Inter-Area Oscillations

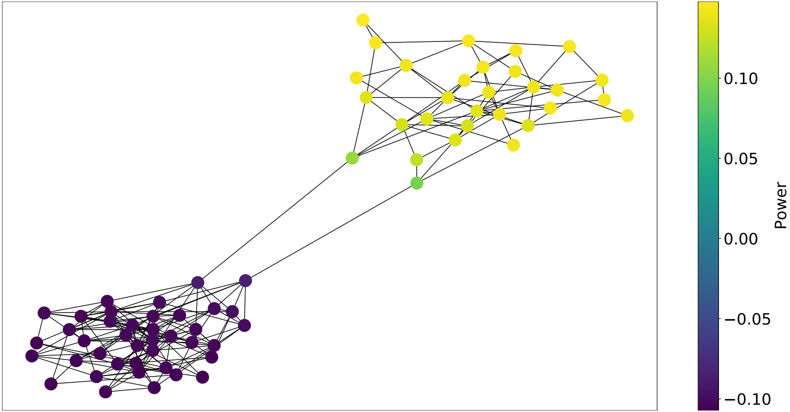
Julian Fritzsch

October 5, 2020

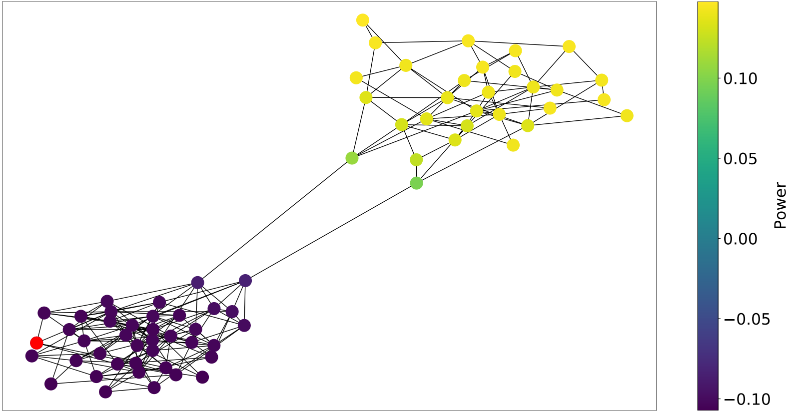
Overview

Motivation

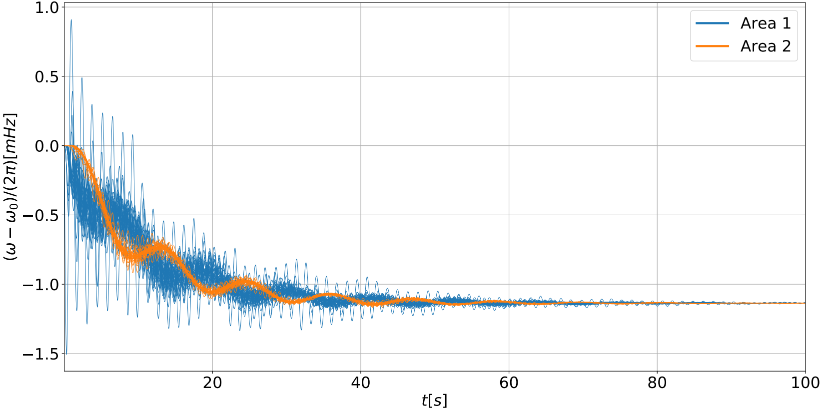
Simple Two Area Network



Simple Two Area Network



Simple Two Area Network



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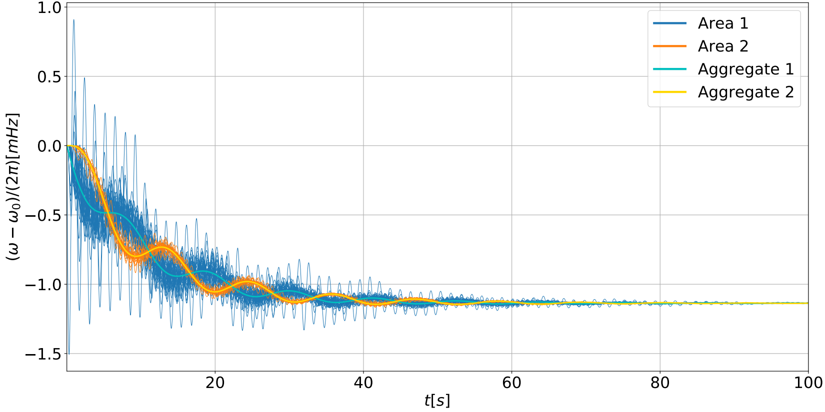

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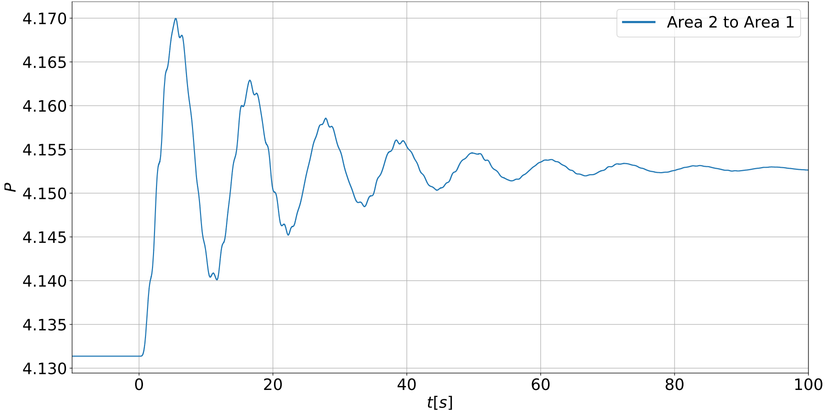

Area Aggregation

Identifying Areas

- aggregate nodes in area if they are "coherent" to r eigenmodes of the system

Peponides '81, Chow '82, Chow '84, Chow '85

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- if two states are coherent the corresponding rows of the eigenvector matrix are equal
- in reality: near coherency
- $x_i/m_i - x_j/m_j = \mathcal{O}(\epsilon)$ if $x(0)$ is in eigenspace of r modes

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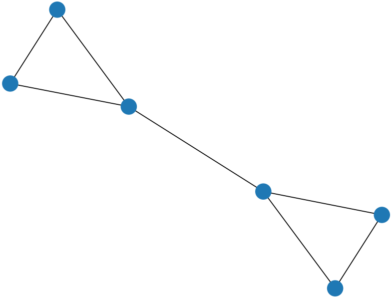
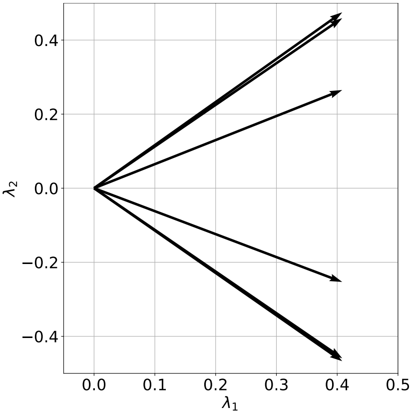
Eigenvalues Six Node System

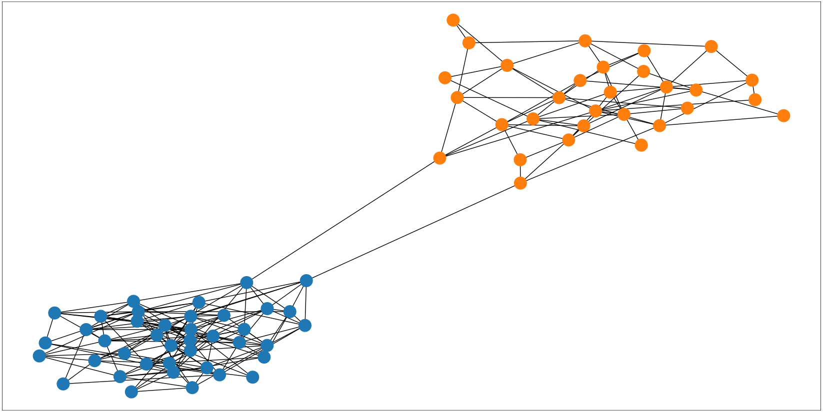




Algorithm

- start with matrix that contains the r smallest eigenvector
- use Gaussian elimination with complete pivoting to find the reference nodes
- calculate projection of remaining nodes onto reference nodes
- node belongs to area with projection closest to one

Examples



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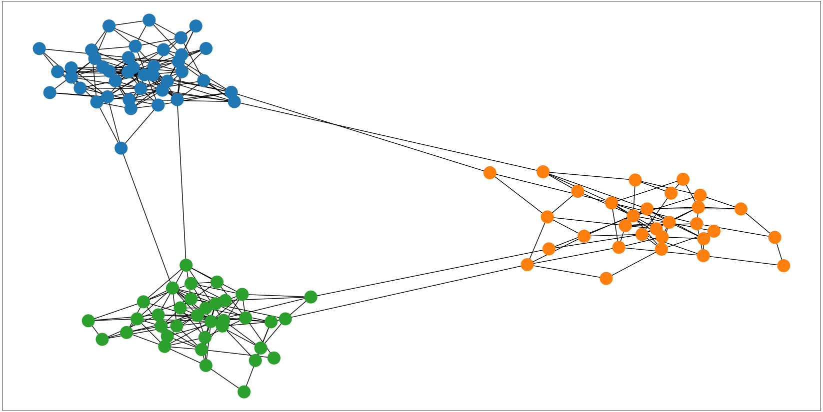

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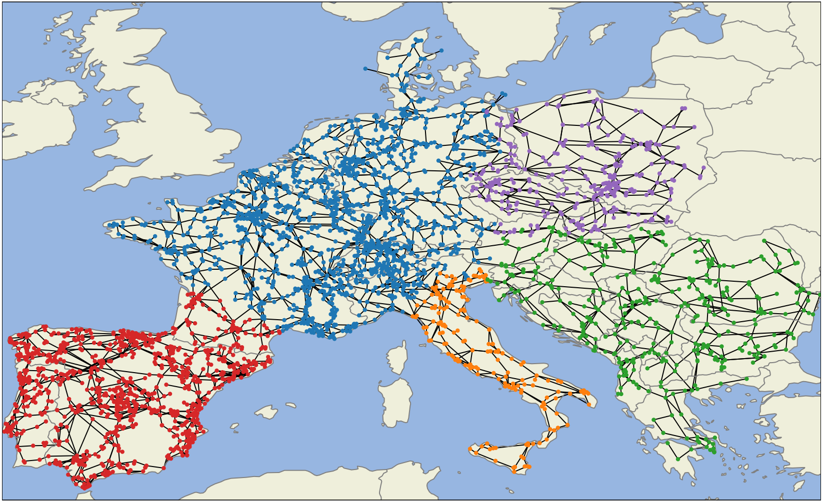

Separation of Time Scales

Idea

System with two time scales
$\rightarrow$ separation into fast and slow variable

Dynamical system:

$$\begin{aligned} \dot{y} &= f(t, y, z, \epsilon) \\ \epsilon\dot{z} &= g(t, y, z, \epsilon) \end{aligned}$$

$\epsilon \ll 1$

Khalil '02

Slow Subsystem

- take limit $\epsilon \to 0$
- fast system converges infinitely fast
- from $g(t, \bar{y}, z, 0) = 0$ find $z = h(t, \bar{y})$

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$\to$ slow subsystem is solution to

$$\dot{\bar{y}} = f(t, \bar{y}, h(t, \bar{y}), 0)$$

Fast Subsystem

- fast variable $z_f = z - h(t, \bar{y})$
- fast time scale $t_f = (t - t_0)/\epsilon$
- $\frac{dz_f}{dt_f} = g(t_0, \bar{y}(t_0), z_f + h(t_0, \bar{y}(t_0)), 0)$
- $z_f(0) = z(t_0) - h(t_0, \bar{y}(t_0))$

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The full solution is:

$$y(t) = \bar{y}(t) + \mathcal{O}(\epsilon)$$
$$z(t) = h(t, \bar{y}) + z_f(t_f) + \mathcal{O}(\epsilon)$$

Aggregation of Power Networks

Setup

Linearized swing equation

$$M\ddot{\theta} + D\dot{\theta} = -L\theta + P$$

Separate Laplacian into internal and external part

$$L = L^I + L^E$$

Area parameter

$$\delta = \frac{\text{strongest connection between two areas}}{\text{nodes in smallest area} \times \text{weakest intra-area connection}} = \frac{\gamma^E}{\underline{m} c^I}$$

Chow '82, Chow '85, Date '91, Romeres '13

Slow and Fast Variables

slow variables: center of inertia angle of area

$$y_\alpha = \frac{\sum_i m_i \theta_i}{\sum_i m_i} = (C_a \theta)_\alpha$$

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$$y_\alpha = \frac{\sum_i m_i \theta_i}{\sum_i m_i} = (C_a \theta)_\alpha$$

fast variables: weighted difference between nodes in area

$$z_\alpha = Q_\alpha \theta^\alpha$$

$$Q_\alpha = \begin{bmatrix} -1 + (m_\alpha - 1)v & 1 - v & -v & \cdots & -v \\ -1 + (m_\alpha - 1)v & -v & 1 - v & \cdots & -v \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 + (m_\alpha - 1)v & -v & -v & \cdots & 1 - v \end{bmatrix}$$

$$v = \frac{m_\alpha - \sqrt{m_\alpha}}{m_\alpha(m_\alpha - 1)}$$

Slow System

- the system can be separated
- defining $\left[\bar{y}, \dot{\bar{y}}\right] = \left[y, \dot{y}/\sqrt{\delta}\right]$
- taking the limit $\delta \to 0$ yields

$$\frac{d}{dt_s}\begin{bmatrix}\bar{y} \\ \dot{\bar{y}}\end{bmatrix} = \begin{bmatrix}0 & I \\ \tilde{A}_0 & -D_1\end{bmatrix}\begin{bmatrix}\bar{y} \\ \dot{\bar{y}}\end{bmatrix} + \begin{bmatrix}0 \\ C_a P - \tilde{A}_{12}\tilde{A}_{22}^{-1} QP\end{bmatrix}$$

For an exact definition of the matrices see Romeres et al. 2013

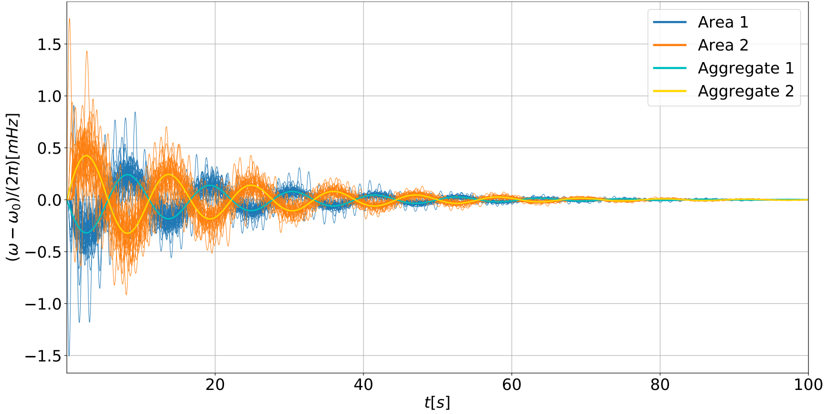
Fast System

the fast variables are given by: $z(t) = z_f(t_f) \underbrace{-\tilde{A}_{22}^{-1}(d\tilde{A}_{21}\bar{y} + \frac{QP}{c^I})}_{h}$

$t_f = t_s/\sqrt{\delta}$

$$\frac{d}{dt_f}\begin{bmatrix} z_f \\ \dot{z}_f \end{bmatrix} = \begin{bmatrix} 0 & \frac{I}{c^I} \\ \tilde{A}_{22} & \frac{-QM^{-1}DQ^T}{c^I} \end{bmatrix}\begin{bmatrix} z_f \\ \dot{z}_f \end{bmatrix}$$

2 Areas With Equal Injection / Consumption



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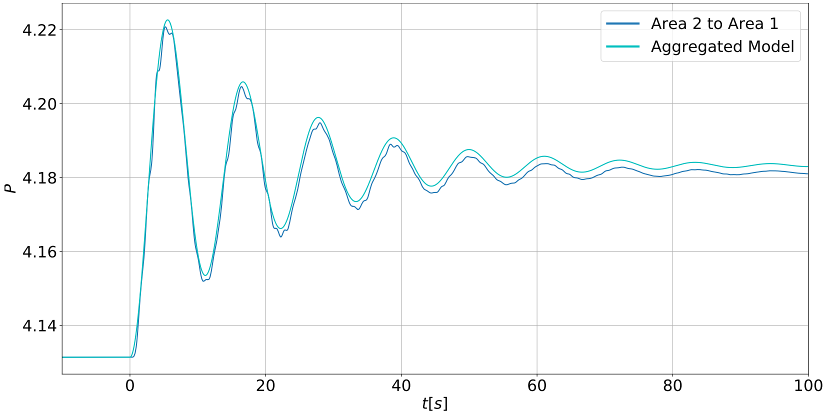

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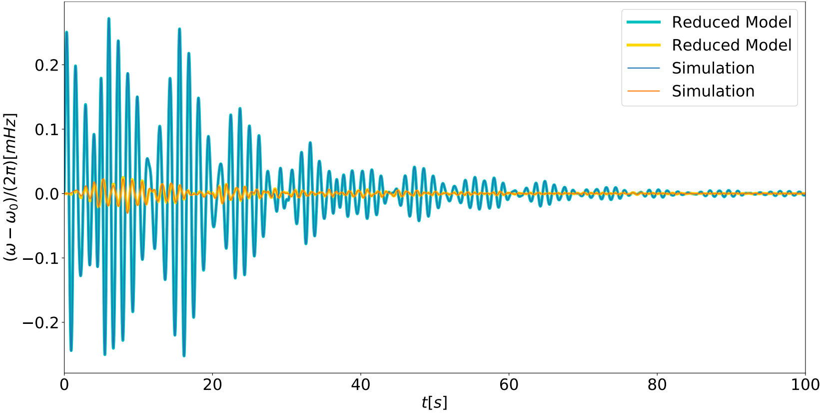
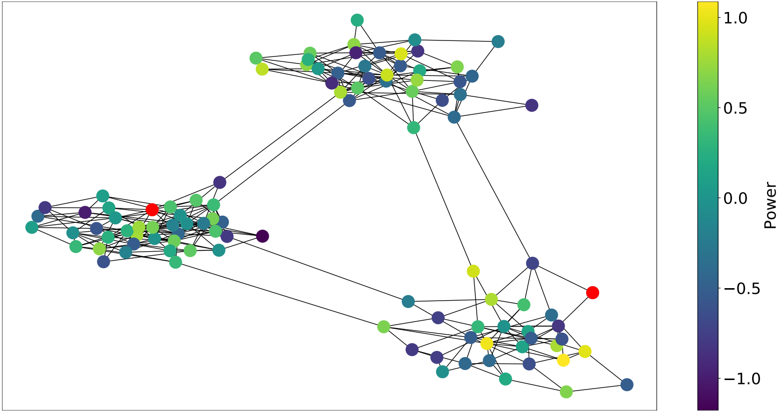


3 Areas With Equal Injection / Consumption



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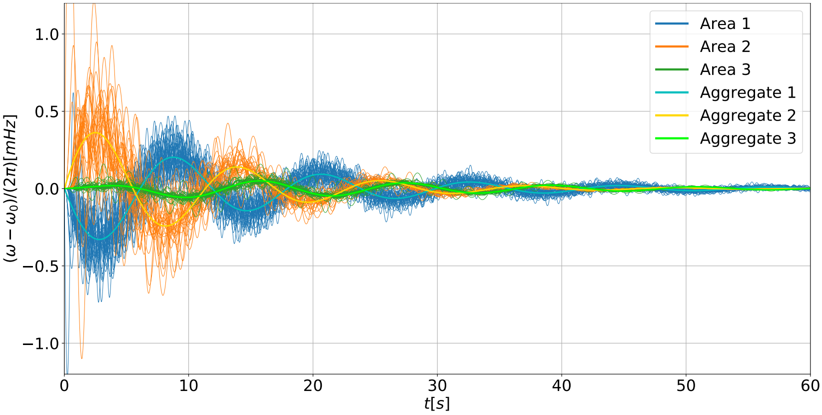

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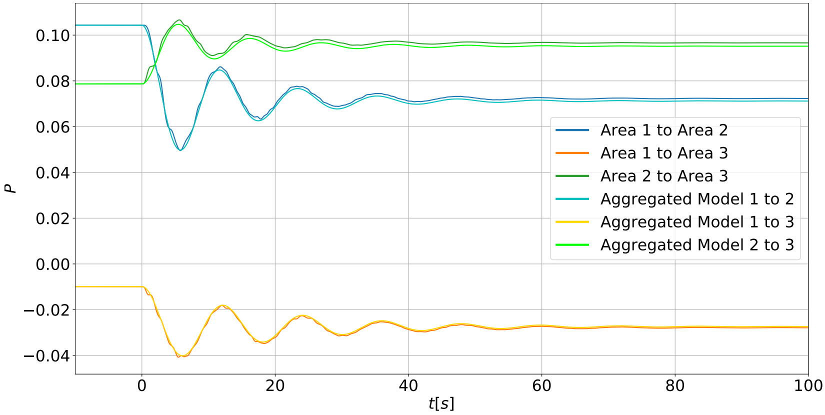
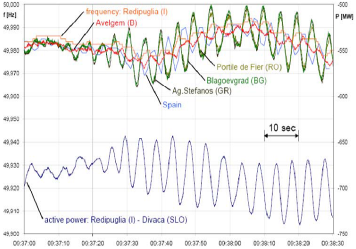
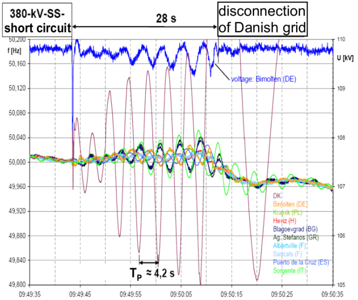


Questions



(a) East-West inter-area oscillation on 01.05.05. Al-Ali et al. 2011



(b) North-South inter-area oscillation on 29.05.07. Al-Ali et al. 2014

- apply previous methods to european grid? Reduction of system?
- how to model unstable oscillations?
- addition of “proper” droop control?