CS 70 Discrete Mathematics and Probability Theory
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1 Contraposition

Prove the statement "if $a+b<c+d$, then $a<c$ or $b<d$".

We will use contrapositive proof. We need to prove: If $a \geq c$ and $b \geq d$, then $a+b \geq c+d$.

Since we know that $a \geq c$ and $b \geq d$, we can sum those two inequalities since they are in the same direction. Their sum is $a+b \geq c+d$.

We proved the contrapositive statement. Therefore the original statement holds.

2 Perfect Square

A *perfect square* is an integer n of the form $n=m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.

We will prove this by direct proof.
Use lemma: If m^2 is odd, then m is odd ($m \in \mathbb{Z}$).
Since the integer m^2 is odd perfect square, m is also odd. Since m is odd, it can be written in form $m=2a+1$. Therefore the odd perfect square integer $m^2=(2a+1)^2$

$$m^2=(2a+1)^2$$
$$=4a^2+4a+1$$
$$=4a(a+1)+1$$

If a is even $a=2b$. Therefore $4(2b)(2b+1)+1 = 8(2b^2+b)+1$ which is in form $8k+1$.
If a is odd $a=2b+1$. Therefore $4(2b+1)(2b+1+1)+1 = 4\cdot 2(2b+1)(b+1)+1$ $=8(2b^2+3b+1)+1$ which is in form $8k+1$.
Since a is always odd or even, m^2 is of the form $8k+1$.

3 Infinite Primes

Prove by contradiction that there are an infinite number of primes.

We will prove this by contradiction.
Assume there are (k) number of primes: $p_1, p_2, p_3, \cdots, p_k$
Note that every number can be written as product of prime factors.
Consider number $r = p_1 \cdot p_2 \cdots p_k + 1$. Number r has at least one prime factor p_i. Therefore $p_i \mid p_1 \cdot p_2 \cdots p_k + 1$ and p_i has to be in our list $p_1, p_2, \cdots p_k$ so $p_i \mid p_1 \cdot p_2 \cdots p_k$.
Since $p_i \mid p_1 \cdots p_k + 1$ and $p_i \mid p_1 \cdots p_k$, $p_i \mid (p_1 \cdots p_k + 1) - (p_1 \cdots p_k)$.
But $(p_1 \cdots p_k + 1) - (p_1 \cdots p_k) = 1$. Therefore $p_i \mid 1$. Then p_i is not a prime number and that is contradiction with our assumption.
We proved the original statement

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4 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.
(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

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Consider two cases

1) Everybody at the party has at least 1 friend
Therefore everyone has $1, 2, 3, \cdots$ or $(n-1)$ friends (you can't be friend with youself). There are $m = n-1$ numbers of friends.
By Pigeonhole Principle since there are only n people and $(n-1)$ different number of friends, at least 2 people have the same number of friends

2) At least one person at the party doesn't have any friends.
Therefore everyone has $0, 1, 2, \cdots$ or $(n-2)$ friends (you can't be friend with yourself and the person with no friends).
There are $(n-1)$ different numbers of friend, which follows the case above.

By discussing case, we proved the statement

solution manual proved this by contradiction