Case study description

You are an analyst in a large company that provides equipment support and vehicle fleet management to its clients. You are responsible for planning the inventory for a warehouse that keeps a stock of machine parts. There are 26 types of items held in the warehouse and each item is labelled with one of the 26 letters of the English alphabet.

Every day, the warehouse receives a single demand to supply a set of 8 items represented by an 8-letter word. All possible words that can be demanded from the warehouse are stored in the file *"words-population.xlsx"* which contains 40,161 English 8-letter words.

You should hold sufficient inventory for 1 day of production – resupply is daily. An unlimited quantity of letters can be held in the warehouse (at zero holding cost).

The Rules:

1. Each successfully completed word sells for £20,000.
2. Failing to supply a word costs you a £10,000 contractual penalty (you are not allowed to supply an "incomplete" word).
3. Each letter held in stock costs £1,000.
4. Once you have decided what inventory to hold, you must re-order to replace exactly the same letters consumed (1 for 1) every night after the day's word has been made, at a cost of £1,000 per letter.
5. In addition to the free (over-night) delivery from the supplier, you may use factory transport to rush collect (only) 1 letter per day at an additional cost of £1,000. That is, in this case the letter still costs £1,000 but you also pay the extra £1,000 for emergency delivery.
6. You have an inventory budget of £250,000.
7. A **game** is equal to 5 words (equivalently, 5 days - a working week).

The manager of the company is interested in three optimisations: profit per game, the number of games won (that is, games with profit) and the number of words created successfully.

TASKS

Task 1: Preliminary Analysis

The goal of this task is to obtain a general picture of the population of words and get an idea what stock of letters would be a reasonable choice as a starting point for further analysis.

Using the entire population of words, you will analyse frequencies of each letter in a word.

a) For each letter, find the range (the minimum and maximum), mean, median and mode for the number of times the letter occurs in a word.

b) For each letter, create a frequency table that shows the distribution of the number of times the letter occurs in a word.

c) What stock of letters guarantees that any single word from the population can be made? Based on the preliminary analysis, what stock of letters would you recommend as a reasonable choice? Justify your answer.

d) Based on the frequency analysis, which word can be seen as an outlier?

Words	1	2	3	4	5	6	7	8	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
ABACTORS	A	B	A	C	T	O	R	S	2	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0
ABBATIAL	A	B	B	A	T	I	A	L	3	2	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
ABANDING	A	B	A	N	D	I	N	G	2	1	0	1	0	0	1	0	1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0

Task 2: Simulation of games

a) For any given fixed stock of letters, implement a simulation of a single game that follows the rules 1 – 7 listed in the description on page 2. Your implementation should show the cost of the stock, the profit for each day and for the whole game (five days) and whether the word demanded on each day was successfully created or not.

b) Create a macro that allows to repeat the simulation of a game any given number of times and returns the relevant results of all the repetitions (please also read Task 3 before creating the macro).

Task 3: Analysis of profit

Suppose that you decide to hold exactly one of each letter of the alphabet in the stock.

Using 3000 repetitions of the game implemented in the macro in Task 2, estimate the following quantities for this stock of letters:

- the average profit per game,
- the average number of words created successfully in a game,
- the proportion of games with profit.

Make a histogram showing the distribution of profit per game and investigate the risk that a game will result in a loss.

Task 4: Optimisation algorithm

a) Using your solutions of tasks 1 – 3, find the stock of letters that maximises the average profit per game.

b) Propose a general iterative algorithm to find the stock that maximises the average profit per game. Describe your algorithm in words.

c) Can you optimise both: the average profit per game and the average number of words created successfully in a game at the same time? Justify your answer.