

Eigenstate thermalization and ensemble equivalence in few-body fermionic systems

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We investigate eigenstate thermalization from the point of view of vanishing particle and heat currents between a few-body fermionic Hamiltonian prepared in one of its eigenstates and an external, weakly coupled Fermi-Dirac gas. The latter acts as a thermometric probe, with its temperature and chemical potential set so that there is neither particle nor heat current between the two sub-systems. We argue that the probe temperature can be attributed to the few-fermion eigenstate in the sense that (i) it varies smoothly with energy from eigenstate to eigenstate, (ii) it is equal to the temperature obtained from a thermodynamic relation in a wide energy range, (iii) it is independent of details of the coupling between the two systems in a finite parameter range, (iv) it satisfies the transitivity condition underlying the zeroth law of thermodynamics and (v) it is consistent with Carnot's theorem. These conditions are essentially independent of the interaction between the few fermions. When the interaction strength is weak, however, orbital occupancies in the few fermion system differ from the Fermi-Dirac distribution so that partial currents from or to the probe will eventually change its state. We find that (vi) above a certain critical interaction strength, orbital occupancies become close to the Fermi-Dirac distribution, leading to a true equilibrium between the few-fermion system and the probe. We conclude that for few-body systems with sufficiently strong interaction, the eigenstate thermalization hypothesis of Deutsch and Srednicki is complemented by ensemble equivalence: individual many-body eigenstates define a microcanonical ensemble that is equivalent to a canonical ensemble.

I. INTRODUCTION

In equilibrium statistical mechanics, observable properties of macroscopic systems are given by ensemble averages over microscopic states. When considering the microcanonical ensemble, the average is taken over states of similar energy. All states in that average contribute with equal weight – this is the postulate of equal a priori probabilities [1]. The process following which almost all initial out-of-equilibrium conditions evolve into equilibrium states well represented by the microcanonical ensemble is called thermalization. Within classical mechanics, thermalization and the emergence of equilibrium statistical mechanics is standardly explained at a microscopic level by dynamical complexity [2] : regardless of their origin, almost all classical trajectories of chaotic dynamical systems eventually look the same as they explore ergodically the constant energy hypersurface in phase space. It is usually accepted that the macroscopic laws of classical statistical mechanics emerge from this ergodicity [3].

The situation is different in quantum mechanics, whose time-evolution cannot by itself lead to the uniform covering of the constant energy manifold typical of the microcanonical measure. Instead it has been postulated that thermalization occurs at the level of individual eigenstates [4, 5], in such a way that expectation values of almost all observables taken over almost any such state in a narrow energy interval give the same result in the thermodynamic limit, tending moreover to the microcanonical average [4–6]. For recent reviews of and details on this *Eigenstate Thermalization Hypothesis* (ETH), see Refs. [7, 8].

A question that naturally arises is whether the ETH is

accompanied by an ensemble equivalence similar to the one prevailing in statistical mechanics [1]. In the spirit of the standard construction of the canonical ensemble from the microcanonical one, it has been shown that tracing over some of the degrees of freedom of a pure quantum state gives, for almost all pure states in a narrow energy interval, a reduced density matrix corresponding to the canonical ensemble [9–11]. A similar procedure showed that typical many-body eigenstates have consistently defined thermodynamic entropies [12]. Taken together with the ETH, this can be interpreted as ensemble equivalence at the level of individual many-body eigenstates. This equivalence requires however a bipartitioning of quantal eigenstates and one may wonder if a temperature can be attributed to individual eigenstates without partitioning. To the best of our knowledge, this question was first asked in Ref. [13], where an ad hoc microcanonical partition function was constructed from the shape of many-body eigenstates and shown to bear similarities with the canonical partition function. In particular, the resulting occupation number $f(E)$ of single-particle orbitals was found to become a smooth, monotonously decreasing function of the energy E for sufficiently strong particle-particle interactions [13, 14].

Motivated by this issue and inspired by the experimental technique of scanning thermal microscopy [15], we show in this manuscript that a temperature consistent with a number of standard thermodynamic criteria can indeed be attributed to individual many-body eigenstates $|A\rangle$ of few-fermion systems, by coupling them weakly to an external Fermi-Dirac gas. The coupling allows for particle exchange between the two systems, and the external Fermi-Dirac gas acts as a probe [16–21]: its temperature T_A and chemical potential μ_A are set so that particle and

heat currents between the few-fermion system, prepared in its eigenstate $|A\rangle$, and the probe vanish. We find that T_A varies smoothly with energy from eigenstate to eigenstate regardless of the strength of the particle-particle interaction, despite the fact that the latter strongly influences the structure of the many-body eigenstates (see Fig. 1). In a large energy range, the obtained temperature is furthermore well approximated by the thermodynamic relation [1]

$$T = (\partial S / \partial E)_X^{-1}, \quad (1)$$

with the entropy S determined by the density of states $\rho(E)$ through Boltzmann's formula

$$S(E) = k_B \ln \rho(E), \quad (2)$$

and the subscript X indicating the set of thermodynamic variables kept constant. In our case, this is the system's volume, via the fixed number of orbitals considered.

It is tempting to attribute the probe temperature T_A to the many-body eigenstate of the few-fermion system to which the probe is connected. Below, we argue that this thermometric definition of eigenstate temperature is indeed consistent with standard thermodynamic definitions in the sense that (i) it varies smoothly from eigenstate to eigenstate and is monotonously increasing with energy, (ii) it is equal to a temperature independently obtained from the thermodynamic relation of Eqs. (1) and (2) in a large energy range, (iii) it is independent of details of the system-probe coupling in a finite parameter range, (iv) it satisfies the transitivity condition underlying the zeroth law of thermodynamics and (v) it is consistent with Carnot's theorem. Quite interestingly, these consistency conditions are valid regardless of the particle-particle interaction in the few fermion system. We argue that ensemble equivalence is however achieved only once a further condition is imposed, that (vi) all partial currents between single-particle orbitals in the few fermion system and the Fermi-Dirac gas vanish. This latter condition ensures that the few fermion state does not eventually change over time, and guarantees that a true equilibrium exists between the two subsystems. We find that condition (vi) is achieved for sufficiently strong particle-particle interaction, $U \gtrsim U_c \sim n^{-3}$, with the number n of fermions, in systems with fixed particle density.

Our numerical results are based on exact diagonalization of few-particle systems, and are therefore limited to systems with up to $n = 8$ fermions. They suggest that the threshold interaction strength U_c above which (iv) is valid, is parametrically similar to the many-body quantum chaos threshold derived in Refs. [22, 23], giving a parametric critical interaction strength going down algebraically with the system size and the number of fermions. Taken together, the conditions (i)–(vi) suggest that small fermionic systems with sufficient but not too strong interaction have many-body eigenstates that not only satisfy the ETH – this was already known – but that each of them defines both a microcanonical and a

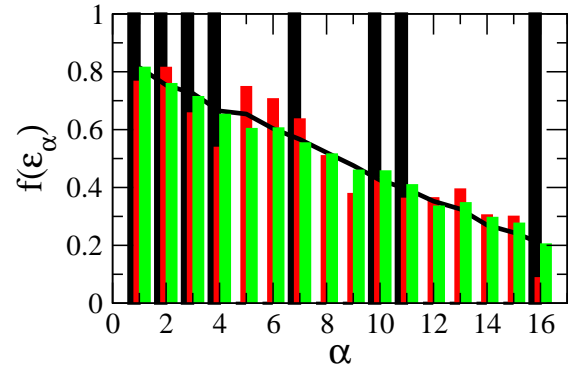


Figure 1: (Color online) Occupation number for the 1001st many-body eigenstate of \mathcal{H}_{sys} (3) with $m = 16$ orbitals and $n = 8$ particles, for $U/\Delta = 0$. [black histogram, with temperature and chemical potential $(T/\Delta, \mu/\Delta) = (5.41, 0.0141)$] 0.1 [red, $(T/\Delta, \mu/\Delta) = (5.43, 0.0139)$] and 0.2 [green, $(T/\Delta, \mu/\Delta) = (5.47, 0.0135)$] The black solid line is the Fermi function corresponding to $(T/\Delta, \mu/\Delta) = (5.47, 0.0135)$. It is slightly unsmooth because the one-body spectrum of (3) is not equidistant.

canonical ensemble which are equivalent to each other in the standard thermodynamic sense.

II. THE MODEL

We consider systems of n interacting, spinless fermions with Hamiltonian given by the two-body random ensemble [24]

$$\mathcal{H}_{\text{sys}} = \sum \epsilon_\alpha c_\alpha^\dagger c_\alpha + \sum U_{\alpha,\beta}^{\gamma,\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta. \quad (3)$$

Here c_α^\dagger and c_α are creation and annihilation operators obeying fermionic anticommutation relations and $\epsilon_\alpha \in [-m\Delta/2, m\Delta/2]$ are m single-particle orbital energies with average spacing Δ . We take ϵ_α as eigenvalues of a $m \times m$ random matrix of the Gaussian orthogonal ensemble [24]. Fermions occupying these single-particle energies interact via a two-body interaction with matrix elements randomly distributed as $U_{\alpha,\beta}^{\gamma,\delta} \in [-U, U]$. The total number of many-body eigenstates of \mathcal{H}_{sys} is $N = m!/n!(m-n)!$ and the corresponding eigenvalues are distributed over a bandwidth $B \simeq n(m-n)\Delta$. In this manuscript we focus on systems at half-filling with $n = m/2$ fermions. Many-body eigenstates of \mathcal{H}_{sys} are linear combinations of totally antisymmetric n -body wavefunctions over m orbitals. Each such many-body eigenstate $|A\rangle$ has single-orbital occupancies $f_A(\epsilon_\alpha) \in [0, 1]$. Occupancies for many-body states corresponding to three different interaction strengths are shown in Fig. 1.

We weakly couple that system to an external noninteracting fermionic gas with Hamiltonian

$$\mathcal{H}_{\text{FD}} = \sum E_i d_i^\dagger d_i. \quad (4)$$

In this Gedankenexperiment, we assume that n consecutive energy eigenvalues E_i , $i = i_0, i_0 + 1, \dots, i_0 + n - 1$, are

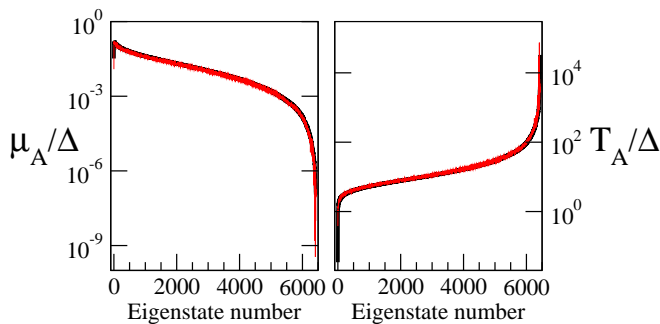


Figure 2: (Color online) Chemical potential (left) and temperature (right) vs. many-body eigenstate number $A \in [1, N/2]$ for a single realization of the two-body randomly interacting fermion model (3) with $m = 16$ orbitals and $n = 8$ fermions and thus $N = 12870$ many-body states. Black dots are for $U/\Delta = 0$, and red dots for $U/\Delta = 0.2$.

equal to the n single-particle energies ϵ_α , $\alpha = 1, 2, \dots, n$. Furthermore, that external fermionic gas is thermalized in the standard textbook way, being e.g. connected to an infinite external reservoir at a tunable temperature T . Consequently, we assume that it is in a mixed state where its single-particle occupancies obey a Fermi-Dirac distribution

$$f_{\text{FD}}(E_i, \mu, T) = \frac{1}{1 + \exp((E_i - \mu)/k_B T)} \quad (5)$$

with chemical potential μ . This Fermi-Dirac gas (FDG) is the probe which will attribute a temperature to each many-body eigenstates of the few interacting fermion system of Eq. (3).

We assume that the tunneling amplitude t between the FDG probe and the few interacting fermion system is small, constant and strictly energy conserving. The few-fermion system is prepared in one of its many-body eigenstates $|A\rangle$. Under our assumptions, the probe-system coupling induces particle (I_A) and heat (J_A) currents between the two subsystems. We approximate them as sums over energy-conserving partial currents,

$$I_A = t^2 \sum_{\alpha} [f_A(\epsilon_\alpha) - f_{\text{FD}}(E_i = \epsilon_\alpha, \mu, T)] \quad (6a)$$

$$J_A = t^2 \sum_{\alpha} (\epsilon_\alpha - \mu) [f_A(\epsilon_\alpha) - f_{\text{FD}}(E_i = \epsilon_\alpha, \mu, T)]. \quad (6b)$$

For each many-body eigenstate $|A\rangle$, I_A and J_A are functions of T and μ . We then tune the temperature $T \rightarrow T_A$ and chemical potential $\mu \rightarrow \mu_A$ of the FDG to ensure that $I_A(\mu_A, T_A) = J_A(\mu_A, T_A) \equiv 0$. If a temperature can at all be attributed to $|A\rangle$, then this temperature is T_A [1].

III. PROBE TEMPERATURE AND CHEMICAL POTENTIAL

Our numerical procedure is the following. We diagonalize exactly the Hamiltonian (3) and calculate its full

set of many-body eigenvectors and the corresponding eigenvalues. We calculate the single-particle occupancies of each eigenstate and use a Newton-Raphson algorithm to obtain the values T_A and μ_A defined by $I_A = J_A = 0$.

Fig. 2 shows results for a single realization of (3) with $n = 8$ fermions on $m = 16$ orbitals, for $U/\Delta = 0$ and $U/\Delta = 0.2$. We see that, first, both temperature and chemical potential vary smoothly from eigenstate to eigenstate in both cases. Second, turning on the interaction has only a small effect on both temperature and chemical potential, despite the fact that it changes the structure of individual many-body eigenstates very significantly, as illustrated in Fig. 1.

We next investigate to what extent the obtained probe temperature corresponds to the thermodynamic relation for the few-fermion system given in Eq. (1). For weak interaction, $U \ll \Delta$, the density of states is Gaussian, $\rho(E) = \rho_0 \exp[-E^2/2\sigma^2]$ with a variance $\sigma^2 \propto n(m-n)$ for dilute systems $n \ll m$ [24]. With Eqs. (1) and (2), this density of states gives a theoretical system temperature [see also Ref. [14]]

$$T_{\text{th}}(E) = -\sigma^2/k_B E. \quad (7)$$

This temperature diverges in the middle of the spectrum and becomes negative at higher temperature, as standardly happens in systems with nonmonotonous density of states [26]. We focus on the lower half of the spectrum, corresponding to positive temperatures.

Fig. 3a confirms that the density of states is Gaussian and that it does not change much as a moderate particle-particle interaction is introduced. Fig. 3b and c further show that in the noninteracting case, the probe temperature is very close to T_{th} of Eq. (7) except in the tail of the many-body density of states. This is due to well-known deviations from Gaussianity there [24]. For the case $m = 16$ and $n = 8$ shown in Fig. 3, the probe temperature differs from T_{th} by less than 10 % for $T_{\text{th}}/\Delta \gtrsim 5.5$ for $U/\Delta = 0$, corresponding to about 90 % of the many-body eigenstates. A moderate interaction has very little impact, except in the middle of the spectrum, where large energy eigenvalues fluctuations increase the discrepancy between the probe temperature and T_{th} . We found that for $U/\Delta = 0.2$, the difference between the two remains below 10 % for $50 \gtrsim T_{\text{th}}/\Delta \gtrsim 5.5$, corresponding to about 80 % of the spectrum. These results do not significantly change upon further increase of the interaction strength as long as $U/\Delta \ll 1$.

IV. FROM PROBE TO EIGENSTATE TEMPERATURE

The results presented so far seem to indicate that the probe temperature gives more than just an operational temperature definition for individual many-body eigenstates. We now argue that this definition satisfies further properties expected of an absolute temperature [1]. First, it is clear from the currents definition, Eqs. (6), that the

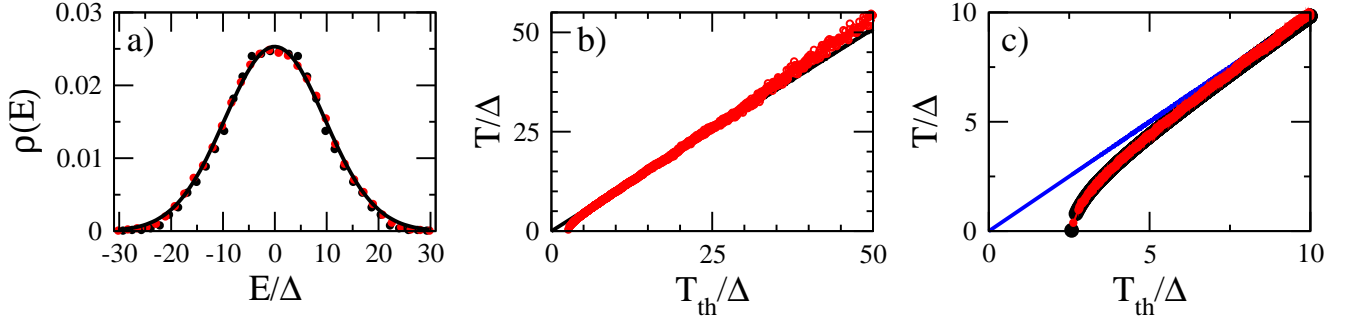


Figure 3: (Color online) a) Many-body density of states. b) Numerically obtained eigenstate temperature T vs. theoretical temperature T_{th} of Eq. (7) for $0 < T_{\text{th}}/\Delta \leq 50$. The interval contains about 90% of the positive temperature eigenstates. c) Same as in panel b) for a restricted range $0 < T_{\text{th}}/\Delta \leq 10$, showing deviations at low energy due to non-Gaussian tails in the density of states [24]. All panels correspond to a single realization of the two-body randomly interacting fermion model (3) with $m = 16$ orbitals, $n = 8$ fermions and $N = 12870$ many-body states. Black dots are for $U = 0$. and red dots for $U = 0.2\Delta$.

temperature definition does not depend on the magnitude of the probe-system coupling amplitude t . This is so only as long as t is small enough, so that Eqs. (6) are valid, but still leaves a sizeable range of parameter t .

Second, the probe temperature is defined by the vanishing of the particle and heat currents between the probe and the system, $I_A(\mu_A, T_A) = J_A(\mu_A, T_A) \equiv 0$. Let us introduce a second many-body state $|B\rangle$ and couple it to the probe while keeping the latter's temperature and chemical potential the same. Let us further suppose then that there is no current between $|B\rangle$ and the probe, $I_B(\mu_A, T_A) = J_B(\mu_A, T_A) \equiv 0$. From Eqs. (6), one straightforwardly concludes that there would be no current either between $|A\rangle$ and $|B\rangle$, $I_{AB} = t^2 \sum_{\alpha} [f_A(\epsilon_{\alpha}) - f_B(\epsilon_{\alpha})] = 0$, $J_{AB} = t^2 \sum_{\alpha} (\epsilon_{\alpha} - \mu) [f_A(\epsilon_{\alpha}) - f_B(\epsilon_{\alpha})] = 0$. The transitivity condition of the zeroth law of thermodynamics is thus satisfied by the probe definition of the temperature.

Third, in the regime of validity of Eqs. (6), the system may temporarily work as a heat engine when the probe is biased away from the equilibrium condition, i.e. $T_A \rightarrow T_A + \delta T$ and $\mu_A \rightarrow \mu_A + \delta \mu$. Assume that all fermions carry an electric charge q . With this bias, both a heat and an electric current flow, with the latter being accompanied by electrical work. The efficiency of the resulting heat engine is given by $\eta = -I_A \delta \mu / J_A$ [25] with, from Eqs. (6)

$$I_A(\delta \mu, \delta T) = -t^2 \sum_{\alpha} [\partial_{\mu} f_{\text{FD}} \delta \mu + \partial_T f_{\text{FD}} \delta T], \quad (8a)$$

$$J_A(\delta \mu, \delta T) = -t^2 \sum_{\alpha} (\epsilon_{\alpha} - \mu) [\partial_{\mu} f_{\text{FD}} \delta \mu + \partial_T f_{\text{FD}} \delta T], \quad (8b)$$

where in both expressions, $f_{\text{FD}} = f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu, T)$ and both δT and $\delta \mu$ are assumed very small to justify the linearization of the Fermi-Dirac distributions. A straightforward calculation gives that the maximal efficiency of the engine is given by

$$\eta_{\text{max}} = \left(\frac{\sqrt{1 + \mathcal{Z}\mathcal{T}} - 1}{\sqrt{1 + \mathcal{Z}\mathcal{T}} + 1} \right) \frac{|\delta T|}{T_A}. \quad (9)$$

The dimensionless figure of merit is given by $\mathcal{Z}\mathcal{T}^{-1} = \mathcal{L}^{(0)}\mathcal{L}^{(2)} / (\mathcal{L}^{(1)})^2 - 1$, with $\mathcal{L}^{(0)} = \sum_{\alpha} \partial_{\mu} f_{\text{FD}}$, $\mathcal{L}^{(1)} = \sum_{\alpha} \partial_T f_{\text{FD}} T_A$ and $\mathcal{L}^{(2)} = \sum_{\alpha} (\epsilon_{\alpha} - \mu_A) \partial_T f_{\text{FD}} T_A$. Eq. (9) defines a relative temperature scale in that η_{max} is a function of the temperature difference δT between system and probe, in agreement with Carnot's theorem.

V. MANY-BODY EIGENSTATE TEMPERATURE

There seem to be good reasons to take the probe temperature as a definition of the many-body eigenstate temperature. Nevertheless, a further condition needs to be satisfied before this is done. In general, $I_B(\mu_A, T_A) = J_B(\mu_A, T_A) \equiv 0$ still allows for partial currents $[f_A(\epsilon_{\alpha}) - f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu, T)] \neq 0$ and $(\epsilon_{\alpha} - \mu)[f_A(\epsilon_{\alpha}) - f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu, T)] \neq 0$ in Eqs. (6). Therefore, the state of the few-fermion system will eventually change, even with a weak, finite system-probe coupling, unless detailed balance conditions are satisfied,

$$[f_A(\epsilon_{\alpha}) - f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu, T)] = 0, \quad \forall \alpha. \quad (10)$$

This of course means that particle occupancies in few-fermion states are given by the Fermi-Dirac distribution. Looking at Fig. 1 we see that this may occur for sufficiently interacting few-fermion systems. To quantify the rate at which Fermi-Dirac like occupancies emerge as the interaction strength increases, we investigate the variance of the partial currents making up the particle and heat currents of Eq. (6),

$$\delta I_A^2 = t^4 \sum_{\alpha} [f_A(\epsilon_{\alpha}) - f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu_A, T_A)]^2, \quad \delta J_A^2 = t^4 \sum_{\alpha} (\epsilon_{\alpha} - \mu_A)^2 [f_A(\epsilon_{\alpha}) - f_{\text{FD}}(E_i = \epsilon_{\alpha}, \mu_A, T_A)]^2. \quad (11)$$

These current variances vanish only when the detailed balance conditions of Eq. (10) are satisfied. When this is the case, orbital occupancies in the few fermion system are given by the Fermi-Dirac distribution, partial currents accordingly vanish and the many-body eigenstate $|A\rangle$ is at equilibrium with the thermometric Fermi-Dirac gas in the usual sense. In particular, the weak coupling between the two subsystems does not change the state of the few fermion system.

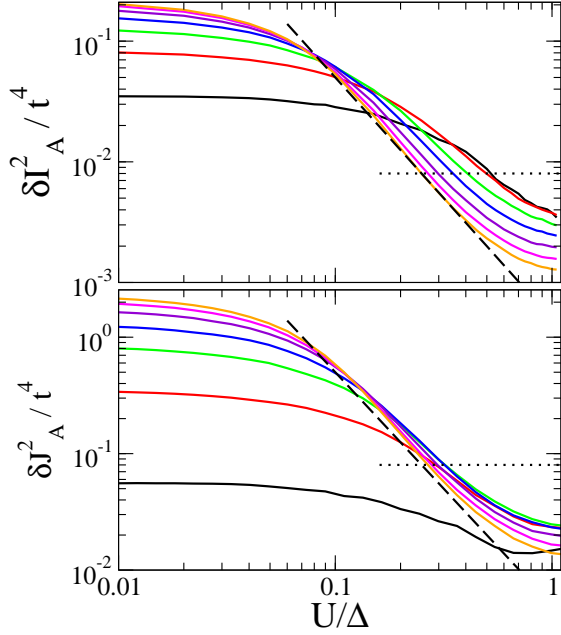


Figure 4: (Color online) Deviation from detailed balance for the particle current (top panel) and the heat current (bottom panel) as a function of the interaction strength U/Δ for 1000 realizations of the two-body randomly interacting fermion model (3) with $m = 12$ orbitals and $n = 6$ fermions and thus $N = 924$ many-body states. Excitation energies above the ground state energy are $\delta E/\Delta = 2$. (black curve), 4. (red), 6. (green), 8. (blue), 10. (violet), 12. (magenta), and 14 (orange). Dashed lines indicate a power-law decay $\propto (U/\Delta)^{-2}$. The dotted line indicates the arbitrarily chosen threshold $\delta I_A^2/t^4 = 8 \times 10^{-3}$ and $\delta J_A^2/t^4 = 8 \times 10^{-2}$ used to define critical interaction strengths U_{c1} and U_{c2} .

Fig. 4 shows δI_A^2 and δJ_A^2 as a function of the normalized interaction strength U/Δ in the few fermion system. It is seen that as U/Δ increases, both variances decrease with rates that depend on the excitation energy above the few fermion ground-state. At high enough excitation energy – though not necessarily too high – we find that $\delta I_A^2, \delta J_A^2 \sim (U/\Delta)^{-2}$ until they saturate at a value depending on both the number of single-particle orbitals and of particles in the few fermion system. We have seen

this behavior for other values of m and n , not shown in Fig. 4.

We are interested in the parametric dependence of the rate at which the current variances vanish. To that end, we define critical interaction strengths U_{c1} and U_{c2} with $\delta I_A^2(U_{c1}) = 8 \times 10^{-3} t^4$ and $\delta J_A^2(U_{c2}) = 8 \times 10^{-2} t^4$. We chose these values, somehow arbitrary, because they correspond to interaction strengths with significantly reduced current variances, but well before their large U , finite-size saturation for all cases considered, $m/n = 2$ and $n = 4, 5, \dots, 8$. Fig. 5 shows the obtained values of U_{c1} (solid circles) and U_{c2} (empty circles) as a function of the normalized excitation energy ϵ/B . At small excitation energy close to the ground-state energy, both U_{c1} and U_{c2} increase with ϵ/B . This reflects the fact that low excitations effectively restrict the number of available single-particle orbitals, which allows a faster transition to a Fermi-Dirac distribution. Furthermore, quasiparticles with small excitation energies carry very little heat current, which explains why U_{c2} is always very small at small excitation energies. For higher excitation energy, both U_{c1} and U_{c2} are monotonously decreasing functions of the number of particles and orbitals, furthermore this holds above an excitation energy that also decreases with the number of particles.

Because of the restricted range of variation of n reachable by exact diagonalization, it is hard to extract a parametric dependence of U_c . Nevertheless, the data shown in Fig. 5 at half filling, $n = m/2$ seem to indicate a behavior $U_{c1} \propto n^{-3}$, consistent with the emergence of quantum chaos reported in Ref. [23]. They also suggest that for

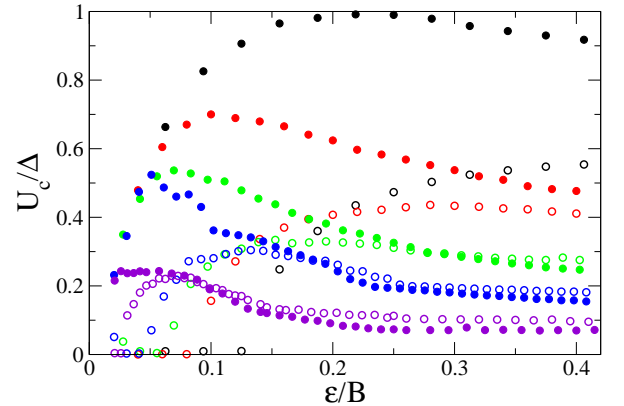


Figure 5: (Color online) Critical interaction strength U_c vs. normalized excitation energy ϵ/B with the many-body band-width $B \simeq n(m-n)\Delta$, for $m = 8, n = 4$ (black), $m = 10, n = 5$ (red), $m = 12, n = 6$ (green), $m = 14, n = 7$ (blue) and $m = 16, n = 8$ (violet). Solid circles give U_{c1} and empty circles give U_{c2} (see text).

based critical interactions become the same. With these data we conjecture that eigenstate thermalization is accompanied by ensemble equivalence, where each individual few-fermion eigenstates exhibits a Fermi-Dirac occupancy distribution, and accordingly defines a canonical ensemble, above a critical interaction strength $U_c \propto n^{-3}$ for systems at half filling. For $n = 8$, this already corresponds to a rather weak interaction strength, $U_c \lesssim 0.1\Delta$.

VI. CONCLUSION

We have shown that few fermion systems have eigenstates with a Fermi-Dirac occupancy of single-particle

orbitals, provided they have a sufficiently strong interaction. Our results indicate that, not too close to the ground-state energy, the critical interaction strength scales parametrically as $U_c \propto n^{-3}$ in systems with constant filling factor. In particular, $U_c \lesssim 0.1\Delta$ with the single-particle orbital spacing Δ , for $n = 8$ fermions on $m = 16$ orbitals. This indicates that for still small systems with, say, $n = 20$, the critical interaction strength is only a fraction of this single-particle orbital spacing. Ensemble equivalence at the level of individual few-body eigenstates is therefore achieved very fast in the number n of fermions, and requires only a weak interaction.

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