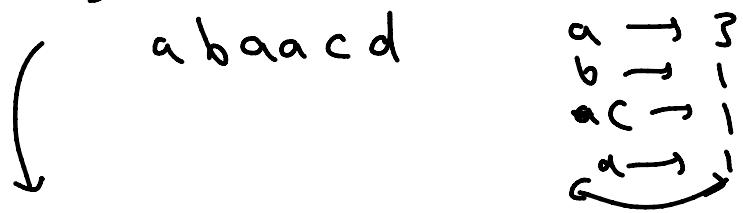


\Rightarrow We want to store key-value pairs.



\rightarrow On avg search $\geq O(1)$
 delete \geq
 insert

array \rightarrow insert \rightarrow end $\rightarrow O(1)$
 search $\rightarrow O(n)$

Direct Address Table :-

→ directly use keys as indexes.

0 - 10,000
 int c[10001]

→ $c[\underline{10}]++;$
 $c[\underline{20}]++;$
 $c[\underline{30}]++;$
 $c[\underline{10}]--;$

insert (10) insert (20)
 search (10) insert (30)
 delete (10) $O(1)$

$c[10] \geq 1 \rightarrow$ true
 $c[10] < 1 \rightarrow$ false

Problems with DAT :-

- ① Only use when values are small integers.
 val $\rightarrow 0 - 10^{12} \rightarrow c[10^{12} + 1] \rightarrow$...

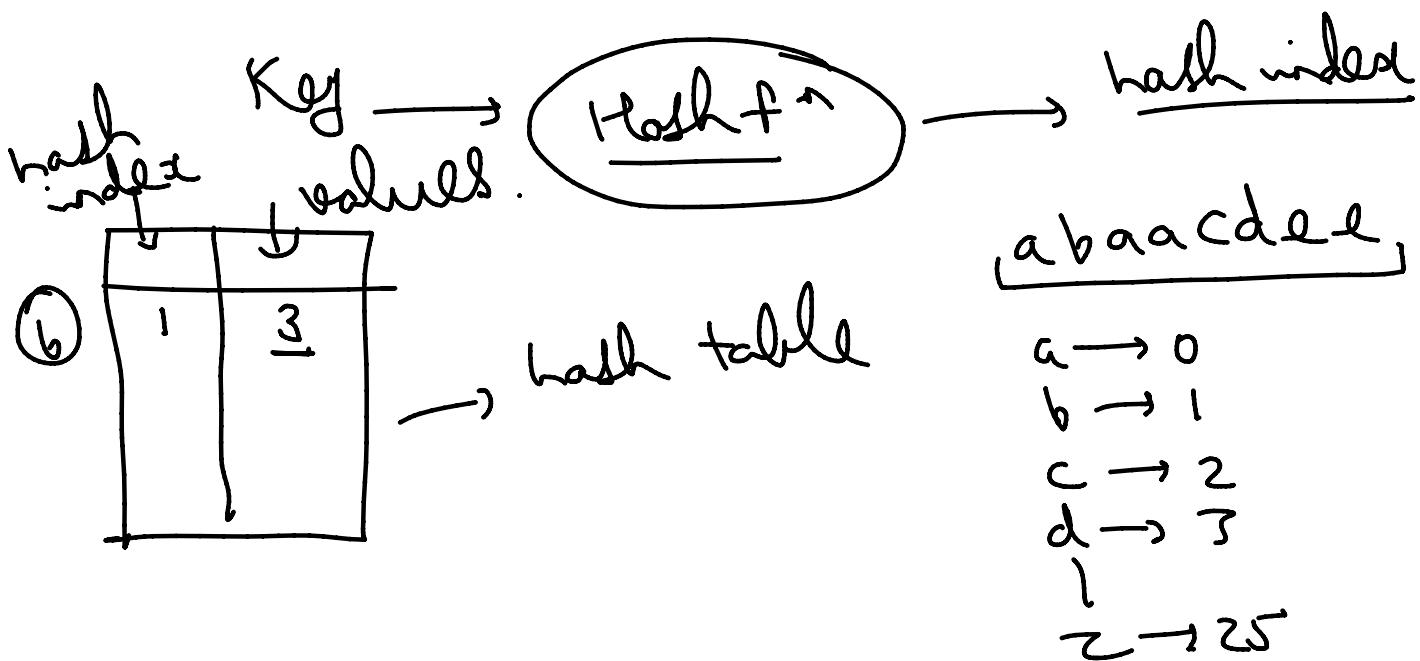
$val \rightarrow 0 - 10^6 \rightarrow C(10)$

② Space wastage $[10000 - 20,000]$ $\{2000\}$

① Cannot use when keys are not int.
e.g.: keys \rightarrow strings
 $"GFBI" \rightarrow 100$

\rightarrow Hashing :-

It is a Technique to convert keys into small int & then use these int further for indexing.



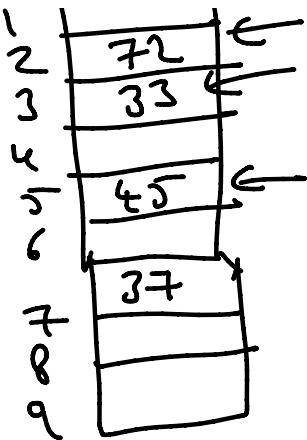
\rightarrow Hash functions :-

① $h(K) = K \% 10;$ \uparrow collision
 $(0-a), \uparrow, \frac{1}{2}, \frac{72}{23} \leftarrow$

Q) $n = 10$ - $(0-a)$, T .
 $i \rightarrow 72, 37, 25, 33, 45, 2$

$$72/10 = 2$$

$$s \rightarrow 72 \quad d \rightarrow 45 \\ 72/10 = 2 \quad 45/10 = 5$$



② Weighted sum :-

$$K = abcd$$

$$h(K) = ((a \times 10^3 + 1) + (b \times 10^2 + 2) + (c \times 10^1 + 3) + (d \times 10^0 + 4)) \% m$$

$m \rightarrow$ size of
hash table.

$$\begin{matrix} a & \rightarrow & 0 \\ b & \rightarrow & 1 \\ c & \rightarrow & 2 \end{matrix}$$

$$\text{Direct sum} : - \frac{acbd}{dacb} \rightarrow \frac{ab}{cd} \rightarrow \frac{24}{12} \rightarrow \frac{(1+2+3+4)}{(2+3+1+4)} \% m$$

⇒ Preferred characteristics of hash fn:-

- ① It should distribute keys uniformly.
- ② Should not be ambiguous. It should return same hash index for same key
- ③ It should be fast.

Ideally, integers \rightarrow constant
strings $\rightarrow O(\text{len})$
.., 1, 0, 1

strings $\rightarrow O(n)$

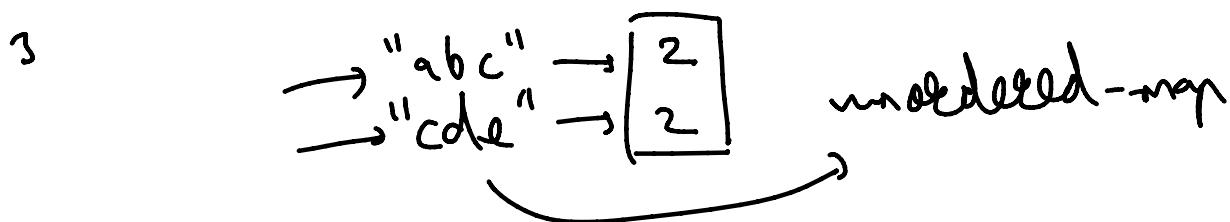
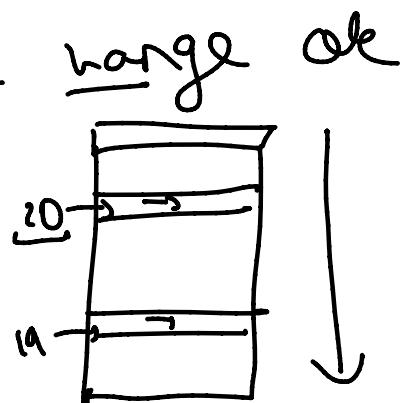
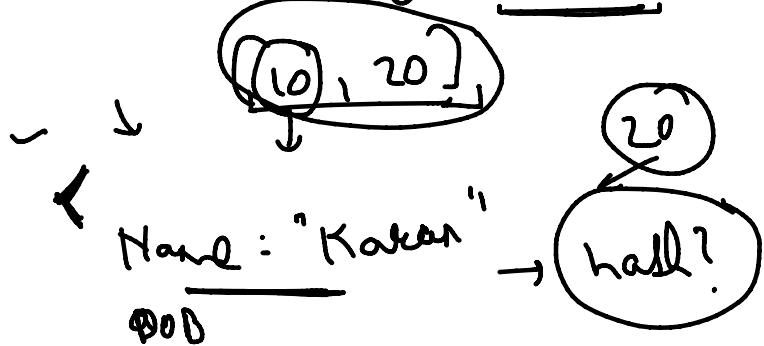
- ④ If hash table size $\rightarrow m$, it should return values 0 to $m-1$.

C++ \rightarrow unordered-map

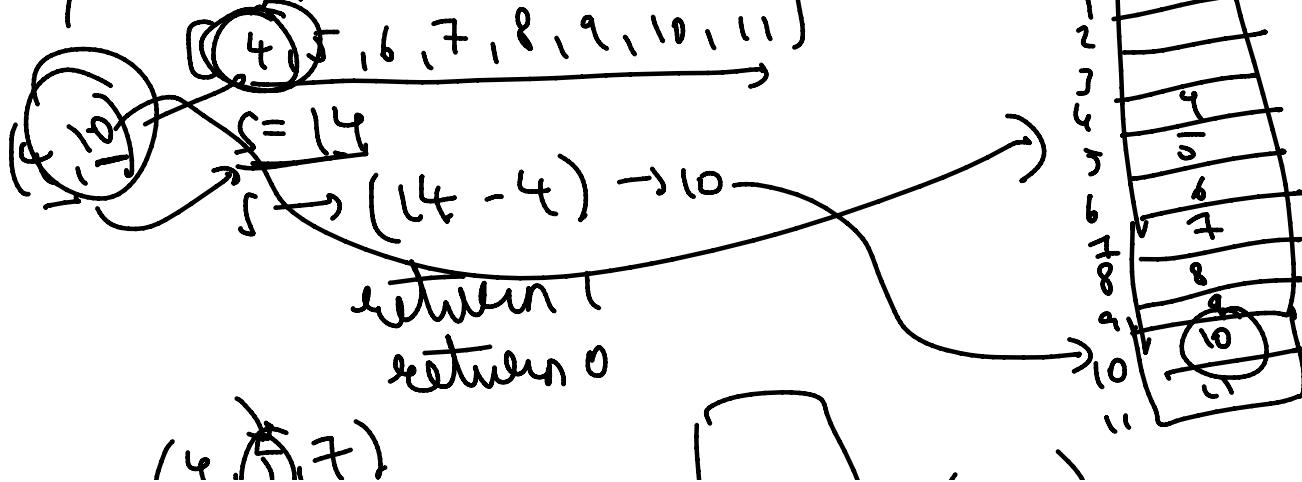
Java \rightarrow HashMap

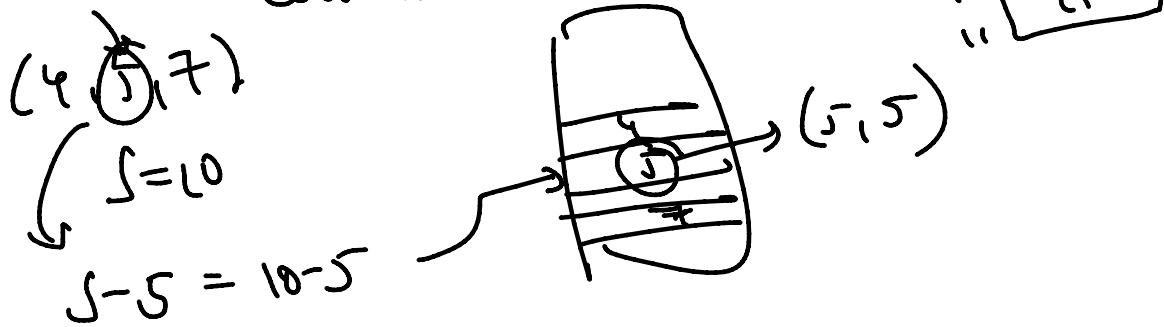
Not used when:-

- ① We are querying a range ok
in sorted order.



Q. $h(K) = K$





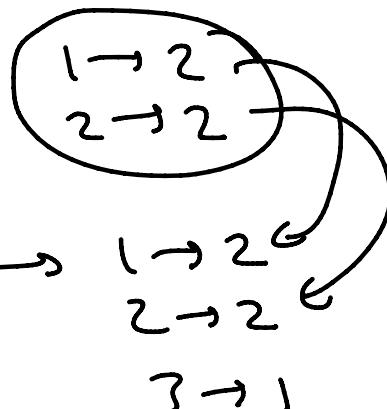
(4, 5, 7) S=10 → 0

(4, 5, 5, 7) S=10 → 1

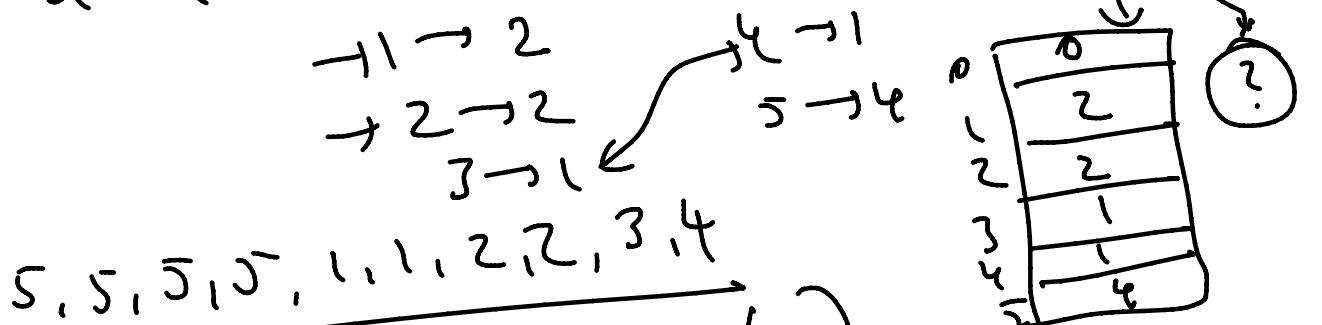
$A = \langle 1, 1, 2, 2 \rangle$

$B = \langle 2, 1, 2, 1, 3 \rangle$

if ($n! = n$)
return 0;



Q1. $a = \langle 1, 3, 1, 2, 2, 5, 5, 5, 5, 4 \rangle$



$\langle \langle 1, 2 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle, \langle 4, 1 \rangle, \langle 5, 4 \rangle \rangle$

~~$\langle \langle 4, 1 \rangle, \langle 1, 3 \rangle, \langle 2, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 5 \rangle \rangle$~~

sort → $\langle \langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 5 \rangle \rangle$

bool comp (int a, int b)

$\left\{ \begin{array}{l} \text{if } (a \leq b) \\ \quad \text{return } T; \\ \text{else} \\ \quad \text{return } F; \end{array} \right.$

3

sort (A, A+n, comp);

$a = \langle 5, 2, 3, 2, 5, 1, 1, 1 \rangle \rightarrow \begin{matrix} 1 \rightarrow 3 \\ 2 \rightarrow 2 \end{matrix}$

$a' = \langle \langle 5, 2 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle, \langle 2, 2 \rangle, \langle 5, 2 \rangle, \langle 1, 1 \rangle, \langle 1, 1 \rangle, \langle 1, 1 \rangle \rangle \rightarrow 2 \begin{matrix} 3 \rightarrow 1 \\ 4 \rightarrow 3 \end{matrix}$

sort

$a'' = \langle \langle 1, 3 \rangle, \langle 1, 3 \rangle, \langle 1, 3 \rangle, \langle 2, 2 \rangle, \langle 2, 2 \rangle, \langle 5, 2 \rangle, \langle 5, 2 \rangle, \langle 5, 2 \rangle, \langle 3, 1 \rangle, \langle 3, 1 \rangle \rangle$

a/ $A_1 = \langle 0, 5, 3, 7, 9, 8, 1, 4, 5, 1, 8 \rangle$

$A_2 = \langle 8, 9, 7, 3, 10 \rangle \rightarrow (3, 7, 8, 9, 10)$

$A_1 = \langle 8, 8, 9, 7, 3, 1, 4, 5, 1, 3 \rangle$

$n(\text{add}(i)) = i;$

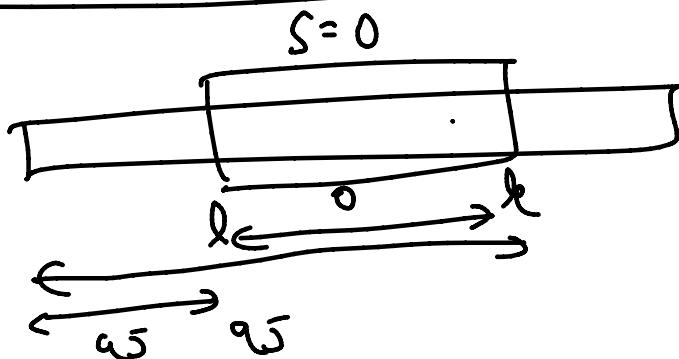
$A_2 = \langle 8, 9, 7, 3, 10 \rangle$



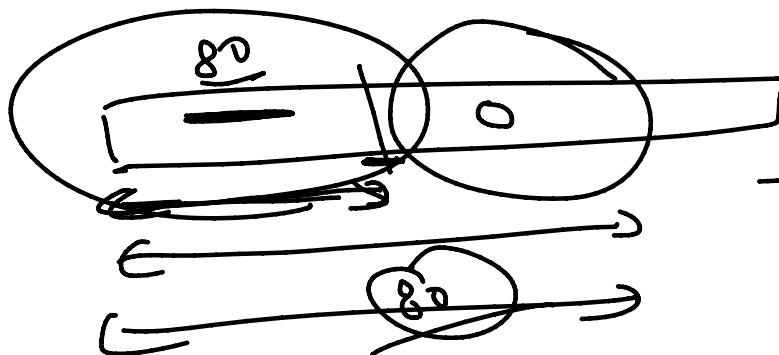
$n_1 = \cup_{i=1}^5 \{(5, 5), (3, 3), (7, 2), (9, 1), (8, 0), (1, 5)\}$

$A_1 = \{ (1, 5), (5, 5), (3, 3), (7, 2), (9, 1), (8, 0), (11, 5), (4, 5), (8, 0) \}$

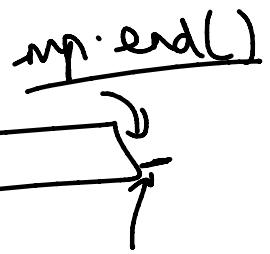
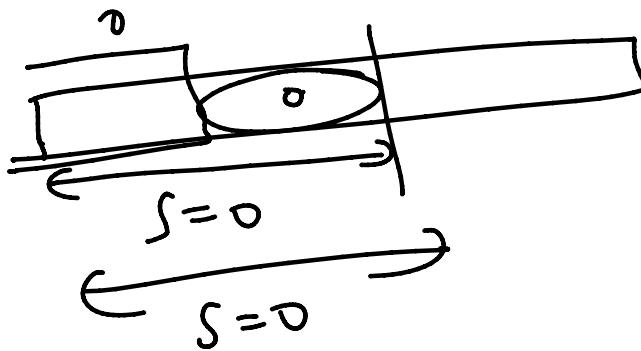
Q1.



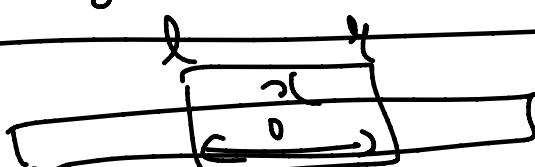
$$\text{pre-sum}(k) = \text{pre-sum}(k-1)$$



$$\text{mp}[80] = \text{true}$$



Q1.



$$\text{pres}[l-1] + \frac{x}{0} = \text{pres}[l]$$





$Q \quad A = [3, 5, 9, 4, 1, 6, 12, 7]$

$(q) \quad (i)$ $\boxed{[1, 2, 3, 4, 5, 6, 7]}$

0	1	2	3	4	5	6	7	8	9	10	11	12
F	T	T	T	T	T	T	T	F	T	F	F	T
↓	↓											
0	1	2	3	4	5	6	7	0	1	0	0	1

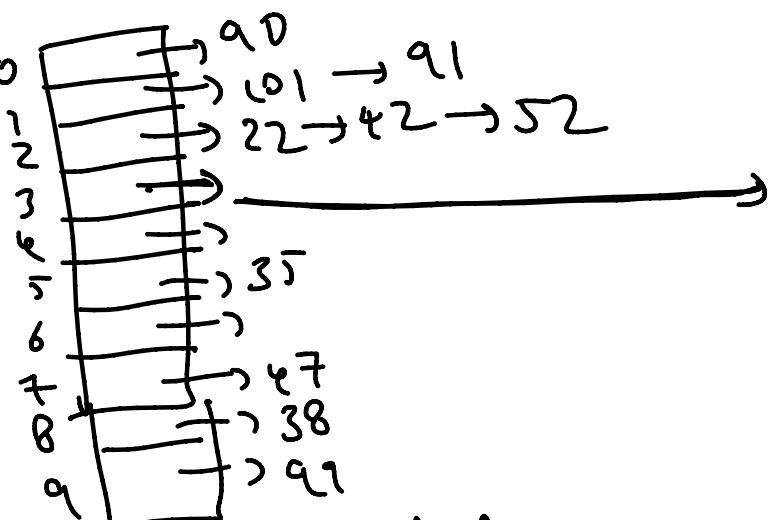
⇒ Collision Handling :-

- Chaining
- Open Addressing

① Chaining :-

$$h(K) = K \% 10;$$

$i \rightarrow 22, 35, 47, 90, 13, 16, 3, 18$
 $42, 38, 99, 101, 91, 52$



Load factor :- $\rightarrow \rightarrow \rightarrow 0.8$ of keys

Load factor :-

$$\alpha = \frac{n}{m}$$

\downarrow
 $n \rightarrow \text{no. of keys}$

$m \rightarrow \text{hash table size}$

assuming uniform distribution

\rightarrow Chaining \rightarrow Linked List, vector :-

insert $\rightarrow O(1) / O(1+\alpha)$

delete $\rightarrow O(1+\alpha)$

search $\rightarrow O(1+\alpha)$

\rightarrow Chaining \rightarrow maintain $\xrightarrow{\text{AVL tree for each node}}$ Balanced BST



α
insert $\geq O(\log \alpha)$
search
delete

