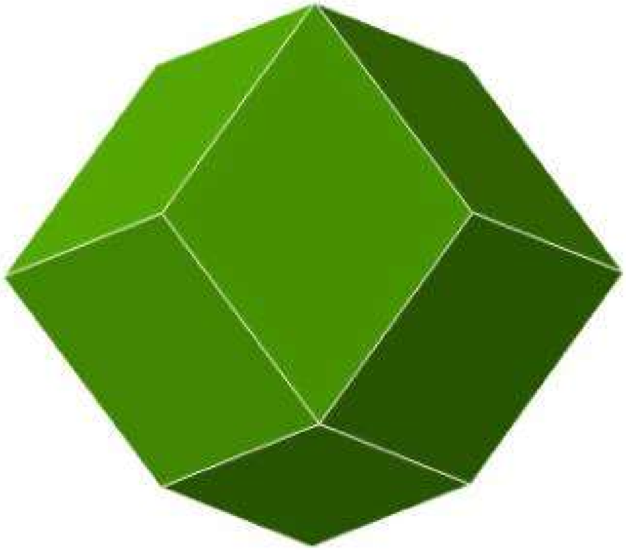
The Rhombic Dodecahedron

Section 1 --- Introduction



The rhombic dodecahedron is a very interesting polyhedron. It figures prominently in Buckminster Fuller's Synergetics.
It has 12 faces, 14 vertices, 24 sides or edges.

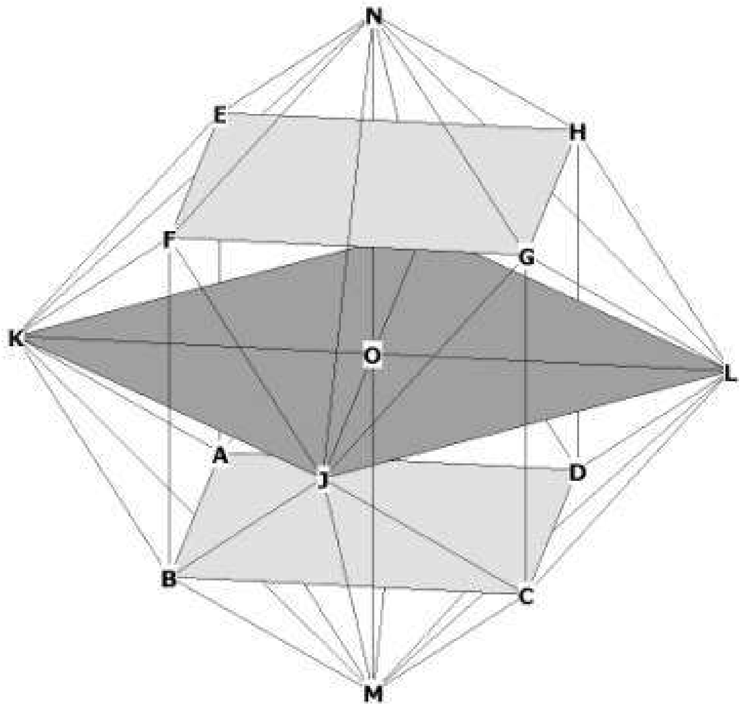


Figure 1

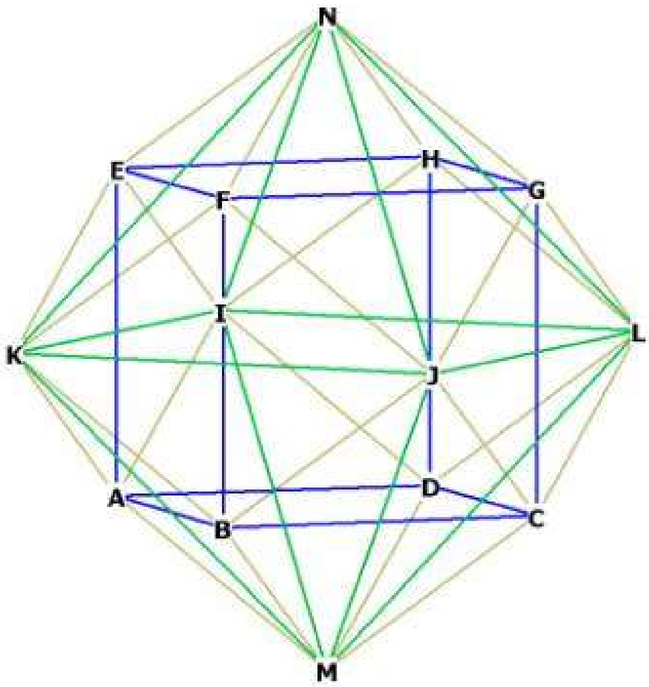


Figure 1A is Figure 1 slightly rotated, showing the edges of rhombic dodecahedron (yellow), octahedron (green) and cube (blue).

Notice in Figures 1 and 1A that the rhombic dodecahedron is composed of diamond faces (for example, NFJG).

The faces are called rhombuses, because they are equilateral parallelograms. In other words, they are square-sided figures with opposite edges parallel to one another.

The rhombic dodecahedron has 8 vertices in the middle that form a cube, (ABCD-EFGH), and the other 6 on the outside which form an octahedron (N-ILJK-M). It is possible to draw a sphere around the cube, and another, larger sphere, around the octahedron. Unlike the 5 regular solids, therefore, not all of the vertices of the rhombic dodecahedron will touch one sphere. We will, in the course of this analysis, find the diameter and radius of each of these spheres.

In Figure 1, I have marked the top and bottom planes of the cube in light gray, and the square plane which serves as the base for the 2 face-bonded pyramids of the octahedron, in dark gray.

You may perceive 3 edges of the cube in this drawing, which connect the top and bottom planes; that is, FB,GC, HD; and also some of the edges of the octahedron, the most visible of which are NJ, NK, NL, MK, MJ, ML. Notice that the edges of the octahedron bisect the diamond faces of the rhombic dodecahedron upon their long axis (for example, NK bisects the long axis of the face NEKF at the upper left). Note also that the short axis segments (that is, EF) are the edges of a cube.

It is important to understand that the outer vertices of the rhombic dodecahedron form an octahedron, as this will be an important part of the analysis.

The rhombic dodecahedron (hereinafter, referred to as r.d.) is a semi-regular polyhedron, in that all of its edges are the same length, yet the angles of its faces differ. Because each face is a parallelogram, there are 2 distinct angles for each face, one which is bisected by the long axis, with an angle less than 90 degrees, the other bisected by the short axis, with an angle greater than 90 degrees. Of course these axes do not actually appear on the face of the polyhedron, I use them here for illustration.

In Figure 2 below, the vertices of the octahedron are the long axis vertices of each face, in this case N and J. The cube vertices are the short axis points, in this case, F and G. This can be more clearly seen by referring to Figure 1.

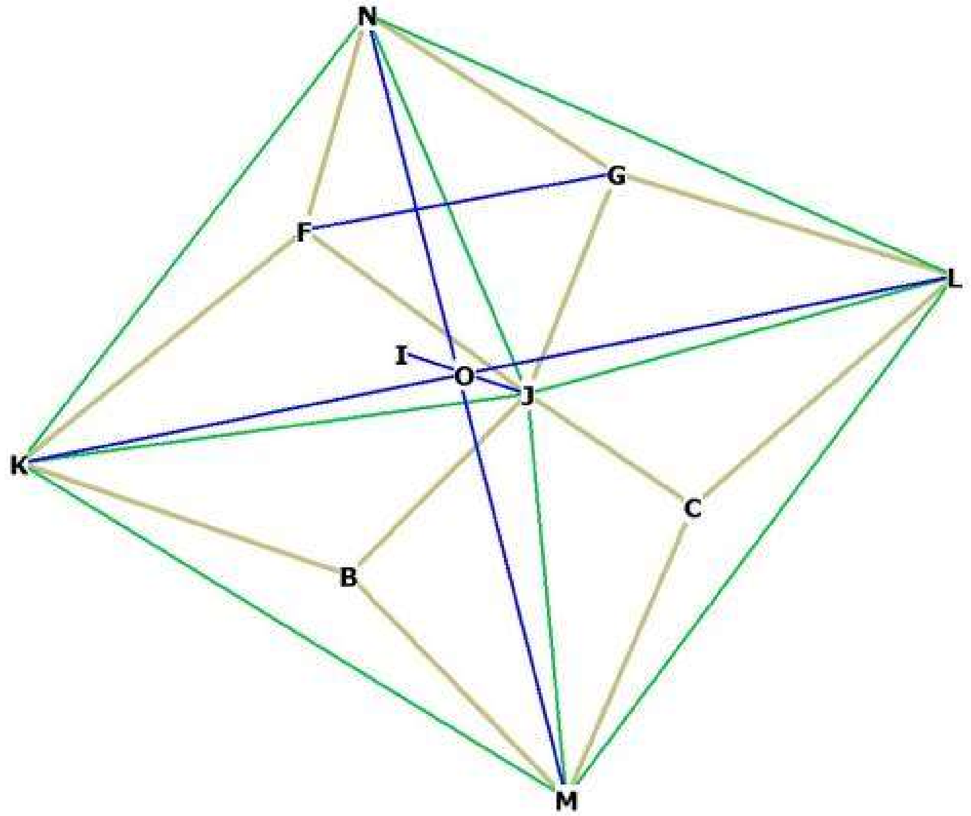


Figure 2

Figure 2 shows the general appearance of the r.d. The front 4 faces of the octahedron can be seen clearly here in green (NKJ, NJL, KJM and JLM). Figure 2 also shows how the short axis vertices of the r.d. (as F, G, B, C) come off the face of the octahedron.

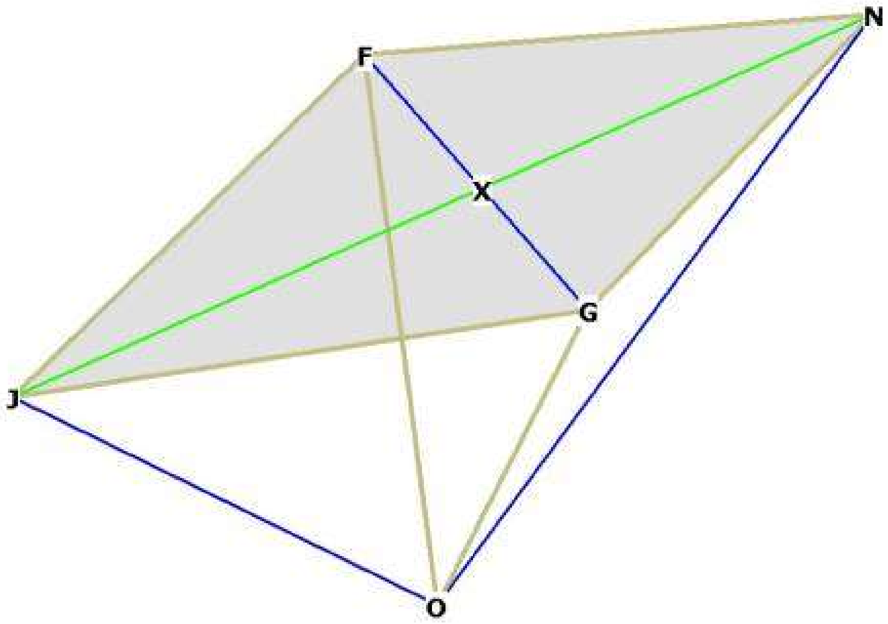


Figure 3

The sides of the rhombic dodecahedron are JFNG, one of the sides of the octahedron is JN. ∡JON is the central angle of the long axis of the r.d. face, and ∡FOG is the central angle of the short axis. O is the centroid of the r.d. and of the octahedron and cube within the r.d. When you build a 3D model of the r.d., it appears that OF = OG = NF = FJ = JG = NG by construction; that is, the distance from the centroid O to any of the 6 short axis vertices of the r.d. faces (the vertices that make the cube, see Figure 1A) are equal in length to the edges of the rhombic dodecahedron. We will prove this later on.

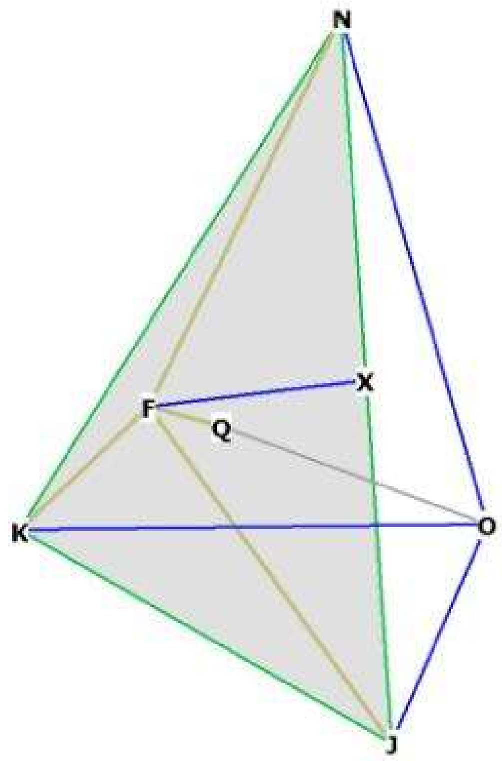


Figure 4

Figure 4 shows the point F, one of the vertices of the rhombic dodecahedron, off one of the faces of the octahedron, NJK. The centroid of the octahedron/r.d. is at O. Q is the center of the octagonal face NJK.

If you build a model of the rhombic dodecahedron with the Zometool you will see at once that the sides of the r.d. come off any of the faces of the octahedron and meet above the center of the octahedron face (for example, at F). F is also the centroid of a tetrahedron with side length = side of the octahedron that can be formed from the face NJK of the octahedron (see Figure 4A below). Tetrahedrons may be formed from any of the faces of the octahedron, and connected in the fashion of Fuller's Isotopic Vector Matrix.

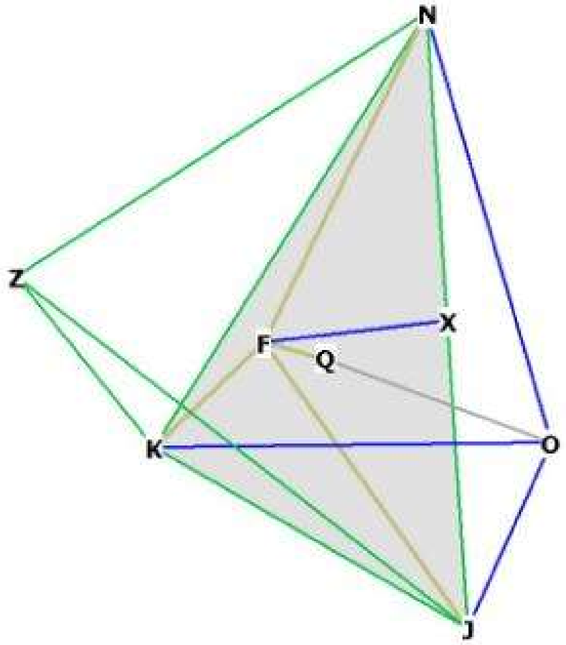


Figure 4A

QF, in Figure 4A, is the distance from the plane of the octahedron/tetrahedron face, to the centroid of any such tetrahedron. XF is the distance from the center of any of the diamond faces of the r.d. to a short axis vertex, in this case, F. OQ is the distance from the centroid to the middle of the face of the octahedron. The centroid of the octahedron is also the centroid of the r.d. (O), which is built around the faces of the octahedron, as shown in Figure 1.

Let's find the volume of the rhombic dodecahedron. As usual, we will use the pyramid.

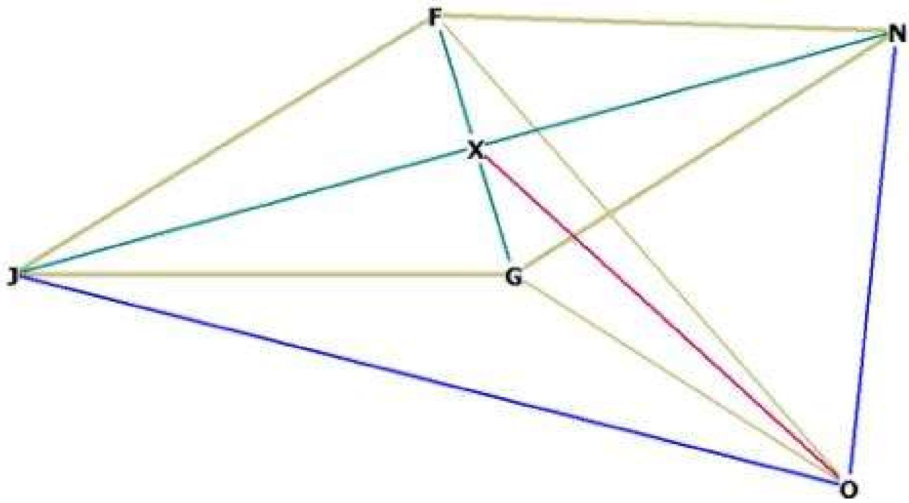


Figure 5 -- JON is 90 °

The height of any pyramid is 1/3 * area of base * height of pyramid.
We need to get the area of each diamond face.
In order to do this we need to take a rather extensive detour in which we will derive lots of interesting information about the rhombic dodecahedron.

Section 2 -- Rhombic Dodecahedron Internals

First off, we need to know the length of the side of the rhombic dodecahedron. In reference to what? We have an octahedron and a cube inside the r.d. We already have the length's of each of these sides, in relation to a sphere that encloses the octahedron and cube. The outer sphere of the r.d. is precisely that sphere that touches all 6 vertices of the octahedron, so we choose the edge of the octahedron as our reference point.
Recall from Octahedron that this relationship is:

$$r = \frac{1}{\sqrt{2}} * \text{side of octahedron}$$

.

Observe that the long axis NJ in Figures 1 and 2 is the side of the octahedron. The distance ON = OJ, in blue, is the radius of the unit sphere that encloses the octahedron.

We have remarked previously that it is possible to build a tetrahedron off of any of the faces of the octahedron that lies within the r.d. , like so:

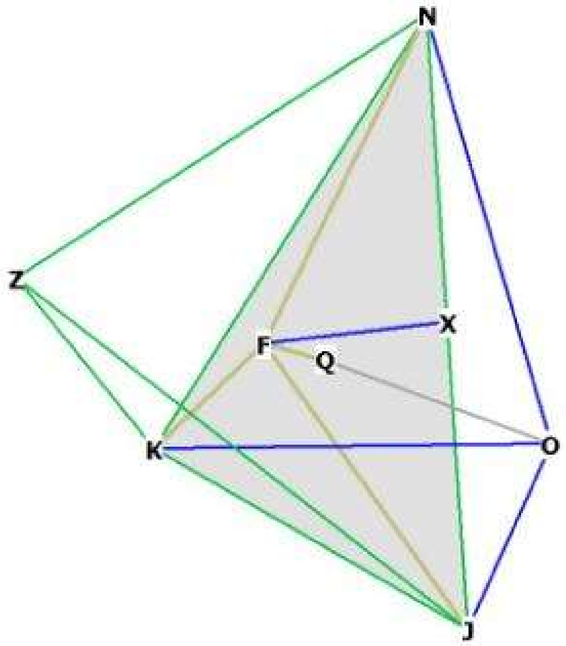


Figure 4A, repeated
Showing the tetrahedron NZKJ, which is built from the face of the octahedron NKJ, and which has edge lengths equal to the edges of the octahedron.

The distance FX in Figure 5 is the distance from the centroid of the tetrahedron to the mid–edge of the tetrahedron. We know from

Tetrahedron that this distance = $\frac{1}{2\sqrt{2}}$ * side of the tetrahedron.

Therefore, FX = $\frac{1}{2\sqrt{2}}$ * side of the octahedron.

Now we can find the side of the r.d. in terms of the octahedron side.

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N
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|_________\
X          F
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Figure 6 (Refer also to Figures 4 and 5).

NX = 1/2 the side of the octahedron, or long-axis of the r.d. face.
NF is the r.d. side.
We will refer to "os" as the side of the octahedron. "os" is equal in length to the side of a tetrahedron whose face is congruent to the octahedron face (See Figure 4A).
"rds" will be the r.d. side.

$$\overrightarrow{NF}^2 = \overrightarrow{NX}^2 + \overrightarrow{FX}^2 = \frac{1}{4}\mathrm{os}^2 + \frac{1}{8}\mathrm{os}^2 = \frac{3}{8}\mathrm{os}^2.$$

$$\mathrm{NF} = \mathrm{rds} = \frac{\sqrt{3}}{2\sqrt{2}}\mathrm{os} = \frac{3}{2\sqrt{6}}\mathrm{os}.$$

$$\mathrm{os} = \frac{2\sqrt{2}}{\sqrt{3}}\mathrm{rds} = \frac{2\sqrt{6}}{3}\mathrm{rds}.$$

While we're at it, lets get FG, the short-axis distance on the r.d. face. FG is visible in Figures 1,2,3 and 5. We have already established

$$\mathrm{FX} = \frac{1}{2\sqrt{2}}\mathrm{os}$$

. Substituting,

$$\mathrm{FX} = \frac{1}{2\sqrt{2}} * \frac{2\sqrt{2}}{\sqrt{3}}\mathrm{rds} = \frac{1}{\sqrt{3}}\mathrm{rds}.$$

$$\text{Since } \mathrm{FG} = 2 * \mathrm{FX},\ \ \mathrm{FG} = \frac{2}{\sqrt{3}}\mathrm{rds}.$$

It is important to establish the distance OF = OG, the distance from the r.d. centroid to the short axis points on the r.d. face. We have shown these in yellow (Figure 3 and Figure 5), indicating their length is equal to the length of the r.d. side. Is this true? Triangle OXF is right, by construction. In order to get OF = OG, we need OX, which also happens to be the height of the r.d. pyramid.

The height of the pyramid can be determined by inspection. From Figures 2 and 4 we see that OX goes from the centroid of the r.d. to the midpoint of the octahedron side. But this distance is exactly 1/2 the octahedron side, as we see clearly in Figure 7.

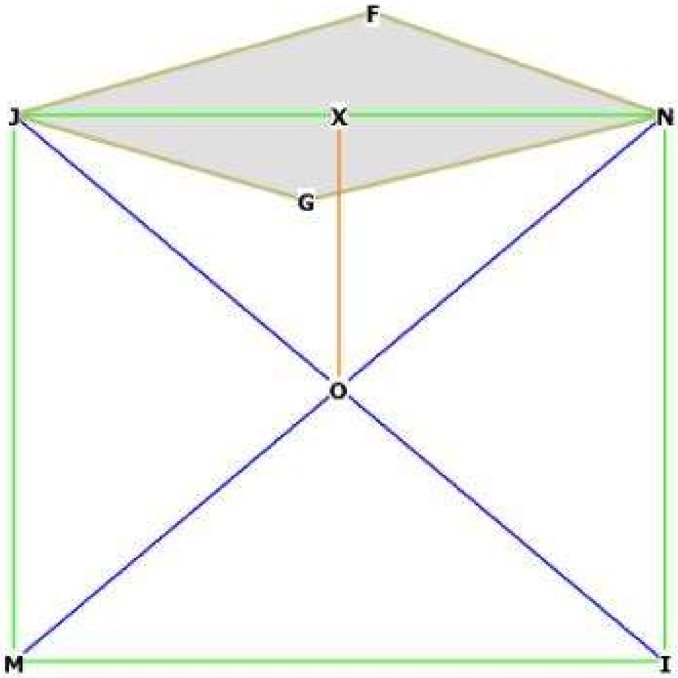

Figure 7, showing the height of the r.d. pyramid (OX) is 1/2 os, the side of the octahedron (NI, JM).

So $h = OX = \frac{1}{2}os$, and $os = \frac{2\sqrt{2}}{\sqrt{3}}rds$, therefore $h = \frac{1}{2} * \frac{2\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}rds.$

Now we have the height of the r.d. pyramid in terms of the r.d. side.

$$XN = \frac{1}{2}os = \frac{1}{2} * \frac{2\sqrt{2}}{\sqrt{3}}rds = \frac{\sqrt{2}}{\sqrt{3}}rds.$$

The triangle OXN is isosceles.

Now that we have calculated OX, it remains to prove that OF = OG. When you build a model of the rhombic dodecahedron you can see this immediately, but lets show it mathematically as well. We will find the result first in terms of the side of the octahedron, then translate this with respect to the side of the r.d.

$$\overrightarrow{OF}^2 = \overrightarrow{OG}^2 = \overrightarrow{OX}^2 + \overrightarrow{XF}^2 = \frac{1}{4}os^2 + \frac{1}{8}os^2 = \frac{3}{8}os^2$$

$$OF = OG = \frac{\sqrt{3}}{2\sqrt{2}}os$$

, which is precisely what we established above.

$$OF = OG = \frac{\sqrt{3}}{2\sqrt{2}} * \frac{2\sqrt{2}}{\sqrt{3}}rds = rds$$

Therefore OF = OG is equal in length to the side of the r.d.

What is the relationship between OQ and FQ? From Tetrahedron and Octahedron we know that:

the distance from the centroid of the tetrahedron to the mid-face = $\frac{1}{2\sqrt{6}}os = FQ.$

the distance from the centroid of the octahedron to the mid-face = $\frac{1}{\sqrt{6}}os = OQ.$

Therefore OQ = 2 * FQ, and we conclude that the face of the octahedron is twice as far away from its centroid, as is the face of the tetrahedron from its centroid.

We also gain the important information that triangle FOG is isosceles.

Now we can get to some important data, that is, the relationship between the side of the r.d. and the radius of the sphere that contains the rhombic dodecahedron. That sphere is, you recall, the sphere that surrounds and touches the vertices of the octahedron (N-ILJK-M in Figure 1 and 1A). We will refer to this radius as r_{outer}. The radius of this sphere is ON = OJ in Figure 8 below. What is ON = OJ?

$\measuredangle$JON is right by construction.

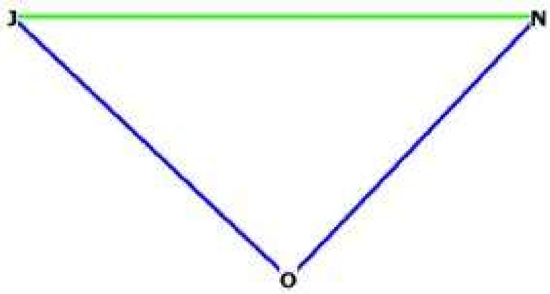

Figure 8

We know that JN = side of the octahedron = os. OJ = ON = r_{outer}, so

$2 * r_{outer}{}^2 = os^2, \; r_{outer}{}^2 = \frac{os^2}{2}.$

$r_{outer} = \frac{1}{\sqrt{2}} os.$

Since we found that $rds = \frac{\sqrt{3}}{2\sqrt{2}} os$, we substitute for os and get,

$r_{outer} = \frac{1}{\sqrt{2}} * \frac{2\sqrt{2}}{\sqrt{3}} rds = \frac{2}{\sqrt{3}} rds.$

Therefore $r_{outer} = \frac{2}{\sqrt{3}} rds$, and $rds = \frac{\sqrt{3}}{2} r_{outer}$.

Now we have expressed the side of the r.d. in terms of the radius of the sphere that contains the rhombic dodecahedron, realizing that this sphere touches only the 6 outer vertices of the r.d. Notice that FG, the short-axis distance across the face of the r.d., (calculated above) , also = $\frac{2}{\sqrt{3}} rds$.

We also know from Figure 3 that the radius of the smaller sphere touching the 6 cube vertices ABCD-EFGH is equal to the side length of the r.d. We may write rds = r_{inner} for this smaller sphere. (Henceforth, we will write “r” for the radius, understanding that r = r_{outer}).

Section 3 - The Resumption of the Volume Calculation

We were, at the end of Section 1, about to find the Volume of the rhombic dodecahedron. To do this we need to recall the height of the pyramid OX (see Figure 7), and find the area of the r.d. face.

By looking at Figure 3 we perceive that the area of the r.d. face can be divided into 2 triangles, each one with base NJ. Both triangles are congruent by side-side-side, and the height, ht, of each is XF = XG.

From above we know that this distance is $\frac{1}{2\sqrt{2}}\text{os}.$

NJ is the side of the octahedron, so NJ = os.

Now we have

$$\text{area}_{1\text{ face triangle}} = \frac{1}{2} * \text{base} * \text{height} = \frac{1}{2} * \text{os} * \frac{1}{2\sqrt{2}}\text{os} = \frac{1}{4\sqrt{2}}\text{os}^2.$$

$$\text{area}_{1\text{ face}} = 2 * \text{area}_{1\text{ face triangle}} = \frac{1}{2\sqrt{2}}\text{os}^2.$$

We want to get all of our data on the r.d. in terms of the r.d. itself, for consistency. Therefore we translate the area from the side of the octahedron, to the side of the r.d.

$$\text{area}_{1\text{ face}} = \frac{1}{2\sqrt{2}}\text{os}^2 = \frac{1}{2\sqrt{2}} * \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2\text{rds}^2 = \frac{1}{2\sqrt{2}} * \frac{8}{3}\text{rds}^2$$

$$= \frac{4}{3\sqrt{2}}\text{rds}^2 = 0.942809042\ \text{rds}^2.$$

Now, finally, we have enough data to calculate the volume of 1 pyramid:

$$\text{Volume}_{1\text{ pyramid}} = \frac{1}{3} * \text{area of base} * \text{h} = \frac{1}{3} * \frac{4}{3\sqrt{2}}\text{rds}^2 * \frac{\sqrt{2}}{\sqrt{3}}\text{rds}$$

$$= \frac{4}{9\sqrt{3}}\text{rds}^3$$

$$\text{Volume}_{\text{r.d.}} = 12 * \text{Volume}_{1\text{ pyramid}} = 12 * \frac{4}{9\sqrt{3}}\text{rds}^3$$

$$\text{Volume}_{\text{r.d.}} = \frac{16}{3\sqrt{3}}\text{rds}^3 = 3.079201436\ \text{rds}^3.$$

What is the surface area of the rhombic dodecahedron?

It is just 12 faces * area of 1 face =

$$12 * \frac{4}{3\sqrt{2}} rds^2 = \frac{16}{\sqrt{2}} rds^2 = 8\sqrt{2} rds^2.$$

$$\text{Surface area}_{\text{rhombic dodecahedron}} = 8\sqrt{2}\ rds^2 = 11.3137085\ rds^2.$$

There are 3 central angles of the rhombic dodecahedron.
In Figure 3, they would be, for example, ∡FOG, ∡FON, ∡JON.
Let's start with ∡FOG.

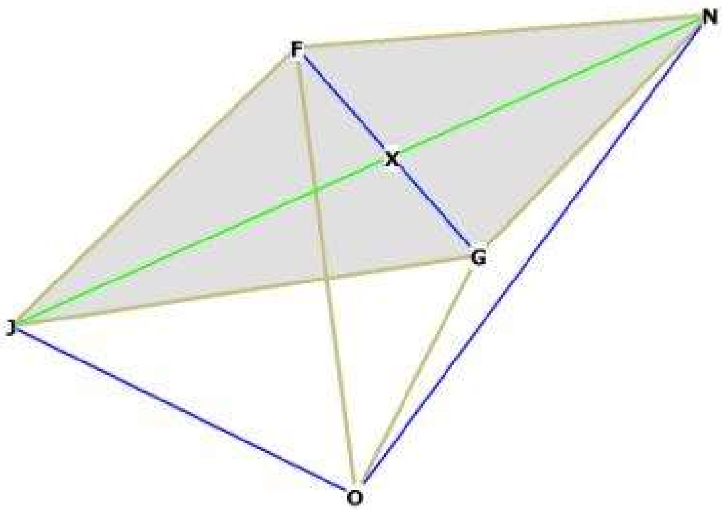

Figure 3, repeated

∡JON is right. ∡OXF, ∡OXG are right.
To find ∡FOG, find ∡FOX. Triangle FOX is right by construction, therefore,

$$\sin(\measuredangle FOX) = FX / FO = \frac{\frac{1}{\sqrt{3}}}{1} = \frac{1}{\sqrt{3}},$$

$$\measuredangle FOX = \arcsin\left(\frac{1}{\sqrt{3}}\right) = 35.26438968°,$$

$$\measuredangle FOG = 2 * \measuredangle FOX = 70.52877936°$$

This is precisely the central angle of the cube! Understandably so, for we already know that the two vertices F and G are two of the vertices of a cube (see Figure 1A).
It is also the dihedral angle of the tetrahedron.

Because the triangle FOG is isosceles,
$\measuredangle$OFG = $\measuredangle$OGF = $\measuredangle$FON = (180° - 70.52877936°) / 2, using the property that the sum of angles of a triangle = 180° .
Therefore $\measuredangle$FON = 54.7356103° (and so does $\measuredangle$FNO).

From Figure 7 we can see immediately that $\measuredangle$JON is right, it being the central angle of the square JMIN. $\measuredangle$JON = 90°

What are the surface angles of the diamond faces of the rhombic dodecahedron?

We know that the short-axis distance across the r.d. face, FG in Figure 3, is $\frac{2}{\sqrt{3}}\text{rds}$.
We know FN = NG = the side of the r.d. = rds.
Triangle FXN is right by construction.
$\measuredangle$FNX is one half the face angle FNG. FX is one- half FG.
So we can write:

$$\sin(\measuredangle FNX) = FX / FN = \frac{1}{\sqrt{3}}.$$

$\measuredangle$FNX = 35.26438968°, so
$\measuredangle$FNG = 70.52877936°.

$\measuredangle$FNG and its counterpart $\measuredangle$FJG are the smaller of the two face angles of the r.d. Now lets get $\measuredangle$JFN.
Once again we use triangle FXN and work with $\measuredangle$XFN, which is one-half the desired angle, JFN.
$\measuredangle$XFN is just 90 - $\measuredangle$FNX, using the property that the sum of all angles in a triangle is 180°.
$\measuredangle$XFN = 54.73561032°, so
$\measuredangle$JFN = 109.4712206°.
This angle, 109.4712206°, is the central angle of the tetrahedron and the dihedral angle of the octahedron. It also is the angle you see when you stand the rhombic dodecahedron on one of its 8 octahedral vertices and look down from above at two intersecting r.d. sides. This polyhedron an all-space filler, meaning that it can be joined with itself, like the cube, to fill any volume without any

space left over.

What is the dihedral angle of the rhombic dodecahedron? To see this calculation, click on
Rhombic Dodecahedron Dihedral Angle

If you examine the r.d. from the inside out with the zometool, you will see it is composed internally of the same rhombi.
To identify these internal rhombuses, lay the rhombic dodecahedron flat on one of its faces. Find 3 r.d. points and the centroid to see the rhombus:

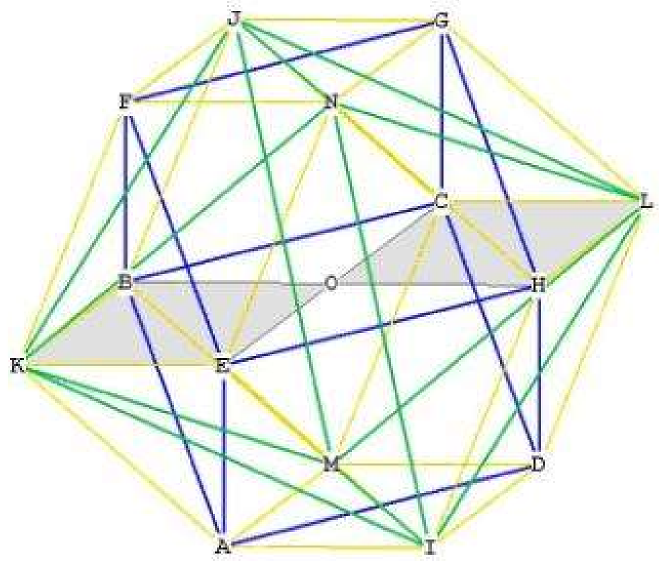

Figure 9 – two of the 12 internal rhombi of the rhombic dodecahedron. O is the centroid.
There are 12 internal rhombi, and 12 external rhombi, each of them identical.

Note that there is another angle on the exterior of the rhombic dodecahedron, and that is the angle that one face plane makes with another face plane as it goes over top of the octahedron within the r.d. This angle is 90°. This can be seen by looking at the long axis diagonals of the octahedron through the rhombi, and following the faces that form around the side of the octahedron. In

Figure 1, this can be seen with the faces NFJG and JBMC. Notice the lines NJ and JM, which are part of the octahedral square, and which bisect the faces along their long axis. $\measuredangle$NJM is a right angle, as is $\measuredangle$NLM and $\measuredangle$NKM. Observe also $\measuredangle$KNL. Again, this property helps the rhombic dodecahedron to be an all-space filler.

Now lets complete the analysis of the rhombic dodecahedron by finding or collecting the distances from the centroid to a short-axis vertex, a long-axis vertex, mid-edge and mid-face. We already have all of this information, except for the distance from the centroid to a mid-edge:

distance from centroid to long-axis r.d. vertex (vertex of the octahedron) = $\frac{2}{\sqrt{3}}\text{rds}$.

distance from centroid to short-axis r.d. vertex (vertex of the cube)

= rds.

distance from centroid to r.d. mid-face (mid-point of octahedron side)

= $\frac{\sqrt{2}}{\sqrt{3}}\text{rds.}$

To find centroid to r.d. mid-edge requires a little work:

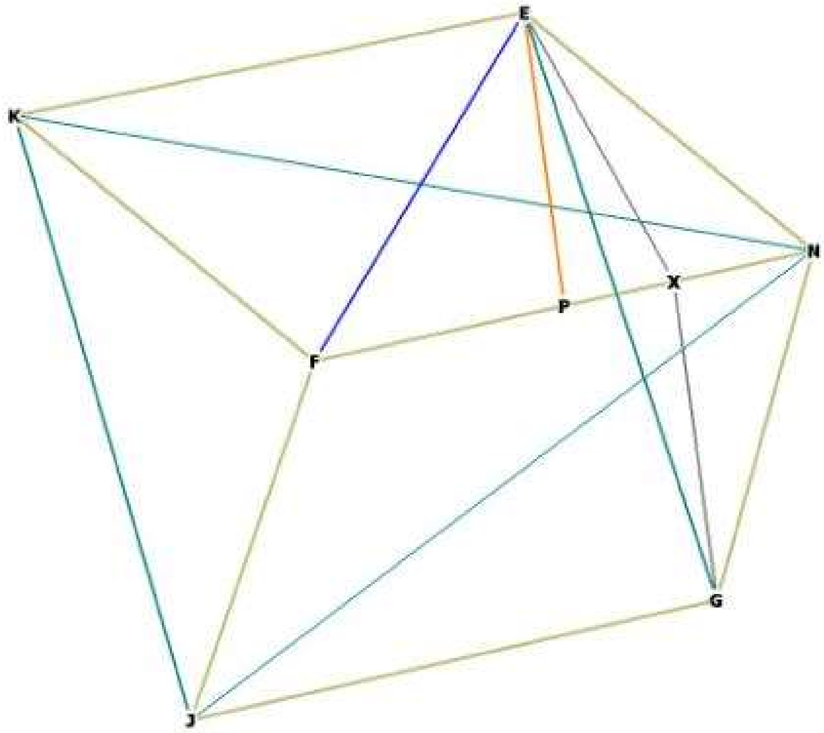

Figure 9 -- showing centroid to mid-edge distance EP.

We have already remarked upon one of the properties of the r.d., that it is composed of 12 internal rhombi identical to the external faces. Those rhombi are divided in half (here, at EF) by the same distance as from the centroid to any of the vertices of the octahedron contained within the r.d. Therefore the r.d. is composed of 24 exterior congruent triangles (here, $\triangle$EFN and $\triangle$EKF), and 24 interior congruent triangles congruent to the exterior ones!

So triangle EFN may substitute for any triangle in the interior of the r.d., and more to the point, the vertex E or F may substitute for the centroid of the r.d., at O. In order to see this clearly, you have to build a 3D model of the rhombic dodecahedron, and I encourage you to do this.
From Rhombic Dodecahedron Dihedral Angle, we know that

$$EX = \frac{2\sqrt{2}}{3}\text{rds.}$$

We know that $\measuredangle$EXN is right by construction, and that EN = rds.
We also know that P is at mid-edge on FN, the side of the r.d.,

therefore NP = 1/2 rds.
We can find XN, and so XP, by writing XP = NP - XN.
We need to find XN.

$$\overrightarrow{XN}^2 = \overrightarrow{EN}^2 - \overrightarrow{EX}^2 = 1\text{rds}^2 - \frac{8}{9}\text{rds}^2 = \frac{1}{9}\text{rds}^2.$$

$$XN = \frac{1}{3}\text{rds}.$$

$$XP = \frac{1}{2}\text{rds} - \frac{1}{3}\text{rds} = \frac{1}{6}\text{rds}.$$

Now we can find EP, the distance from centroid to mid-edge.

$$\overrightarrow{EP}^2 = \overrightarrow{EX}^2 + \overrightarrow{XP}^2 = \frac{8}{9}\text{rds}^2 + \frac{1}{36}\text{rds}^2 = \frac{33}{36}\text{rds}^2.$$

$$EP = \frac{\sqrt{33}}{6}\text{rds} = 0.957427108\ \text{rds}.$$

The Dihedral Angle of the Rhombic Dodecahedron

When calculating dihedral angles, it is vital to ensure that the angle chosen is an accurate representation of the intersection of the 2 planes. In the rhombic dodecahedron (r.d.), this is a little tricky.

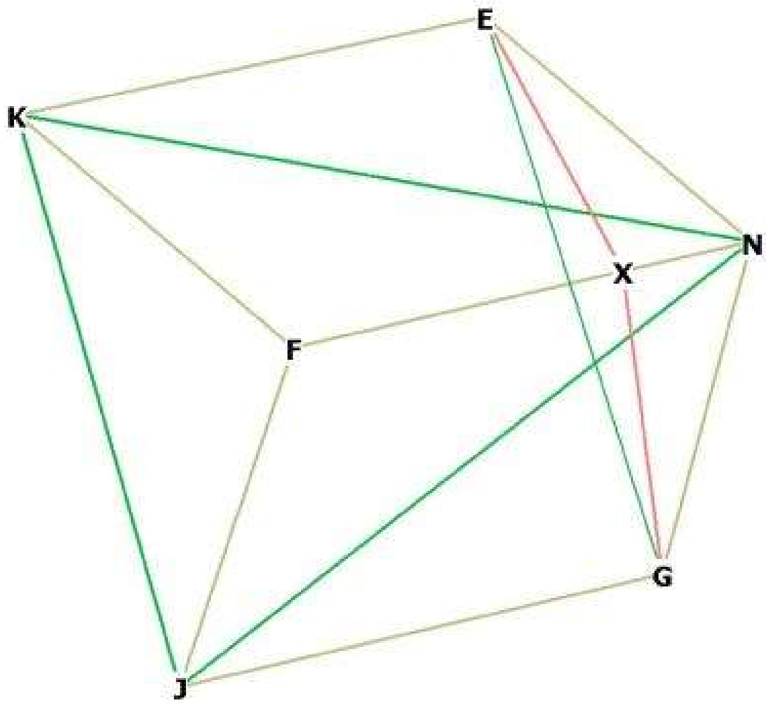

Figure 1 -- The r.d. dihedral angle

The dihedral angle of the r.d. is the angle EXG. This angle is a geometrically accurate description of the intersection of any 2 planes of the r.d., in this case, the 2 planes KENF and JGNF. The lines EX and GX are constructed such that ∡NXG and ∡NXE are right. ∡ EXG is a "roof" over the 2 planes which exactly describes the dihedral angle.

We will be working with triangle EXG .

It's easier to see this if you construct a model of the rhombic dodecahedron, and lay it on one of its edges.

I have shown EG in green, indicating it is equal to the side of the octahedron (referred to as os). We have to show this.

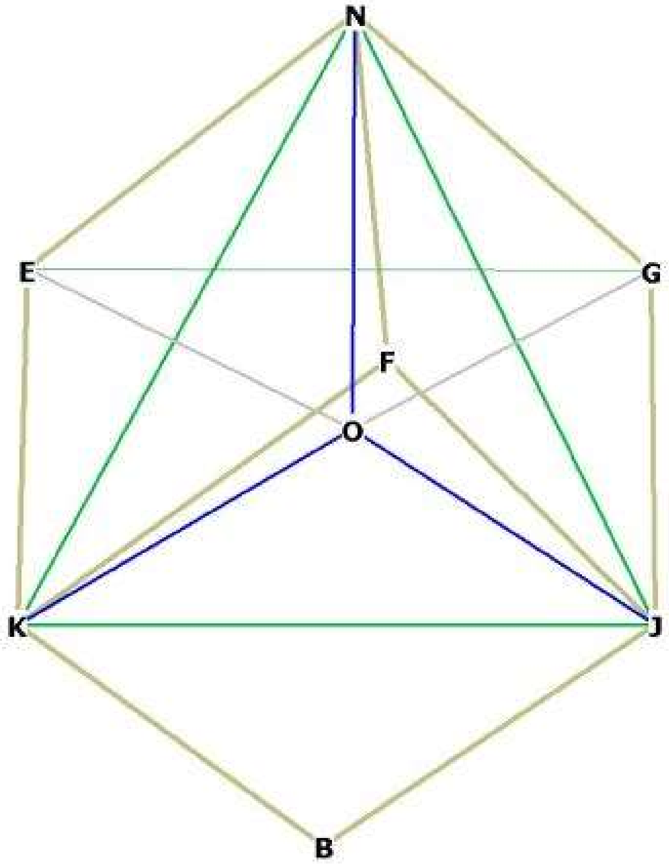

Figure 2 O is the centroid. NKJ is one of the faces of the octahedron. ON, OK and OJ are radii of the outer sphere that touch the 6 octahedral vertices. OE and OG touch the short axis vertices of the r.d.

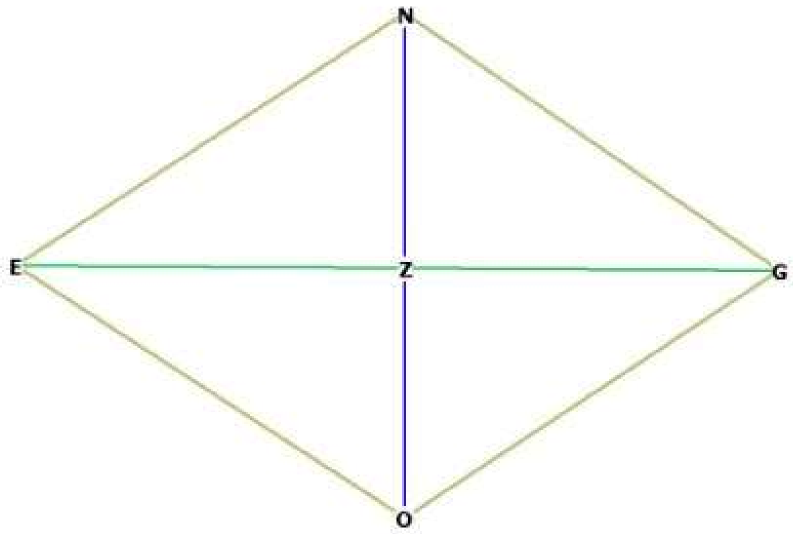

Figure 3 -- another view showing how EG is found. NEOG is one of 12 internal rhombi of the rhombic dodecahedron.

OE and OG are the sides of the rhombic dodecahedron, or rds. We have previously shown that the triangles OEN and OGN are isosceles and congruent (the vertices may have different names, but the triangles are the same!) NZE is right by construction
To obtain EG, simply re-do the calculation we did previously,

$$\overrightarrow{EZ}^2 = \overrightarrow{EN}^2 - \overrightarrow{NZ}^2,$$

$$\overrightarrow{EZ}^2 = rds^2 - \frac{1}{3}rds^2 = \frac{2}{3}rds^2$$

$$EZ = \frac{\sqrt{2}}{\sqrt{3}}\ rds.$$

$$EG = 2 * EZ, \quad EG = \frac{2\sqrt{2}}{\sqrt{3}}rds.$$

which has been obtained previously as os.
Therefore EG is identical in length to the side of the octahedron.

This internal rhombus, NEOG, is identical to the rhombus of the r.d. face. The r.d. is composed, internally and externally, of identical rhombi, which gives it the quality of an all-space filler.

We have established EG as the side of the octahedron. Now it remains to determine GX and XB.

Look at Figure 1 again. The triangle ENX is right by construction, and we know the angle ENX, it being the long-axis angle of the r.d. face, as $2 * \arcsin(\frac{1}{\sqrt{3}})$, or 70.52877936 degrees.

We also know EN = side of the r.d, or rds.

So we write sin(∡ENX) = EX / EN = EX / rds.

EX = rds * sin(70.52877936) = 0.942809042 * rds = $\frac{2\sqrt{2}}{3}$ rds.

(Note: EX = $\frac{1}{\sqrt{3}}$ os).

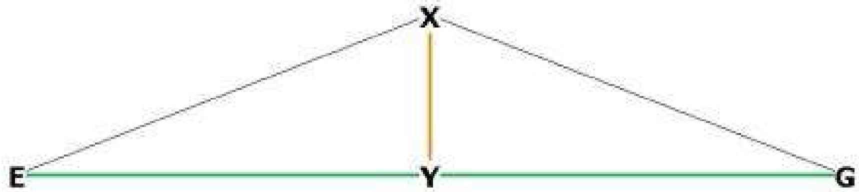

Figure 4 -- showing the dihedral angle in simplified fashion

Now, EX and EY are known. We need to get ∡EXY.

We know EG is the side of the octahedron, or os, and it is $\frac{2\sqrt{2}}{\sqrt{3}}$ rds and so EY is $\frac{1}{2}$ that.

sin (∡EXY) = EY / EX = $\dfrac{\frac{\sqrt{2}}{\sqrt{3}}\text{rds}}{\frac{2\sqrt{2}}{3}\text{rds}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$.

∡EXY = $\arcsin(\frac{\sqrt{3}}{2})$ = 60°

Therefore ∡ EXG = 2 * ∡EXY = 120°

Dihedral angle of the rhombic dodecahedron = 120°

Rhombic Dodecahedron Reference Charts
(included in the book)

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