

# Linked List

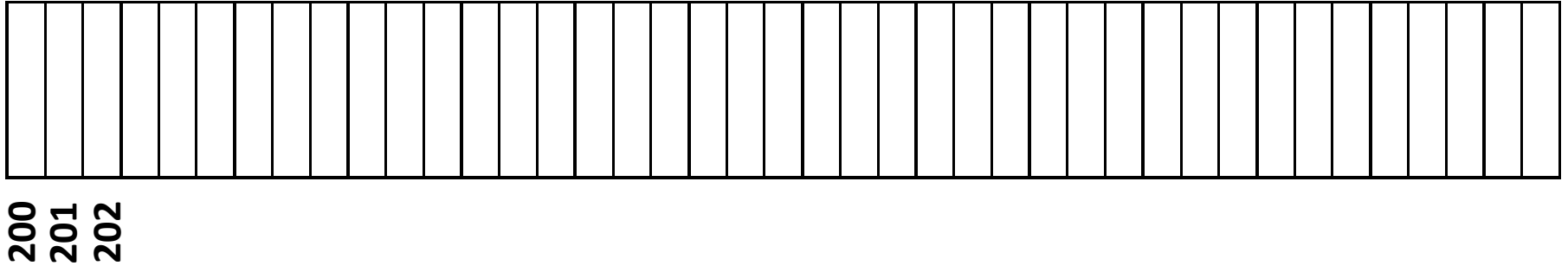
# Limitations of Dynamic Array



Heap Memory

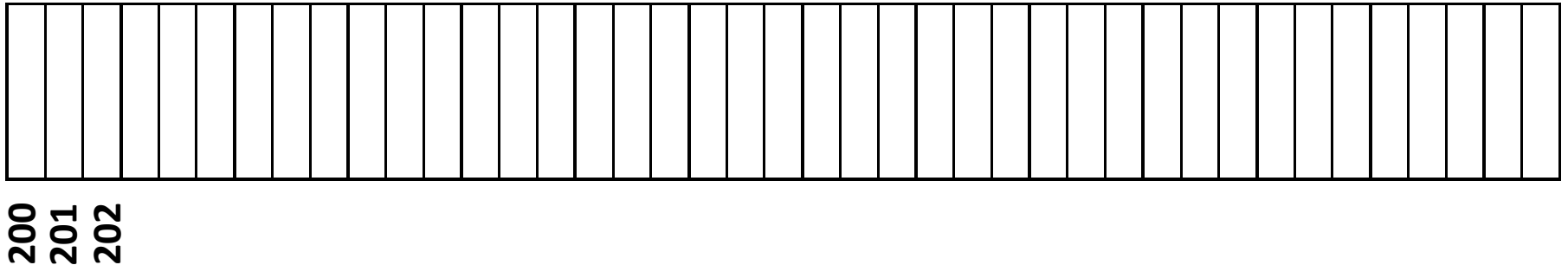
# Limitations of Dynamic Array

## Heap Memory (Horizontal View)



# Limitations of Dynamic Array

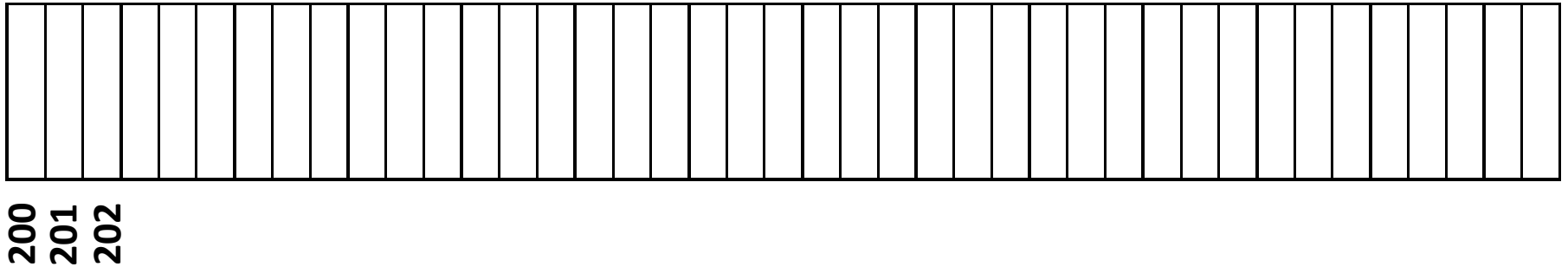
Heap Memory (Horizontal View)



- **Memory Manager** (A part of operating systems) manages the memory.
  - Keeps track of free space
  - Allocates space on request from the programs

# Limitations of Dynamic Array

Heap Memory (Horizontal View)

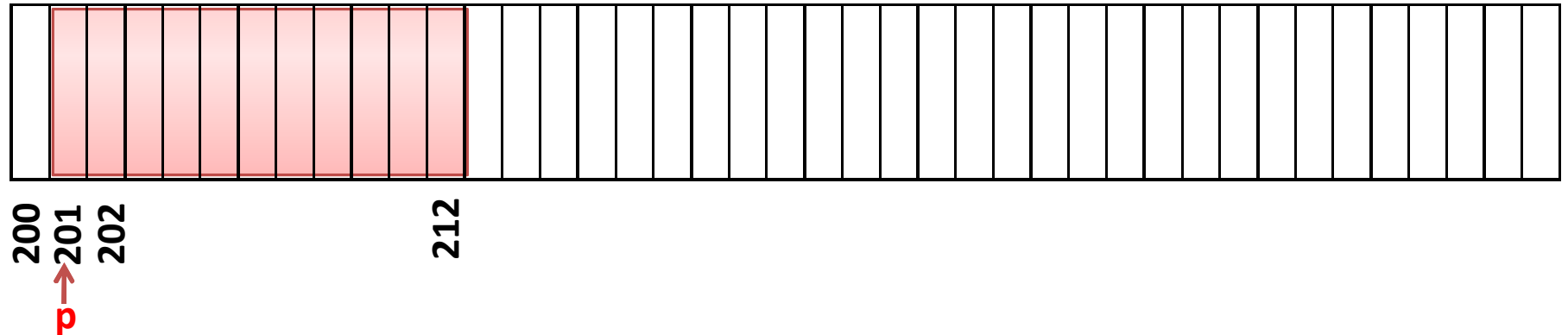


**Consider a program segment:**

```
int *p=(int*)malloc(3*sizeof(int));
```

# Limitations of Dynamic Array

Heap Memory (Horizontal View)

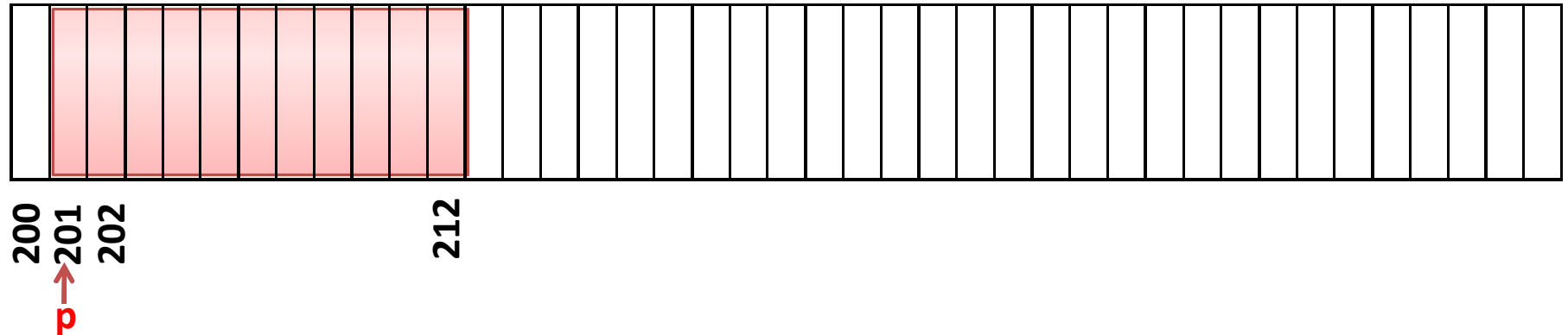


Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
```

# Limitations of Dynamic Array

Heap Memory (Horizontal View)



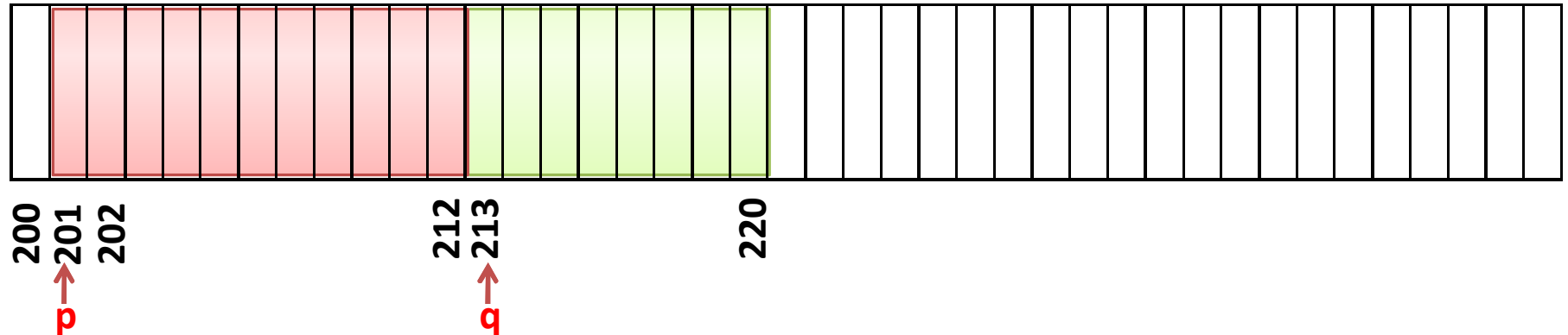
Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
```

```
float *q=(float*)malloc(sizeof(float)); // float size is 8 bytes
```

# Limitations of Dynamic Array

Heap Memory (Horizontal View)



Consider a program segment:

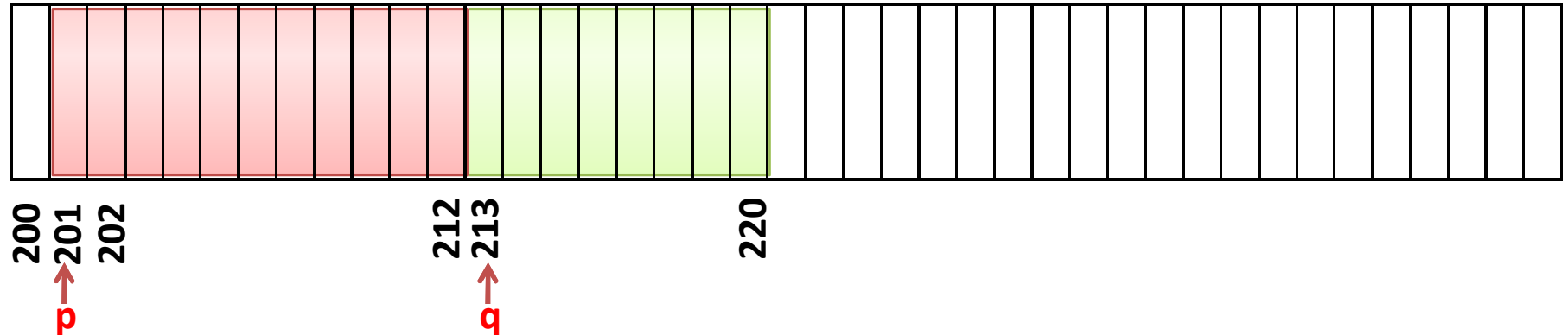
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```
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# Limitations of Dynamic Array

Heap Memory (Horizontal View)

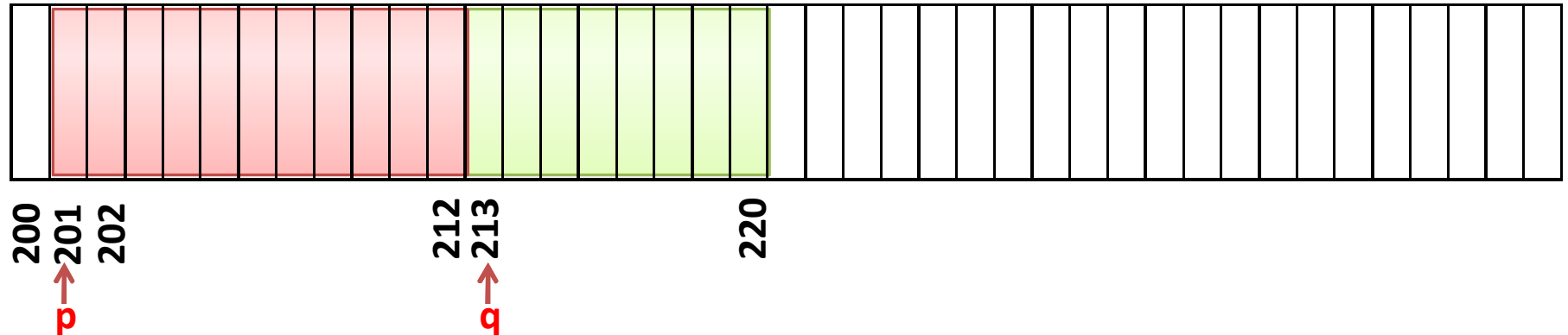


Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
float *q=(float*)malloc(sizeof(float)); // float size is 8 bytes
int *r=(int*)malloc(4*sizeof(int)); // int size is 4 bytes
```

# Limitations of Dynamic Array

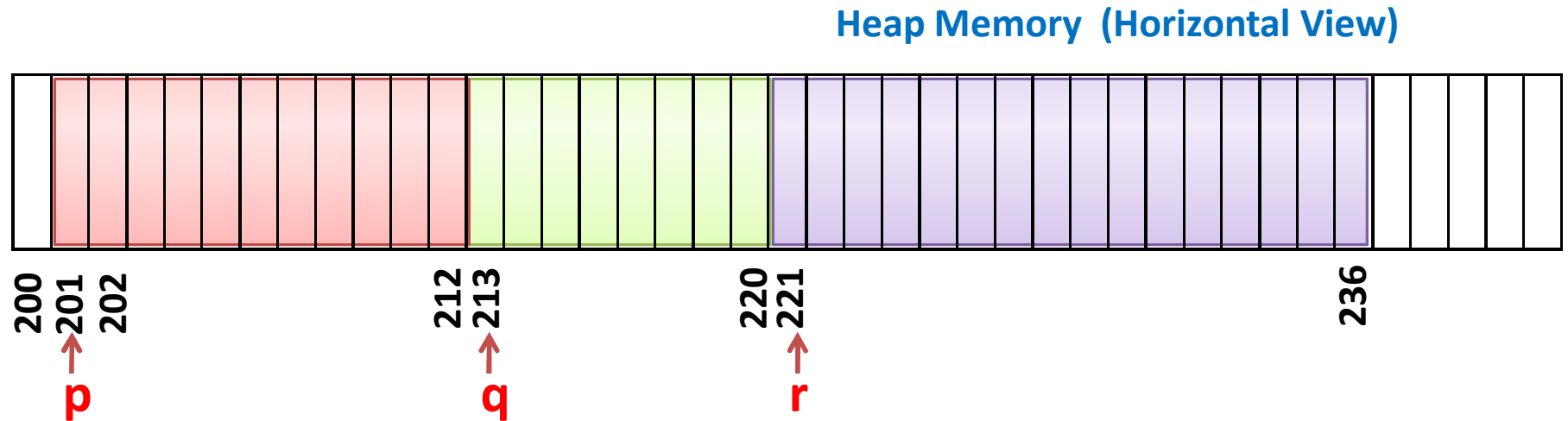
Heap Memory (Horizontal View)



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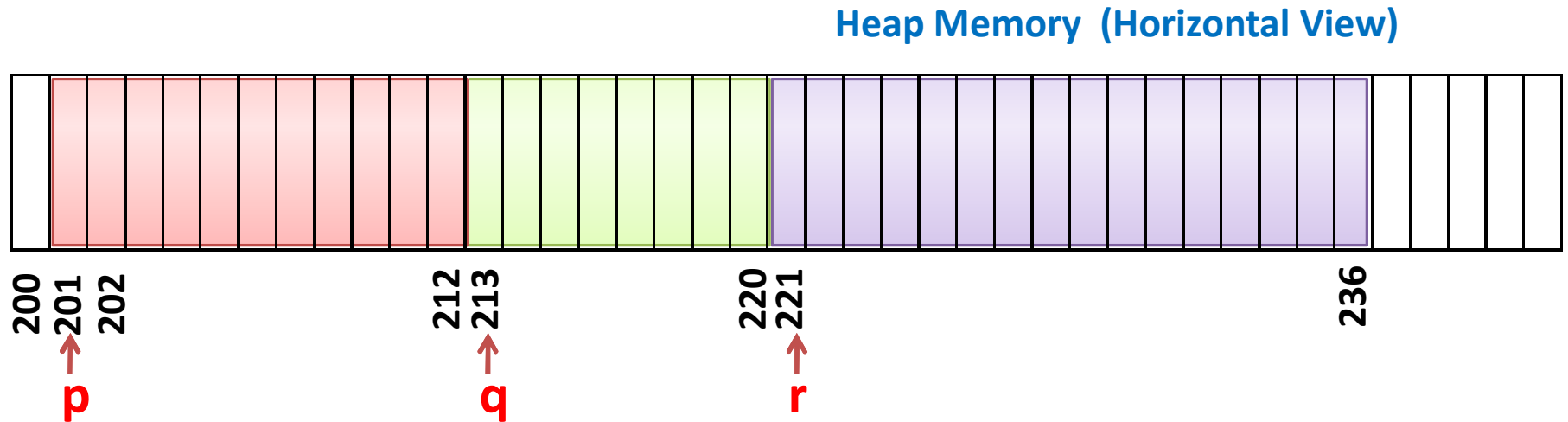
# Limitations of Dynamic Array



Consider a program segment:

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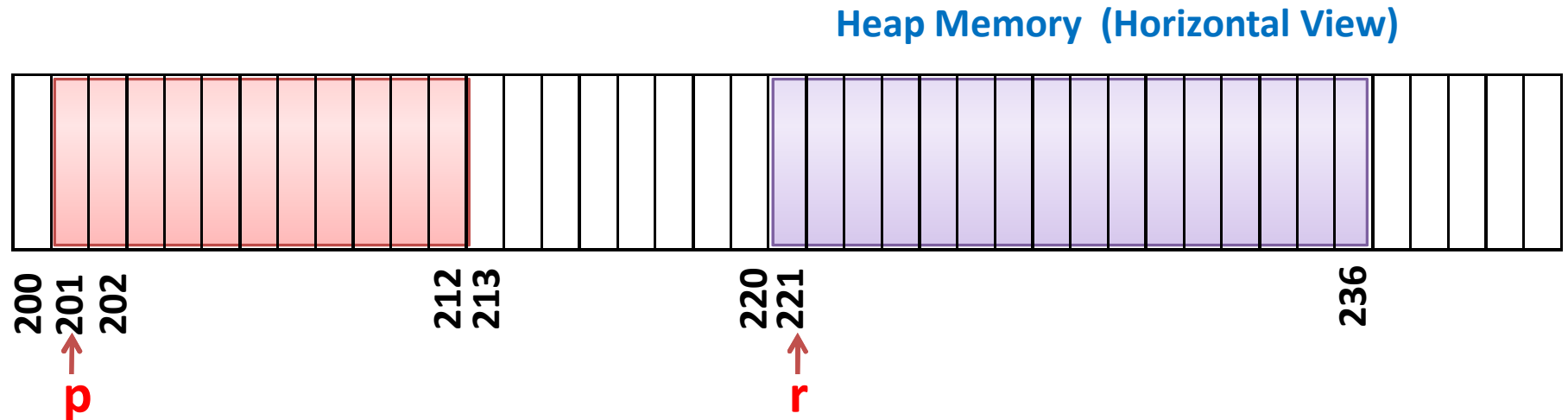
# Limitations of Dynamic Array



Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
float *q=(float*)malloc(sizeof(float)); // float size is 8 bytes
int *r=(int*)malloc(4*sizeof(int)); // int size is 4 bytes
free(q)
```

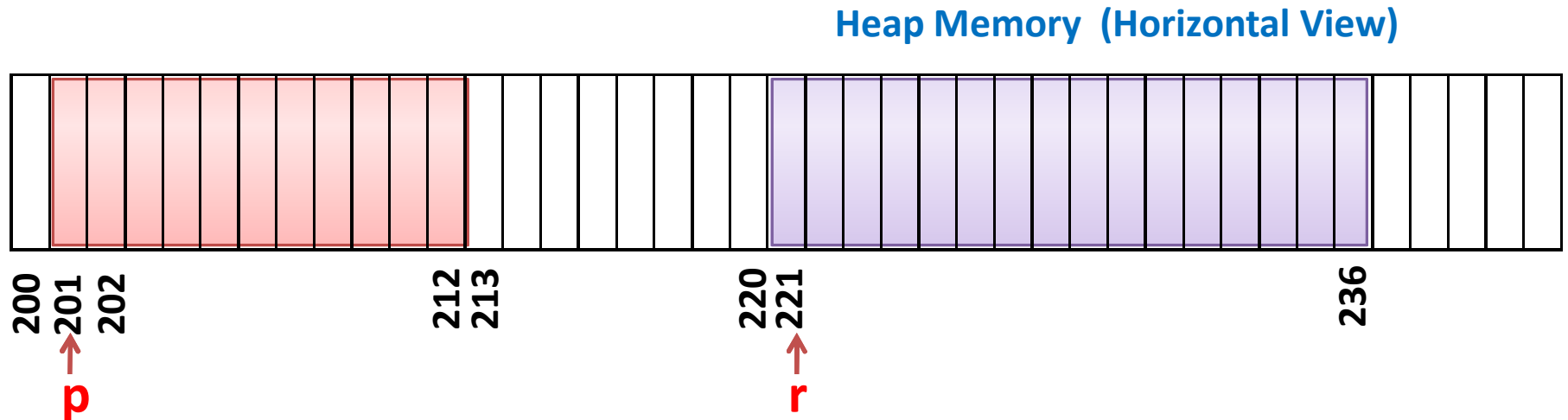
# Limitations of Dynamic Array



Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
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free(q)
```

# Limitations of Dynamic Array



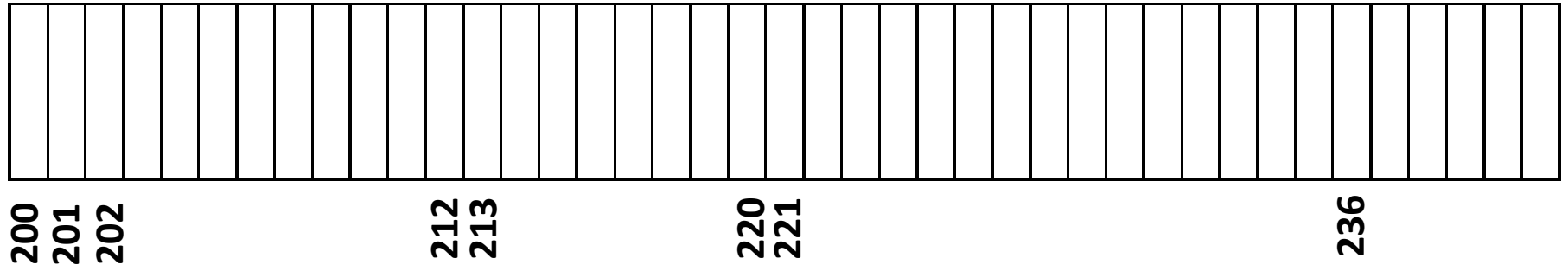
Consider a program segment:

```
int *p=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
float *q=(float*)malloc(sizeof(float)); // float size is 8 bytes
int *r=(int*)malloc(4*sizeof(int)); // int size is 4 bytes
free(q)
int *s=(int*)malloc(3*sizeof(int)); // int size is 4 bytes
```

**Although free memory is more than required  $3 \times 4 = 12$  bytes, yet it cannot be allocated as it is not contiguous. Returns a null pointer.**

# Solution: Linked List

Heap Memory (Horizontal View)

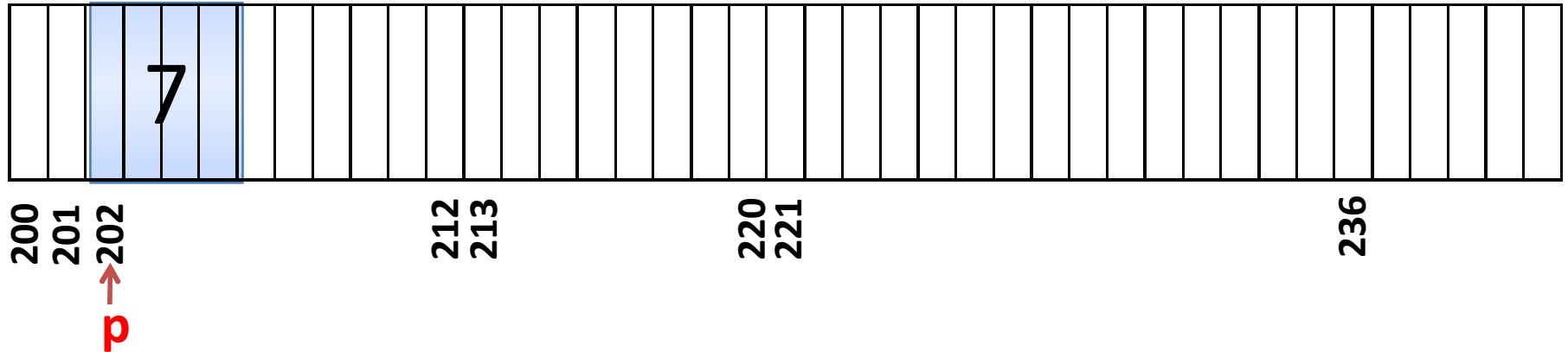


**Suppose we need to store a list of 4 integers: 7, 10, 5, 9.**

Instead of asking memory manager for an integer array of 4 elements, these 4 integers can be stored one at a time in memory in different places.

# Solution: Linked List

Heap Memory (Horizontal View)

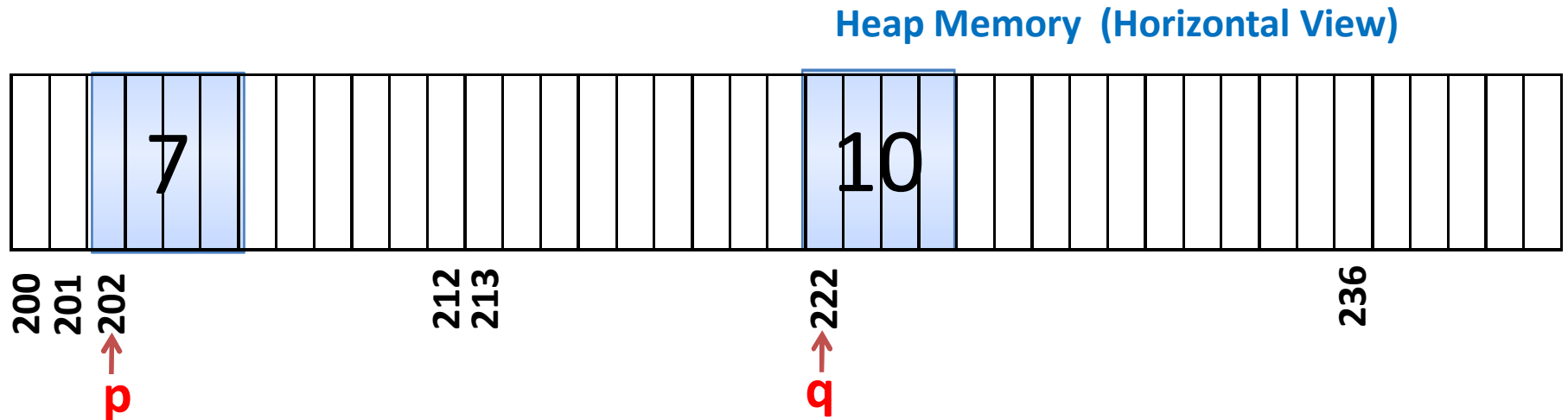


Suppose we need to store a list of 4 integers: 7, 10, 5, 9.

```
int *p=(int*)malloc(sizeof(int)); *p=7;
```



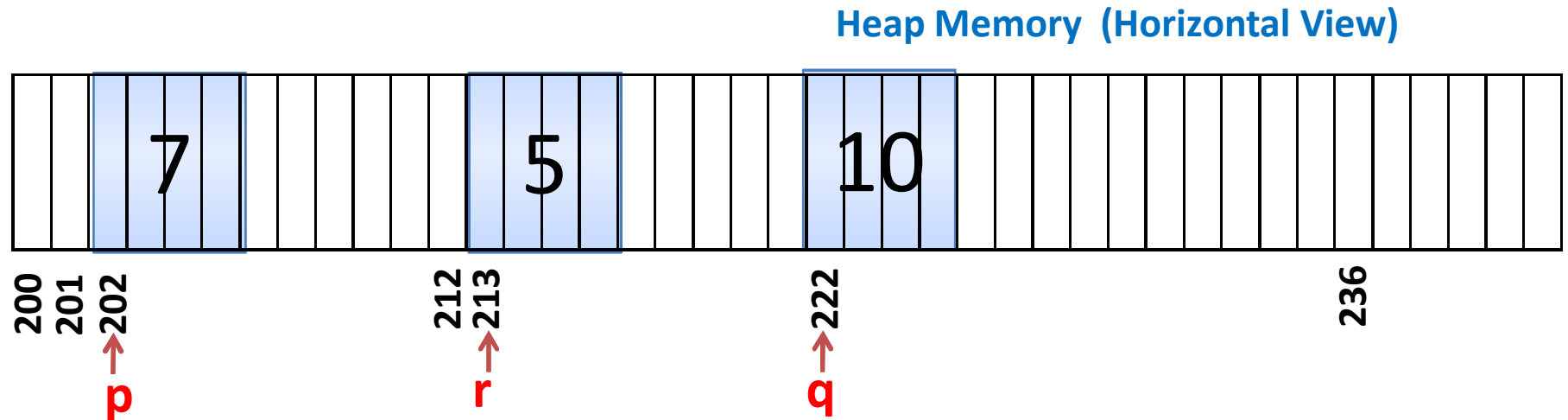
# Solution: Linked List



Suppose we need to store a list of 4 integers: 7, 10, 5, 9.

```
int *p=(int*)malloc(sizeof(int)); *p=7;  
int *q=(int*)malloc(sizeof(int)); *q=10;
```

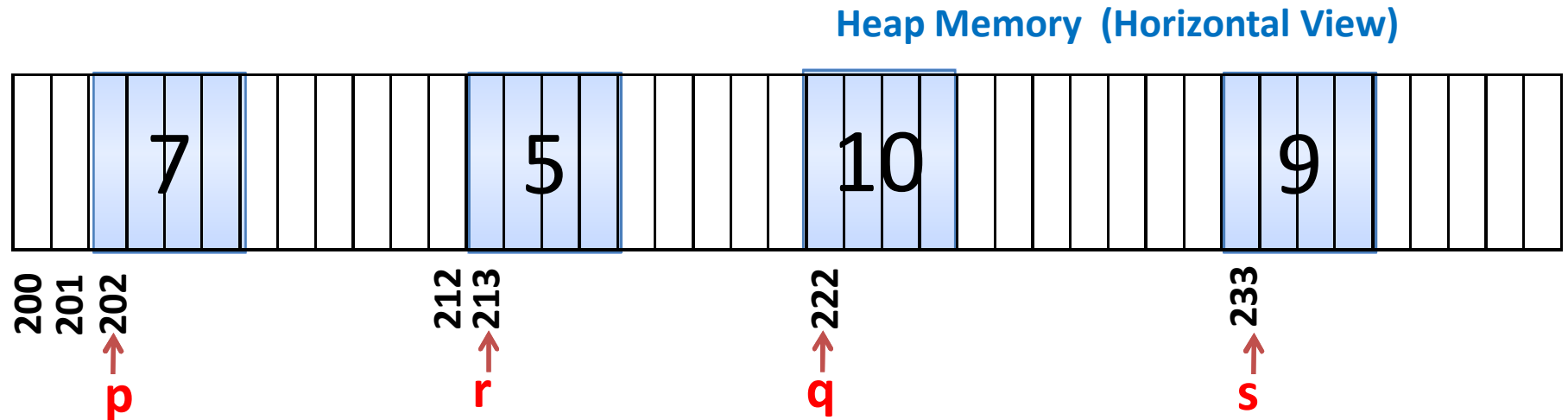
# Solution: Linked List



Suppose we need to store a list of 4 integers: 7, 10, 5, 9.

```
int *p=(int*)malloc(sizeof(int)); *p=7;  
int *q=(int*)malloc(sizeof(int)); *q=10;  
int *r=(int*)malloc(sizeof(int)); *r=5;
```

## Solution: Linked List

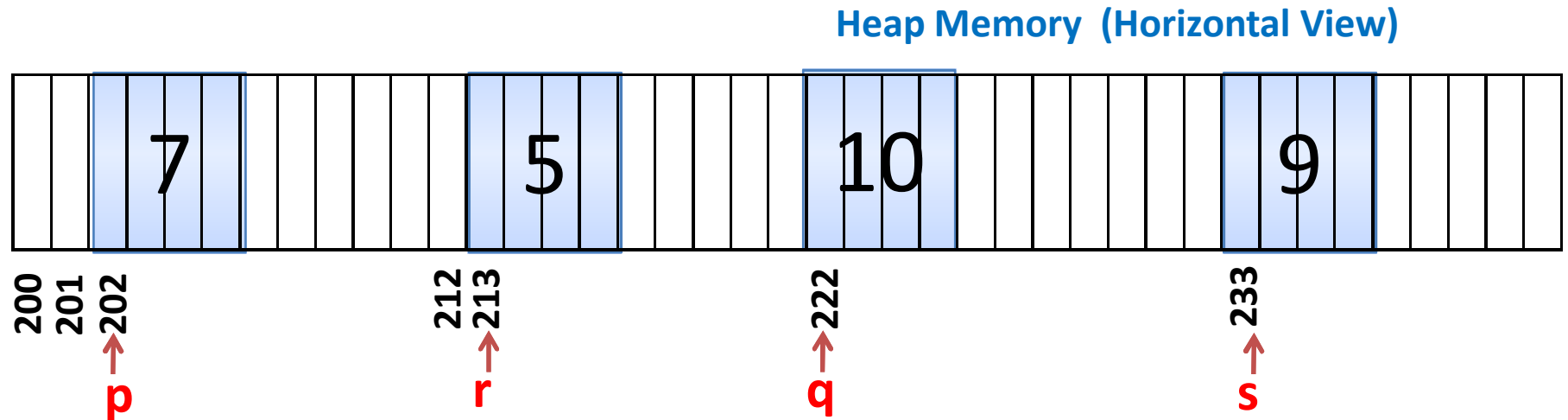


**Suppose we need to store a list of 4 integers: 7, 10, 5, 9.**

```
int *p=(int*)malloc(sizeof(int)); *p=7;
int *q=(int*)malloc(sizeof(int)); *q=10;
int *r=(int*)malloc(sizeof(int)); *r=5;
int *s=(int*)malloc(sizeof(int)); *s=9;
```

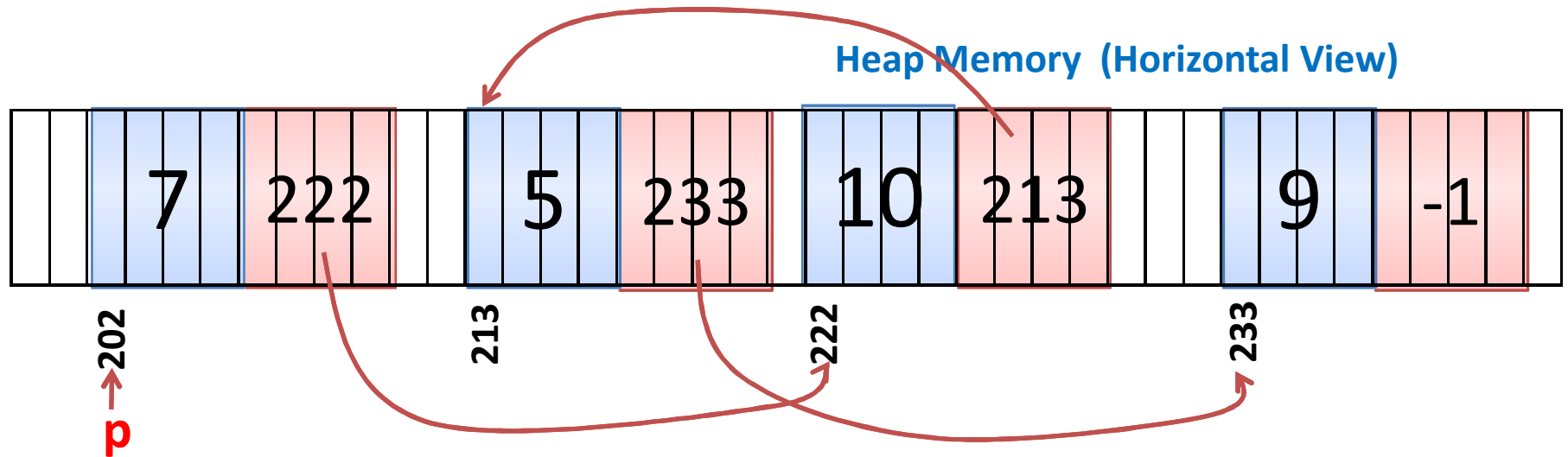
**Non-contiguous allocation:** Hence it is not possible to traverse the list directly as done in array.

## Solution: Linked List



To traverse the elements of the list, one should store the address of the next element in the list with each element.

## Solution: Linked List



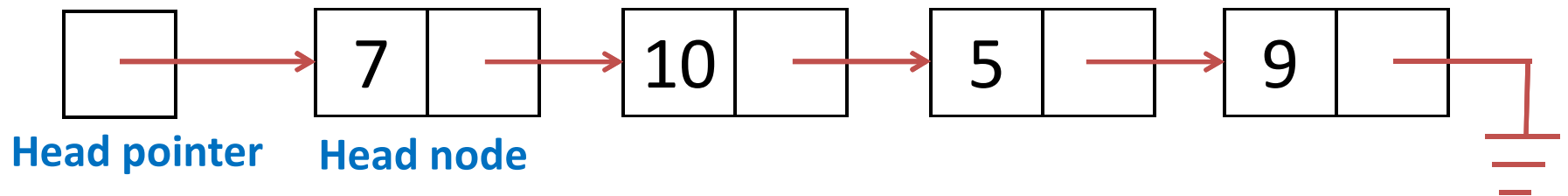
To traverse the elements of the list {7, 10, 5, 9}, one should store the address of the next element (**pointer to next element**) in the list with each element.

It is sufficient to know the address of (pointer to) the first node (also called as **head node**) to retrieve all the elements of the list.

The last node points to null (represented by -1 here).

# Logical View of Singly Linked List

A linked list contains a sequence of **nodes**.



Each node contains a **value**.

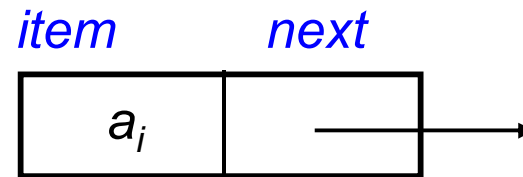
Each node contains the address (**link**) of the next node.

The last node contains a **null link**.

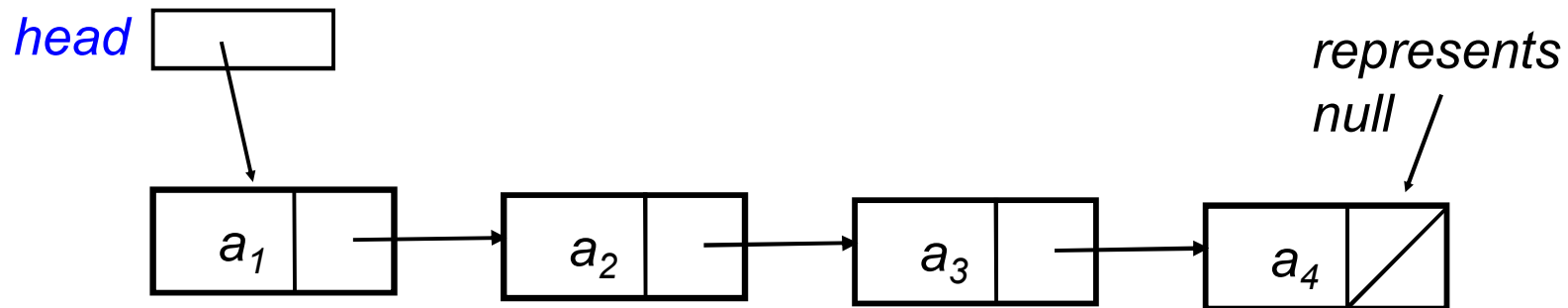
The list is identified by a **head pointer** (link to the **head node**)

# Linked List Approach

- Main problem of array is the slow deletion/insertion since it has to shift items in its *contiguous* memory
- **Solution:** linked list where items need *not be contiguous* with nodes of the form



- Sequence (list) of four items  $\langle a_1, a_2, a_3, a_4 \rangle$  can be represented by:



# Pointer-Based Linked Lists

- A node in a linked list is usually a struct

```
struct Node
```

```
{ int item;
```

```
  Node *next;
```

```
}; //end struct
```



A node

- A node is dynamically allocated

```
Node *p;
```

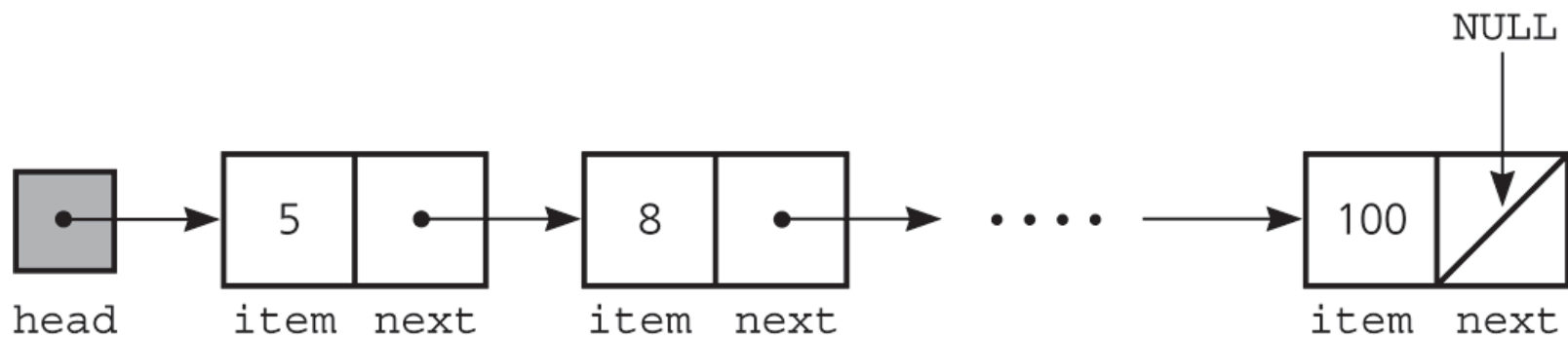
```
p = malloc(sizeof(Node));
```



# Pointer-Based Linked Lists

- The head pointer points to the first node in a linked list
- If head is *NULL*, the linked list is empty
  - head=NULL
- head=malloc(sizeof(Node) )

# A Sample Linked List



# Traverse a Linked List

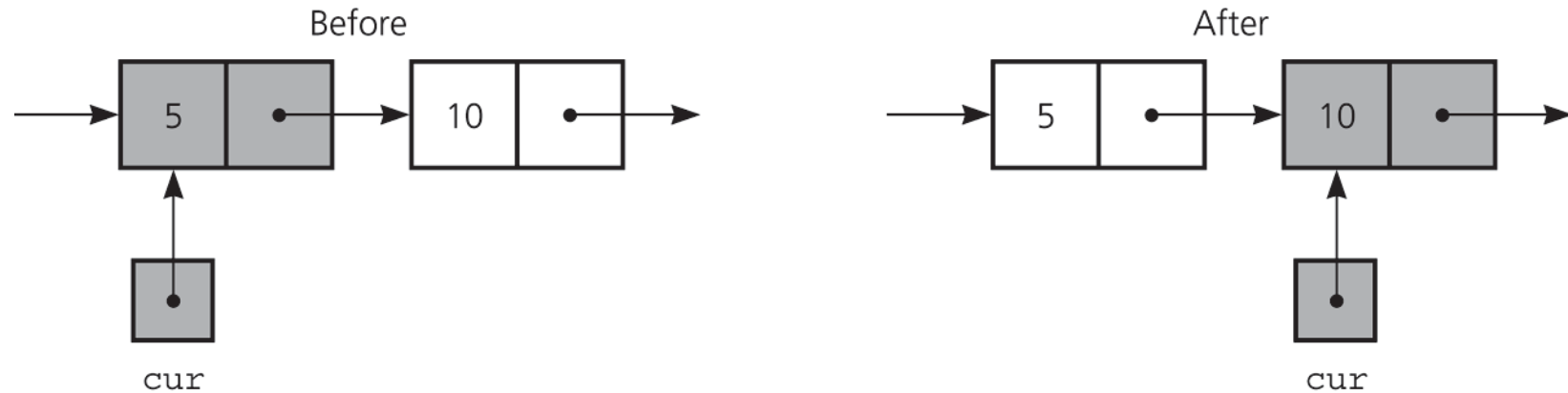
- Reference a node member with the -> operator

```
p->item;
```

- A traverse operation visits each node in the linked list
  - A pointer variable cur keeps track of the current node

```
for (Node *cur = head;  
      cur != NULL; cur = cur->next)  
    x = cur->item;
```

# Traverse a Linked List



The effect of the assignment  $cur = cur \rightarrow next$

# Delete a Node from a Linked List

- Deleting an interior/last node

```
prev->next=cur->next;
```

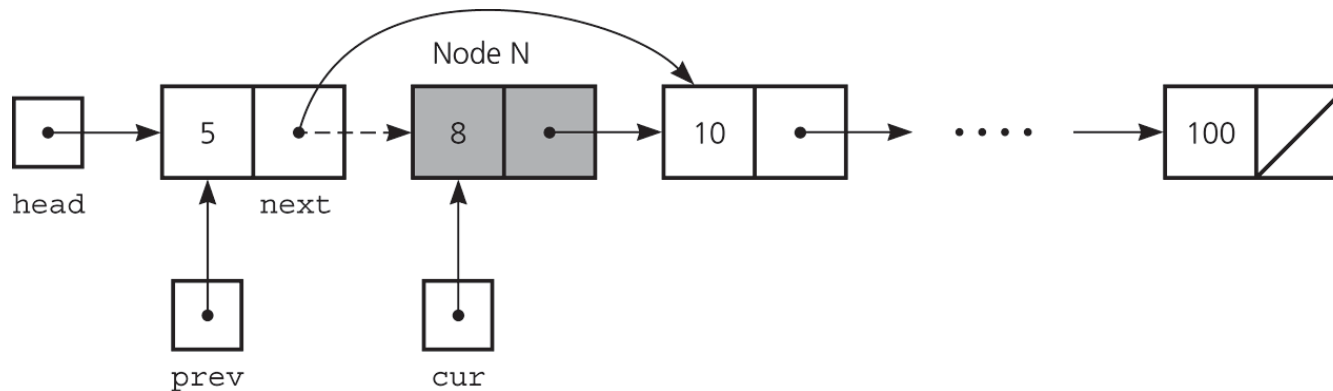
- Deleting the first node

```
head=head->next;
```

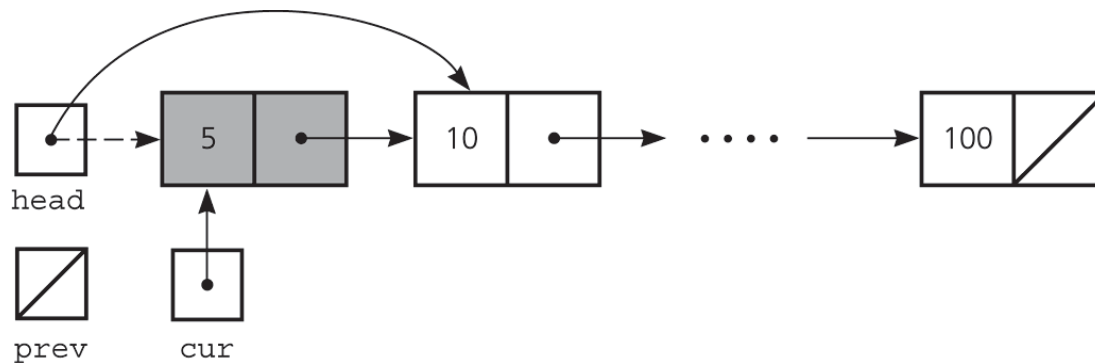
- Return deleted node to system

```
free (cur) ;
```

# Delete a Node from a Linked List



**Deleting a node from a linked list**



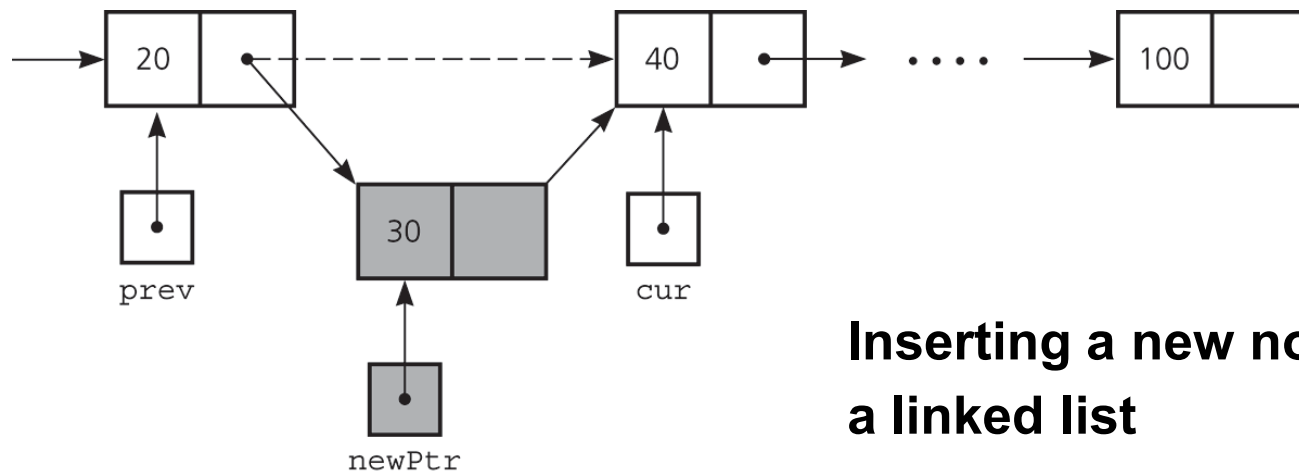
**Deleting the first node**

# Insert a Node into a Linked List

- To insert a node between two nodes

```
newPtr->next = cur;
```

```
prev->next = newPtr;
```

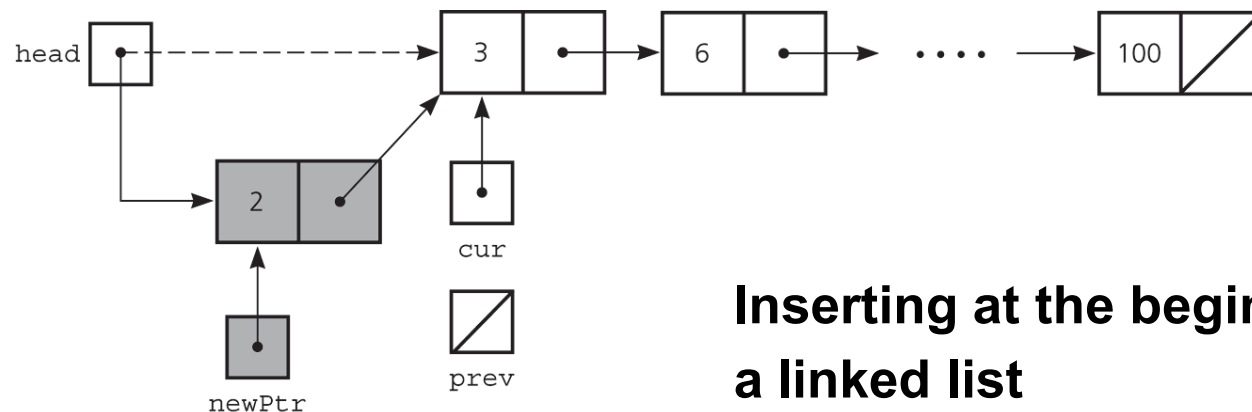


# Insert a Node into a Linked List

- To insert a node at the beginning of a linked list

```
newPtr->next = head;
```

```
head = newPtr;
```



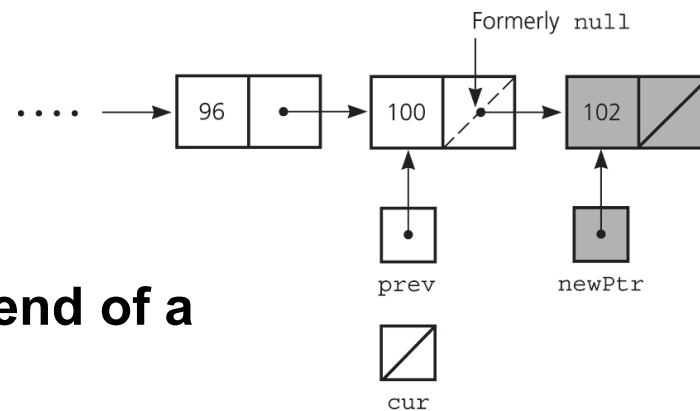
**Inserting at the beginning of a linked list**



# Insert a Node into a Linked List

- Inserting at the end of a linked list

```
newPtr->next = cur;  
prev->next = newPtr;
```



**Inserting at the end of a linked list**

# Look up

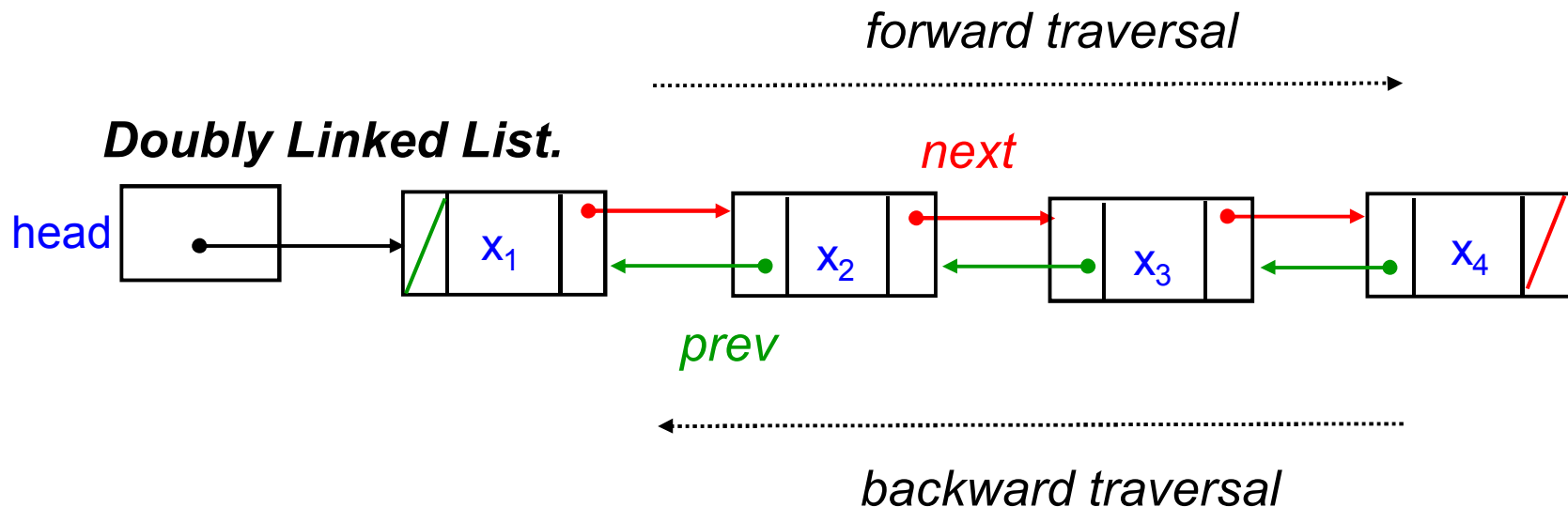
```
BOOLEAN lookup (int x, Node *L)
{ if (L == NULL)
    return FALSE
  else if (x == L->item)
    return TRUE
  else
    return lookup(x, L->next);
}
```

# An ADT Interface for List

- Functions
  - isEmpty
  - getLength
  - insert
  - delete
  - Lookup
  - ...
- Data Members
  - head
  - Size
- Local variables to member functions
  - cur
  - prev

# Doubly Linked Lists

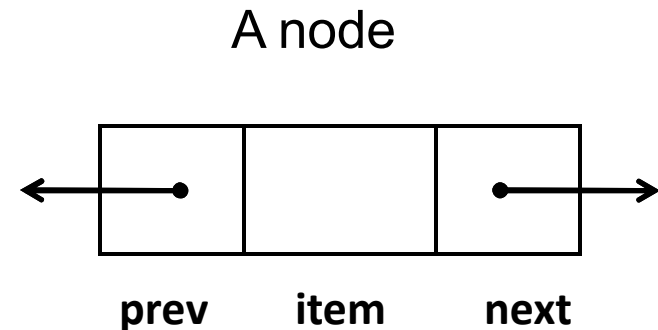
- Frequently, we need to traverse a sequence in BOTH directions efficiently
- ***Solution***: Use doubly-linked list where each node has two pointers



# Doubly Linked Lists

- A node in a doubly linked list is usually a struct

```
struct Node  
{ int item;  
  Node *prev;  
  Node *next;  
}; //end struct
```

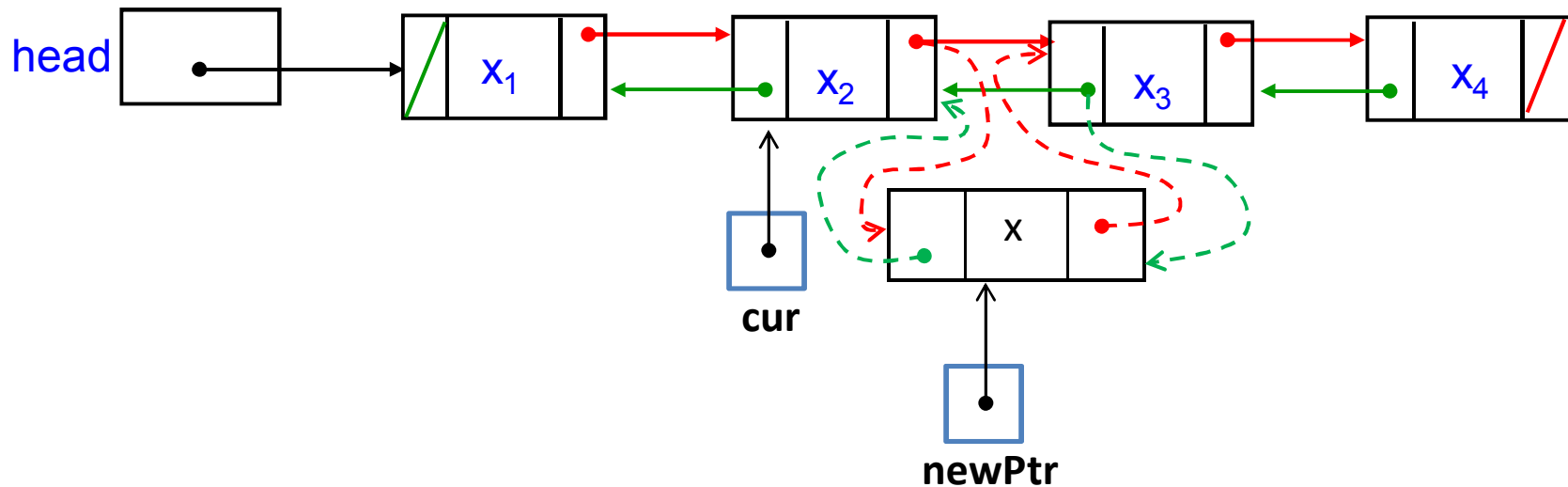


- A node is dynamically allocated

```
Node *p;  
p = malloc(sizeof(Node));
```

# Inserting a Node

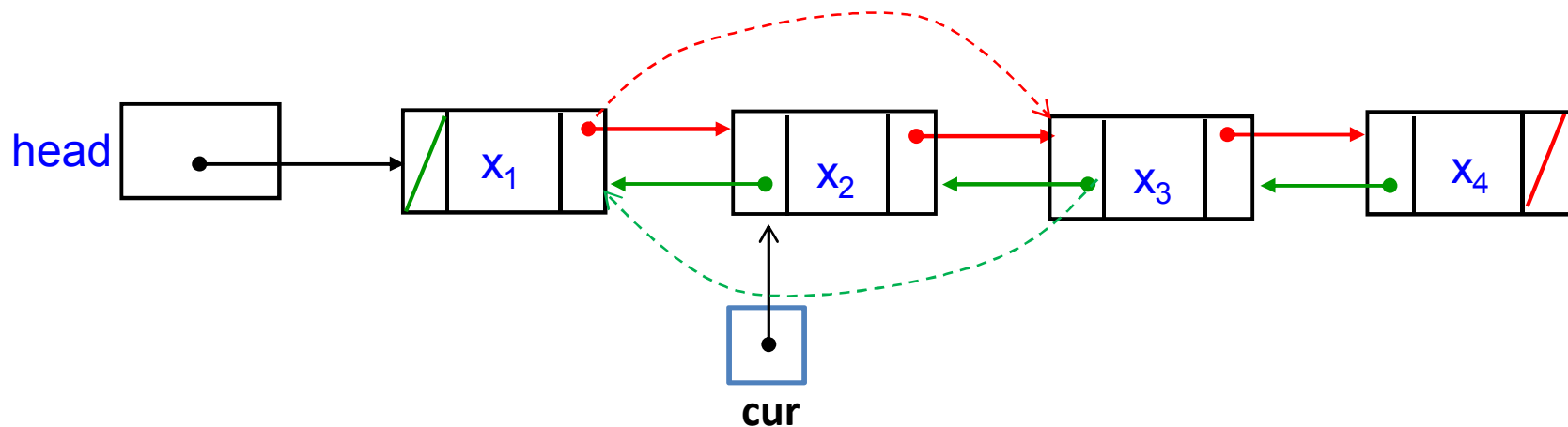
- Insert a node in the middle (After cur)



```
newPtr->next = cur->next;  
newPtr->prev = cur;  
cur->next->prev = newPtr;  
cur->next = newPtr;
```

# Deleting a Node

- Delete a node from middle

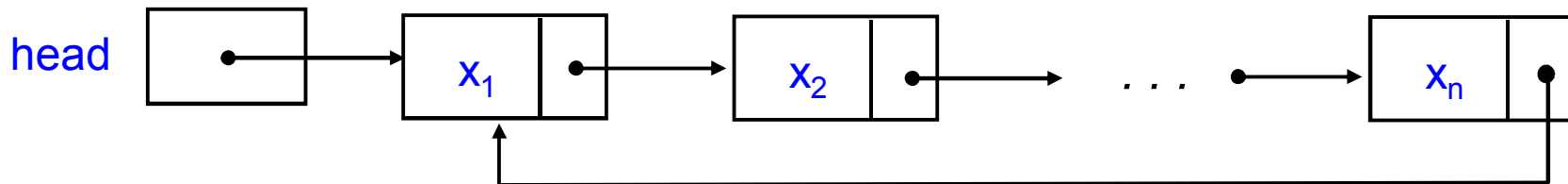


```
cur->prev->next = cur->next;  
cur->next->prev = cur->prev;  
free(cur);
```

# Circular Linked Lists

- May need to cycle through a list repeatedly, e.g. round robin system for a shared resource.
- ***Solution***: Have the last node point to the first node (Head).

## ***Circular Linked List.***





# Try in Lab

- Insert a node into a sorted linked lists at the correct position.
- Concatenate two singly linked lists.
- Concatenate two doubly linked lists.
- Concatenate two circular linked lists.

# Array Vs. Linked List

## Cost of Accessing $i^{\text{th}}$ Element

### Array

Accessed by index

Constant

$O(1)$

### Linked List

Accessed by traversing

Best case:  $O(1)$

Average/Worst case:  $O(n)$

# Array Vs. Linked List

## Memory Requirement

### Array

Fixed size

3	7	4	-	-
---	---	---	---	---

Unused

Suffers from fragmentation.

- Contiguous Memory may not be available

### Linked List

No unused memory

Extra space for pointers

- Negligible if data part is big

Suffers less from fragmentation.

- Small blocks are likely to be available

# Array Vs. Linked List

## Cost of inserting an element

### Array

No traversal but shifting

Beginning –  $O(n)$

Middle –  $O(n)$

End –  $O(1)$  // If array is not full

$O(n)$  // If array is full

// Reallocation required

### Linked List

Traversal but no shifting

Beginning –  $O(1)$

Middle –  $O(n)$

End –  $O(n)$

# Array Vs. Linked List

## Cost of deleting an element

### Array

No traversal but shifting

Beginning –  $O(n)$

Middle –  $O(n)$

End –  $O(1)$  // If array is not full

### Linked List

Traversal but no shifting

Beginning –  $O(1)$

Middle –  $O(n)$

End –  $O(n)$

# Array Vs. Linked List

## Ease of Use

### **Array**

Easy to implement

### **Linked List**

Complex use of pointers

Care should be taken to free up the memory for deleted nodes.

# Polynomial Representation

- What is it?

An example of a single variable polynomial:

$$4x^6 + 10x^4 - 5x + 3$$

$$4x^6 + 0.x^5 + 10x^4 + 0.x^3 + 0.x^2 + (-5)x^1 + 3x^0$$

Remark: the order of this polynomial is 6  
(look for highest exponent)

# Polynomial ADT

- Why call it an Abstract Data Type (ADT)?

A single variable polynomial can be generalized as:

$$f(x) = \sum_{i=0}^n a_i x^i$$



# Polynomial ADT (continued)

...This sum can be expanded to:

$$a_n x^n + a_{(n-1)} x^{(n-1)} + \dots + a_1 x^1 + a_0$$

Notice the two visible data sets namely: (**C** and **E**), where

- **C** is the **coefficient** set [Real values:  $a_0$  to  $a_n$ ].
- **E** is the **exponent** set object [Integer values:  $0$  to  $n$ ].

# Polynomial ADT (continued)

- Now what?

By definition of a data types:

A set of values and a set of allowable operations on those values.

We can now operate on this polynomial the way we like...

# Polynomial ADT (continued)

- What kinds of operations?

Here are the most common operations on a polynomial:

- Add & Subtract
- Multiply
- Differentiate
- Integrate
- etc...

# Polynomial ADT (continued)

- Why implement this?

Calculating polynomial operations by hand can be very cumbersome. Take **differentiation** as an example:

$$d(23x^9 + 18x^7 + 41x^6 + 163x^4 + 5x + 3)/dx$$

$$= (23*9)x^{(9-1)} + (18*7)x^{(7-1)} + (41*6)x^{(6-1)} + \dots$$

# Polynomial ADT (continued)

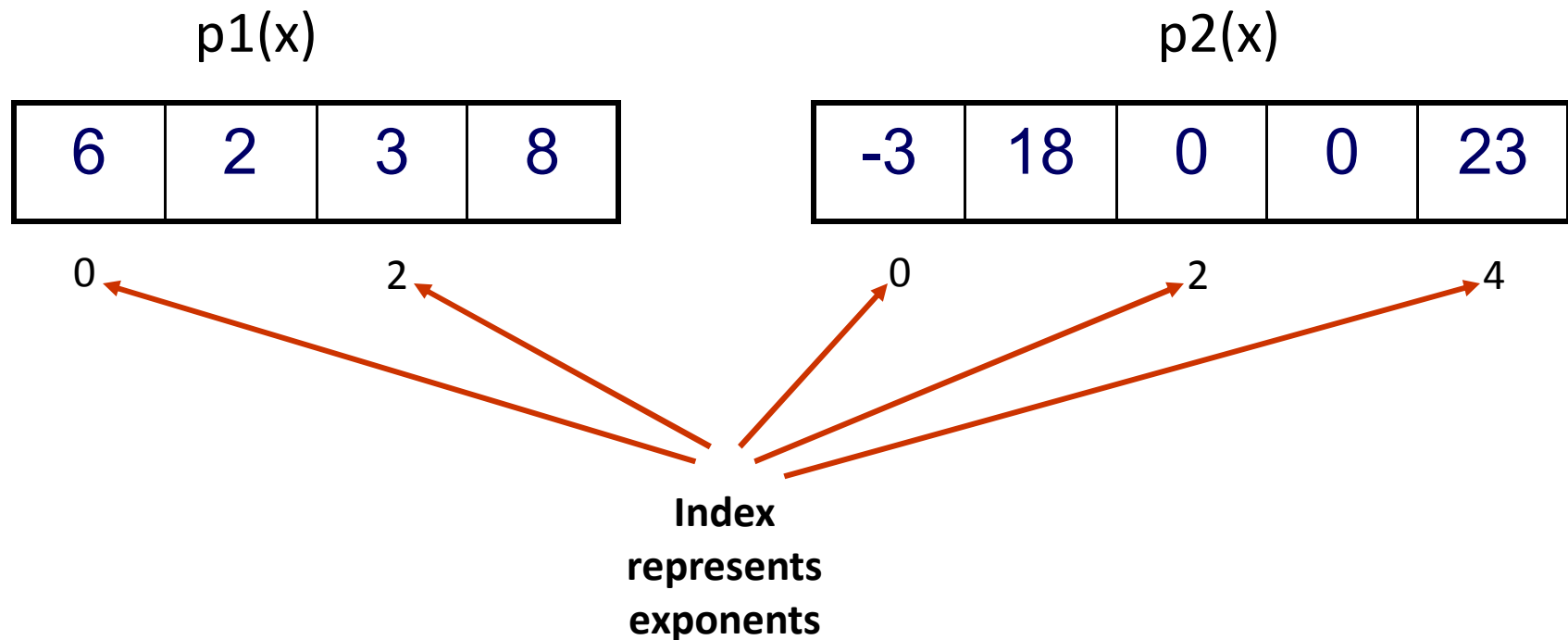
- How to implement this?

There are different ways of implementing the polynomial ADT:

- Array (not recommended)
- Linked List (preferred and recommended)

# Polynomial ADT (continued)

- Array Implementation:
- $p1(x) = 8x^3 + 3x^2 + 2x + 6$
- $p2(x) = 23x^4 + 18x - 3$



# Polynomial ADT (continued)

- This is why arrays aren't good to represent polynomials:
- $p_3(x) = 16x^{21} - 3x^5 + 2x + 6$

6	2	0	0	-3	0	.....	0	16
0	1	2	3	4	5		20	21

WASTE OF SPACE!

# Polynomial ADT (continued)

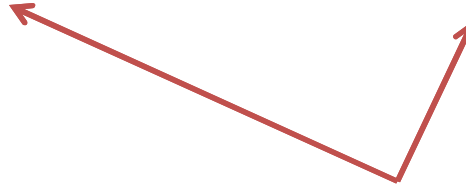
- Advantages of using an Array:
  - only good for **non-sparse** polynomials.
  - ease of storage and retrieval.
- Disadvantages of using an Array:
  - have to allocate array size ahead of time.
  - huge array size required for sparse polynomials. Waste of space and runtime.



# Polynomial ADT (continued)

- Sparse Polynomial
- $p_4(x) = 6x^{102} - 13x^{14} + 22x^2 - 2$

-2	0	22	...	...	-13	...	...	...	0	6
0	1	2	3	-----13	14	15	-----100		101	102



**Huge wastage of space**

# Polynomial ADT using Linked List

## Node Representation

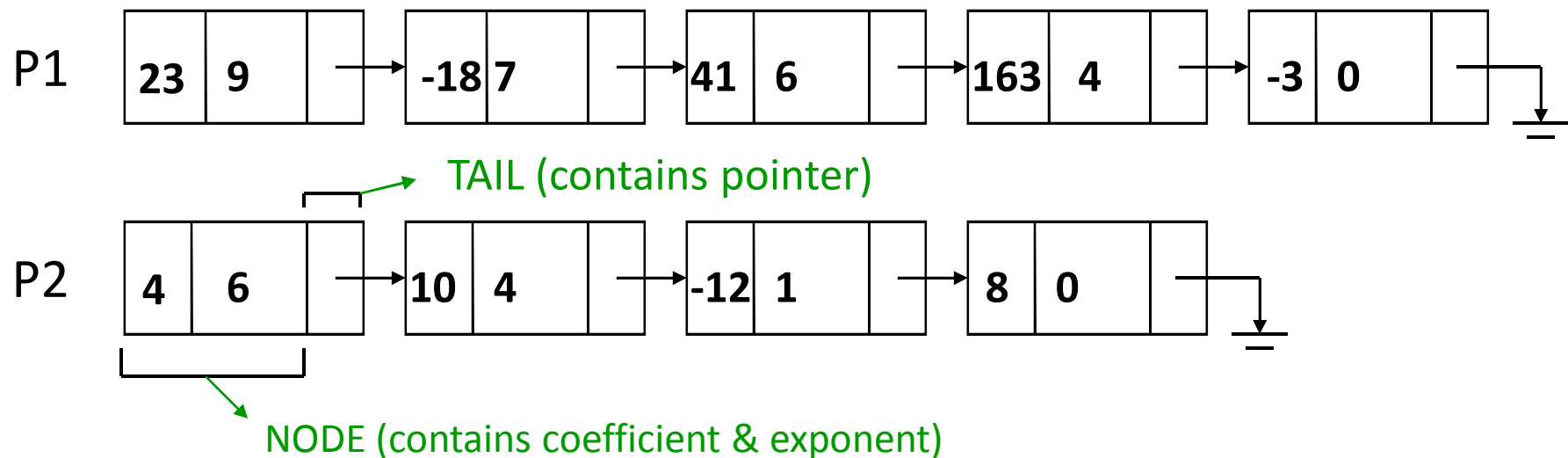
```
struct polynode
{
    double coeff;
    int expon;
    polynode* next;
};
```

```
typedef struct polynode *polyptr;
```

coeff	expon	next
-------	-------	------

# Polynomial ADT (continued)

- Linked list Implementation:
- $p1(x) = 23x^9 - 18x^7 + 41x^6 + 163x^4 - 3$
- $p2(x) = 4x^6 + 10x^4 - 12x + 8$



# Polynomial ADT (continued)

- Advantages of using a Linked list:
  - save space (don't have to worry about sparse polynomials) and easy to maintain
  - don't need to allocate list size initially and can declare nodes (terms) only as needed

# Adding Two Polynomials (Linked Lists)

- Given two polynomials:

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8 - 8x^7 + 163x^4 + 5$$

# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9$$

# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8$$

# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8 + (-18+10)x^7$$



# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8 - 8x^7$$

# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8 - 8x^7 + 163x^4$$

# Adding Two Polynomials (Linked Lists)

- How it works

$$p1(x) = 23x^9 - 18x^7 + 163x^4 - 3$$

$$p2(x) = 4x^8 + 10x^7 + 8$$

**After addition**

$$p3(x) = 23x^9 + 4x^8 - 8x^7 + 163x^4 + (-3+8)$$

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# Polynomial ADT (continued)

- Adding polynomials using a Linked list representation: (storing the result in p3)

To do this, we have to break the process down to cases:

- **Case 1: exponent of p1 > exponent of p2**
  - Copy node of p1 to end of p3.
  - $p1 = p1 \rightarrow next$ ;
- **Case 2: exponent of p1 < exponent of p2**
  - Copy node of p2 to end of p3.
  - $p2 = p2 \rightarrow next$ ;
- **Case 3: exponent of p1 = exponent of p2**
  - Create a new node in p3 with the same exponent and with the sum of the coefficients of p1 and p2.
  - $p1 = p1 \rightarrow next$ ;  $p2 = p2 \rightarrow next$ ;

# Polynomial ADT (continued)

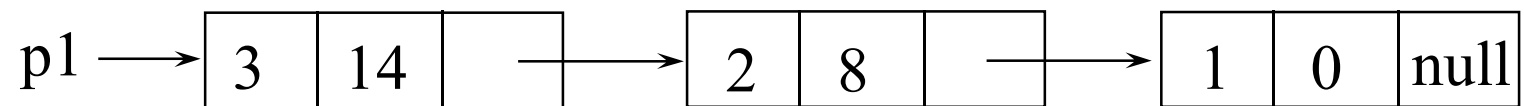
- Adding polynomials using a Linked list representation: (storing the result in p3)

To do this, we have to break the process down to cases:

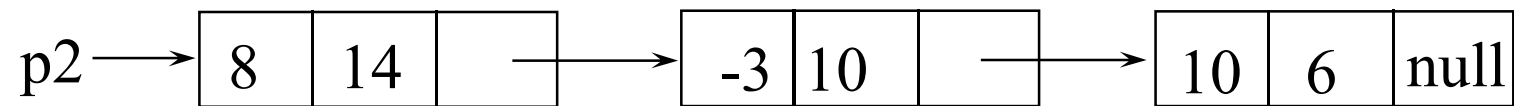
- **Case 4: p1 == NULL**
  - Copy entire p2 to end of p3.
- **Case 5: p2 == NULL**
  - Copy entire p1 to end of p3.

# Example

$$p1 = 3x^{14} + 2x^8 + 1$$

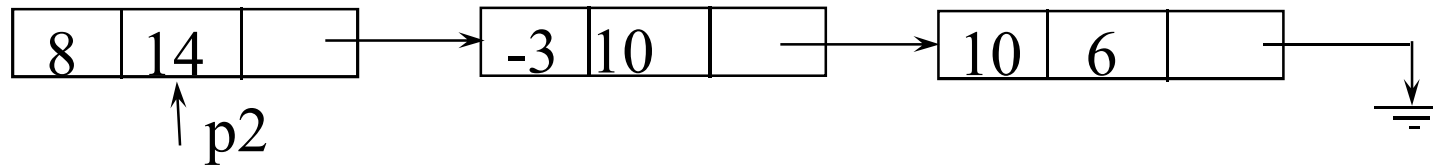
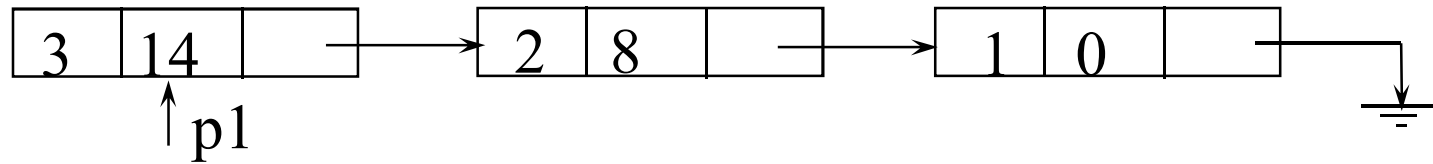


$$p2 = 8x^{14} - 3x^{10} + 10x^6$$

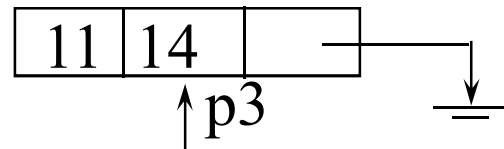




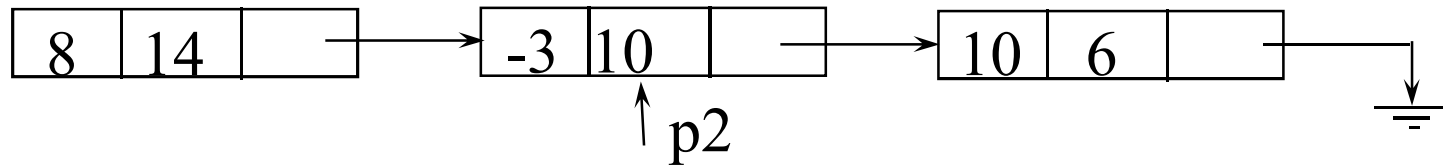
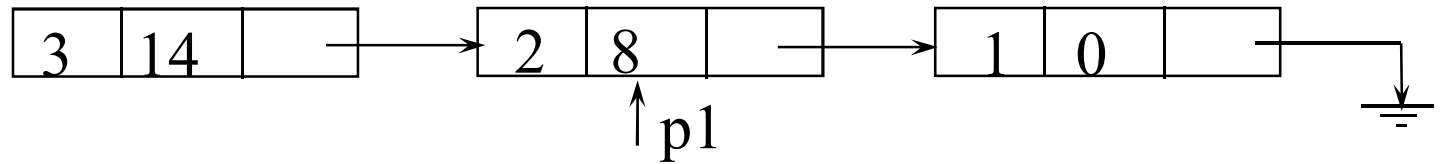
# Adding Two Polynomials



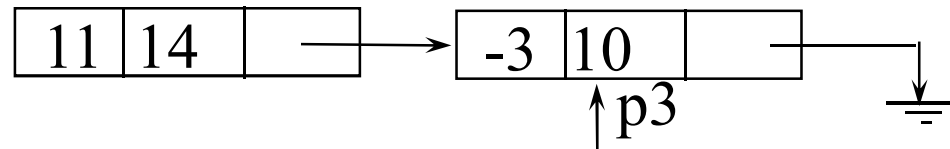
$p_1 \rightarrow \text{expon} == p_2 \rightarrow \text{expon}$



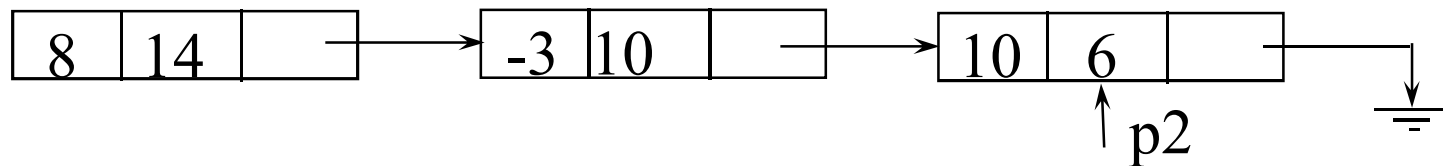
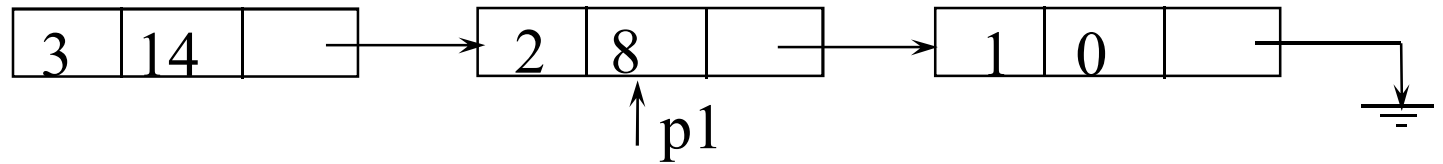
# Adding Two Polynomials



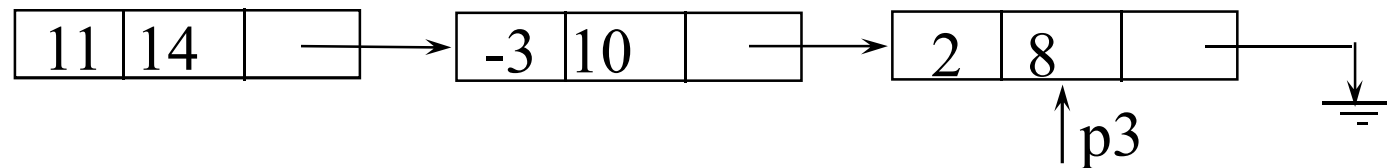
$p1 \rightarrow \text{expon} < p2 \rightarrow \text{expon}$



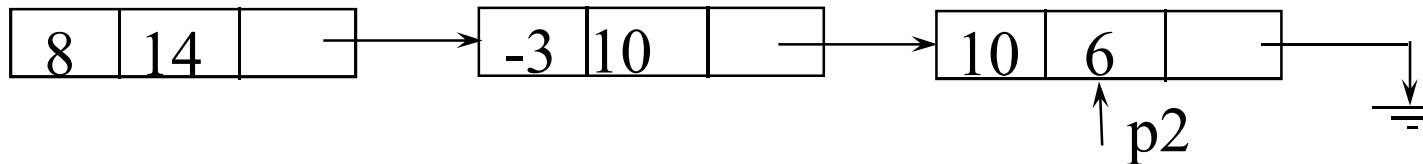
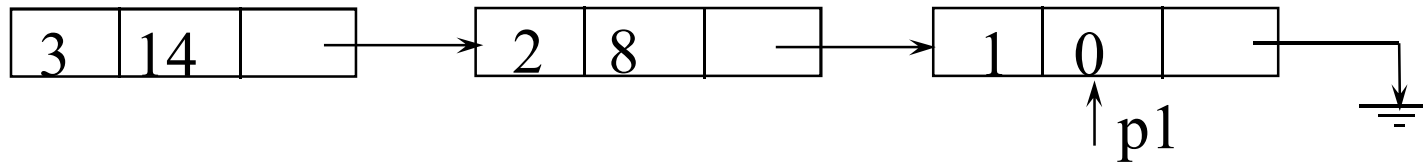
# Adding Two Polynomials



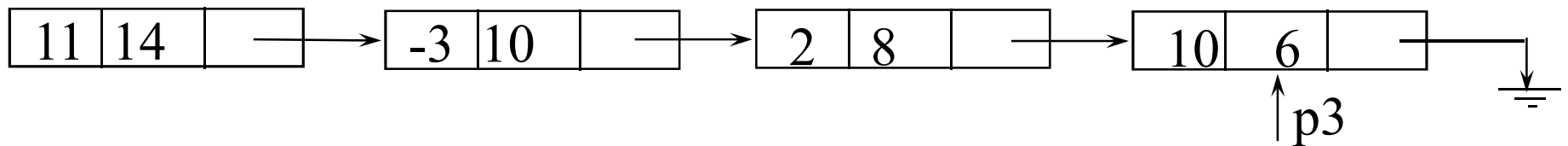
$p1 \rightarrow \text{expon} > p2 \rightarrow \text{expon}$



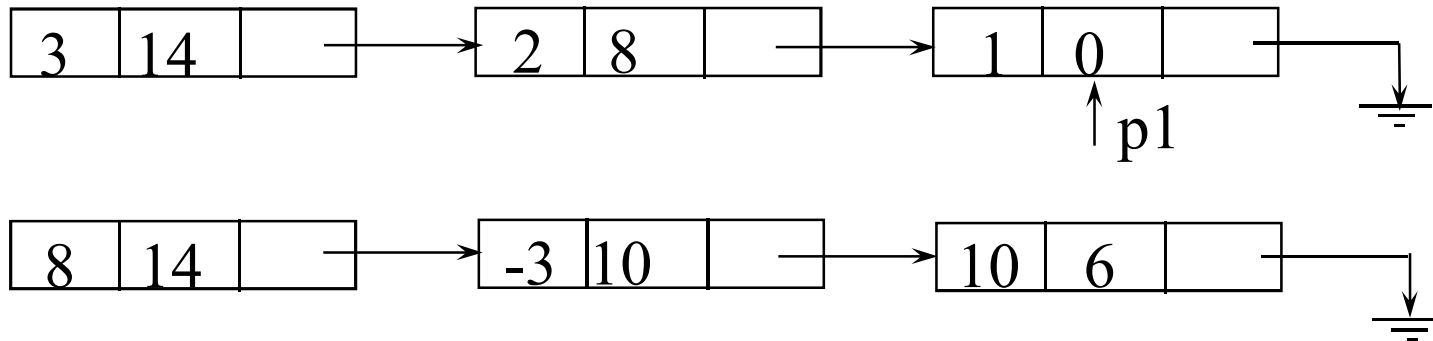
# Adding Two Polynomials



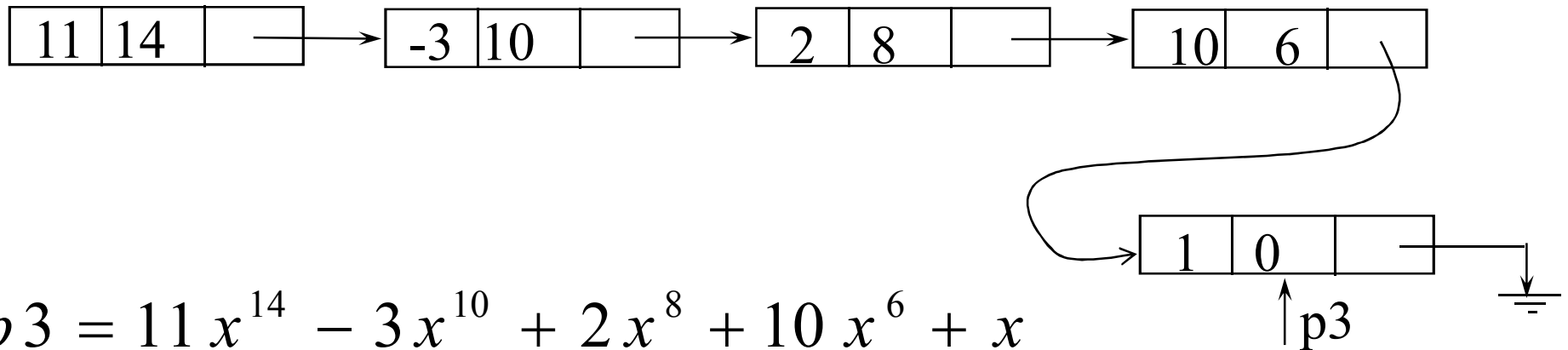
p1->expon < p2->expon



# Adding Two Polynomials



p2 = NULL



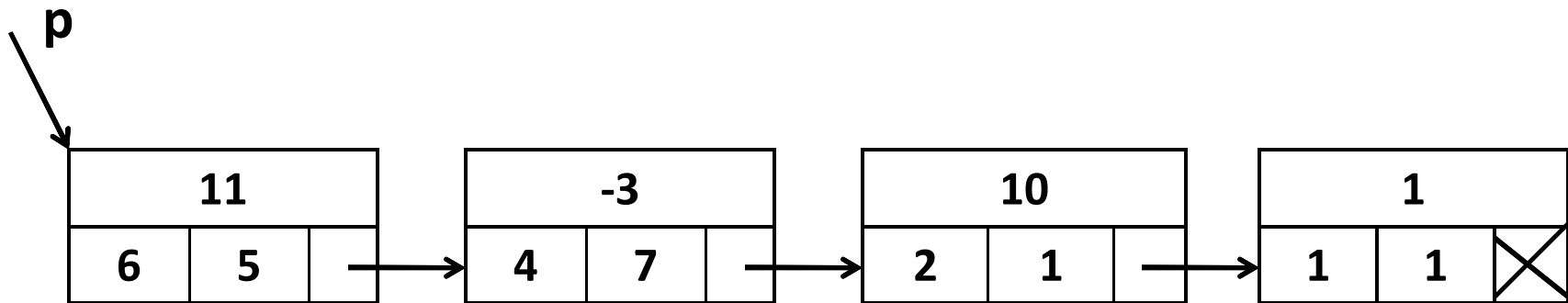
- **Home task**

Represent a two-variable polynomial using linked list.

$$4x^{10}y^7 - 10x^7y^6 + 13x^6y^6 - 23xy^2 + 62$$

# Representing two-variable polynomial

$$p = 11x^6y^5 - 3x^4y^7 + 10x^2y + xy$$



# Representing two-variable polynomial

## Another way

$$p = 11x^6y^5 - 3x^4x^7 + 10x^2y + xy$$

$x \searrow y \rightarrow$		1	2	3	4	5	6	7
1	<b>1</b>	0	0	0	0	0	0	0
2	<b>10</b>	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	<b>-3</b>
5	0	0	0	0	0	0	0	0
6	0	0	0	0	<b>11</b>	0	0	0

**Sparse Matrix**



# Sparse Matrix Representation

In a sparse matrix, most entries are zeroes:

$$\begin{bmatrix} -1 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 4 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 6 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

# Sparse Matrix Representation

Many problems require large sparse matrices.

For example, a college may have 1000 students and 500 different courses.

We can have a matrix  $M$  of size  $1000 \times 500$ , where  $M(i,j)$  represents the marks obtained by student  $i$  in course  $j$ . If  $M(i,j)=0$ , it means student  $i$  has not taken course  $j$ .

Each student takes only few courses. This will be sparse matrix with most of the elements 0.

Store as an ordinary array?  $1000 \times 500 = 500000$  entries  
Impractical (usually).

# Sparse Matrix Representation using Linked List

**Idea:** Only allocate memory for the entries that are non-zero.

**Mechanism:**

Store each non-zero entry in a node.

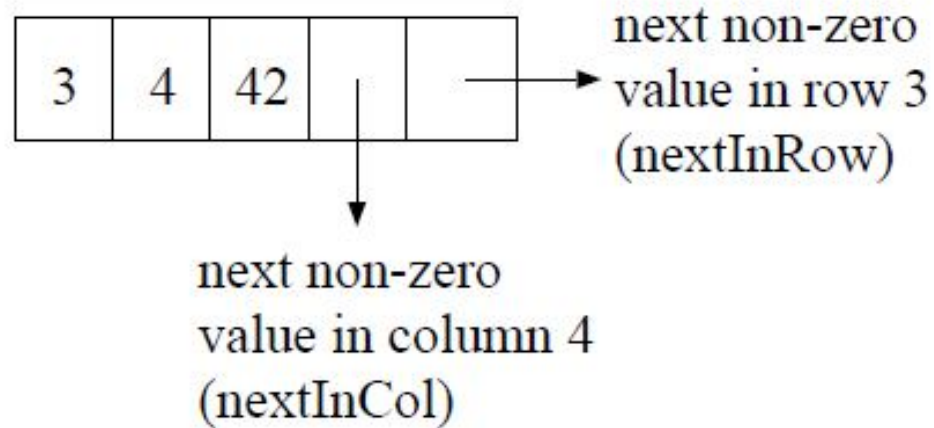
Use the heap to get the space for non-zero entries.

Link non-zero entries by row and by column.

Each node will need to know its row number and column number and (non-zero) value.

# An Example Node

For example, row 3 and column 4 (i.e., entry [3,4]) contains 42:



```
struct Node{  
    int row, col, value;  
    Node* NextInCol;  
    Node* NextInRow;  
}
```

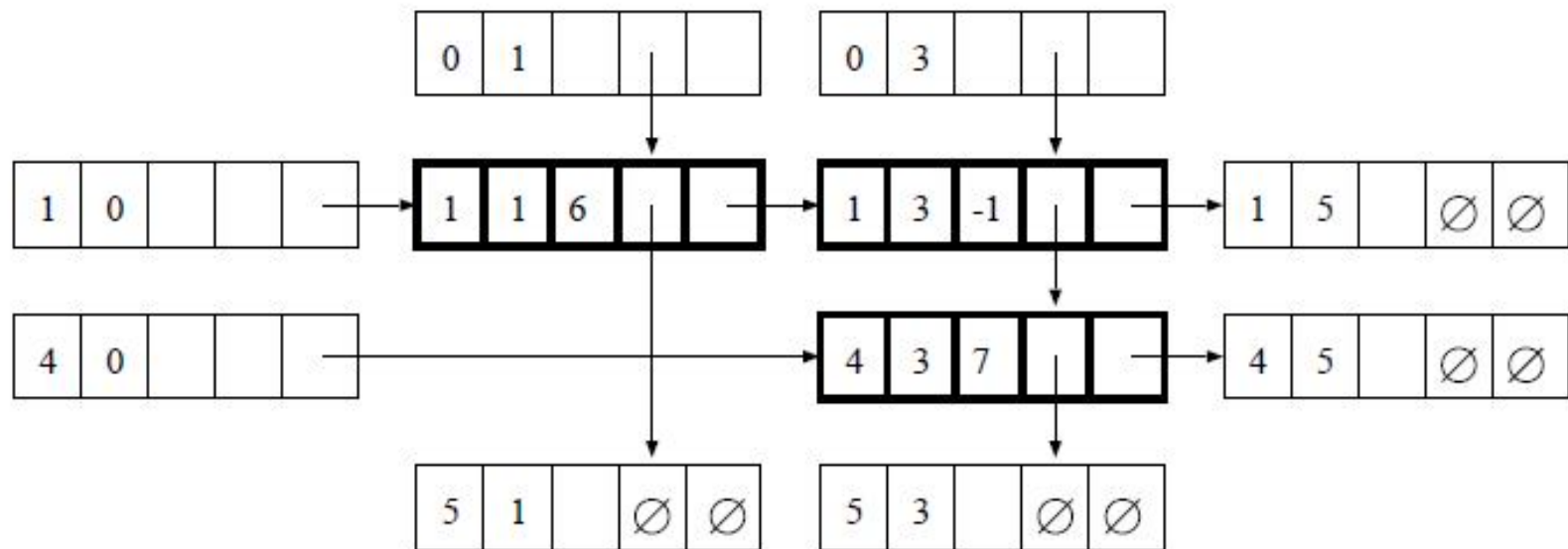
# Row and Columns

- A row or a column is a linked list of non-zero entries.
- We will use plain linked lists (not circular or doubly-linked) with dummy nodes.
- The dummy header nodes would contain row or column number 0.
- The dummy trailer nodes would contain row number **numRows+1** or column number **numCols+1**.

# Example

Example:

$$\begin{bmatrix} 6 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix}$$



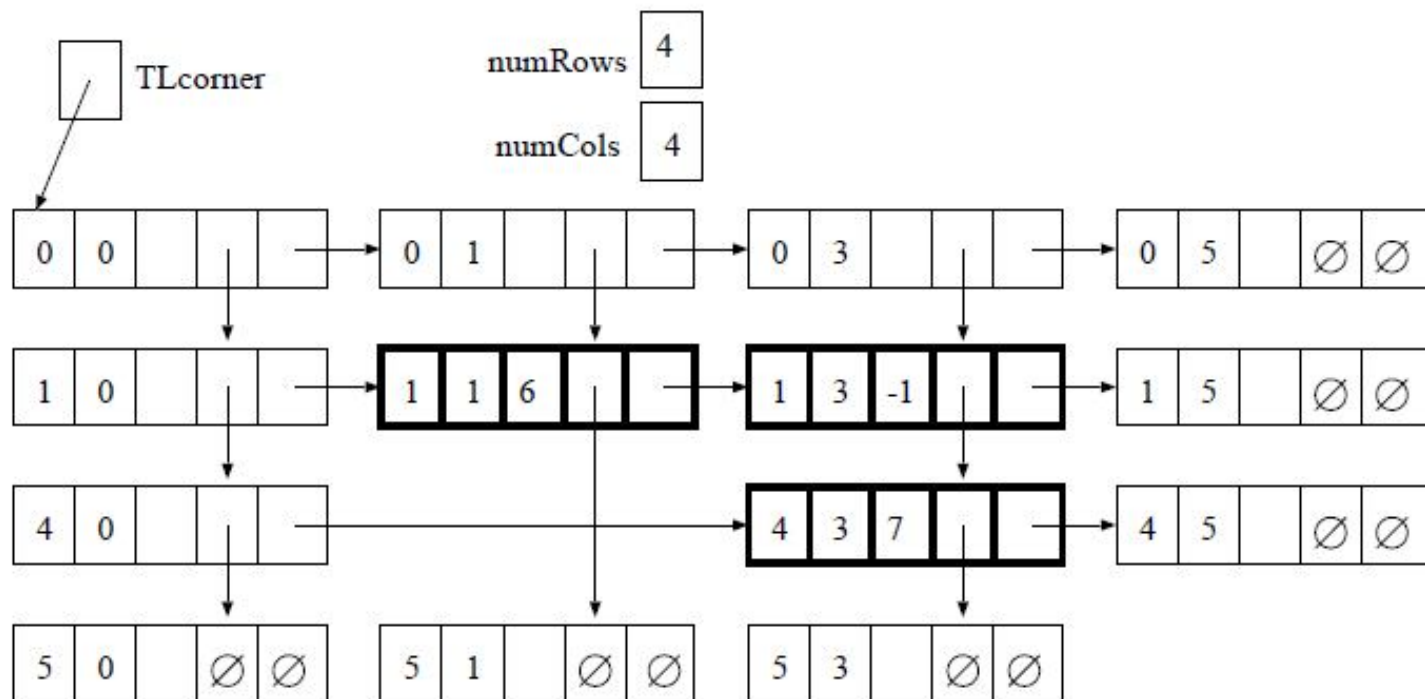
# Searching and Element

- We will need to search for particular rows or columns.
- **Idea:** Link together “row 0”, which is a list of all the available columns, and link together “column 0”, which is a list of all the available rows.
- How about dummy nodes for those lists?  
Good idea.

# Example

Example (re-examined):

$$\begin{bmatrix} 6 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix}$$





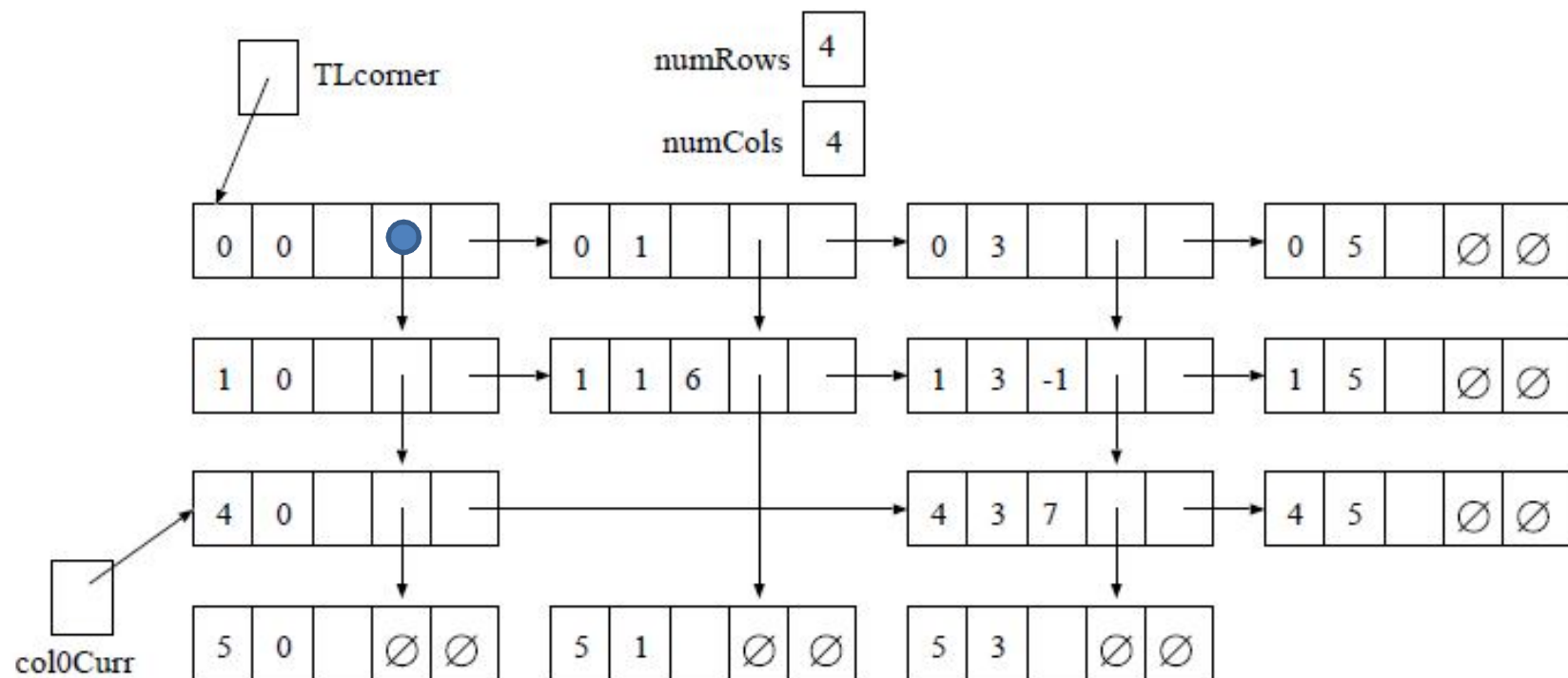
# General Search Strategy

## Search for [R,C] element

- Search down dummy column 0 looking for the row list for row R (or search across dummy row 0 looking for the column list for column C).
  - If the row list (or column list) doesn't exist, then all entries in that row (or column) are 0.
- Then search along row R looking for a node in column C (or search down column C looking for a node in row R).
- If the node you're looking for doesn't exist, then the value of that entry is 0.
- We always search an ordered list.

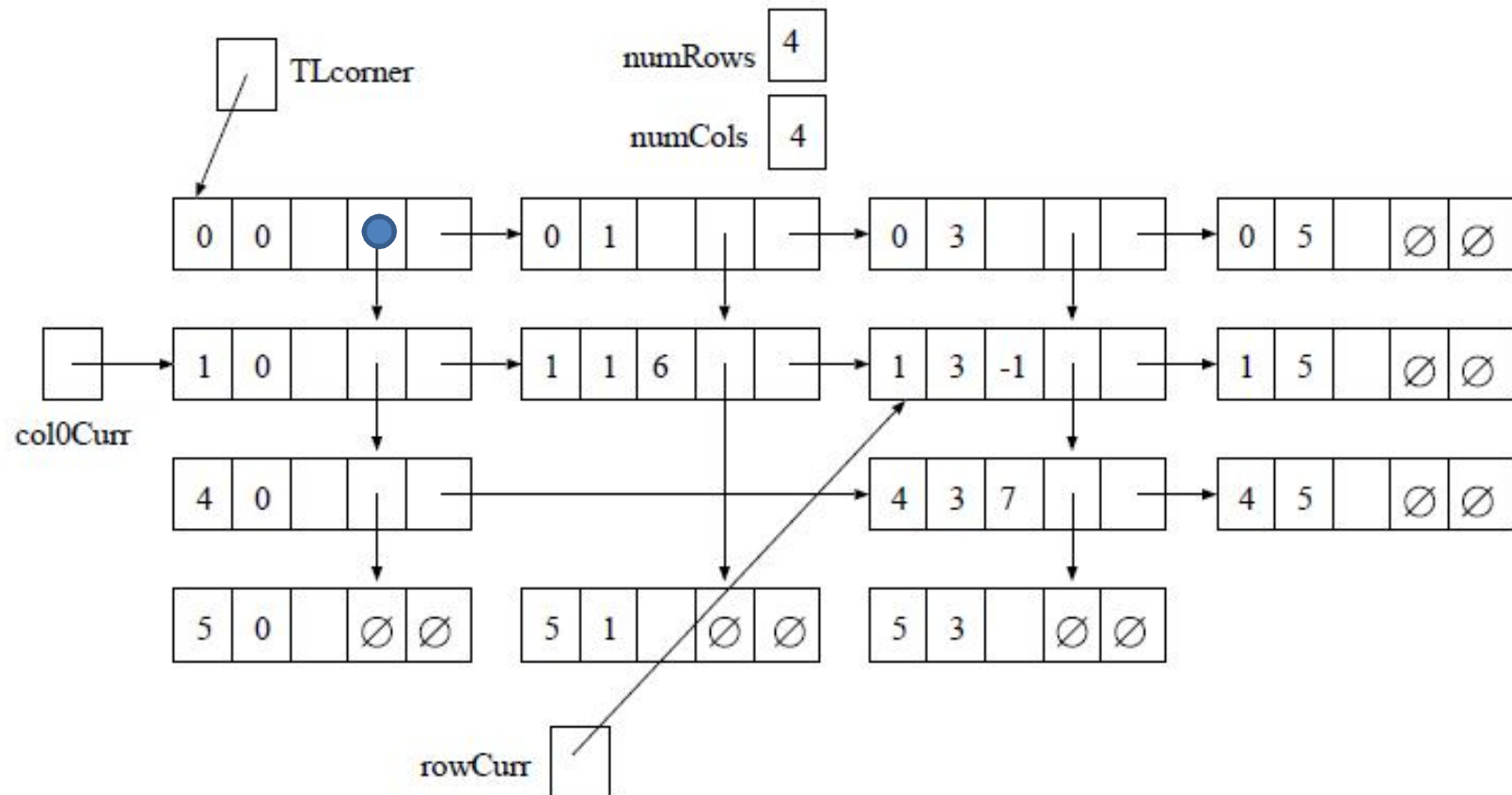
# Example

We can see that the value of entry  $[3,2]$  is zero because there is no list for row 3:



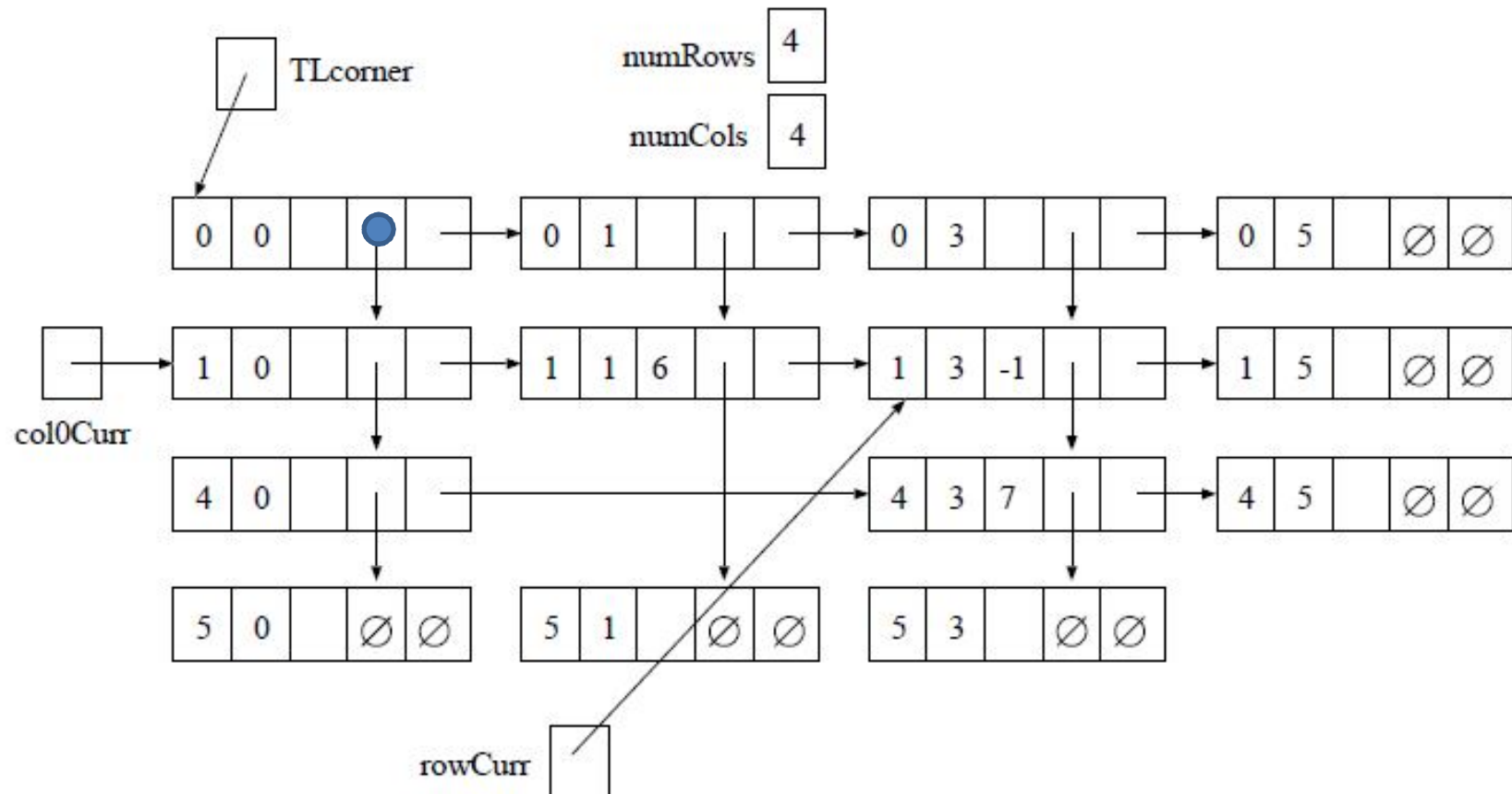
# Example

We can see that the value of entry  $[1,2]$  is zero because, although there is a list for row 1, the list does not contain a node in column 2:



# Example

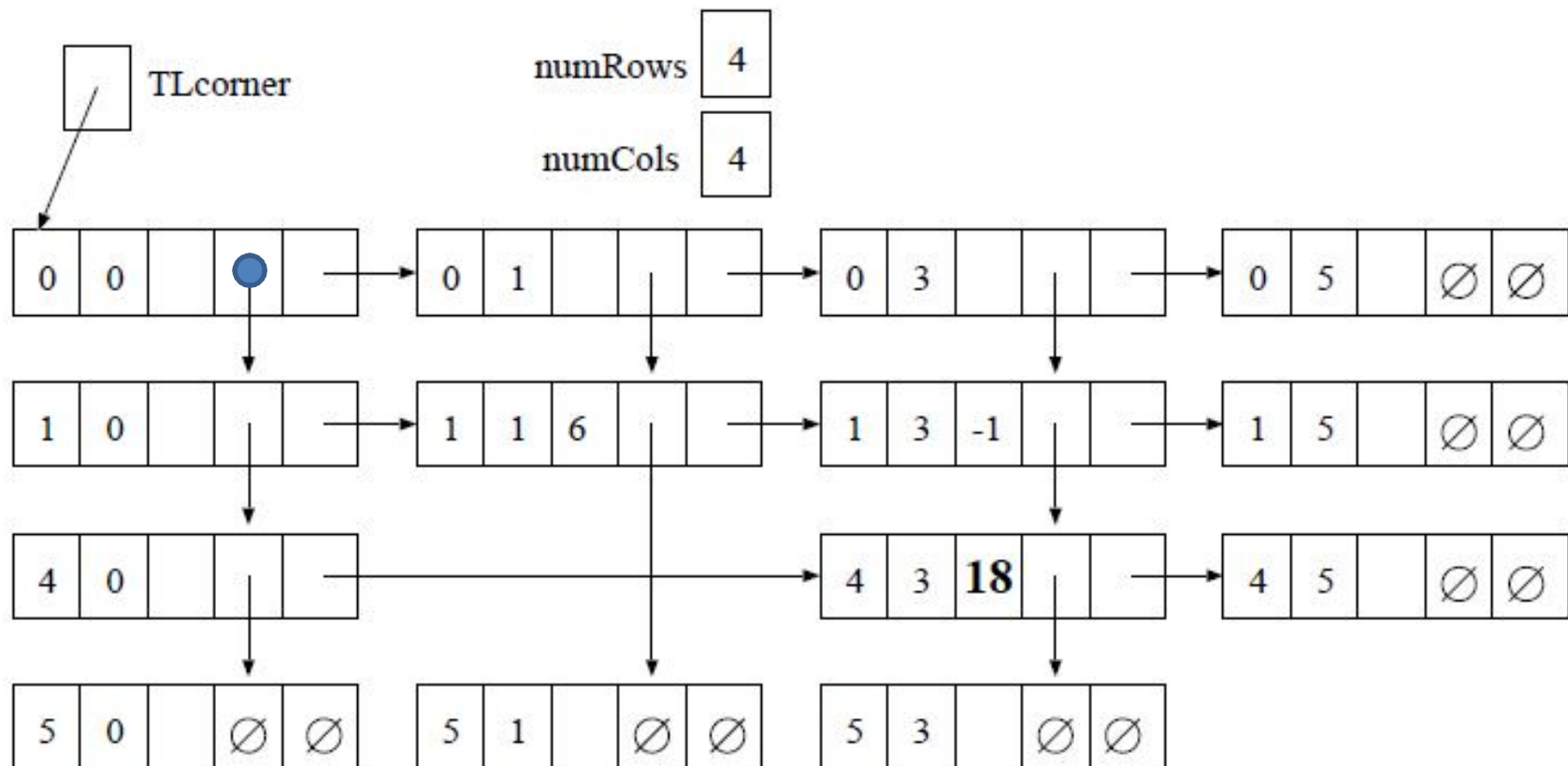
We can see that the value of entry  $[1,3]$  is  $-1$  because there is a list for row 1, it contains a node in column 3 and that node contains the value  $-1$ .



# Setting a new value

If `newValue` is not 0 and a node already exists for entry `[R,C]`, simply change the value in the existing node.

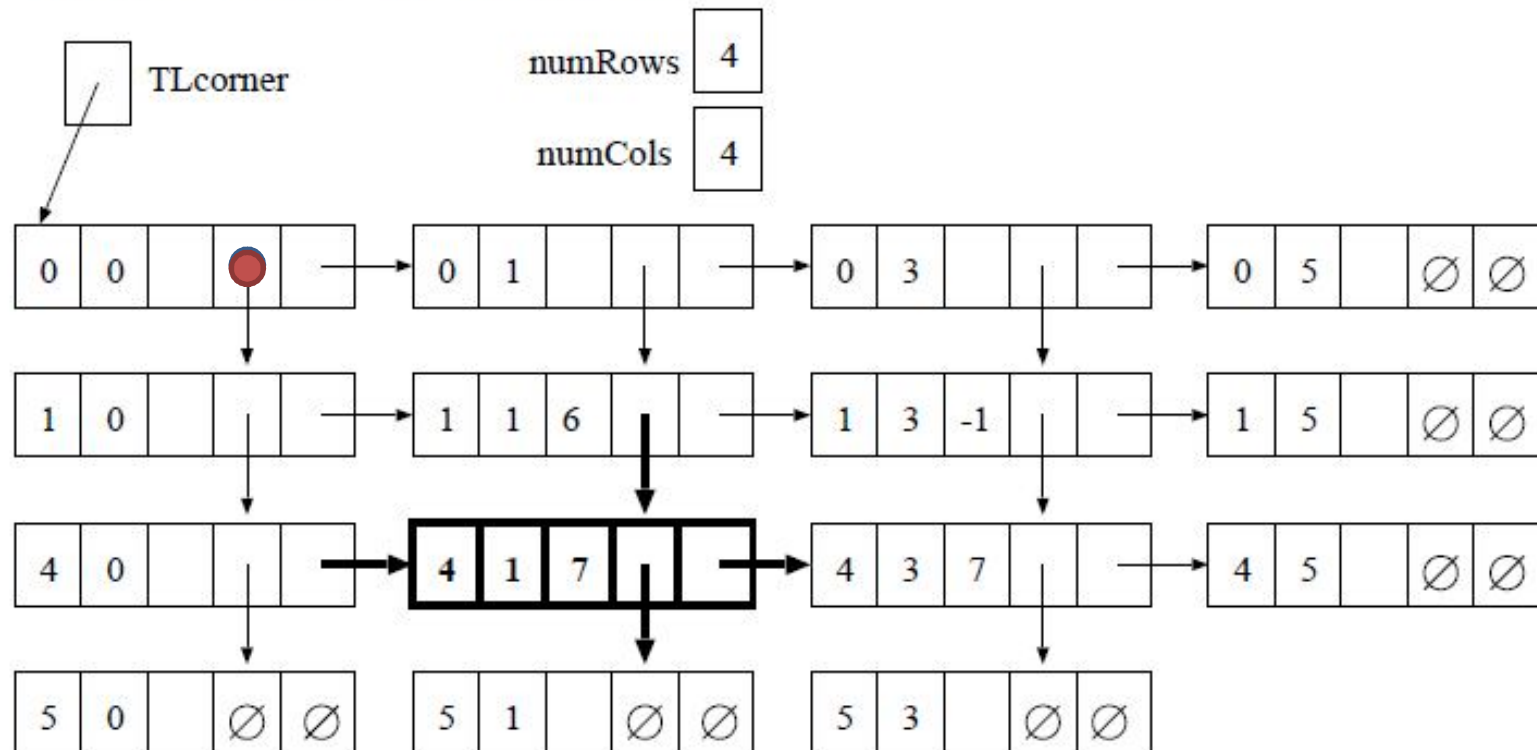
`setValue(4,3,18)` causes the following change:



# More Example

If `newValue` is not 0, but no node exists for entry  $[R,C]$ , add a new node for entry  $[R,C]$ . If lists for both row  $R$  and column  $C$  already exist, simply link the new node into those lists in the appropriate positions:

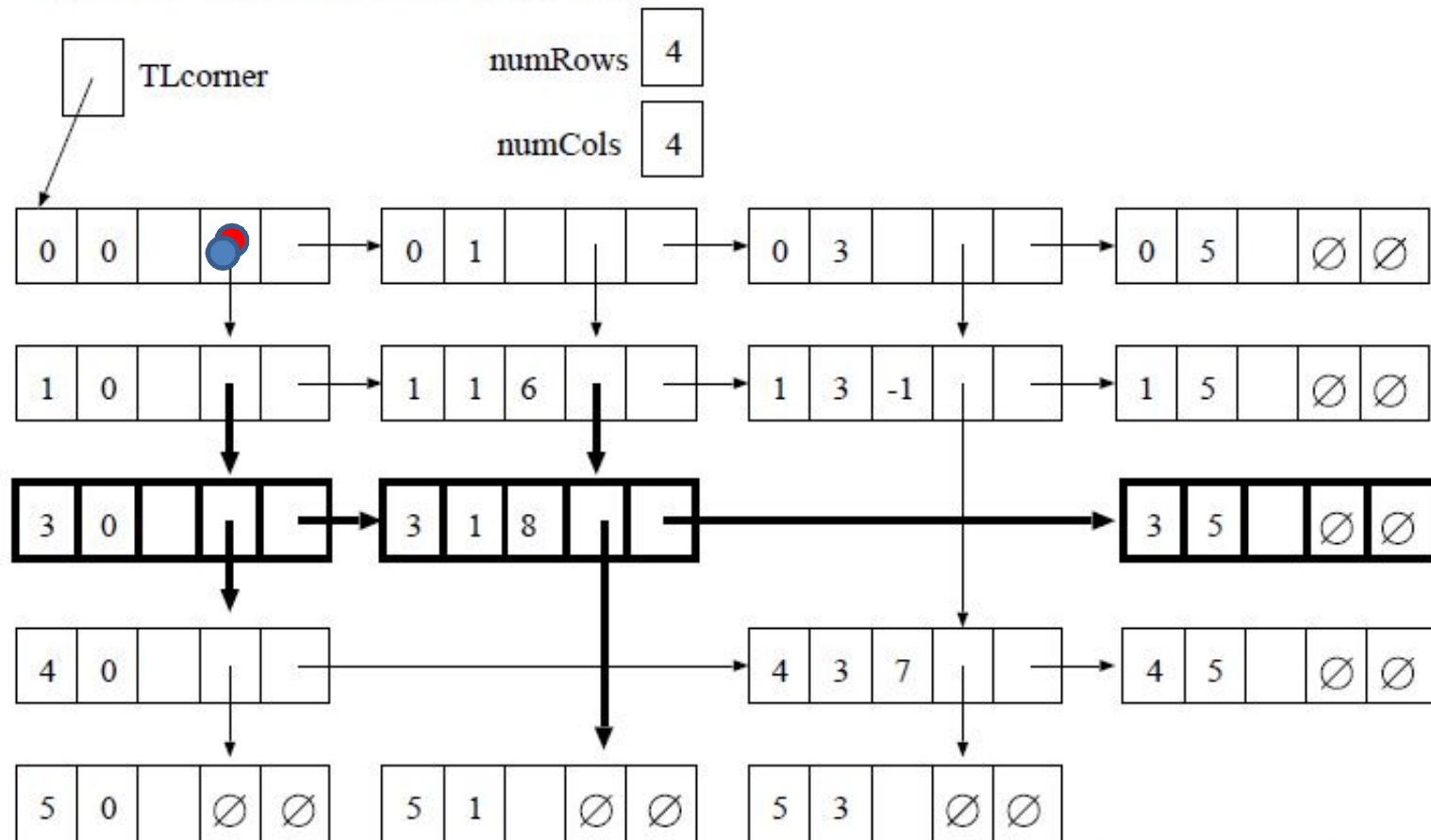
`setValue(4,1,7)` causes the following changes:



# More Example

If `newValue` is not 0, no node exists for entry  $[R,C]$ , but no list exists for row  $R$ , then add a new node for entry  $[R,C]$  and create a new row list for row  $R$ .

`setValue(3,1,8)` causes the following changes:





# Example

If `newValue` is not 0, you might have to add a new list for column C.

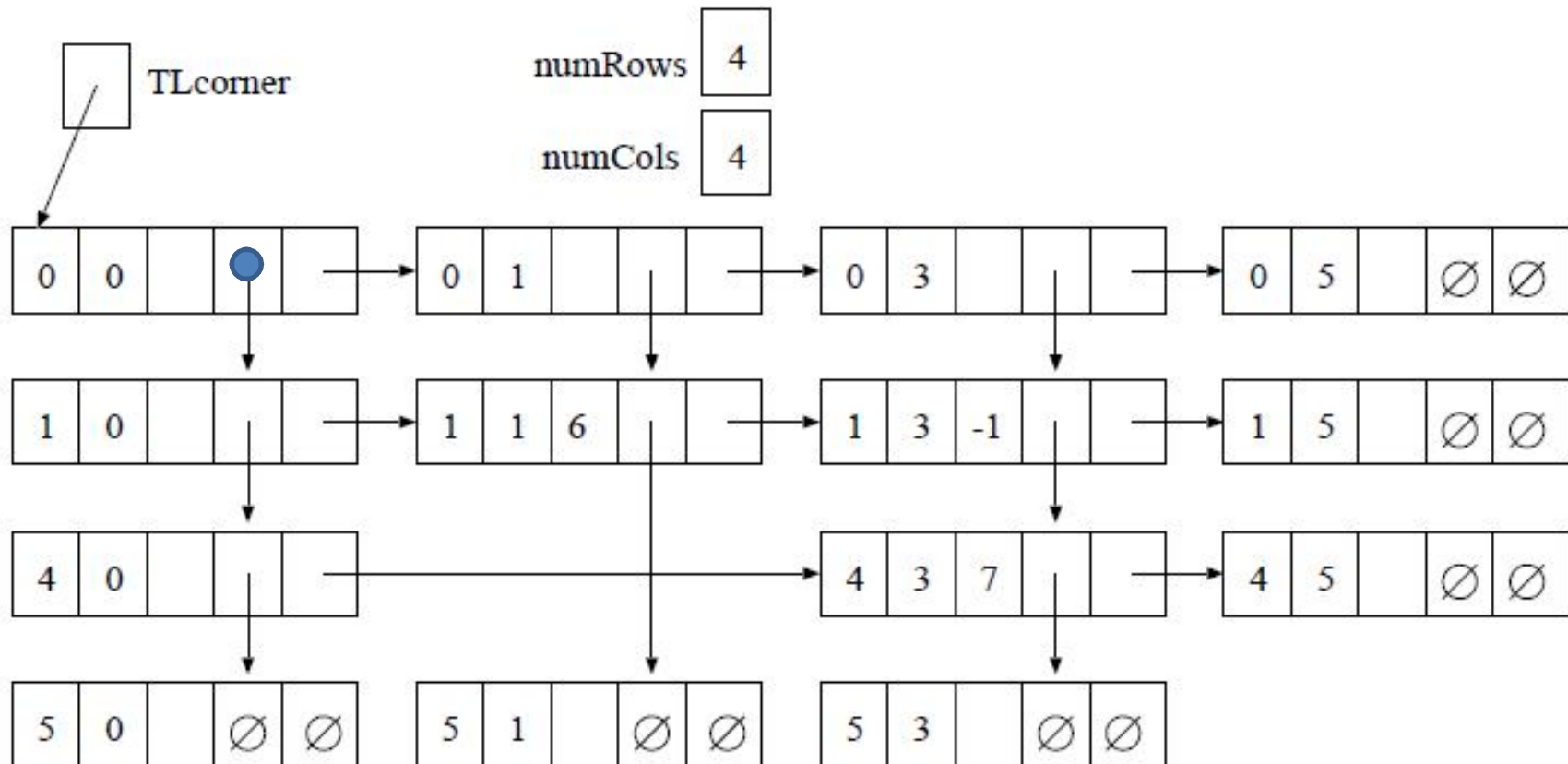
You might have to add new lists for both row R and column C.  
(Pictures not shown.)



# Example

If `newValue` is 0 and there is no node for entry `[R,C]`, then do nothing!

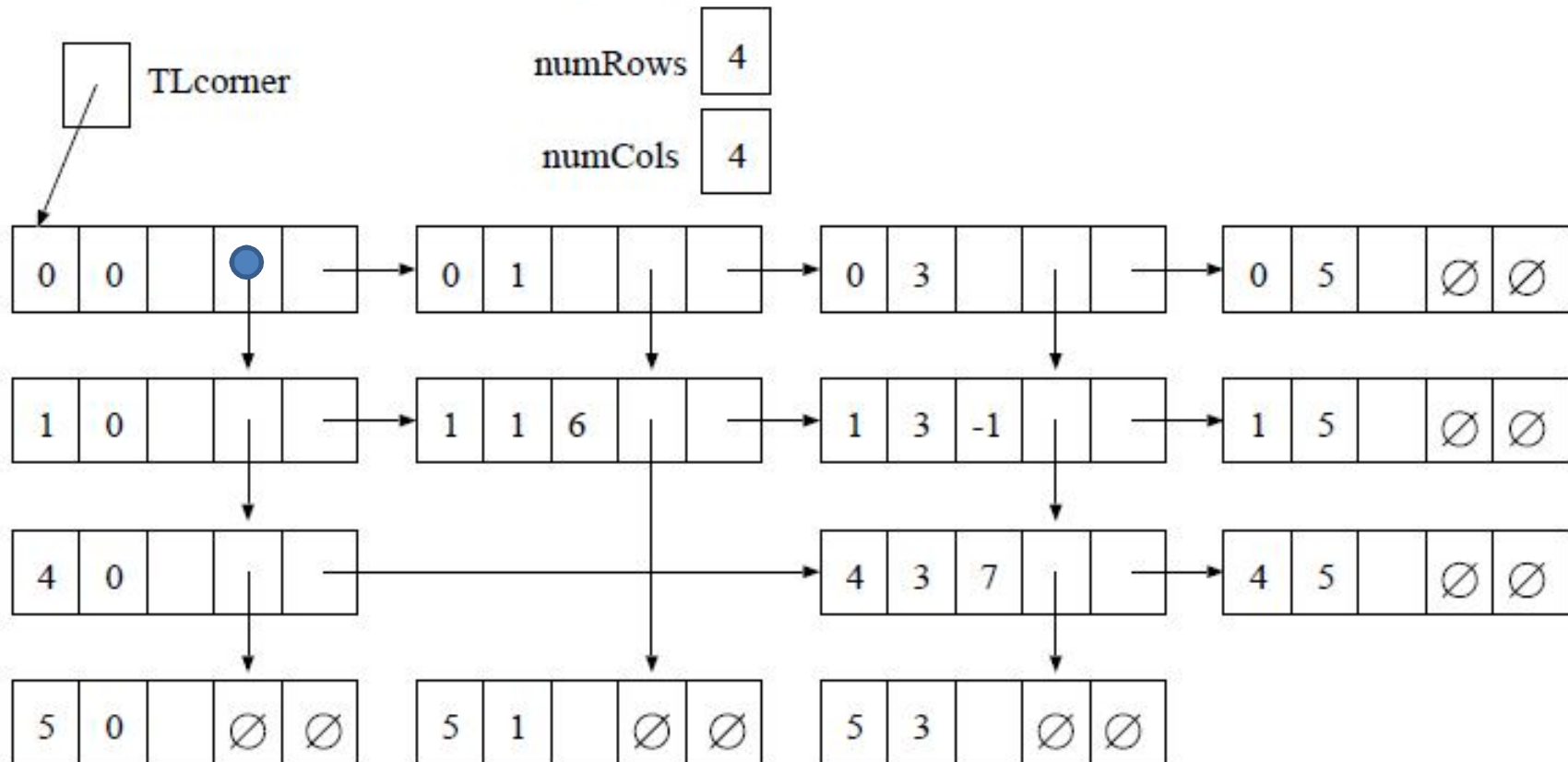
`setValue(3,2,0)` does nothing:



# Example

If `newValue` is 0 and there is a node for entry `[R,C]`, then delete that node. If row `R` and column `[C]` both contain other non-zero entries, simply unlink that node from the row list and the column list.

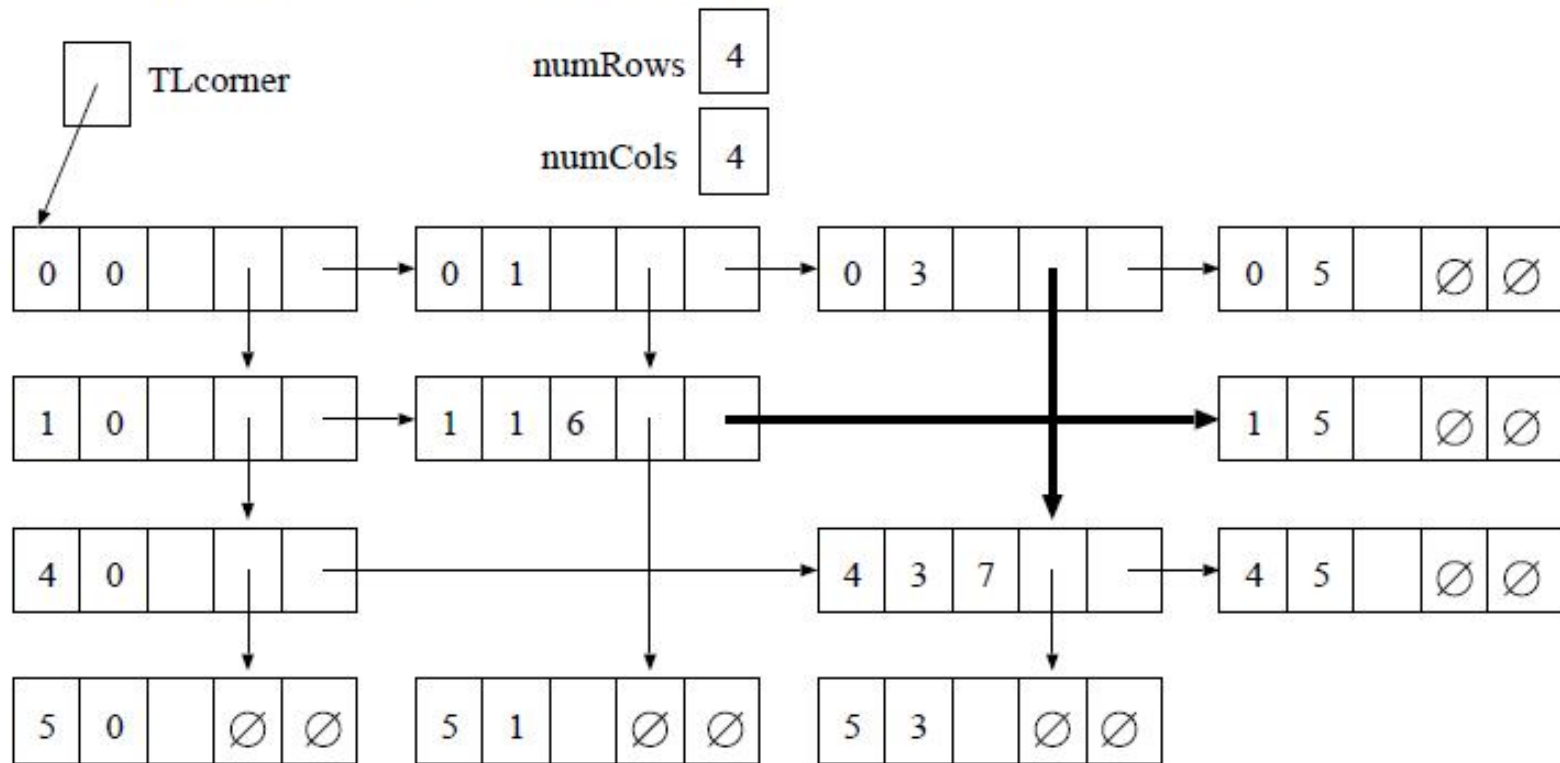
`setValue(1,3,0)` causes the following changes:



# Example

If `newValue` is 0 and there is a node for entry  $[R,C]$ , then delete that node. If row  $R$  and column  $[C]$  both contain other non-zero entries, simply unlink that node from the row list and the column list.

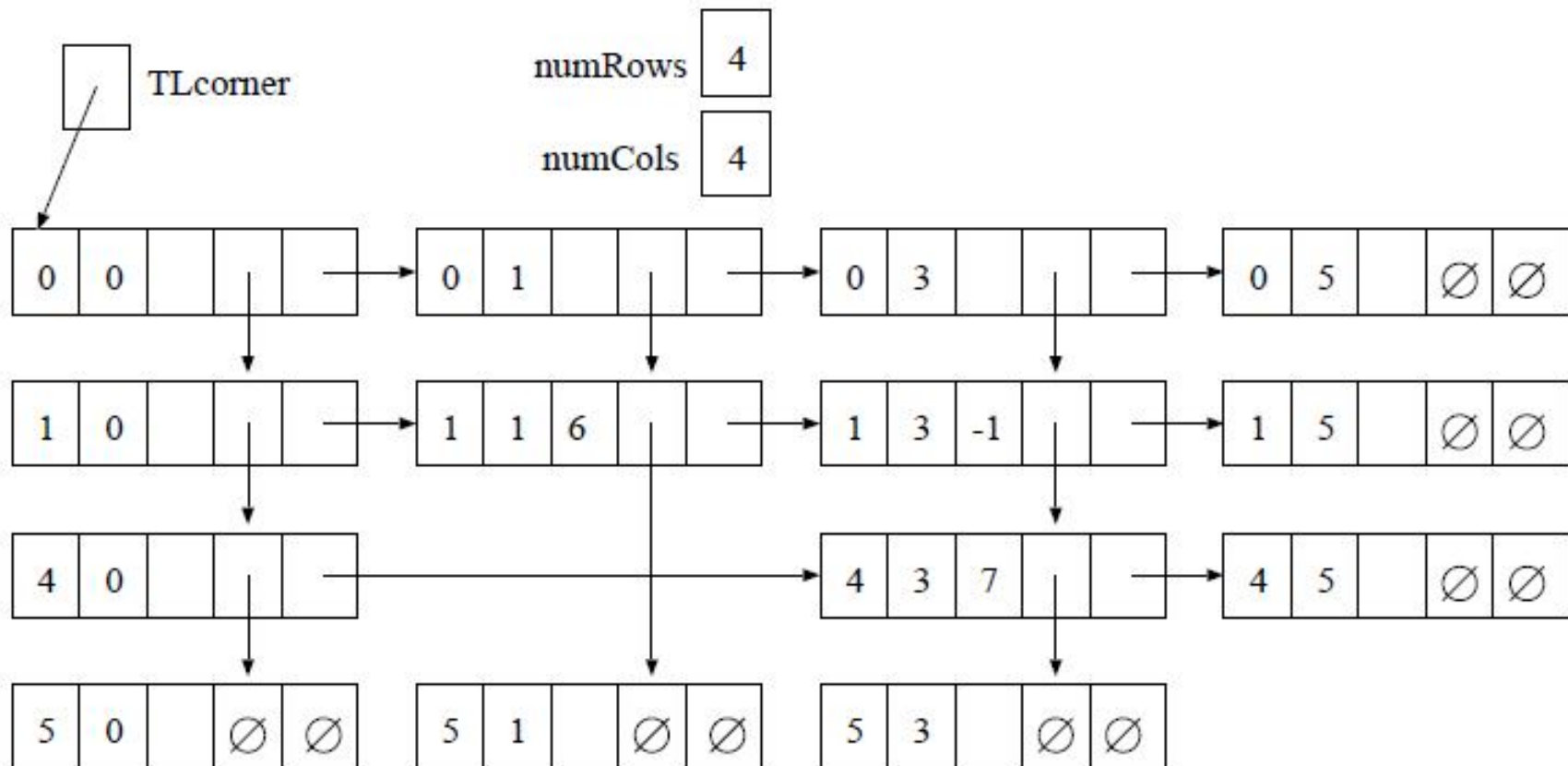
`setValue(1,3,0)` causes the following changes:



# Example

If `newValue` is 0, a node for entry  $[R,C]$  exists, but row  $R$  contains no other non-zero entries, then you have to delete the entire row  $R$ .

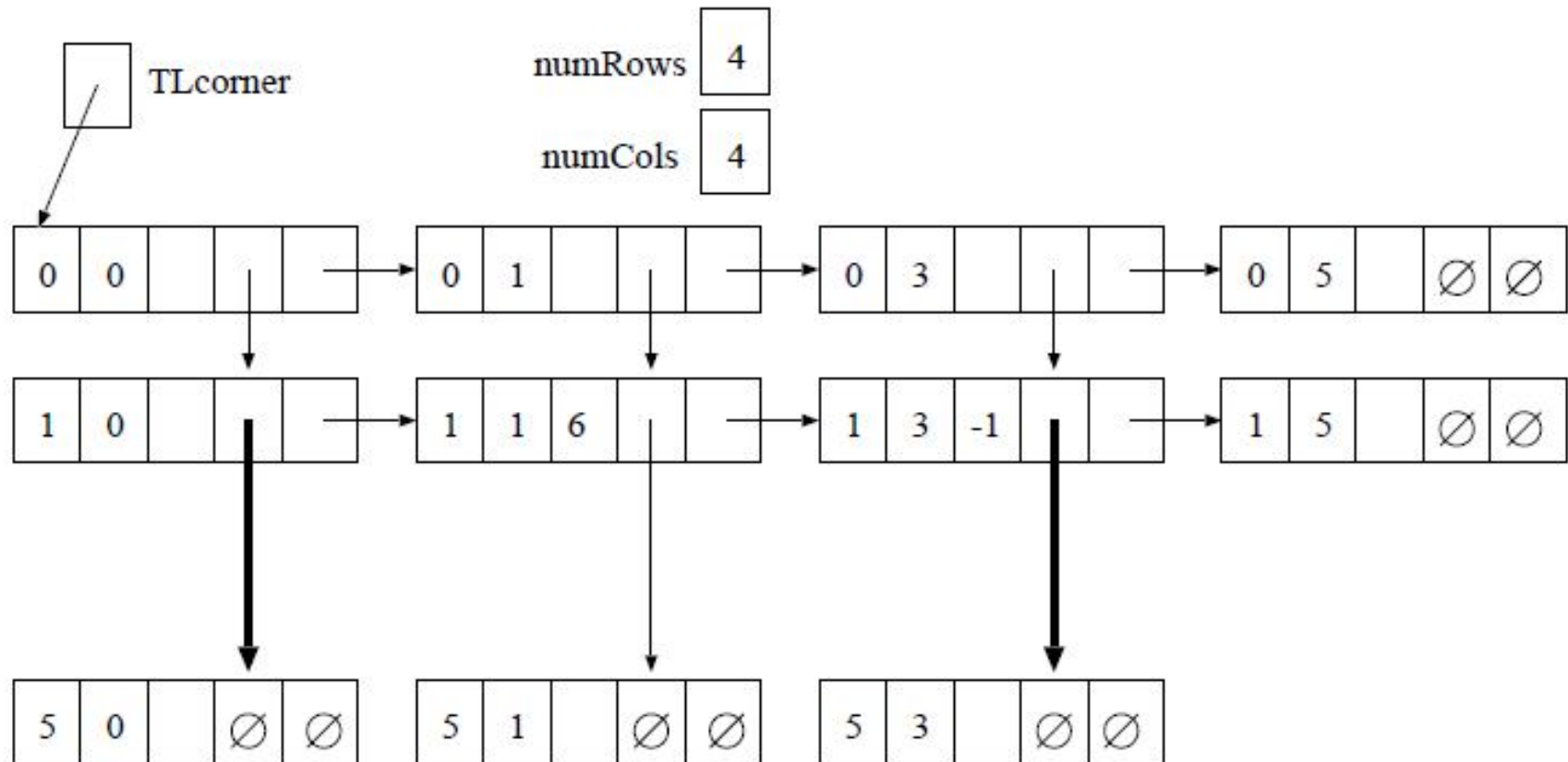
`setValue(4,3,0)`



# Example

If `newValue` is 0, a node for entry  $[R,C]$  exists, but row  $R$  contains no other non-zero entries, then you have to delete the entire row  $R$ .

`setValue(4,3,0)` causes the following changes:



# Example

- If newValue is 0 and you delete the only non-zero value in column C, then you have to delete the column list for column C.
- You might have to delete both the list for row R and the list for column C.

(Pictures omitted.)