7. $$U_t = \int_0^t e^{-\lambda(t-s)} \, dB_s$$

Denotamos por $Z_t = \int_0^t e^{+\lambda s} \, dB_s$.

Así, $U_t = f(t, Z_t)$, con $f(t,x) = x e^{-\lambda t}$.

Aplicamos la fórmula de Itô en su forma diferencial:

$$df(t,x) = f_x(t,x) \, dZ_t + \left(f_t(t,x) + \frac{1}{2} f_{xx}(t,x) \right) dt$$

$$\left. \begin{array}{l} f_x(t,x) = e^{-\lambda t}, \quad f_{xx}(t,x) = 0, \\ f_t(t,x) = -\lambda e^{-\lambda t} = -\lambda f(t,x) \end{array} \right| \Rightarrow$$

$$\Rightarrow df(t,x) = dU_t = -\lambda f(t,x) \, dt + e^{-\lambda t} dZ_t.$$

$$\Rightarrow dU_t = -\lambda Z_t e^{-\lambda t} dt + e^{-\lambda t} dZ_t$$

$$U_t = Z_t e^{-\lambda t} \quad y \quad dZ_t = e^{\lambda t} dB_t$$

$$\Rightarrow \underline{dU_t = -\lambda U_t \, dt + dB_t}.$$

(8). $Y_t = \operatorname{sen}(B_t)$.

$$f(t,x) = \operatorname{sen} x \quad \begin{cases} f_x(t,x) = \cos x \\ f_{xx}(t,x) = -\operatorname{sen} x. \end{cases}$$

$$dY_t = df(t,x) = \left(f_t(t,x) + \frac{1}{2} f_{xx}(t,x)\right)dt + f_x(t,x)\, dB_t$$

$$= -\frac{1}{2}\operatorname{sen}(B_t)\, dt + \cos(B_t)\, dB_t.$$

(9). $dS_t = \mu dt + \sigma dB_t$.

Sea $Y_t = g(S_t)$, $g(S_t, t) = 2 + t + e^{S_t}$

$Y_t = f(t,x) = g(t, S_t)$.

Lema de Itô, aplicado a g y a S_t

$$\cancel{dY_t = dg(t,S_t) = \left(g_t(t,x) + \frac{1}{2} g_{xx}(t,x)\right)dt + g_x(t,x)(dS_t)^2}$$

$$dg(t,S_t) = g_t(t,S_t)\, dt + g_x(t,S_t)\, dS_t + \frac{1}{2} g_{xx}(t,S_t)(dS_t)^2 =$$

$$= 1 \cdot dt + e^{S_t} dS_t + \frac{1}{2} e^{S_t} (dS_t)^2 =$$

$$= \underline{dt + e^{S_t}(\mu dt + \sigma dB_t) + \frac{1}{2}\sigma^2 e^{S_t} dt}.$$

siendo $(dt)^2 \simeq 0$, $(dt \cdot dB_t) \simeq 0$ $(dB_t)^2 = dt$.