Transient Primary Control Effort of AC Electric Power Networks under Line Contingencies [1]

Philippe Jacquod and Tommaso Coletta

School of Engineering, University of Applied Sciences of Western Switzerland, CH-1950 Sion, Switzerland

I. Abstract

We evaluate the robustness of high voltage AC power grids under a line fault. We find that, within the DC power flow approximation, the transient primary control effort necessary to restore synchrony under such a fault depends on whether the faulted line connects two passive, two active buses or one active to one passive bus. In all cases, performance measures depend quadratically on the original load on the faulted line multiplied by an inertia-dependent prefactor. When the faulted line is connected to at least one passive bus, this inertia-dependent prefactor further depends on the topology of the network. Rather unexpectedly, we find that faulting moderately loaded lines often leads to large response due to the topology dependent prefactor.

Approach based on $\mathcal{L}_\in$-norms applied to fluctuating feed-in so far [2, 3, 4, 5, 6], but not to fault line before this !

II. Power Network Model and Performance Measures

Swing equations in the DC approximation

$$\boldsymbol{M}\ddot{\boldsymbol{\theta}} = -\boldsymbol{D}\dot{\boldsymbol{\theta}} + \boldsymbol{P} - \boldsymbol{L_b}\boldsymbol{\theta}$$

- inertia and damping coefficient matrices : $\boldsymbol{M} = \mathrm{diag}(\{m_i\})$ and $\boldsymbol{D} = \mathrm{diag}(\{d_i\})$, $i = 1, \ldots N$
- power injection vector $\boldsymbol{P}$
- N-bus network modeled by Laplacian matrix $\boldsymbol{L_b}$

Instantaneous line fault

$\boldsymbol{L_b}(t) = \boldsymbol{L_b}^{(0)} - \delta(t)\,\tau\, b_{\alpha\beta}\boldsymbol{e}_{(\alpha,\beta)}\boldsymbol{e}_{(\alpha,\beta)}^\top$
$\boldsymbol{e}_l \in \mathbb{R}^N$ with $(\boldsymbol{e}_l)_i = \delta_{il}$, $i, l = 1, \ldots N$
$\boldsymbol{e}_{(l,q)} = \boldsymbol{e}_l - \boldsymbol{e}_q$

Angle and frequency deviations

Synchronous operational state : $(\boldsymbol{\theta}^\star, \boldsymbol{\omega}) := (\boldsymbol{L_b}^\dagger\boldsymbol{P}, 0)$
$\boldsymbol{\varphi}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^\star$: angle deviation from synchronous operational state
$\boldsymbol{\omega}(t) = \dot{\boldsymbol{\varphi}}(t)$: frequency deviation from synchronous operational state

Performance measures

$$\mathcal{P} = \int_0^\infty [\,\boldsymbol{\varphi}^\top \boldsymbol{\omega}^\top]\boldsymbol{Q}\begin{bmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\omega}\end{bmatrix}\mathrm{d}t\,, \quad \boldsymbol{Q} = \begin{bmatrix}\boldsymbol{Q}^{(1,1)} & 0\\ 0 & \boldsymbol{Q}^{(2,2)}\end{bmatrix}$$

$\boldsymbol{Q}^{(1,1)} = 0$ and $\boldsymbol{Q}^{(2,2)}$ chosen to test property of specific interest

Here we focus on case $\boldsymbol{Q}^{(1,1)} = 0$ and $\boldsymbol{Q}^{(2,2)} = \boldsymbol{D}$ a.k.a. primary control effort [4]

Assume constant ratio of damping to inertia, $d_i/m_i = \gamma$ $\forall i$
!!! This means in particular that $m_i \neq 0$ $\forall i$, i.e. all buses have inertia
⇒ Consider Kron reduced network with only rotating machines left (Kron-reduce all other buses) (active buses : not Kron-reduced; passive buses : Kron-reduced)

III. General method

(A) Swing equations relative to the operating point

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}} = -\boldsymbol{D}\dot{\boldsymbol{\varphi}} - \boldsymbol{L_b}\boldsymbol{\varphi} + \delta(t)\,\tau\, b_{\alpha\beta}\boldsymbol{e}_{(\alpha,\beta)}\boldsymbol{e}_{(\alpha,\beta)}^\top(\boldsymbol{\theta}^\star + \boldsymbol{\varphi}) \tag{1}$$

(B) Formal solution to Eq. (1) with the initial condition $(\overline{\boldsymbol{\varphi}}(0), \overline{\boldsymbol{\omega}}(0)) = (0, 0)$:

$$\begin{bmatrix}\overline{\boldsymbol{\varphi}}(t)\\ \overline{\boldsymbol{\omega}}(t)\end{bmatrix} = e^{\boldsymbol{A}t}\underbrace{\begin{bmatrix}0\\ \boldsymbol{M}^{-1/2} b_{\alpha\beta}\,\tau\,\boldsymbol{e}_{(\alpha,\beta)}\boldsymbol{e}_{(\alpha,\beta)}^\top\boldsymbol{\theta}^\star\end{bmatrix}}_{\boldsymbol{B}} \tag{2}$$

with

$$\boldsymbol{A} = \begin{bmatrix}0 & \mathbb{I}\\ -\boldsymbol{M}^{-1/2}\boldsymbol{L_b}\boldsymbol{M}^{-1/2} & -\boldsymbol{M}^{-1}\boldsymbol{D}\end{bmatrix} \tag{3}$$

(C) Performance measure

$$\mathcal{P} = \boldsymbol{B}^\top\boldsymbol{X}\boldsymbol{B}\,, \tag{4}$$

with the *observability Gramian* $\boldsymbol{X} = \int_0^\infty e^{\boldsymbol{A}^\top t}\boldsymbol{Q}^{\mathrm{M}}e^{\boldsymbol{A}t}\mathrm{d}t$, and

$$\boldsymbol{Q}^{\mathrm{M}} = \begin{bmatrix}\boldsymbol{M}^{-1/2}\boldsymbol{Q}^{(1,1)}\boldsymbol{M}^{-1/2} & 0\\ 0 & \boldsymbol{M}^{-1/2}\boldsymbol{Q}^{(2,2)}\boldsymbol{M}^{-1/2}\end{bmatrix} \tag{5}$$

(D) Lyapunov Equation

For asymptotically stable systems (i.e. when all eigenvalues of $\boldsymbol{A}$ have negative real part) $\boldsymbol{X}$ satisfies the *Lyapunov equation*

$$\boldsymbol{A}^\top\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A} = -\boldsymbol{Q}^{\mathrm{M}} \tag{6}$$

References

[1] T. Coletta and Ph. Jacquod, arXiv:1711.10348v2 (2018).

[2] E. Tegling, B. Bamieh and D.F. Gayme, IEEE Transactions on Control of Network Systems **2**, 254 (2015).

[3] M. Siami and N. Motee, IEEE Transactions on Automatic Control **61**, 4055 (2016).

[4] B.K. Poolla, S. Bolognani and F. Dörfler, IEEE Transactions on Automatic Control **62**, 6209 (2017).

[5] F. Paganini and E. Mallada, Proceedings of the 55th Allerton Conference on Communication, Control, and Computing, 324 (2017).

[6] M. Tyloo, T. Coletta, and Ph. Jacquod, Phys. Rev. Lett. **120**, 084101 (2018).

IV. Strategy

⇒ Pick a $\boldsymbol{Q} \to \boldsymbol{Q}^{\mathrm{M}}$ of interest

⇒ Solve Eq. (6) for $\boldsymbol{X}$ with $\boldsymbol{A}$ in Eq. (3)

⇒ Calculate $\mathcal{P}$ in Eq. (4) with $\boldsymbol{B}$ in Eq. (2).

V. Results

Assumption of $m_i \neq 0$ forces us to consider Kron-reduced network
But line fault is in original, true physical network
⇒ Evaluate impact of true line fault on Kron reduced network
⇒ Separately consider three different cases with faulted line with block Laplacian between active (with index g) and passive (c) nodes
(i) between two active nodes

$$\mathcal{P} = \frac{P_{\alpha,\beta}^2\,\tau^2}{2}(m_\alpha^{-1} + m_\beta^{-1})\,, \tag{7}$$

(ii) between two passive nodes

$$\mathcal{P} = \frac{P_{\alpha,\beta}^2\,\tau^2}{2}\frac{\sum_{i\in\mathcal{N}_g} m_i^{-1}\Big[\boldsymbol{e}_{(\alpha,\beta)}^\top \boldsymbol{L_b}^{(cc)^{-1}}\boldsymbol{L_b}^{(cg)}\hat{\boldsymbol{e}}_i\Big]^2}{[1 - b_{\alpha\beta}\boldsymbol{e}_{(\alpha,\beta)}^\top[\boldsymbol{L_b}^{(cc)}]^{-1}\boldsymbol{e}_{(\alpha,\beta)}]^2}\,, \tag{8}$$

(iii) between one active and one passive node

$$\mathcal{P} = \frac{P_{\alpha,\beta}^2\,\tau^2}{2}\frac{\sum_{i\in\mathcal{N}_g} m_i^{-1}\Big[\delta_{i\alpha} + \hat{\boldsymbol{e}}_\beta^\top \boldsymbol{L_b}^{(cc)^{-1}}\boldsymbol{L_b}^{(cg)}\hat{\boldsymbol{e}}_i\Big]^2}{[1 - b_{\alpha\beta}[\boldsymbol{L_b}^{(cc)}]^{-1}_{\beta\beta}]^2}\,, \tag{9}$$

with $P_{\alpha,\beta}$ the original power flow on the faulted line, with capacity $b_{\alpha\beta}$, and $\mathcal{N}_\}$ is the set of active nodes.

VI. Numerical results

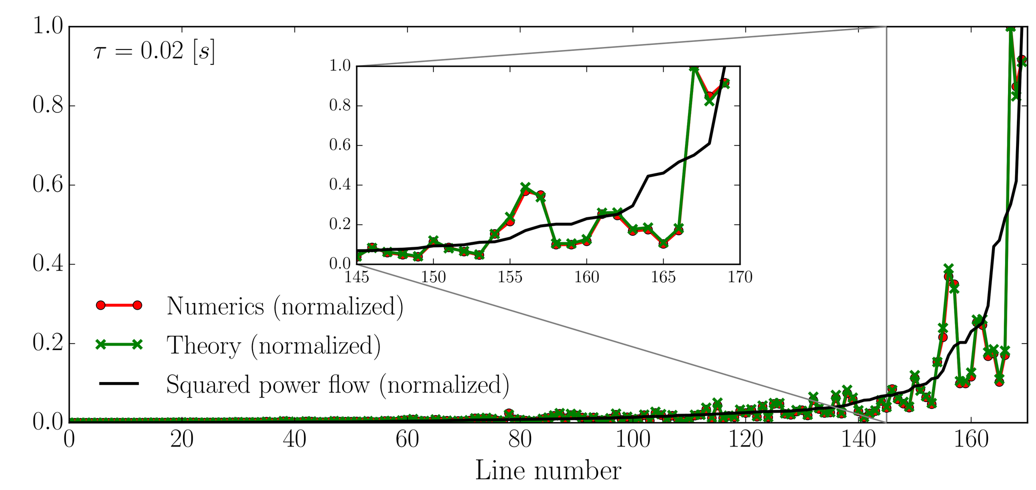

Fig. 1: Normalized primary control effort performance measure $\mathcal{P}/\mathcal{P}_{\max}$ (red), theoretical prediction (green), and square of the power flowing on the line prior to the fault (black) for line contingencies. Faulted transmission lines are ordered according to the power flowing on the line prior to the fault. Some initially moderately loaded lines induce the largest primary effort.

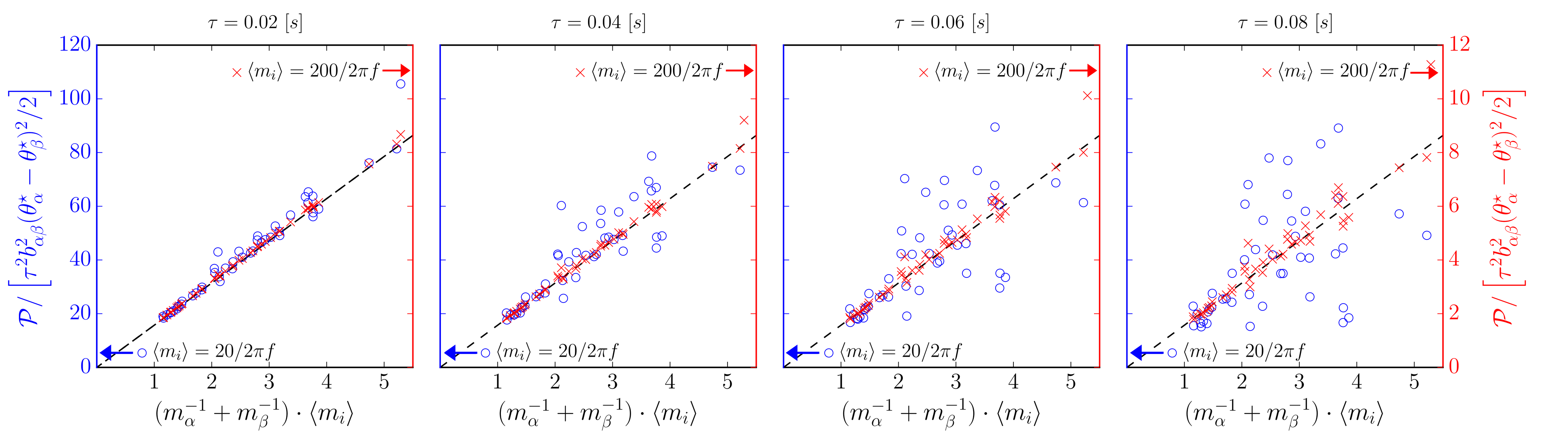

Fig. 2: Transient primary control effort for a line contingency as a function of the resistance distance separating the nodes of the faulted line. Each data point corresponds to the fault of a line connecting two generators in the physical network. Simulation parameters: IEEE 118-bus test case.

LaTeX TikZposter