

## Sqrt Decomposition

#1 N-size array

Q-op

\* range sum query

\* point update

value 5 10 ~~9~~ 2 1  
idx 0 1 2 3 4

$\sum [1, 3] \rightarrow 9$

idx=2  
value=9

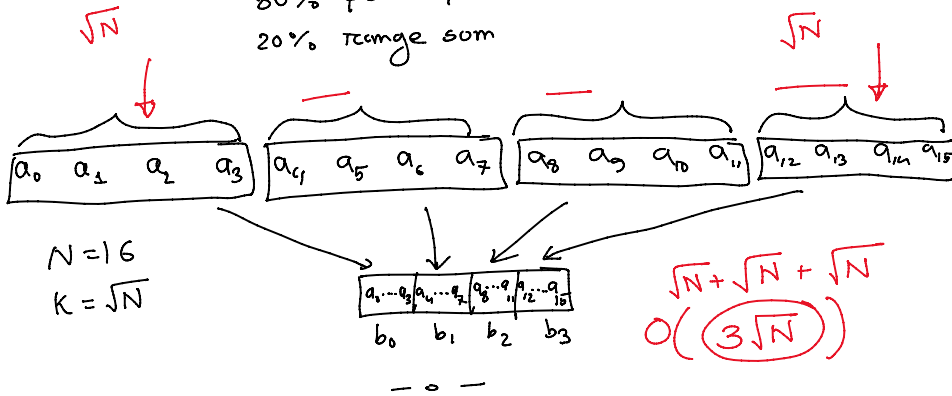
$\sum [1, 3] \rightarrow 21$

$1 \leq N \leq 5 \times 10^4$

$1 \leq Q \leq 10^5$

80% point update

20% range sum

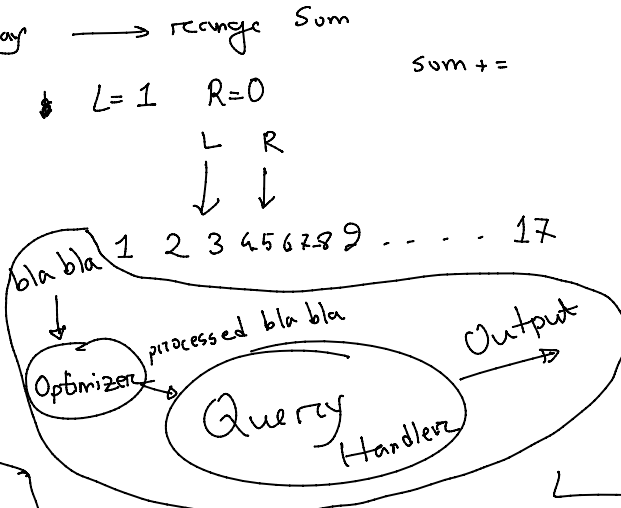


## Mo's ordering

#2 N-size array  $\rightarrow$  range Sum

cost	Query
16	[1, 16]
12	[2, 5]
12	[3, 16]
11	[4, 6]
11	[5, 16]
10	[6, 7]
11	[8, 16]
83	

cost	Query
9	1, 9
4	2, 12
8	3, 5
7	5, 10
15	13, 18
43	



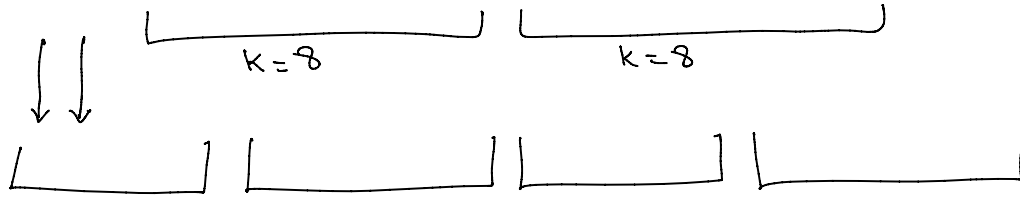
cost	Query
6	[2, 5] 5
3	[4, 6] 16
13	[1, 16] 6
2	[3, 16] 16
12	[6, 7]
10	[5, 16]
3	[8, 16]

$a: [L, R]$

bool cmp(Q a, Q b)

```

{
    b_a = a.L / k;
    b_b = b.L / k;
    if (b_a == b_b) return a.R < b.R;
    return b_a < b_b;
}
    
```



$O(\text{Left} + \text{Right})$

$$RM = n \times \frac{n}{k}$$

$$LM = qk$$

$$O(n\sqrt{n} + q\sqrt{n})$$

$$\approx O((n+q)\sqrt{n})$$

$$\approx O(n\sqrt{n})$$

$$O\left(\frac{n^2}{k} + qk\right)$$

$$\approx O\left(\frac{n^2}{k} + nk\right)$$

$$\frac{n^2}{k} = nk$$

$$\Rightarrow n = k^2$$

$$\therefore k = \sqrt{n}$$

$$f(k) = \frac{n^2}{k} + nk$$

$$\Rightarrow f'(k) = -\frac{n^2}{k^2} + n$$

$$\Rightarrow 0 = -\frac{n^2}{k^2} + n$$

$$\Rightarrow \frac{n^2}{k^2} = n$$

$$\Rightarrow \frac{n}{k^2} = 1 \Rightarrow k^2 = n$$

$$\therefore k = \sqrt{n}$$

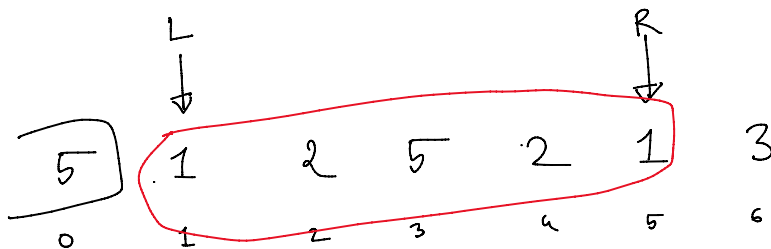
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#3:

N-size array

$[L, R] \rightarrow \# \text{ distinct value}$

$$f(L, R) = f_1(R) - f_1(L-1)$$



$f$	$v$
1	11
2	11
3	
4	
5	1

$$f(R) - f(L-1)$$

$$uv = 3$$

$$[1, 5] \rightarrow [0, 2]$$

$$[0, 2] \rightarrow [1, 5]$$