Theory of Computation

Chapter 03
The Church-Turing Thesis

Introduction to the Theory of Computation, 3rd Ed, Michael Sipser
Introduction to Automata Theory Languages and Computation, 2nd, Hopcroft, Motwani, and Ullman
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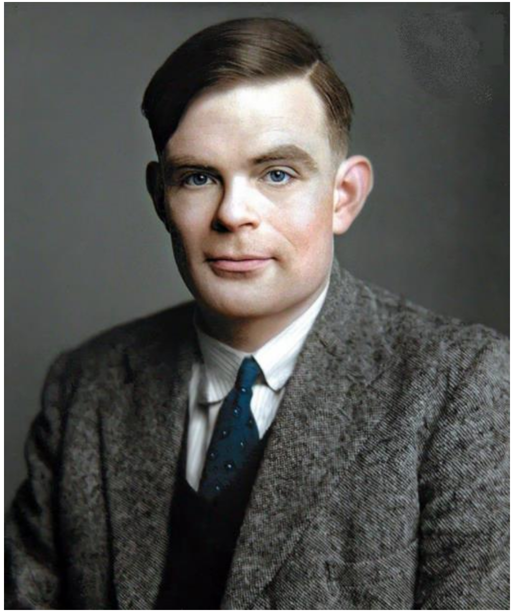
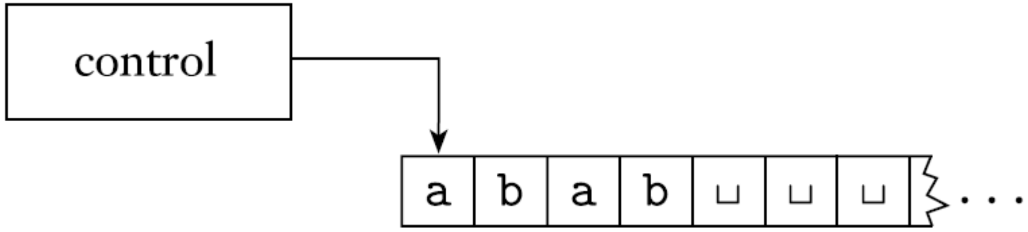
Turing Machines (TM)

Alan Turing says - "We can only see a short distance ahead, but we can see plenty there that needs to be done."



Alan Mathison Turing
Born: 23 June 1912
Maida Vale, London, England

Schematic of a Turing Machines (TM)



Properties of TM

- A Turing machine can both write on the tape and read from it.
- The read–write head can move both to the left and to the right.
- The tape is infinite.
- The special states for rejecting and accepting take effect immediately.

Example Turing Machine, M1

Language, $B = \{ w\#w \mid w \in \{0,1\}^* \}$

M1 = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, reject . Cross off symbols as they are checked to keep track of which symbols correspond.
2. When all symbols to the left of the # have been crossed off, check for any remaining symbols to the right of the #. If any symbols remain, reject; otherwise, accept ."

Example Turing Machine, M1

Turing machine M1 computing on input 011000#011000

```
0 1 1 0 0 0 # 0 1 1 0 0 0 ⊔ ...
x 1 1 0 0 0 # 0 1 1 0 0 0 ⊔ ...
x 1 1 0 0 0 # x 1 1 0 0 0 ⊔ ...
x 1 1 0 0 0 # x 1 1 0 0 0 ⊔ ...
x x 1 0 0 0 # x 1 1 0 0 0 ⊔ ...
x x x x x x # x x x x x x ⊔ ...
                          accept
```

Formal Definition of a Turing Machine (TM)

DEFINITION 3.3

A ***Turing machine*** is a 7-tuple, $(Q, \Sigma, \Gamma, \delta, q_0, q_{\mathrm{accept}}, q_{\mathrm{reject}})$, where Q, Σ, Γ are all finite sets and

1. Q is the set of states,
2. Σ is the input alphabet not containing the ***blank symbol*** $\sqcup$,
3. Γ is the tape alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$,
4. $\delta \colon Q \times \Gamma \longrightarrow Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}\}$ is the transition function,
5. $q_0 \in Q$ is the start state,
6. $q_{\mathrm{accept}} \in Q$ is the accept state, and
7. $q_{\mathrm{reject}} \in Q$ is the reject state, where $q_{\mathrm{reject}} \neq q_{\mathrm{accept}}$.

Configuration of a Turing Machine (TM)

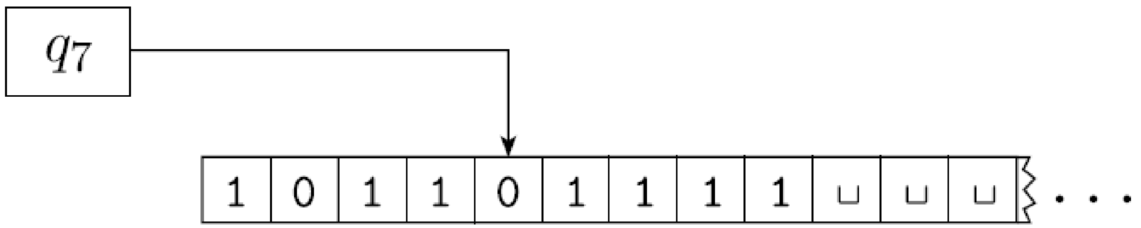


FIGURE 3.4
A Turing machine with configuration $1011q_701111$

✓ C1 **yields** C2 – go from C1 to C2 in a single step

Configuration of a Turing Machine (TM)

- start configuration
- accepting configuration
- rejecting configuration
- halting configurations

Configuration of a Turing Machine (TM)

- A Turing machine M accepts input w if a sequence of configurations $C_1, C_2, \ldots, C_k$ exists, where
  1. C_1 is the start configuration of M on input w,
  2. each C_i yields C_{i+1}, and
  3. C_k is an accepting configuration.

Turing-recognizable

DEFINITION 3.5

Call a language ***Turing-recognizable*** if some Turing machine recognizes it.[1]

Turing-decidable

DEFINITION 3.6

Call a language ***Turing-decidable*** or simply ***decidable*** if some Turing machine decides it.[2]

Higher Level Descriptions - M2

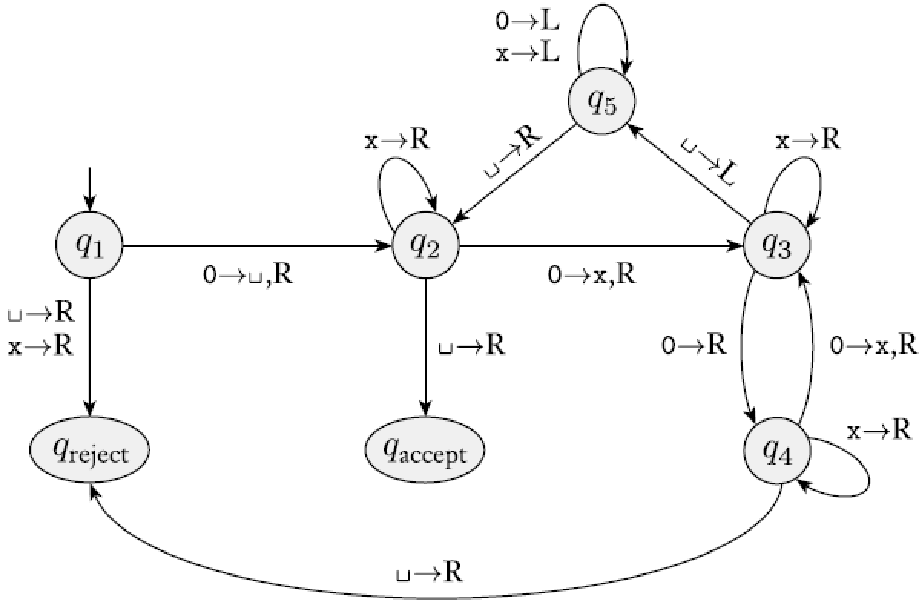
- A = { 0^{2^n} | n ≥ 0}

M_2 = "On input string w:

1. Sweep left to right across the tape, crossing off every other 0.
2. If in stage 1 the tape contained a single 0, *accept*.
3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, *reject*.
4. Return the head to the left-hand end of the tape.
5. Go to stage 1."

Formal Description - M2

- $A = \{ 0^{2^n} \mid n \geq 0 \}$



Higher Level Descriptions - M1

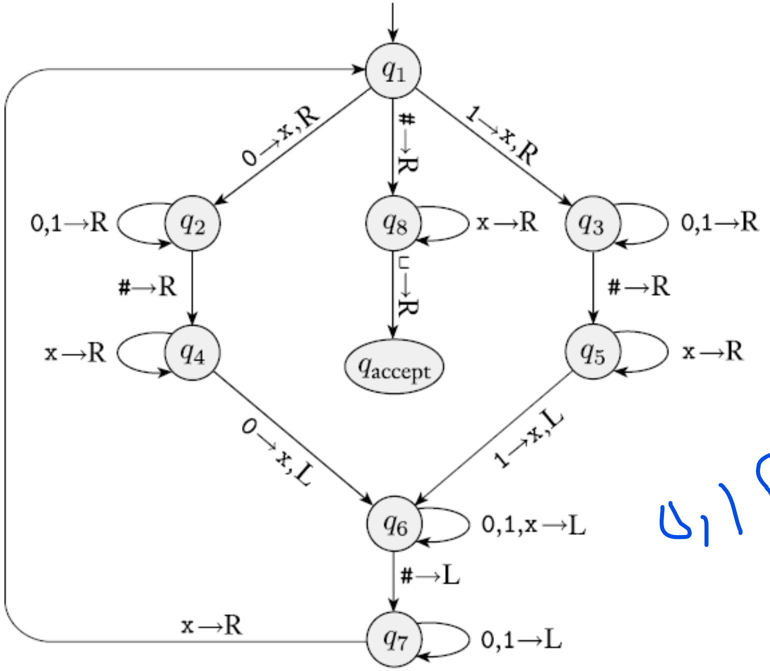
- $B = \{ w\#w \mid w \in \{0,1\}^* \}$

M1 = "On input string w:

1. Zig-zag across the tape to corresponding positions on either side of the # symbol to check whether these positions contain the same symbol. If they do not, or if no # is found, reject . Cross off symbols as they are checked to keep track of which symbols correspond.
2. When all symbols to the left of the # have been crossed off, check for any remaining symbols to the right of the #. If any symbols remain, reject; otherwise, accept ."

Formal Description - M1

- B = { w#w| w ∈ {0,1}∗ }



0,1??

Higher Level Descriptions – M3

- $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$.

M_3 = "On input string w:

1. Scan the input from left to right to determine whether it is a member of $a^+b^+c^+$ and *reject* if it isn't.
2. Return the head to the left-hand end of the tape.
3. Cross off an a and scan to the right until a b occurs. Shuttle between the b's and the c's, crossing off one of each until all b's are gone. If all c's have been crossed off and some b's remain, *reject*.
4. Restore the crossed off b's and repeat stage 3 if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, *accept*; otherwise, *reject*."

Higher Level Descriptions – M3

- E = $\{\#x_1\#x_2\# \cdots \#x_\ell \mid$ each $x_i \in \{0,1\}*$ and $x_i \neq x_j$ for each $i \neq j\}$.

M_4 = "On input w:

1. Place a mark on top of the leftmost tape symbol. If that symbol was a blank, *accept*. If that symbol was a #, continue with the next stage. Otherwise, *reject*.
2. Scan right to the next # and place a second mark on top of it. If no # is encountered before a blank symbol, only x_1 was present, so *accept*.

Higher Level Descriptions – M3

- $E = \{\#x_1\#x_2\# \cdots \#x_\ell \mid$ each $x_i \in \{0,1\}*$ and $x_i \neq x_j$ for each $i \neq j\}$.

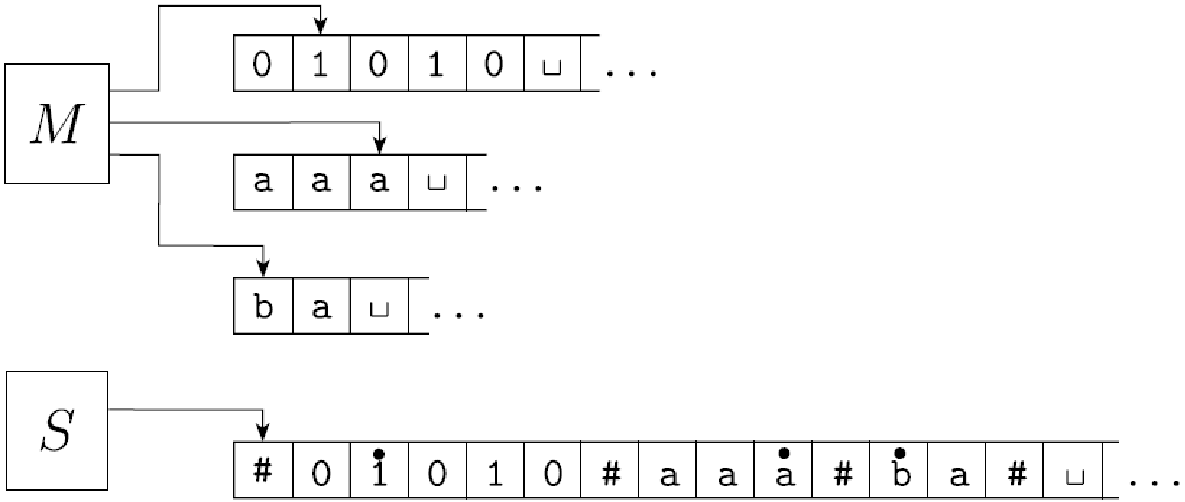
3. By zig-zagging, compare the two strings to the right of the marked #s. If they are equal, *reject*.

4. Move the rightmost of the two marks to the next # symbol to the right. If no # symbol is encountered before a blank symbol, move the leftmost mark to the next # to its right and the rightmost mark to the # after that. This time, if no # is available for the rightmost mark, all the strings have been compared, so *accept*.

5. Go to stage 3."

Variants of Turing Machines

- Multitape Turing machine



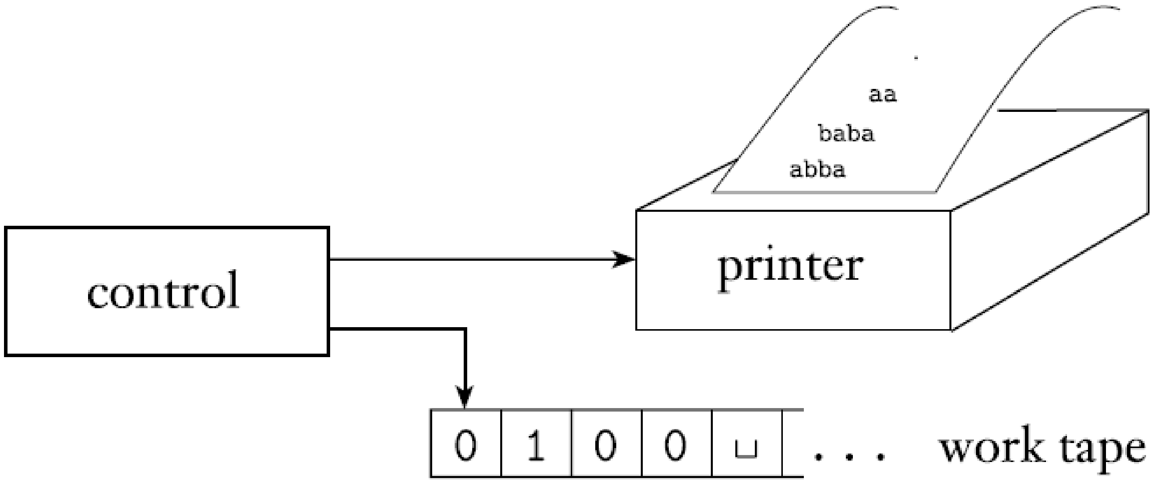
Variants of Turing Machines

- Nondeterministic Turing Machines

$\delta : Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L,R\})$.

Variants of Turing Machines

- Enumerators



Interesting Story

ARTICLE 3.3 The Definition of Algorithm - Self Study

END OF SLIDES

THANK YOU ☺