CS 61B: Lecture 21
Wednesday, March 12, 2014

Today's reading: Goodrich & Tamassia, Sections 9.1, 9.2, 9.5-9.5.1.

DICTIONARIES

Suppose you have a set of two-letter words and their definitions. You want to be able to look up the definition of any word, very quickly. The two-letter word is the _key_ that addresses the definition.

Since there are 26 English letters, there are 26 * 26 = 676 possible two-letter words. To implement a dictionary, we declare an array of 676 references, all initially set to null. To insert a Definition into the dictionary, we define a function hashCode() that maps each two-letter word (key) to a unique integer between 0 and 675. We use this integer as an index into the array, and make the corresponding bucket (array position) point to the Definition object.

```
public class Word {
  public static final int LETTERS = 26, WORDS = LETTERS * LETTERS;
  public String word;

  public int hashCode() {                  // Map a two-letter Word to 0...675.
    return LETTERS * (word.charAt(0) - 'a') + (word.charAt(1) - 'a');
  }
}

public class WordDictionary {
  private Definition[] defTable = new Definition[Word.WORDS];

  public void insert(Word w, Definition d) {
    defTable[w.hashCode()] = d;               // Insert (w, d) into Dictionary.
  }

  Definition find(Word w) {
    return defTable[w.hashCode()];               // Return the Definition of w.
  }
}
```

What if we want to store every English word, not just the two-letter words? The table "defTable" must be long enough to accommodate pneumonoultramicroscopicsilicovolcanoconiosis, 45 letters long. Unfortunately, declaring an array of length 26^45 is out of the question. English has fewer than one million words, so we should be able to do better.

Hash Tables (the most common implementation of dictionaries)

Suppose n is the number of keys (words) whose definitions we want to store, and suppose we use a table of N buckets, where N is perhaps a bit larger than n, but much smaller than the number of _possible_ keys. A hash table maps a huge set of possible keys into N buckets by applying a _compression function_ to each hash code. The obvious compression function is

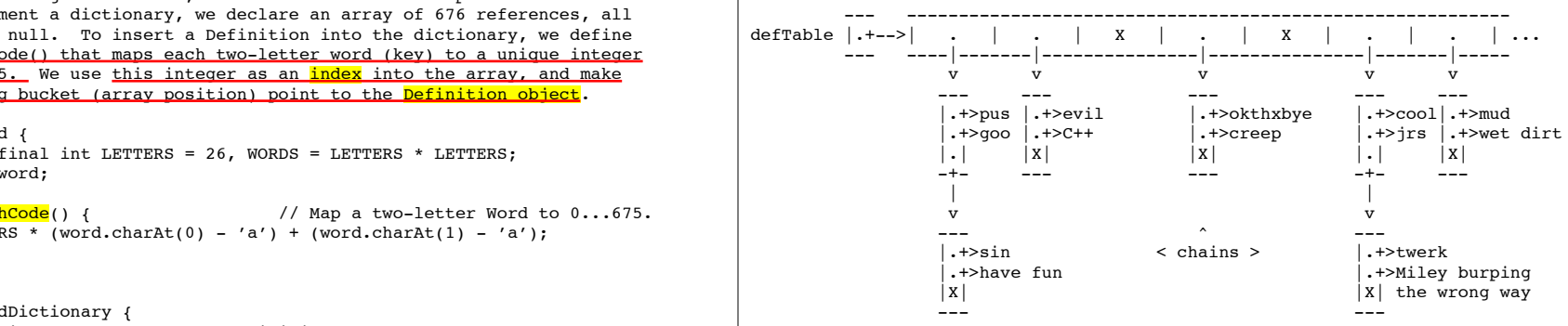
  h(hashCode) = hashCode mod N.

Hash codes are often negative, so remember that mod is not the same as Java's remainder operator "%". If you compute hashCode % N, check if the result is negative, and add N if it is.

With this compression function, no matter how long and variegated the keys are, we can map them into a table whose size is not much greater than the actual number of entries we want to store. However, we've created a new problem: several keys are hashed to the same bucket in the table if h(hashCode1) = h(hashCode2). This circumstance is called a _collision_.

How do we handle collisions without losing entries? We use a simple idea called _chaining_. Instead of having each bucket in the table reference one entry, we have it reference a linked list of entries, called a _chain_. If several keys are mapped to the same bucket, their definitions all reside in that bucket's linked list.

Chaining creates a second problem: how do we know which definition corresponds to which word? The answer is that we must store each key in the table with its definition. The easiest way to do this is to have each listnode store an _entry_ that has references to both a key (the word) and an associated value (its definition).



Hash tables usually support at least three operations. An Entry object references a key and its associated value.

```
public Entry insert(key, value)
  Compute the key's hash code and compress it to determine the entry's bucket.
  Insert the entry (key and value together) into that bucket's list.
public Entry find(key)
  Hash the key to determine its bucket.  Search the list for an entry with the
  given key.  If found, return the entry; otherwise, return null.
public Entry remove(key)
  Hash the key to determine its bucket.  Search the list for an entry with the
  given key.  Remove it from the list if found.  Return the entry or null.
```

What if two entries with the same key are inserted? There are two approaches.
(1) Following Goodrich and Tamassia, we can insert both, and have find() or remove() arbitrarily return/remove one. Goodrich and Tamassia also propose a method findAll() that returns all the entries with a given key.
(2) Replace the old value with the new one, so only one entry with a given key exists in the table.
Which approach is best? It depends on the application.

WARNING: When an object is stored as a key in a hash table, an application should never change the object in a way that will change its hash code. If you do so, the object will thenceforth be in the wrong bucket.

The _load factor_ of a hash table is n/N, where n is the number of keys in the table and N is the number of buckets. If the load factor stays below one (or a small constant), and the hash code and compression function are "good," and there are no duplicate keys, then the linked lists are all short, and each operation takes O(1) time. However, if the load factor grows too large (n >> N), performance is dominated by linked list operations and degenerates to O(n) time (albeit with a much smaller constant factor than if you replaced the hash table with one singly-linked list). A proper analysis requires a little probability theory, so we'll put it off until near the end of the semester.

Hash Codes and Compression Functions

Hash codes and compression functions are a bit of a black art. The ideal hash code and compression function would map each key to a uniformly distributed random bucket from zero to N - 1. By "random", I don't mean that the function is different each time; a given key always hashes to the same bucket. I mean that two different keys, however similar, will hash to independently chosen integers, so the probability they'll collide is 1/N. This ideal is tricky to obtain.

In practice, it's easy to mess up and create far more collisions than necessary. Let's consider bad compression functions first. Suppose the keys are integers, and each integer's hash code is itself, so hashCode(i) = i.

Suppose we use the compression function h(hashCode) = hashCode mod N, and the number N of buckets is 10,000. Suppose for some reason that our application only ever generates keys that are divisible by 4. A number divisible by 4 mod 10,000 is still a number divisible by 4, so three quarters of the buckets are never used! Thus the average bucket has about four times as many entries as it ought to.

The same compression function is much better if N is prime. With N prime, even if the hash codes are always divisible by 4, numbers larger than N often hash to buckets not divisible by 4, so all the buckets can be used.

For reasons I won't explain (see Goodrich and Tamassia Section 9.2.4 if you're interested),

```
h(hashCode) = ((a * hashCode + b) mod p) mod N
```

is a yet better compression function. Here, a, b, and p are positive integers, p is a large prime, and p >> N. Now, the number N of buckets doesn't need to be prime.

I recommend always using a known good compression function like the two above. Unfortunately, it's still possible to mess up by inventing a hash code that creates lots of conflicts even before the compression function is used. We'll discuss hash codes next lecture.