Linear (or Ordered) Lists ADT

instances are of the form

$$(e_0, e_1, e_2, ..., e_{n-1})$$

where e_i denotes a list element

 $n \ge 0$ is finite

list size is n

Linear Lists

$$L = (e_0, e_1, e_2, e_3, ..., e_{n-1})$$

relationships

e₀ is the zero'th (or front) element

 e_{n-1} is the last element

e_i immediately precedes e_{i+1}

Linear List Examples/Instances

```
Students in a course =
   (Jack, Jill, Abe, Henry, Mary, ..., Judy)
Exams =
   (exam1, exam2, exam3)
Days of Week = (S, M, T, W, Th, F, Sa)
Months = (Jan, Feb, Mar, Apr, ..., Nov, Dec)
```

Linear List Operations—size()

determine list size

$$L = (a,b,c,d,e)$$

$$size = 5$$

Linear List Operations—get(theIndex)

get element with given index

$$L = (a,b,c,d,e)$$

$$get(0) = a$$

$$get(2) = c$$

$$get(4) = e$$

$$get(-1) = error$$

$$get(9) = error$$

Linear List Operations—indexOf(theElement)

determine the index of an element

$$L = (a,b,d,b,a)$$

$$indexOf(d) = 2$$

$$indexOf(a) = 0$$

$$indexOf(z) = -1$$

Linear List Operations—remove(theIndex)

remove and return element with given index

$$L = (a,b,c,d,e,f,g)$$

remove(2) returns c

and L becomes (a,b,d,e,f,g)

index of d,e,f, and g decrease by 1

Linear List Operations—remove(theIndex)

remove and return element with given index

$$L = (a,b,c,d,e,f,g)$$

Linear List Operations—add(theIndex, theElement)

add an element so that the new element has a specified index

$$L = (a,b,c,d,e,f,g)$$

add(0,h) => L = (h,a,b,c,d,e,f,g)index of a,b,c,d,e,f, and g increase by 1

Linear List Operations—add(theIndex, theElement)

$$L = (a,b,c,d,e,f,g)$$

add(2,h) => L = (a,b,h,c,d,e,f,g)index of c,d,e,f, and g increase by Iadd(10,h) => erroradd(-6,h) => error

Data Structure Specification

- ☐ Language independent
 - ➤ Abstract Data Type

Linear List Abstract Data Type

```
AbstractDataType LinearList
 instances
   ordered finite collections of zero or more elements
 operations
   isEmpty(): return true iff the list is empty, false otherwise
   size(): return the list size (i.e., number of elements in the list)
   get(index): return the indexth element of the list
   index O f(x): return the index of the first occurrence of x in
          the list, return -1 if x is not in the list
   remove(index): remove and return the indexth element,
       elements with higher index have their index reduced by 1
   add(theIndex, x): insert x as the indexth element, elements
       with the Index >= index have their index increased by 1
    output(): output the list elements from left to right
```

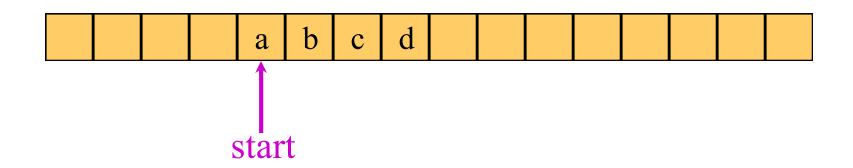
Implementation of Linear List

By Arrays
By Linked Lists

Array Implementation of List

1D Array Representation In C

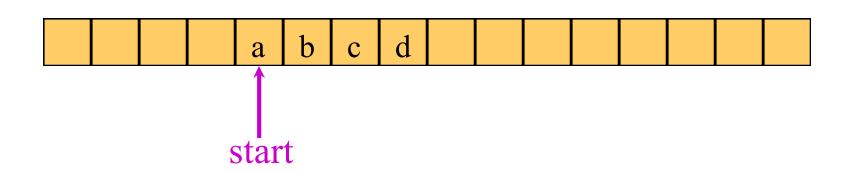
Memory



- 1-dimensional array x = [a, b, c, d]
- map into contiguous memory locations

Space Overhead

Memory

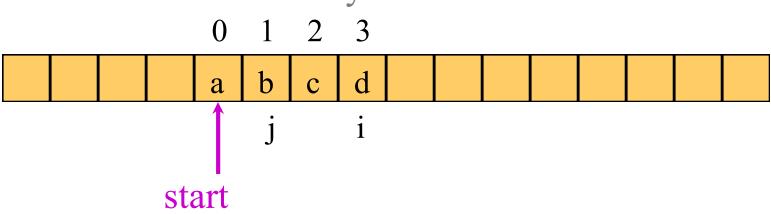


space overhead = 4 bytes for start

(excludes space needed for the elements of x)

Space Overhead

Memory



- location(x[i]) = start + w.i
 - w = size of each element

• location(x[i]) = location(x[j]) + w.(i-j)

2D Arrays

The elements of a 2-dimensional array a declared as:

```
int a[3][4];
```

may be shown as a table

```
      a[0][0]
      a[0][1]
      a[0][2]
      a[0][3]

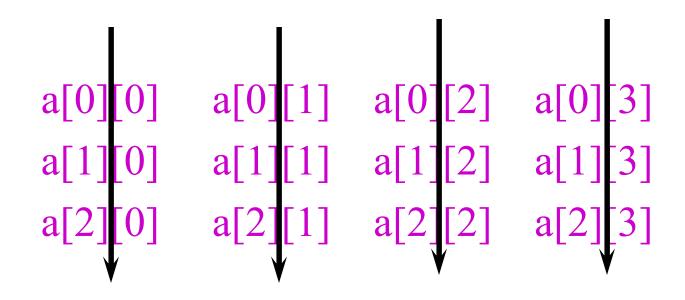
      a[1][0]
      a[1][1]
      a[1][2]
      a[1][3]

      a[2][0]
      a[2][1]
      a[2][2]
      a[2][3]
```

Rows Of A 2D Array

$$a[0][0]$$
 $a[0][1]$ $a[0][2]$ $a[0][3]$ row 0
 $a[1][0]$ $a[1][1]$ $a[1][2]$ $a[1][3]$ row 1
 $a[2][0]$ $a[2][1]$ $a[2][2]$ $a[2][3]$ row 2

Columns Of A 2D Array



column 0 column 1 column 2 column 3

2D Array Representation

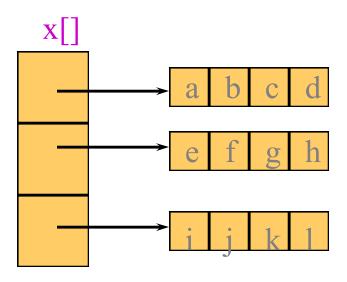
2-dimensional array x

view 2D array as a 1D array of rows

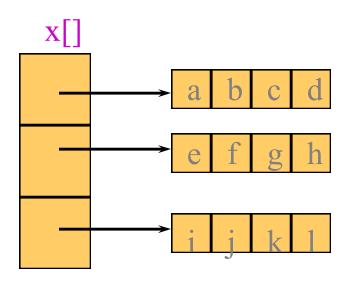
$$x = [row0, row1, row 2]$$

 $row 0 = [a,b,c,d]$
 $row 1 = [e, f, g, h]$
 $row 2 = [i, j, k, 1]$
and store as 4 1D arrays

2D Array Representation In C



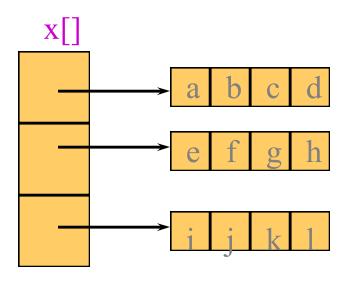
Space Overhead



space overhead = space required by the array x[]

- = 3 * 4 bytes
- = 12 bytes
- = number of rows x 4 bytes

Array Representation In C



- This representation is called the **array-of-arrays** representation.
- Requires contiguous memory of size 3, 4, 4, and 4 for the 4 1D arrays.
- 1 memory block of size number of rows and number of rows blocks of size number of columns

Row-Major Mapping

• Example 3 x 4 array:

```
abcdefghijkl
```

- Convert into 1D array by collecting elements by rows.
- Within a row elements are collected from left to right.
- Rows are collected from top to bottom.
- We get $\{a, b, c, d, e, f, g, h, i, j, k, 1\}$

row 0	row 1	row 2	• • •	row i		
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Locating Element x[i][j]

row 0 row 1 row 2 ... row i

- assume x has r rows and c columns
- each row has c elements
- i rows to the left of row i
- so ic elements to the left of x[i][0]
- so x[i][j] is mapped to position
 ic + j of the 1D array

Column-Major Mapping

```
abcdefghijkl
```

- Convert into 1D array by collecting elements by columns.
- Within a column elements are collected from top to bottom.
- Columns are collected from left to right.
- We get $\{a, e, i, b, f, j, c, g, k, d, h, l\}$

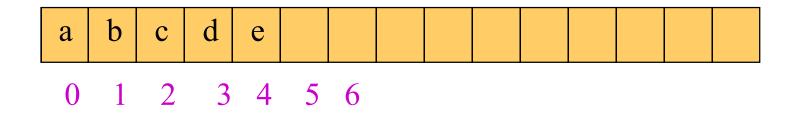
Locating Element x[i][j]

col 0 col 1 col 2 ... col i

- assume x has r rows and c columns
- each column has r elements
- j columns to the left of column j
- so jr elements to the left of x[0][j]
- so x[i][j] is mapped to position jr + i of the 1D array

Linear List Array Representation

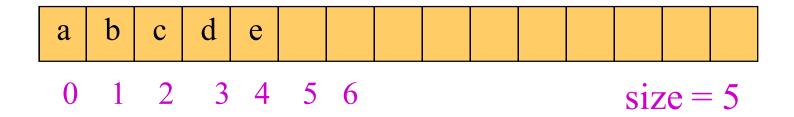
use a one-dimensional array element[]



$$L = (a, b, c, d, e)$$

Store element i of list in element[i].

Representation

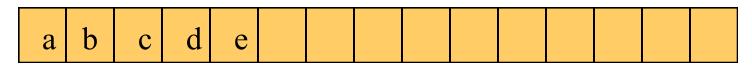


put element i of list in element[i]

use a variable size to record current number of elements

Add An Element

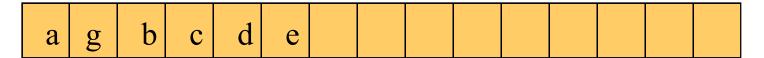




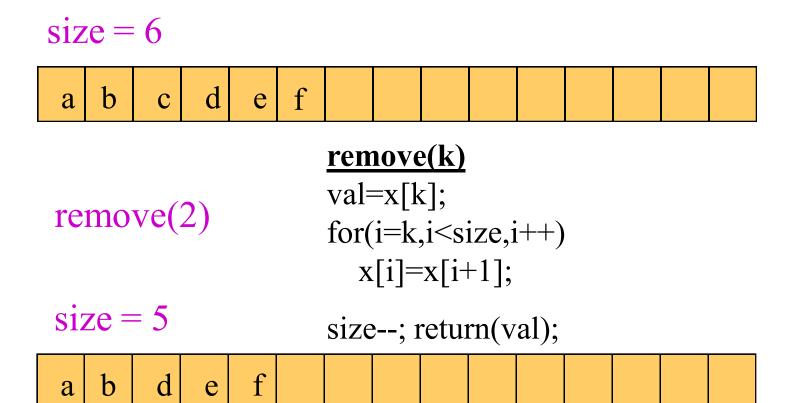
add(k,val)

add(1,g) for(i=size-1,i>=k,i--)
$$x[i+1]=x[i];$$

$$size = 6$$
 $x[k]=val; size++;$



Remove An Element



Disadvantage

Need to estimate the maximum demand (r and c) **Solution:** Dynamic Memory Allocation

Need **contiguous** memory of size rc.

Solution: Linked Lists