Military Institute of Science and Technology
Department of Computer Science and Engineering
Mathematical Analysis for Computer Science
CSE-313
Practice Problem on Chapter 1 and 2

1. **Let Qn be the minimum number of moves needed to transfer a tower of n disks from A to B if all moves must be clockwise |that is, from A to B, or from B to the other peg, or from the other peg to A. Also, let Rn be the minimum number of moves needed to go from B back to A under this restriction. Prove that**

$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1} + 1, & \text{if } n > 0; \end{cases} \qquad R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n + Q_{n-1} + 1, & \text{if } n > 0. \end{cases}$$

2. **If W is the minimum number of moves needed to transfer a tower of n disks from one peg to another when there are four pegs instead of three, show that**

$$W_{n(n+1)/2} \leqslant 2W_{n(n-1)/2} + T_n \,, \qquad \text{for } n > 0.$$

   (Here T = 2^n - 1 is the ordinary three-peg number.)

3. **Suppose there are 2n people in a circle; the first n are “good guys" and the last n are “bad guys." Show that it is always an integer m (depending on n) such that, if we go around the circle executing every mth person, all the bad guys are first to go. (For example, when n = 3 we can take m = 5; when n = 4 we can take m = 30.)**

4. **🧊n = $\sum_{0<=k<=n}$ K^3 = $\frac{n^2(n+1)^2}{4}$ Prove it,**

   i) By induction
   ii) Using Perturbation
   iii) Using Repertoire