The Price of Synchrony in a Power Grid with Fluctuating Feed-In

Philippe Jacquod

Condynet-Potsdam 12.6.2017

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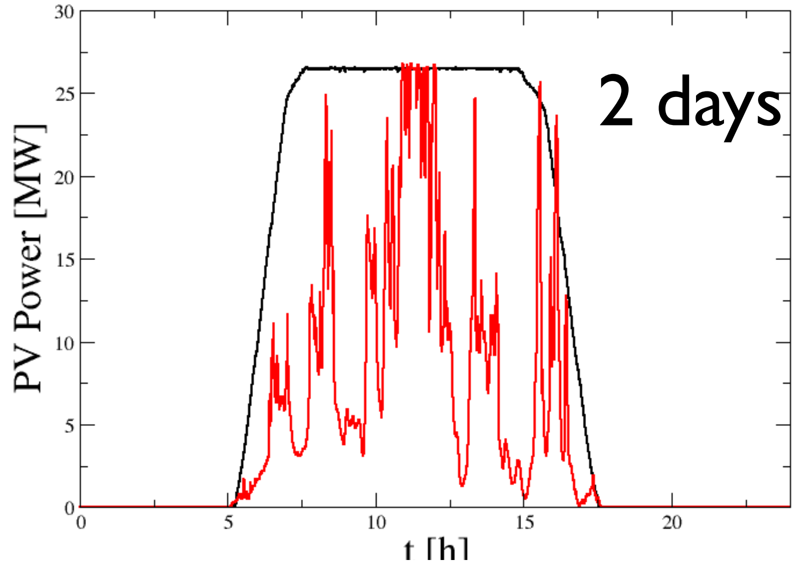
16:00 – 18:00 **Science Session II: Structural Aspects**

- Prof. Marc Timme (MPIDS):
  "Nonlinear Rerouting and Propagation of Perturbations in Power Grids"
- Prof. Jörg Raisch (TU Berlin):
  "A Framework for Hierarchical Control of Power Systems"
- Prof. Martin Braun (Fraunhofer IWES):
  "New Operational Strategies and Planning Approaches"
- Prof. Philippe Jacquod (HES-SO, Schweiz):
  "The Price of Synchrony in a Power Grid with Fluctuating Feed-In"

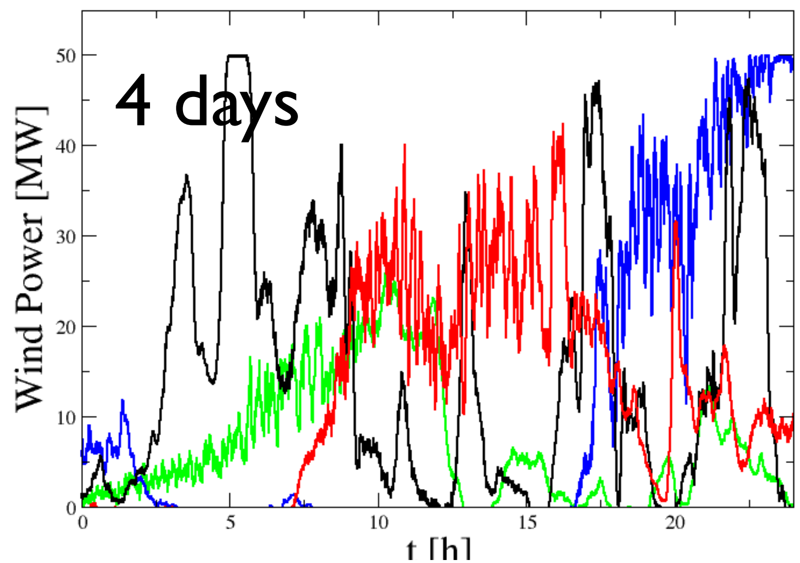
New renewables - fluctuating Feed-in

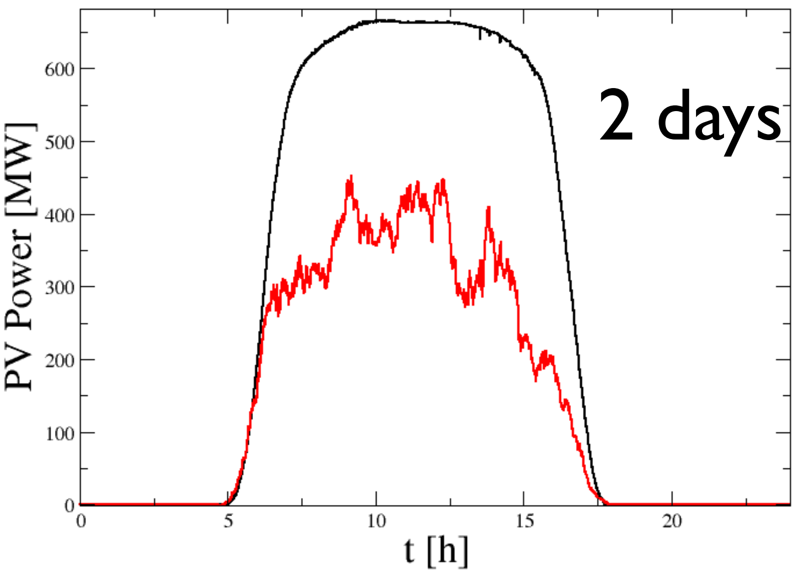
PV

Wind

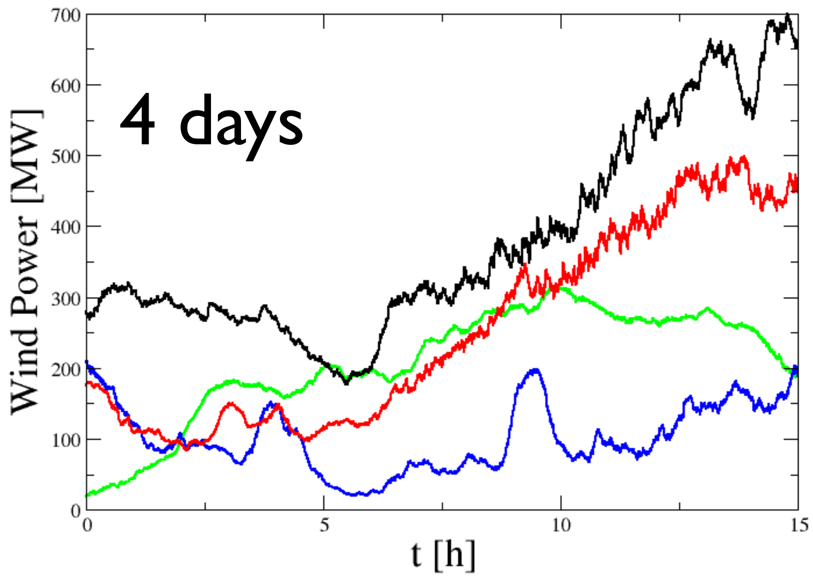


Individual plants





Regional aggregation (AZ+NM)

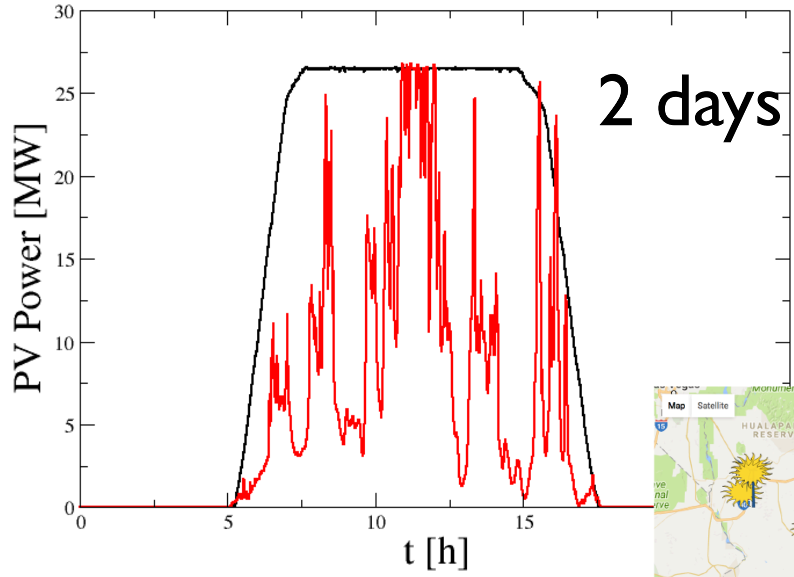


sveri.energy.arizona.edu/ thanks to Will Holmgren for data

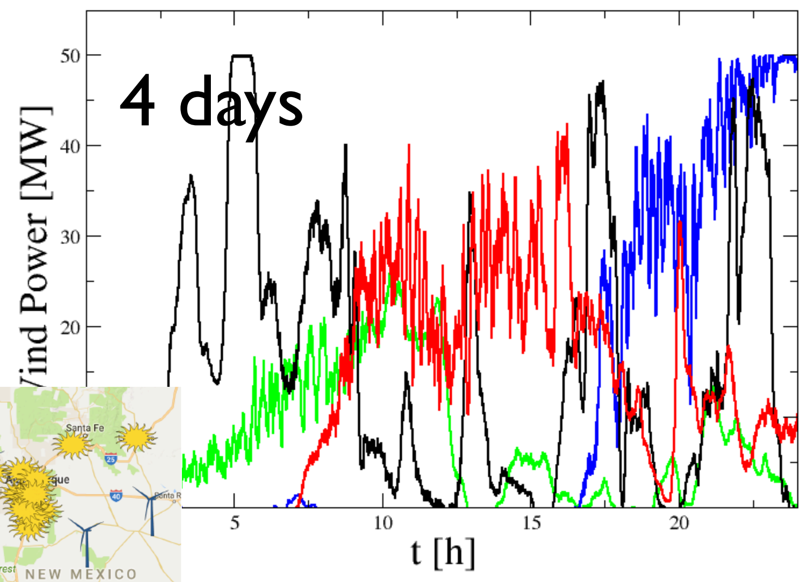
New renewables - fluctuating Feed-in

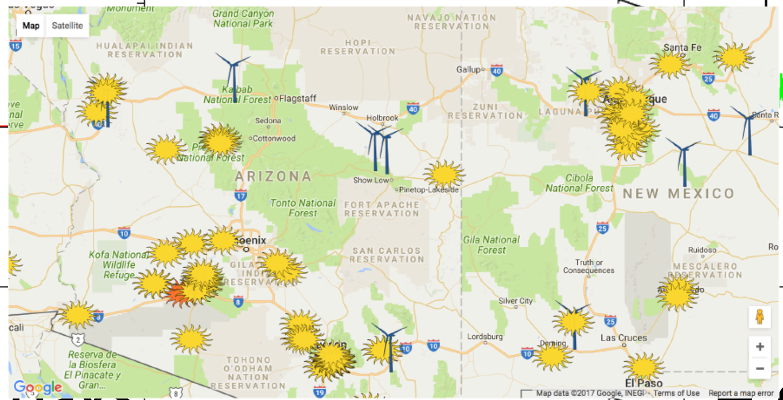
PV

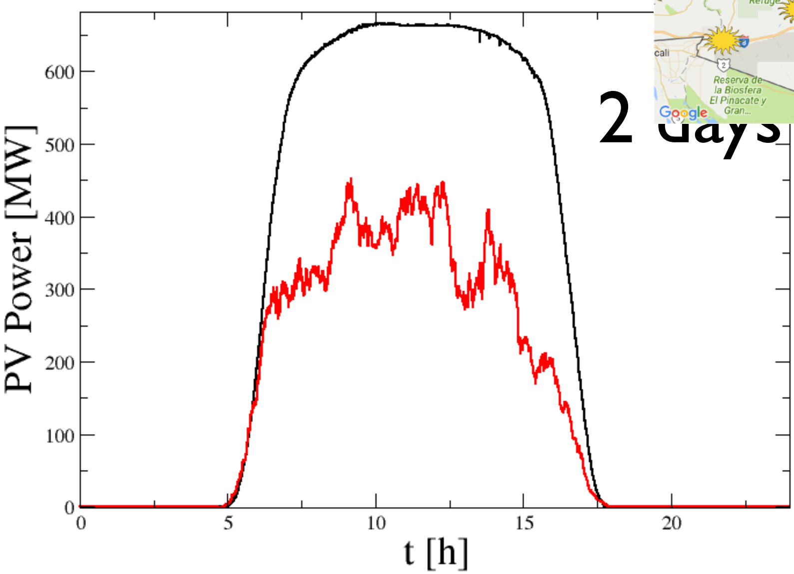
Wind



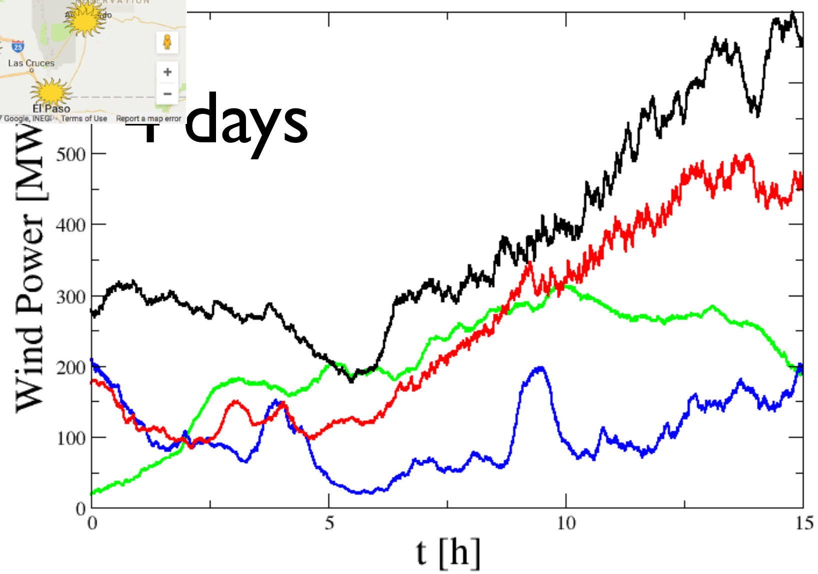
Individual plants







Regional aggregation (AZ+NM)



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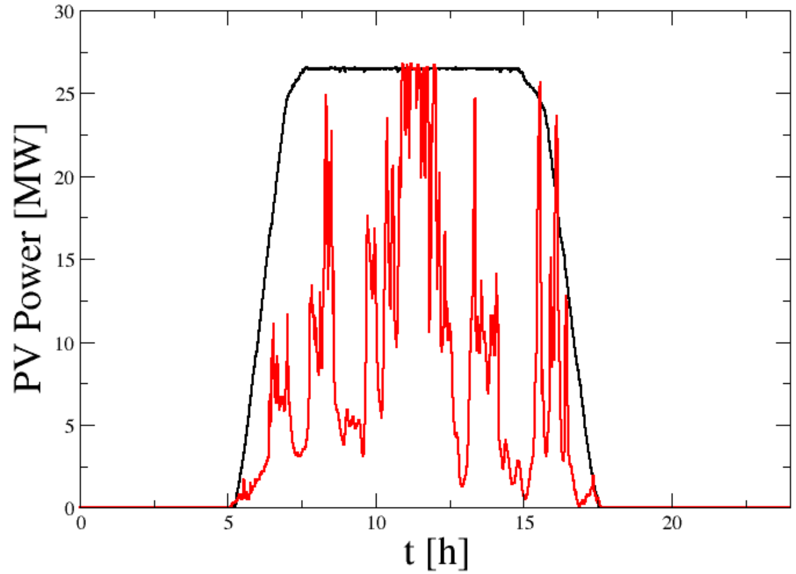
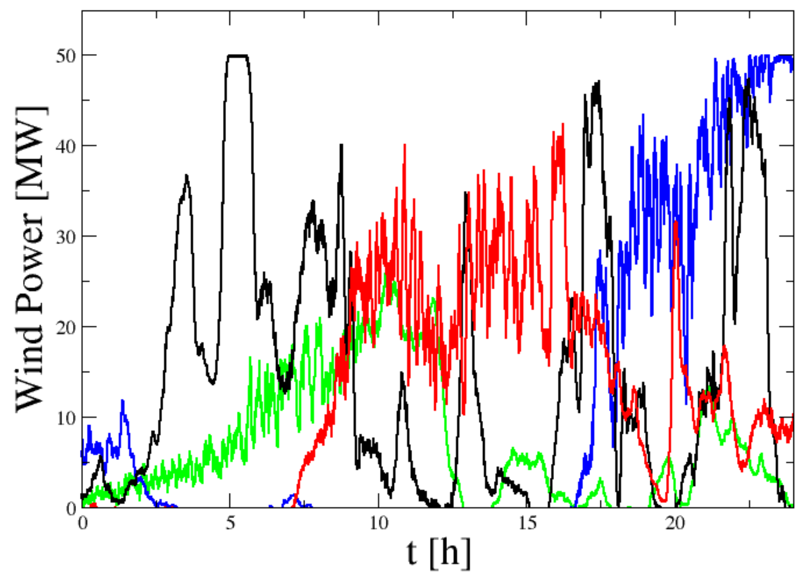
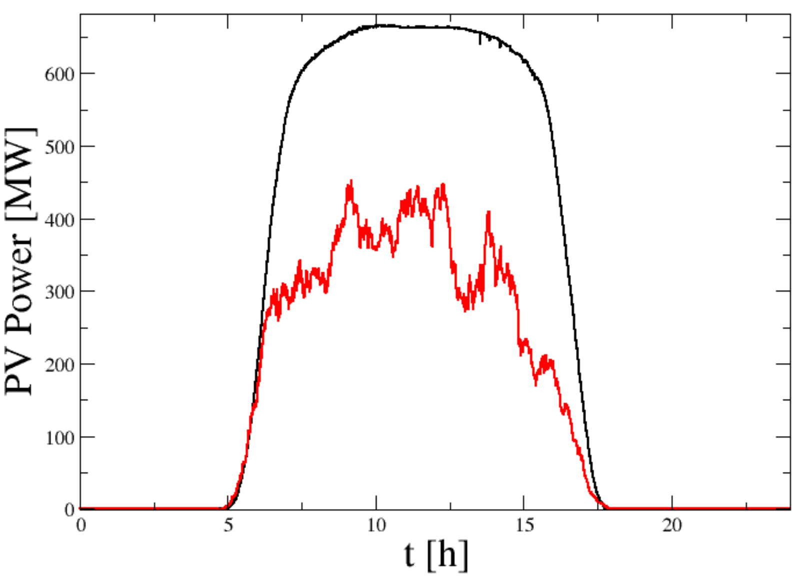
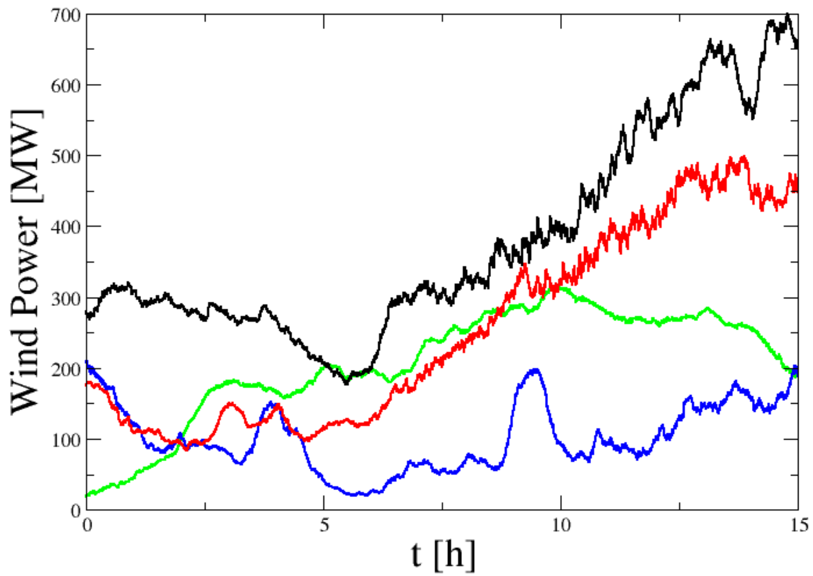




Relative flucs.
$dP/P \sim P^{-1/2}$





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Steady-state vs. dynamics of power systems

Steady-state : power flow Eqs.

$$P_i = \sum_j |V_i V_j| \left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right]$$

- i : node/bus index
- PV-buses : production
- PQ-buses : consumption
- 1 “slack-bus”

Dynamics: swing Eqs. (neglect voltage variations from now on)

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum \left[B_{ij}\sin(\theta_i - \theta_j) + G_{ij}\cos(\theta_i - \theta_j)\right]$$

Steady-state vs. dynamics of power systems

Dynamics: swing Eqs. (neglect voltage variations from now on)

$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum [B_{ij}\sin(\theta_i - \theta_j) + G_{ij}\cos(\theta_i - \theta_j)]$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$

$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$

$$P_i^{(0)} = \sum B_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})$$

The Question : Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

*validity of the expansion close to fixed point ?
*"work" or "energy cost" to stay near there ?

Brownian motion - Langevin equation

P Langevin
1872-1946

Problem : particle in viscous medium subjected to random noise

Diff. eq. for velocity

$$\boxed{\dot{v}(t) = -\frac{\gamma}{m} v(t) + \xi(t)}$$

viscosity

(Gaussian) random force field characterized by

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi(t_1) \xi(t_2) \rangle = \xi_0^2 \, e^{-\chi |t_1 - t_2|}$$

Solution :

$$\boxed{v(t) = e^{-\frac{\gamma}{m} t} \, v(0) + e^{-\frac{\gamma}{m} t} \int_0^t e^{\frac{\gamma}{m} t'} \, \xi(t') \, \mathrm{d}t'}$$

viscosity damping
of initial velocity

???

Brownian motion - Langevin equation

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Diff. eq. for velocity

$$\boxed{\dot{v}(t) = -\frac{\gamma}{m} v(t) + \xi(t)}$$

viscosity

(Gaussian) random force field characterized by

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi(t_1)\xi(t_2) \rangle = \xi_0^2 \, e^{-\chi|t_1-t_2|}$$

Solution : investigate the velocity distribution

$$\langle v(t) \rangle = e^{-\frac{\gamma}{m}t} \, v(0)$$

$$\langle v^2(t) \rangle = e^{-2\frac{\gamma}{m}t} \, v^2(0) + e^{-\frac{2\gamma}{m}t} \, \xi_0^2 \iint_0^t e^{\frac{\gamma}{m}(t_1+t_2)} \, e^{-\chi|t_1-t_2|} \, \mathrm{d}t_1 \, \mathrm{d}t_2$$

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viscosity

(Gaussian) random force field characterized by

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi(t_1)\xi(t_2) \rangle = \xi_0^2 \, e^{-\chi|t_1 - t_2|}$$

Solution : characterize the velocity distribution

$$\lim_{t\to\infty} \langle v(t) \rangle = 0$$

$$\lim_{t\to\infty} \langle v^2(t) \rangle = \frac{\xi_0^2}{(\gamma/m + \chi)\gamma/m}$$

Brownian motion - Langevin equation

P Langevin
1872-1946

Problem : particle in viscous medium subjected to random noise

Diff. eq. for velocity

$$\boxed{\dot{v}(t) = -\frac{\gamma}{m} v(t) + \xi(t)}$$

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Kinetic energy “price” (work from random force)

$$\frac{m}{2} \int_0^T \langle v^2(t) \rangle \mathrm{d}t = \frac{m \xi_0^2}{2} \left[\frac{T}{(\gamma/m + \chi)\gamma/m} - \frac{e^{-2\gamma T/m} - 1}{2(\gamma/m - \chi)\gamma^2/m^2} + \frac{2(e^{-(\gamma/m + \chi)T} - 1)}{(\gamma/m + \chi)(\gamma^2/m^2 - \chi^2)} \right]$$

Some related works

Kinetic energy "price" (work from random force)

$$\frac{m}{2}\int_0^T \langle v^2(t)\rangle \mathrm{d}t = \frac{m\xi_0^2}{2}\left[\frac{T}{(\gamma/m+\chi)\gamma/m} - \frac{e^{-2\gamma T/m}-1}{2(\gamma/m-\chi)\gamma^2/m^2} + \frac{2(e^{-(\gamma/m+\chi)T}-1)}{(\gamma/m+\chi)(\gamma^2/m^2-\chi^2)}\right]$$

The Price of Synchrony:
Evaluating the Resistive Losses in Synchronizing Power Networks

Emma Sjödin, Bassam Bamieh and Dennice F. Gayme

$$\|H\|_{\mathcal{H}_2}^2 = \int_0^\infty \mathrm{E}\left\{y^*(t)y(t)\right\}\ \mathrm{d}t$$

Quadratic form of angle differences, angle deviations, frequencies…

Optimal Placement of Virtual Inertia in Power Grids

Bala Kameshwar Poolla Saverio Bolognani Florian Dörfler*

June 20, 2016

Coherency performance metric $\int_0^\infty \left\{\sum_{i,j=1}^n a_{ij}(\theta_i(t)-\theta_j(t))^2 + \sum_{i=1}^n s_i\,\omega_i^2(t)\right\}dt$

Some related works

Kinetic energy "price" (work from random force)

$$\frac{m}{2}\int_0^T \langle v^2(t)\rangle \mathrm{d}t = \frac{m\xi_0^2}{2}\left[\frac{T}{(\gamma/m+\chi)\gamma/m} - \frac{e^{-2\gamma T/m}-1}{2(\gamma/m-\chi)\gamma^2/m^2} + \frac{2(e^{-(\gamma/m+\chi)T}-1)}{(\gamma/m+\chi)(\gamma^2/m^2-\chi^2)}\right]$$

The Price of Synchrony:
Evaluating the Resistive Losses in Synchronizing Power Network

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Method restricted to $\delta P(t) \sim \delta(t - t_0)$?

Optimal Placement of Virtual Inertia in Power Grids

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June 20, 2016

Power grid with fluctuating feed-in

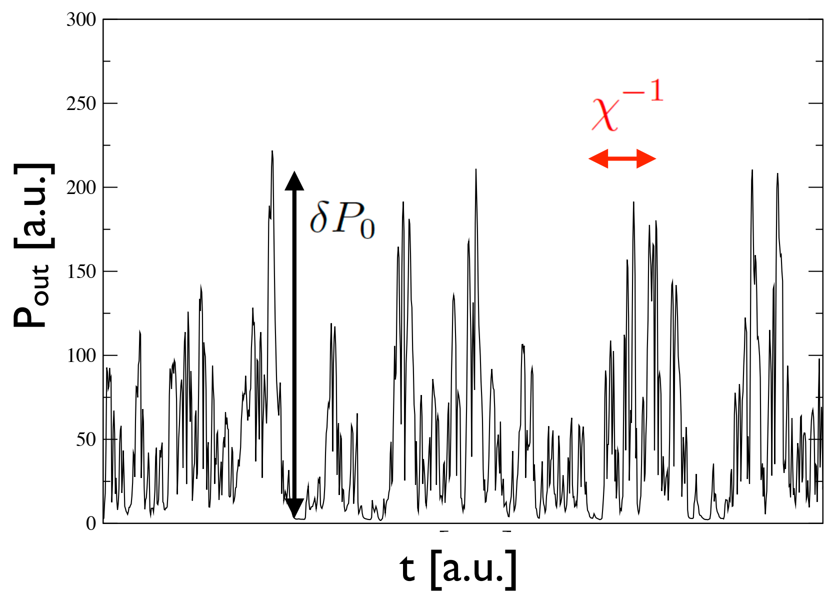
Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$

$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$

$$P_i^{(0)} = \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)})$$



Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

A: (i) linearize the dynamics about a fixed-point solution

$$\delta\dot{\vec{\theta}} = \delta\vec{P} + \mathcal{M}\delta\vec{\theta}$$

(ii) expand angles over eigenmodes of stability matrix

→ *get equation for coefficients of expansion !*

$$\mathcal{M}\vec{\phi}_\beta = \lambda_\beta\vec{\phi}_\beta$$

$$\delta\vec{\theta}(t) = \sum_\beta c_\beta(t)\vec{\phi}_\beta$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

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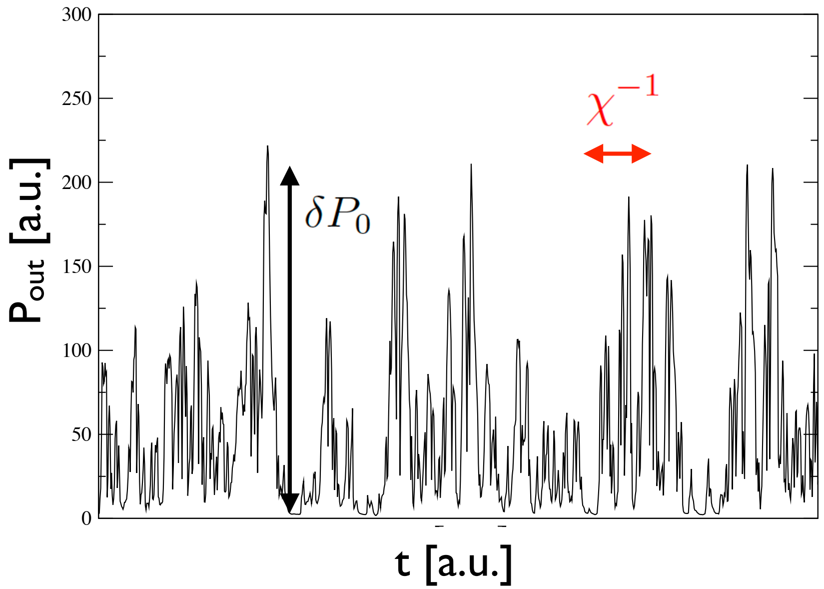
$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



Can one characterize $\delta\theta_i(t)$ given $\delta P_i(t)$?

A: (i) linearize the dynamics about a fixed-point solution

$$\rightarrow \quad \delta\dot{\theta}_i = \delta P_i - \sum B_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \, (\delta\theta_i - \delta\theta_j)$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

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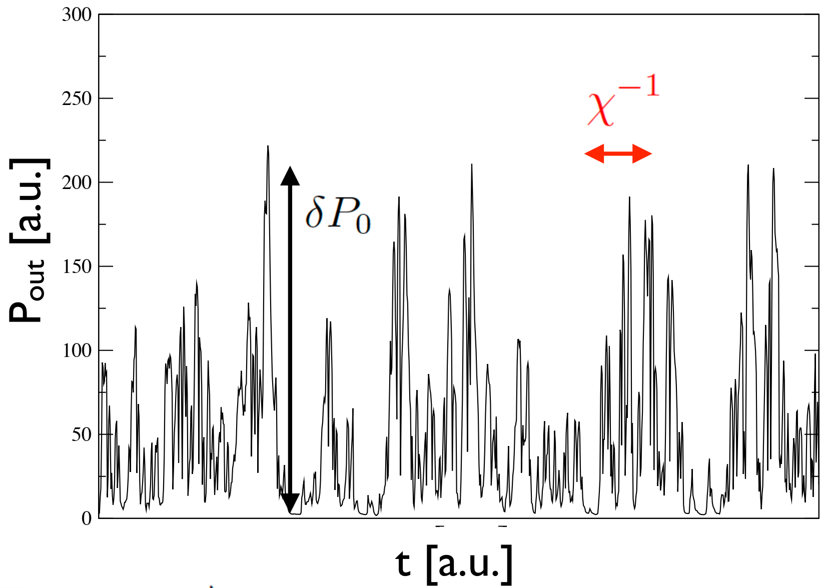


equation for coefficients of expansion $\delta\vec{\theta}(t) = \sum_\beta c_\beta(t)\vec{\phi}_\beta$

Langevin equation : $$\dot{c}_\alpha(t) = \lambda_\alpha c_\alpha(t) + \delta\vec{P}(t) \cdot \vec{\phi}_\alpha$$

gives exponential decay
of deviation (usual)

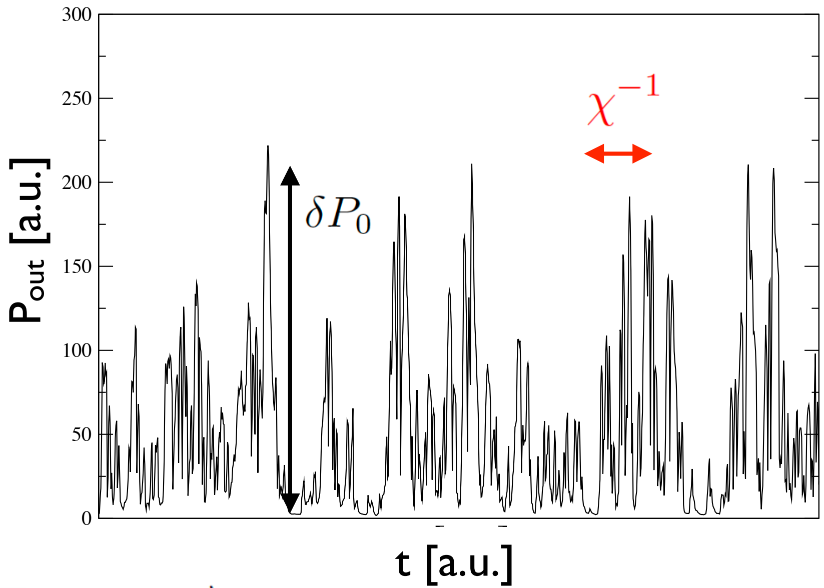
fluctuations about
exponential decay

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



equation for coefficients of expansion $\delta\vec{\theta}(t) = \sum_\beta c_\beta(t) \vec{\phi}_\beta$

$$\boxed{c_\alpha(t) = e^{\lambda_\alpha t} c_\alpha(0) + e^{\lambda_\alpha t} \int_0^t e^{-\lambda_\alpha t'} \, \delta\vec{P}(t') \cdot \vec{\phi}_\alpha \mathrm{d}t'}$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)$$

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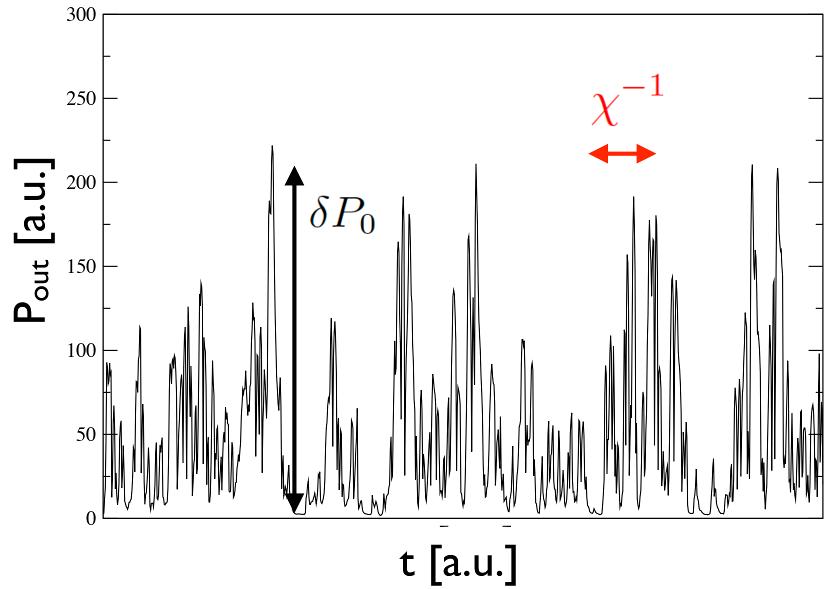


Example of fluctuations $\langle \delta P_i(t) \rangle = 0$ $\langle \delta P_i(t_1) \delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-\chi |t_1 - t_2|}$

- No spatial correlation
- Characteristic time χ^{-1}

$\longrightarrow$ $\langle c_\alpha(t) \rangle = 0$

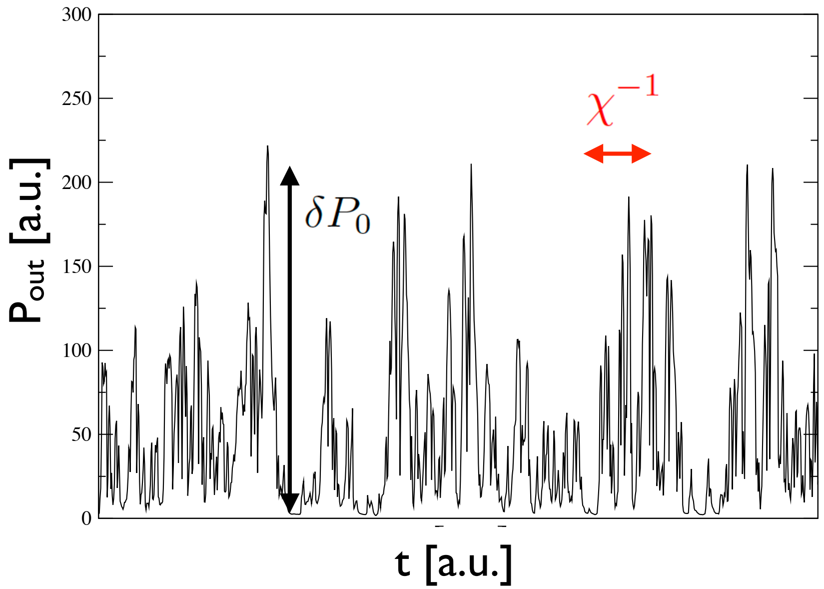
$\longrightarrow$ $\langle c_\alpha^2(t) \rangle = e^{2\lambda_\alpha t} \iint_0^t e^{-\lambda_\alpha (t_1 + t_2)} \langle (\delta \vec{P}(t_1) \cdot \vec{\phi}_\alpha) (\delta \vec{P}(t_2) \cdot \vec{\phi}_\alpha) \rangle \mathrm{d}t_1 \mathrm{d}t_2$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\dot{\theta}_i = P_i - \sum B_{ij} \sin(\theta_i - \theta_j) + G_{ij} \cos(\theta_i - \theta_j)$$

$$\boxed{\begin{aligned} P_i(t) &= P_i^{(0)} + \delta P_i(t) \\ \theta_i(t) &= \theta_i^{(0)} + \delta\theta_i(t) \\ P_i^{(0)} &= \sum B_{ij} \sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$



Example of fluctuations

$$\langle \delta P_i(t) \rangle = 0$$

$$\langle \delta P_i(t_1) \delta P_j(t_2) \rangle = \delta P_0^2 \delta_{ij} e^{-\chi |t_1 - t_2|}$$

- No spatial correlation
- Characteristic time χ^{-1}

$$\boxed{\begin{aligned} &\langle \delta\theta_i(t) \rangle = 0 \\ &\lim_{t\to\infty} \langle \delta\vec{\theta}(t)^2 \rangle = \delta P_0^2 \sum_\alpha \frac{\sum_{\text{noisy } i} |\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha - \chi)} \\ &\lim_{t \ll \lambda_\alpha^{-1}, \chi^{-1}} \langle \delta\vec{\theta}(t)^2 \rangle = \delta P_0^2 \sum_{\text{noisy } i} |\phi_{i,\alpha}|^2 \, t^2 \end{aligned}}$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\delta\dot{\theta}_i = \delta P_i - \sum B_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})\,(\delta\theta_i - \delta\theta_j)$$

$$P_i(t) = P_i^{(0)} + \delta P_i(t)$$

$$\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t)$$

$$P_i^{(0)} = \sum B_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})$$

(i) Validity of expansion $\lim_{t\to\infty}\langle\delta\vec{\theta}(t)^2\rangle = \delta P_0^2 \sum_\alpha \frac{\sum_{\text{noisy } i} |\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha - \chi)} \ll 1$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\delta\dot{\theta}_i = \delta P_i - \sum B_{ij} \cos(\theta_i^{(0)} - \theta_j^{(0)})\,(\delta\theta_i - \delta\theta_j)$$

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(i) Validity of expansion

$$\lim_{t\to\infty} \langle \delta\vec{\theta}(t)^2 \rangle = \delta P_0^2 \sum_\alpha \frac{\sum_{\text{noisy } i} |\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha - \chi)} \ll 1$$

(ii) Dissipation (small angles)

$$\int_0^T \sum g_{ij}\,(\delta\theta_i - \delta\theta_j)^2\,\mathrm{dt} \simeq \gamma\delta P_0^2 \sum_{\alpha,\text{noisy } i} |\phi_{i,\alpha}|^2 \frac{T}{\chi - \lambda_\alpha}$$

$$\gamma \equiv g_{ij}/b_{ij}$$

Power grid with fluctuating feed-in

Angle dynamics (simplified; stay tuned)

$$\delta\dot{\theta}_i = \delta P_i - \sum B_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)})\,(\delta\theta_i - \delta\theta_j)$$

$$\boxed{\begin{aligned} &P_i(t) = P_i^{(0)} + \delta P_i(t) \\ &\theta_i(t) = \theta_i^{(0)} + \delta\theta_i(t) \\ &P_i^{(0)} = \sum B_{ij}\sin(\theta_i^{(0)} - \theta_j^{(0)}) + G_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}) \end{aligned}}$$

(i) Validity of expansion $\quad \lim_{t\to\infty}\langle\delta\vec{\theta}(t)^2\rangle = \delta P_0^2 \sum_\alpha \frac{\sum_{\text{noisy } i}|\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha - \chi)} \ll 1$

(ii) Dissipation $\quad \int_0^T \sum g_{ij}\,(\delta\theta_i - \delta\theta_j)^2\,\mathrm{dt} \simeq \gamma\delta P_0^2 \sum_{\alpha,\text{noisy } i} |\phi_{i,\alpha}|^2 \frac{T}{\chi - \lambda_\alpha}$

(iii) Kinetic energy price $\quad \langle\delta\dot{\vec{\theta}}^2\rangle \simeq \sum_\alpha \left[\delta P_0^2 \sum_{\text{noisy } i} |\phi_{i,\alpha}|^2(1 + 2\lambda_\alpha) + \lambda_\alpha^2\langle c_\alpha^2\rangle\right]$

Power grid with fluctuating feed-in

(i) Validity of expansion
$$\lim_{t\to\infty} \langle \delta\vec{\theta}(t)^2 \rangle = \delta P_0^2 \sum_\alpha \frac{\sum_{\text{noisy } i} |\phi_{i,\alpha}|^2}{\lambda_\alpha(\lambda_\alpha - \chi)} \ll 1$$

(ii) Dissipation
$$\int \sum g_{ij} \left(\delta\theta_i - \delta\theta_j\right)^2 \mathrm{d}t \simeq \gamma \delta P_0^2 \sum_{\alpha, \text{noisy } i} |\phi_{i,\alpha}|^2 \frac{T}{\chi - \lambda_\alpha}$$

(iii) Kinetic energy price
$$\langle \delta\dot{\vec{\theta}}^2 \rangle \simeq \sum_\alpha \left[\delta P_0^2 \sum_{\text{noisy } i} |\phi_{i,\alpha}|^2 (1 + 2\lambda_\alpha) + \lambda_\alpha^2 \langle c_\alpha^2 \rangle \right]$$

All these expression depend on eigenvalues and eigenvectors of the stability matrix ~ weighted Laplacian of network

Main question : are modes with slow relaxation excited by noise ?

Connection between e-values and extension of e-vectors

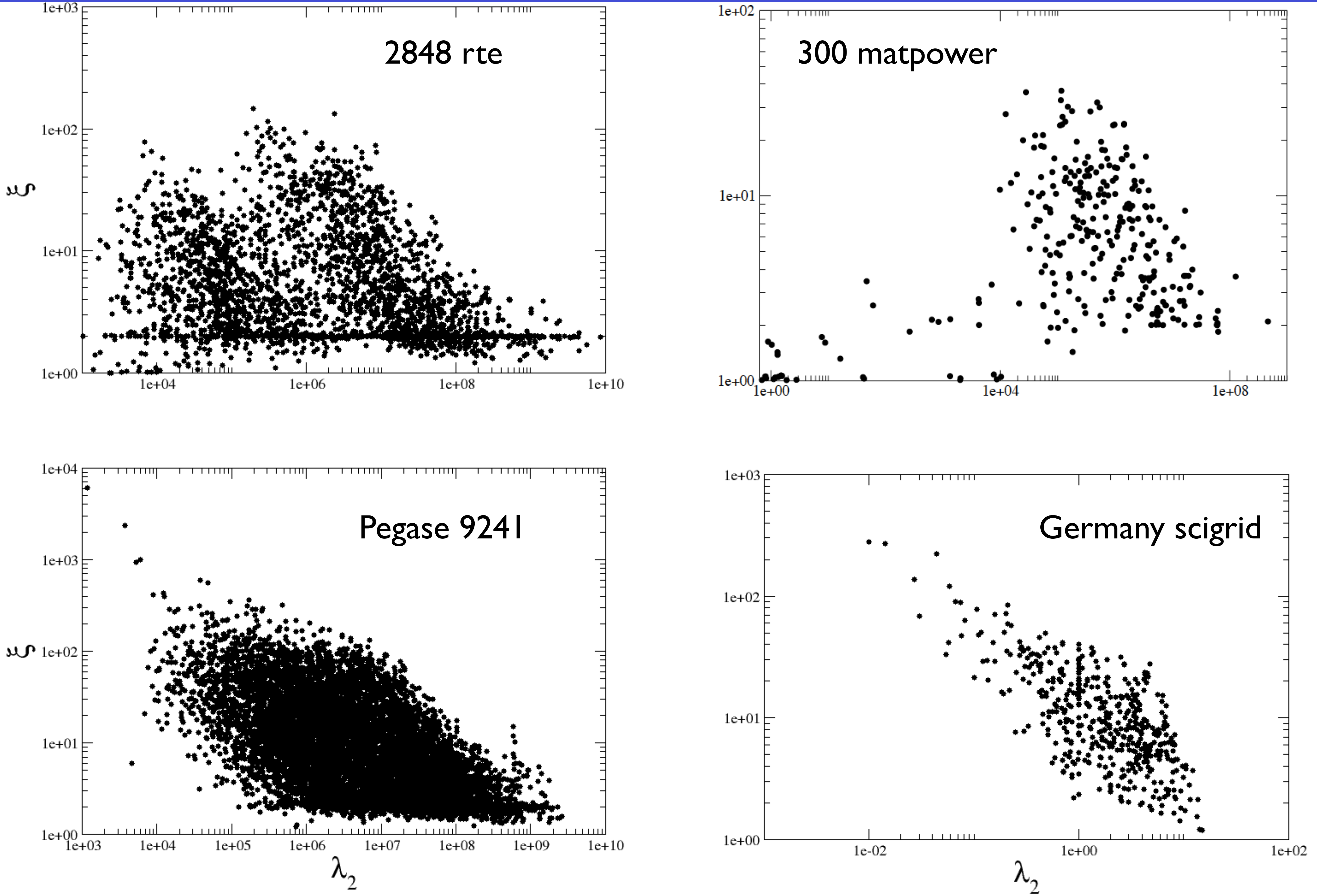
Extension of modes characterized by participation ratio

$$\xi_\alpha = \left(\sum_i |\phi_{\alpha,i}|^4 \right)^{-1}$$

~1 for localized modes

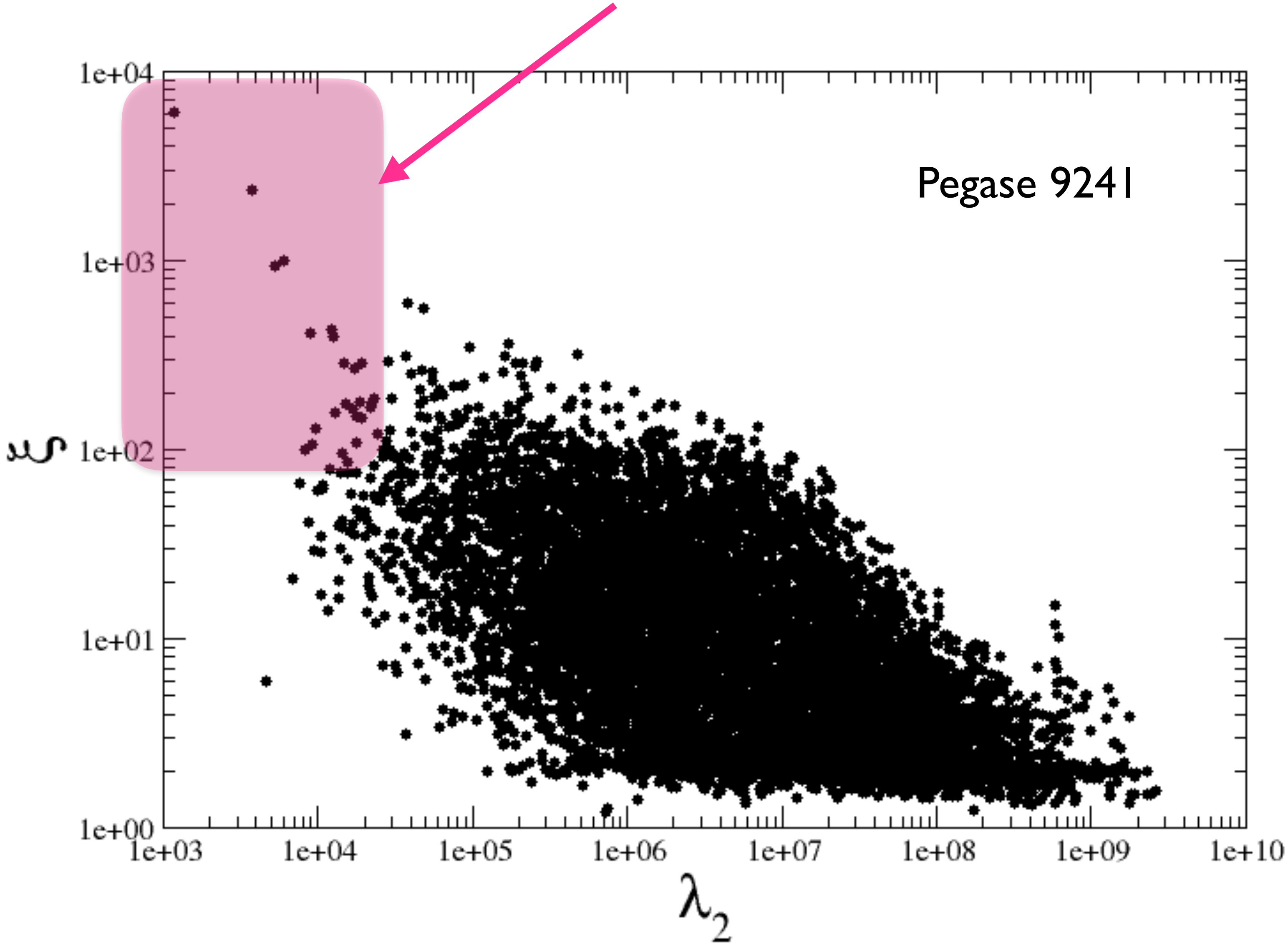
~N for extended modes

Connection between e-values and extension of e-vectors



Connection between e-values and extension of e-vectors

Modes with slower relaxation
are extended - hard to avoid them !



With inertia

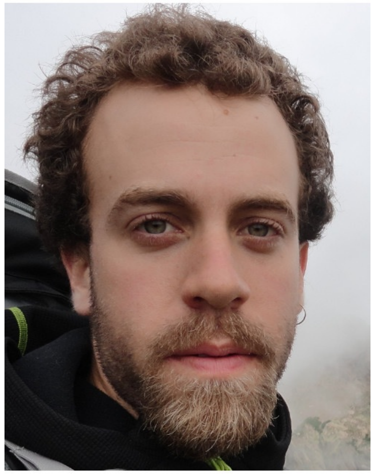
$$I_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum [B_{ij}\sin(\theta_i - \theta_j) + G_{ij}\cos(\theta_i - \theta_j)]$$

$$\boxed{\lim_{t\to\infty} \langle \delta\vec{\theta}(t)^2\rangle + \langle \delta\vec{\omega}(t)^2\rangle = \delta P_0^2 \sum_\alpha \frac{[\mathrm{Re}(\Lambda_\alpha) - \chi] \sum_{\text{noisy } j} \phi_{j,\alpha,L}\, \phi_{j,\alpha,R} / D_{j-N}^2}{\mathrm{Re}(\Lambda_\alpha)(\Lambda_\alpha^* - \chi)(\Lambda_\alpha - \chi)}}$$

Work in progress…

The team

Tommaso Coletta, postdoc



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