QUESTIONS

Q1. Report

You work for a consultancy company in the UK. Your company uses High Performance Computing to solve problems for clients in areas such as: big data analytics in health care, oil industry or defense.

Your boss asks you to write a 6 page document to outline the strategy for high performance computing for the company. This document should review

- applications where high performance computing is required.
- the different possible supercomputers: multi-core, multi-node, shared and distributed memory, or GPUs..
- The different possible programming models for parallel machines.
- The possible use of cloud computing, such as services from Amazon.
- The CEO is very interested in sustainability, so the document should address that. .
- The report should include references in an appropriate format..

Q2. Strings handling

The goal of this question is to do some research on strings manipulation in python.

Create a class called StringContainer, initialised it with a string containing only spaces and alpha-numerical characters, which contains the following instance variables:

- the string itself.
- the length of the string.

Include the following methods:

- *split*: returns a list of the words in the string.
- *check_occurence(str)*: returns true if *str* is contained in the string and False if not.
- *get_frequency(str)*: returns the number of occurence of the string *str* in the whole message:
  e.g if the message is ‘Hello Hello World 18’, and *str* is ‘Hello‘, the function should return ’2’.
- *replace(str, newstr)*: replaces every occurence of the string *str* by *newstr*.
- *merge(obj)*: appends the string contained in the object *obj* to the string of the instance of StringContainer.
- *mirror*: returns the string in the reverse order.

The class initialisation should take care of Exceptions. Provide your code together with a few illustrative examples of each of the functions to demonstrate theirs functionality.

Q3. Guitar string simulation

In this exercise you will study the motion of a guitar string. An ideal string has no dissipation (the string vibrate forever) and the amplitude variation are assumed to be small.

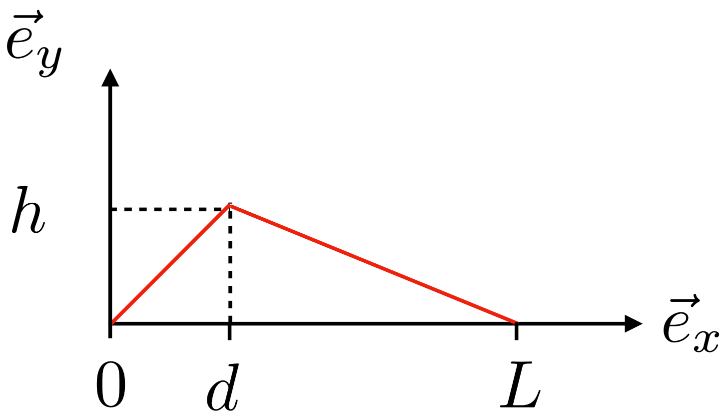

Figure Q3 (1): Initial plucked deformation of a guitar string, $y(x, 0) = f(x)$.

When no forces are applied to the string, the string is along the x-axis and has a length L. The string motion is assumed to occur in one plane which is characterised by the two axis $\mathbf{e}_x$ and $\mathbf{e}_y$. At any time, the motion of string is thus described by a function $y(x, t)$. The cartesian coordinate system used in this exercise is illustrated for a particular initial condition is illustrated in **Q3 (1)**.

With those assumption the second Newton's Law can be applied and the equation of motion for y reads :

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0, \quad (1)$$

and the initial deformation is given by $y(x, 0) = f(x)$ where $f(0) = f(L) = 0$. We assume that initial velocity of the string vanishes everywhere, i.e $\frac{\partial y}{\partial t}(x, 0) = 0$ for all x.The coefficient c has the dimension of a velocity and can be shown to be related to the string tension T (in N) and to the linear mass density μ (in kg.m^{-1}) by the relation $c = (T/\mu)^{1/2}$.

The general solution is

$$y(x, t) = \lim_{nmax\to\infty} \sum_{n=1}^{nmax} A_n \sin k_n x \cos \omega_n t \quad (2)$$

with $k_n = n\pi/L$ and $\omega_n = ck_n$. The coefficients A_n are related to the initial condition by

$$A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\mathrm{d}x \tag{3}$$

One example of initial condition, is a "linear" pluck at a position d along the string of height h as illustrated in **Q3 (1)**. In that case the function f reads:

$$f(x) = \begin{cases} \frac{hx}{d}, & 0 \leq x \leq d \\ \frac{(L-x)h}{L-d}, & d < x \leq L \end{cases} \tag{4}$$

In that case the coefficients A_n can be computed analytically and read:

$$A_n = \frac{2hL^2}{\pi^2 n^2 d(L-d)}\sin\left(\frac{n\pi d}{L}\right) \tag{5}$$

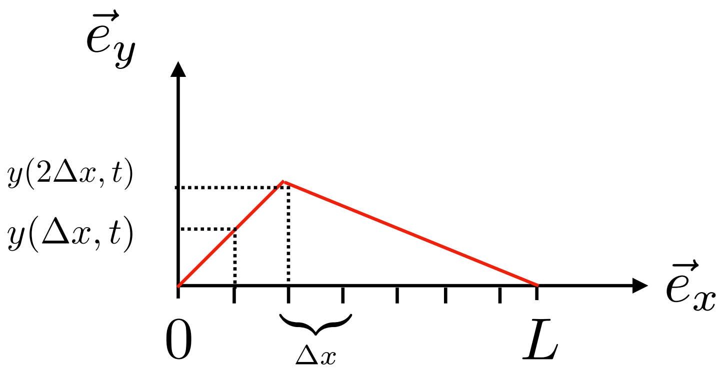

Figure Q3 (2): Discretisation of the problem:The position of the string at time t is approximated by a set of Nx value separated by a distance ΔX.

In order to solve numerically the equation (1), the position must be discretised on a grid of Nx equally spaced points so that the step size is $\Delta x = L/Nx$, as illustrated in **Q3 (2)**. The time dependence of y is also approximated with a discrete number of equally spaced time steps of Δt. If ΔX and Δt are small enough, it can then be shown that the approximate solution $y_i^j = y(j\Delta x, i\Delta t)$ can be obtained by using iteratively the algorithm 1 which relates y_{i+1}^j to y_i^j and y_{i-1}^j. The position of the string can for instance be stored in a numpy array of size $(Nt, Nx+1)$.

You can use for the final plots $Nx = 200$, $d = L/10$, $L = 1$ m and $h = 0.5 \times 10^{-2}$ m throughout this exercise. While developping your program it is a good idea to reduce Nx so that the code is quick enough. It is convenient to set c through the relation $c = 2Lf$ where f is the frequency of the first mode, you can for instance use $f = 440\ \mathrm{Hz}$ (the musical note $\mathbf{A}$).To ease the analysis of the convergence of the algorithm we will in practice restrict ourselves to $dt = dx/c$. The residual, defined by $r(i\Delta t) = r_i = \sum_j (y_i^j - y_{\mathrm{analytical}}|_i^j)^2$, is a standard measure of the difference between the numerical and the analytical solution.

- Write a function that returns the analytical solution using equation (2) and (5) for a vector of positions x as a vector, for a given value of the parameters, for a given $nmax$, and for a given time t.
- Plot the initial condition function $f(x)$ and the analytical solution for $t = 0$ and check that they do match.
  You can truncate the estimation of the analytical solution to $nmax = 1000$.
- Define a function that implements the algorithm (1).
- Use the algorithm implemented in the previous question to predict the motion of the string for a number of time steps $Nsteps > 1000$. Compute and plot the residual as a function of time.
- Produce two animations illustrating the motion of the string:
  - for the initial condition (4)
  - for the initial condition $f_n(x) = A\sin(xn\pi/L)$ for $A = 0.5 \times 10^{-2}$ and $n = 3$ (a so-called stationary wave solution).

END OF QUESTIONS