Note 0: REVIEW OF SETS AND NOTATION

- **REVIEW OF SETS**
  - set = well defined collections of objects
  - objects = elements = members
    - x is member of A: $x \in A$
    - x is not member of A: $x \notin A$
  - A and B are equal if they have the same elements: $A = B$
  - order and repetition doesn't matter
  - set of rational numbers: $\mathbb{Q}$ or $\{\frac{a}{b} \mid a,b \text{ are integers}, b \neq 0\}$
  - set of all items which satisfy this condition
    - {item | condition on that item}

=> CARDINALITY
  - = size of set: $|A|$
  - if A is finite, its cardinality must be non-negative integer
  - there is unique set with cardinality 0
    - = empty set, ϕ

=> SUBSETS AND PROPER SUBSETS
  - A is subset of B: $A \subseteq B$
    - if every element of set A is also in set B
  - B is superset of A: $B \supseteq A$
  - proper subset
    - if $A \subseteq B$, but $A \neq B$ -> $A \subset B$
  - properties
    - empty set is a proper subset of any nonempty set: $\phi \subset A$
    - empty set is a subset of every set B: $\phi \subseteq B$
    - every set A is a subset of itself: $A \subseteq A$

=> INTERSECTIONS AND UNIONS
  - intersection: $A \cap B$
    - disjoint sets if $A \cap B = \phi$
  - union: $A \cup B$
  - properties
    - -> $A \cup B = B \cup A$
    - -> $A \cup \phi = A$
    - -> $A \cap B = B \cap A$
    - -> $A \cap \phi = \phi$

=> RELATIVE COMPLEMENT

- relative complement of A in B : $B-A$, $B\setminus A$
  = set difference between B and A
- set of elements in B, but not in A $\{x \in B \mid x \notin A\}$
  -> $\{1,2,3\}-\{3,4,5\}=\{1,2\}$
- properties
  - -> $A-A=\phi$
  - -> $A-\phi=A$
  - -> $\phi-A=\phi$
- generally $A-B \neq B-A$

=> SIGNIFICANT SETS

- $\mathbb{N}$ ... natural numbers $\{0,1,2...\}$
- $\mathbb{Z}$ ... integers
- $\mathbb{Q}$ ... rational
- $\mathbb{R}$ ... real
- $\mathbb{C}$ ... complex

=> PRODUCTS AND POWER SETS

- Cartesian product = cross product : $A \times B$
- set of all pairs whose first component is an element of A and second element of B
  - -> $A=\{1,2,3\}$, $B=\{u,v\}$
    - -> $A \times B = \{(1,u),(2,u),(3,u),(1,v),(2,v),(3,v)\}$
- power set of S : $\mathcal{P}(S)$
  - set of all subsets of S: $\{T \mid T \subseteq S\}$
    - -> $S=\{1,2,3\}$
      - -> $\mathcal{P}(S)=\{\{\},\{1\},\{2\},...,\{2,3\},\{1,2,3\}\}$
  - if $|S|=k$, then $|\mathcal{P}(S)|=2^k$

• REVIEW OF MATHEMATICAL NOTATION

=> SUMS AND PRODUCTS

$$\sum_{i=1}^{n} i = 1+2+...+n$$

$$\sum_{i=m}^{n} f(i) = f(m)+f(m+1)+...+f(n)$$

$$\prod_{i=m}^{n} f(i) = f(m)\cdot f(m+1)\cdot \ldots \cdot f(n)$$

$$\prod_{i=1}^{n} i = 1\cdot 2\cdot \ldots \cdot n$$

=> UNIVERSAL AND EXISTENTIAL QUALIFIERS

-> for all: $\forall$

-> there exists: $\exists$

$(\exists_n \in \mathbb{N})(n \text{ is prime})$

- for any claim C - statement $(\exists x \in \phi)(C(x))$ is false
  - statement $(\forall x \in \phi)(C(x))$ is true
    ↳ vacuously or trivially true

• QUANTIFIERS AND IMPLICATIONS

-> $\forall x \forall y\, P(x,y) \Rightarrow \forall y \forall x\, P(x,y)$ True

$\forall x \forall y$... for all x and y in our universe

-> $\exists x \exists y\, P(x,y) \Rightarrow \exists y \exists x\, P(x,y)$ True

$\exists x \exists y$... there exists x and y

-> $\exists x \forall y\, P(x,y) \Rightarrow \forall y \exists x\, P(x,y)$ True

$\exists x \forall y$... there exists x for every y

$\forall y \exists x$... for every y there exists x

-> note this implication is False

$\forall x \exists y\, P(x,y) \Rightarrow \exists y \forall x\, P(x,y)$