Formula
Newton's Forward Difference formula
$p = \frac{x - x_0}{h}$ $y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \cdot \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \cdot \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \cdot \Delta^4 y_0 + \ldots$

Find the value of y at x = -1

x	y
0	1
1	0
2	1
3	10

Solution: Newton's forward difference interpolation method to find solution

Newton's forward difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

The value of x at which we want to find the f(x):x=-1

h = x1 - x0 = 1 - 0 =1

p= (x - x0) / h = (-1 - 0) / 1= -1

Newton's forward difference interpolation formula is

$y(x)= y_0 + p.\Delta y_0 + [\ p(p-1) / 2!\ ] \cdot\Delta^2 y_0 + [\ p(p-1)(p-2) / 3!\ ] \cdot\Delta^3 y_0$

y(-1)= 1 + (-1)×-1 + [-1(-1-1) / 2] ×2 + [-1(-1-1)(-1-2) / 6] ×6

y(-1)= 1 + 1 + 2 - 6

y(-1)= -2

Solution of y(-1) = -2 using newton's forward interpolation method

Find solution of the following problem using Newton's Backward Difference formula

x	y
0	1
1	0
2	1
3	10

x = 4

Solution: The values for x and y is presented in the table below:

x	0	1	2	3
y	1	0	1	10

Newton's backward difference interpolation method to find solution

Newton's backward difference table is

x	**y**	∇y	$\nabla 2y$	$\nabla 3y$
0	1			
		-1		
1	0		2	
		1		**6**
2	1		**8**	
		9		
3	**10**			

The value of x at which we want to find the value of $y=f(x)$:$x=4$

$h= x1-x0 =1-0 = 1$

$p= (x-xn)/h =4-3/1= 1/1 = 1$

Newton's backward difference interpolation formula is

$y(x)=yn + p\nabla yn + [p(p+1)/2!]\cdot\nabla 2yn + [p(p+1)(p+2)/3!]\cdot\nabla 3yn$

$y(4)=10+1\times 9+[1(1+1)/2]\times 8+[1(1+1)(1+2)/6]\times 6$

$y(4)=10+9+8+6$

$y(4)=33$

Solution of newton's backward interpolation method for $y(4)=33$

Formula

Langrange's formula

$$y(x) = \frac{(x - x_1)(x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)} \times y_0 + \frac{(x - x_0)(x - x_2)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots(x_1 - x_n)} \times y_1$$
$$+ \frac{(x - x_0)(x - x_1)(x - x_3)\ldots(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\ldots(x_2 - x_n)} \times y_2 + \ldots + \frac{(x - x_0)(x - x_1)\ldots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)\ldots(x_n - x_{n-1})} \times y_n$$

Find Solution using Lagrange's formula

x	f(x)
300	2.4771
304	2.4829
305	2.4843
307	2.4871

x = 301

Solution:

The value of table for x and y

x	300	304	305	307
y	2.4771	2.4829	2.4843	2.4871

Lagrange's Interpolating Polynomial

The value of x at which we want to find f(x):x=301

Langrange's formula is

$$f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \times y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \times y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \times y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \times y_3$$

$$y(301) = \frac{(301 - 304)(301 - 305)(301 - 307)}{(300 - 304)(300 - 305)(300 - 307)} \times 2.4771 + \frac{(301 - 300)(301 - 305)(301 - 307)}{(304 - 300)(304 - 305)(304 - 307)} \times 2.4829 + \frac{(301 - 300)(301 - 304)(301 - 307)}{(305 - 300)(305 - 304)(305 - 307)} \times 2.4843 + \frac{(301 - 300)(301 - 304)(301 - 305)}{(307 - 300)(307 - 304)(307 - 305)} \times 2.4871$$

$$y(301) = \frac{(-3)(-4)(-6)}{(-4)(-5)(-7)} \times 2.4771 + \frac{(1)(-4)(-6)}{(4)(-1)(-3)} \times 2.4829 + \frac{(1)(-3)(-6)}{(5)(1)(-2)} \times 2.4843 + \frac{(1)(-3)(-4)}{(7)(3)(2)} \times 2.4871$$

$$y(301) = \frac{-72}{-140} \times 2.4771 + \frac{24}{12} \times 2.4829 + \frac{18}{-10} \times 2.4843 + \frac{12}{42} \times 2.4871$$

$$y(301) = 2.4786$$

Solution of the polynomial at point 301 is y (301) =2.4786