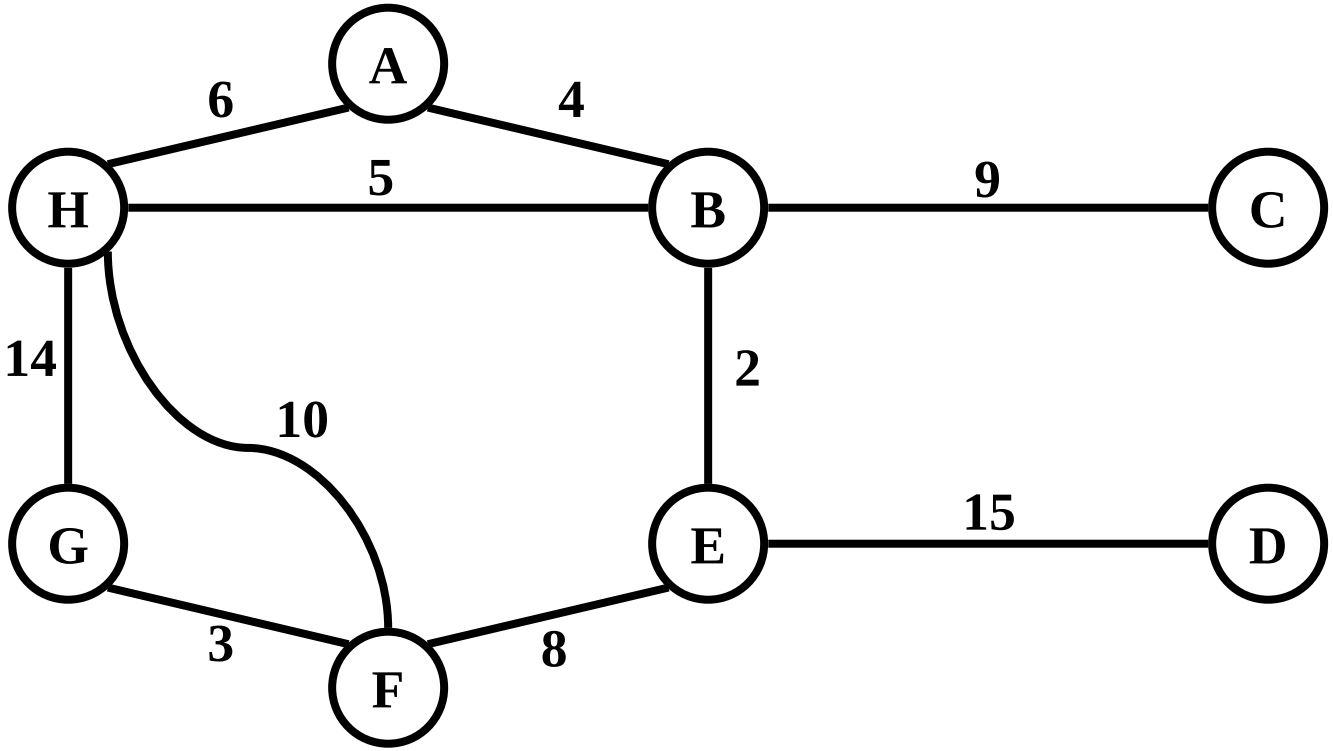
GREEDY METHOD
KRUSKAL'S ALGORITHM

Minimum Spanning Tree

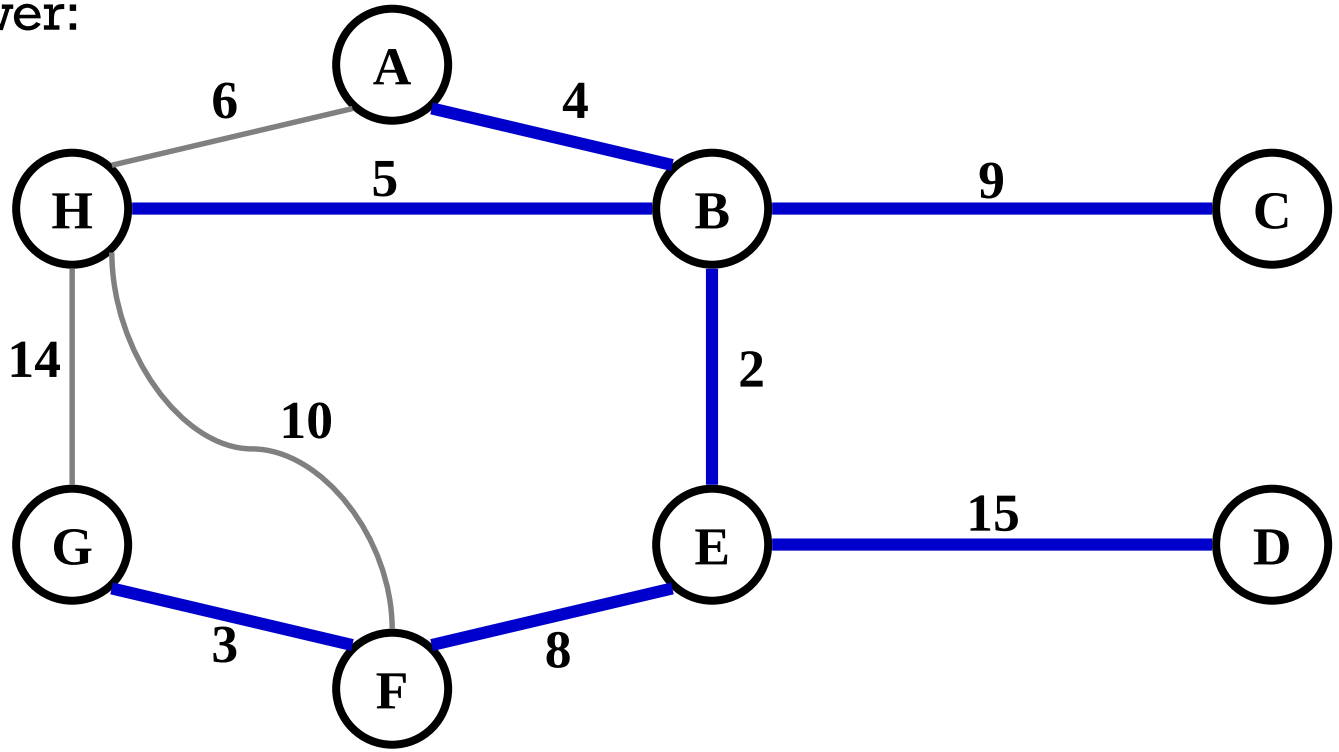
MINIMUM SPANNING TREE

- *Which edges form the minimum spanning tree (MST) of the graph as shown below?*



MINIMUM SPANNING TREE

- Answer:



KRUSKAL'S ALGORITHM

Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which

- form a tree that includes every vertex
- has the minimum sum of weights among all the trees that can be formed from the graph

HOW KRUSKAL'S ALGORITHM WORKS

- It falls under a class of algorithms called greedy algorithms that find the local optimum in the hopes of finding a global optimum.
- We start from the edges with the lowest weight and keep adding edges until we reach our goal.
- The steps for implementing Kruskal's algorithm are as follows:
  1. Sort all the edges from low weight to high
  2. Take the edge with the lowest weight and add it to the spanning tree. If adding the edge created a cycle, then reject this edge.
  3. Keep adding edges until we reach all vertices.

DISJOINT-SET | UNION-FIND

Any minimum spanning tree algorithm revolves around checking if adding an edge creates a loop or not. The most common way to find this out is an algorithm called Union Find. The Union-Find algorithm divides the vertices into clusters and allows us to check if two vertices belong to the same cluster or not and hence decide whether adding an edge creates a cycle.

Need to support following operations:

- MakeSet(x): $S = S \cup \{\{x\}\}$
- Union(S_i, S_j): $S = S - \{S_i, S_j\} \cup \{S_i \cup S_j\}$
- FindSet(x): return $S_i \in S$ such that $x \in S_i$

- Before discussing implementation details, we look at application: MSTs

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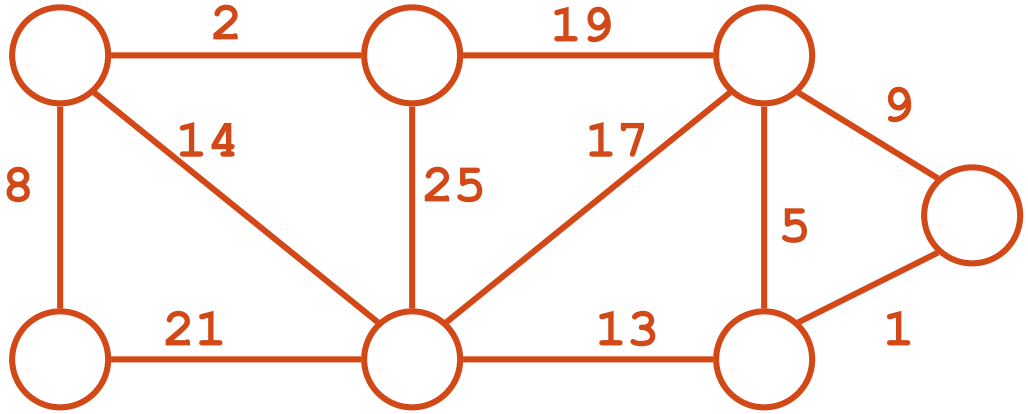


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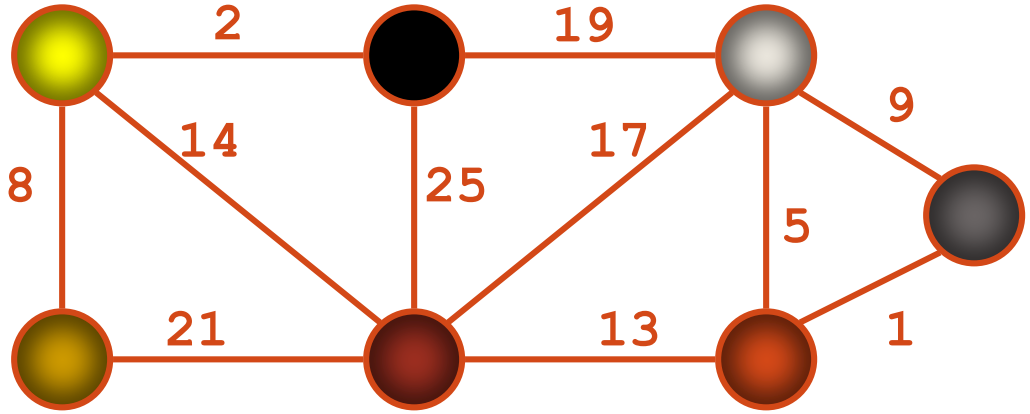


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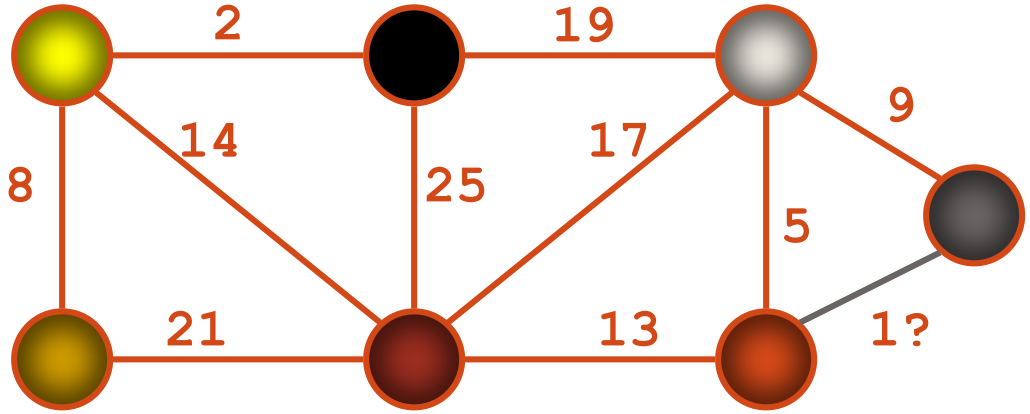


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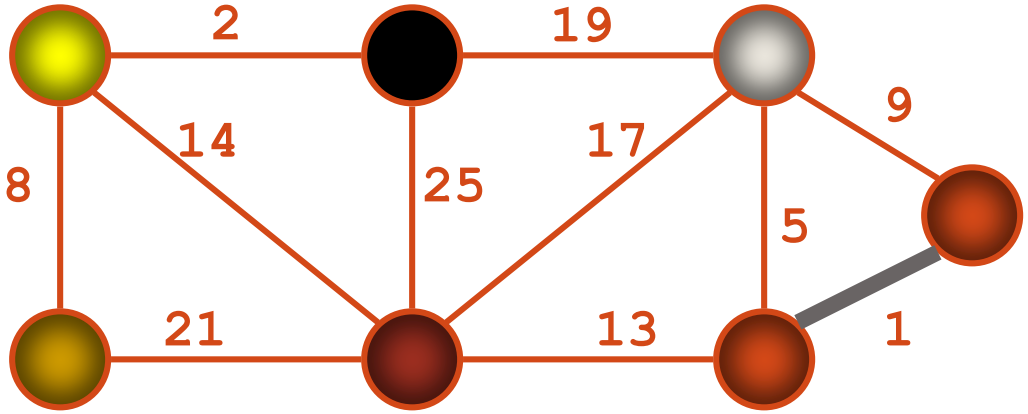


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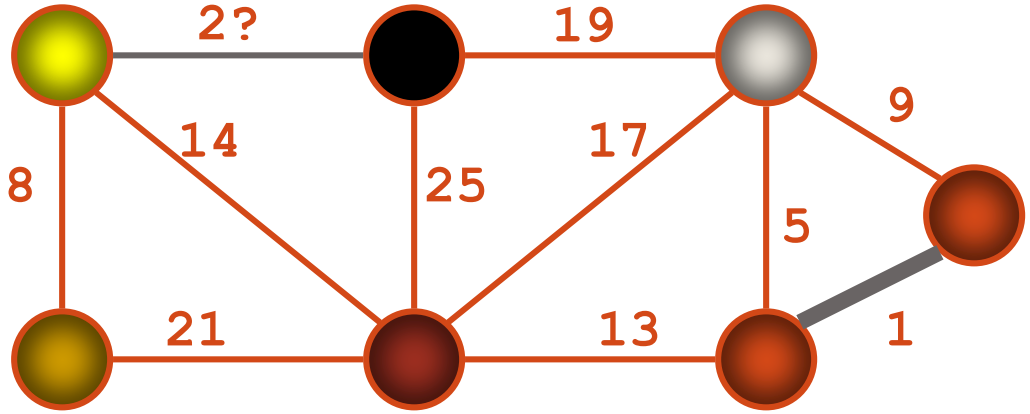


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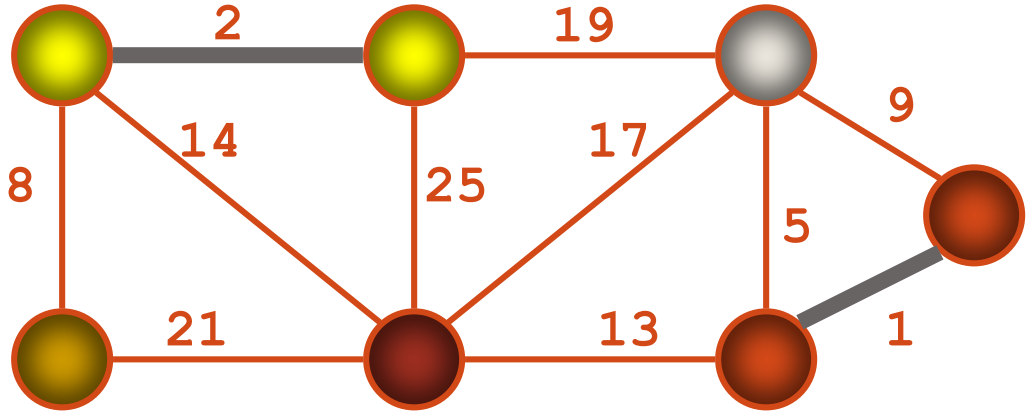


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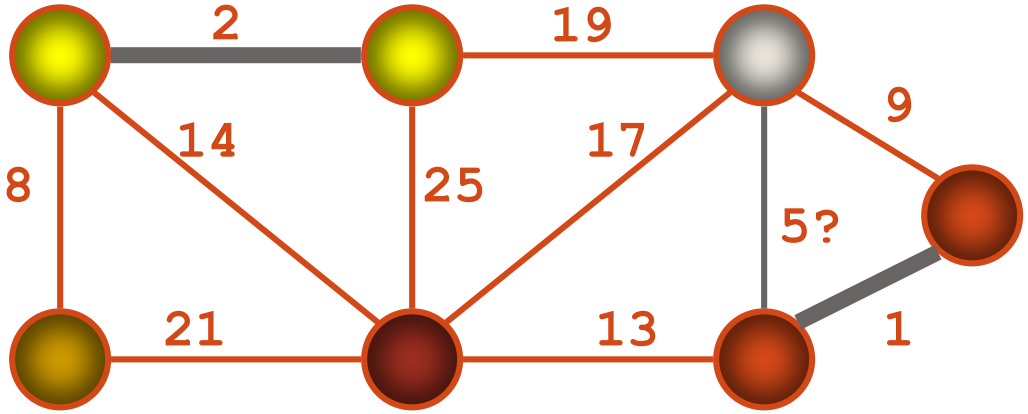


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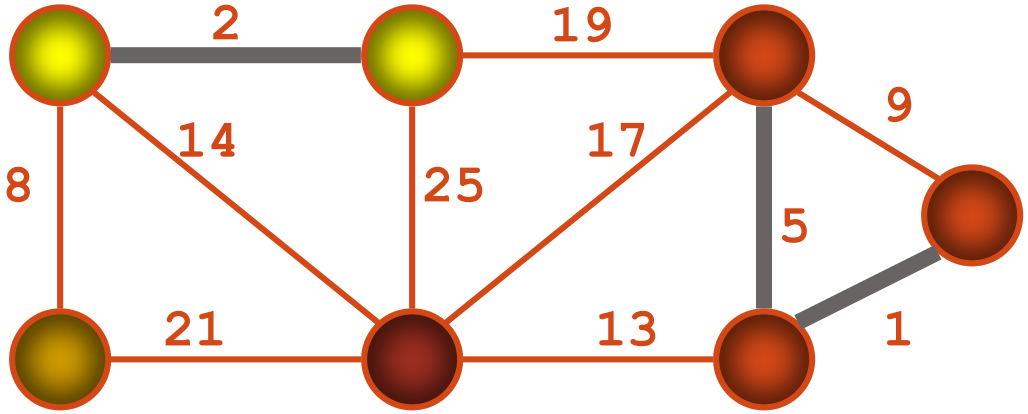


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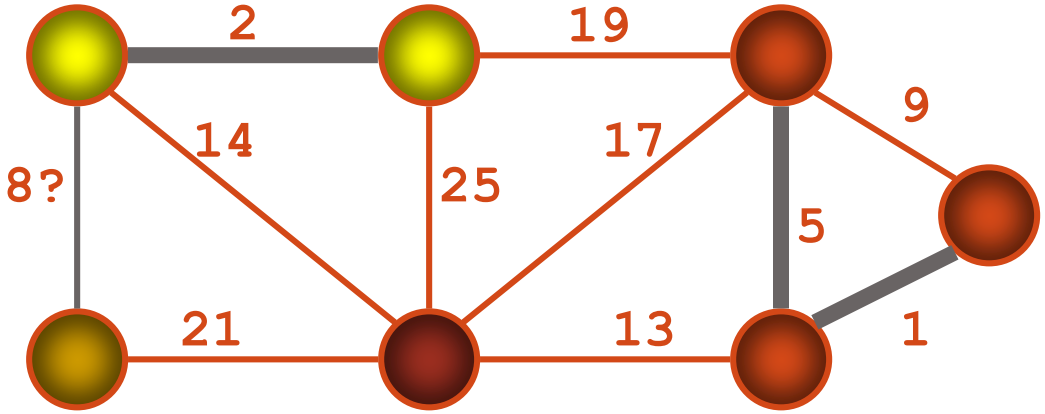


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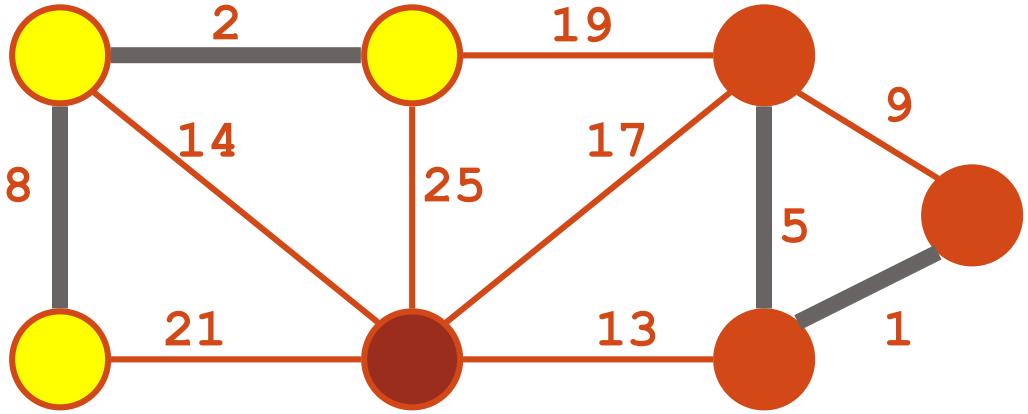


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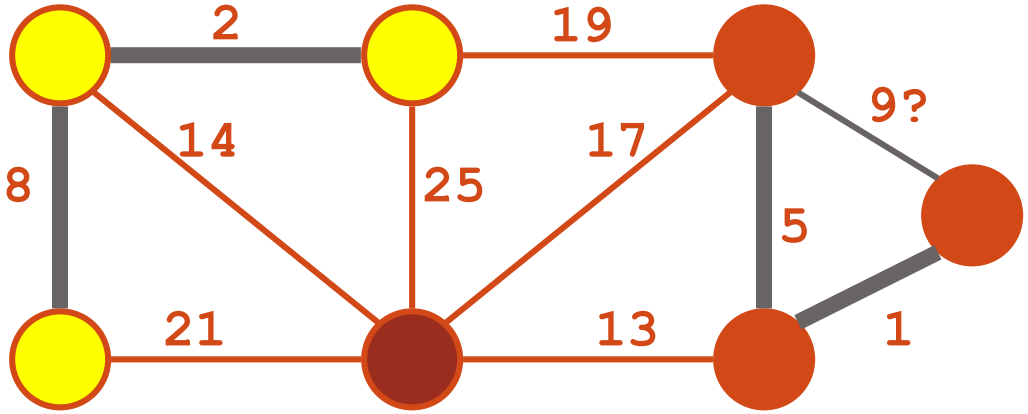


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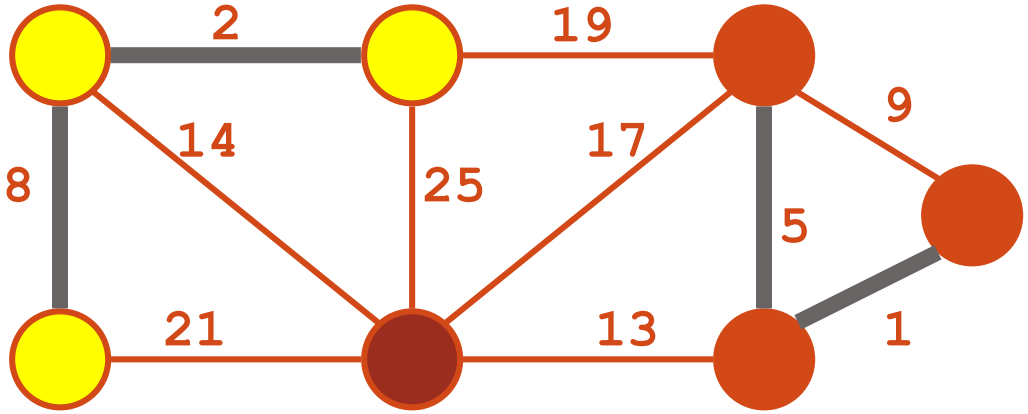


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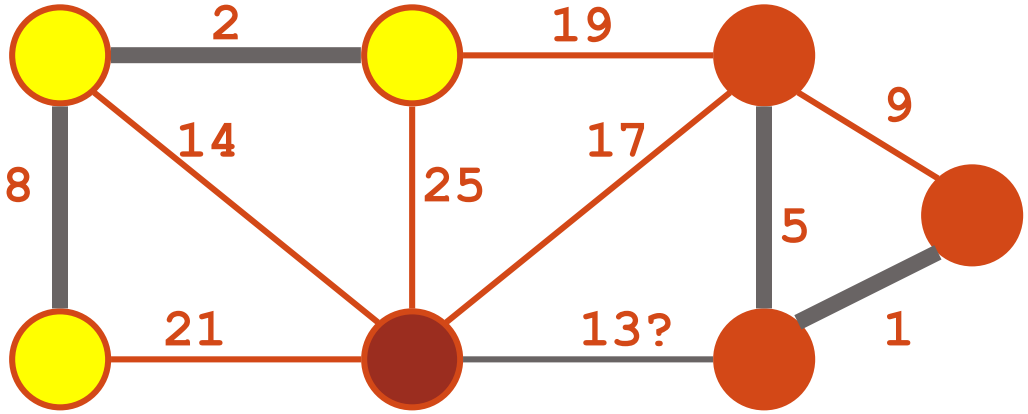


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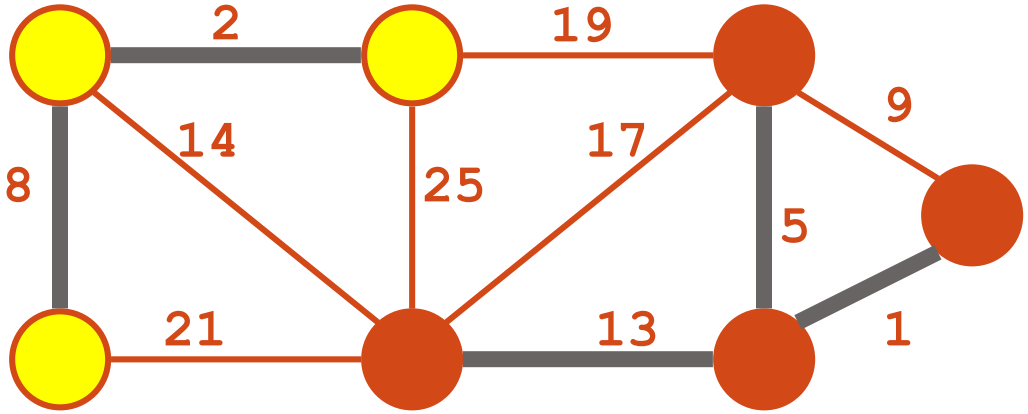


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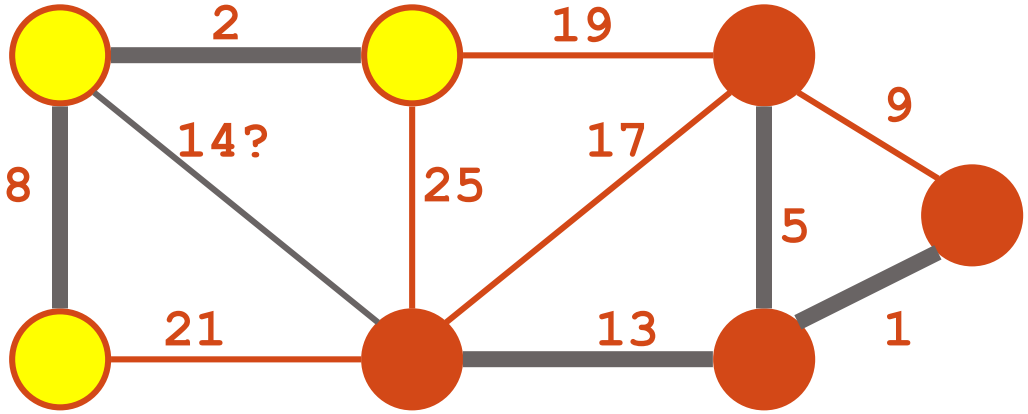


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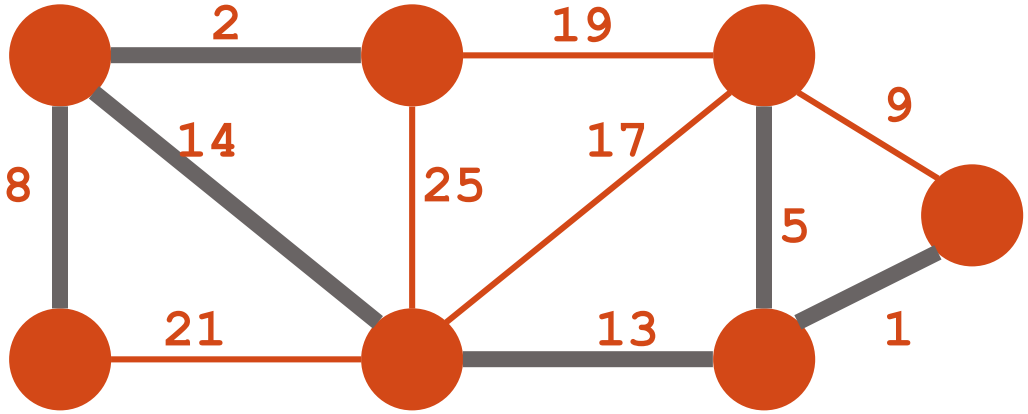


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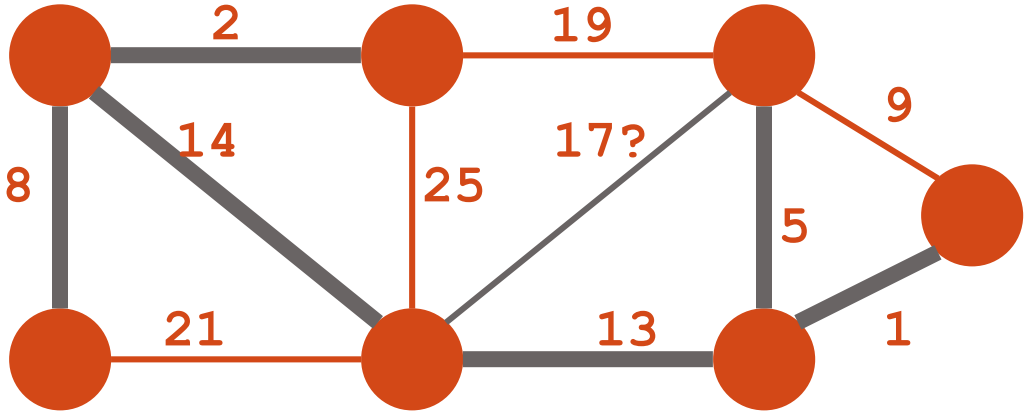


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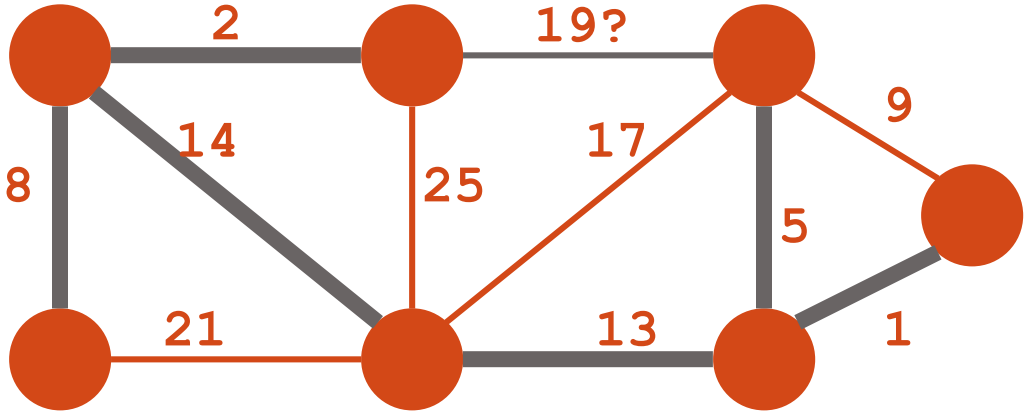


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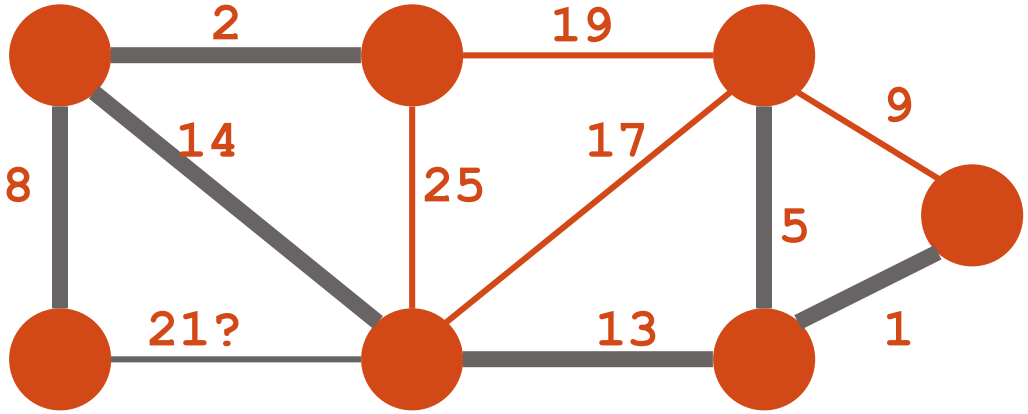


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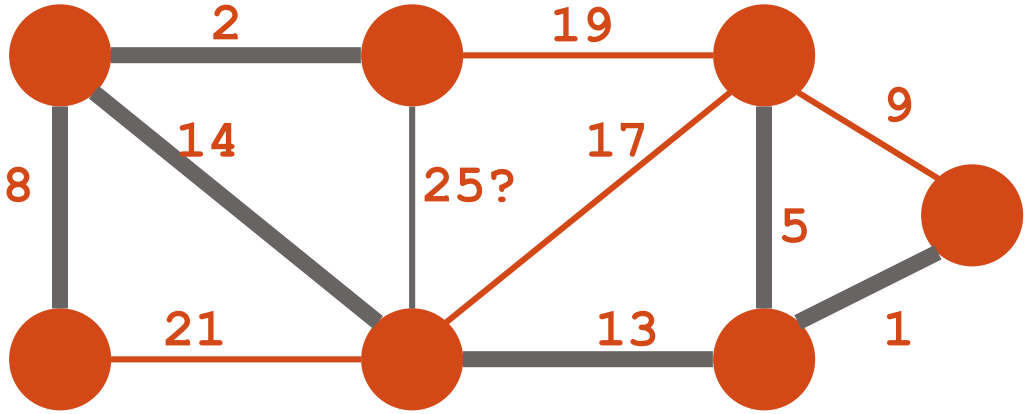


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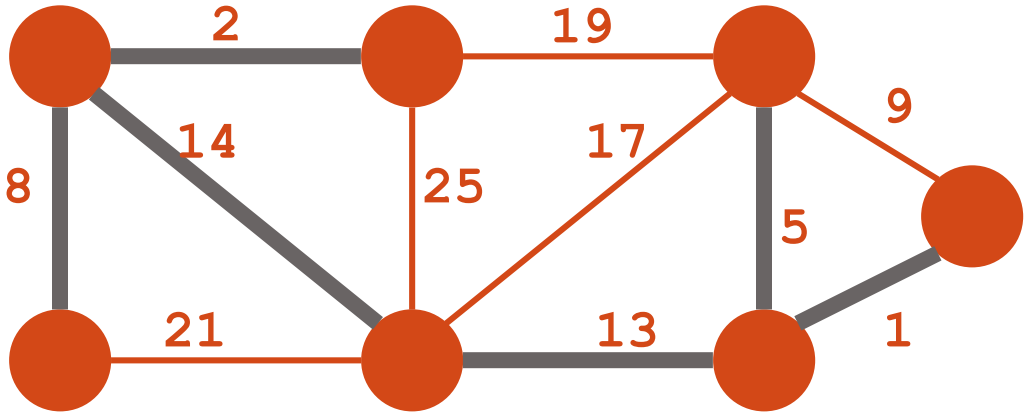


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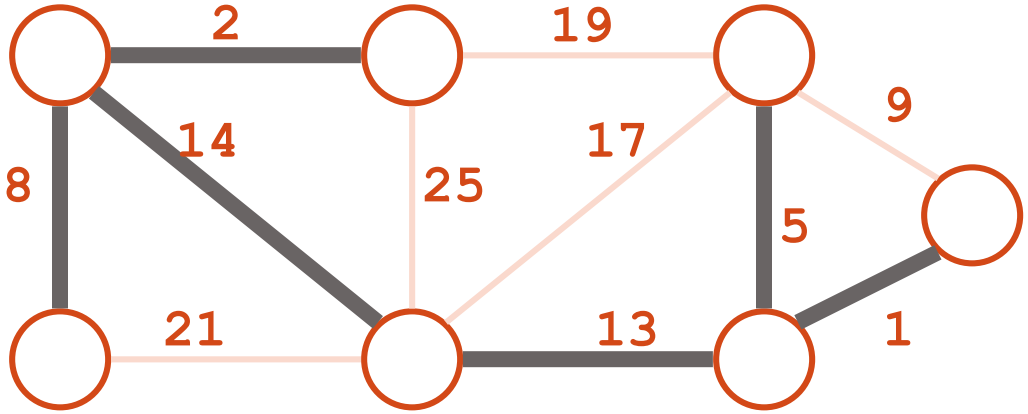


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KRUSKAL'S ALGORITHM: APPLICATION

- To design networks like telecommunication networks, water supply networks.
- To find paths in the map
- In order to layout electrical wiring
- In computer network (LAN connection)

EXAMPLE GRAPH