CS 70 Discrete Mathematics and Probability Theory
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DIS 1A

1 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

$(\exists x \in \mathbb{R})(x \notin \mathbb{Q})$ or $(\exists x \in \mathbb{R})\neg(x \in \mathbb{Q})$

✓ -> True

- for example $\sqrt{2}$ is real but not rational number

(b) All integers are natural numbers or are negative, but not both.

$(\forall x \in \mathbb{Z})((x \in \mathbb{N} \vee x<0) \wedge \neg(x \in \mathbb{N} \wedge x<0))$

✓ -> True

- integers are either ≥ 0 xor <0, therefore they are either natural xor negative number

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

$(\forall x \in \mathbb{N})(6|x) \Rightarrow (2|x \vee 3|x)$

✓ -> True

- if natural number is divisible by 6, then it is in the form $6k$ where $k \in \mathbb{N}$
- since $6k = 2 \cdot 3k$, 2 divides $6k$ and 3 divides $6k$

(d) $(\forall x \in \mathbb{R})\ (x \in \mathbb{C})$

All real numbers are complex

✓ -> True

- set of real number is a subset of complex numbers

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

If integer is divisible by 2 or is divisible by 3, then it is divisible by 6.

✓ -> False

- 4 is divisible by 2 (so the first part of implication is true) and it is not divisible by 6 -> counterexample

(f) $(\forall x \in \mathbb{N})\ ((x>7) \implies ((\exists a, b \in \mathbb{N})\ (a+b=x)))$

If natural number x is ~~bigger~~ larger than 7, then there exist natural numbers a and b such their sum is equal to x.

If natural number is larger than 7, then it can be written as a sum of two other natural numbers natural numbers

✓ -> True

- since $x>7$, then $x-1$ is also $\mathbb{N}$ and $1 \in \mathbb{N}$, let $a=1$ and $b=x-1$

-> $a+b = 1+x-1 = x$

2 DeMorgan's Law

Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

$\neg(A \vee B) \equiv \neg A \wedge \neg B$

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$\neg A \wedge \neg B$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

✓

A	B	$A \wedge B$	$\neg A$	$\neg B$	$\neg(A \wedge B)$	$\neg A \vee \neg B$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

$\neg(A \wedge B) \equiv \neg A \vee \neg B$

3 XOR

The truth table of XOR is as follows.

A	B	A XOR B
F	F	F
F	T	T
T	F	T
T	T	F

1. Express XOR using only $(\wedge, \vee, \neg)$ and parentheses.

✓ $(A \vee B) \wedge \neg(A \wedge B)$

2. Does (A XOR B) imply $(A \vee B)$? Explain briefly.

✗ ~~No~~

~~→ when A = True and B = True, then A xor B = False and A ∨ B = True~~

Yes

$(A \text{ xor } B) = T \Rightarrow ((A=T) \wedge (B=F)) \vee ((A=F) \wedge (B=T)) \Rightarrow A \vee B = T$

3. Does $(A \vee B)$ imply (A XOR B)? Explain briefly.

✗ ~~Yes~~

~~→ when A ∨ B is true then A xor B is true, that is according to truth table below~~

A	B	$A \vee B$	A xor B
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

No

when $(A=T) \wedge (B=T)$, then $(A \vee B) = T$ but $(A \text{ xor } B) = F$

4 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

True

✓ - for all can be switched if they are adjacent since $\forall x \forall y$ means for all x and y in our universe

(b) $\exists x \exists y P(x,y) \implies \exists y \exists x P(x,y)$.

True

✓ - there exist can be switched if they are adjacent since $\exists x \exists y$ means there exist x and y in our universe

(c) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

False

✓ - $\forall$ and $\exists$ can't be switched if they are adjacent because $\forall x \exists y$ means for all x there exist y and $\exists y \forall x$ means there exist y for every x

- counterexample
  - All student have favorite class $\not\Rightarrow$ there is a class that all students consider their favorite

(d) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

~~False~~

✗ ~~- $\forall$ and $\exists$ can't be switched if they are adjacent (reason above)~~

~~- counterexample~~
~~- There is a professor that likes all of his students $\not\Rightarrow$ All students likes this professor~~

True

$\exists x \forall y$ ... there exist x for every y

$\forall y \exists x$ .... for every y there exists x