# Performance Measures in Electric Power Networks under Line Contingencies


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## 1: Model and Motivation

- **Objective: Quantify the magnitude of the transient excursion of an asymptotically stable, high voltage AC electric network operating state subject to perturbations.**

Swing equations in the DC approximation

$$\boldsymbol{M}\ddot{\boldsymbol{\theta}} = -\boldsymbol{D}\dot{\boldsymbol{\theta}} + \boldsymbol{P} - \boldsymbol{L_b}\boldsymbol{\theta}, \tag{1}$$

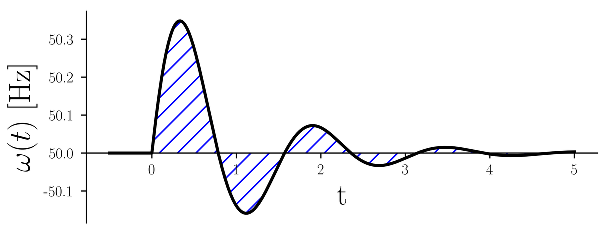
- Neglect line conductances, constant voltage magnitudes.
- $\boldsymbol{M} = \mathrm{diag}(\{m_i\})$, and $\boldsymbol{D} = \mathrm{diag}(\{d_i\})$, where $m_i$ and $d_i$ are the inertia and damping coefficients of the $i^{\mathrm{th}}$ synchronous machine.
- $\boldsymbol{L_b}$ network Laplacian of line susceptances $b_{ij}$, $\boldsymbol{P}$ power injections.

Assume asymptotically stable operating state, $(\boldsymbol{\theta}^\star, \boldsymbol{\omega}) := (\boldsymbol{L_b}^\dagger \boldsymbol{P}, 0)$, subject to additive or multiplicative perturbations. $\boldsymbol{\theta}(t) = \boldsymbol{\theta}^\star + \boldsymbol{\delta\theta}(t)$, and $\boldsymbol{\omega}(t) = \boldsymbol{\delta}\dot{\boldsymbol{\theta}}(t)$,

$$\boldsymbol{M}\boldsymbol{\delta}\ddot{\boldsymbol{\theta}}(t) = -\boldsymbol{D}\boldsymbol{\delta}\dot{\boldsymbol{\theta}}(t) - \boldsymbol{L_b}\boldsymbol{\delta\theta}(t) + \boldsymbol{\delta p}(t) - \boldsymbol{\delta L_b}(t)\boldsymbol{\delta\theta}(t), \tag{2}$$

Assess the transient evaluating **integral quadratic performance measures**

$$\mathcal{P} = \int_0^\infty \left[\, \boldsymbol{\delta\theta}^\top \; \boldsymbol{\omega}^\top \,\right] \boldsymbol{Q} \begin{bmatrix} \boldsymbol{\delta\theta} \\ \boldsymbol{\omega} \end{bmatrix} \mathrm{d}t\,, \quad \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}^{(1,1)} & 0 \\ 0 & \boldsymbol{Q}^{(2,2)} \end{bmatrix} \tag{3}$$




Specific performance measures investigated in the literature: transmission losses [2, 3], primary control effort [4], network coherence [5].

## 2: Impulse perturbation formalism

- For impulse (Dirac-$\delta$) additive perturbations $\boldsymbol{\delta p}(t) = \delta(t)\boldsymbol{p}_0$, the performance measures $\mathcal{P}$ can be evaluated in terms of the observability Gramian $\boldsymbol{X}$

$$\mathcal{P} = \boldsymbol{B}^\top \boldsymbol{X} \boldsymbol{B} \tag{4}$$

where

$$\boldsymbol{X} = \int_0^\infty e^{\boldsymbol{A}^\top t} \boldsymbol{Q} e^{\boldsymbol{A}t} \mathrm{d}t\,, \quad \boldsymbol{A} = \begin{bmatrix} 0 & \mathbb{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{L_b} & -\boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1}\boldsymbol{p}_0 \end{bmatrix} \tag{5}$$

- For asymptotically stable systems, $\boldsymbol{X}$ is obtained solving the Lypanuov equation $\boldsymbol{A}^\top\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A} = -\boldsymbol{Q}$.

**Our contribution: Solution for generic performance measures**

- We obtain analytic expressions for the observability Gramian in terms of the eigenvectors, $T_{\mathrm{M}}$, and eigenvalues, $\lambda_i^{\mathrm{M}}$, of $\boldsymbol{M}^{-1/2}\boldsymbol{L_b}\boldsymbol{M}^{-1/2}$, valid for generic performance measures and $d_i/m_i = \mathrm{Cst}$.
- Frequency based performance measure

$$X_{ij}^{(2,2)} = \sum_{l,q=1}^{N} (T_{\mathrm{M}})_{il}(T_{\mathrm{M}}^\top)_{qj}(\boldsymbol{T}_{\mathrm{M}}^\top \boldsymbol{M}^{-1/2}\boldsymbol{Q}^{(2,2)}\boldsymbol{M}^{-1/2}\boldsymbol{T}_{\mathrm{M}})_{lq} \times \left[\frac{\gamma(\lambda_l^{\mathrm{M}} + \lambda_q^{\mathrm{M}})}{2\gamma^2(\lambda_l^{\mathrm{M}} + \lambda_q^{\mathrm{M}}) + (\lambda_q^{\mathrm{M}} - \lambda_l^{\mathrm{M}})^2}\right], \tag{6}$$

- Phase based performance measure

$$X_{ij}^{(2,2)} = \sum_{l,q=1}^{N} (T_{\mathrm{M}})_{il}(T_{\mathrm{M}}^\top)_{qj}(\boldsymbol{T}_{\mathrm{M}}^\top \boldsymbol{M}^{-1/2}\boldsymbol{Q}^{(1,1)}\boldsymbol{M}^{-1/2}\boldsymbol{T}_{\mathrm{M}})_{lq} \times \left[\frac{2\gamma}{2\gamma^2(\lambda_l^{\mathrm{M}} + \lambda_q^{\mathrm{M}}) + (\lambda_q^{\mathrm{M}} - \lambda_l^{\mathrm{M}})^2}\right], \tag{7}$$

- Eqs. (6) and (7) relate the **perfomance measures to the network topology.**

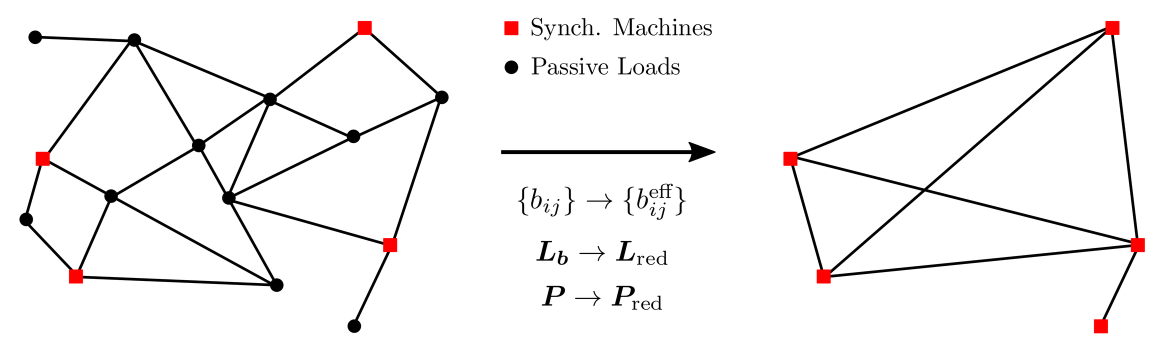
## 3: Line contingencies as multiplicative perturbations

- We investigate line contingencies with very short clearing times $\boldsymbol{L_b}(t) = \boldsymbol{L_b} - \delta(t) b_{\alpha\beta}\boldsymbol{e}_{(\alpha,\beta)}\boldsymbol{e}_{(\alpha,\beta)}^\top$.
- Relative to the nominal operating point, $\boldsymbol{\theta}(t) = \boldsymbol{\theta}^\star + \boldsymbol{\delta\theta}(t)$, the swing equations (2) becomes

$$\boldsymbol{M}\boldsymbol{\delta}\ddot{\boldsymbol{\theta}} = -\boldsymbol{D}\boldsymbol{\delta}\dot{\boldsymbol{\theta}} - \boldsymbol{L_b}\boldsymbol{\delta\theta} + \delta(t) b_{\alpha\beta}\boldsymbol{e}_{(\alpha,\beta)}\boldsymbol{e}_{(\alpha,\beta)}^\top(\boldsymbol{\theta}^\star + \boldsymbol{\delta\theta})\,. \tag{8}$$

with initial condition $(\boldsymbol{\delta\theta}_g(0), \boldsymbol{\omega}_g(0)) = (0, 0)$.

- For **impulse line fault**, this **multiplicative perturbation can be mapped to an additive perturbation**. We can apply the observability Gramian formalism!
- Perform Kron reduction to eliminate passive nodes (resistive loads), effective reduced network consisting only of synchronous machines [6].




→ Swing equations for synchronous machines $\boldsymbol{M}\ddot{\boldsymbol{\theta}}_g = -\boldsymbol{D}\dot{\boldsymbol{\theta}}_g + \boldsymbol{P}_{\mathrm{red}} - \boldsymbol{L}_{\mathrm{red}}\boldsymbol{\theta}_g$,

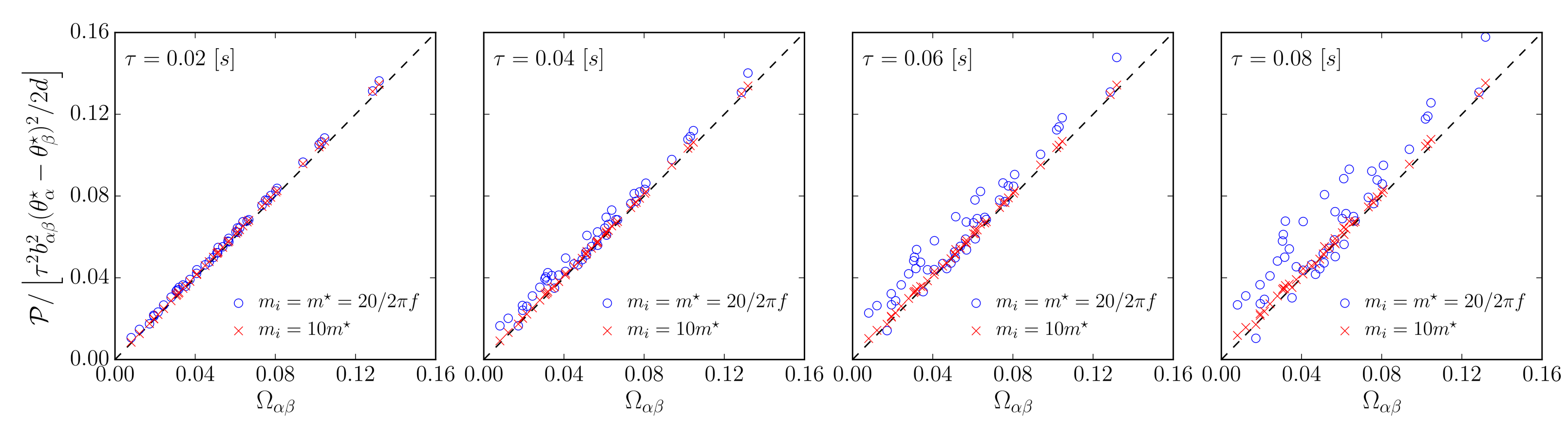
→ Map line contingencies in the physical network to the reduced network, affects $\boldsymbol{P}_{\mathrm{red}}$, $\boldsymbol{L}_{\mathrm{red}}$.

→ 3 types of line contingencies: sm-sm, sm-p, p-p.

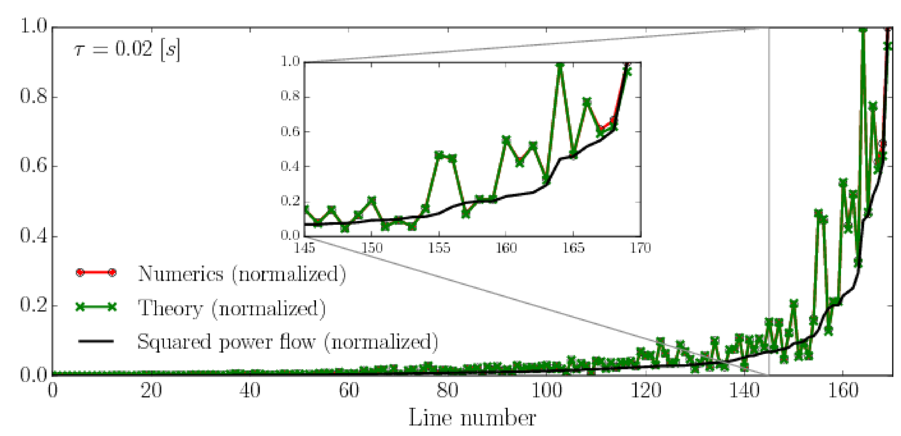
- In numerical simulations replace Dirac-$\delta$ impulse by box function $\Theta(t)\Theta(\tau - t)$ and vary the fault clearing time $\tau$.

## 4: Network coherence

- We consider the coherence measure, $\mathcal{P} = \int_0^\infty \sum_i \delta\theta_i(t)^2\,\mathrm{d}t$, and assume $(m_i = m, d_i = d \forall i)$
- For lines connecting two synchronous machines $\mathcal{P} = \frac{[b_{\alpha\beta}(\theta_{g,\alpha}^\star - \theta_{g,\beta}^\star)]^2}{2d}\Omega_{\alpha\beta}$. It depends on the square of the power flowing on the line prior to the fault, times the resistance distance.

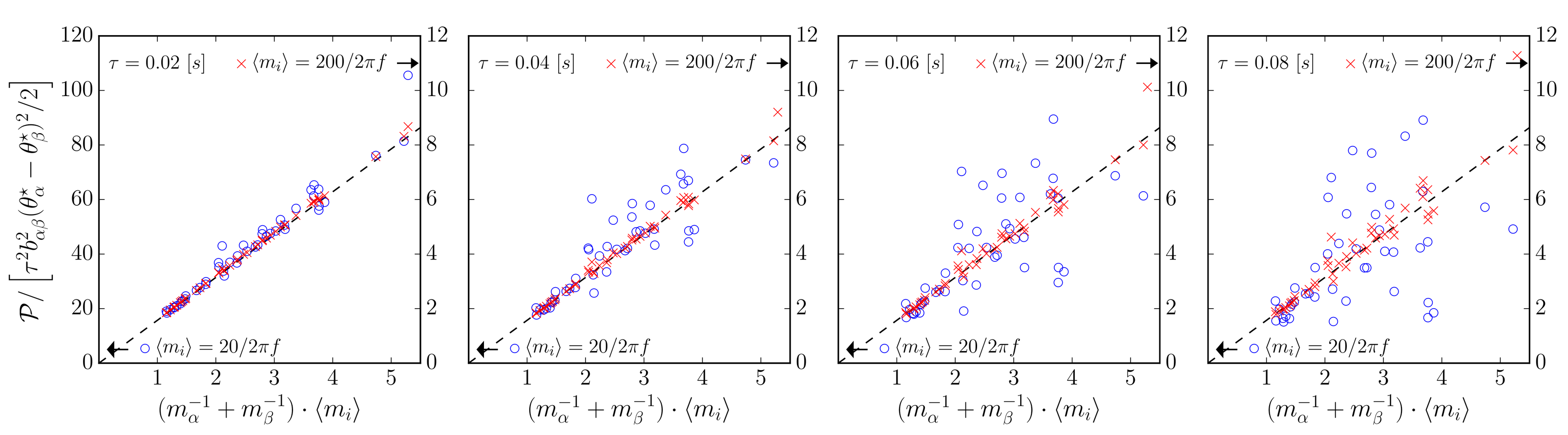



- For other line contingencies, the topological factor is more involved and depends on the topology of the network of passive nodes.
- IEEE-118 Kron reduced network.

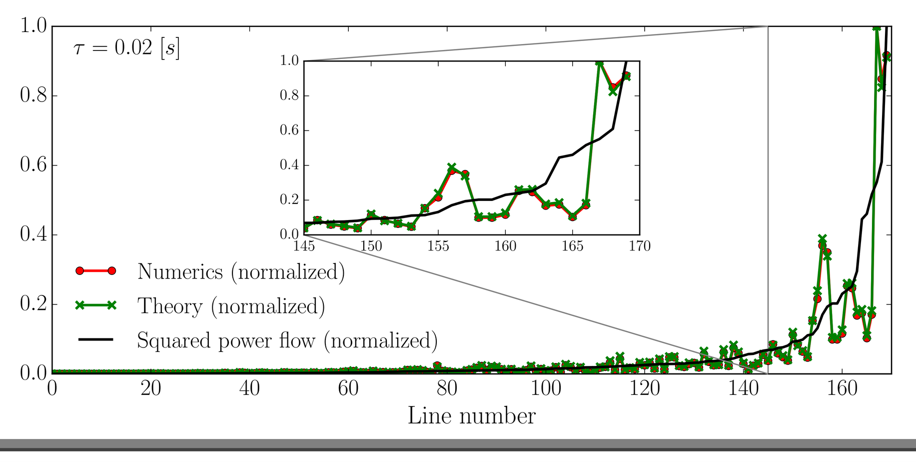



## 5: Primary control effort

- We consider the primary control effort measure, $\mathcal{P} = \int_0^\infty \sum_i d_i\omega_i(t)^2\,\mathrm{d}t$
- For lines connecting two synchronous machines $\mathcal{P} = \frac{[b_{\alpha\beta}(\theta_{g,\alpha}^\star - \theta_{g,\beta}^\star)]^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_\beta}\right)$. It depends on the square of the power flowing on the line prior to the fault, times the sum of the inverse inertias at the ends of the faulted line.




- For other line contingencies, the inertia dependent factor is more involved and depends on the topology of the network of passive nodes.
- IEEE-118 Kron reduced network.




## 6: Conclusion

We showed how impulse multiplicative perturbations such as line contingencies can be mapped to additive perturbations allowing for a treatment of integral performance measures in terms of the observability Gramian formalism. We performed dynamical simulations of line contingencies having finite clearing times. Our numerical results indicate that our analytical predictions hold for line contingencies lasting over few cycles of the AC frequency. **The magnitude of the integral response is not a monotonic function of the square of the power flowing on the line prior to the fault. It also depends on the network topology, via the resistance distance, and on the distribution of inertias.**

**Open & other related issues**

→ Extend the observability Gramian formalism to inhomogenous synchrous machine parameters.

→ Response to noisy perturbations beyond white noise (visit Mr. Tyloo's poster).

## References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