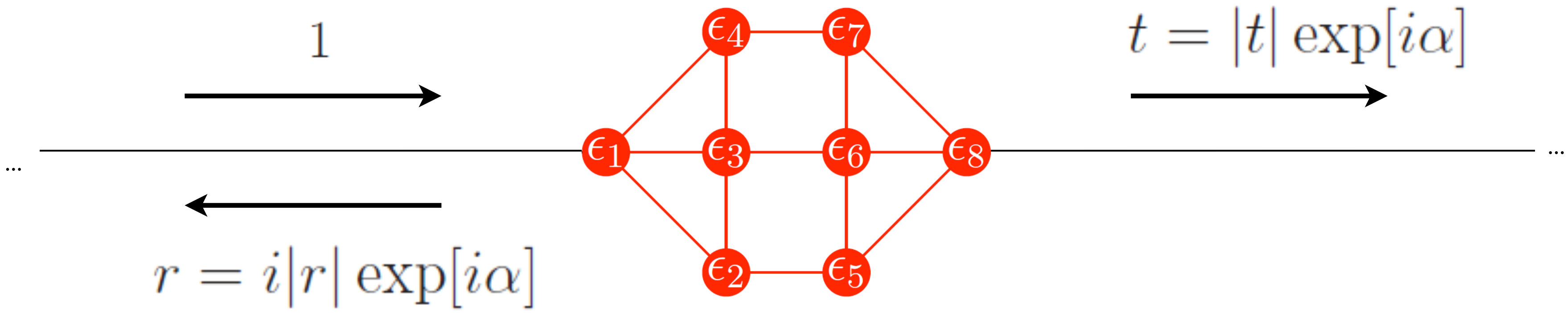
Transmission Phase through Finite-sized Electron Systems

Philippe Jacquod - HES-SO

PJ and M Büttiker, PRB 88, 241409(R) (2013)
R Molina et al., PRB 88, 045419 (2013)
PJ, R Whitney, J Meair, and M. Büttiker, PRB 86, 155118 (2012)
R Molina, R Jalabert, D Weinmann, and PJ, PRL 108, 076803 (2012)
JP Bergfield, PJ and CA Stafford, PRB 82, 205405 (2010)
R Whitney and PJ, PRL 103, 247002 (2009); EPL 91, 67009 (2010)

Scattering Phases

Extended scatterer connected to two semi-infinite 1D conductors



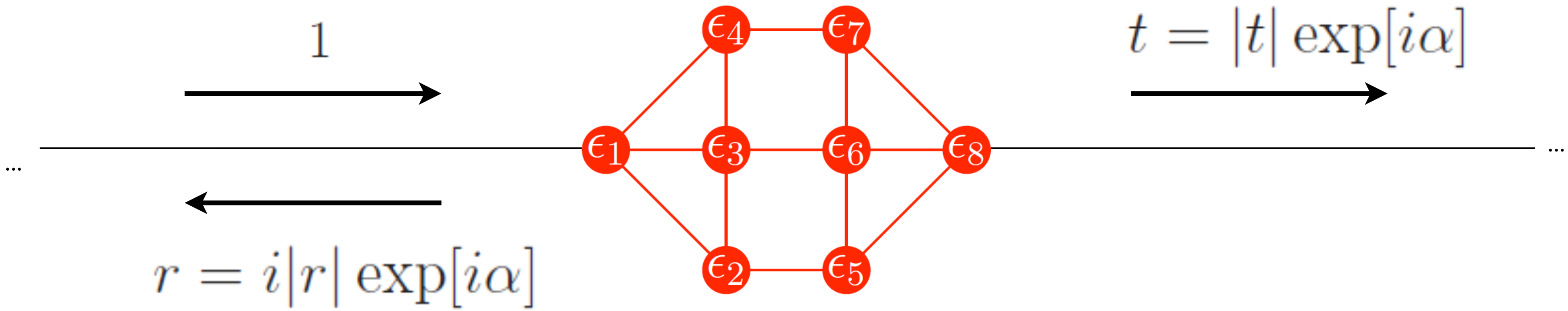
What is α? Can we measure it ?

Certainly not with the conductance (Landauer-Büttiker)

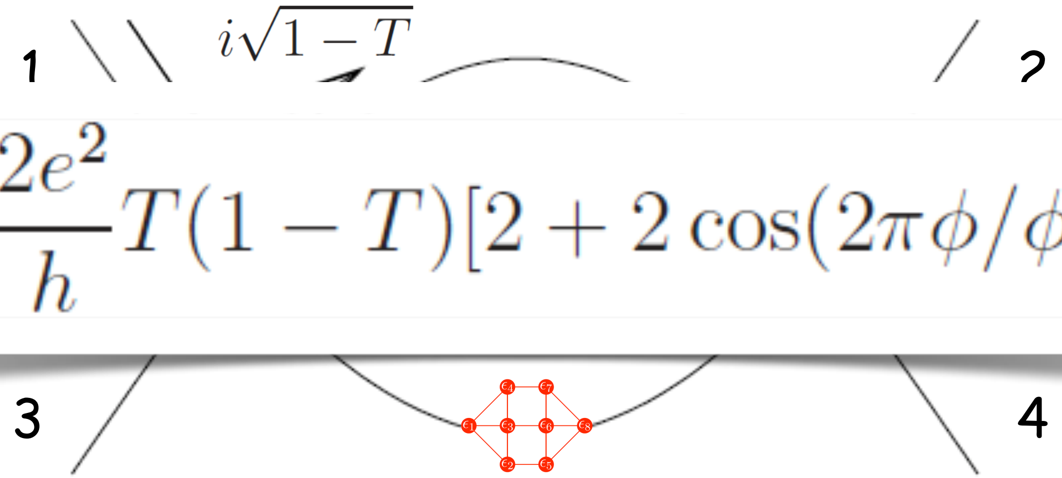
$$G = \frac{2e^2}{h}|t|^2 = \frac{2e^2}{h}T$$

Scattering Phases

Extended scatterer connected to two semi-infinite 1D conductors

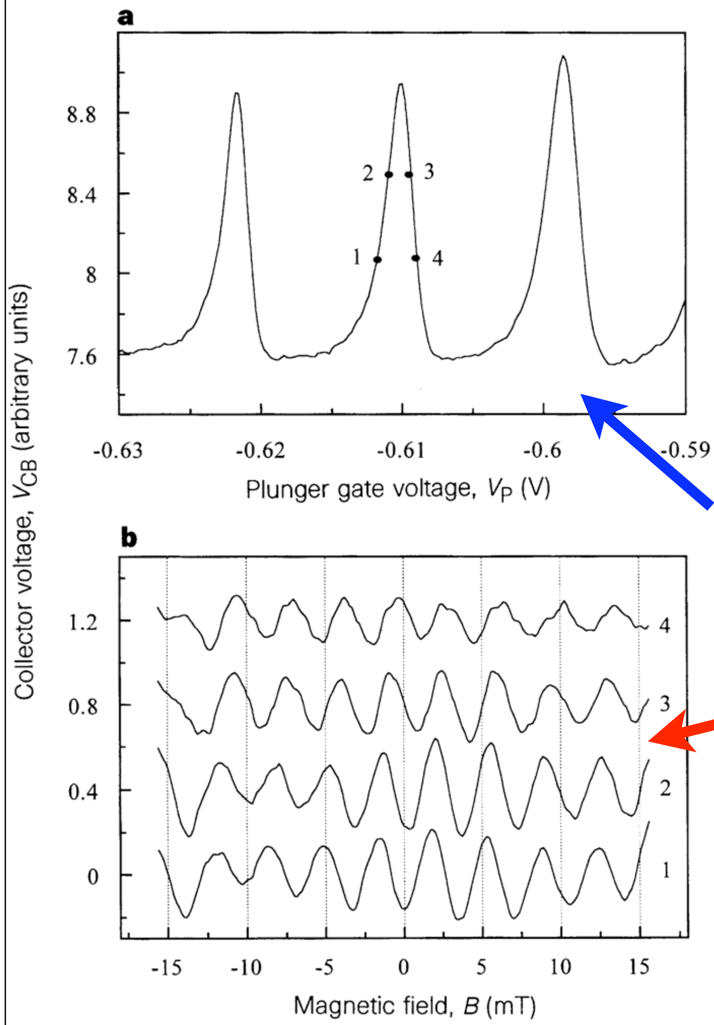


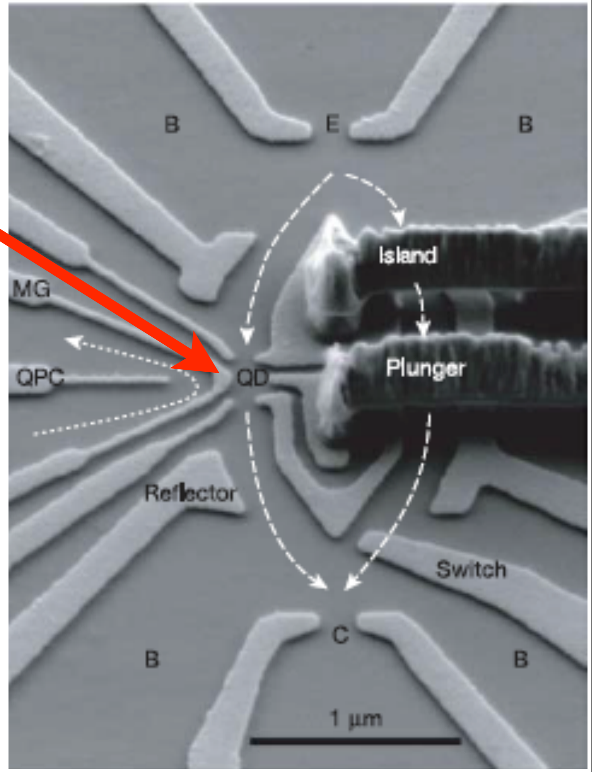
Solution : embedding into an interferometer



$$G_{41} = \frac{2e^2}{h} T(1-T)[2 + 2\cos(2\pi\phi/\phi_0 + \alpha)]$$

Measuring Scattering Phases (Heiblum & Co)





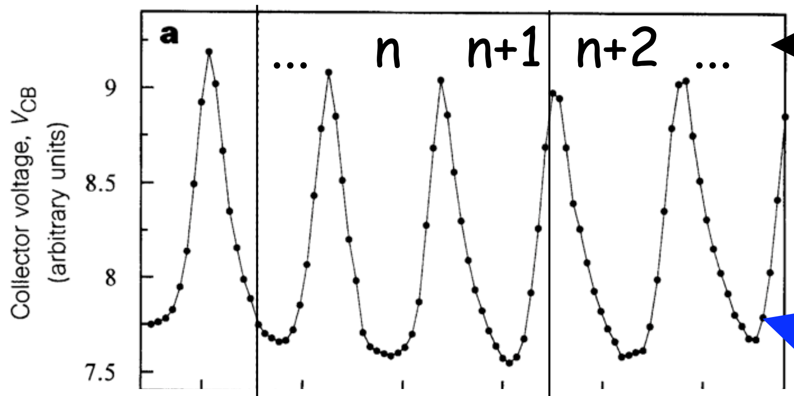
Quantum dot
in Aharonov-Bohm
interferometer

Transmission through
interferometer
*vs. chem. potential
on the dot
*vs. magnetic field
at 4 different charging - determines β_1 in

$$g = g_0 + \sum_n g_n \cos(2\pi n \phi/\phi_0 + \beta_n)$$

Yacoby et al., PRL '95
Schuster et al., Nature '97
Avinum-Kalish et al., Nature '05

Measuring Scattering Phases



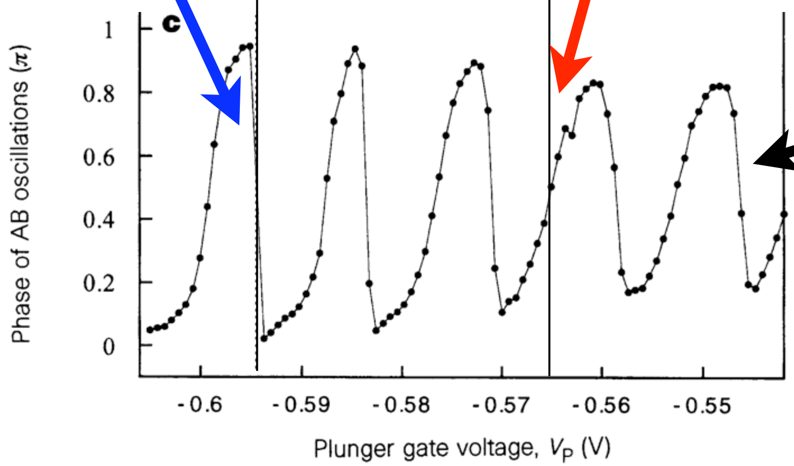
# of electrons on the dot

Transmission through interferometer vs. charging of dot
~adding electrons one by one
~Coulomb-blockade resonances

Succession of:

phase slips in between each two resonance

phase jump at each resonance



Transmission phase β_1
vs. charging of the dot

Yacoby et al., PRL '95
Schuster et al., Nature '97
Avinum-Kalish et al., Nature '05

Friedel Sum Rule

"The transmission phase at energy E jumps by π each time E crosses a bound state in the scatterer."

DoS vs. Green's fcn

$$\rho(\omega) = \frac{1}{\pi}\mathrm{Im}\ \mathrm{Tr}\left[\hat{G}^a(\omega)\right]$$

Fisher-Lee relation

$$S_{\alpha,\beta}(\omega) = \delta_{\alpha,\beta} - 2i\sqrt{\Gamma_\alpha\Gamma_\beta}G^r_{j_\alpha,j_\beta}(\omega)$$

Define the phase

$$\theta_F(\omega) = \mathrm{ImlnDet}\left[\omega - \hat{H}_{dot} - \hat{\Sigma}^a(\omega)\right]$$

-> $\dfrac{\partial\theta_F}{\partial\omega} = \pi\rho(\omega)$ and thus $\boxed{N_{dot} = \dfrac{1}{\pi}\theta_F(E_F)}$

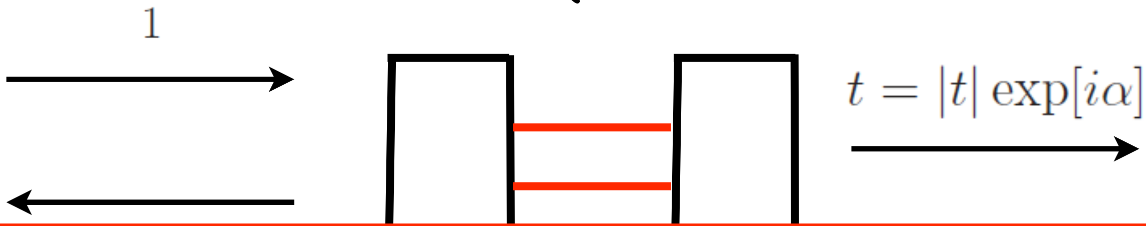
Also $\theta_F(\omega) = \dfrac{1}{2i}\mathrm{lnDet}\left[\hat{S}(\omega)\right]$ is a scattering phase

Levy Yeyati and Büttiker '00

Friedel Sum Rule

"The transmission phase at energy E jumps by π each time E crosses a bound state in the scatterer."

Ex.: resonant tunnel barriers (isolated resonances; 1D)

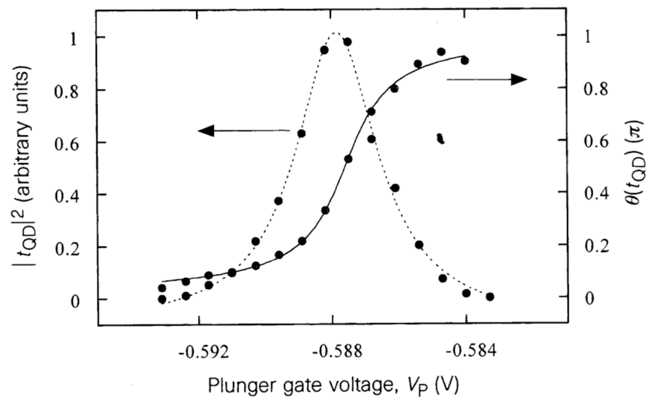


We still need to understand the phase slips in between each consecutive resonance !

Br

$$E - E_n + i\Gamma/2$$

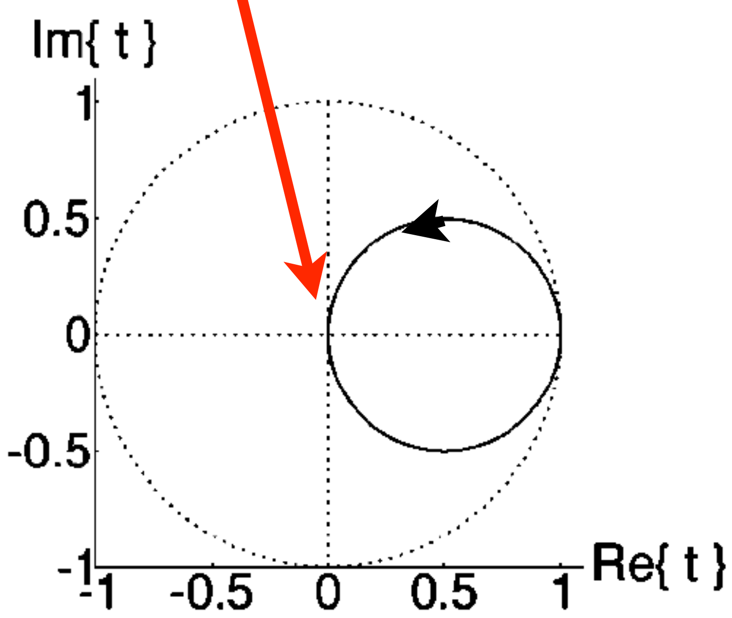
$$\boxed{\alpha(E) = \alpha_n + \arctan[2(E - E_n)/\Gamma]}$$



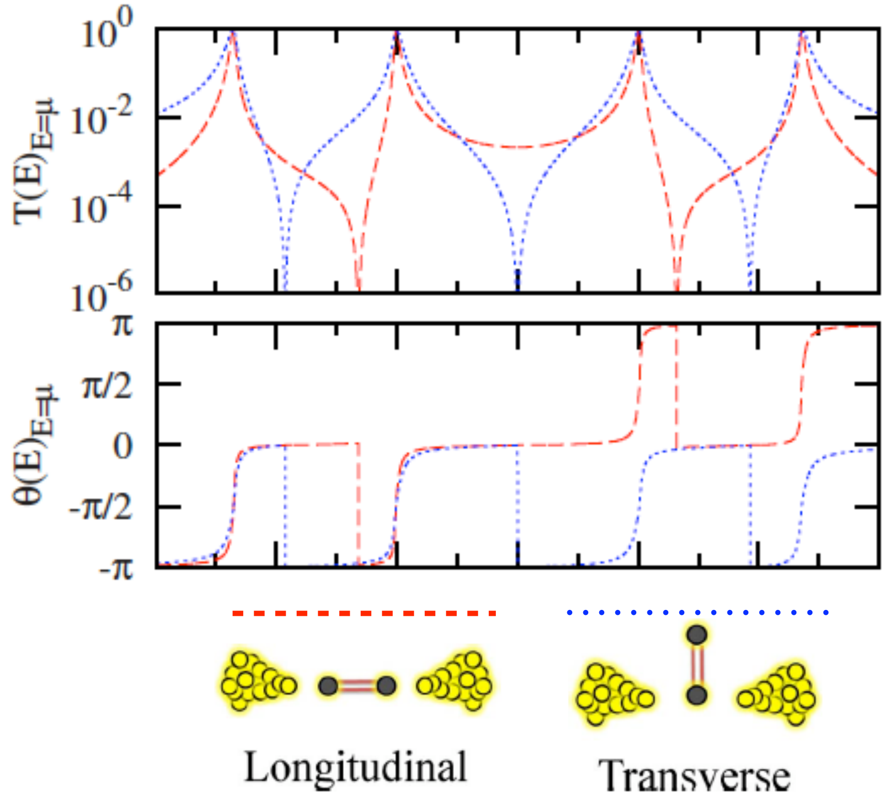
Taniguchi and Büttiker '99; Lee '99

Scattering Phase and Transmission Zeros

Smooth vanishing of $t = |t| \exp[i\alpha]$



Phase jump of π !



May occur, e.g. in molecular transport via interference of two different transmission paths

Taniguchi and Büttiker '99; Lee '99

Bergfield, PJ, Stafford, PRB '10

Condition for Transmission Zeros

- Connection to 1D external leads -> via only two points

- Fisher-Lee relation

$$t(\epsilon) = -2i(\Gamma_l \Gamma_r)^{1/2} G_{1M}(\epsilon)$$

- Green's function

$$G(\epsilon) = (\epsilon - H_{\mathrm{D}} - \Sigma(\epsilon))^{-1}$$

Green's function from site 1 to site M of the scatterer

$\Sigma(\epsilon)$: self-energy due to the leads

H_D : dot's Hamiltonian

➡ Vanishing of t is equivalent to

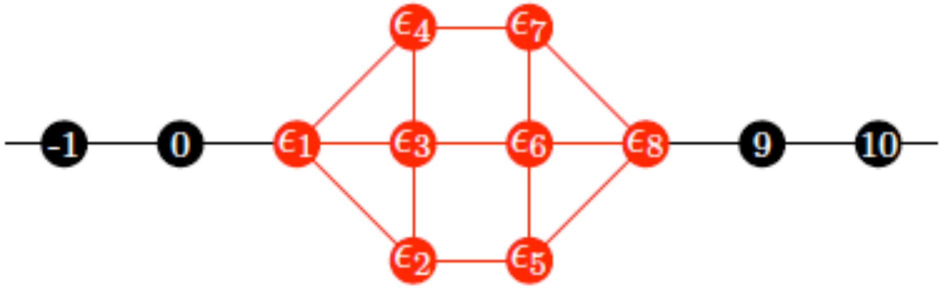
$$C_{1M}\{\epsilon - H_D - \Sigma(\epsilon)\} = 0$$

[$C_{1M}\{A\}$: cofactor of the element (1,M) of the matrix A]

Levy Yeyati and Büttiker, PRB '00

Condition for Transmission Zeros

➡ Vanishing of t is equivalent to



$$\mathcal{C}_{1M}\left\{\epsilon - H_D + \Sigma(\epsilon)\right\} = 0$$

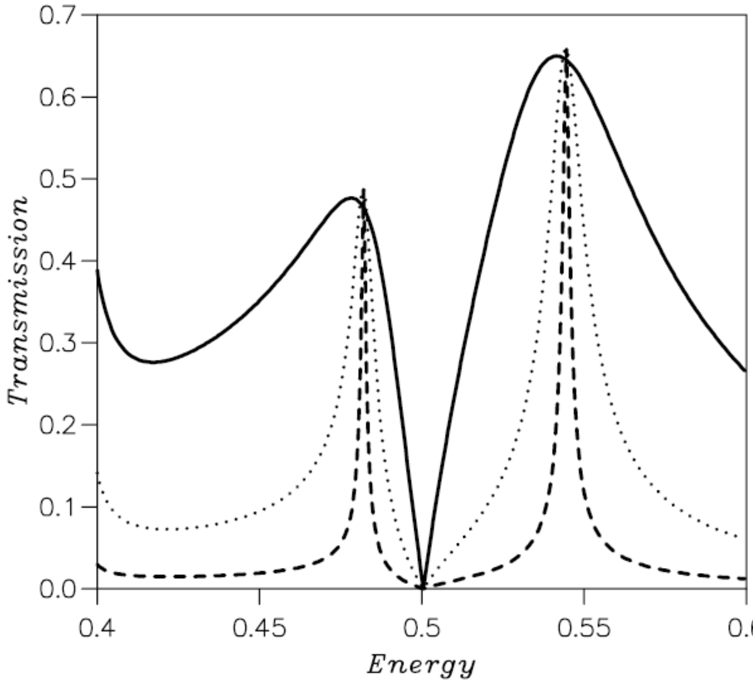
Self-energy due to leads vanish except at 1 and M

➡ cofactor does not depend on it!

$$\mathcal{C}_{1M}\{\epsilon - H_D\} = 0$$

➡ Occurrence of zeros depends on the isolated dot only !

➡ Relate zeros to wave functions of the isolated dot



Levy Yeyati and Büttiker, PRB '00

Condition for Transmission Zeros

➡ Vanishing of t is equivalent to

$$\mathcal{C}_{1M}\{\epsilon - H_D\} = 0$$

➡ Relate zeros to wave functions of the isolated dot

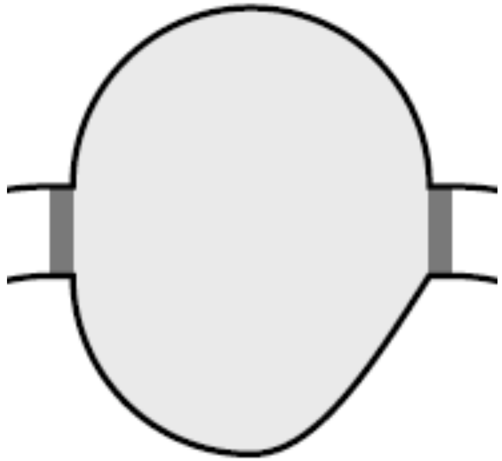
How can we systematically have $D_m > 0$?

$$G_{1M}(\epsilon) = \sum_m \psi_m(1)\psi_m(M)/[\epsilon - \epsilon_m + \cancel{i(\Gamma_l + \Gamma_r)}]$$

➡ Condition for zero between resonance #m and #m+1 :

$$D_m = \psi_m(1)\psi_m(M)\psi_{m+1}(1)\psi_{m+1}(M) > 0$$

Condition for Transmission Zeros



Experimental dots are ballistic and irregularly shaped

Use statistical method to calculate average of D_m

$$\langle D_m \rangle = \langle \psi_m(1)\psi_m(M)\psi_{m+1}(1)\psi_{m+1}(M)\rangle$$
$$\simeq \langle \psi_m(1)\psi_m(M)\rangle \langle \psi_{m+1}(1)\psi_{m+1}(M)\rangle$$

Berry '77 : wavefunctions in ballistic systems have long-range correlations

$$\boxed{\langle \psi_m(1)\psi_m(M)\rangle = J_0(k_m|\mathbf{r}_1 - \mathbf{r}_M|)/A}$$

Note: to be compared with exp. small correlations in diffusive systems !

$$\langle \psi_m(1)\psi_m(M)\rangle \sim \exp[-k_m \ell]$$

Condition for Transmission Zeros

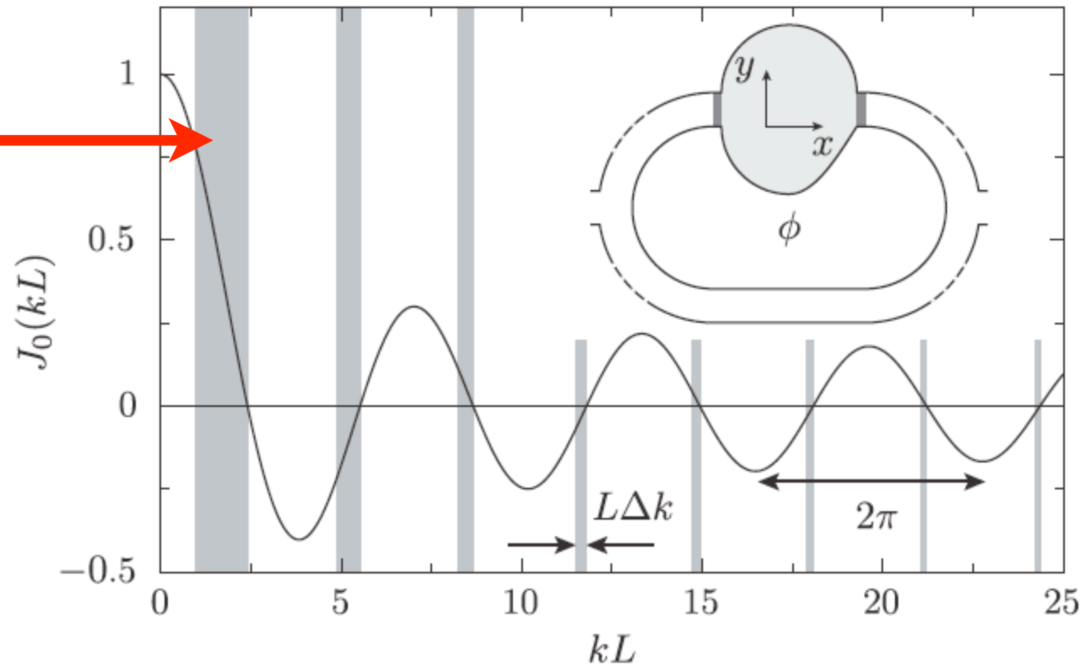
$$\langle D_m \rangle = J_0(k_m|\mathbf{r}_1 - \mathbf{r}_M|)J_0(k_{m+1}|\mathbf{r}_1 - \mathbf{r}_M|)/A^2$$

Fluctuations are small (not shown) -> neglect them

When $\langle D_m \rangle > 0$ -> t=0 somewhere between resonance #m and #m+1

Grey regions have $\langle D_m \rangle < 0$

(change in sign of Bessel function from k_m to k_{m+1})



Molina, Jalabert, Weinmann and PJ, PRL '12

Condition for Transmission Zeros

$$\langle D_m \rangle = J_0(k_m|\mathbf{r}_1 - \mathbf{r}_M|)J_0(k_{m+1}|\mathbf{r}_1 - \mathbf{r}_M|)/A^2$$

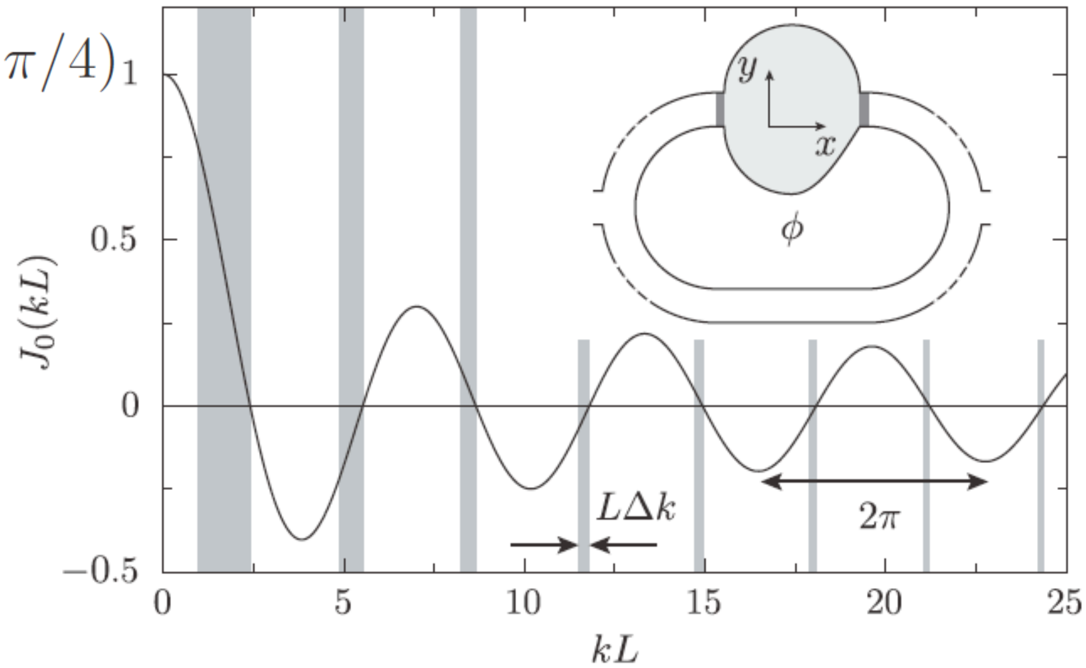
Take finally 2D spacing + asymptotic of Bessel function

$$k_{m+1} - k_m \simeq \pi/(kL^2)$$

$$J_0(kL) \simeq \sqrt{2/(\pi kL)}\cos(kL - \pi/4)$$

➡ Probability not to have a transmission zero is

$$\mathcal{P} \simeq (kL)^{-1}$$



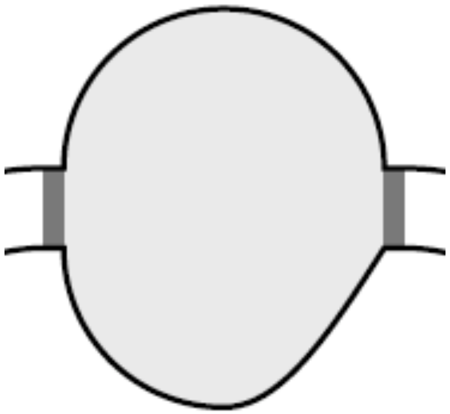
Molina, Jalabert, Weinmann and PJ, PRL '12

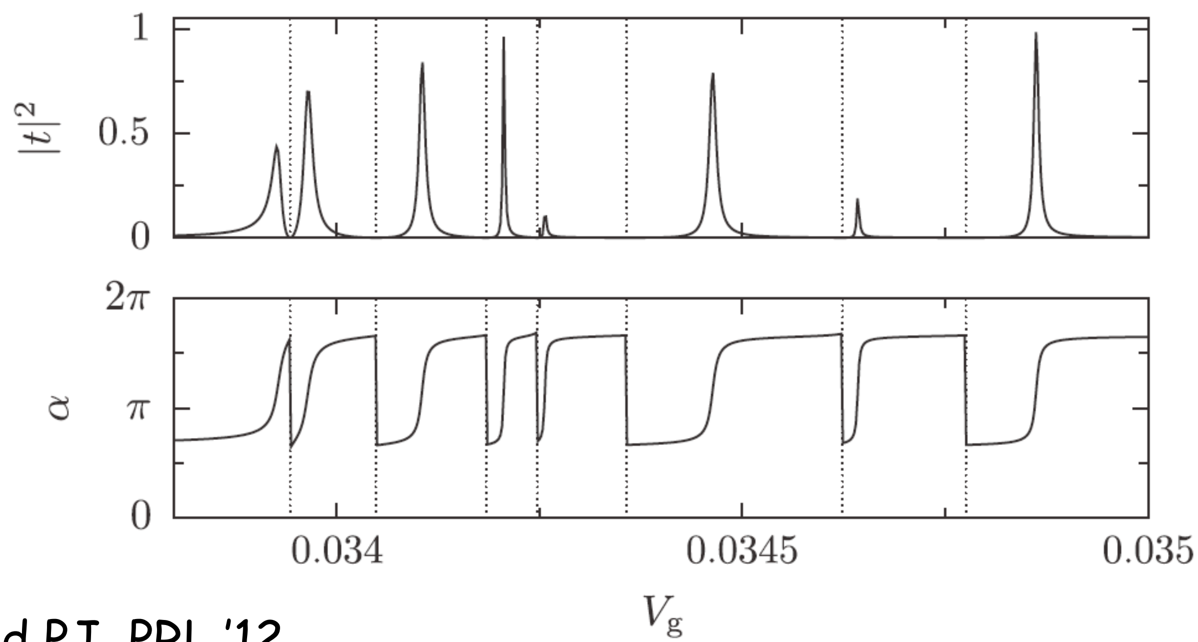
Condition for Transmission Zeros

$$\langle D_m \rangle = J_0(k_m|\mathbf{r}_1 - \mathbf{r}_M|)J_0(k_{m+1}|\mathbf{r}_1 - \mathbf{r}_M|)/A^2$$

➡ Probability NOT to have a transmission zero is $\mathcal{P} \simeq (kL)^{-1}$

Numerics on chaotic dot at kL ~ 100





Molina, Jalabert, Weinmann and PJ, PRL '12

Semiclassical Crossover

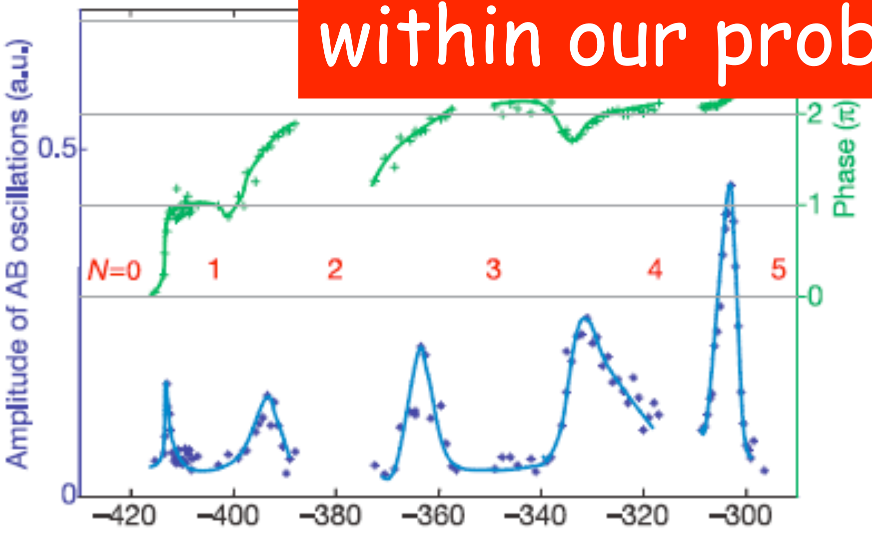
Crossover from 'mesoscopic' to 'universal' phase for electron transmission in quantum dots

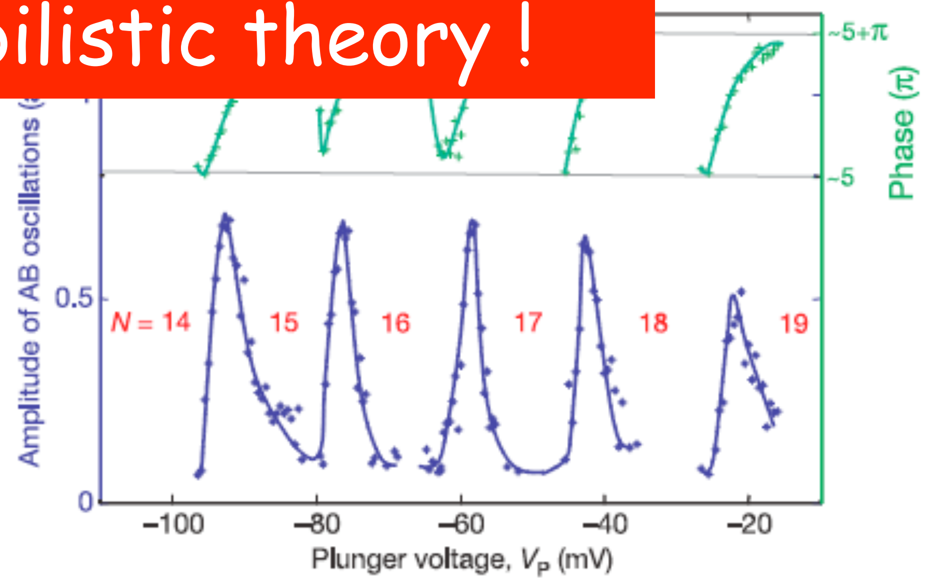
M. Avinun-Kalish[1], M. Heiblum[1], O. Zarchin[1], D. Mahalu[1] & V. Umansky[1]

Few electrons ~ small kL
missing phase slips
aka mesoscopic behavior

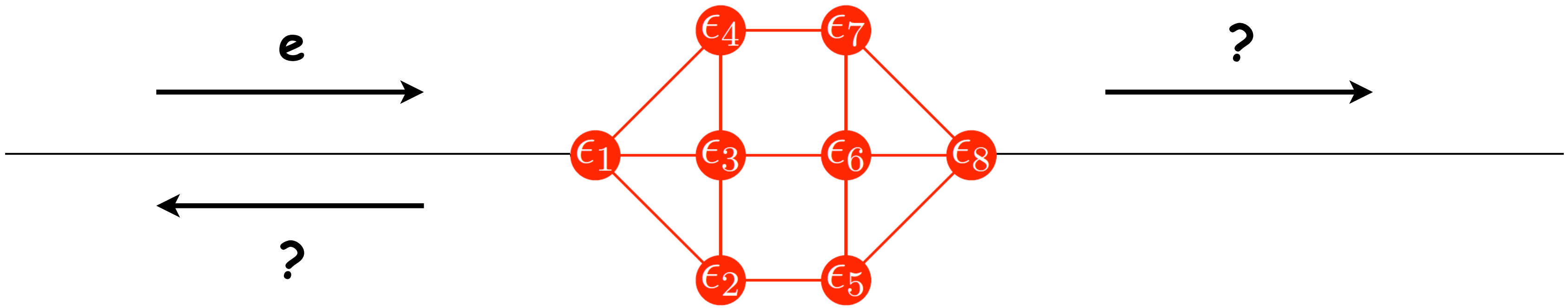
More electrons ~ large kL
systematic phase slips
aka universal behavior

This crossover can be understood within our probabilistic theory !





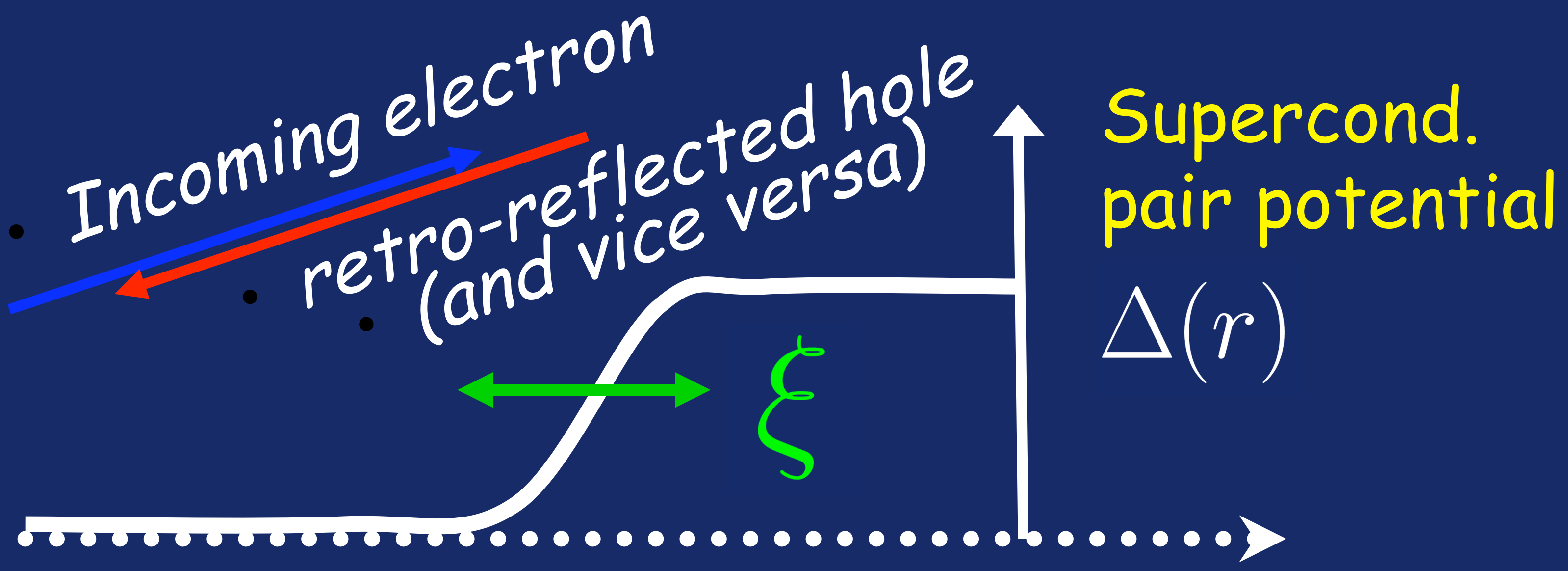
Scattering Phases w. superconductor



What is Andreev reflection

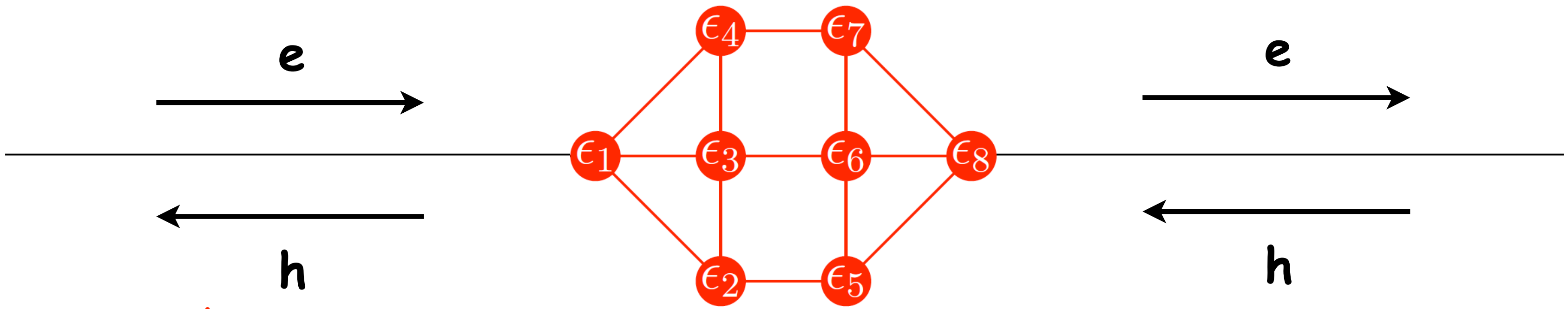
- low-energy electron quasiparticle approaches superconductor from normal region

"Charge-reversing retro-reflection"



Andreev, '64; sidenote : Andreev reflection ~ Hawking radiation

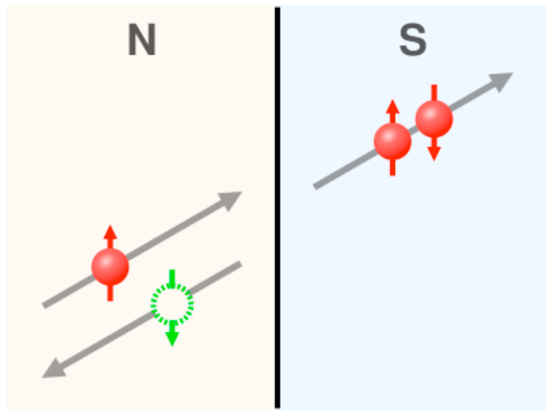
Scattering Phases w. superconductor



Superconductivity :

➡ electrons enter by pair only

➡ Andreev reflection of e into holes
+injection of a Cooper pair



➡ Friedel sum rule -> transmission phase = 0 (mod 2 π)

$$\boxed{N_{dot} = \frac{1}{\pi}\theta_F(E_F)}$$

Good news : Friedel sum rule holds!
Andreev interferometer; Whitney and PJ, '09, '10

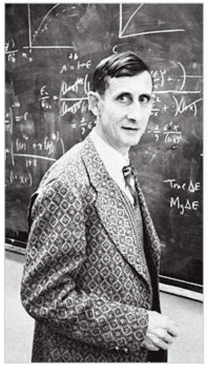
Bad news : ...kind of trivially...

Symmetry classes - the 10-fold way

"Dyson's 3-fold way + particle-hole symmetry"

Time-reversal symmetry ⟷ Antiunitary symmetries
Particle-hole symmetry ⟷ $P^2, T^2 = -1, 0, 1$

3x3=9 and two possibilities for P=T=0 -> 10-fold way



F Dyson

J Verbaarschot

Symmetry class		TRS	PHS	SLS
Wigner-Dyson	A (unitary)	0	0	0
	AI (orthog.)	+1	0	0
	AII (sympl.)	−1	0	0
Chiral	AIII (unitary)	0	0	1
	BDI (orthog.)	+1	+1	1
	CII (sympl.)	−1	−1	1
Altland-Zirnbauer	D	0	+1	0
	C	0	−1	0
	DIII	−1	+1	1
	CI	+1	−1	1

M Zirnbauer

A Altland

Classification of nontrivial topological states vs. symmetries

PHYSICAL REVIEW B **78**, 195125 (2008)

Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]

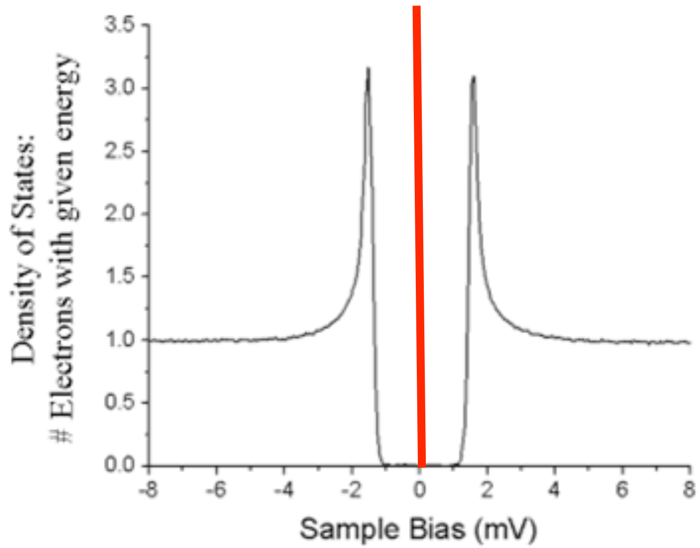
		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
Standard	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
(Wigner-Dyson)	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	−1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
(sublattice)	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	−1	−1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	−1	0	-	$\mathbb{Z}$	-
	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

Topological superconductor : definition

"Material that is superconducting in the bulk - with a quasiparticle excitation gap - but with zero-energy gapless surface states."

Those surface states are Majorana fermions

-> possibility to have odd # of electrons in a SC



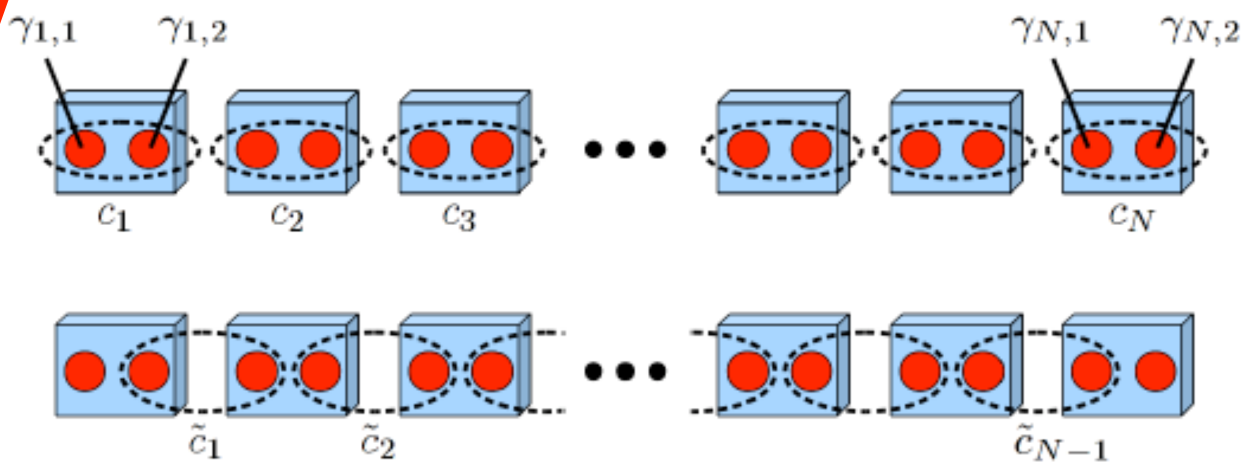
E Majorana

Kitaev's chain : 1D p-wave superconductor

$$\mathcal{H}_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c. \right)$$

Majorana operators

$$\gamma_{i,1} = c_i^\dagger + c_i,$$
$$\gamma_{i,2} = i\left(c_i^\dagger - c_i\right)$$



$$\mathcal{H}_{\text{chain}} = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1}$$

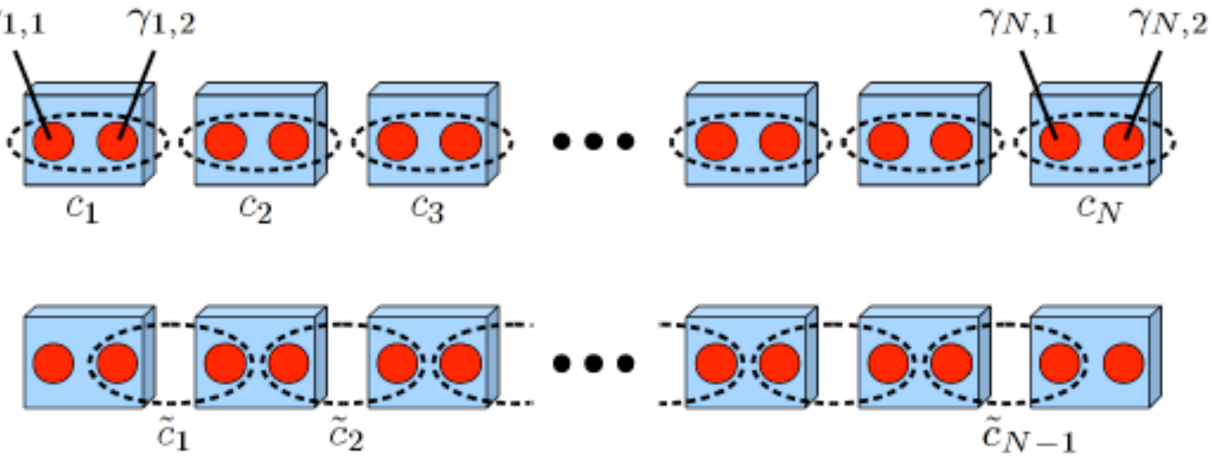
(special case μ=0 and Δ=t)

Kitaev's chain : 1D p-wave superconductor

$$\mathcal{H}_{\text{chain}} = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1}$$

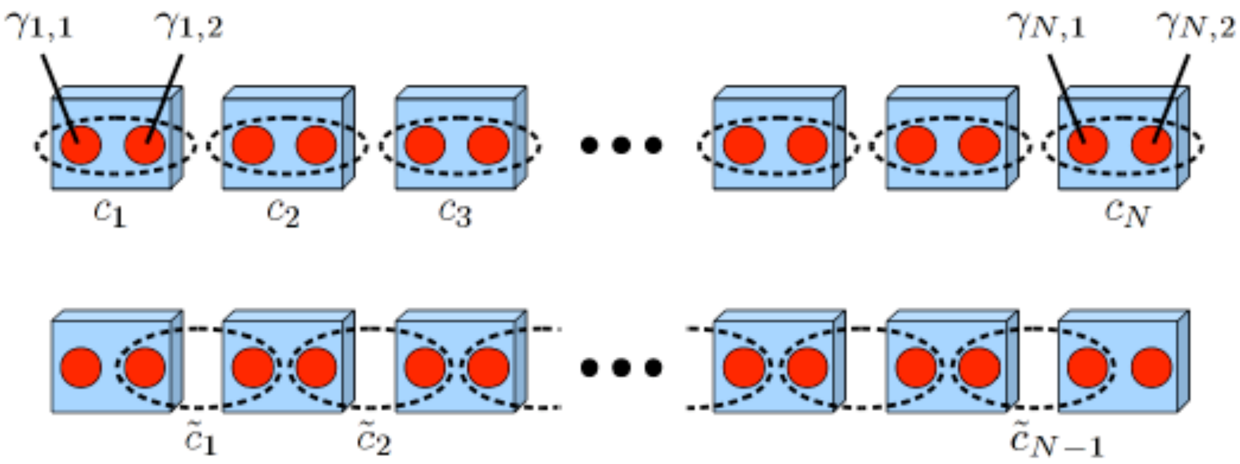
Fermion operators

$$\boxed{\tilde{c}_i = (\gamma_{i+1,1} + i\gamma_{i,2})/2}$$



$$\mathcal{H}_{\text{chain}} = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i$$

Kitaev's chain : 1D p-wave superconductor



$$\mathcal{H}_{\text{chain}} = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i$$

Absence of $\tilde{c}_M = (\gamma_{N,2} + i\gamma_{1,1})/2$!!!

-> zero-energy state

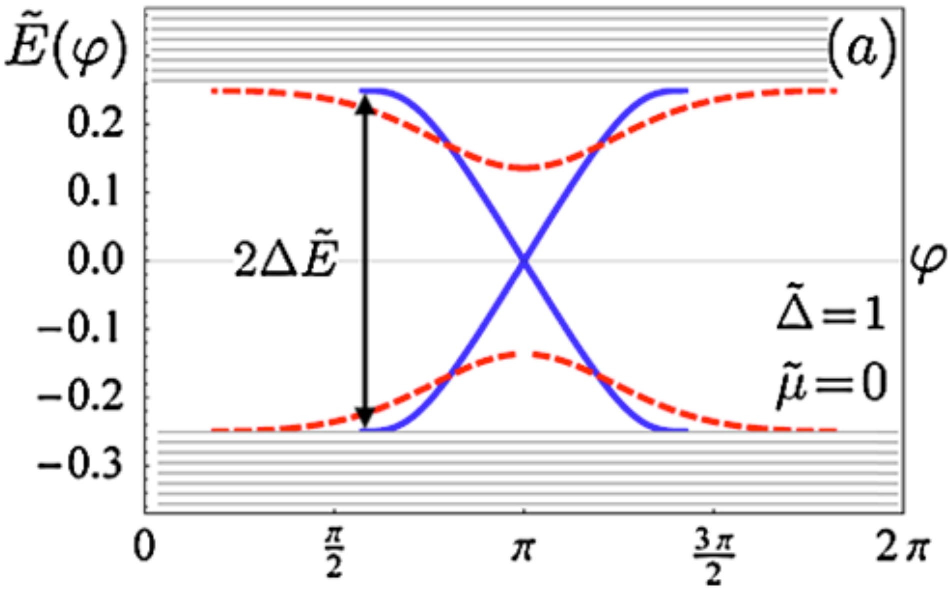
("two half-fermions")

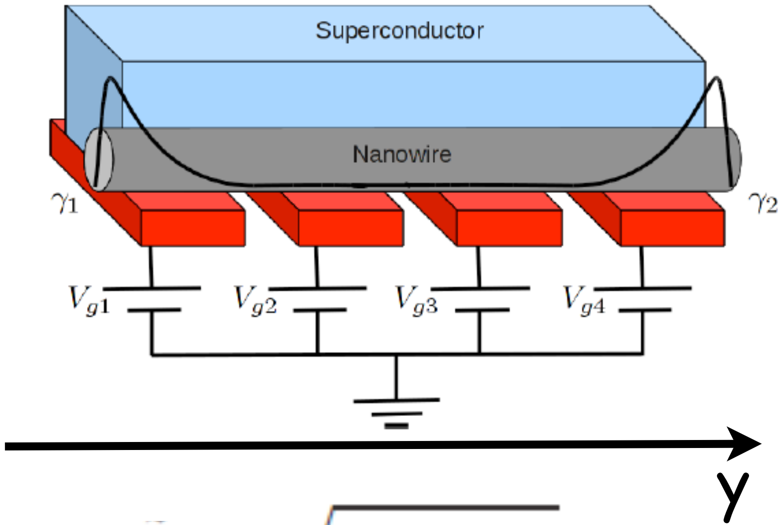
Majorana platforms : how to generate a 1D p-wave superconductor

- Fu and Kane: Interface between SC and TI

- Lutchyn, Sau, Sarma; Oreg, Refael, von Oppen :
  Proximity-coupled nanowire with spin-orbit

$$H = (p_y^2/2m - \mu)\tau_z + u p_y \sigma_z \tau_z + B_Z \sigma_x + \Delta \tau_x$$





$$|B| > \sqrt{\Delta^2 + \mu^2}$$

Majorana fermions in transport

Electron Teleportation via Majorana Bound States in a Mesoscopic Superconductor

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(Received 23 October 2009; published 2 February 2010)

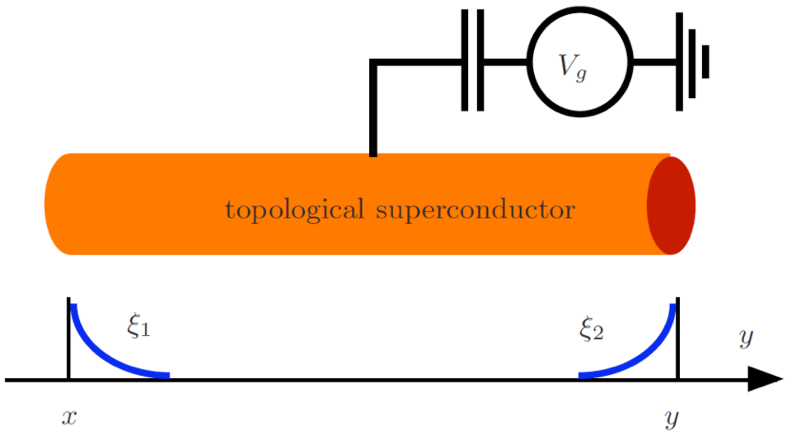
Zero-energy Majorana bound states in superconductors have been proposed to be potential building blocks of a topological quantum computer, because quantum information can be encoded nonlocally in the fermion occupation of a pair of spatially separated Majorana bound states. However, despite intensive efforts, nonlocal signatures of Majorana bound states have not been found in charge transport. In this work, we predict a striking nonlocal phase-coherent electron transfer process by virtue of tunneling in and out of a pair of Majorana bound states. This teleportation phenomenon only exists in a mesoscopic superconductor because of an all-important but previously overlooked charging energy. We propose an experimental setup to detect this phenomenon in a superconductor–quantum-spin-Hall-insulator–-magnetic-insulator hybrid system.

I.e. Green's function

sign change with n->n+1

$$G^{e,o}(x, t \to \infty; y, 0) = \mp i \xi_2^*(y) \xi_1(x) \sim O(1)$$

"long-range coherence"
-> tunneling of single electron
-> needs fixed parity



Friedel sum rule

-> Connection between a scattering phase and the number of occupied states in the scatterer

-> For Coulomb blockade, this works for the transmission phase

$$N_{dot} = \frac{1}{\pi}\theta_F(E_F)$$

i.e. additional phase of π when n->n+1

Not observable for trivial superconductor (N=2,4,6,...)

Observable for topological superconductor:

-: MS empty

+: MS occupied

$$G^{e,o}(x, t \to \infty; y, 0) = \mp i \xi_2^*(y)\xi_1(x)$$

[Fisher-Lee connection $t_{ab} = -2\, i\, (\Gamma_a \Gamma_b)^{1/2}\, G_{ab}$]

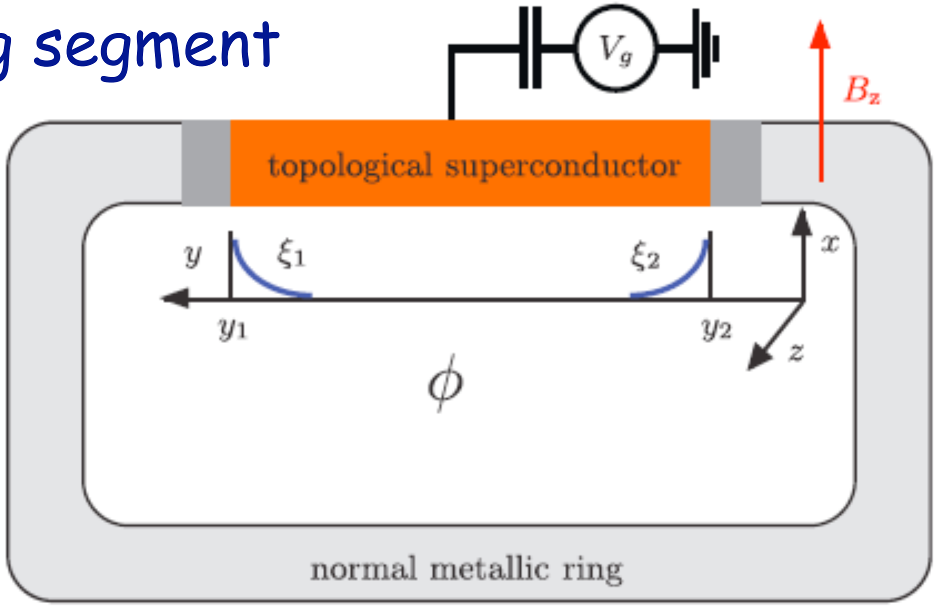
e: MS empty

o: MS occupied

Pj and M Büttiker, prb '13

The setup :

Normal metallic ring interrupted by a Coulomb blockaded superconducting segment



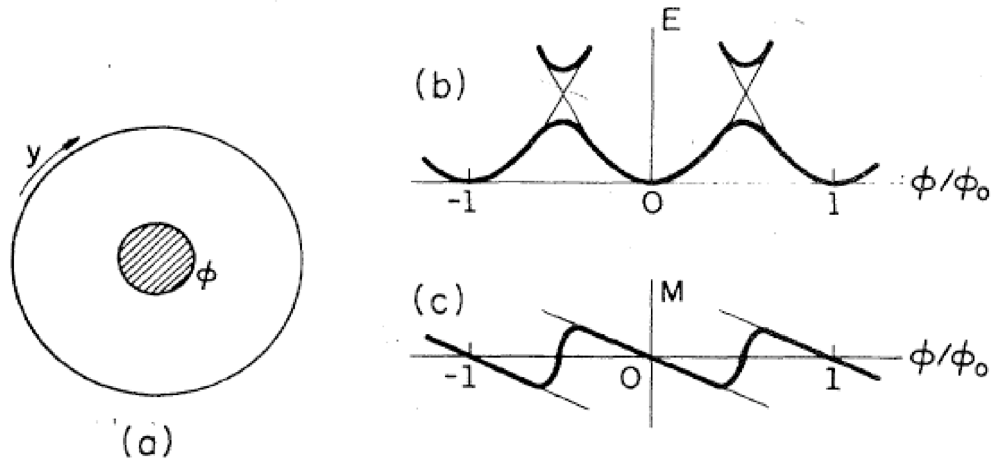
-> fix parity on the SC :

- n, n+2, n+4... electrons in the trivial phase
- n, n+1, n+2, n+3... electrons in the topological phase

Signatures of Majorana fermions in persistent currents

General idea :

-ring pierce by B-flux
-low enough T
-QM: p -> p-eA



-> ground-state current with period h/p q
(p integer; q charge of the transferred particle)

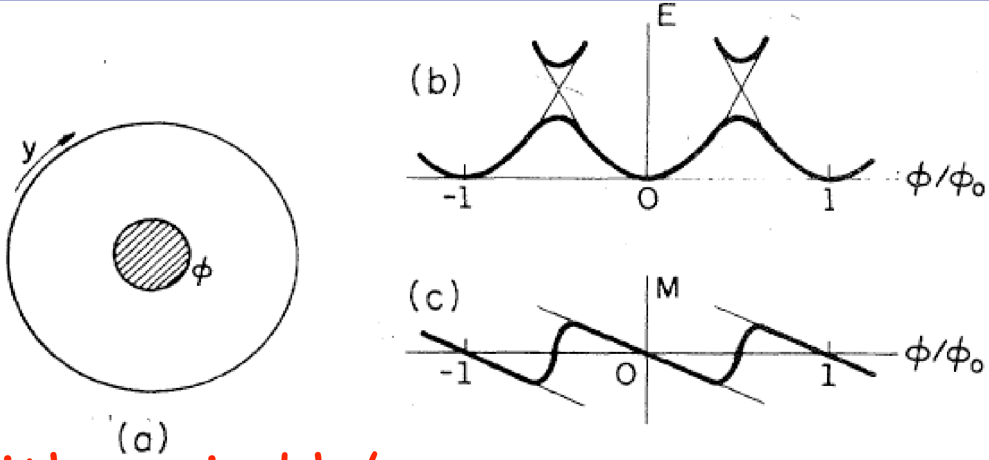
Büttiker and Klapwijk : ring with superconducting segment

$L \gg \xi$: h/pq=h/2e, h/4e ..

$L < \xi$: h/pq=h/e, h/2e ...

Signatures of Majorana fermions in persistent currents

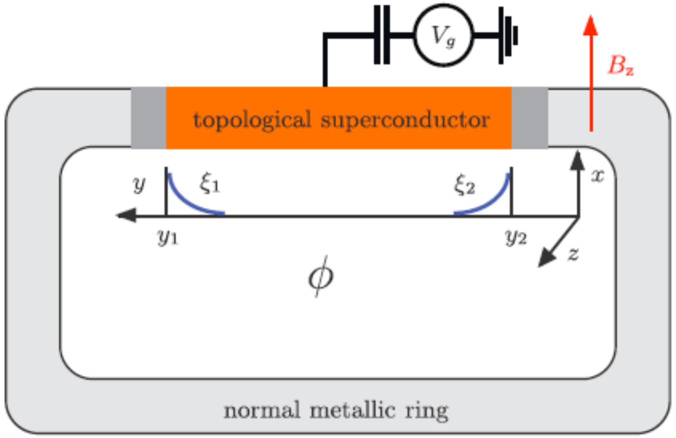
General idea :



-> ground-state current with period h/p q
(p integer; q charge of the transferred particle)

Can one detect the presence of Majorana bound states via the periodicity of persistent currents ?

Signatures of Majorana fermions in persistent currents



Effective Hamiltonian

$$H = H_{\text{ring}} + \delta(f^\dagger f - 1/2) + (\lambda_1 c_{\text{L}}^\dagger f + \text{H.c.})$$
$$+ [-i\lambda_2(-1)^{f^\dagger f} c_{\text{R}}^\dagger f \exp(i\varphi) + \text{H.c.}].$$

C : fermions on the ring   f : fermion on the topological SC

λ : hopping on SC from left/right

δ : energy difference between N and N+1 states (tunable)

$\phi = \hbar\varphi/e$

Signatures of Majorana fermions in persistent currents

Additional projection onto
|# e in ring, # e on SC> = |M,n> and |M-1,n+1>

$$\boxed{H_{\mathrm{red}} = \begin{pmatrix} \epsilon_M & \tilde{\lambda}_1 - i\tilde{\lambda}_2(-1)^{n_0} e^{i\varphi} \\ \tilde{\lambda}_1 + i\tilde{\lambda}_2(-1)^{n_0} e^{-i\varphi} & \delta \end{pmatrix}}$$

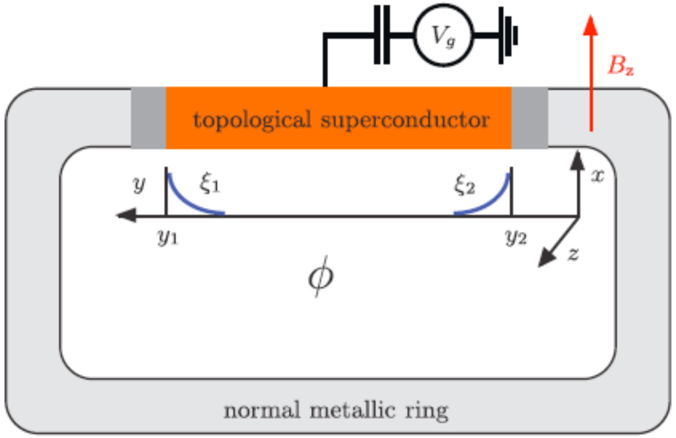
$$\Big\downarrow \; I(\varphi) = -(e/\hbar)\partial_\varphi E_-$$

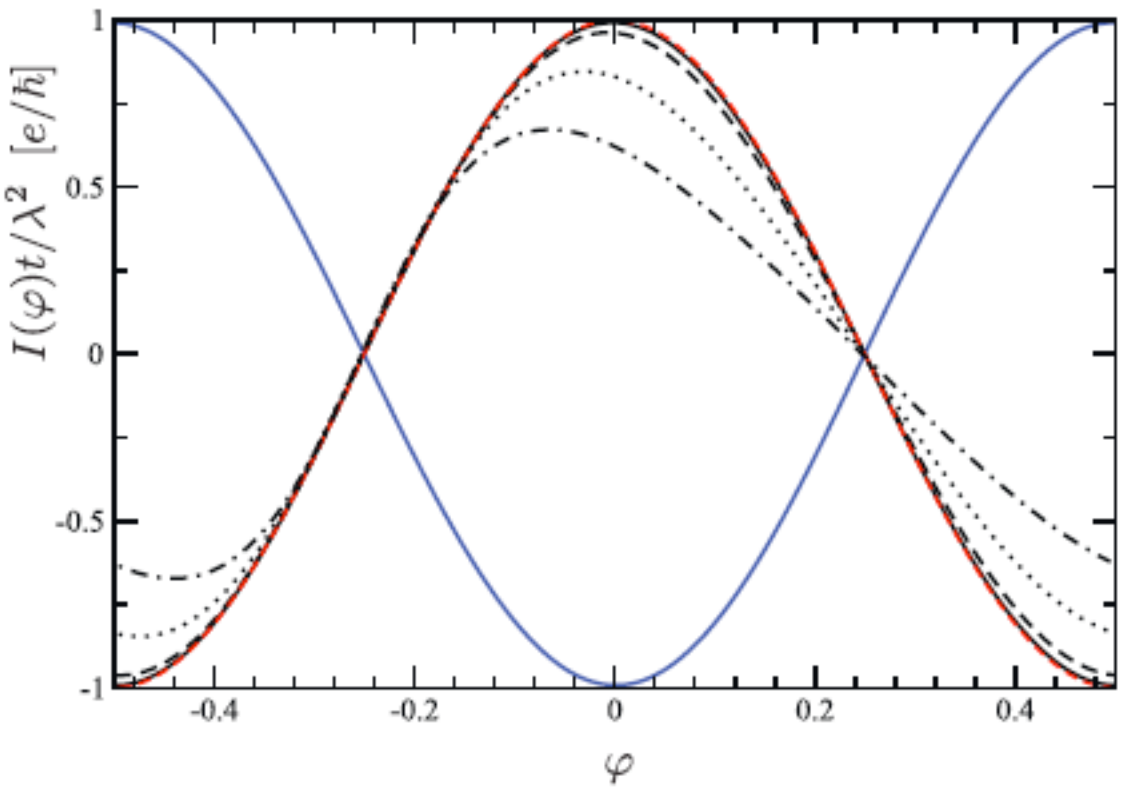
$$\boxed{I(\varphi) = \frac{e}{\hbar} \frac{(-1)^{n_0}\tilde{\lambda}_1\tilde{\lambda}_2 \cos\varphi}{\sqrt{(\epsilon_M - \delta)^2/4 + \tilde{\lambda}_1^2 + \tilde{\lambda}_2^2 + 2\tilde{\lambda}_1\tilde{\lambda}_2(-1)^{n_0}\sin\varphi}}}$$

Signatures of Majorana fermions in persistent currents

$$I(\varphi) = \frac{e}{\hbar} \frac{(-1)^{n_0} \tilde{\lambda}_1 \tilde{\lambda}_2 \cos\varphi}{\sqrt{(\epsilon_M - \delta)^2/4 + \tilde{\lambda}_1^2 + \tilde{\lambda}_2^2 + 2\tilde{\lambda}_1 \tilde{\lambda}_2 (-1)^{n_0} \sin\varphi}}$$

(i) finite current at zero flux
(ii) parity-dependence
(iii) h/e harmonics despite SC





$$\phi = \hbar\varphi/e$$

Free energy symmetry

Generally: $I(\phi) = -\partial_\phi \mathcal{F}$ with free energy even in B-field

How can one get a finite I(0) ?

Answer : $\mathcal{F}(\phi, B_Z) = \mathcal{F}(-\phi, -B_Z)$
i.e. F even in total field (flux + Zeeman)

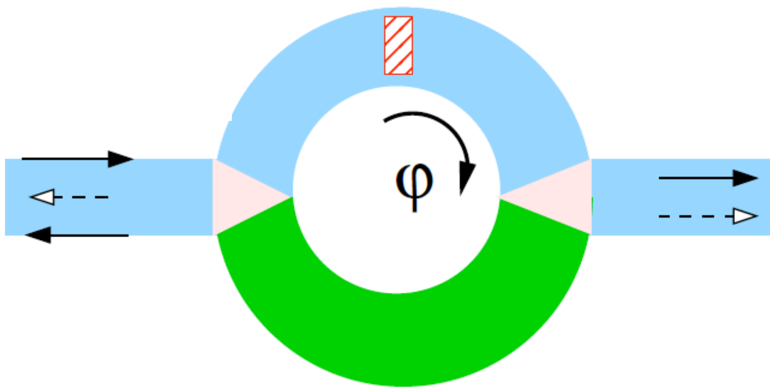
Proof : take wire hamiltonian

$$H = (p_y^2/2m - \mu)\tau_z + u p_y \sigma_z \tau_z + B_Z \sigma_x + \Delta \tau_x$$

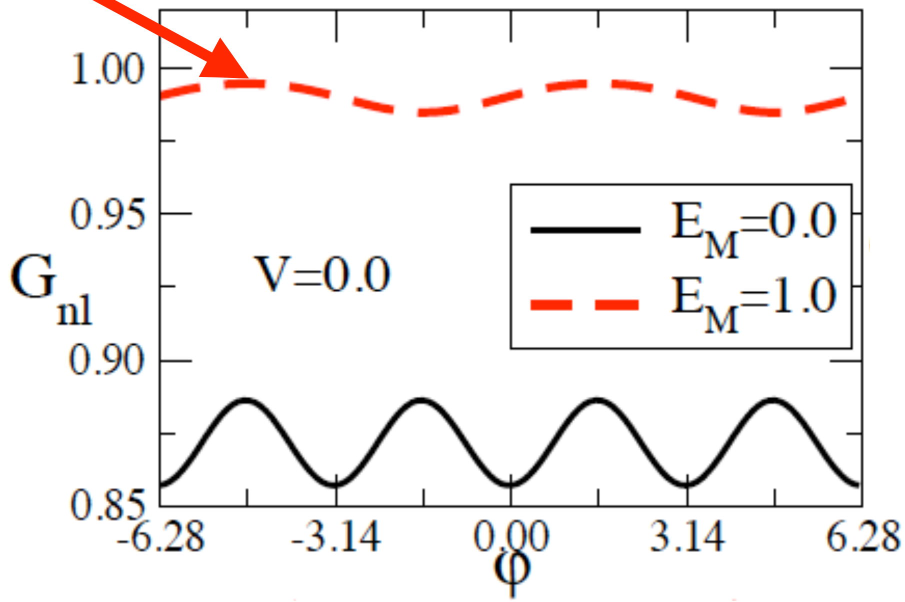
$B_Z \to -B_Z$ is equivalent to space inversion in y-direction
~ interchanges Majorana operators
~ $\phi \to -\phi$

Onsager symmetry

Aharonov-Bohm conductance setup



Fixed B_Z -> antisymmetric conductance



But NOT a violation of Onsager, i.e.

$$G(\phi, B_Z) = G(-\phi, -B_Z)$$

From Benjamin and Pachos prb 2010

In Memoriam Markus Büttiker (1950-2013)