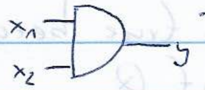
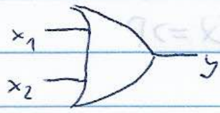
Note 1: PROPOSITIONAL LOGIC

• PROPOSITIONAL LOGIC

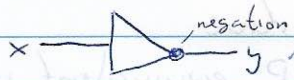
- proposition = statement which is either true or false
  - $\boxed{\text{EX}}$: $\sqrt{3}$ is irrational, $1+1=5$ → propositions
    $2+2$, $x^2+3x=5$ → not propositions
- propositional formulae = propositions joined together to form more complex statements
  - → conjunction: and
    $P \land Q$



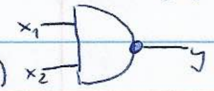
  - → disjunction: or
    $P \lor Q$



  - → negation: not
    $\neg P$



  - logic gates used in circuits
    - all can be simulated using just NAND gate
      $y = \neg(x_1 \land x_2)$



- important to use parenthesis to mark what order the operators are to be applied in

⇒ TRUTH TABLES

- check if two formulaes are equivalent
- propositional formulae with logical operators $\land, \lor, \neg$ are fully expressive
  - we can create a formula that is true exactly on those inputs where the truth table is
- how to construct formulae from table

P	Q	P ∧ Q
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	φ
F	F	T
F	T	T
T	F	T
T	T	F

$$\varphi = ((\neg P)\land(\neg Q))\lor((\neg P)\land Q)\lor(P\land(\neg Q))$$

⇒ DE MORGAN'S LAWS

- two equivalencies

$$\boxed{\begin{aligned}\neg(P\land Q) &\equiv (\neg P)\lor(\neg Q)\\ \neg(P\lor Q) &\equiv (\neg P)\land(\neg Q)\end{aligned}}$$

$\equiv$ ... is equivalent to

- because of the first equivalency, we need only $\neg$ and $\land$ for formulae to be fully expressive
  - we can't remove all $\land$ and $\lor$ simultaneously

⇒ IMPLICATIONS

$P \Rightarrow Q$

- P implies Q, if P then Q

P	Q	$P \Rightarrow Q$	$\neg P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- P... hypothesis, antecedent
- Q... conclusion, consequent
- vacuously true
  - implication is true because hypothesis is false
- P if and only if Q

  $P \Rightarrow Q$ and $Q \Rightarrow P$

  $P \Leftrightarrow Q$

- contrapositive

  $\neg Q \Rightarrow \neg P$, equivalent with $P \Rightarrow Q$

- converse

  $Q \Rightarrow P$

• QUANTIFIERS

- EX: For all natural numbers n, n^2+n+41 is prime
  - → proposition - there is underlying universe we are working in
- statements are quantified over universe
- two quantifiers: $\forall, \exists$
- in finite universe, we can express existentially and universally quantified propositions without quantifiers
  - use disjunctions and conjuctions respectively
  - EX: $U = \{1,2,3,4\}$

    $\exists x P(x)$ is equivalent to $P(1) \vee P(2) \vee P(3) \vee P(4)$

    $\forall x P(x)$ is equivalent to $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$
- Quantifiers don't commute
  - EX: All students have a favorite class. × There is a class that all students consider their favorite

    $(\forall s \in S)(\exists c \in C)$ (c is s's favorite class)

    × $(\exists c \in C)(\forall s \in S)$ (c is s's favorite class)

⇒ DE MORGAN'S LAWS

- equivalents for quantifiers

$$\boxed{\begin{aligned} \neg(\forall x\, P(x)) &\equiv \exists x\, (\neg P(x)) \\ \neg(\exists x\, P(x)) &\equiv \forall x\, (\neg P(x)) \end{aligned}}$$

- **LOGICAL EQUIVALENCE**
  - => $\forall x\,(P(x) \land Q(x)) \equiv \forall x\,P(x) \land \forall x\,Q(x)$
  - => $\exists x\,(P(x) \lor Q(x)) \equiv \exists x\,P(x) \lor \exists x\,Q(x)$
  - -> these aren't equivalent
    - => $\forall x\,(P(x) \lor Q(x)) \not\equiv \forall x\,P(x) \lor \forall x\,Q(x)$
    - => $\exists x\,(P(x) \land Q(x)) \not\equiv \exists x\,P(x) \land \exists x\,Q(x)$