Military Institute of Science and Technology

Department of Computer Science & Engineering
Subject: Numerical Analysis Sessional (**CSE – 214)**

Exp. No.-4

Name of the Exp: Interpolating a table of data by Newton's forward and backward difference interpolation formula and Lagrange's Interpolation formula

Introduction:

In the field of numerical analysis, **interpolation is a type of estimation**, a method of constructing new data points within the range of a discrete set of known data points.
In engineering and science, one often has a number of data points, which is obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required **to interpolate, i.e., estimate** the value of that function for an intermediate value of the independent variable.

Let a set of tabular values $(x_0, y_0), (x_1, y_1)........(x_n, y_n)$ satisfying the relation $y = f(x)$ **,** where the explicit nature of f(x) is not known, it requires to find a simpler function say $\varphi(x)$ such that $f(x)$ and $\varphi(x)$ agree at the set of tabulated points. Such a process is called Interpolation. If $\varphi(x)$ is a polynomial then the process is called the interpolating polynomial.

Objective of the Experiment:

1. To get introduced with different interpolation formulae.
2. To write a program to find out the value of y at a point x from a given tabular points by Newton's Forward and backward difference Interpolation formula for equally spaced points
3. To write a program in order to find out the value of y at a point x from a given tabular points by Lagrange's interpolation formula for equally or not equally spaced points.

Theory:

Interpolation with evenly spaced data points by Newton's forward and backward difference formulae:

For the points at the beginning of Tabular data: Let there are n+1 number of data points, $\mathbf{(x_0\,,\,y_0),\,(x_1\,,y_1)\ldots\ldots\ldots(x_n\,,y_n)}$ given. When values of x are at equal distance and the value of x, for which the value of y is to be determined, is at the beginning of the given data table then use ***Newton's forward difference interpolation Formula*** to find the value of y, which is

$$y_n(x) = y_o + p\Delta y_o + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots + \frac{p(p-1)(p-2)\ldots\ldots\ldots(p-n+1)}{(n)!}\Delta^n y_0$$

… … …(1)

Where, $x = x_0 + ph$ **,** h = difference between two successive values of x.

Table-1: Forward difference Table

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

Where, $\Delta y_0 = y_1 - y_0; \Delta y_1 = y_2 - y_1; \Delta y_2 = y_2 - y_3; \Delta^2 y_0 = \Delta y_1 - \Delta y_0; \Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$ and so on **… … … (2)**

For the points at the ending of Tabular data: Let there are n+1 number of data points, (x0, y0), (x1,y1)…(xn,yn) are given, when values of x are at equal distance and the value of x, for which the value of y is to be determined, is at the end of the given data table then use ***Newton's backward difference interpolationFormula*** in order to find out the polynomial y which is

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots\dots\dots\dots + \frac{p(p+1)(p+2)\dots\dots\dots(p+n-1)}{(n)!}\nabla^n y_n$$

… … … (3)

Where, $x = x_n + ph$ **,** h=difference between two successive values of x.

Values of ∇y_n , $\nabla^2 y_n$, $\nabla^3 y_n$ …. $\nabla^n y_n$ can be found from the following backward difference **Table 2**

Table-2: Backward difference Table

x	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
x_1	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$
x_3	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_4	y_4		$\nabla^2 y_5$			
		∇y_5				
x_5	y_5					

Where,

$\nabla y_1 = y_1 - y_0$, $\nabla y_2 = y_2 - y_1$;

$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$;

$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

and so on … **… … (4)**

Thus, from the above table and from (2) and (4), it is clear that same number occurs in the same position whether it is forward or backward difference table.

Interpolation with unevenly spaced points using Lagrange's formula

Newton's interpolation Formulae is not applicable where values of x are unequally spaced.

In that case Lagrange's interpolation formula is applicable, which is,

$$y(x) = \frac{(x - x_1)(x - x_2)\dots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\dots(x_0 - x_n)} \times y_0 + \frac{(x - x_0)(x - x_2)\dots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\dots(x_1 - x_n)} \times y_1$$
$$+ \frac{(x - x_0)(x - x_1)(x - x_3)\dots(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\dots(x_2 - x_n)} \times y_2 + \dots + \frac{(x - x_0)(x - x_1)\dots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)\dots(x_n - x_{n-1})} \times y_n$$

Practice Problems:

1. The deflection d measured at various distances x from one end of a cantilever is given by:

x	0.0	0.2	0.4	0.6	0.8	1.0
d	0.0000	0.0347	0.1173	0.2160	0.2987	0.3333

Find value of d when x is 0.567

2. Write a program to find out y (10) and y (1) for the following tabular data

x	x_0=3	x_1=4	x_2=5	x_3=6	x_4=7	x_5=8	x_6=9
y	y_0=2.7	y_1=6.4	y_2=12.5	y_3=21.6	y_4=34.3	y_5=51.2	y_6=72.9

3. Write a program to find out y (1.5) for the following tabular data

x	x_0=0	x_1=1	x_2=2	x_3=4
y	y_0=2	y_1=5	y_2=9	y_3=12

4. Write a program to find out y (4.5) for the following tabular data:

x	x_0=1	x_1=2	x_2=3	x_3=4	x_4=5
y	y_0=2	y_1=5	y_2=9	y_3=12	y_4=20

THE NEW MATLAB FUNCTIONS USED IN THIS PROGRAM

1. factorial(N)

Calculates the factorial of N

ie. factorial(N)=1*2*3*4………N

2. length (C)

If C is a vector, n = length(C); returns the size of the longest dimension of C. which is the same as its length. Examples:

C= [16479]

n = length(C)

3. diff(Y,i)

If Y is a vector, diff(Y) calculates differences between adjacent elements of Y. Then diff(Y) returns a vector, one element shorter than Y, of differences between adjacent elements: [Y(2)-Y(1) Y(3)-Y(2) ... Y(n)-Y(n-1)]

diff(Y,i) applies diff recursively i times, resulting in the ith difference. Thus, diff(Y,2) is the same as diff(diff(Y)).

Reference Book:

1. Introductory Methods of Numerical Analysis: by S.S. Sastry.
2. Numerical Methods -Gerald/Wheetley