Bubble Meter Performance and Feed Throat Area Calculation for a Laser-Powered Rocket Engine

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This document provides clarification on the results of the bubble meter and explains how these results can be used to determine the flow speed, mass flow, and critical area A^*. The application of this study focuses on the argon flow entering a combustion chamber and exiting via a nozzle.

Chapter 1: Flow Regulator Results

Volume Flow Calculation

Given that 1 mole of an ideal gas occupies 22.4 liters and the molar mass of argon is 40 g/mol, the volume flow rate (Q) for a mass flow rate ($\dot{m}$) of 0.6 g/s can be calculated as follows:

$$\dot{n} = \frac{\dot{m}}{M} = \frac{0.6\ \mathrm{g\,s^{-1}}}{40\ \mathrm{g\,mol^{-1}}} = 0.015\ \mathrm{mol\,s^{-1}} \tag{1}$$

$$Q = \dot{n} \times 22.4\ \mathrm{L\,mol^{-1}} = 0.015\ \mathrm{mol\,s^{-1}} \times 22.4\ \mathrm{L\,mol^{-1}} = 0.336\ \mathrm{L\,s^{-1}} \tag{2}$$

Thus, the volume flow rate is $0.336\ \mathrm{L\,s^{-1}}$.

Results for Air

The flow regulator was tested with air at pressures of 50 psi and 100 psi. The results are summarized in the following table:

Table 1: Flow rate of air through the regulator at different rotations and pressures.

Rotation increments	**50 psi** ($\mathrm{L\,s^{-1}}$)	**100 psi** ($\mathrm{L\,s^{-1}}$)
0.50	0.026	0.048
1.00	0.039	0.067
1.10	0.048	0.085
1.20	0.065	0.125
1.30	0.089	0.168
1.40	0.116	0.221
1.50	0.140	0.253
1.60	0.161	0.306
1.66	0.172	0.336
1.70	0.184	0.345
1.80	0.204	0.381
1.90	0.228	0.425
2.00	0.244	0.460
2.42	0.336	0.656

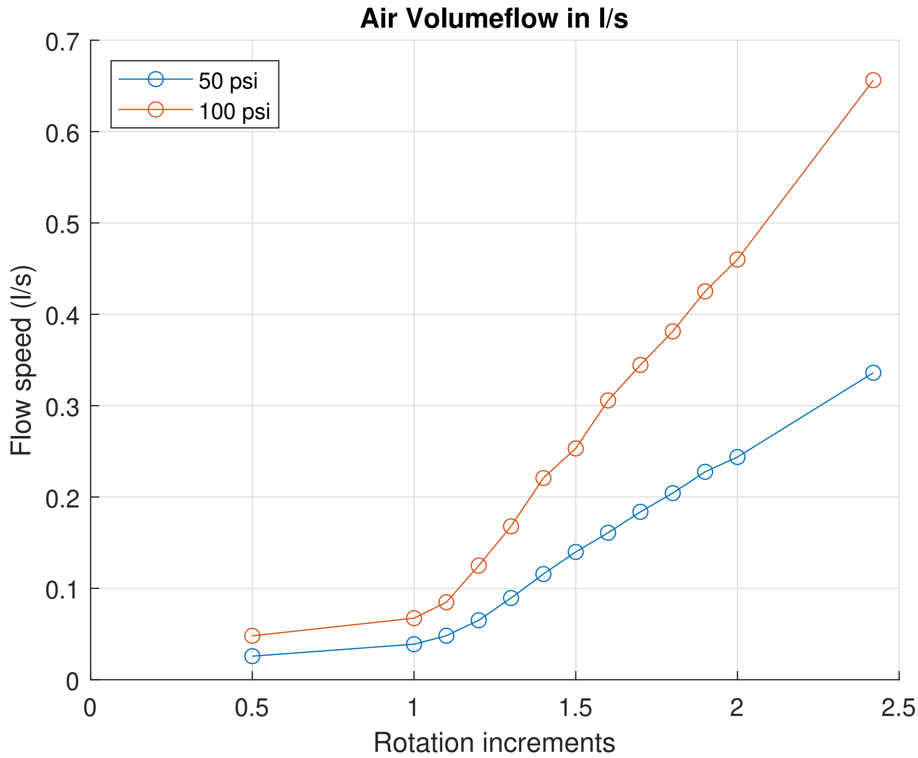

Figure 1: Flow rate vs Rotation for the flow regulator at 50 psi and 100 psi for air.

It can be concluded that the needle valve acts linearly to pressure increase.

Results for Argon

The flow regulator was tested with argon at pressures of 2000 kPa and 5000 kPa. The results are summarized in the following table:

Table 2: Flow rate of argon through the regulator at different rotations and pressures.

Rotation increments	**2000 kPa** ($\mathrm{L\,s^{-1}}$)	**5000 kPa** ($\mathrm{L\,s^{-1}}$)
0.50	0.12	0.25
1.00	0.17	0.37
1.10	0.20	0.51
1.20	0.27	0.67
1.30	0.39	1.24
1.40	0.49	1.52
1.50	0.61	1.88
1.60	0.73	2.29
1.66	0.80	2.54
1.70	0.90	2.84
1.80	1.05	3.20
1.90	1.14	3.41
2.00	1.28	3.99

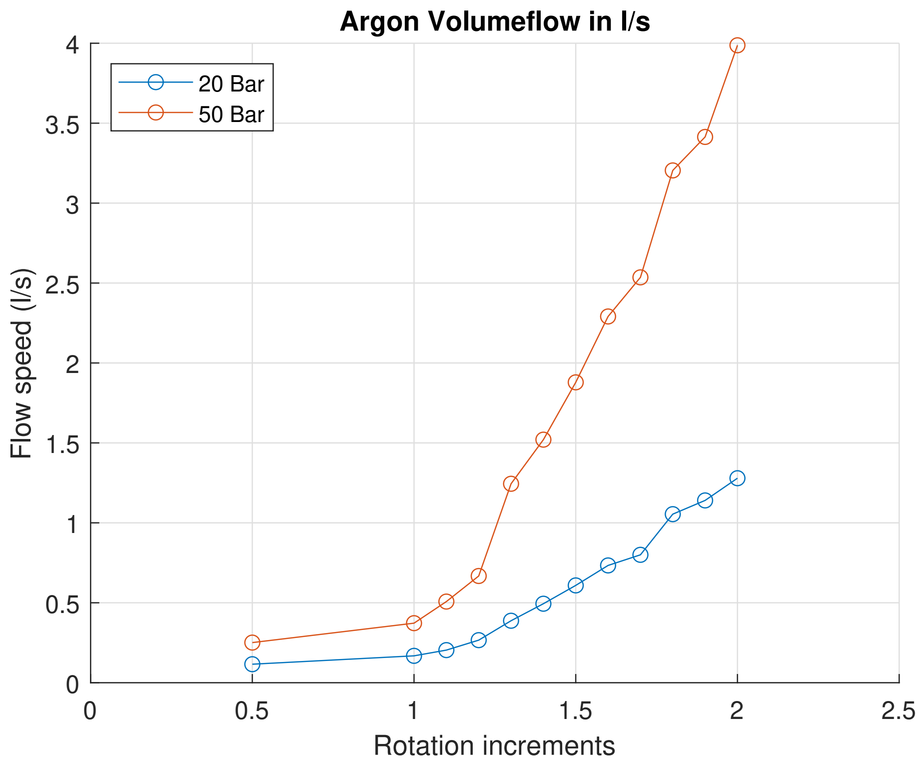

Figure 2: Flow rate vs Rotation for the flow regulator at 2000 kPa and 5000 kPa for argon.

The same linear relationship holds for argon as well.

Summary

The results demonstrate that the needle valve exhibits a linear response to pressure increases for both air and argon. The required volume flow rates can be achieved by adjusting the needle valve rotations accordingly.

Chapter 2: Calculation of A^* for Argon System

Parameters and Introduction

The critical area A^* is calculated based on the mass flow rate of 0.6 g/s of argon. The parameters used in the calculation are:

- Pressure at needle valve entry (P_0): 5000 kPa
- Temperature (T): 293 K
- Pressure in the combustion chamber (P_c): 2000 kPa
- Mass flow ($\dot{m}$): 0.6 g/s
- Gas constant for argon (R): 208.13 J/(kg·K)
- Specific heat ratio for argon ($\gamma = 1.67$)

Calculation of A^* for the test setup

The critical area A^* can be calculated using the equation for choked flow:

$$\dot{m} = A^* \cdot P_0 \cdot \sqrt{\frac{\gamma}{R \cdot T}} \cdot \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3}$$

Rearranging for A^*:

$$A^* = \frac{\dot{m}}{P_0 \cdot \sqrt{\frac{\gamma}{R \cdot T}} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{4}$$

Substituting the values:

$$\begin{aligned} A^* &= \frac{0.0006}{5000 \times 10^3 \cdot \sqrt{\frac{1.67}{208.13 \cdot 293}} \cdot \left(\frac{2}{1.67+1}\right)^{\frac{1.67+1}{2(1.67-1)}}} \\ A^* &= 4.071 \times 10^{-8}\,\mathrm{m}^2 \\ A^* &= 0.04071\,\mathrm{mm}^2 \end{aligned} \tag{5}$$

Volume Flow and Density at Given Pressures

The volume flow ($\dot{V}$) can be calculated using the ideal gas law:

$$\dot{V} = \frac{\dot{m} \cdot R \cdot T}{P_0} \tag{6}$$

Substituting the values:

$$\begin{aligned} \dot{V} &= \frac{0.0006 \cdot 208.13 \cdot 293}{5000 \times 10^3} \\ \dot{V} &= 7.318 \times 10^{-6}\,\mathrm{m}^3/\mathrm{s} \\ \dot{V} &= 0.00732\,\mathrm{l/s} \end{aligned} \tag{7}$$

The density (ρ) of argon at $P_0 = 5000$ kPa and 293 K can be calculated as:

$$\rho = \frac{P_0}{R \cdot T} \tag{8}$$

$$\begin{aligned} \rho &= \frac{5000 \times 10^3}{208.13 \cdot 293} \\ \rho &= 81.99\,\mathrm{kg/m}^3 \end{aligned} \tag{9}$$

Rotation increments for different opening areas

With all the previous information, the opening area of the needle valve can be calculated for different opening positions. This is done by calculating the mass flow rate for all the volume flows at different increments and then calculating the opening area using Equation 4. This results in the table displayed below.

We calculate the opening area for each rotation increment provided in the argon table at a given upstream pressure ($P_0 = 5000$ kPa).

Table 3: Calculated Opening Area for Needle Valve at Different Rotations (Upstream Pressure = 5000 kPa and Outlet Pressure = 100 kPa)

Increment	Volume Flow (L/s)	Mass Flow (g/s)	Area (mm^2)	Diameter (mm)
0.50	0.25	0.41	0.028	0.188
1.00	0.37	0.61	0.041	0.229
1.10	0.51	0.84	0.057	0.269
1.20	0.67	1.10	0.075	0.308
1.30	1.24	2.03	0.138	0.419
1.40	1.52	2.49	0.169	0.464
1.50	1.88	3.08	0.210	0.517
1.60	2.29	3.76	0.255	0.570
1.66	2.54	4.17	0.283	0.600
1.70	2.84	4.66	0.317	0.635
1.80	3.20	5.25	0.357	0.674
1.90	3.41	5.59	0.380	0.696
2.00	3.99	6.54	0.445	0.752

Summary

These results are computed for the test setup and not for the experiment. This will later be compared to how this can be adapted to the experiment. For the test setup, $P_0 = 5000\,\text{kPa}$ is used as the incoming pressure, and after the needle valve, the flow is released into the atmosphere with a pressure of $P_a = 100\,\text{kPa}$. Because of this high-pressure difference, only a very low volume flow is required to fulfill the requirement of the mass flow rate. Later on, the volume flow and mass flow will remain the same, but the A^* will change due to the different pressure conditions.

Chapter 3: Calculation of A^* for the Experiment

In this chapter, we will calculate the critical area (A^*) for a needle valve with an inlet pressure (P_0) of 50 bar, which exits into a compression chamber with a pressure (P_c) of 20 bar. The temperature in the chamber is considered to be 1200 K.

Parameters and Formula

To calculate A^*, we use the pressure difference ($P_0 - P_c$) instead of just the upstream pressure. This approach is valid because the flow rate through the valve is driven by the pressure difference between the inlet and the outlet, which directly influences the velocity of the gas and the resulting mass flow rate. Also, from now on, the mass flow rate is slightly adjusted to meet the required mass flow rate of the experiment.

The general equation for calculating the critical area (A^*) is:

$$A^* = \frac{\dot{m}}{(P_0 - P_c) \cdot \sqrt{\frac{\gamma}{R \cdot T}} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{10}$$

The given parameters are:

- Mass flow rate, $\dot{m} = 0.641\,\text{g/s} = 0.000641\,\text{kg/s}$
- Upstream pressure, $P_0 = 50\,\text{bar} = 5 \times 10^6\,\text{Pa}$
- Downstream pressure, $P_c = 20\,\text{bar} = 2 \times 10^6\,\text{Pa}$
- Pressure difference, $\Delta P = P_0 - P_c = 30\,\text{bar} = 3 \times 10^6\,\text{Pa}$
- Specific heat ratio for argon, $\gamma = 1.67$

- Gas constant for argon, $R = 208.13\,\mathrm{J/(kg{\cdot}K)}$
- Temperature in the chamber, $T = 1200\,\mathrm{K}$

Calculation of A^*

Substituting these values into the formula:

$$\begin{aligned}
A^* &= \frac{0.000641}{3 \times 10^6 \cdot \sqrt{\frac{1.67}{208.13 \cdot 1200}} \cdot \left(\frac{2}{1.67+1}\right)^{\frac{1.67+1}{2(1.67-1)}}} \\
A^* &= 1.4695 \times 10^{-7}\,\mathrm{m}^2 \\
A^* &= 0.147\,\mathrm{mm}^2
\end{aligned} \tag{11}$$

This results in a diameter for A^* of $d = 0.4326\,\mathrm{mm}$

Summary

The calculated critical area A^* for the needle valve, with an inlet pressure of 50 bar and exiting into a compression chamber with a pressure of 20 bar at a temperature of 1200 K, is $0.147\,\mathrm{mm}^2$. This area ensures the desired mass flow rate of 0.641 g/s under the given conditions. Understanding this critical area is crucial for accurately controlling the flow rates and achieving efficient performance in the laser-powered rocket engine.

Chapter 4: Conclusion and Recommendation

Conclusion

The document demonstrated that the needle valve exhibits a linear response to pressure increases for both air and argon, with the required volume flow rates achievable by adjusting the needle valve rotations. The critical area A^* for the test and experimental setups was calculated, showing that the flow rates can be accurately controlled by considering the pressure differences and temperature conditions.

Recommendation

For future experiments, it is recommended to use a rotation increment of approximately 1.33 to achieve an A^* of 0.147 mm^2 with a diameter $d = 0.433$ mm and a mass flow of 0.641 g/s. This way, all the requirements are met. Keep close monitoring of the pressure in the combustion chamber to ensure it stays at 20 bar. If the pressure increases, this could indicate that the nozzle exit is dirty and therefore has a narrower diameter than supposed. For checking the calculations, the MATLAB script used is included in the appendix.