Rate of Change of Frequency under line contingencies

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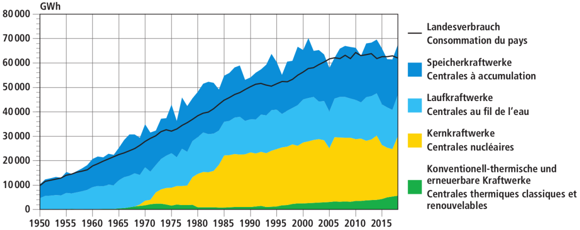
R. D., M. Tyloo, and P. Jacquod, *arXiv preprint* **1906.05698** (2019)

Hes·so // VALAIS WALLIS
School of Engineering

Where is HES-SO?

Motivation



OFEN, Statistique Suisse de l'électricité 2018.

IEA, Global EV Outlook 2019 (www.iea.org/publications/reports/globalevoutlook2019/).

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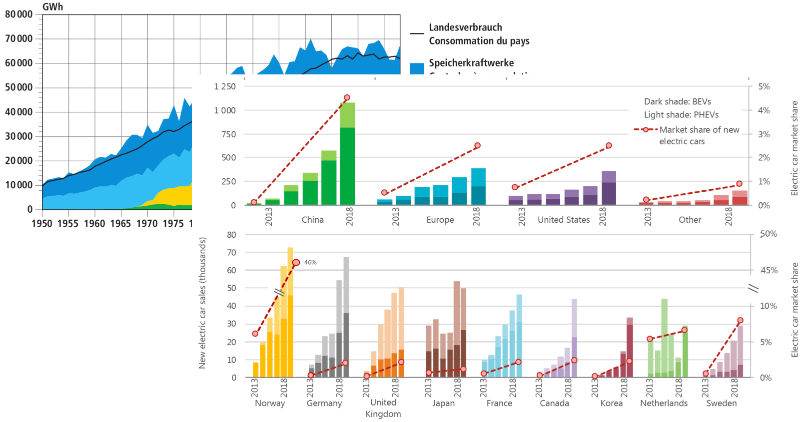

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Motivation

What is the impact of a given contingency?

What are the critical elements in a grid?

How to identify (efficiently) critical operating states?

The Swing Equations

We consider:

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j b_{ij}(\theta_i - \theta_j)\,, \qquad i \in \{1, ..., n\}\,,$$

m_i: inertia, d_i: damping, b_{ij}: susceptance, P_i: generation/load.

$$M\ddot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} = \mathbf{P} - \mathbb{L}\boldsymbol{\theta}\,,$$

$M = \mathrm{diag}(\mathbf{m})$, $D = \mathrm{diag}(\mathbf{d})$, $\mathbb{L}$ Laplacian matrix.

Shorthand notation: $\omega_i := \dot{\theta}_i$.

A. R. Bergen and V. Vittal, *Power System Analysis* (Prentice Hall, 2000).

Analytical solution

Assume $m_i \equiv m$, $d_i \equiv d$, and consider angle deviations

$$\boldsymbol{\delta\theta}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*\,, \qquad \boldsymbol{\theta}^* = \mathbb{L}^\dagger \mathbf{P}_0\,, \qquad \mathbf{P}(t) = \mathbf{P}_0 + \boldsymbol{\delta}\mathbf{P}(t)\,.$$

$$m\ddot{\boldsymbol{\delta\theta}} + d\dot{\boldsymbol{\delta\theta}} = \boldsymbol{\delta}\mathbf{P}(t) - \mathbb{L}\boldsymbol{\delta\theta}\,.$$

Expanding on the eigenmodes of $\mathbb{L}$:

$$\mathbb{L}\mathbf{u}^{(\alpha)} = \lambda_\alpha \mathbf{u}^{(\alpha)}\,, \qquad \boldsymbol{\delta\theta}(t) = \sum_{\alpha=1}^{n} c_\alpha(t)\mathbf{u}^{(\alpha)}\,.$$

M. Tyloo and P. Jacquod, *Phys. Rev. E* **100** 032303 (2019).

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$$\boxed{m\ddot{c}_\alpha(t) + d\,\dot{c}_\alpha(t) = \boldsymbol{\delta}\mathbf{P}(t)\cdot\mathbf{u}^{(\alpha)} - \lambda_\alpha c_\alpha(t)\,, \quad \alpha = 1, ..., n\,.}$$

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Analytical solution:

$$c_\alpha(t) = m^{-1} e^{-(\gamma+\Gamma_\alpha)t/2} \int_0^t e^{\Gamma_\alpha t_1} \int_0^{t_1} \boldsymbol{\delta}\mathbf{P}(t_2)\cdot\mathbf{u}^{(\alpha)} e^{(\gamma-\Gamma_\alpha)t_2/2} dt_2 dt_1\,.$$

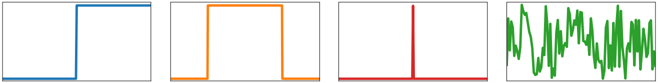
M. Tyloo and P. Jacquod, *Phys. Rev. E* **100** 032303 (2019).

Contingencies

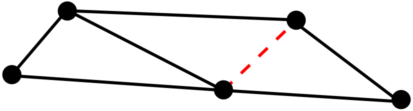
Swing Equations, $i \in \{1, ..., n\}$:

$$M\ddot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} = \mathbf{P} - \mathbb{L}\boldsymbol{\theta}\,.$$

Nodal perturbations: additive, $\mathbf{P} \to \mathbf{P} + \delta\mathbf{P}$.



Line perturbations: multiplicative, $\mathbb{L} \to \mathbb{L} - \beta \mathbf{e}_{ij}\mathbf{e}_{ij}^\top$.



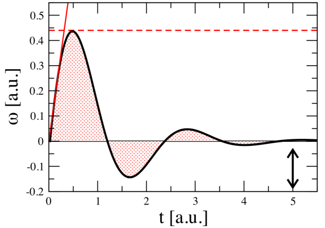
Measures of the impact

Transmission losses: $\mathcal{L}_2$-norm of angle deviations.

Primary control effort: $\mathcal{L}_2$-norm of frequency deviations.

Nadir: $\mathcal{L}_\infty$-norm of frequency deviations.

RoCoF: $\mathcal{L}_\infty$-norm of the time derivative of the frequency.



$$\sup_t |\dot{\omega}(t)|$$

E. Tegling, B. Bamieh, and D. F. Gayme, *IEEE Trans. Control Netw. Syst.* **2** 254 (2015).

T. W. Grunberg and D. F. Gayme, *IEEE Trans. Control Netw. Syst.* **5**, 456 (2018).

B. K. Poolla, S. Bolognani, and F. Dörfler, *IEEE Trans. Autom. Control* **62** 6209 (2017).

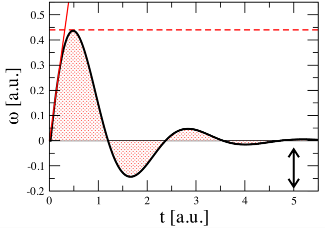
F. Paganini and E. Mallada, *Proc. of the 55th ACCC* (2017).

T. Coletta and P. Jacquod, *IEEE Trans. Control Netw. Syst.* Early access (2019).

The RoCoF

Maximal local RoCoF:

$$\mathrm{RoCoF} = \max_i \|\dot{\omega}_i(t)\|_\infty \,.$$



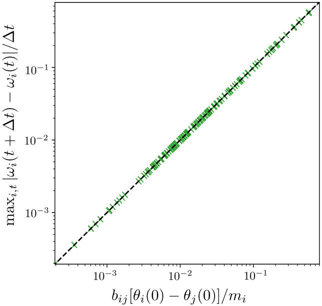
RoCoF is maximal at $t = 0^+$.

$$\boldsymbol{\omega}(0) = 0\,, \qquad \mathbf{P} = \mathbb{L}\,\boldsymbol{\theta}(0)\,, \qquad \mathbb{L}^* = \mathbb{L} - b_{ij}\mathbf{e}_{ij}\mathbf{e}_{ij}^\top\,,$$

$$M\dot{\boldsymbol{\omega}}(0) + D\boldsymbol{\omega}(0) = \mathbf{P} - \mathbb{L}^*\boldsymbol{\theta}(0)\,,$$

$$\Longrightarrow\ \dot{\omega}_k = (\delta_{ik} - \delta_{jk})\frac{b_{ij}(\theta_i - \theta_j)}{m_k}\,.$$ $\rightarrow$ RoCoF at nodes i and j.

Numerics (IEEE 118-Bus)



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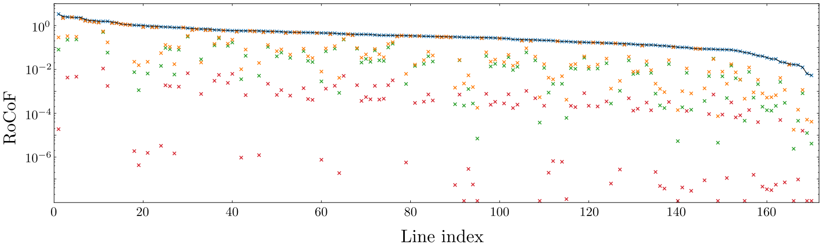


Black line: theory.

x: 100% inertia at loads, RoCoF at all nodes.

x: 100% inertia at loads, RoCoF at generators only.

x: 1% inertia at loads, RoCoF at generators only.

x: 0% inertia at loads, RoCoF at generators only.

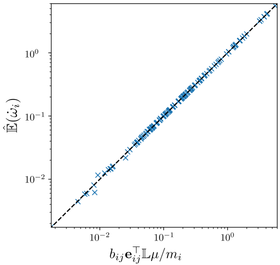
Including uncertainties

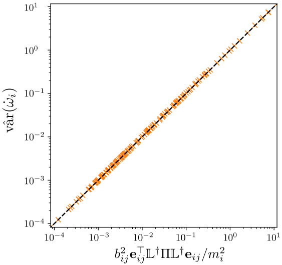
Statistics on generation and loads:

$$\mathbb{E}\left[P_k\right] = \mu_k\,, \qquad \mathbb{E}\left[(P_k - \mu_k)(P_\ell - \mu_\ell)\right] = \Pi_{k\ell}\,.$$

One gets:

$$\mathbb{E}(\dot{\omega}_i) = \frac{b_{ij}}{m_i}\mathbf{e}_{ij}^\top \mathbb{L}^\dagger \boldsymbol{\mu}\,, \qquad \mathrm{var}(\dot{\omega}_i) = \frac{b_{ij}^2}{m_i^2}\mathbf{e}_{ij}^\top \mathbb{L}^\dagger \Pi \mathbb{L}^\dagger \mathbf{e}_{ij}\,.$$





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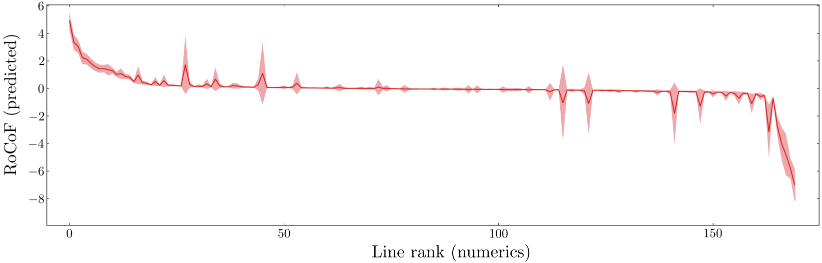


Conclusion

The RoCoF after a line loss is:

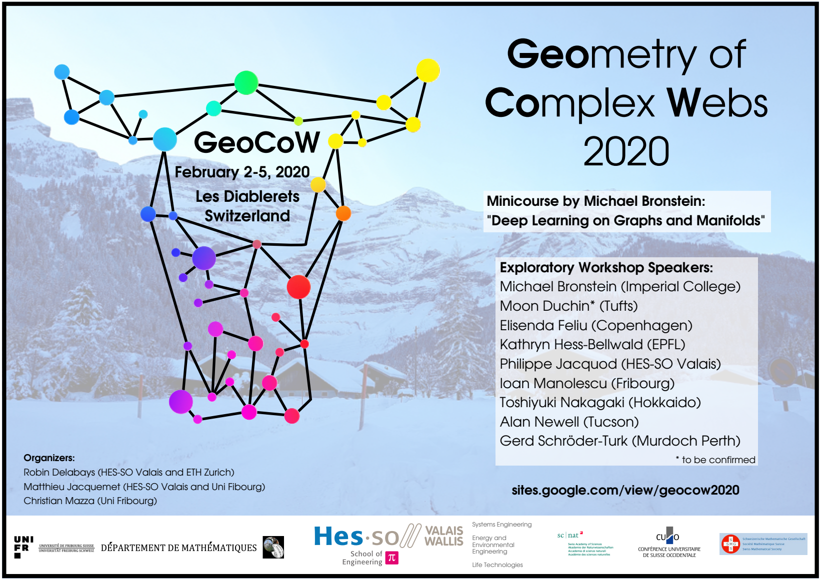
- proportional to the flow on the line;
- inversely propotional to the inertia of the node where it is measured.

If we have only statistics on the power injections, we derive statistics on the RoCoFs.

Consequences:

- The most loaded lines are the most critical (expected);
- Less inertia means more critical systems, but...

Caveat: We assume inertia at every nodes, which is not true (yet...).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