Linear stability and the Braess paradox in Coupled Oscillator networks and Electric power grids

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AC electric networks as a motivation

- Collective dynamics

  → Coupled rotating machines operated at a synchronous frequency 50/60Hz

- Constantly assessing the grid
  - → Thermal limits
  - → N-1 feasibility
  - → Transient stability
  - → Voltage stability

Braess paradox in AC networks

- Does more Cu make the network better?

  → Impact of line addition on minimal transmission capacity (i.e. oscillator coupling) required for synchrony

- Braess paradox (traffic networks)

  → Nonlinear effect

- Testing network upgrades against

  → Power rerouting

  → Stability of solutions

- Predictive w/r/t the position of the line addition

D. Witthaut & M. Timme, New J. of Phy. 14, 083036 (2012)
D. Braess, Unternehmensforschung 12, 258 (1968)
T. Nishikawa & A.E. Motter, PRE 73, 065106 (2006)

From AC electric networks to coupled oscillators

- Nodes $V_j = |V_j| e^{i(\omega t + \theta_j(t))}$ $\omega = 50/60\mathrm{Hz}$
- Line admittance $Y_{jk} = G_{j,k} + iB_{j,k}$
- Standard approximations $G_{jk} \ll B_{jk}$, $|V_k| \equiv V_0$
- Power flows (i.e. Kirchoff's laws)

$$P_i = \sum_{j \sim i} K_{i,j} \sin(\theta_i - \theta_j) \qquad K_{i,j} = B_{i,j} V_0^2$$

- Swing equations

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_{j \sim i} K_{i,j} \sin(\theta_i - \theta_j)$$

Linear stability condition

- Linearize dynamics $\theta_i(t) \to \theta_i^{(0)} + \delta\theta_i(t)$

$$\dot{\delta\vec{\theta}} = M(\{\theta_i^{(0)}\})\delta\vec{\theta}$$

- Stability matrix, weighted Laplacian

$$M_{ij} = \begin{cases} K_{ij}\cos(\theta_i^{(0)} - \theta_j^{(0)}) & i \neq j \\ \sum_{l\sim i} K_{il}\cos(\theta_i^{(0)} - \theta_l^{(0)}) & i = j \end{cases}$$

- Stability $\leftrightarrow$ negative definite spectrum $\lambda_i, \vec{u}^{(i)}$

$\longrightarrow$ Lyapunov exponents (timescales to equilibrium)

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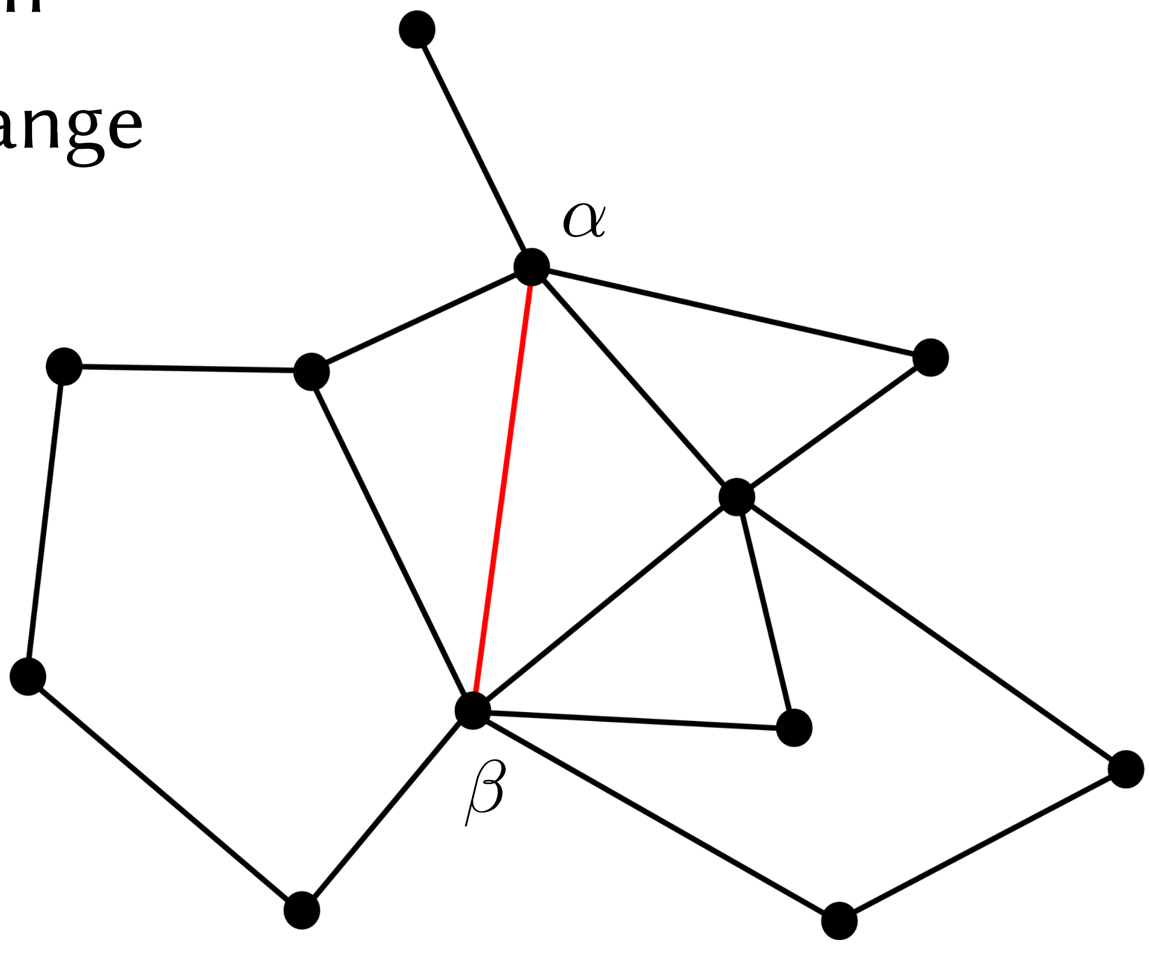
Line addition

- New transmission path

  $\longrightarrow$ connectivity change

- Phases $\{\theta_i\}$ readapt

- Power rerouting



Perturbative line addition

- New line with capacity $\delta \ll K$
- Angle correction $\theta_i^{(0)} \to \tilde{\theta}_i \approx \theta_i^{(0)} + \mathcal{O}(\delta/K)$

  $\longrightarrow$ Power rerouting $P_{ij} = K \sin(\theta_i - \theta_j)$
- Correction to the stability matrix $M \to M + \Delta M$

  $\longrightarrow$ New coefficients

  $\longrightarrow$ Correction of existing weights
- Corrected Lyapunov exponent

$$\lambda_2 \to \lambda_2 + \Delta\lambda_2 \qquad\qquad \Delta\lambda_2 = \vec{u}^{(2)\top} \cdot \Delta M \cdot \vec{u}^{(2)}$$

Perturbative line addition

- Correction of the Lyapunov exponent

$$\Delta\lambda_2 = -\delta\cos\theta_{\alpha,\beta}\left[u_\alpha^{(2)} - u_\beta^{(2)}\right]^2 + \delta\sin\theta_{\alpha,\beta}\sum_{\langle i,j\rangle} f_{ij}^{\alpha,\beta}\left[u_i^{(2)} - u_j^{(2)}\right]^2$$

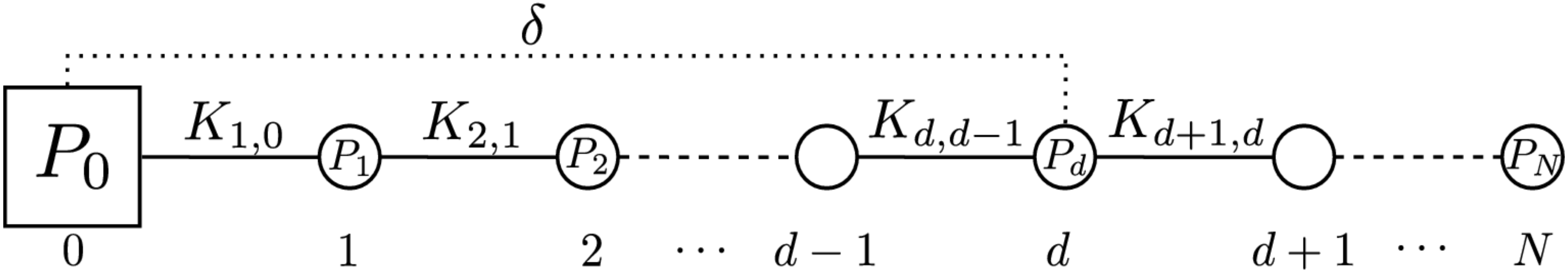
- Discuss the sign of $\Delta\lambda_2$

  $\Delta\lambda_2 < 0$ enhanced stability

  $\Delta\lambda_2 > 0$ reduced stability

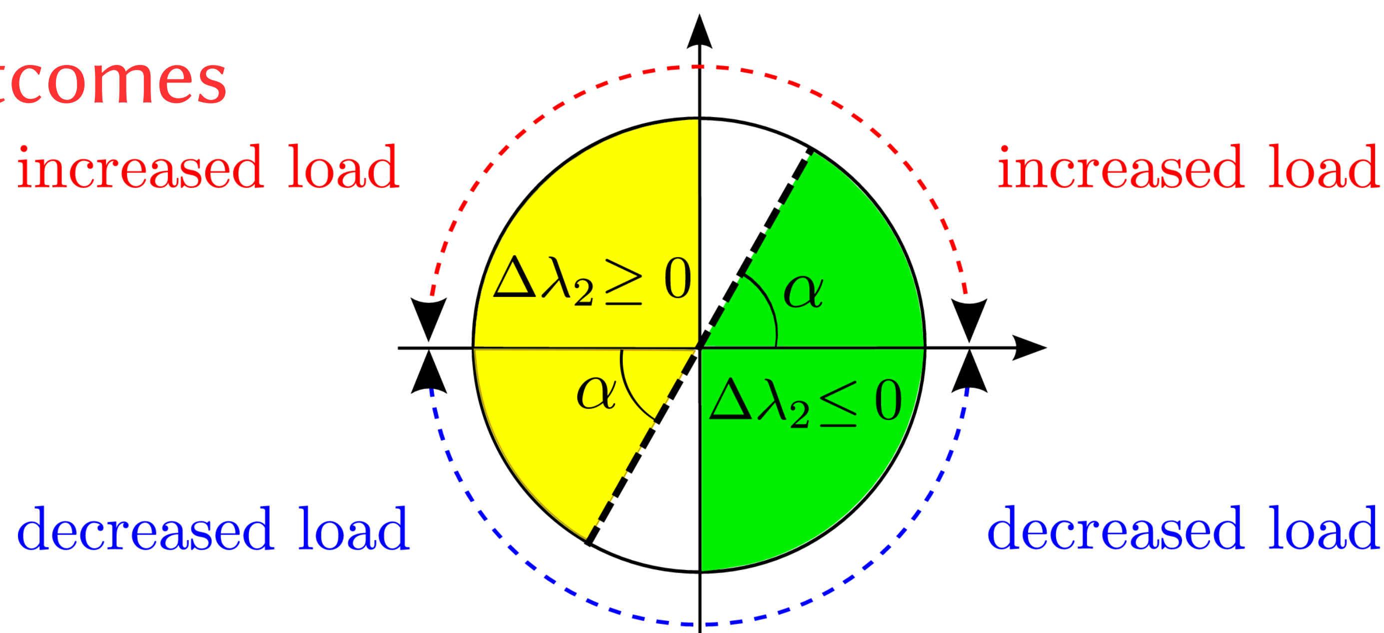
- What if $\theta_\alpha - \theta_\beta \approx 0, \pi$

The chain model



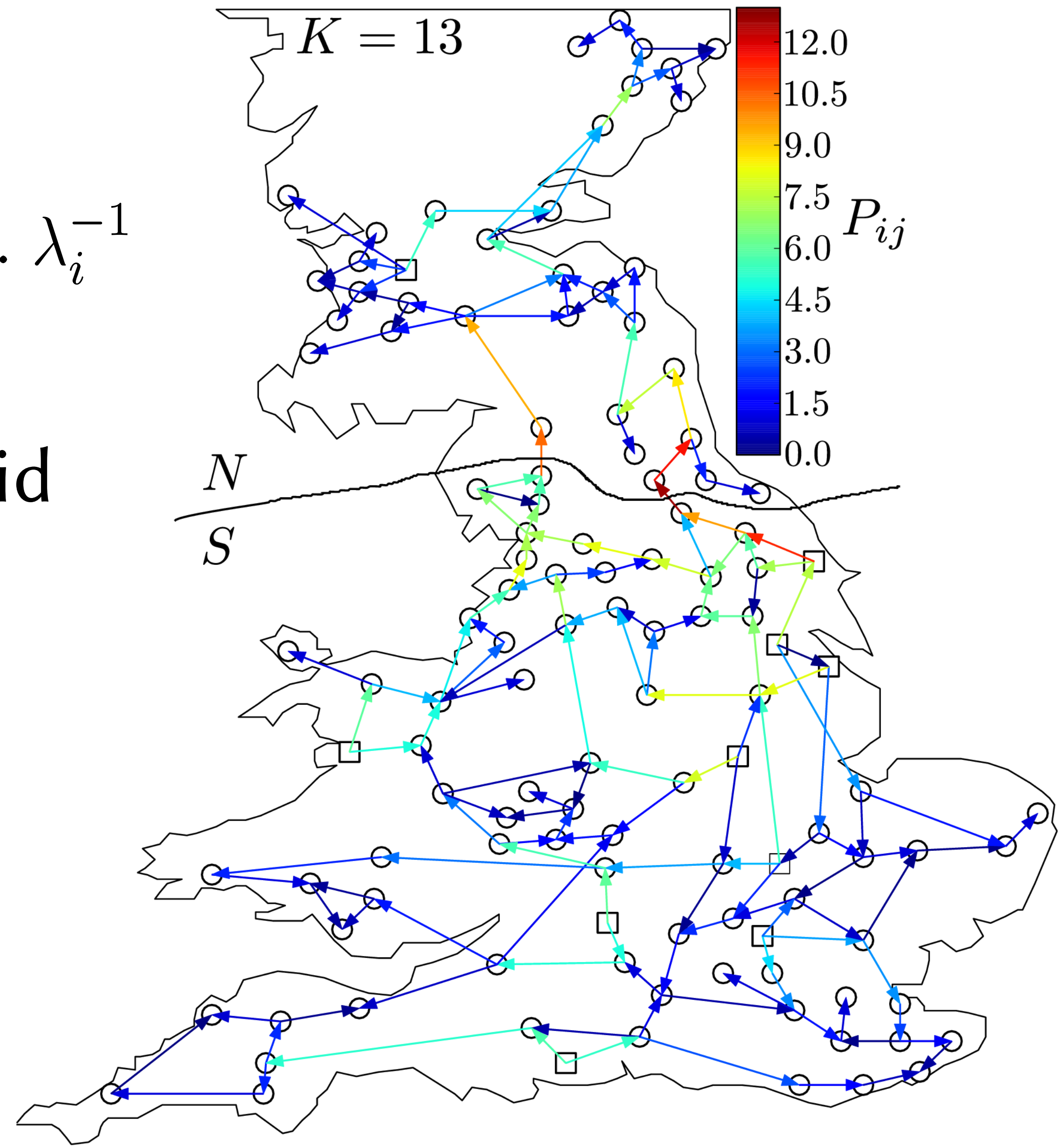
- $P_{0d} = -\delta \sin(\theta_d - \theta_0)$
- Tridiagonal stability matrix, sign of $\Delta\lambda_2$ depends only on $\theta_d - \theta_0$

$\longrightarrow$3 Braess outcomes



Transmission girds

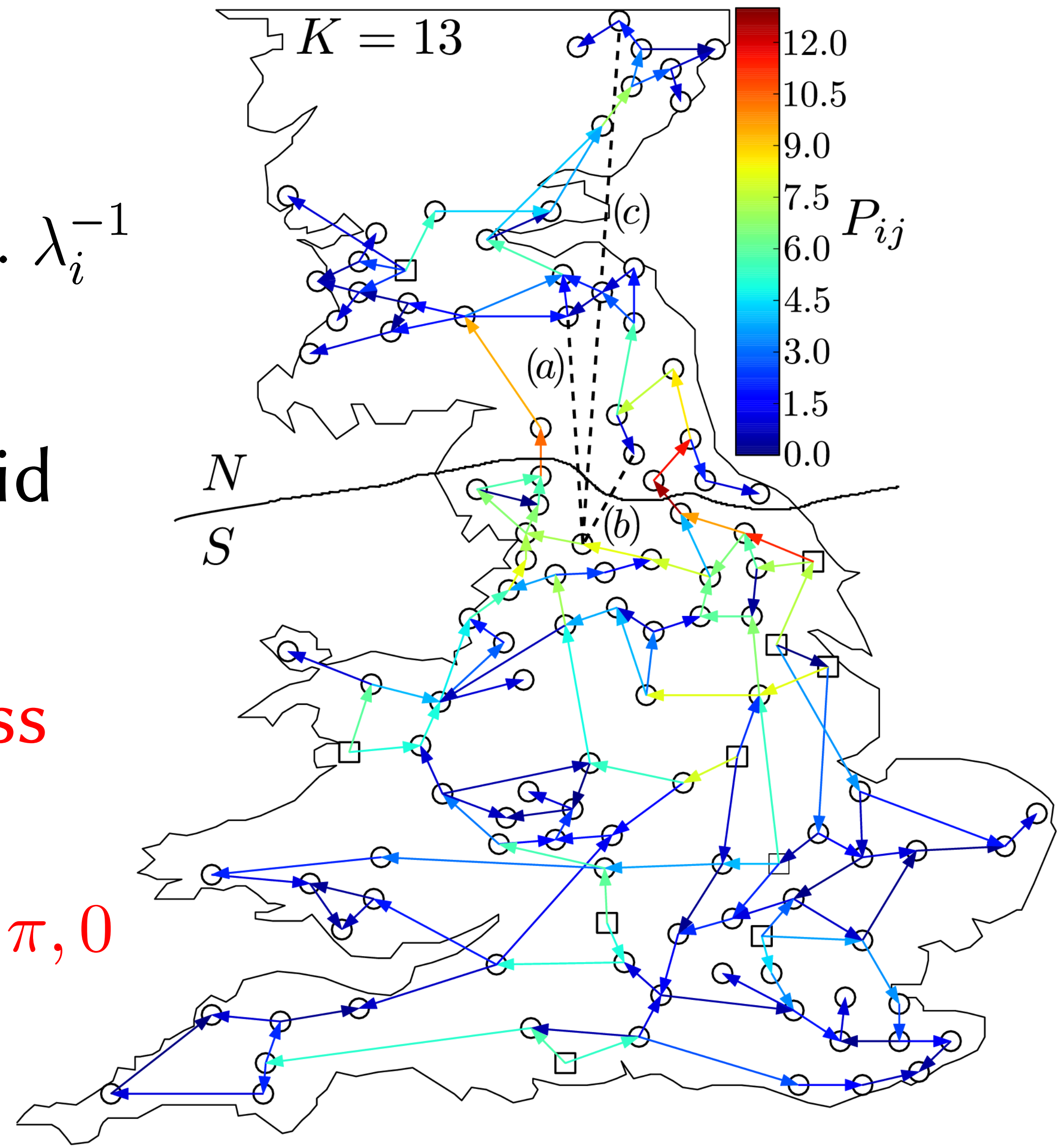
- Angle correction
  - $\longrightarrow$ Laplacian pseudoinv. λ_i^{-1}
  - $\longrightarrow$ Eigenvectors of M
- Numerics on the UK grid
  - $\longrightarrow$ Preferential axis



D. Witthaut & M. Timme, New J. of Phy. 14, 083036 (2012)

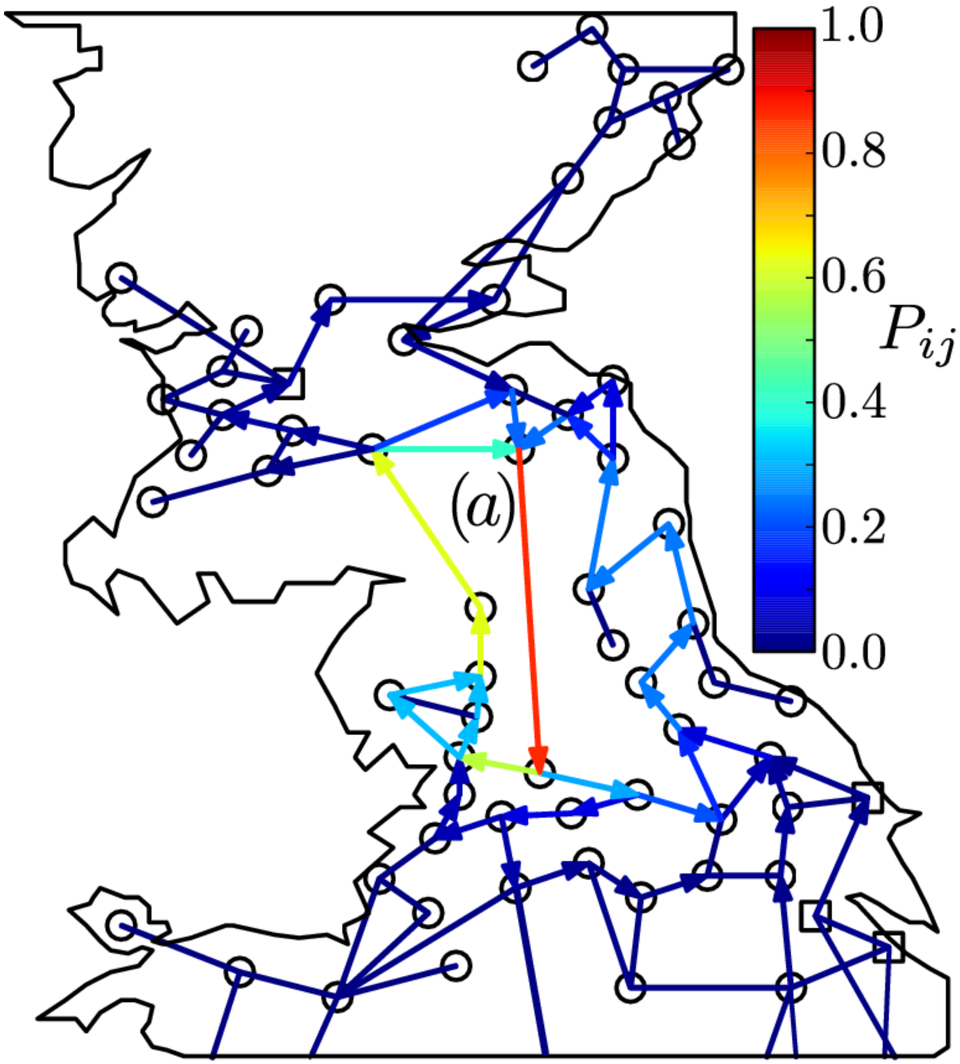
Transmission girds

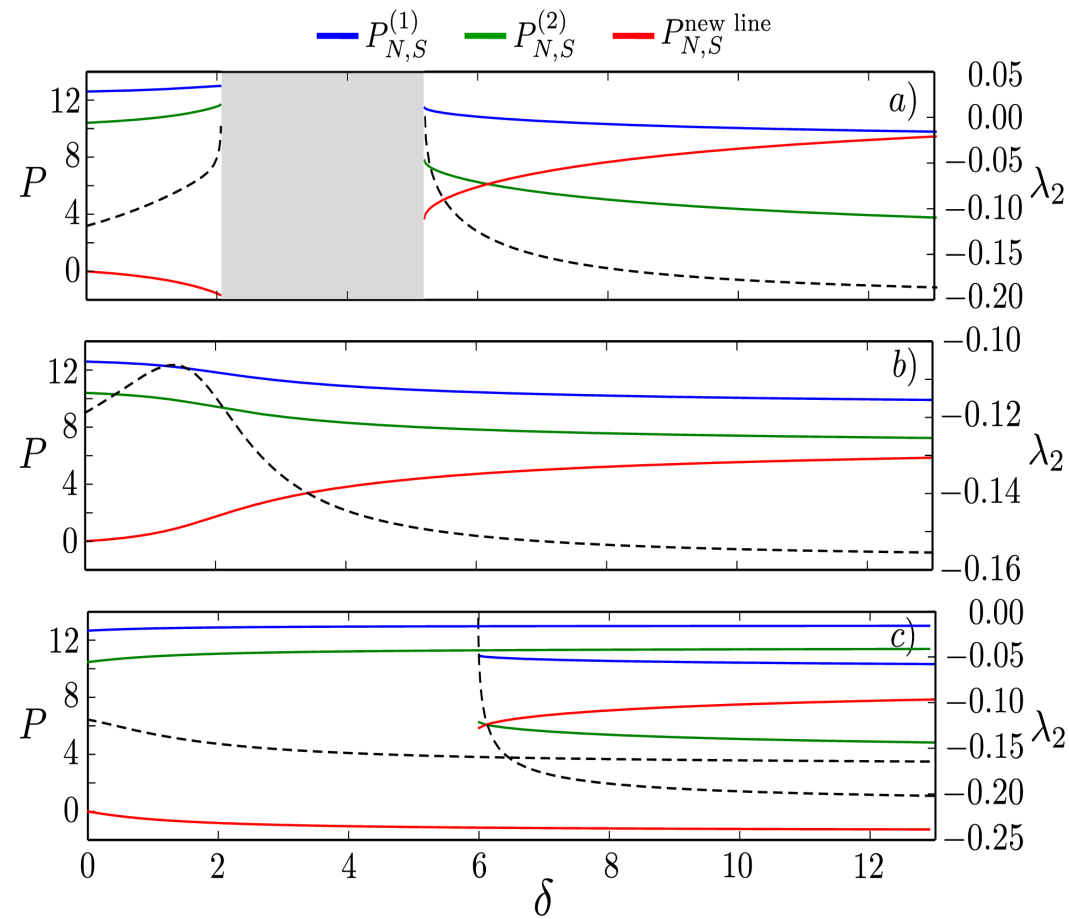
- Angle correction
  - $\longrightarrow$ Laplacian pseudoinv. λ_i^{-1}
  - $\longrightarrow$ Eigenvectors of M
- Numerics on the UK grid
  - $\longrightarrow$ Preferential axis
- Can we identify 3 Braess scenarios?
  - $\longrightarrow$ Limits when $\theta_{\alpha,\beta} \approx \pi, 0$



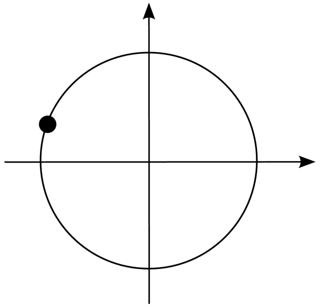
T. Coletta & Ph. Jacquod PRE 93, 032222 (2016)

Transmission girds





$$\theta_N - \theta_S \approx 0.9\pi$$



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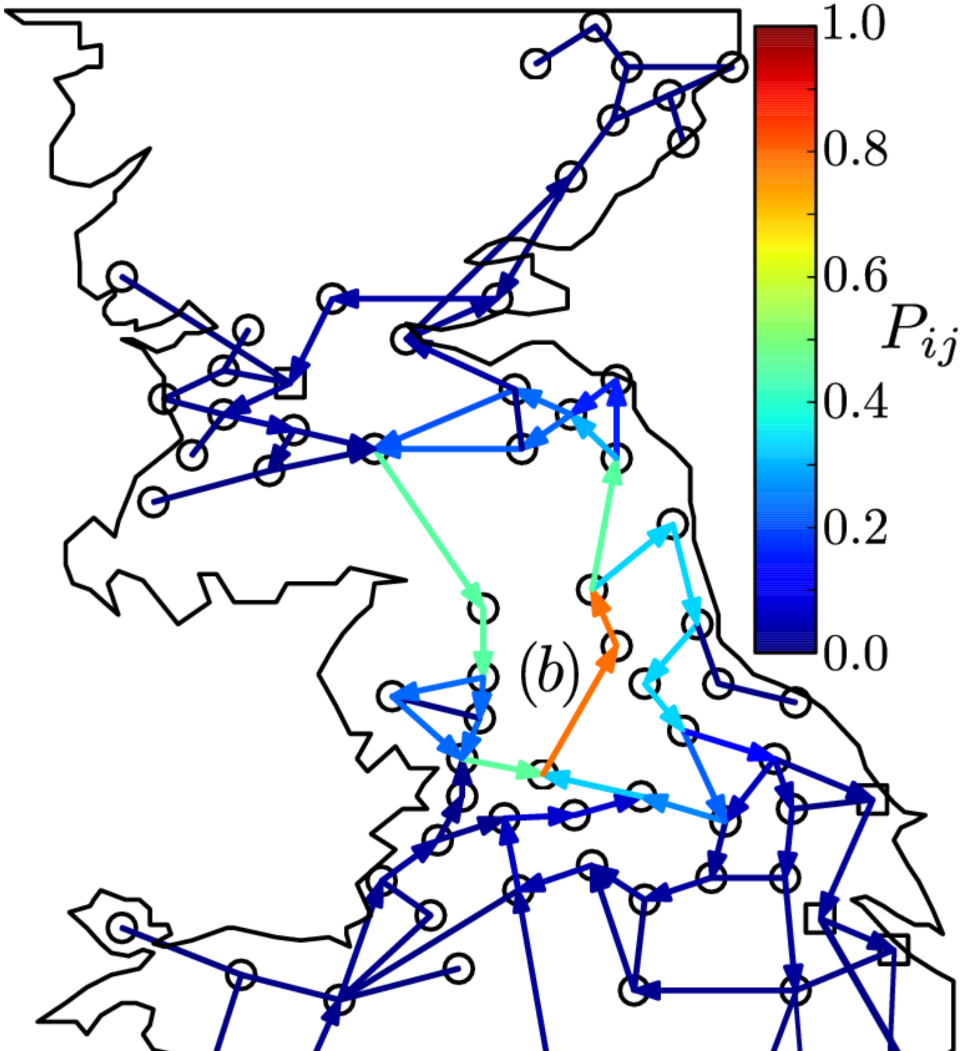
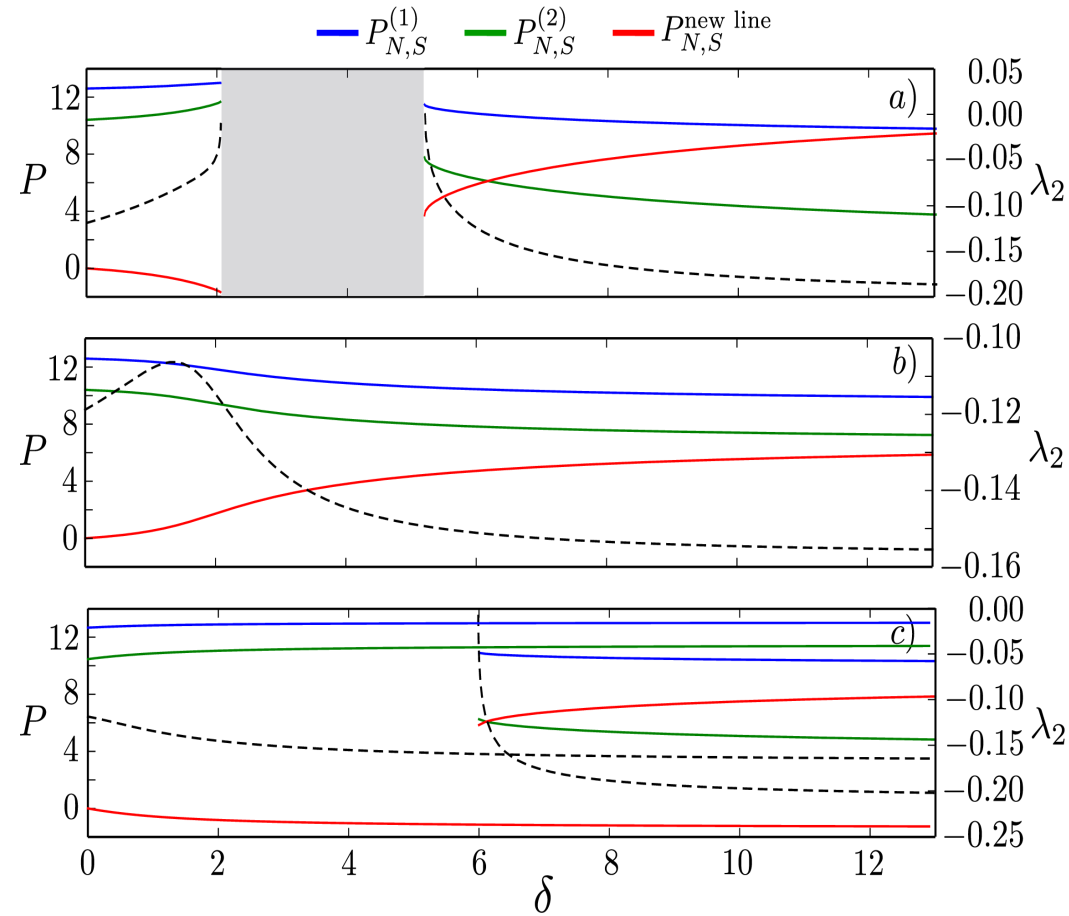
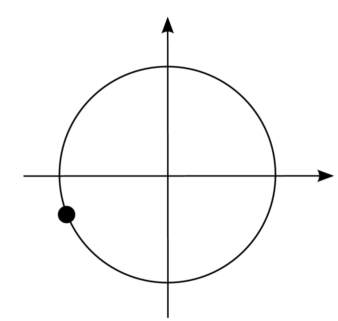




$$\theta_N - \theta_S \approx -0.9\pi$$



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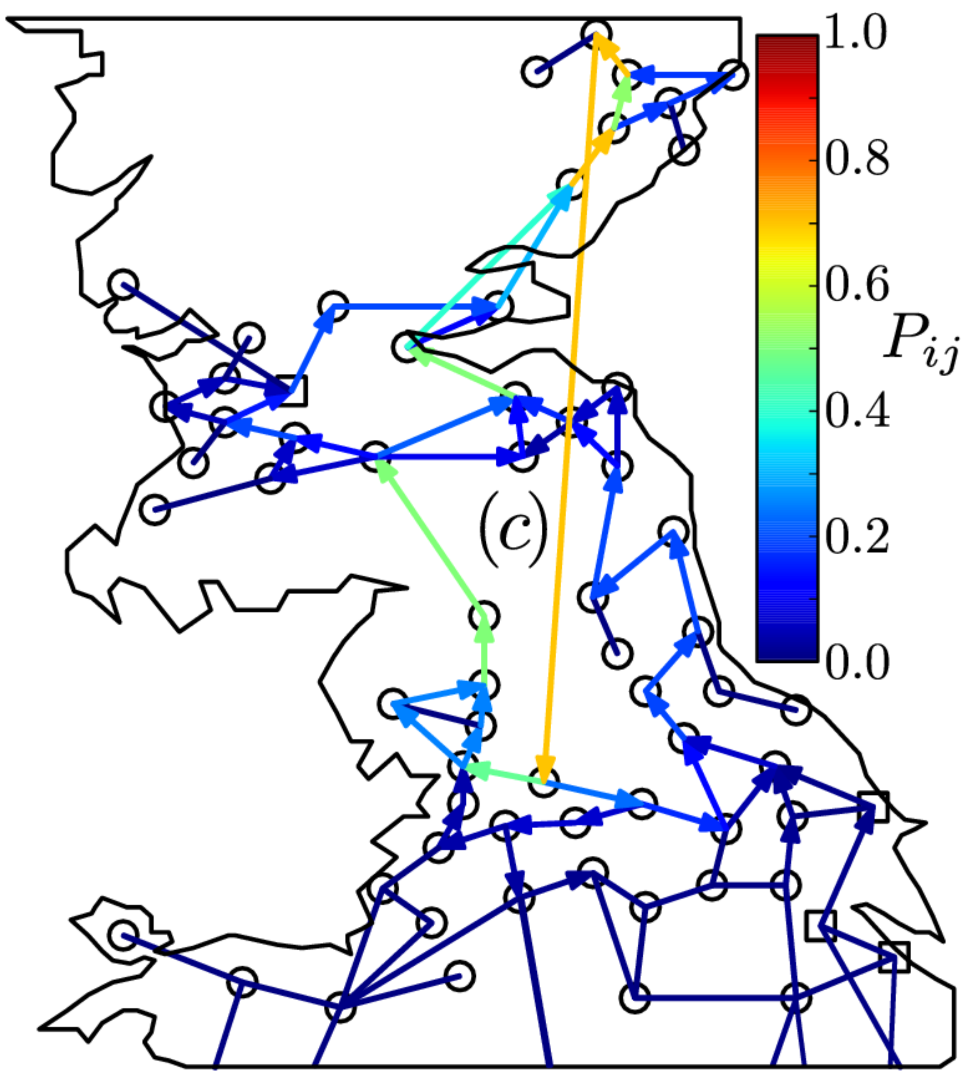
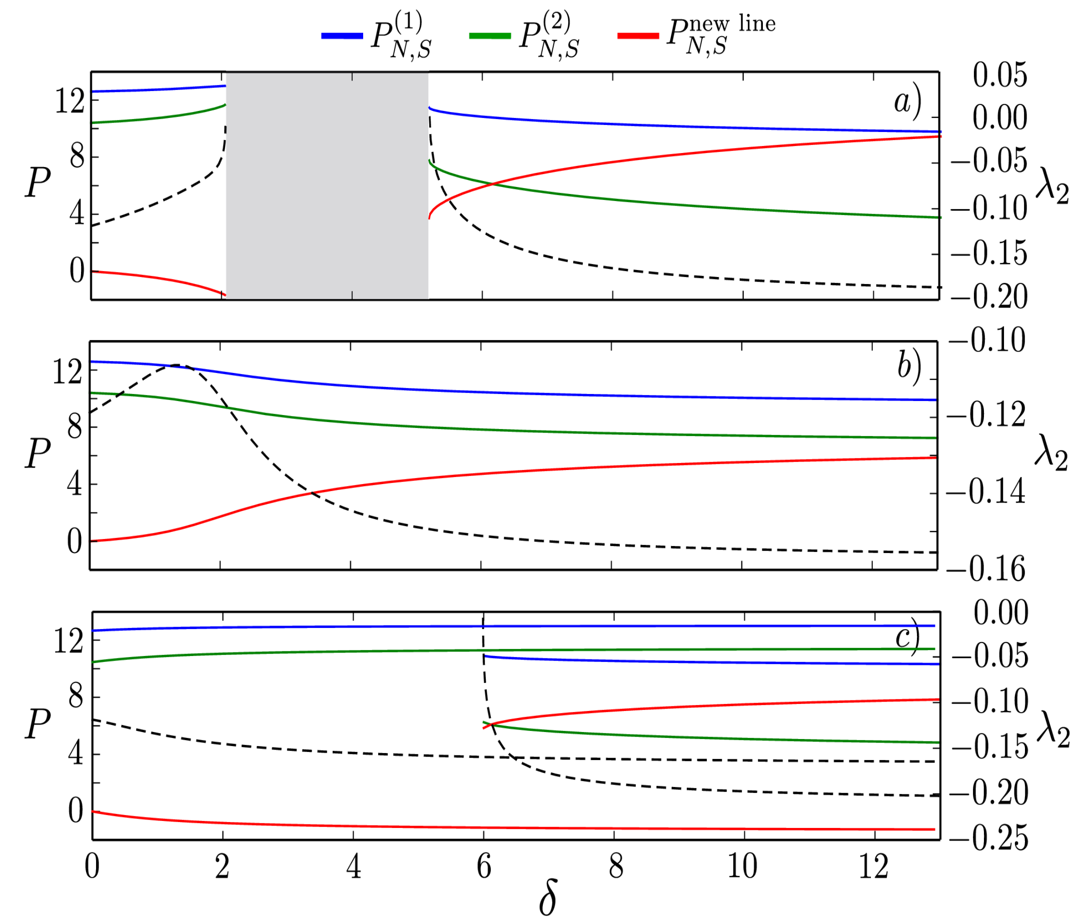
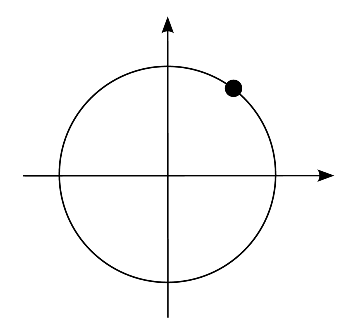




$$\theta_N - \theta_S \approx 0.3\pi$$



R. Delabays, T. Coletta & Ph. Jacquod, J. Mat. Phy. 57, 032701 (2016)
T. Coletta, R. Delabays, I. Adagideli & Ph. Jacquod arXiv:1605.07925 (2016)

Conclusion

- Identification of 4 scenarios resulting from line addition depending on

  $\longrightarrow$ enhanced/reduced linear stability

  $\longrightarrow$ increase/decrease transmission load

- Chain model $\leftrightarrow$ analytical understanding
- Perturbative approach gives an insight to complex network topologies
- Solutions that differ by loop flows coexist

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