Morning

Formal Language And Automata

- Languages
- Autamata
- Computation

Language

What is a language ?

This is a sentence.

This is also a sentence.

……

So we have

{ sentence 1, sentence 2, sentence 3, …… }

the set of sentences $\Leftrightarrow$ Language

Sentence and Alphabet

- sentence/string = sequence of symbols chosen

  from the alphabet Σ

  example : Mouse love rice.

- alphabet = set of symbols

  example : ASCII, 中文国标, $\Sigma = \{0,1\}$

  symbols $\Rightarrow$ sentences $\Rightarrow$ language

Rules/Grammar

- rules = by which sentence is generated

  example : rules for English

  <sentence> → <noun-phrase><predicate>

  <noun-phrase> → <article><noun>

  <predicate> → <verb>

  <article> → a|an|the

  <noun> → wolf|sheep

  <verb> → love|eat

Examples of language

example 1.1

alphabet : $\{0,1\}$

$L=\{w|w$ consists of 0's and 1's,

rule

and end with 0 $\}$

$L=\{0,00,10,000,010,100,110,0000,\ldots\ldots\}$

$11111100 \in L$, $1 \notin L$, $0001 \notin L$, $20 \notin L$

Examples of language

example 1.2

rule

rule

$$L=\{\, 0^n 1^n \mid n \geq 0 \,\}$$

alphabet = {0,1}

$$L=\{\varepsilon,\ 01,\ 0011,\ 000111,\ 00001111,\ 0000011111,\ \ldots\ldots\}$$

example 1.3 The empty language

$$L = \{\ \} = \phi$$

L : contains no string

example 1.4

$L = \{ w \mid w \text{ is a sentence in English} \}$

Bolt won two gold medals in Daegu. √

The sheep eats grass . √

The grass eats sheep . ?

♣ Formal language focus on form, not meaning

String operations

$w = a_1 a_2 \ldots\ldots a_n$ abba

$v = b_1 b_2 \ldots\ldots b_n$ bbbaaa

Concatenation

$wv = a_1 a_2 \ldots\ldots a_n b_1 b_2 \ldots\ldots bn$ abbabbbaaa

Reverse

$w^R = a_n a_{n-1} \ldots\ldots a_1$

Another operation

$$w^n = \underbrace{w\,w\,\ldots\ldots\,w}_{n}$$

- $w = abb \Rightarrow w^2 = abbabb, \;\; w^3 = abbabbabb$

- definition : $w^0 = \varepsilon$

Operations on languages

- The usual set operations

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$$

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \text{ and } w \in L_2 \}$$

$$L_1 - L_2 = \{ w \mid w \in L_1 \text{ and } w \notin L_2 \}$$

- Reverse

$$L^R = \{ w^R \mid w \in L \}$$

Example $L = \{ a^n b^n \mid n \geq 1 \} \Rightarrow L^R = \{ b^n a^n \mid n \geq 1 \}$

Operations on languages

- Concatenation

$$L_1L_2 = \{ wv \mid w \in L_1 \text{ and } v \in L_2 \}$$

Example $L = \{ a^nb^n \mid n \geq 1 \}$, $K = \{ b^na^n \mid n \geq 1 \}$

$LK = \{a^nb^n b^na^n \mid n \geq 1 \}$ ✗

$LK = \{a^nb^n b^ma^m \mid n \geq 1 , m \geq 1 \}$

$L^2 = ?$

example 1.5

$L=\{ w \in \{0, 1\}^* \mid \text{all 0's precede all 1's in } w \}$

$L=\{ \varepsilon, 0, 1, 00, 01, 11, 000, 001, 011, 111, \ldots\ldots \}$

example 1.6

$\mathcal{L}_1=\{w\in\{0,1\}^*|$ no prefix of w contains 1$\}$

$=\{\varepsilon$, 0, 00, 000, 0000, 00000, 000000, ……$\}$

$\mathcal{L}_2=\{w\in\{0,1\}^*|$ no prefix of w starts with 1$\}$

$=\{\varepsilon$, 0,00,01,000,001,010,011,0000,0001, ……$\}$

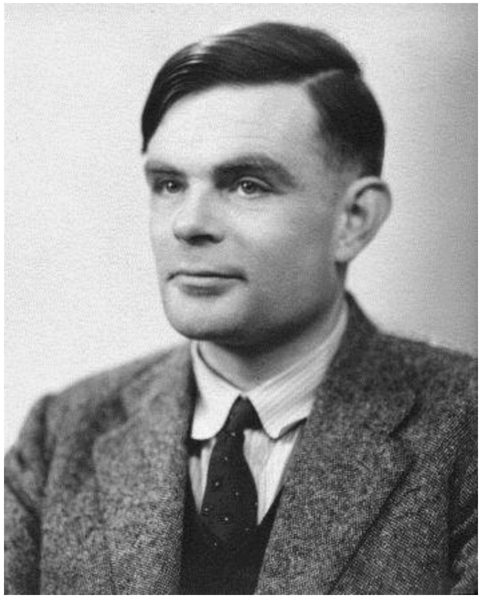
$=\{w\in\{0,1\}^*|$ the first character of w is 0 $\} \cup \{\varepsilon\}$

$\mathcal{L}_3=\{w\in\{0,1\}^*|$ every prefix of w starts with 1$\}$

$= \phi$

Automata

- Alan MarthisonTuring
  - On Computable Numbers With an Application to the Entscheidungs Problem
  - Turing Machine

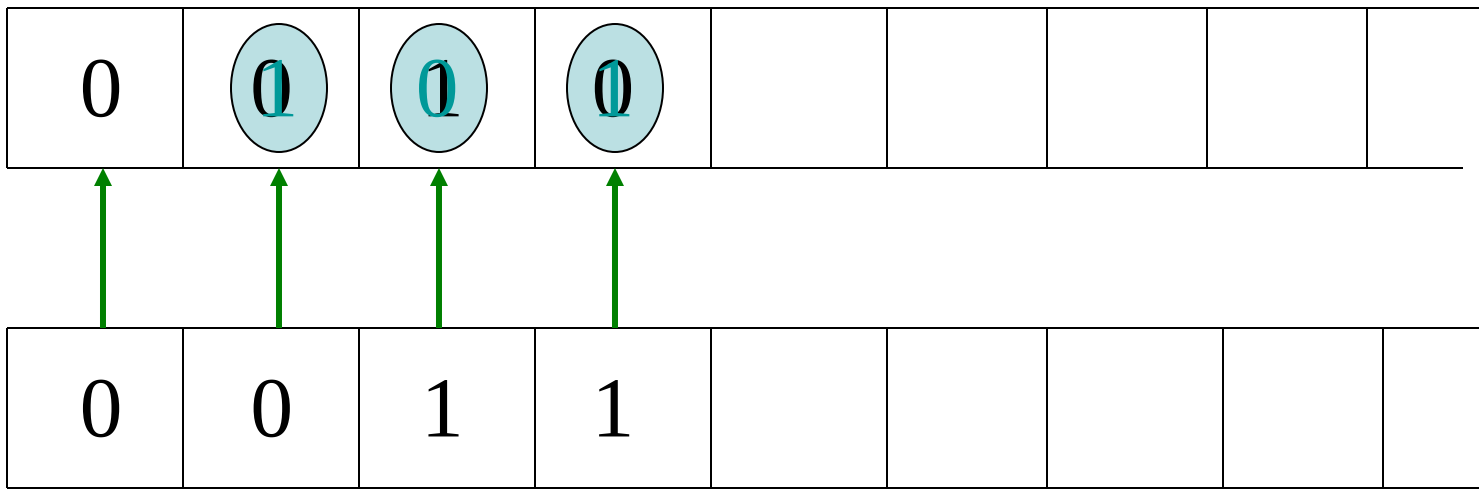


Automata

- Finite Automata
  - Deterministic Finite Automata
  - Non-deterministic Finite Automata
- Push Down Automata

Computation

$$\begin{array}{r} 2 \\ + \quad 3 \\ \hline 5 \end{array} \qquad\qquad \begin{array}{r} 0\,0\,1\,0 \\ + \quad 0\,0\,1\,1 \\ \hline 0\,1\,0\,1 \end{array}$$



Computation for computer

In our life

Coded

In computer

Process

0011101100001011 1100100111100
0011101100001011 11001001111000
00111011000010 11001001111

Produce

Automaton

Language

(formal description)

(set of strings)

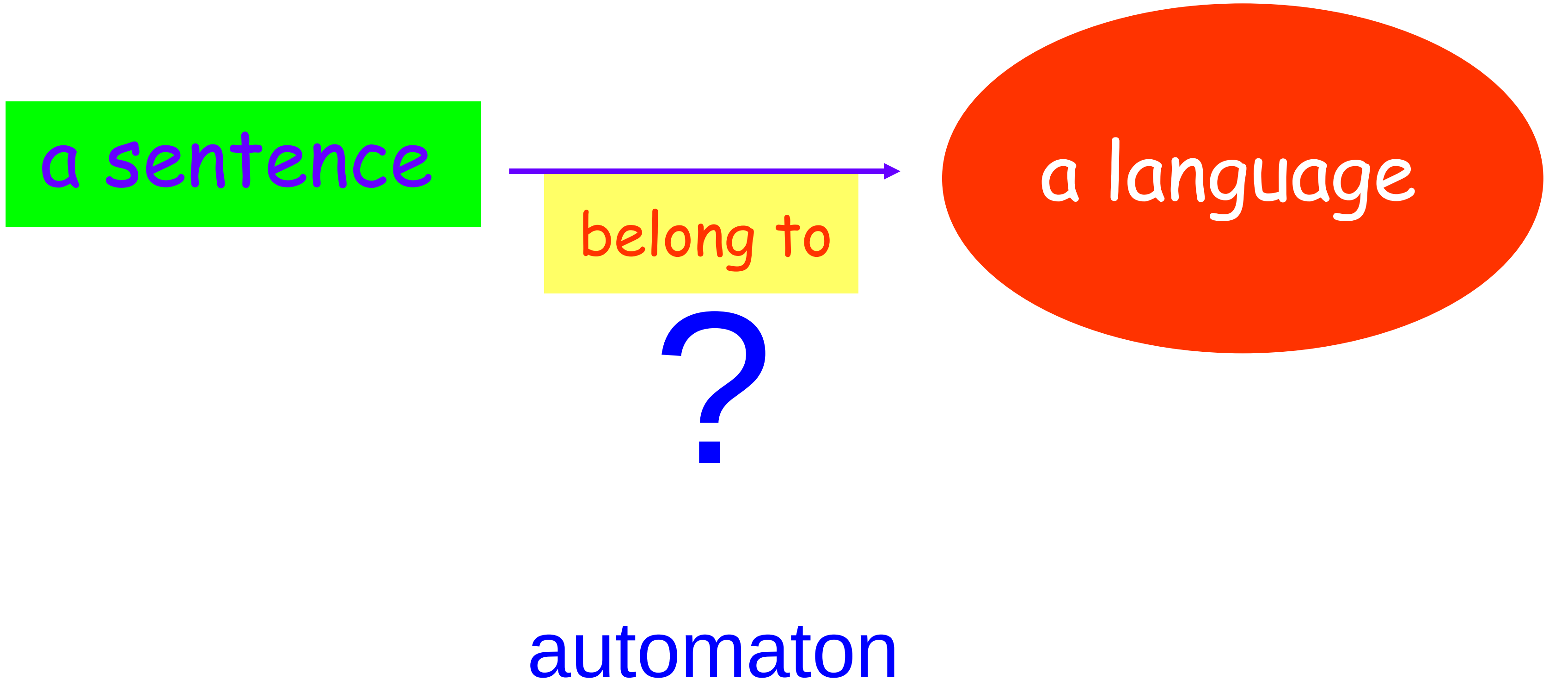
Computation

- Computable Problems
  - write a program to solve
- Intractable Problems
  - find someway to work around

Undecidable Problem

```
main ( )
{
    Int n, total, x, y, z ;
    scanf(“%d”, &n);
    total=3;
    while(1){
        for(x=1;x<=total-2;x++)
            for(y=1;y<=total-x-1;y++){
                z=total-x-y;
                if(exp(x,n)+exp(y,n)==exp(z,n))
                    printf(“hello,world\n”);
            }
        total++;
    }
}
```

Undecidable Problem



Content

Automata		Languages		Grammars
Construction		Properties		Design
Finite Automaton	→	Regular Language	←	Regular Expression
↓	Recognize	↓	Generate	↓
Push Down Automaton	→	Context Free Language	←	Context Free Grammar
↓		↓		↓
Turing Machine	→	Recursively Enumerable	←	(Phrase Grammar)

Text book

1. Introduction to Automata Theory , Languages , and Computation (Third Edition)

—— John E. Hopcroft
Rajeev Motwani
Jeffrey D. Ullman

Text book

2. An Introduction to Formal Languages and Automata (Third Edition)

—— Peter Linz

Goal

1. Understanding “theoretical” concepts

------ method of formal description

2. Get a sense of how to reason formally

3. Improving reading ability with English

Homework

- All exercises listed on qq-group
- Need not submit
- Check by free talk
- Discussions in group

Honor and Collaboration

- Collaboration is strongly encouraged
- Solutions must be written independently
- Responsible for Understanding and explaining

Exam

- Only final exam

- Open exam

  You are allowed to refer to text-book, class handouts, and notes during the exam

- Closed exam

  Nothing allowed except one pen

Grading Policy

- Homework : 40%
- Final exam : 60%
- Addition : 10 points
  - Paper
  - Presentation
  - Participation

Instructor

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Good good study

day day up !