Simple if beautiful...

...but is it useful?

Robin Delabays

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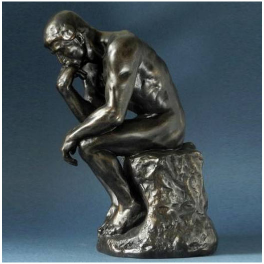
Hes·so VALAIS WALLIS
School of Engineering

Representing reality

Reasons can be:

Representing reality



Reasons can be:

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Representing reality

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- aesthetic;
- ...

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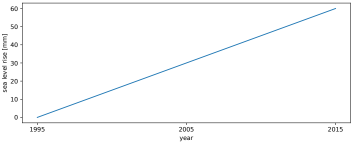

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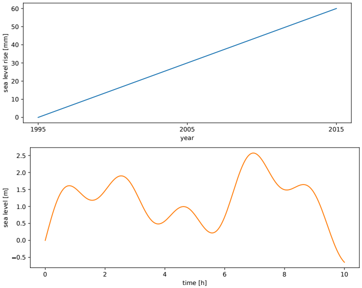

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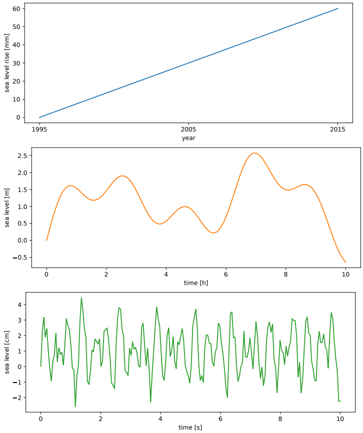

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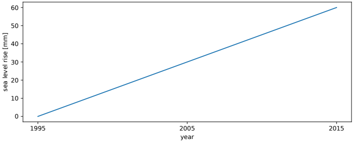

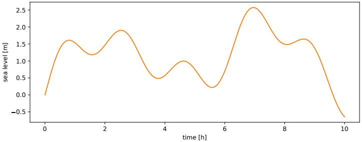

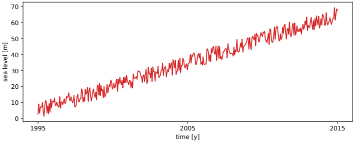

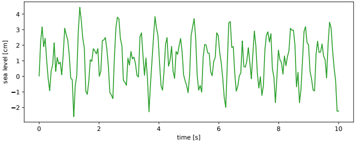

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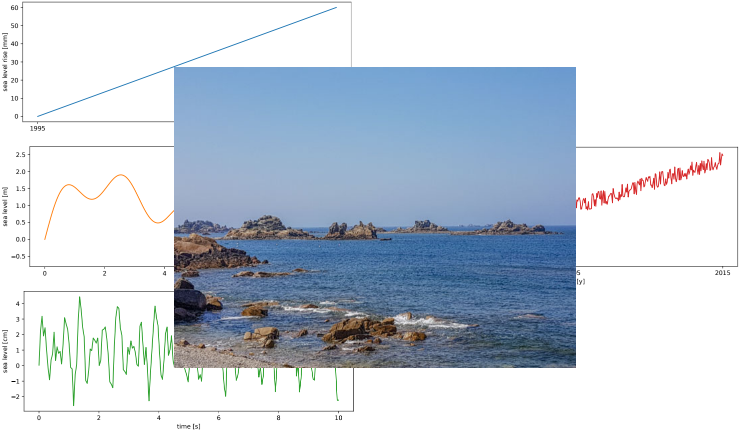

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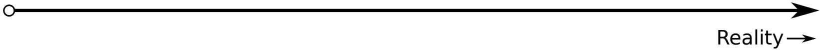

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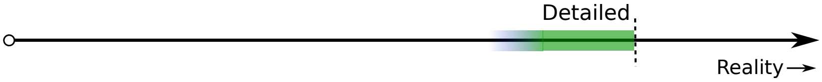
- Focused research;



Accuracy range

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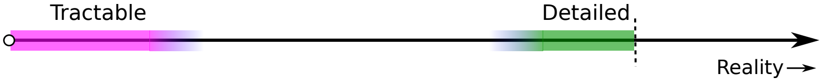

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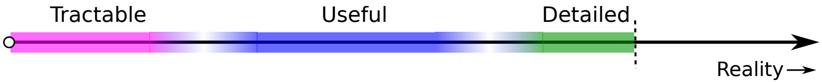

The Power Flow Equations (1)

Power flow in AC grid:

- Apparent, active, and reactive power: $S = P + iQ$;
- Complex voltage, amplitude, and phase: $E = Ve^{i\theta}$;
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Then the power balance at j:

$$S_j = \sum_{k \neq j} S_{jk} = \sum_{k \neq j} E_j y_{jk}^* (E_j^* - E_k^*) = \sum_k E_j E_k^* Y_{jk}^*$$

The Power Flow Equations (2)

Using:

- $S_j = P_j + iQ_j$;
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The Swing Dynamics

Voltage phase dynamics at each point j:

$$\begin{aligned} m_j\ddot{\theta}_j + d_j\dot{\theta}_j &= P_j^{\mathrm{mec}} - P_j^{\mathrm{el}} \\ &= P_j^{\mathrm{mec}} - \sum_k V_j V_k \left[B_{jk}\sin(\theta_j - \theta_k) + G_{jk}\cos(\theta_j - \theta_k)\right] . \end{aligned}$$

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But these equations are hard to solve/integrate!

$\Longrightarrow$ numerics

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So what do we do?

Simplifying assumptions

In high voltage power grids:

- Conductance is small compared to susceptance $\implies G \approx 0$;
- Phase differences are small $\implies \cos(\theta_j - \theta_k) \approx 1$;
- Voltage amplitudes are almost constant $\implies V_j \approx cte$.

Simplifying assumptions

$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]$$

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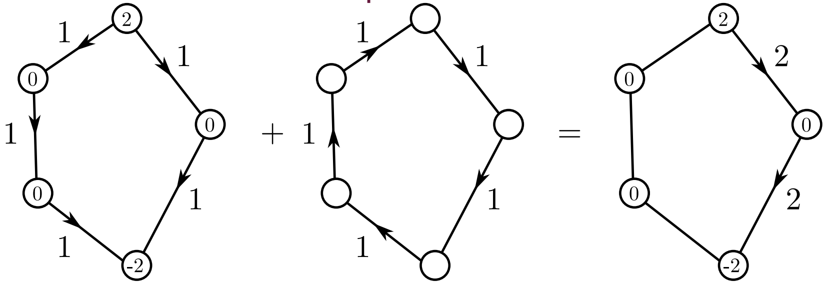
Number of solutions - Loop flows



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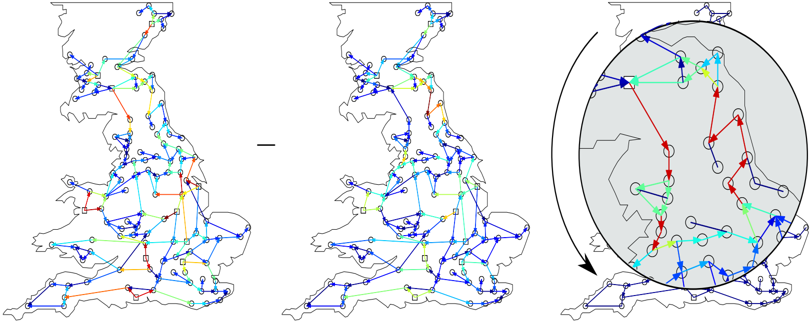

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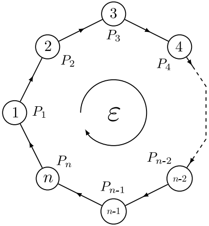

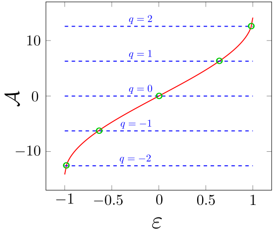

$$q = \frac{1}{2\pi} \sum_{j=1}^{n_k} |\theta_j - \theta_{j+1}|_{2\pi} \in \mathbb{Z}\,.$$

R. D., T. Coletta, and P. Jacquod, *J. Math. Phys.* **57** (2016).

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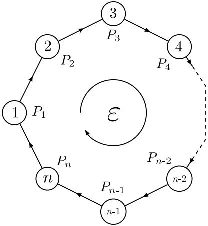

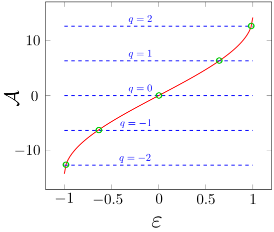

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$$\mathcal{N} \leq \prod_{k=1}^{c} \left[2 \cdot \mathrm{Int}\left(n_k/4\right) + 1\right].$$

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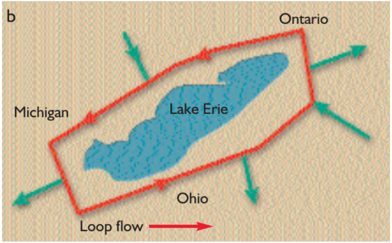
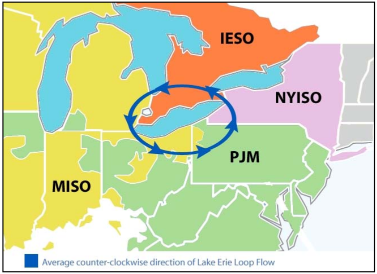
Number of solutions - Lake Erie loop flow





E. J. Lerner, *Industrial Physicist* **9** (2003).

S. G. Whitley, *Technical Report; New York Independent System Operator* (2008).

Performance measures

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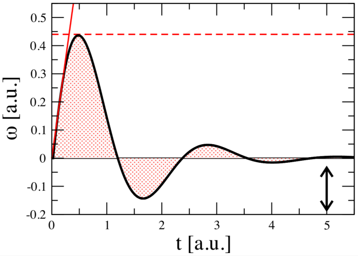
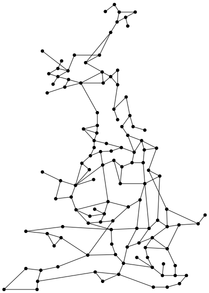
2. Choose a metric: $\|\ldots\|_p = \|\ldots\|_1,\ \|\ldots\|_2,\ \|\ldots\|_\infty,\ \ldots$
3. Compute:

$$\|x_i\|_p = \frac{1}{T}\left[\int_0^T |x_i(t)|^p dt\right]^{1/p}.$$



A few concepts of graph theory

A **graph** is a set of **vertices/nodes** $V = \{1, ..., n\}$ and a set of **edges/lines** $E \subset V \times V$, which are pairs of vertices.



Its **Laplacian matrix**,

$$L_{ij} = \begin{cases} d_i \,, & \text{if } i = j \,, \\ -1 \,, & \text{if } (i,j) \in E \,, \\ 0 \,, & \text{otherwise,} \end{cases}$$

with eigenvalues $0 = \lambda_1 < \lambda_2 \leq ... \leq \lambda_n$.

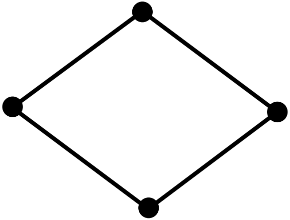
The **Kirchhoff indices**

$$Kf_1 = n \sum_{j \geq 2} \frac{1}{\lambda_j} \,, \qquad Kf_2 = n \sum_{j \geq 2} \frac{1}{\lambda_j^2} \,, \qquad Kf_m = n \sum_{j \geq 2} \frac{1}{\lambda_j^m} \,.$$

Example

$$V = \{1, 2, 3, 4\}$$
$$E = \{(1, 2), (1, 4), (2, 3), (3, 4)\}$$

$$L = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix}$$



$$Kf_1 = 5\,, \qquad Kf_2 = 2.25\,.$$

Quadratic performance measures

Considering $\delta\theta_i$ (resp. $\delta\omega_i$) and $\|...\|_2$ gives additional losses (resp. primary control effort):

M. Tyloo, T. Coletta, and P. Jacquod, *Phys. Rev. Lett.* **120** (2018).

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For short perturbations,

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and for long perturbations,

$$\mathcal{P}_1 \sim Kf_2$$

$$\mathcal{P}_2 \sim Kf_1\,.$$

M. Tyloo, T. Coletta, and P. Jacquod, *Phys. Rev. Lett.* **120** (2018).

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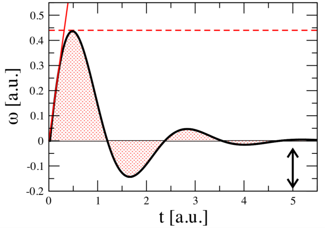
RoCoF under line contingency

Considering $\delta\dot{\omega}_i$ and $\|...\|_\infty$ gives the Rate of Change of Frequency.

After line (i,j) is lost,

$$\max_{k,t} |\delta\dot{\omega}_k| = |\delta\dot{\omega}_i(0^+)| \, .$$



R. D., M. Tyloo, and P. Jacquod, *Chaos* **29** (2019).

Concluding remark

The results presented are mathematically exact, but at the price of simplifying the model.

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