Calculating Methods of Quantitative in Sheep Herds

At the very beginning, there are some basic theories and assumptions that must be clarified.

A. Neighbors. The neighbors of one agent are related with the searching radius. Agents located within a circle range are defined as neighbors of the one at the center point.

B. Sheep herd is a time variant system. As a result, all quantitative should be functions of time *t*. Researchers should redefine the time and length unit of the dataset, which will be treated as the standard SI through calculating process.

C. Speed angle is the same as sheep orientation. It is within range (-Pi, Pi].

- **Density & Integrated conditional density**

Density is described as the agent number per unit area of whole group. It is only a function of time. The convex hull of all agents is used to calculate area.

$$\rho(t) = \frac{N(t)}{A(t)}$$

Where *N(t)* is the number of all agents, *A(t)* is convex hull area.

Integrated conditional density is a function of both time and radius. It only counts the average neighbors within radius *R* of some agents. Those sampled agents are usually close to the center of the whole group.

$$\tau(R,t) = \frac{1}{n_c}\sum_{i=1}^{n_c}\frac{N_i(R,t)}{\pi R^2}$$

Where n_c is the number of sampled agents.

- **Polarization & Order Parameter**

Order Parameter is used to describe the degree of alignment of the group. It can be defined as the normalized magnitude of the summed velocity.

$$\psi(t) = \frac{\sum \boldsymbol{v}(t)}{\sum|\boldsymbol{v}(t)|}$$

If group is perfectly aligned, ψ=1; randomly oriented, ψ=0.

Polarization has similar physical meaning to Order Parameter. But their calculation formulas are different.

$$p(t) = \sqrt{\langle\cos\phi_i\rangle_i^{\,2} + \langle\sin\phi_i\rangle_i^{\,2}}$$

- **Average Velocity & Local Average Velocity**

Average Velocity contains both the average orientation and average speed of whole group.

$$Ave_{ori}(t) = \frac{1}{N}\sum_{i=0}^{N}\xi_i$$

$$Ave_{speed}(t) = \frac{1}{N}\sum_{i=0}^{N}v_i$$

Local Average Velocity (*LAV*) is a function of R and t.

$$R = \alpha \cdot \langle\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}\rangle_i$$

$$(x_0, y_0) = (\langle x_i\rangle, \langle y_i\rangle)$$

$$LAV(R, t, i) = \frac{1}{n_{ci}}\sum_{j=0}^{n_{ci}}\boldsymbol{V}_{i,j}$$

$$n_{ci} = count(dis_{neighbors,i} < R)$$

Where α is adjustable. $\boldsymbol{V}_{i,j}$ means velocity of the j^{th} neighbor of i^{th} agent.

- **Nearest Neighbor Distance**

Nearest neighbor distance (DNN) is a function of t.

$$DNN(t) = \frac{1}{N}\sum_{i=0}^{N}d_i$$

$$d_i = \min(D_{i,j})$$

$$D_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

- **Density Angular Distribution**

Density Angular Distribution (*DAD*) only analyze agents who are close to the center of group. The radius used for this selection is defined as:

$$r = R|_{\alpha=0.5}$$

R is defined in *Average Velocity & Local Average Velocity* part.

$$\theta_{ij} = \theta\left(D_{i,j} < r\right)$$

$$DAD(k, t) = \frac{count(\xi_k < \theta_{ij} < \xi_{k+1})}{0.5 \cdot (\xi_{k+1} - \xi_k) \cdot r^2}$$

$$\xi_{k+1} - \xi_k = \frac{2\pi}{N_k}\ ,\quad \xi_0 = -\pi$$

N_k is adjustable.

- **Velocity Gradient**

Gradient field of velocity value. Function of x, y and t. Discrete calculating method: $quan_{ij}$ means one quantitative of the j[th] nearest neighbors of agent i. $j \leq 8$.

$$\delta v_{ij} = v_{ij} - v_i$$

$$\delta l_{ij} = \sqrt{(x_{ij} - x_i)^2 + (y_{ij} - y_i)^2}$$

$$\xi_{ij} = acrtan2(y_i - y_{ij}, x_i - x_{ij})$$

$$grad_{x,i} = \sum_j \left(\frac{\delta v_{ij}}{\delta l_{ij}} \cdot \cos \xi_{ij} \right)$$

$$grad_{y,i} = \sum_j \left(\frac{\delta v_{ij}}{\delta l_{ij}} \cdot \sin \xi_{ij} \right)$$

- **Vorticity**

Neighbors of agent i are defined in the same way as that in *Local Average Velocity* part. And α is also adjustable.

$$\delta \boldsymbol{V}_{ij} = \boldsymbol{V}_{ij} - \boldsymbol{V}_i$$

$$\delta \boldsymbol{L}_{ij} = (x_{ij} - x_i, y_{ij} - y_i)$$

$$\boldsymbol{Vor} = \frac{1}{N_i} \sum_{j \in N_i} \frac{\delta \boldsymbol{L}_{ij} \times \delta \boldsymbol{V}_{ij}}{\left|\delta \boldsymbol{V}_{ij}\right|}$$

- **Divergence**

When neighbors, $\delta \boldsymbol{V}_{ij}$ and $\delta \boldsymbol{L}_{ij}$ are the same as those in *Vorticity* part, divergence can be defined as:

$$\boldsymbol{Div} = \frac{1}{N_i} \sum_{j \in N_i} \delta \boldsymbol{L}_{ij} \cdot \delta \boldsymbol{V}_{ij}$$

- **Pairwise Distribution**

Pairwise Distribution (PD) counts numbers of pairs whose distance are between d_k and d_{k+1}.

$$PD = \frac{count(d_k < D_{i,j} < d_{k+1})}{\pi({d_{k+1}}^2 - {d_k}^2)}$$

$$d_{k+1} - d_k = \frac{d_{max}}{N_d} \quad , \quad d_0 = \frac{d_{max}}{N_d}$$

$$d_{max} = \max(D_{i,j})$$

Where $D_{i,j}$ is defined in *Nearest Neighbor Distance* part.

- **Correlation**

As the name indicates, Correlation is proportional to the discrete convolution of two quantitative.

$$Cor_{1,2}(n) = \frac{1}{n_{1,2}} \sum_{t=-\infty}^{\infty} \tilde{q}_1(t) \cdot \tilde{q}_2(n-t)$$

$$\tilde{q}_i = q_i - \bar{q}_i$$

$$n_{1,2} = count(\ \tilde{q}_1(t)\ has\ number\ \&\&\ \tilde{q}_2(n-t)\ has\ number\ )$$