TOPIC 7: (Number Theory II) Vieta Formulas and Vieta Jumping:

Homework Problems:

1) Find the sum of the roots, real and non-real, of the polynomial $x^{2001} + (\frac{1}{2} - x)^{2001}$.

2) The equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$ has three real roots. Find the sum of those three roots.

3) If a, b are positive integers so that exactly one of them is a multiple of 5 show that then $a^4 + 4b^4$ is not a multiple of 5.

4) If a, b are positive integers and $(ab - 1) \mid (a^2 + b^2)$ then $q = (a^2 + b^2) / (ab - 1) = 5$.

5) (Optional) If n, m are odd, positive integers with $n > m$ and $(n^2 - m^2 + 1) \mid (n^2 + 1)$ then $n^2 - m^2 + 1$ is a square.

Hint: Let $a = (n+m)/2$ and $b = (n-m)/2$ and proceed as in Example 6.