TOPIC 3: The Extremal Principle (Homework Problems):

Homework Problems:

1) Prove that there are no solutions in pairwise different positive integers x, ,y, z, t so that
$$x^x + y^y = z^z + t^t.$$

2) Show that the equation $x^2 + y^2 + z^2 = 2xyz$ has no integral solutions except $x = y = z = 0$.

3) We are given n points in the plane such that among any three of them there are always two whose distance is less than or equal to 1. Show that there are two circles of radius 1 which together contain all points.

4) Seven students have at least 100 coins among them. No two students have the same number of coins. Show that there are 3 students who have together at least 50 coins.