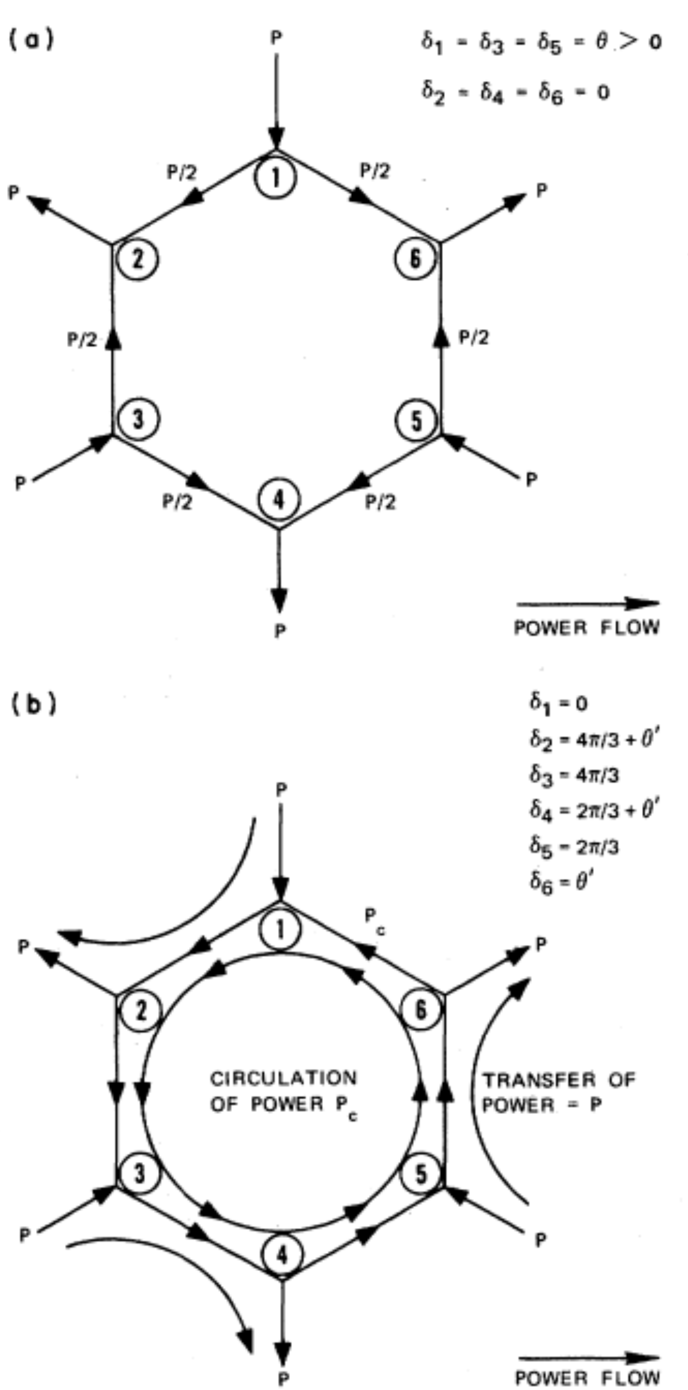
Loop Flows in High Voltage AC Power Grids

Philippe Jacquod

colls. : R Delabays
T Coletta
I Adagideli (Sabanci - Istanbul)

How many stable solutions to the power flow equations ?



Korsak IEEE Trans. Power Appar. Syst. (1972)

Different solutions vs. loop currents

From the example networks above, it is evident that stable load flows are not necessarily unique, and that power flows, following a transient, for example, could possibly lock into a stable configuration in which power circulates in one or more loops of the network, with great increase in losses as a result.

Referee comment

Gerd Lüders (Yale University, New Haven, Conn.): This paper illustrates, through an example, that a power system can have more than one stable load flow solution or stable singular point. The study of the number of stable singular points is important:

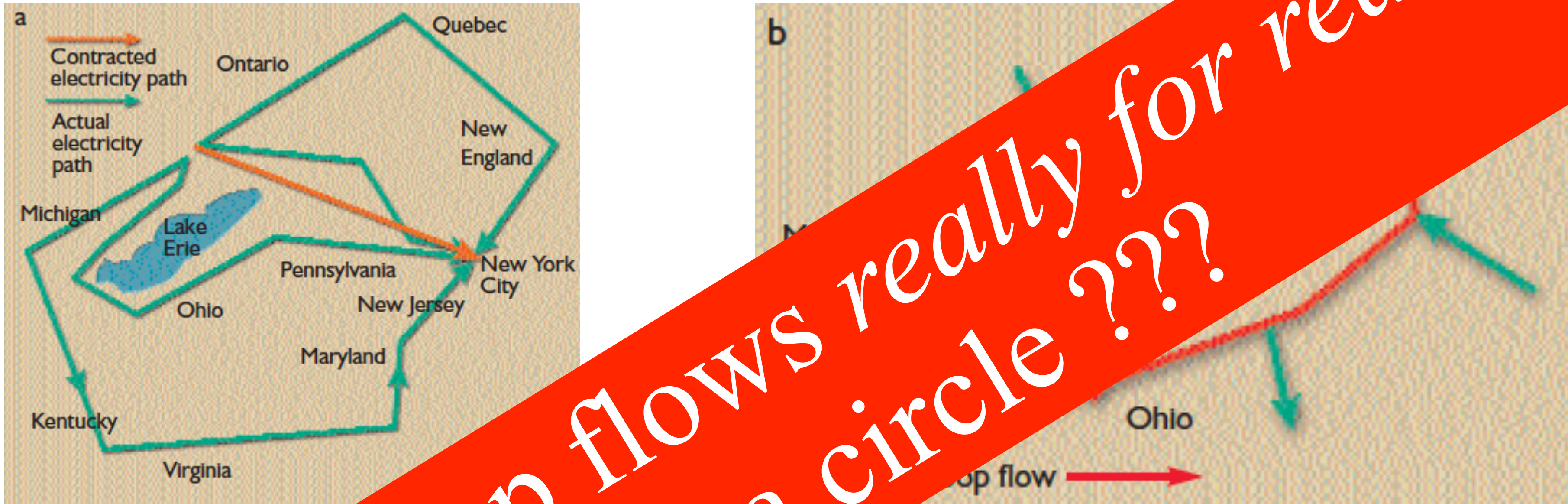
1) for the operation of a power system, because one would be interested in operating the system on that stable singular point which would maximize its stability and minimize its losses, and

2) for applying Lyapunov's Method to transient stability studies of power systems, because this method requires a thorough knowledge of the singular points of a power system [9].

$$\sum^{\text{loop}} \delta_{ij} = \pm 2\pi k$$

See also: Janssens and Kamagate '03

Loop flows



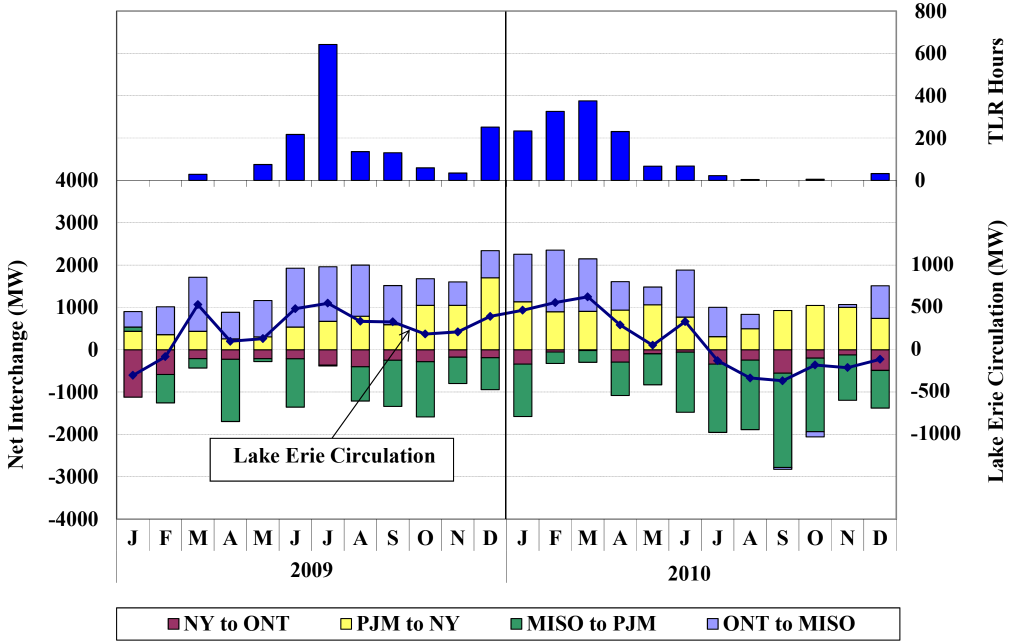
Are such loop flows really for real ?
1GW in a circle ???

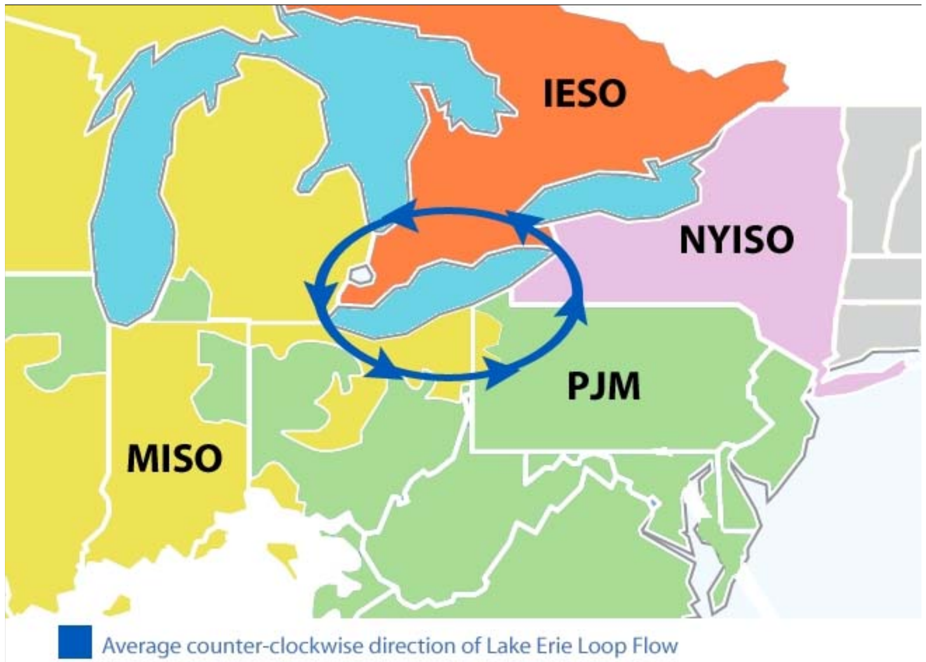
“Where the … ge geographical obstacles, such as the Rocky
Mount … eat Lakes in the East, loop flows around the
ob … drive as much as 1 GW of power in a circle, *taking up*
tra … city without delivering power to consumers.”

Casazza, Electrical World (1998)
Lerner, The industrial physicist (2003)

Loop flows

"Electric power does not follow a specified path but divides among transmission routes based on Kirchhoff's laws and network conditions at the time. This pattern results in a phenomenon called circulating power of which there are two types : loop flow and parallel flow."





Casazza, Electrical World (1998)
Lerner, The industrial physicist (2003)
Lake Erie Loop Flow Mitigation - A report from the New York Independent System Operator (2008)
Patton, Pallas, van Schaick, Chen, State of the Market Report for the NYISO Markets (2010)
US DOE, National Electric Transmission Congestion Study (2015)

Steady-State (balanced) AC transport : Power flow equations

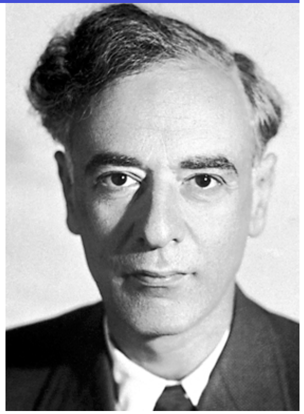
$$P_i = \sum_j |V_i V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]$$

$$Q_i = \sum_j |V_i V_j| \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right]$$

- i : node/bus index
- PV-buses : production
- PQ-buses : consumption
- 1 “slack-bus”

Solution state entirely determined by $V_i = |V_i| e^{i\theta_i}$
~discrete complex function on the plane

Vortices in superfluids and superconductors



L Landau
1908-1968

- Landau theory of superfluidity/superconductivity :

complex order parameter $\psi(\mathbf{x}) = |\psi(\mathbf{x})|\, e^{i\theta(\mathbf{x})}$

super-current $\mathbf{J}(\mathbf{x}) = \dfrac{\hbar|\psi(\mathbf{x})|^2}{m_s} \nabla\theta(\mathbf{x})$

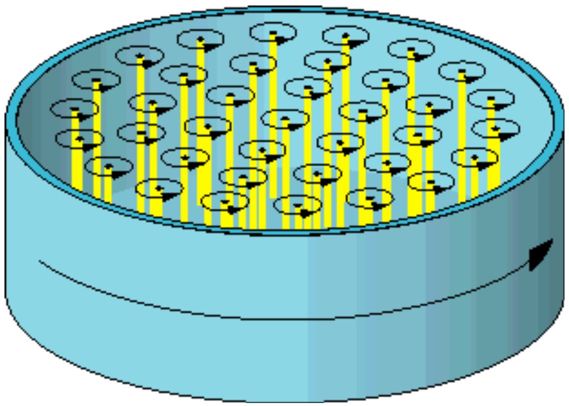
- Two cases :

(i) $|\psi(\mathbf{x})| > 0$ everywhere $\longrightarrow$ $\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 0$

(ii) $|\psi(\mathbf{x})| = 0$ somewhere $\longrightarrow$ $\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 2\pi q$

q=integer : "winding number"

~number of times phase goes around unit circle
as one travels around a loop in space



-> quantization of circulation ~vortex
(super-current around any loop)

$$\oint_C \mathbf{J} \cdot \mathrm{d}\mathbf{l} = \frac{\hbar|\psi(\mathbf{x})|^2}{m_s}(\theta_+ - \theta_-) = 2\pi q \frac{\hbar|\psi(\mathbf{x})|^2}{m_s}$$

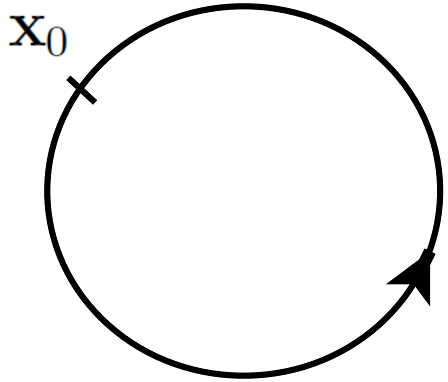
Th. : Onsager '49, Feynman '55

Quantization of circulation

$$\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 2\pi q$$

- Origin : superfluid described by complex function $\psi(\mathbf{x}) = |\psi(\mathbf{x})|\, e^{i\theta(\mathbf{x})}$
  its phase must be uniquely defined

→ move the particle around any closed contour
single-valuedness $\theta(\mathbf{x} \to \mathbf{x}_0) = \theta(\mathbf{x}_0) + 2\pi q$



q : "winding number"
~number of times phase goes around unit circle as one travels around a loop in space

- Meaning and consequences
  (i) Can we contract the curve to a single point ?
  Yes : q=0 ; No : q= any integer
  (ii) It is hard to unwind q -> q'
  "Topological protection"

- Translated into AC power grid language (Lübner '72, Janssens and Kamagate '03)

$$\sum_{i \in \mathrm{loop}} \left|\theta_{i+1} - \theta_i\right|_{2\pi} = 2\pi q$$

Superfluidity and -conductivity vs. AC electric power systems

	Superconductor	high voltage AC power grid
State	$\Psi(\mathbf{x}) = \lvert\Psi(\mathbf{x})\rvert e^{i\theta(\mathbf{x})}$	$V_i = \lvert V_i \rvert e^{i\theta_i}$
Current / power flow	$I_{ij} = I_c \sin(\theta_i - \theta_j)$ DC Josephson current	$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$ Power flow; lossless line approx.
Circulation	$\oint_C \nabla\theta \cdot \mathrm{d}\mathbf{l} = 2\pi q$	$\sum_{i \in \text{loop}} \lvert\theta_{i+1} - \theta_i\rvert_{2\pi} = 2\pi q$

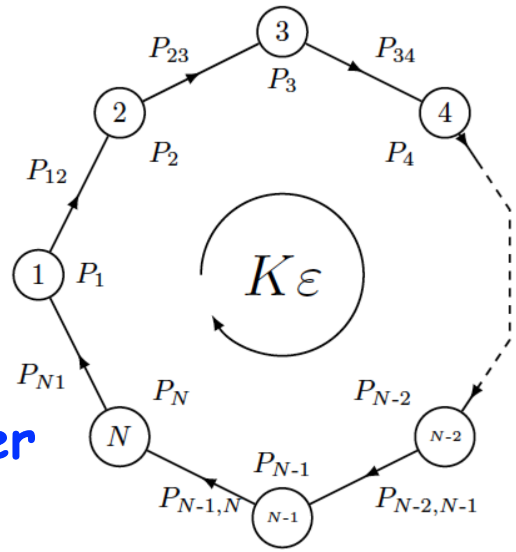
Circulating loop flows

*Thm: Different solutions to the following power-flow problem

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16

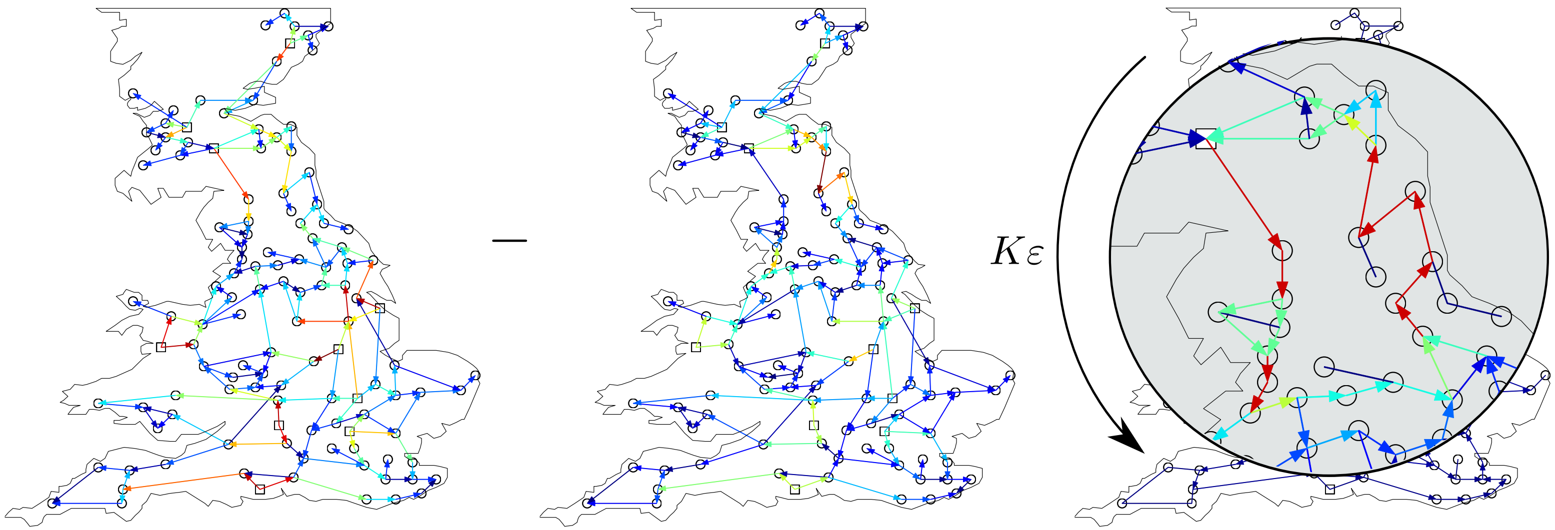


*Voltage angle uniquely defined

→ $q = \sum_i |\theta_{i+1} - \theta_i|_{2\pi} / 2\pi \in \mathbb{Z}$ **~topological winding number**

→ discretization of these loop currents **~vortex flows**

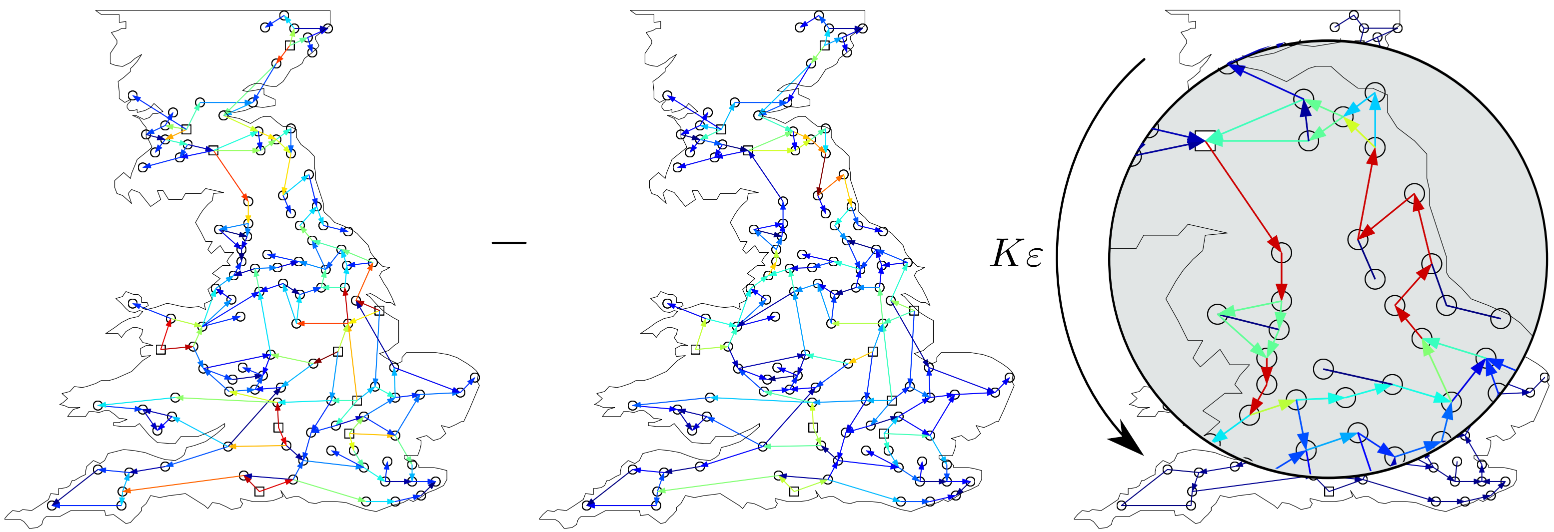
Janssens and Kamagate '03



Circulating loop flows

PF solutions with different loop flows exist, but…

(i) how many of them are there ?
(ii) how can one identify them beforehand ?
how can one find them numerically ?
(iii) how are they generated ?



Circulating loop flows

PF solutions with different loop flows exist, but…

(i) how many of them are there ?

R Delabays (previous talk):

upper bound OK, but seems rather high

→ *try numerics*

Delabays, Coletta, and PJ, J Math Phys '16
Delabays, Coletta, and PJ, to appear in J Math Phys '17

Circulating loop flows

PF solutions with different loop flows exist, but…

(ii) how can one identify them beforehand ?
how can one find them numerically ?

Numerically finding circulating loop flows

Algorithm to determine different solutions numerically

(i) use iterative method (RK on swing; NR on power flow…)

(ii) use different initial states

brute force : random initial state ?
…time-consuming to get states with finite q

Numerically finding circulating loop flows

Algorithm to determine different solutions numerically

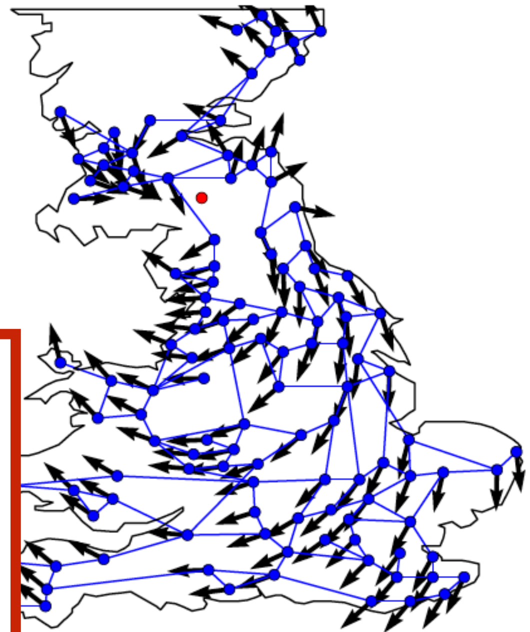
(i) use iterative method (RK on swing; NR on power flow…)

(ii) construct vortex-carrying initial state

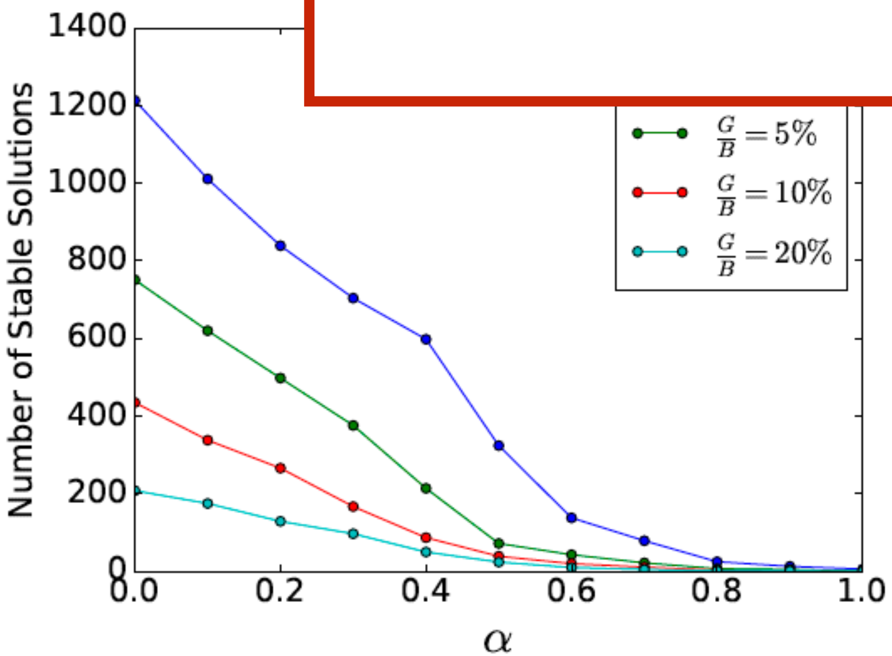
θ

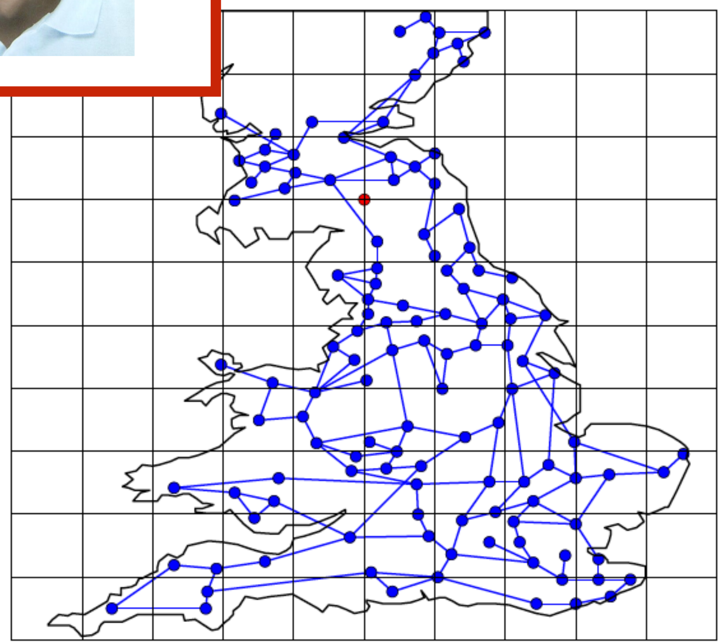
(iii) iterate and

(iv) pick another



See Melvyn Tyloo's poster





Circulating loop flows

PF solutions with different loop flows exist, but…

(iii) how do they appear ? how are they generated ?

Problem : topological protection
i.e. discrete jump $q \longrightarrow q' \neq q$ requires to unwind a large number of voltage angles

Coletta, Delabays, Adagideli and PJ, New J Phys. '16

Circulating loop flows

PF solutions with different loop flows exist, but…

(iii) how do they appear ? how are they generated ?

Solution :

(i) line tripping
(ii) line tripping and reclosing
(iii) dynamical phase slip (loss of stability of q state)
(iv) fluctuating productions

Coletta, Delabays, Adagideli and PJ, New J Phys. '16

Generation of vortex flow by line tripping

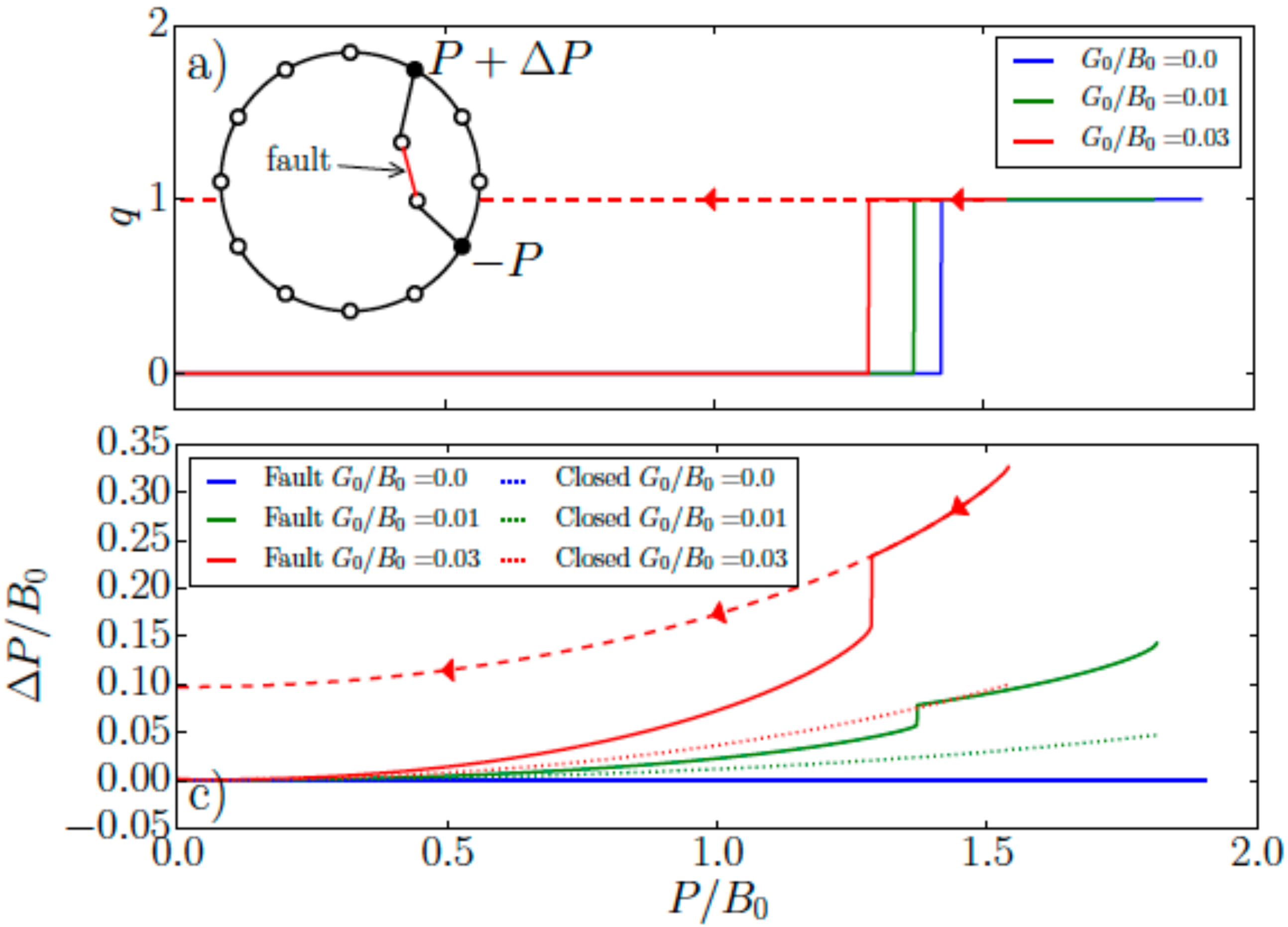
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

Line tripping at $P/B_0 > 1.3$

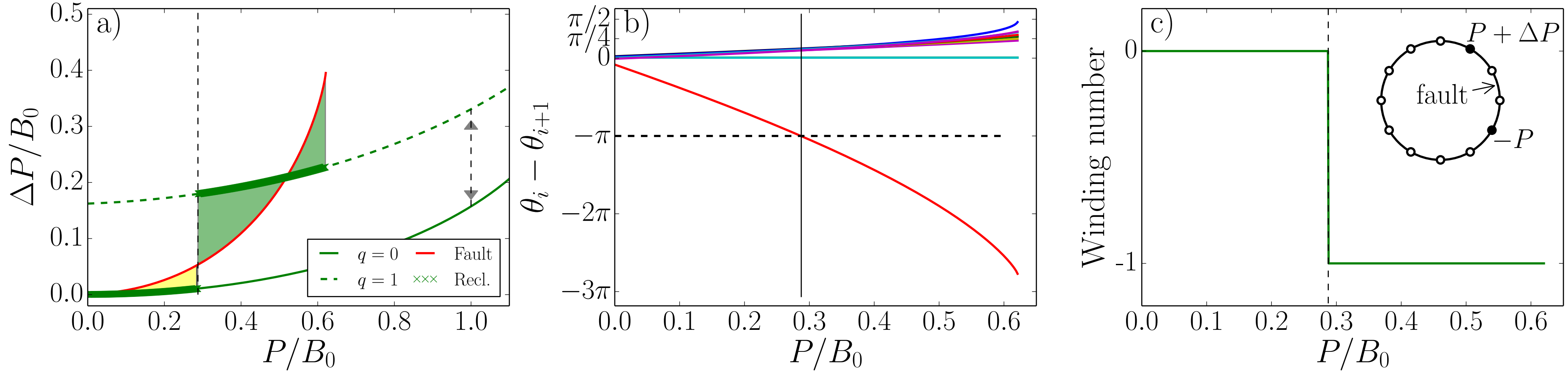
→ q=1

Vortex state characterized by
-hysteresis
= **topological protection**
-higher losses



Coletta, Delabays, Adagideli and PJ '16

Generation of vortex flow by line tripping and reclosure



- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

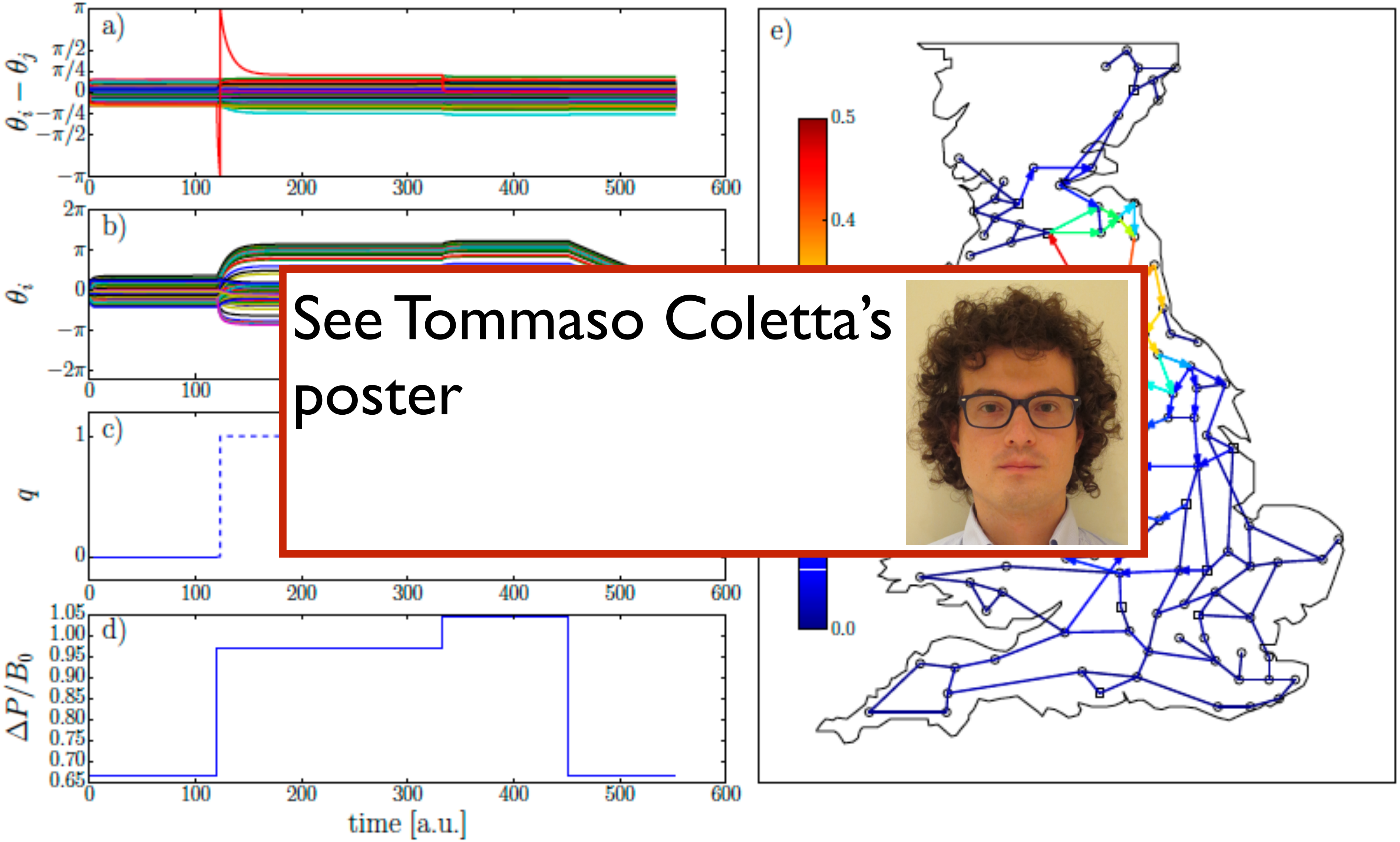
Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

Coletta, Delabays, Adagideli and PJ '16; Janssens and Kamagate '03

Hes·so// VALAIS WALLIS

Generation of vortex flow by line tripping and reclosure



See Tommaso Coletta's poster

Coletta, Delabays, Adagideli and PJ, NJP '16

Thank you !

Coletta and PJ, Phys Rev E 93, 032222 (2016)

Delabays, Coletta, and PJ, J Math Phys 57, 032701 (2016)

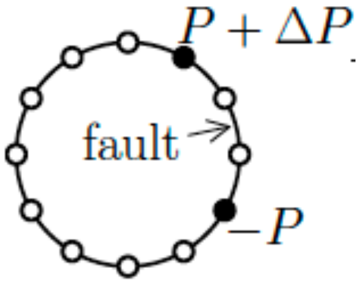
Coletta, Delabays, Adagideli and PJ, New J Phys '16 (to appear)

Delabays, Coletta, and PJ, arXiv:1609.02359, submitted to J Math Phys

Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

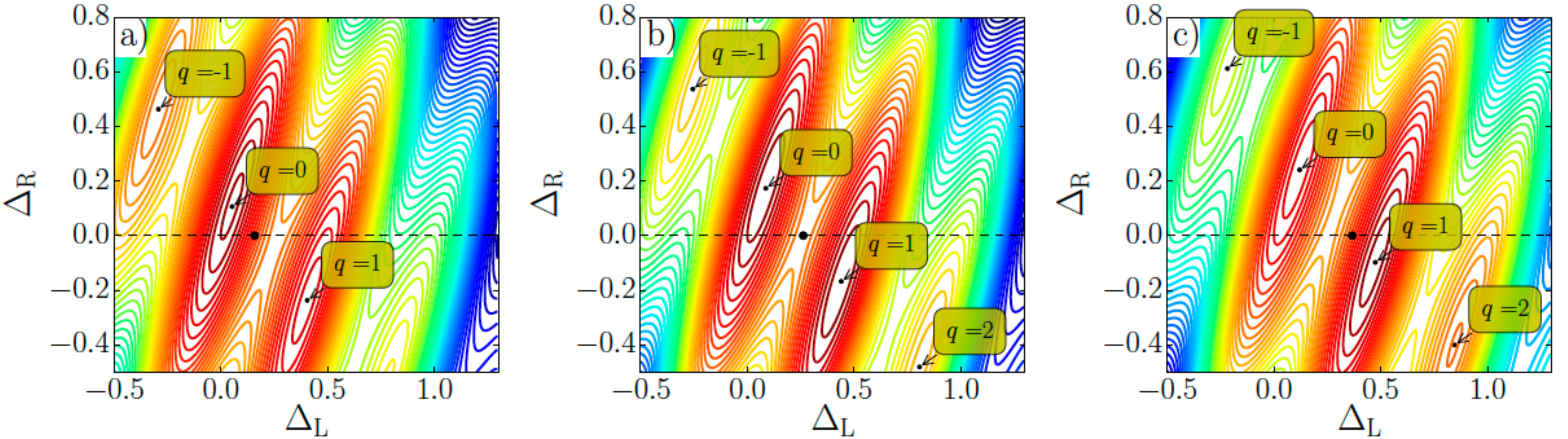
$$\mathcal{V}(\{\theta_i\}) = -\sum_l P_l \theta_l - \sum_{\langle l,m\rangle} B_0 \cos(\theta_l - \theta_m)$$



van Hemmen and Wreskinski '93

*In our case

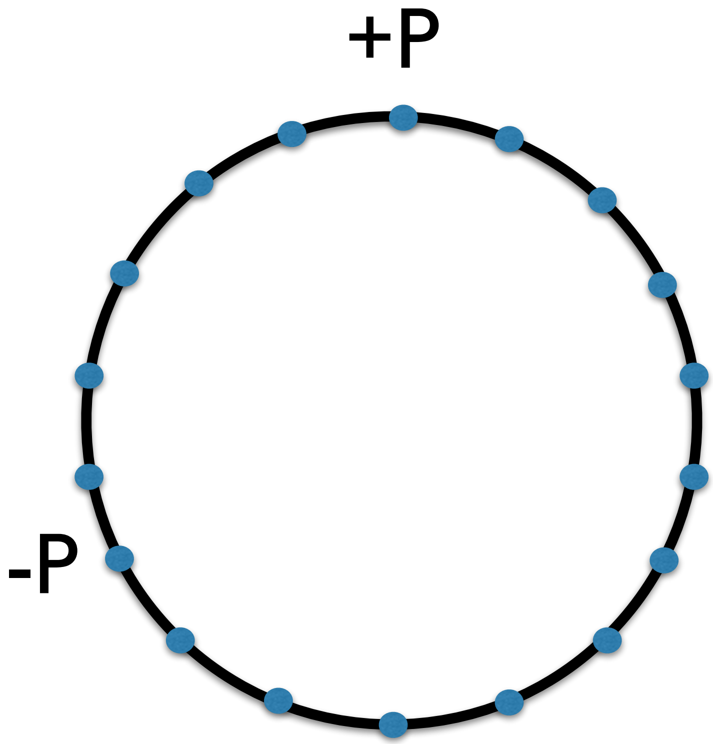
$$\mathcal{V}(\Delta_{\rm L}, \Delta_{\rm R}) = -N_{\rm L} P \Delta_{\rm L} - N_{\rm L} B_0 \cos\Delta_{\rm L} - (N_{\rm R}-1) B_0 \cos\Delta_{\rm R} - B_0 \cos(N_{\rm L}\Delta_{\rm L} - (N_{\rm R}-1)\Delta_{\rm R})$$

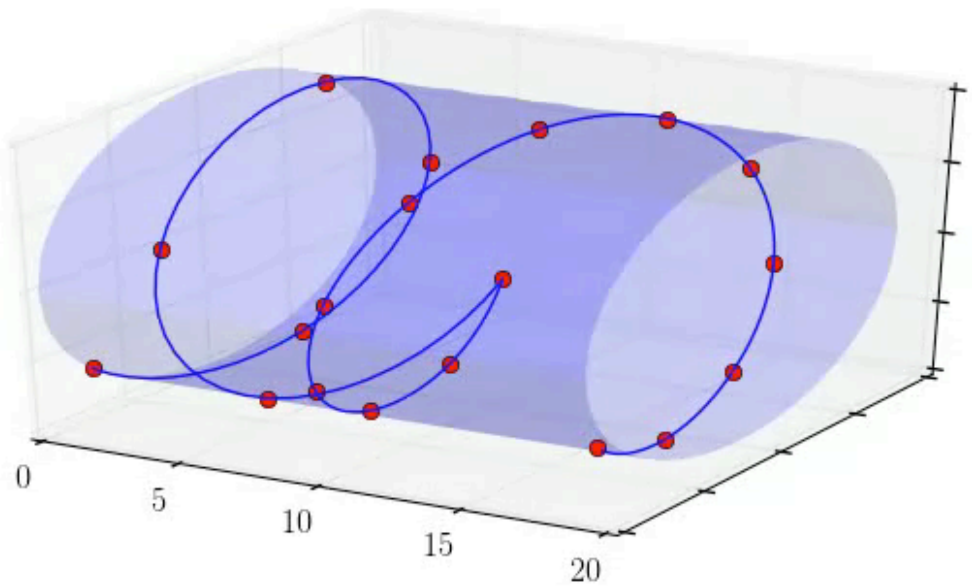


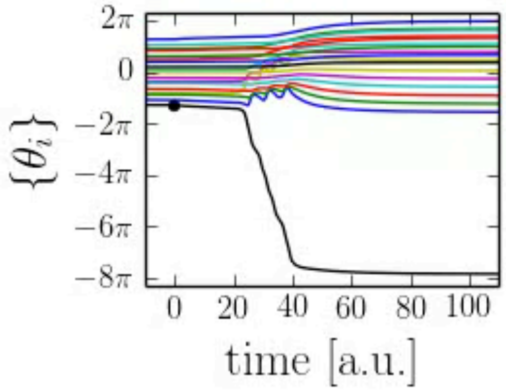
$P \approx 0.159 B_0$ $\qquad$ $P = B_0 \sin(\pi/12) \approx 0.259 B_0$ $\qquad$ $P \approx 0.359 B_0$

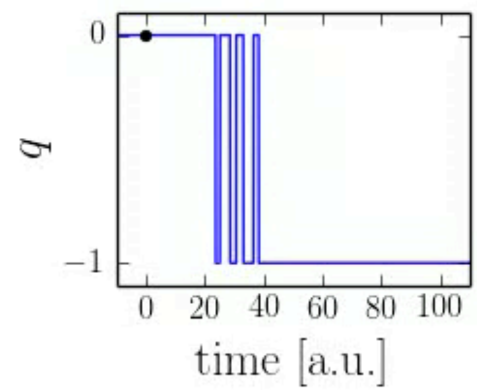
Coletta, Delabays, Adagideli and PJ, NJP '16

Dynamical generation of vortex flows









Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham ’01
Matveev, Larkin and Glazman ‘02