HW 0

5 ASYMPTOTIC NOTATION

5a For each pair of functions $f(n)$ and $g(n)$ state whether

- $f(n) = O(g(n))$
- $f(n) = \Omega(g(n))$
- $f(n) = \theta(g(n))$

$\Rightarrow f(n) = n$ and $g(n) = n^2 - n$

✓ $f(n) = O(g(n))$

$\Rightarrow f(n) = n^2$ and $g(n) = n^2 + n$

✓ $f(n) = \theta(g(n))$

$\Rightarrow f(n) = 8n$ and $g(n) = n \log n$

✓ $f(n) = O(g(n))$

$\Rightarrow f(n) = 2^n$ and $g(n) = n^2$

✓ $f(n) = \Omega(g(n))$

$\Rightarrow f(n) = 3^n$ and $g(n) = 2^{2n}$

✓ $f(n) = \theta(g(n))$

NOTE $2^{2n} = (2^2)^n = 4^n$

5b For each of the following, state the order of growth using θ notation

$\Rightarrow f(n) = 50$

✓ $f(n) = \theta(1)$

$\Rightarrow f(n) = n^2 - 2n + 3$

✓ $f(n) = \theta(n^2)$

$\Rightarrow f(n) = n + \ldots + 3 + 2 + 1$

⊗ $f(n) = \theta(n)$

$\frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \Rightarrow \theta(n^2)$

$\Rightarrow f(n) = n^{100} + 1.01^n$

✓ $f(n) = \theta(1.01^n)$

$\left(\frac{1.01^n}{n^{100}}\right)' = \frac{1.01^n \ln 1.01}{100\, n^{99}} = c \cdot \frac{1.01^n}{n^{99}}$

$\Rightarrow f(n) = n^{1.1} + n \log n$

✓ $f(n) = \theta(n^{1.1})$

$\frac{n^{1.1}}{n \log n} = \frac{n \cdot n^{0.1}}{n \cdot \log n} = \frac{n^{0.1}}{\log n}$

$\left(\frac{n^{0.1}}{\log n}\right)' = \frac{0.1\, n^{-0.9}}{\frac{1}{n \ln(10)}} = 0.1 \frac{1}{n^{0.9}} \cdot \frac{n \ln(10)}{1}$

$= c \cdot \frac{n}{n^{0.9}} = c \cdot n^{0.1}$