Vortex Flows in High Voltage AC Power Grids

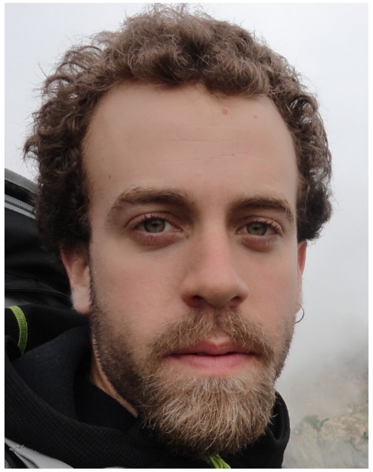
Philippe Jacquod

Dynamics Days - 06.06.2016

Delabays, Coletta, Adagideli and PJ, arXiv:1605.07925
Delabays, Coletta and PJ, J Math Phys 57, 032701 (2016)

The team

Tommaso Coletta, postdoc

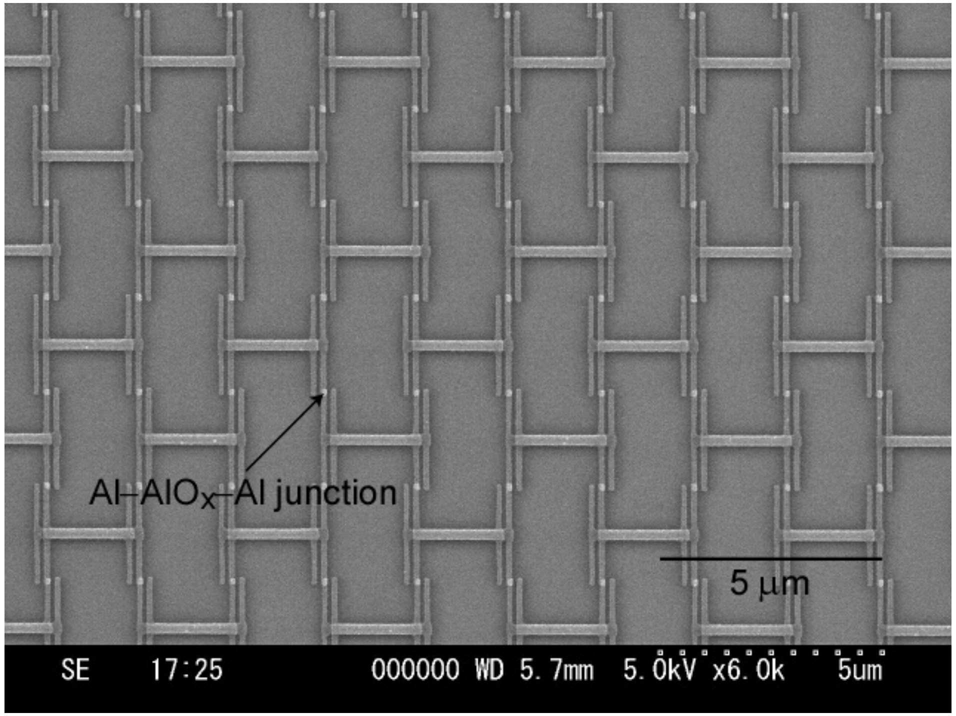


Robin Delabays, grad student
(talk earlier in the session)

Inanc Adagideli (Sabanci)

Take-home message

Profound, unexpected similarities between Josephson junction arrays and high voltage AC power grids !



Takahide, Yagi, Kanda, Ootuka, and Kobayashi
Phys. Rev. Lett. 85, 1974 (2000)

Hes·so // VALAIS WALLIS

Superconductivity vs. electric power systems !

	Superconductor	high voltage AC power grid
State	$\Psi(\mathbf{x}) = \|\Psi(\mathbf{x})\|e^{i\theta(\mathbf{x})}$	$V_i = \|V_i\|e^{i\theta_i}$
Current / power flow	$I_{ij} = I_c \sin(\theta_i - \theta_j)$ Josephson current	$P_{ij} \simeq B_{ij} \sin(\theta_i - \theta_j)$ Power flow; lossless line approx.
winding # $q=\Sigma_i\|\theta_{i+1}-\theta_i\|/2\pi$	Flux quantization Persistent currents	Circulating loop flows

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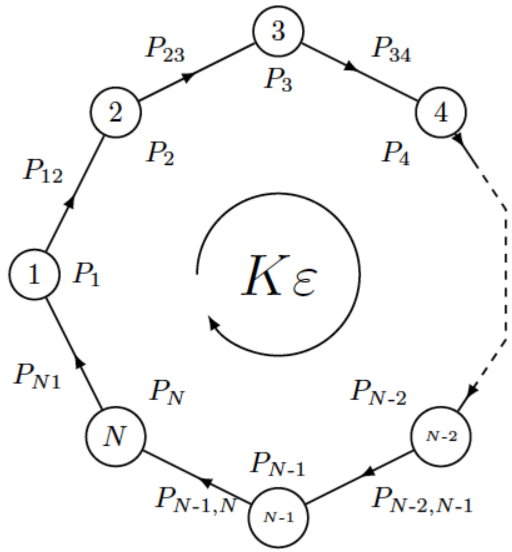
Circulating loop flows

*Thm: Different solutions to the following power-flow problem (AC Kirchhoff)

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

may differ only by circulating loop current(s) **in any network**

Dörfler, Chertkov, Bullo, PNAS '13; Delabays, Coletta and PJ, JMP '16



*Voltage angle uniquely defined

→ $q = \Sigma_i |\theta_{i+1} - \theta_i| / 2\pi \in \mathbb{Z}$ **~topological winding number**

→ "quantization" of these loop currents **~vortex flows**

Janssens and Kamagate '03

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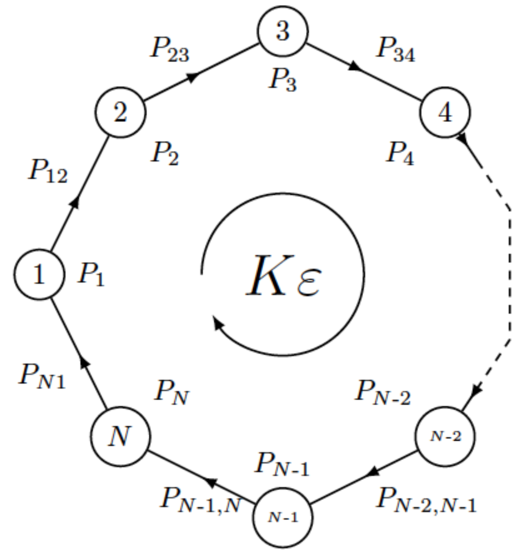

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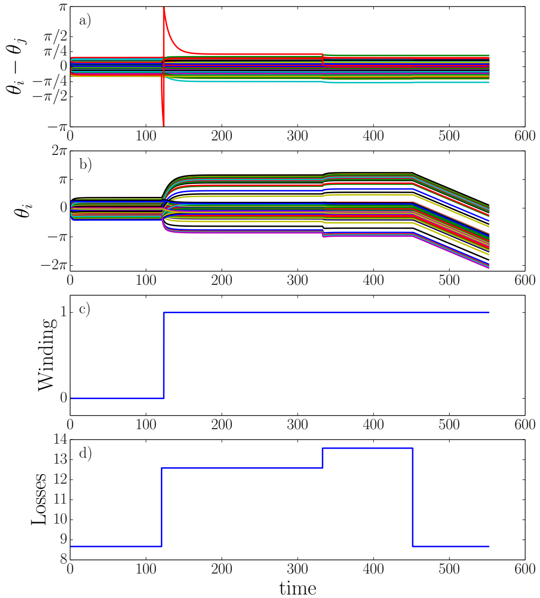

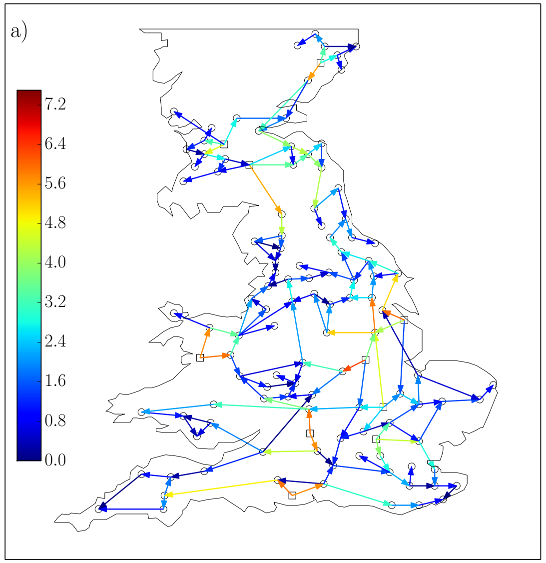

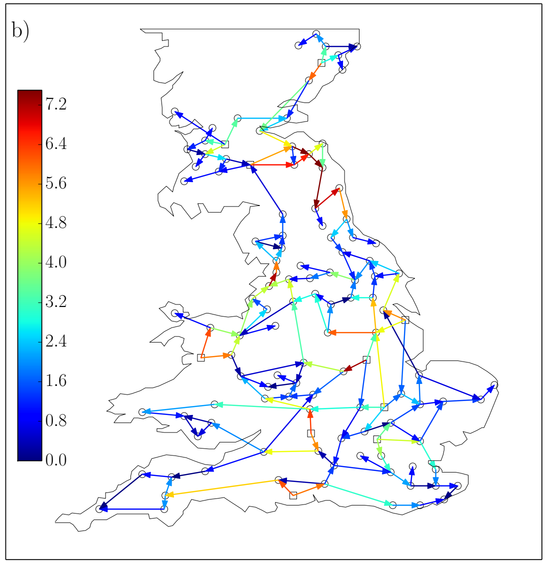

Talk by R Delabays earlier in the session

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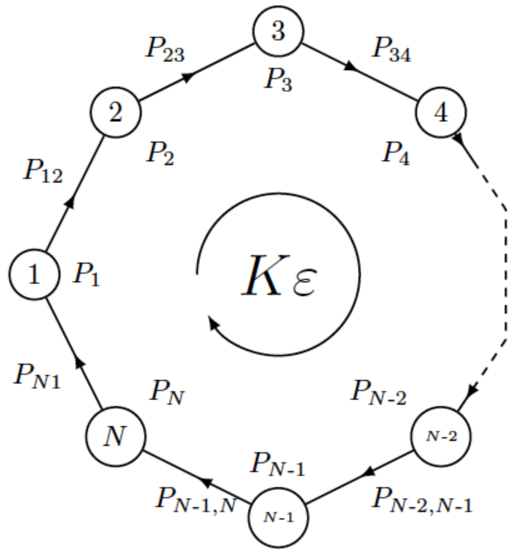

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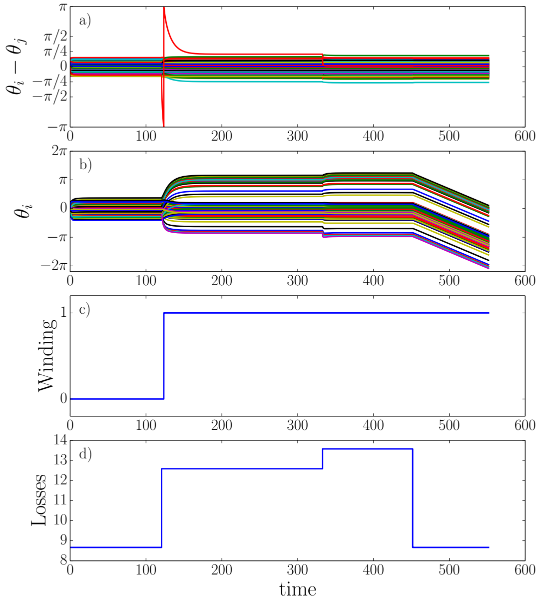

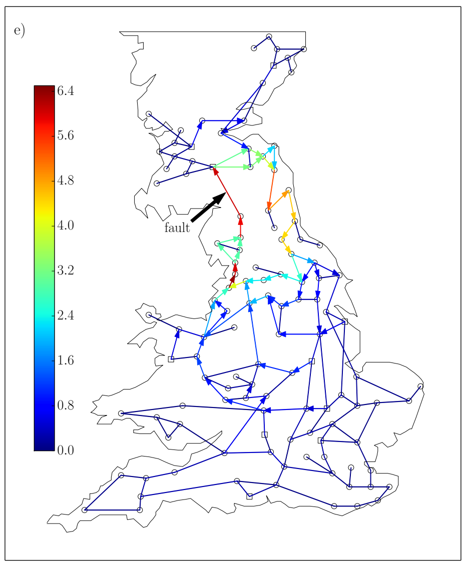

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Vortex flows in AC power grids vs. superconductivity

- Vortex/circulation creation in SC via B-field

Can one create vortex flows in AC power grids ?
How ?

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

Vortex flows in AC power grids vs. superconductivity

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Can one create vortex flows in AC power grids ?
How ?

*Three mechanisms : *dynamical phase slip*
**line tripping*
**line tripping and reclosure*

- Superconductors are dissipationless
- AC power grids are dissipative ~ they have a finite resistance

Do vortex flows survive the presence of dissipation ?

YES ! They are robust against moderate amounts of dissipation

Power flow equations - approximations

- Admittance dominated by its imaginary part for large conductors ~ high voltage

→ neglect conductance
neglect voltage drops

Power injected at bus l

$$P_l = \sum_m \tilde{B}_{lm} \sin(\theta_l - \theta_m)$$

Sum of power flowing out of l

A.k.a. lossless line approximation

- Incorporating dissipation to leading order $\tilde{G}_{lm}/\tilde{B}_{lm} \ll 1$

$$\rightarrow \quad P_l = \sum_{m \neq l} \left(\tilde{B}_{lm} \sin(\theta_l - \theta_m) + \tilde{G}_{lm}[1 - \cos(\theta_l - \theta_m)] \right)$$

- Ohmic losses

$$\Delta P = \sum_l P_l = \sum_{l,m} \tilde{G}_{lm} \; [1 - \cos(\theta_l - \theta_m)] > 0$$

Generation of vortex flow by line tripping

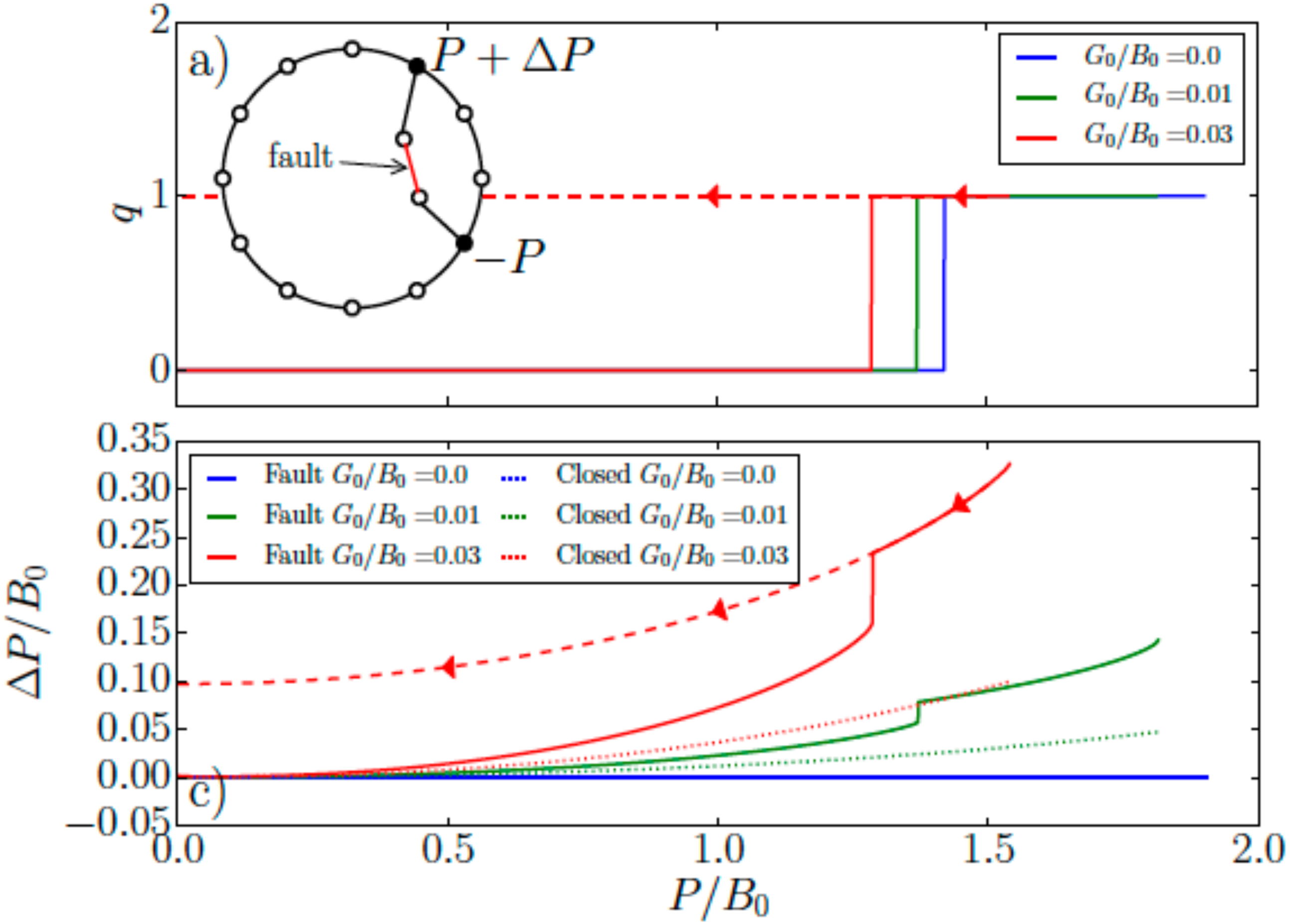
*Power grids are meshed - path redundancy
*Power is still supplied after one line trips
*Consider line tripping in an asymmetric, three-path circuit
*all winding numbers vanish initially

Power redistribution can lead to vortex flow with $q=\Sigma_i|\theta_{i+1}-\theta_i|/2\pi > 0$

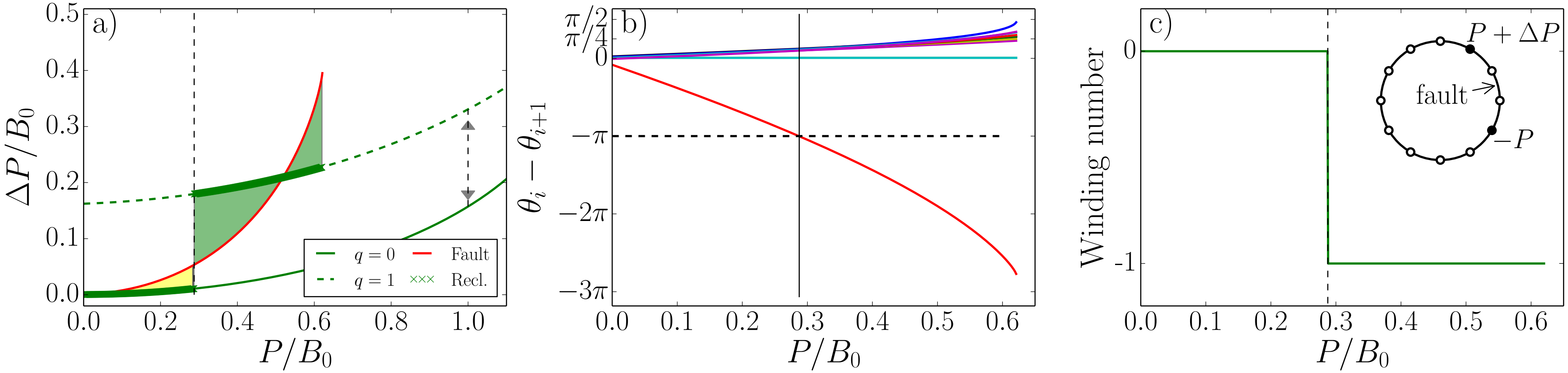
Line tripping at $P/B_0 > 1.3$

→ q=1

Vortex state characterized by
-hysteresis
  = **topological protection**
-higher losses



Generation of vortex flow by line tripping and reclosure



- Single-loop network
- One producer, one consumer
- One line trips, is later reclosed

Vortex formation for $P/B_0 > 0.26$

Vortex formation for $|\theta_{i+1}-\theta_i|>\pi$ (two ends of faulted line)

Janssens and Kamagate '03

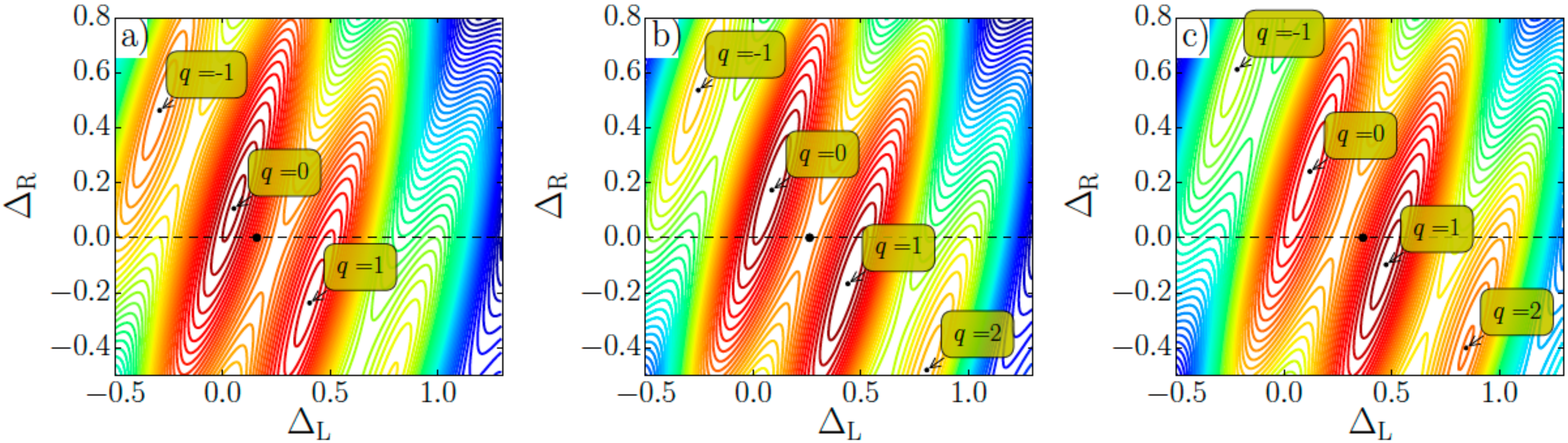
Generation of vortex flow by line tripping and reclosure

*Lyapunov function - defines basins of attraction for different solutions

$$\mathcal{V}(\{\theta_i\}) = -\sum_{l} P_l \theta_l - \sum_{\langle l,m\rangle} B_0 \cos(\theta_l - \theta_m)$$

van Hemmen and Wreskinski '93

*Steady-state solutions have $\nabla \mathcal{V} = 0$



$P \approx 0.159 B_0$ $\quad$ $P = B_0 \sin(\pi/12) \approx 0.259 B_0$ $\quad$ $P \approx 0.359 B_0$

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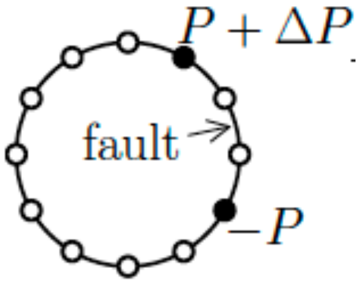
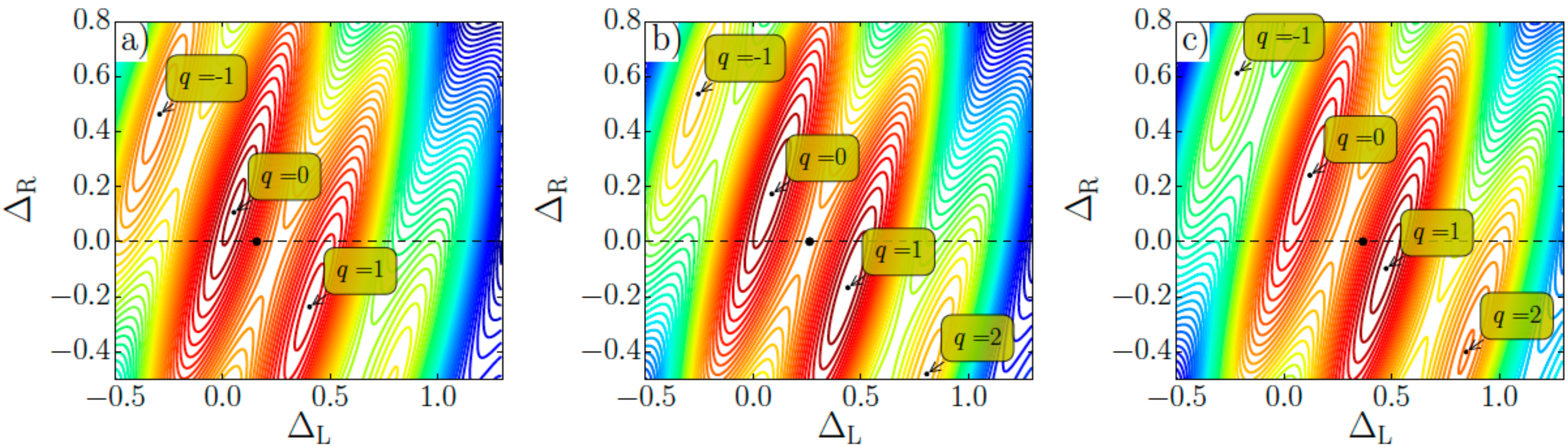


van Hemmen and Wreskinski '93

*In our case

$$\mathcal{V}(\Delta_{\rm L}, \Delta_{\rm R}) = -N_{\rm L} P \Delta_{\rm L} - N_{\rm L} B_0 \cos \Delta_{\rm L} - (N_{\rm R} - 1) B_0 \cos \Delta_{\rm R} - B_0 \cos(N_{\rm L}\Delta_{\rm L} - (N_{\rm R} - 1)\Delta_{\rm R})$$



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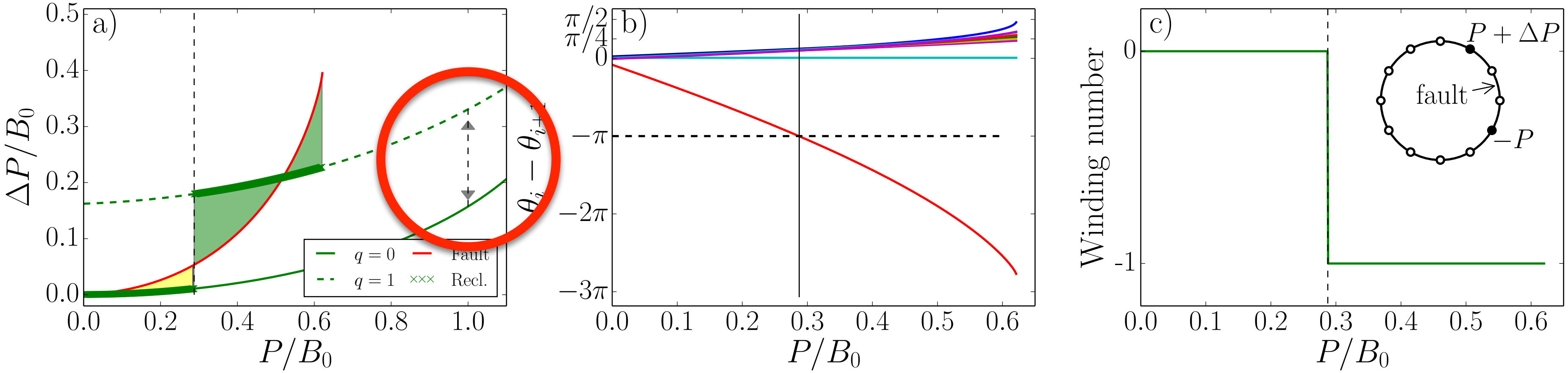
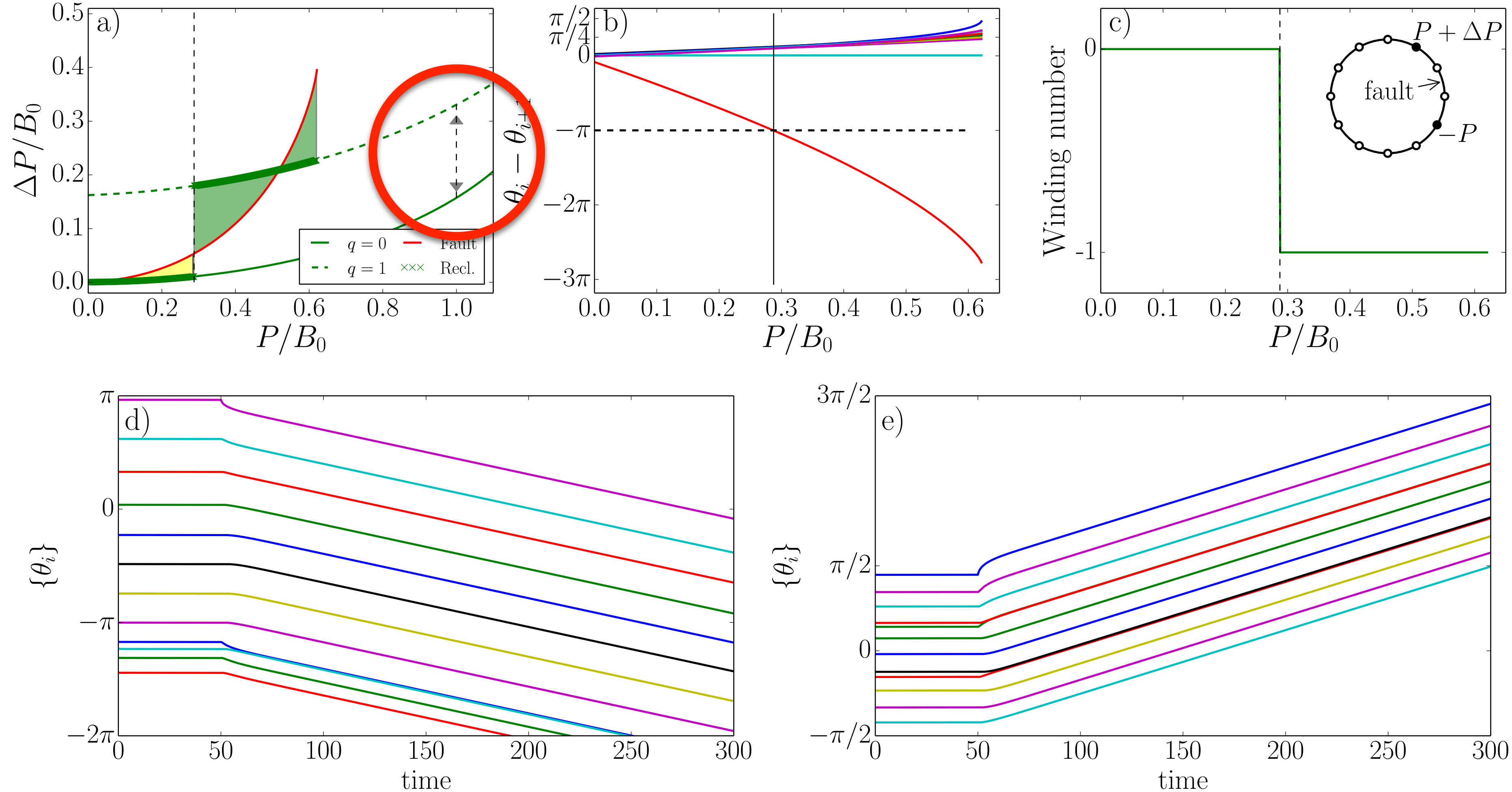


Try to kill / create vortex by adapting ΔP ?

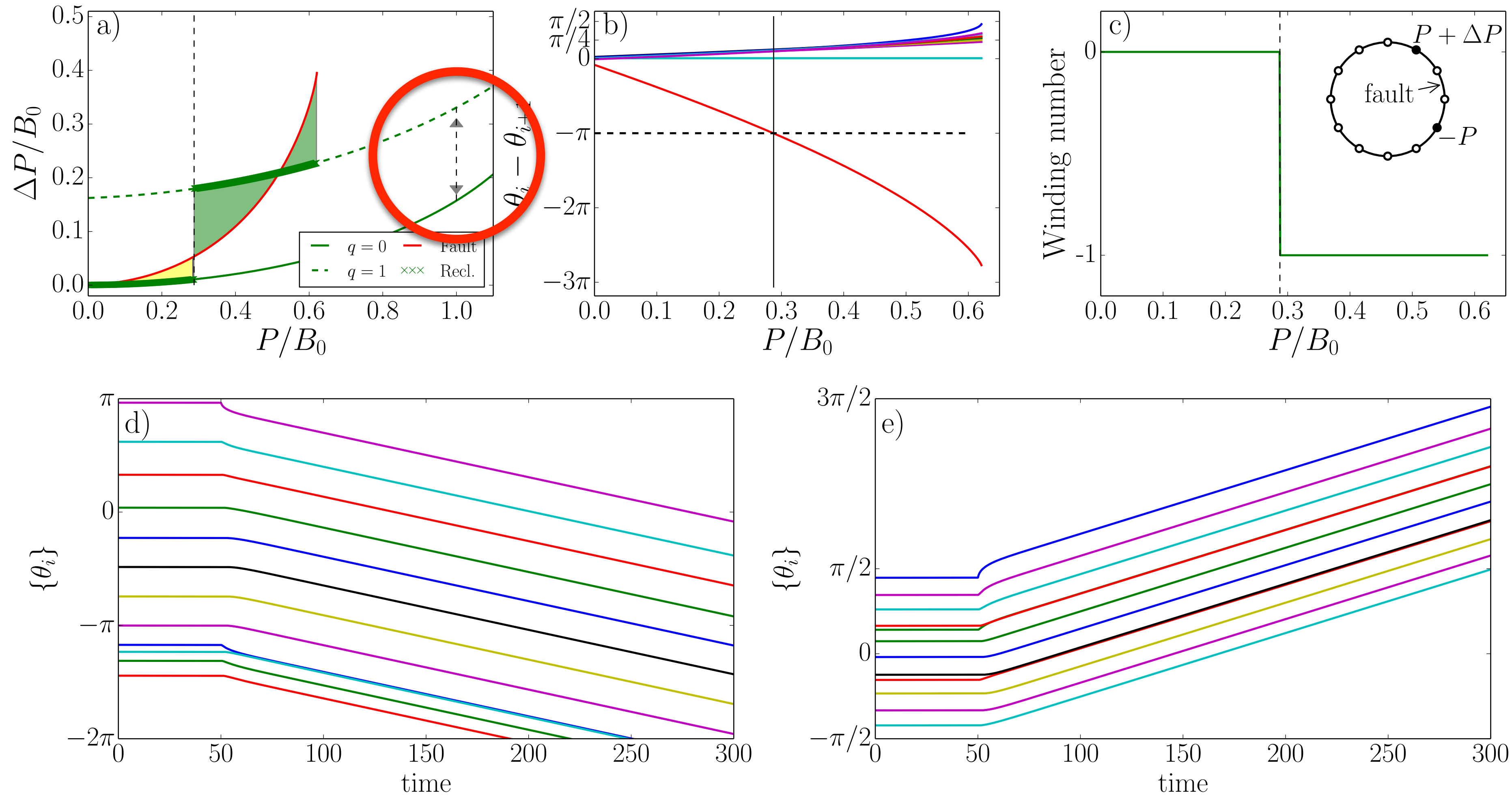
Generation of vortex flow by line tripping and reclosure



!! Cannot kill nor create vortex by adapting ΔP !!

Instead one changes the grid's frequency

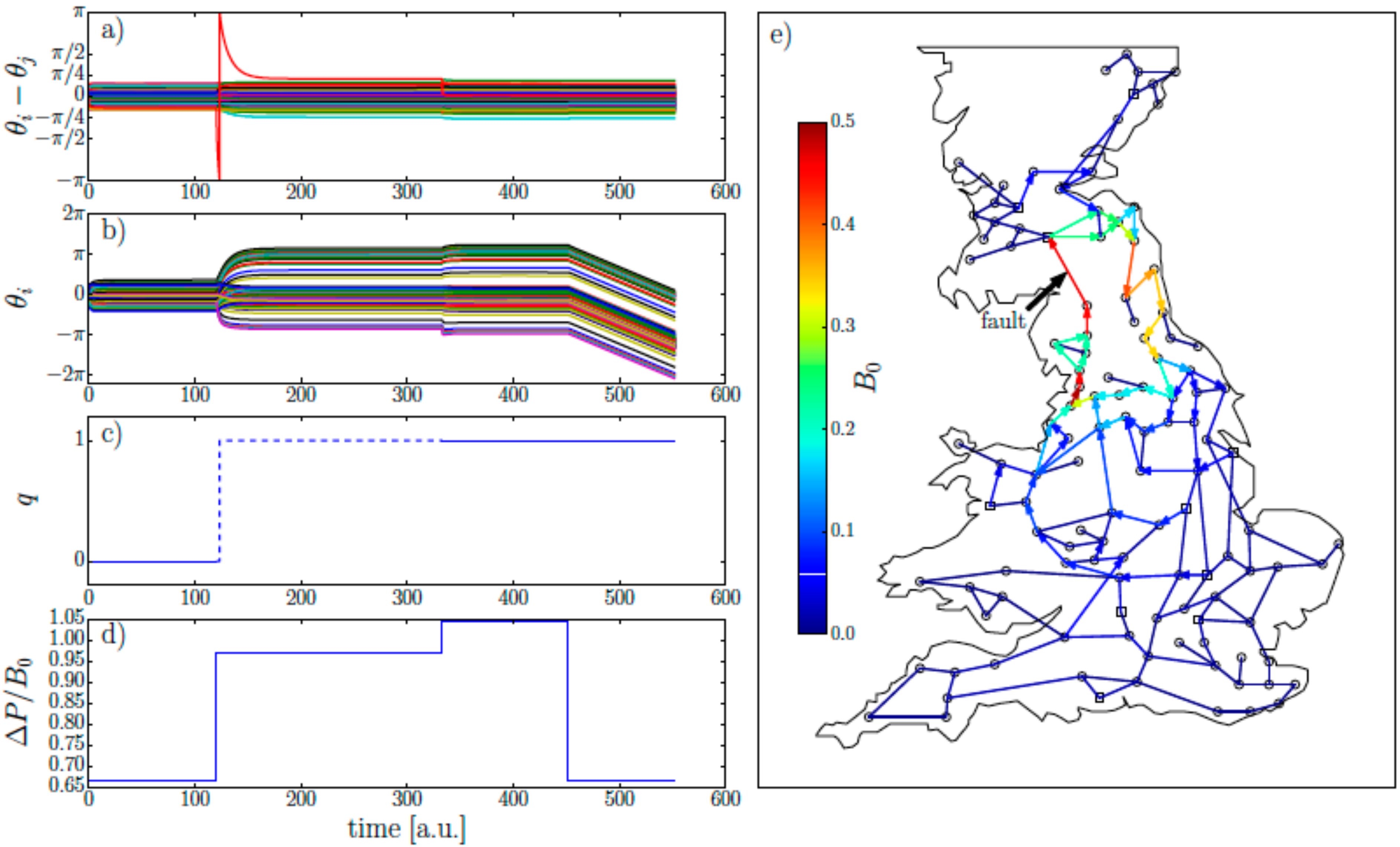
Generation of vortex flow by line tripping and reclosure



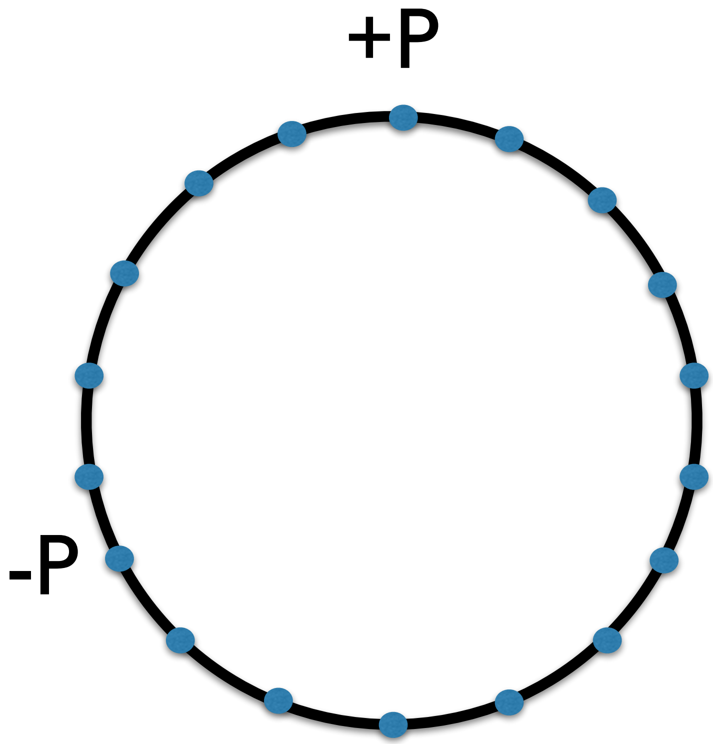
!! Cannot kill nor create vortex by adapting ΔP !!

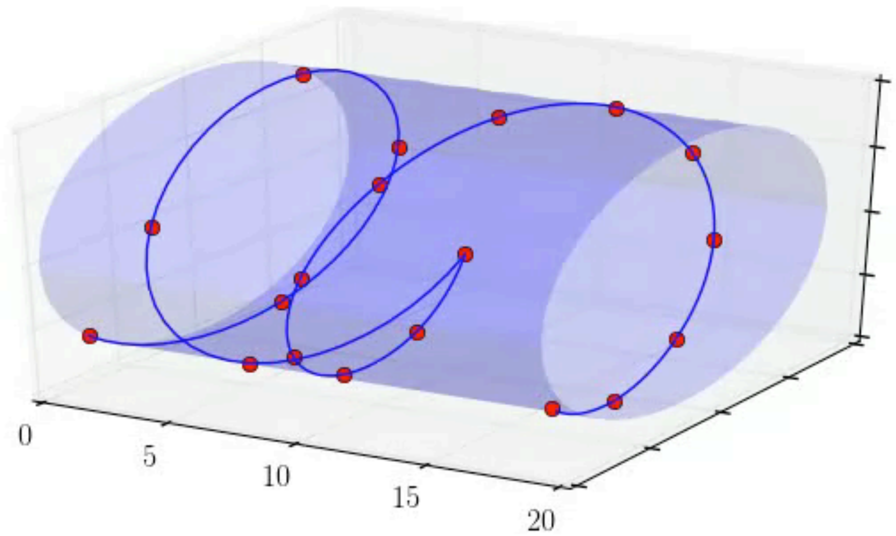
!! Topological protection !!

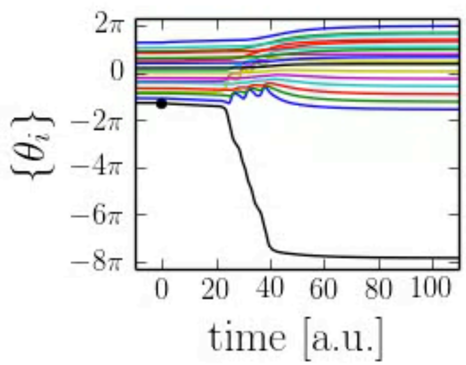
Generation of vortex flow by line tripping and reclosure

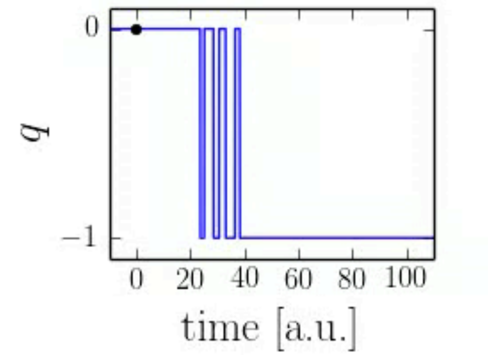


Dynamical generation of vortex flows









Similar to quantum phase slips in JJ arrays
Lau, Markovic, Bockrath, Bezryadin and Tinkham '01
Matveev, Larkin and Glazman '02