Theory of Computation

Chapter 02

Non Context- Free Languages

Introduction to the Theory of Computation, 3rd Ed, Michael Sipser
Introduction to Automata Theory Languages and Computation, 2nd, Hopcroft, Motwani, and Ullman
Last modified 28/10/2020

Oct, 2020
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Non-regular Language

Let's consider the languages:

1. *C = {w| w has an equal number of 0s and 1s}, and*
2. *D = {w| w has an equal number of occurrences of 01 and 10 as substrings}.*

Pumping Lemma for **Non-regular Language**

THEOREM 1.70

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Pumping Lemma for **Non-Context-Free Languages**

THEOREM 2.34

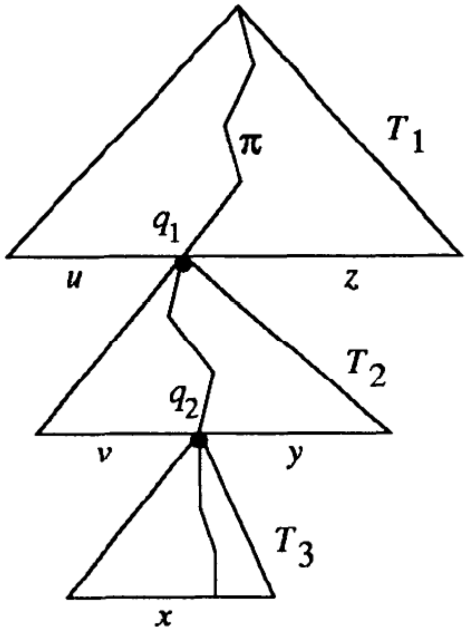
Pumping lemma for context-free languages If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in A$,

2. $|vy| > 0$, and

3. $|vxy| \leq p$.

Pumping Lemma for **Non-Context-Free Languages**



Pumping Lemma for **Non-Context-Free Languages**

- To prove that a language B is not context-free
  1. First assume that B is context-free in order to obtain a contradiction.
  2. Then use the pumping lemma to guarantee the existence of a pumping length **p** such that all strings of length **p** or greater in B can be pumped.
  3. Next, find a string **s** in B that has length **p** or greater but that cannot be pumped.
  4. Finally, demonstrate that **s** cannot be pumped by considering all ways of dividing s into u, v, x, y, and z for each such division, finding a value **i** where $uv^ixy^iz \notin B$.
  5. The existence of s contradicts the pumping lemma if B were context-free. Hence B cannot be context-free.
- Finding s takes a bit of creative thinking.

Non-Context Free Proof: Example-1

- Show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context free.

Proof:

Let, B is context free. So, it can be divided into uvxyz

Cases:

1. both v and y contain only one type of alphabet symbol.
2. either v or y contains more than one type of symbol

Non-Context Free Proof: Example-2

- Show that the language $C = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is not context free.

Proof:

Let, C is context free. So, it can be divided into uvxyz, Let $s = a^p b^p c^p$ is a string in C

Cases:

1. both v and y contain only one type of alphabet symbol.
   # a's do not appear in v or y
   # b's do not appear in v or y
   # c's do not appear in v or y
2. either v or y contains more than one type of symbol

Non-Context Free Proof: Example-3

- Show that the language D = {ww| w ∈ {0,1}*}. is not context free.

Proof:

Let, D is context free. So, it can be divided into uvxyz

String S = $\mathbf{0^p1\ 0^p1}$

$$\overbrace{\underbrace{000\cdots000}_{u}\ \underbrace{0}_{v}\ \underbrace{1}_{x}}^{0^p1}\ \overbrace{\underbrace{0}_{y}\ \underbrace{000\cdots0001}_{z}}^{0^p1}$$

- The string $\mathbf{0^p1^p0^p1^p}$ seems to capture more of the "essence" of Language D.

END