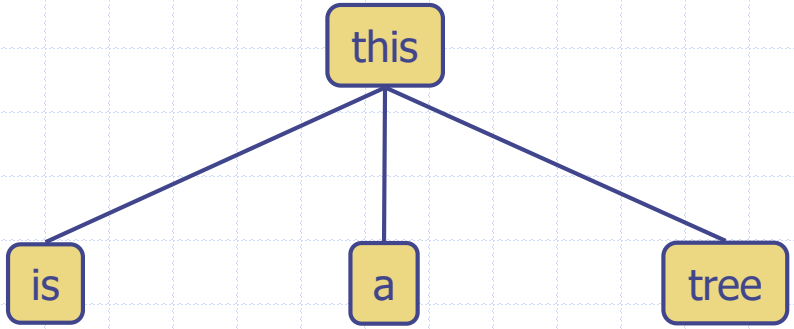
Trees



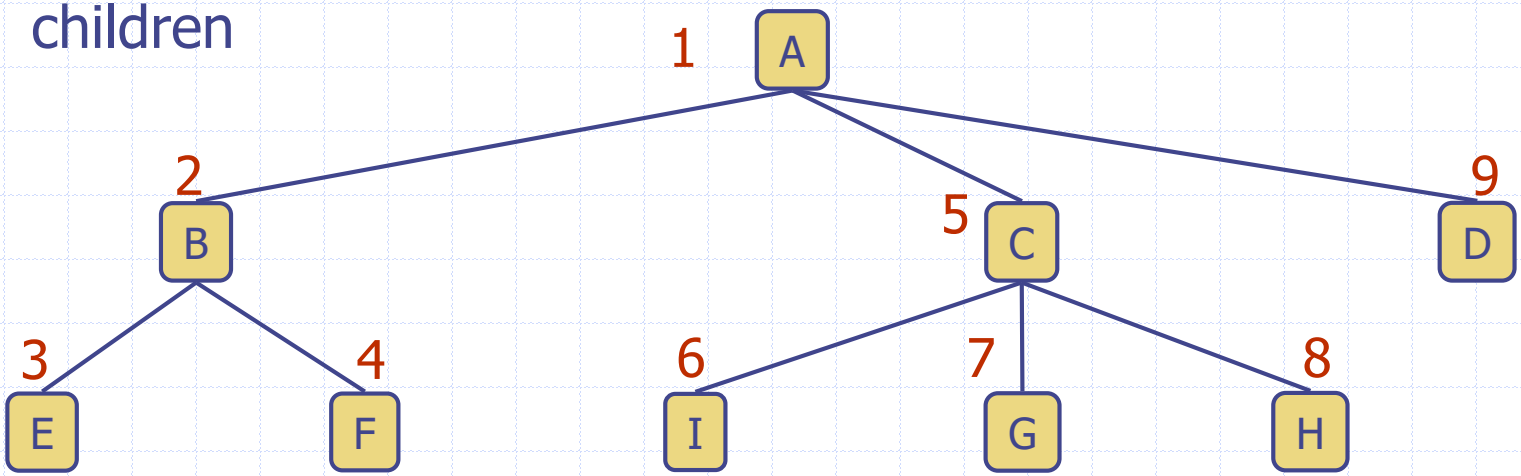
Traversal of Trees

- A traversal of a tree T is a systematic way of visiting all the nodes of T
- Traversing a tree involves visiting the root and traversing its subtrees
- There are the following traversal methods:
  - Preorder Traversal
  - Postorder Traversal
  - Inorder Traversal (of a binary tree)

Preorder Traversal

- In a preorder traversal, a node is visited before its descendants
- If a tree is ordered, then the subtrees are traversed according to the order of the children

Algorithm *preOrder*(*v*)
  visit(*v*)
  for each child *w* of *v*
    preorder (*w*)

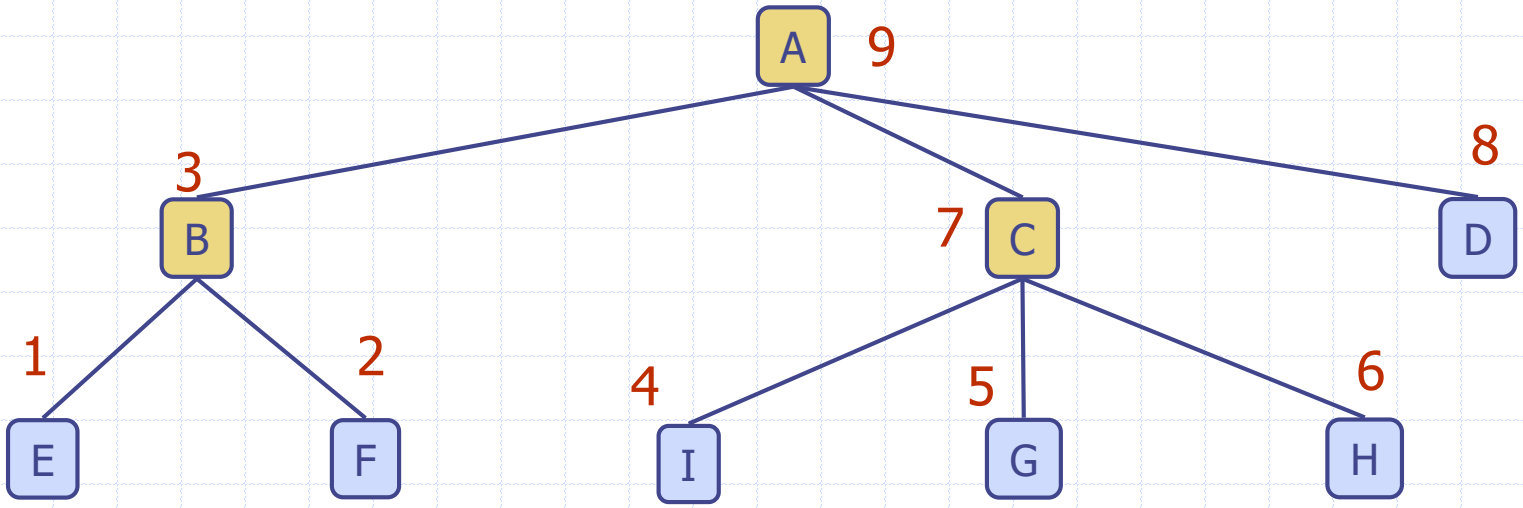


Preorder: *ABEFCIGHD*

Postorder Traversal

- In a postorder traversal, a node is visited after its descendants

Algorithm ***postOrder(v)***
  for each child ***w*** of ***v***
    postOrder **(*w*)**
  visit**(*v*)**

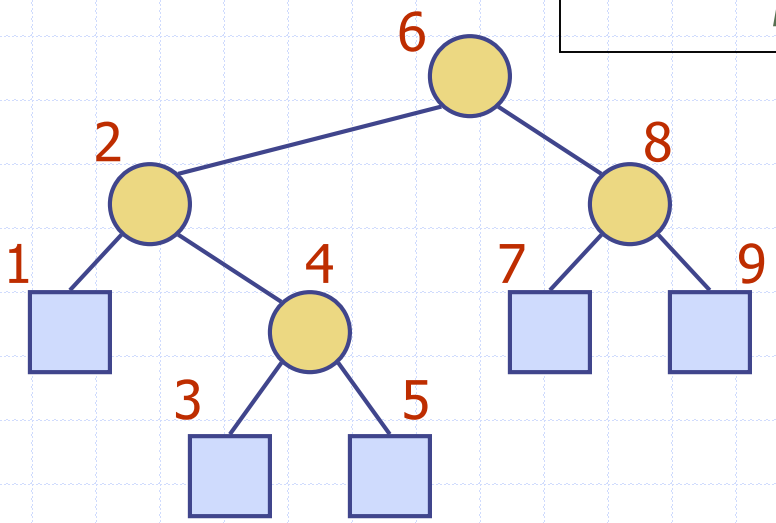


Postorder*: EFBIGHCDA*

Inorder Traversal

- In an inorder traversal a node is visited after its left subtree and before its right subtree

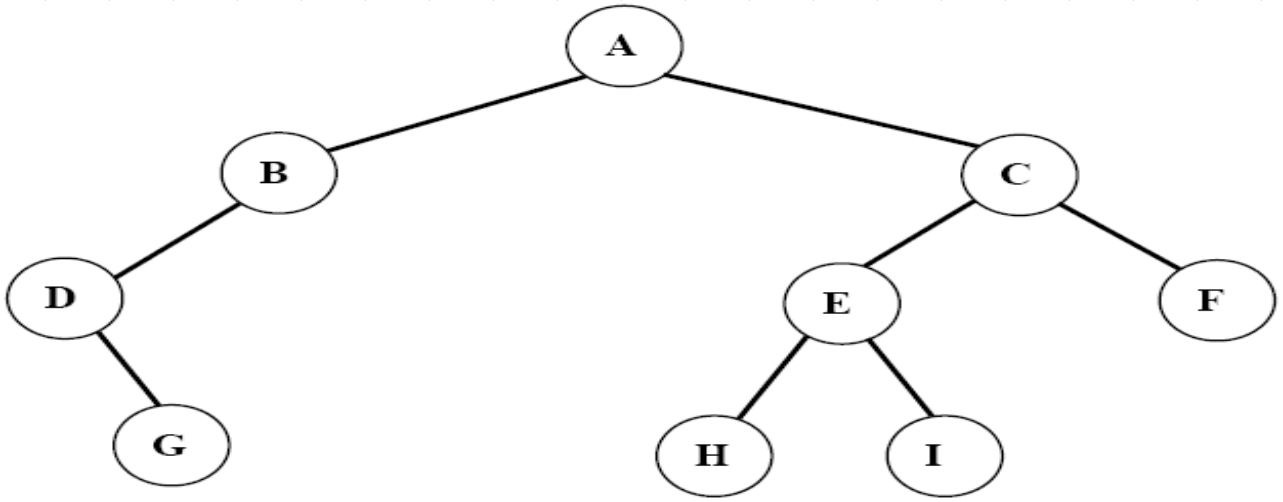
Algorithm *inOrder*(*v*)
  if *isInternal* (*v*)
    inOrder (*leftChild* (*v*))
  visit(*v*)
  if *isInternal* (*v*)
    inOrder (*rightChild* (*v*))



Inorder Traversal

Traversing a binary tree in *inorder*

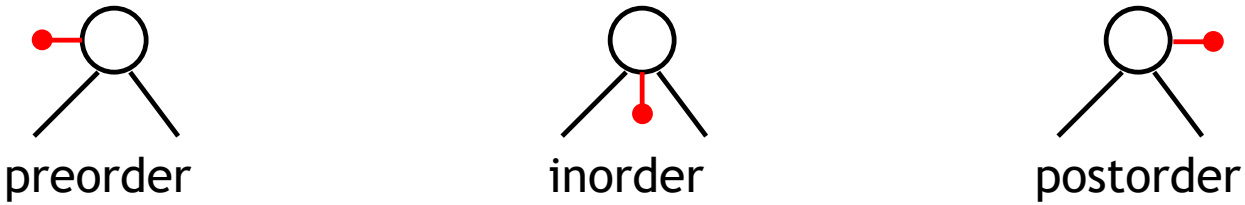
1. Traverse the ***left subtree*** in inorder.
2. Visit the ***root***.
3. Traverse the ***right subtree*** in inorder.



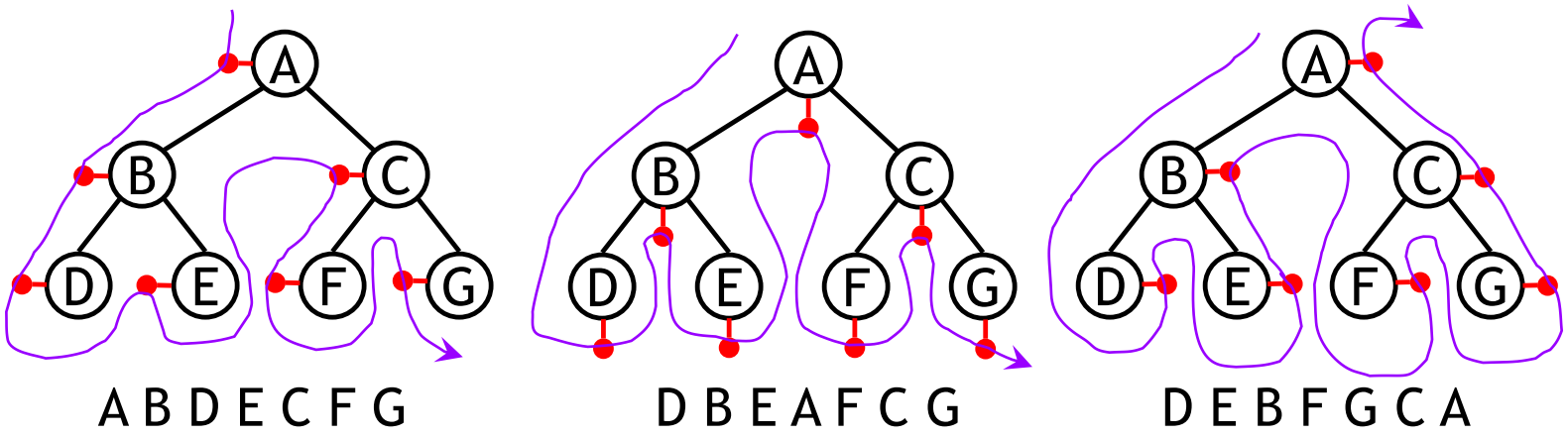
Inorder*: DGBAHEICF*

Tree traversals using "flags"

- The order in which the nodes are visited during a tree traversal can be easily determined by imagining there is a "flag" attached to each node, as follows:

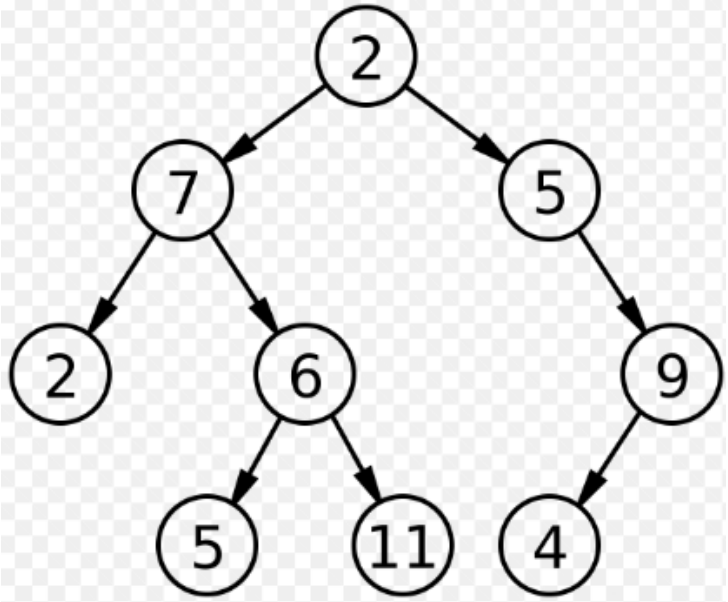


- To traverse the tree, collect the flags:



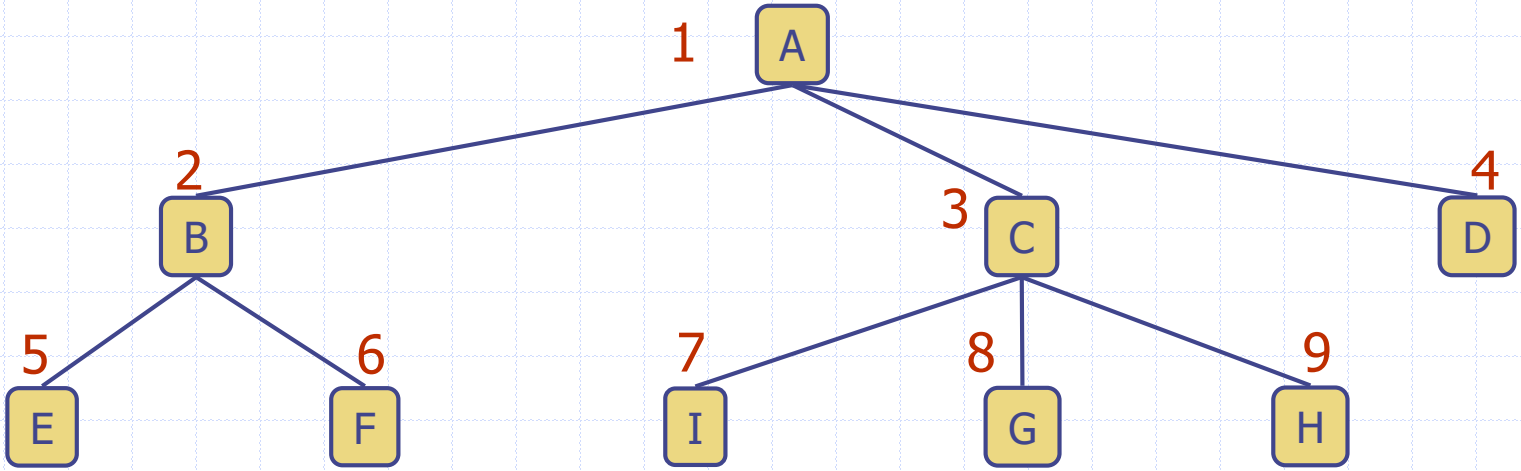
Exercise

- Perform pre-order, post-order and in-order types of traversal on the following tree



Level Order Traversal

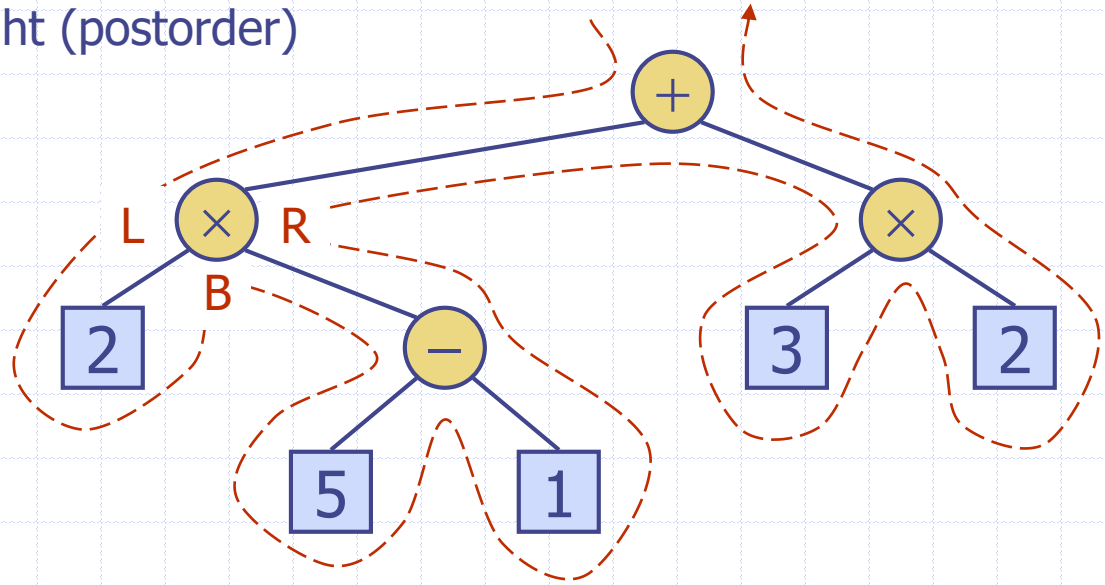
- In a level order traversal, every node on a level is visited before going to a lower level



Level order*: ABCDEFIGH*

Euler Tour Traversal

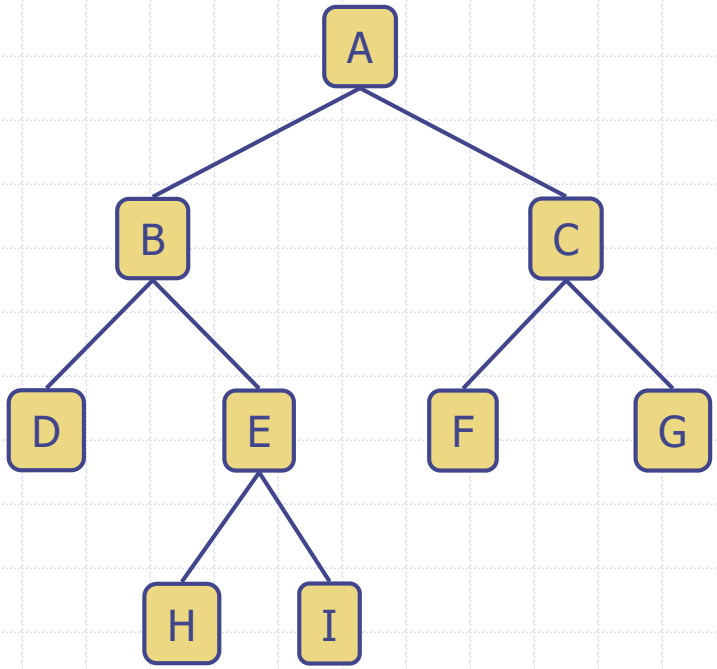
- Generic traversal of a binary tree
- Includes a special cases the preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



(Proper) Binary Tree

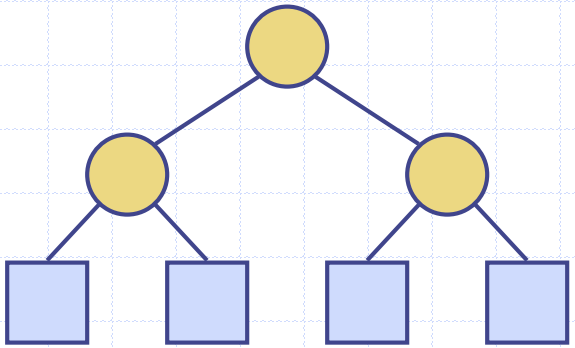
- A (proper) binary tree is a tree with the following properties:
  - Each internal node has two children
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a (proper) binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree

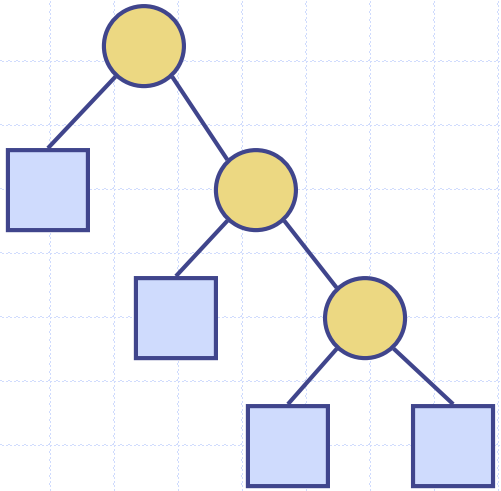
- Applications:
  - arithmetic expressions
  - decision processes
  - searching



Properties of Binary Trees

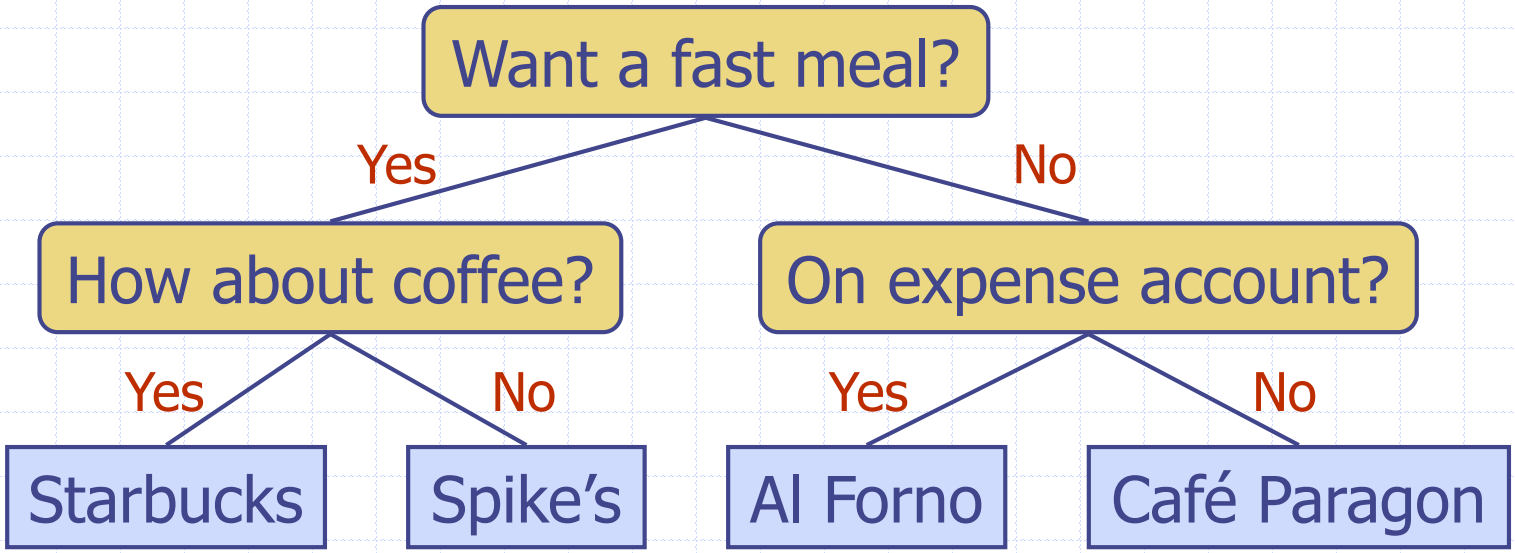
- Let T be a (proper) binary tree with n nodes, and let h denote the height of T. Then T has the following properties.
  - The number of external (leaf) nodes in T is at least h+1 and at most 2^h.
  - The number of internal nodes in T is at least h and at most 2^h-1.
  - The number of nodes in T is at least 2h+1 and at most $2^{h+1}-1$.
  - The height h of T satisfies $\log(n+1) \leq h \leq (n-1)/2$.
- Proof:





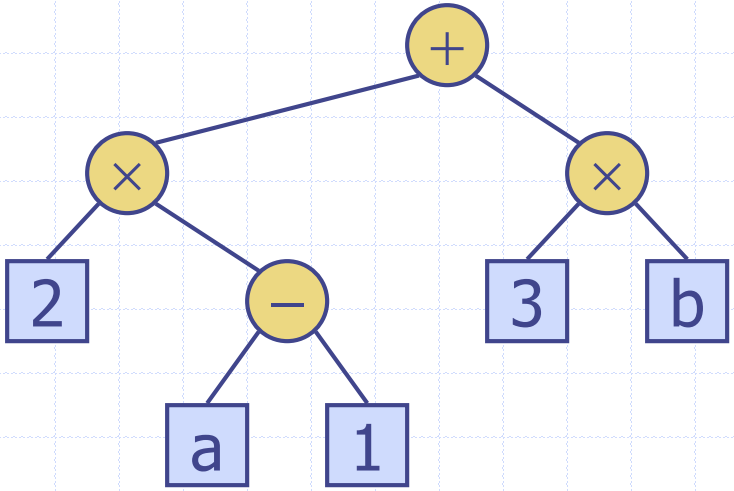
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$

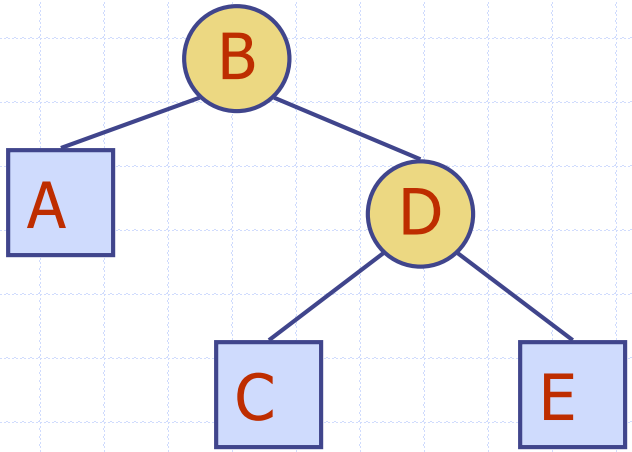


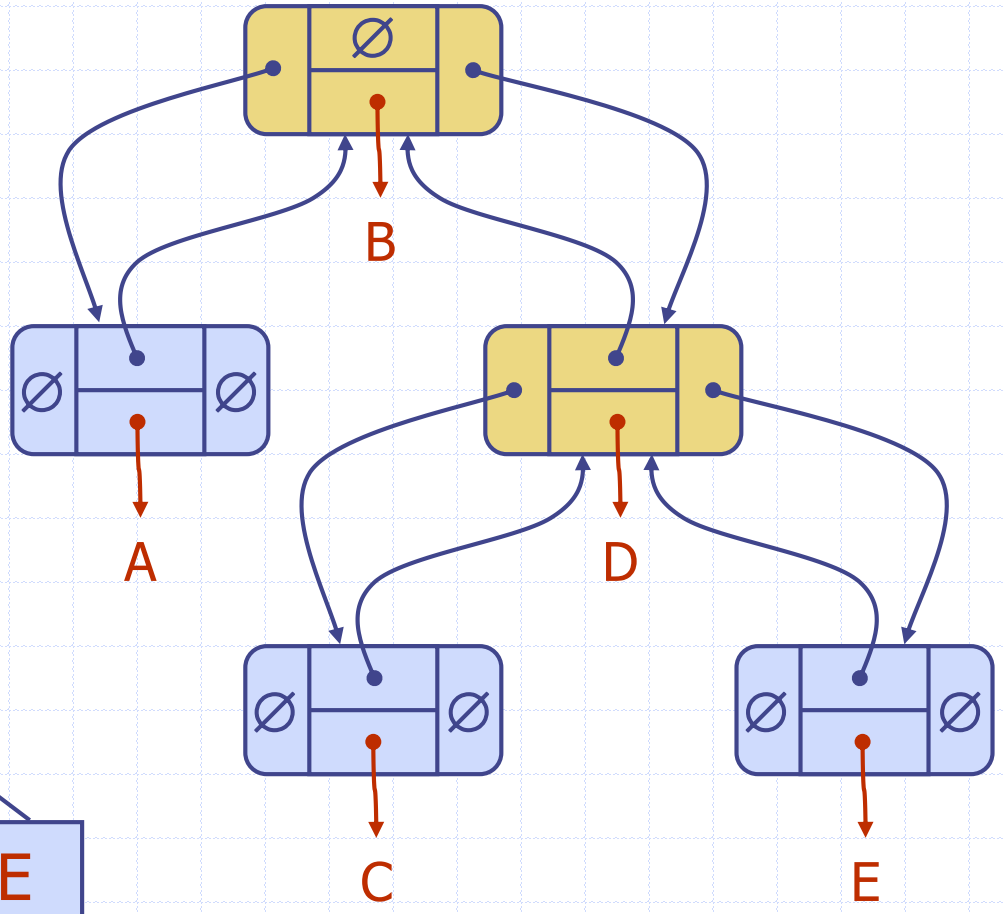
BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position leftChild(v): returns the left child of v
  - position rightChild(v) : returns the right child of v
  - position sibling(v) : returns the sibling of node v

Linked Structure for Binary Trees

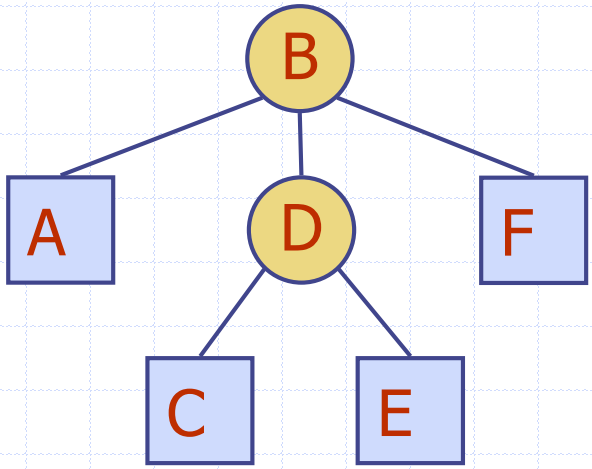
- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node

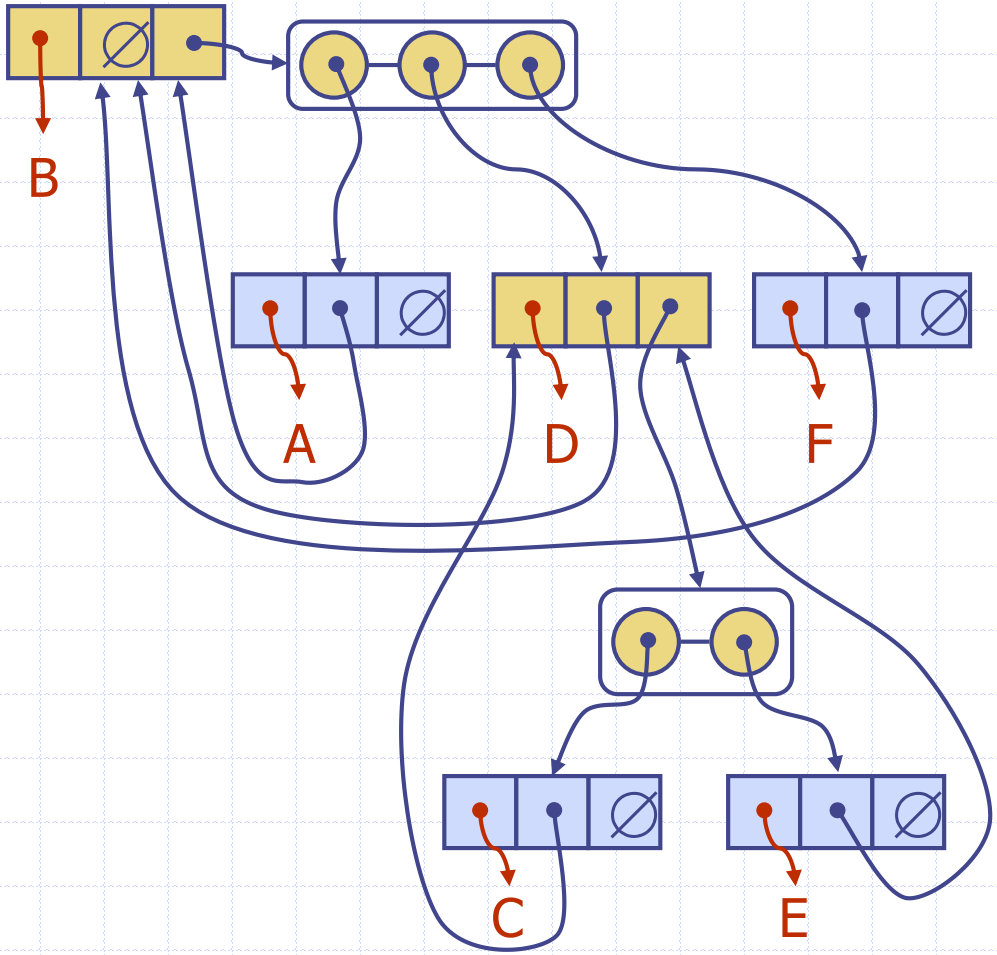




Linked Structure for General Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Children Container: Sequence of children nodes



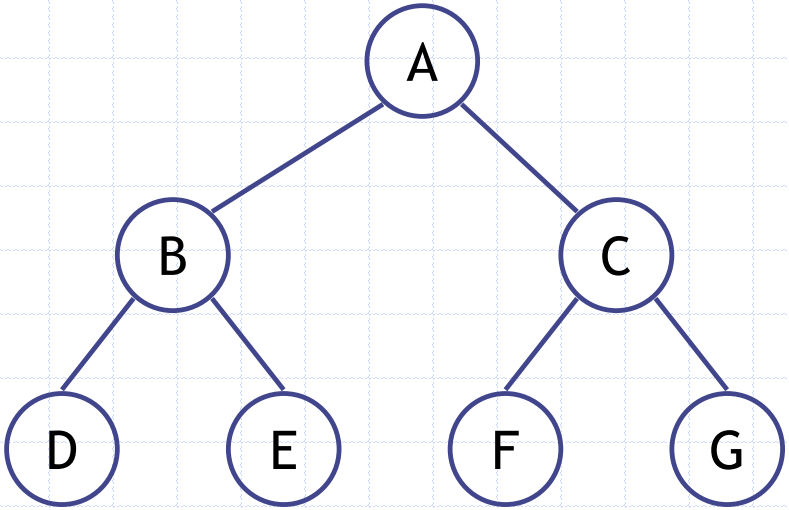


Construction of Binary Tree from order

Preorder: A B D E C F G

Inorder: D B E A F C G

Construction of Binary Tree



Exercise

Construct A Binary Tree from the orders below

Preorder: ABDEHICFG

Inorder: DBHEIAFCG

Solution