomputation, it is applicable to all situations in which recomputation is used.

Consider a situation during search as shown in the left part of Figure 36.5. There, the entire left subtree emanating from the root node (colored in orange) has been explored. When exploration continues for the right subtree, each time a node is recomputed, a clone of the root node is made immediately followed by a commit operation for the second alternative. Hence it is much better to compute a new root node for the entire right subtree and perform the corresponding commit operation just once. This optimization is referred to as last alternative optimization (LAO) [33, Chapter 7]. Figure 36.5 shows the new root of the search tree after performing LAO. Note that no space needs to be stored for the previous root node as it will never be used again for recomputation.

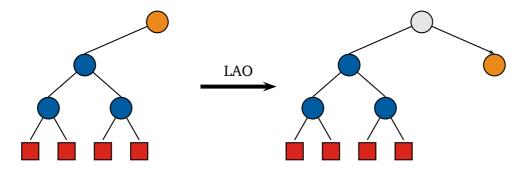


Figure 36.5: Last alternative optimization (LAO)

```
[DOWNLOAD]
DFS USING FULL RECOMPUTATION AND LAO \equiv
 class Edge {
    ► TEST FOR LAST ALTERNATIVE
 };
 Space* dfs(Space* s, Space*& r, Edge* p) {
    switch (s->status()) {
    case SS_BRANCH:
      {
        Edge e(s,p);
        if (Space* t = dfs(e.commit(s),r,&e))
          return t;
        e.next();
        ► PERFORM LAO
        return dfs(e.recompute(r),r,&e);
      }
    }
 }
```

Figure 36.6: Depth-first search using full recomputation and LAO

Figure 36.6 shows a search engine implementing left-most depth-first search using full recomputation and LAO. Only very few aspects have changed compared to the engine for full recomputation without LAO from Section 36.1. An important change is that the root space is now passed by reference: this is necessary as the root space changes during exploration.

The Edge class is extended by a function la() that tests whether an edge happens to be a last alternative:

```
TEST FOR LAST ALTERNATIVE ≡
bool la(void) const {
   return (p == NULL) && (a == 1);
}
```

The actual optimization is performed just before exploring the second alternative:

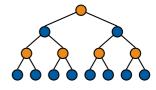
```
if (e.la()) {
   Space* t = r;
   if (e.commit(t)->status() == SS_FAILED)
     return NULL;
   r = t->clone();
   return dfs(t,r,NULL);
}
```

The space t serves as a temporary reference. After performing the commit() operation on t for the second alternative, it is tested whether the new root node is already failed. The new root space becomes the clone of t: this is important as a clone typically requires less memory than a space on which constraint propagation has been performed (see Section 35.2).

# 36.5 Hybrid recomputation

Exploration based on copying alone or based on full recomputation is unrealistic. As already described in Section 7.2, Gecode's search engines use hybrid recomputation: they create a clone now and then to limit the amount of recomputation.

This section describes hybrid recomputation where the amount of recomputation is limited by the *commit distance*  $c_d$ : during recomputation at most  $c_d$  commit() operations are executed. Hybrid recomputation with commit distance  $c_d=2$  where orange nodes are nodes that store a clone is sketched to the right.



The additional clones are stored in a field c in the Edge class shown in Figure 36.7. Initially, the field c does not store a clone. The intuition is that the clone c (if not NULL) is a clone that corresponds to the node to which the edge leads (please remember that edges lead upwards to the root of the search tree).

```
[DOWNLOAD]
DFS USING HYBRID RECOMPUTATION ≡
 const unsigned int c_d = 5;
 class Edge {
 protected:
    ...
    Space* c;
 public:
    Edge(Space* s, Edge* e)
      : p(e), ch(s->choice()), a(0), c(NULL) {}
    ► CREATE CLONE
    ▶ PERFORM RECOMPUTATION
 };
 Space* dfs(Space* s, Edge* p, unsigned int d) {
    switch (s->status()) {
    case SS_BRANCH:
        ► STORE CLONE IF NEEDED
        ► EXPLORE FIRST ALTERNATIVE
      }
   }
 Space* dfs(Space* s) {
    return dfs(s,NULL,c_d);
 }
```

Figure 36.7: Depth-first search using hybrid recomputation

The function clone() stores a clone of a space s in an edge:

```
CREATE CLONE ≡
  void clone(Space* s) {
   c = s->clone();
}
```

Recomputation as performed by the recompute() function now continues to search the path of edges until an edge that stores a clone is found:

```
PERFORM RECOMPUTATION ≡
Space* recompute(void) const {
   return commit((c != NULL) ? c->clone() : p->recompute());
}
```

It must be guaranteed that there is always at least one edge in a path that stores a clone, otherwise recomputation would crash (see below).

The function dfs() that implements exploration takes the additional argument d (for distance) which defines how many commit() operations would be needed to recompute the current space. If d reaches the limit  $c_d$  (for simplicity,  $c_d$  is a constant as defined in Figure 36.7), a new clone must be stored in the current edge. Hence, the code for branching starts by checking whether a clone must be stored for the current edge e:

```
STORE CLONE IF NEEDED ≡
Edge e(s,p);
if (d >= c_d) {
  e.clone(s); d=0;
}
```

The initial call to the function dfs() that implements exploration takes  $c_d$  as value for d. By this, it is guaranteed that the first edge stores a clone (that is, a clone that corresponds to the root node of the search tree).

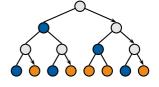
Exploring the alternatives is as before, the only change is that the incremented distance d+1 is passed as additional argument:

```
EXPLORE FIRST ALTERNATIVE ≡

if (Space* t = dfs(e.commit(s),&e,d+1))

return t;
```

LAO as described for full recomputation (Section 36.4) can be added analogously to hybrid recomputation. Therefore we do not present hybrid recomputation with LAO. However, how LAO performs with hybrid recomputation is sketched to the right: gray nodes correspond to nodes where a clone had been stored, while orange nodes correspond to nodes where a clone has been moved due to LAO.



# 36.6 Adaptive recomputation

Consider the situation when a search engine using recomputation encounters a failed space during exploration. Then it is quite likely that as exploration continues, more failed spaces are found during search. This is due to the fact that some decision made during branching (that is, some alternative) has lead to failure. It is quite likely that the wrong decision from which search must recover is somewhere on the path to the root node.

That means that search now must explore the entire subtree starting from the wrong decision. In this situation it would be advantageous if additional clones were available for exploring the entire subtree. With other words, after encountering failure, search should become more pessimistic by investing into the creation of additional clones to speed up further exploration.

Adaptive recomputation [33, Chapter 7] optimizes recomputation in the case of failures: an additional clone is created during recomputation. The idea is that on a path of length n without any clone, an additional clone is placed in the middle of that path. This additional clone then speeds up further recomputation (which is likely to occur as has been argued above).

Adaptive recomputation is controlled by a parameter called *adaptive distance*  $a_d$ : only if  $n \ge a_d$  an additional clone is created. This avoids creating an excessive amount of clones.

Adaptive recomputation is sketched in Figure 36.8. The only change compared to hybrid recomputation (see Section 36.5) is the implementation of recompute(). The argument n is the length of the path to the next clone. When a clone is found, d is set to  $\lfloor \frac{n}{2} \rfloor$  provided that  $n \geq a_d$ . Otherwise d is set to n (its old value) which also prevents that an additional clone is created. For the edge that is d commit() operations away from the current space, a new clone is stored on the path, provided that the space s is not failed.

The space s can be failed as has been discussed in Section 36.2.2. The search engine here takes a rather simplistic approach to this situation: it just does not store a clone. A real-life engine would take more benefit from the information that a space on the path is actually failed: it would immediately discard the entire path below the failed space, see Chapter 37.

Another optimization that a real-life search engine would employ is to not place a clone in a position where it could be moved by last alternative optimization, see again Chapter 37.

```
[DOWNLOAD]
DFS USING ADAPTIVE RECOMPUTATION \equiv
  const unsigned int c_d = 5;
  const unsigned int a_d = 2;
  class Edge {
  protected:
    Space* recompute(unsigned int n, unsigned int& d) {
      if (c != NULL) {
        d = (n >= a_d) ? n/2 : n;
        return commit(c->clone());
      } else {
        Space* s = p->recompute(n+1,d);
        if ((d == n) && (s->status() != SS_FAILED))
          clone(s);
        return commit(s);
      }
    Space* recompute(unsigned int& d) {
      return recompute(1,d);
    }
  };
```

Figure 36.8: Depth-first search using adaptive recomputation

# 37

# An example engine

This chapter puts all techniques for search and recomputation from Chapter 35 and Chapter 36 together. It presents a realistic search engine that can be used to search for several solutions.

**Overview.** Section 37.1 sketches the design of the example depth-first search engine to be used in this chapter. How the engine is implemented is shown in Section 37.2. Section 37.3 details how exploration is implemented, whereas Section 37.4 details how recomputation is implemented for the search engine.

## 37.1 Engine design

The example engine to be developed in this chapter implements depth-first search using hybrid and adaptive recomputation with full last alternative optimization. It provides an interface similar to the interface of Gecode's pre-defined search engines: it is initialized with a space (even though the search engine presented here does not make a clone for simplicity) and provides a next() function that returns a space for the next solution or returns NULL if there are no more solutions.

The outline of the search engine is shown in Figure 37.1. To keep things simple, the values for the commit distance c\_d and adaptive distance a\_d are constants. Furthermore, the search engine uses an array of fixed size to implement the path of edges for recomputation. In case the size of the array is exceeded during exploration, an exception of type StackOverflow is thrown. A real-life engine would of course use a dynamic data structure such as a C++ vector.

## 37.2 Engine implementation

Figure 37.2 shows the class Engine implementing depth-first search. The engine maintains a path p of edges for recomputation (to be explained below), a current space s, and a distance d. The current space s can be NULL. If the current space s is not NULL, then the path p corresponds to s. If the space is NULL, the search engine uses recomputation to compute the next space needed for exploration.

Figure 37.1: Depth-first search engine

```
SEARCH ENGINE ≡
  class Engine {
  protected:
    Path p;
    Space* s;
    unsigned int d;
  public:
    Engine(Space* r) : s(r), d(c_d) {}
    Space* next(void) {
      do {
        while (s != NULL)
          ► EXPLORATION
        while ((s == NULL) && p.next())
          s = p.recompute(d);
      } while (s != NULL);
      return NULL;
   ~Engine(void) {
      delete s;
   }
  };
```

Figure 37.2: Implementation of depth-first search engine

The distance d describes the number of commit operations needed for recomputation. It is initialized to c\_d to force the immediate creation of a clone on the path (analogous to the search engine in Section 36.5).

**Exploration mode.** The engine operates in two modes: *exploration mode* and *recomputation mode*. It operates in exploration mode while the current space s is not NULL. Exploration mode continues until the current space s becomes failed or solved. In both cases, the current space is set to NULL.

If the space s becomes solved, it is returned as a solution by the next() function. If the next() function is called again, then the situation is exactly the same as for failure: the current space is NULL and the engine switches to recomputation mode. Exploration is detailed in Section 37.3.

**Recomputation mode.** In recomputation mode, the engine tries to recompute the current space. The next() function of the path p moves the path to the next alternative. If there is a next alternative (the search space has not yet been completely explored), the next() function of a path returns **true**. The recompute() function tries to recompute the current space s according to the path p. Due to adaptive recomputation, the recompute() function might update the distance d and might actually fail to recompute a space that corresponds to the current path (in which case it returns NULL). Recomputation is detailed in Section 37.4.

If no more alternatives are to be tried (that is, the next() function of the path p has returned **false**), the next() function of the search engine terminates by returning NULL.

# 37.3 Exploration

The search engine continues in exploration mode while the current space s is different from NULL and executes the code shown in Figure 37.3. In case the current space s is failed, it is discarded, s is set to NULL, and the engine switches to recomputation mode. The same is true if the engine finds a solution, however it garbage collects branchers on the solution found and returns it. With another invocation of the next() function, the engine will operate in recomputation mode.

**Edge implementation.** Figure 37.4 shows how an edge is implemented. The implementation is analogous to the edge classes used in Chapter 36. Edges support choices with an arbitrary number of alternatives, the test la() whether an edge is at its last alternative takes the number of alternatives of the choice into account.

Rather than having a default constructor and a destructor, edges use the init() and reset() functions. This is more convenient as edges are maintained in an array implementing a stack, see below for details.

```
EXPLORATION ≡
  switch (s->status()) {
  case SS_FAILED:
    delete s; s = NULL;
    break:
  case SS_SOLVED:
      Space* t = s; s = NULL;
      (void) t->choice();
      return t;
  case SS_BRANCH:
    if (d >= c_d) {
      p.push(s,s->clone()); d=1;
    } else {
      p.push(s,NULL); d++;
    }
  }
```

Figure 37.3: Implementation of exploration

**Path implementation.** Figure 37.5 shows how a path of edges is implemented. The array e stores the edges of the path. The array implements a stack of edges and the unsigned integer n defines the number of edges that are currently on the stack. The edge at position n-1 of the array of edges e corresponds to the top of the stack.

**Pushing edges on the path.** During exploration, the engine pushes new edges on the path p as shown in Figure 37.3. If the distance d has reached the commit distance  $c_d$ , the engine pushes an edge to the path that has an additional clone and resets the distance d accordingly.

Pushing an edge checks for stack overflow and initializes the field of the edge array that corresponds to the top of stack as follows:

```
void push(Space* s, Space* c) {
   if (n == n_stack)
     throw StackOverflow("Path::push");
   e[n].init(s,c); e[n].commit(s);
   n++;
}
```

Note that the push operation also performs the commit() operation on the current space s that corresponds to the edge just pushed onto the path.

```
EDGE ≡
  class Edge {
  protected:
    const Choice* ch;
    unsigned int a;
    Space* c;
  public:
    void init(Space* s, Space* c0) {
      ch = s->choice(); a = 0; c = c0;
    Space* clone(void) const {
      return c;
    void clone(Space* s) {
      c = s->clone();
    void next(void) {
      a++;
    bool la(void) const {
      return a+1 == ch->alternatives();
    void commit(Space* s) {
      s->commit(*ch,a);
    ► PERFORM LAO
    void reset(void) {
      delete ch; ch=NULL; delete c; c=NULL;
   }
  };
```

Figure 37.4: Implementation of edges

Figure 37.5: Implementation of path of edges

# 37.4 Recomputation

In recomputation mode, the engine uses operations to move the engine to the next alternative and to perform recomputation of a space corresponding to the current path.

**Move to next alternative.** Moving to a next alternative discards all edges from the path that are already at their last alternative (that is, the function la() returns true). If the engine finds an edge with remaining alternatives, it moves the edge to the next alternative. If no edges are left, the function next() returns **false** as follows:

```
MOVE TO NEXT ALTERNATIVE ≡
bool next(void) {
    while (n > 0)
        if (e[n-1].la()) {
        e[--n].reset();
        } else {
        e[n-1].next(); return true;
        }
    return false;
}
```

**Perform recomputation.** The recompute() function shown in Figure 37.6 performs recomputation. LAO and adaptive recomputation are orthogonal optimizations and are discussed later. First, i is initialized such that it points to the closest edge on the path that has a clone. Then, s is initialized to a clone of the edge's clone and the distance d is updated accordingly. Finally, all commit() operations between i and n are performed on s.

Figure 37.6: Implementation of recomputation

**Last alternative optimization.** Before actually starting recomputation, the recompute() function checks whether it can perform LAO. It checks whether the last edge of the path can perform LAO (in which case t is different from NULL) as follows:

```
PERFORM LAO ≡
  if (Space* t = e[n-1].lao()) {
    e[--n].reset();
    d = c_d;
    return t;
}
```

The edge is removed from the path and the distance d is set to  $c_d$  to force the immediate creation of a new clone when the engine continues in exploration mode.

LAO for an edge checks whether the edge is at the latest alternative and whether the edge stores a clone:

```
PERFORM LAO =
    Space* lao(void) {
        if (!la() || (c == NULL))
            return NULL;
        Space* t = c; c = NULL;
        commit(t);
        return t;
    }
```

If this is the case, the clone from the edge is removed and is committed to the last alternative.

**Adaptive recomputation.** If the current distance d reaches the adaptive distance a\_d, recomputation tries to perform adaptive recomputation as follows:

```
if (d >= a_d) {
   unsigned int m = i + d/2;
   for (; i < m; i++)
      e[i].commit(s);
      SKIP OVER LAST ALTERNATIVES
      CREATE ADDITIONAL CLONE
}</pre>
```

The value of m is the middle between the position of the clone i and the position of the last edge on the path. The position m is a candidate position where the additional clone might be stored. All commit operations for edges between the clone and edge at position m are executed.

It is entirely pointless to store the additional clone at an edge that is already at its last alternative (this is what LAO is all about). Hence, adaptive recomputation skips over all edges that are already at their last alternative as follows:

```
SKIP OVER LAST ALTERNATIVES ≡
for (; (i < n) && e[i].la(); i++)
e[i].commit(s);</pre>
```

An additional clone for an edge is only created if the edge is not already the topmost edge of the path:

```
CREATE ADDITIONAL CLONE ≡
if (i < n-1) {
    ▶ PERFORM PROPAGATION
    e[i].clone(s);
    d = n-i;
}</pre>
```

After storing the clone, the distance d is adapted accordingly.

Before being able to create a clone, adaptive recomputation performs constraint propagation by executing the status() function of the space s as follows:

```
PERFORM PROPAGATION \( \)
    if (s->status() == SS_FAILED) {
        delete s;
        for (; i < n; n--) {
              e[n-1].reset(); d--;
        }
        return NULL;
    }
}</pre>
```

If constraint propagation leads to a failed space (see Section 36.2.2), all edges below the failed space are discarded and recomputation returns NULL to signal that recomputation did not succeed in recomputing a space for the current path.

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# Changelog

- Released for Gecode 3.7.3 (2012-03-20).
- Released for Gecode 3.7.2 (2012-02-22).
- Added tip that compilers for Qt and Gecode must match (see Tip 2.10) (2011-11-10, thanks to Pavel Bochman).
- Released for Gecode 3.7.1 (2011-10-10).
- Explained semantics of n-ary implication (see Section 4.4.4) (2011-10-06).
- Released for Gecode 3.7.0 (2011-08-31).
- Added links to the Global Constraint Catalog (GCCAT, [5]) (2011-08-22).
- Documented membership constraints (see Section 4.4.2) (2011-08-22).
- Documented number of values constraints (see Section 4.4.9) (2011-08-17).
- Fixed error in explanation of value precedence constraint for multiple values (see Section 4.4.19) (2011-08-17, thanks to Chris Mears).
- Added missing information on creating a variable implementation disposer (see Section 31.4) (2011-08-15, thanks to Gustavo Gutierrez).
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- Released for Gecode 3.6.0 (2011-07-15).
- Documented precede constraint (see Section 5.2.8) (2011-07-13).
- Explained that constraint post functions are clever in that they select a good propagator (see Section 4.3) (2011-07-08, thanks to Kish Shen).
- Documented precede constraint (see Section 4.4.19) (2011-06-30).
- Documented nooverlap constraint (see Section 4.4.16) (2011-06-07).
- Documented path constraint for Hamiltonian paths (see Section 4.4.17) (2011-06-07).
- Moved graph and scheduling constraints to integer module (see Section 4.4.17 and Section 4.4.18) (2011-05-26).
- Fixed example for count constraint (2011-05-03, thanks to Kish Shen).
- Added Tip 2.9 about the library path to the compilation instructions (2011-03-28, thanks to Gabriel Hjort Blindell, Flutra Osmani).

- Added pointers to MiniModel reference documentation (2011-03-24).
- Added archiving for choices and branchers (2011-03-14).
- Adapted to new names for set channeling constraints (2011-02-22).
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- Removed discussion of limited discrepancy search (2010-10-09).
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- Documented the binpacking constraint (Section 4.4.15) (2010-10-06).
- Added explanation how to initially schedule a propagator using advisors (see Tip 24.2) (2010-10-05, thanks to Chris Mears).
- Added installation and compilation instructions (moved and eexpanded from the reference documentation) (Section 2.6) (2010-10-05).
- Explain that variables are re-selected during branching (Tip 7.1) (2010-09-02, thanks to Kish Shen).
- Explain branch filter functions (Section 7.1.4) (2010-09-01, thanks to Felix Brandt).
- Documented that variable implementation views are parametric with respect to variables (Section 33.2) (2010-08-31).
- Documented that the variable base class is VarImpVar (Section 32.1) (2010-08-31).
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- Released for Gecode 3.4.0 (first complete version) (2010-07-26).
- Added a how to read section (Section 1.3) and overview material to each chapter and part (2010-07-21).
- Added the part on programming search engines (Part S) (2010-07-20).
- Added the part on programming variables (Part V) (2010-07-02).
- Explain that the compiler defines the platform used on Windows (Tip 2.8) (2010-06-17, thanks to Dan Scott).
- Explained that variables do not have init() functions as the are not needed in Tip 4.2 (2010-06-04).

- Fixed typo in Section 7.2.2 (2010-05-10, thanks to Andreas Karlsson).
- Documented new MiniModel for set constraints (Section 6.1) and adapted to other MiniModel changes (2010-05-07).
- Added more case studies (2010-05-06).
- Documented new operations on argument arrays (Section 4.2.2) (2010-05-06).
- Only use absolute URLs as not all PDF viewers honor the base URL (2010-04-11).
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- Described that linear expressions can freely mix integer and Boolean variables and that also sum expressions are supported (Section 6.1) (2010-02-01).
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- Explained integer shared arrays for element (Tip 4.9) (2009-11-01).
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- Documented element constraint for matrix interface (Section 6.2) (2009-06-08).
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- Documented parallel search (Section 7.3) (2009-05-12).
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- Documented script commandline driver (Section 3.3, Chapter 9) (2009-04-20).
- Documented wait post functions (Section 4.5, Section 5.3) (2009-04-09).
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- Fix for gcc compilation instructions in Section 2.3.3 (2009-03-26, thanks to Roberto Castañeda).

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- Generate shorter inter-document references to avoid problems with some PDF viewers (2009-03-23, thanks to Håkan Kjellerstrand).
- Initial release for Gecode 3.0.0 (2009-03-13).

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