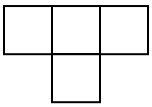
TOPIC 2: The Invariance Principle (Homework Problems):

Homework Problems:

1) Start with the integers 1, … , 4n-1. In one move you may replace any two integers by their difference. Prove that an even integer will be left after 4n-2 moves.

2) (AIME 2007I.14)
A sequence is defined over non-negative integral indexes in the following way:
$a_0 = a_1 = 1$ and $a_{n+1}a_{n-1} = a_n^2 + 2007$.
Find an invariant for this sequence.

3) Can a 10x10 chessboard be covered with 25 T-tetrominoes?



4) The number 2^{29} has 9 digits all different. Which digit is missing? Don't google the number or do other naughty things! Show your reasoning!