Military Institute of Science and Technology

Department of Computer Science & Engineering

Subject: Numerical Analysis Sessional (**CSE – 214)**

Exp. No.-5 **Date**-9th Aug, 2019

Name of the Exp.: Numerical Integration Formulae (Trapezoidal and Simpson's 1/3 rule) for Equidistant x coordinates.

Introduction:

The general problem of numerical integration may be stated as follows. Given a set of data points (x_o,y_o), (x_1,y_1), …….(x_n,y_n) of a function y=f(x), where f(x), is not known explicitly, it is required to compute the value of the definite integral

$$I = \int_a^b y.dx = \int_a^b f(x).dx$$

Replace f(x) by an interpolating polynomial φ(x) and then integrating on it, an approximate value of definite integration can be obtained. Thus, different integration formulae can be obtained depending upon the type of interpolation formula used. In this experiment, trapezoidal Rule and Simpson's 1/3 rule for numerical integration have to be used which are derived from Newton's forward difference Interpolation formula.

Objectives:

1. To integrate a function y=f(x), when it is known.
2. To integrate y=f(x), from a set of data points, when y=f(x) is not known.
3. To compare the two different rules for Numerical Integration: Trapezoidal and Simpson's 1/3 rule.
4. To observe the increase of the accuracy of the result with the decrease of the width of a strip taken account for integration.

Theory:

If a set of n+1 data points (x_o, y_o), (x_1, y_1),………….. (x_n, y_n) of a function y=f(x) is given where the function y=f(x) is not explicitly known, then the definite Integral

$$I = \int_a^b y.dx = \int_a^b f(x).dx ....(1)$$

can be obtained by approximating y by Newton's forward difference formula. Thus we obtain

$$I = \int_a^b y.dx = \int_{xo}^{x_n} [y_o + p\Delta y_o + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + ...].dx$$

………………….(2)

Here the interval [a,b] has been divided into n equal subintervals or strip such that $a = x_o < x_1 < x_2 < .... < x_n = b$. Clearly $x_n = x_o + nh$

Since, x= x_o+ph, dx=h.dp, hence the above integral becomes

$$I = h\int_0^n [y_o + p\Delta y_o + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + ...].dp$$

……………….(3)

Which gives on simplification

$$I = \int_{X_O}^{X_n} y.dx = nh[y_o + \frac{n}{2}\Delta y_o + \frac{n(2n-3)}{12}\Delta^2 y_o + \frac{n(n-2)^2}{24}\Delta^3 y_o + ...]$$

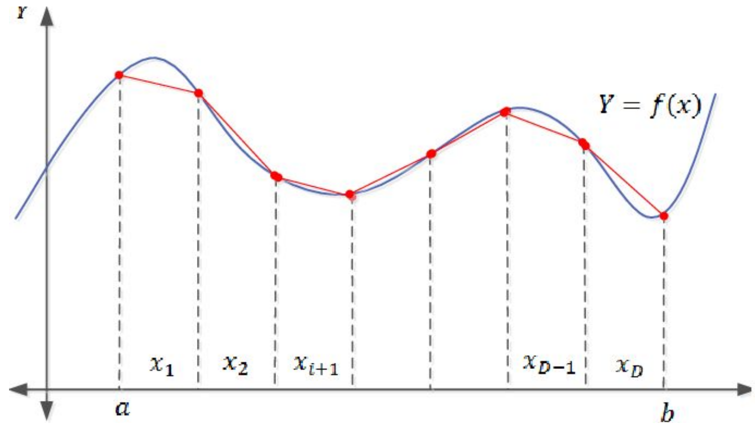
………………..(4)

The equation (4) is called **General Quadrature Formula for Numerical Integration of y=f(x)** in a range from $x=x_o$ to $x=x_n$.

*Since this general formula has been derived by integrating **Newton's Forward Difference Interpolation Formula**, so, the values of **x coordinates will be at equidistant**.*
From this general formula (4), we can obtain different integration formulae by putting n=1, 2, 3, 4, …., etc.

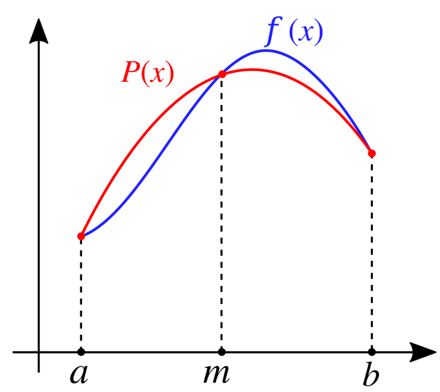
Trapezoidal Rule:

By putting **n=1**, in the General Quadrature formula (4), we get the **Trapezoidal Rule for Numerical Integration**, which is,



$$\int_{X_o}^{X_n} y.dx = (h/2)*[y_o + 2*(y_1 + y_2 + y_3 + ...y_{n-1}) + y_n]$$

where, h is the difference between two successive values of x.



Simpson's 1/3 Rule

By putting **n=2,** in the General Quadrature formula (4) , we get the **Simpson's 1/3 Rule for Numerical Integration,** which is,

$$\int_{X_o}^{X_n} y.dx = (h/3)*[y_o + 4*(y_1 + y_3 + y_5 + ...y_{n-1}) + 2*(y_2 + y_4 + y_6 + ...y_{n-2}) + y_n]$$

where, h is the difference between two successive values of x.
Note:It should be noted that this rule requires the division of the whole range into an even number of subintervals of width h.

Problems:

1. Write programs to evaluate the numerical value of I by both Trapezoidal and Simpson's 1/3 Rule and compare the result.

   a) $I = \int_0^1 e^{-x} dx$ for h = 0.1

   b) $I = \int_{X_o}^{X_n} y.dx = \int_0^{\frac{\pi}{2}} \sin x.dx$ for h=π/4, π/8.

   c) $I = \int_{X_o}^{X_n} y.dx = \int_0^1 \frac{1}{1+x}.dx$ for h=0.5, 0.25, 0.125.

   Also show that the accuracy of the result increases with the decrease in the value of h.

2. For the following data points obtain an integration of a function y=f(x) using Trapezoidal and Simpson's 1/3 Rule and compare the results.

x	x0=1	x1=2	x2=3	x3=4	x4=5	x5=6	x6=7
y	y0=2	y1=5	y2=10	y3=17	y4=26	y5=37	y6=50