

C-Programming and DS

Today Class Topics

Memory Representation and Address calculation of

- ✓ 1D Array
- ✓ 2D Array
- ✓ 3D Array
- ✓ Sparse Matrix
- ✓ Recursion
- ✓ Types of Recursion

LTM
UTM



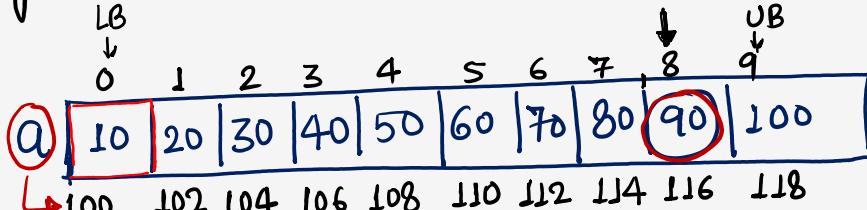
Array :-

- 1) Similar items stored
- 2) Contiguous memory allocation
- 3) Random access O(1) time complexity
- 4) Array index always starts from 0

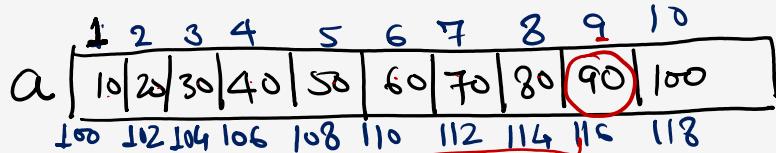
~~GATE CSE~~
~~DS → C~~



1D-Array:- $\text{int } a[0-9] = a[10] = \{10, 20, 30, \dots, 100\}$,



$$\begin{aligned}\text{loc}(a[8]) &= \text{Base}(a) + 8 * \text{SOE} \\ &= 100 + 8 * 2 = 116\end{aligned} \Rightarrow O(1)$$



$$\begin{aligned}\text{loc}(a[9]) &= \text{Base}(a) + (9 - 1) * \text{SOE} \\ &= 100 + 8 * 2 \Rightarrow 116 \Rightarrow O(1)\end{aligned}$$

$\text{int } a[10],$

Length of Array = $UB - LB + 1$
 $9 - 0 + 1 = 10$

SOE = S = Size of each element
 $\text{int}, S \Rightarrow 2 \text{ bytes}$

Question: $a[-67 \dots 769]$, BA = 2000, size of each element = 8 byte, loc($a[636]$) = ?

$\begin{matrix} \uparrow \\ LB \\ UB \end{matrix}$

$$\text{Total elements in the Array} = 769 - -67 + 1 = 837$$

$$\begin{aligned} \text{loc}(a[636]) &= BA + (636 - -67) * 8 = \underline{\underline{7624}} \\ &= \cancel{2000} + \cancel{8} \end{aligned}$$



Question: $a[-39 \dots +599]$, $BA = 6500$, $\text{Size of element} = 7 \text{ byte}$, $\text{loc}(a[546]) = ?$

$$\text{loc}(a[\underline{546}]) = 6500 + (546 - -39) * 7 = \underline{\underline{10595}}$$

$a[\underline{lb_1} \dots \underline{ub_1}]$, BA , S

$$\boxed{\text{loc}(a[i]) = BA + (\underline{i - lb_1}) * S}$$

$$= \underline{\underline{BA + i * S}}$$



NOTE:- $a[LB \dots UB]$, BA, ~~Sizeofelement(SOE)~~ = S ,

$$\text{Loc}(a[i]) = \text{BA} + (i - LB) * S$$

Why array index always start with 0 ?

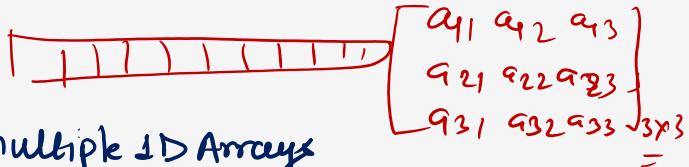
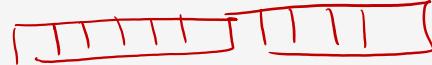


2D Array :-

Ex: $\text{int } a[1 \dots 4][1 \dots 5]$

$\text{int } a[4][5];$

$a[0 \dots 3][0 \dots 4]$



- 1) No. of Rows (n_r) = $ub_1 - lb_1 + 1 = 4 - 1 + 1 = 4$ ✓
- 2) No. of Columns (n_c) = $ub_2 - lb_2 + 1 = 5 - 1 + 1 = 5$ ✓
- 3) 2D-Array will be stored in memory in the forms of multiple 1D Arrays
- 4) 2D-Array elements will be stored in the memory in 2-different ways

✓ a) Row major order (RMO)

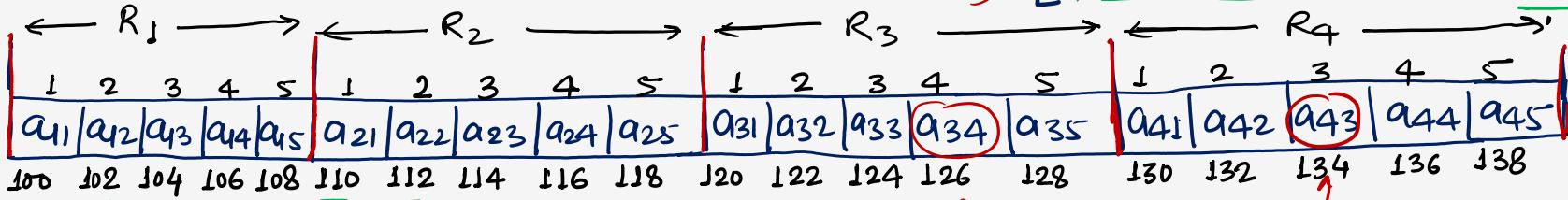
✓ b) Column major order (CMO)

C-Language by default follows row major order



Row major order (RMO) :-

	1	2	3	4	5
1	<u>a_{11}</u>	<u>a_{12}</u>	<u>a_{13}</u>	<u>a_{14}</u>	<u>a_{15}</u>
2	a_{21}	a_{22}	a_{23}	a_{24}	<u>a_{25}</u>
3	a_{31}	a_{32}	a_{33}	<u>a_{34}</u>	a_{35}
4	<u>a_{41}</u>	<u>a_{42}</u>	<u>a_{43}</u>	<u>a_{44}</u>	<u>a_{45}</u>



i) $\text{loc}(a[4][3]) = ? \quad 100 + [(4-1)*5 + (3-1)] + 2 = 134$

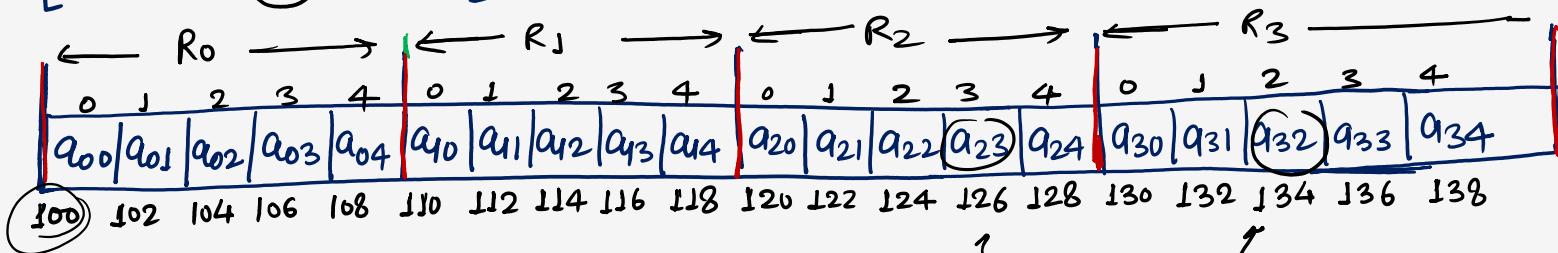
$$\begin{matrix} & (4-1) * 5 + (3-1) \\ \uparrow & \uparrow \\ lb_1 & lb_2 \end{matrix} + 2 = 134$$

ii) $\text{loc}(a[3][4]) = ? \quad 100 + [(3-1)*5 + (4-1)] + 2 = 126$

$$\begin{matrix} & (3-1) * 5 + (4-1) \\ \uparrow & \uparrow \\ lb_1 & lb_2 \end{matrix} + 2 = 126$$

	0	1	2	3	4
0	a ₀₀	a ₀₁	a ₀₂	a ₀₃	a ₀₄
1	a ₁₀	a ₁₁	a ₁₂	a ₁₃	a ₁₄
2	a ₂₀	a ₂₁	a ₂₂	a ₂₃	a ₂₄
3	a ₃₀	a ₃₁	a ₃₂	a ₃₃	a ₃₄

4x5



i) $\text{loc}(a[3][2]) = ? \quad 100 + (3+5+2)*2 = 134 \Rightarrow \underline{\underline{OU}}$

ii) $\text{loc}(a[2][3]) = ? \quad 100 + (2+5+3)*2 = 126$

Question: $a[-69 \dots +398] [-19 \dots +699]$, $BA = 7800$, $S = 9$ byte, $\text{loc}(a[361][673]) = ?$

Solⁿ :- \uparrow \uparrow \uparrow \uparrow
 lb₁ ub₁ lb₂ ub₂

$$n_r = ub_1 - lb_1 + 1 = 398 - -69 + 1 =$$

$$n_c = ub_2 - lb_2 + 1 = 699 - -19 + 1 = \underline{\underline{719}}$$

So E

$$\begin{aligned} \text{loc}(a[\underline{361}][\underline{673}]) &= 7800 + [(361 - -69) * \overset{\uparrow}{719} + (673 - -19)] * \overset{\uparrow}{9} \\ &= \underline{\underline{2796558}} \quad \text{Ans} \end{aligned}$$



Question: $a[-378 \dots +716][-428 \dots +658]$, BA = 8200, SOE = 8

$$\text{loc}(a[699][599]) = ?$$

$$n_c = 658 - -428 + 1 =$$

$$\text{loc}(a[\underline{699}][599]) = 8200 + [(699 - -(378) * n_c + (599 - -428)] * 8$$

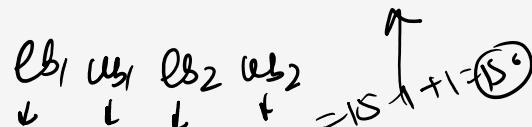
$\uparrow \quad \downarrow$
 $lb_1 \quad lb_2$

NOTE:- $A[\underbrace{lb_1 \dots ub_1}_{\text{Row}}][\underbrace{lb_2 \dots ub_2}_{\text{Col}}]$, BA, SOE = S ,

$$n_r = ub_1 - lb_1 + 1$$

$$n_c = ub_2 - lb_2 + 1$$

$$\text{loc}(a[i][j]) = \underline{\text{BA}} + [(i-lb_1)*n_c + (j-lb_2)] * S$$



Ex:- $A[1 \dots 10][1 \dots 15]$ of integer. Assume that each integer take one byte , Base(A)=100,

Find $\text{loc}[A[i][j]] = ?$

- a) $15i + j + 84$
- b) $15j + i + 84$
- c) $10i + j + 89$
- d) $10j + i + 89$

$$\begin{aligned}
 &= 100 + [(i-1)*15 + (j-1)] * 1 \\
 &= 100 + 15i - 15 + j - 1 \\
 &= 100 + 15i + j - 16 \\
 &\quad \cancel{15i + 5 + 84}
 \end{aligned}$$



Column major order (CMO) :-

	1	2	3	4	5
1	a ₁₁	a ₁₂	a ₁₃	a ₁₄	a ₁₅
2	a ₂₁	a ₂₂	a ₂₃	a ₂₄	a ₂₅
3	a ₃₁	a ₃₂	a ₃₃	a ₃₄	a ₃₅
4	a ₄₁	a ₄₂	a ₄₃	a ₄₄	a ₄₅

	C ₁				C ₂				C ₃				C ₄				C ₅			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	a ₁₁	a ₂₁	a ₃₁	a ₄₁	a ₁₂	a ₂₂	a ₃₂	a ₄₂	a ₁₃	a ₂₃	a ₃₃	a ₄₃	a ₁₄	a ₂₄	a ₃₄	a ₄₄	a ₁₅	a ₂₅	a ₃₅	a ₄₅

100 102 104 106 108 110 112 114 116 118 120 122 124 126 128 130 132 134 136 138

i) loc(a[4][3]) = ? $100 + [(3-1) \times 4 + (4-1)] * 2 = 122$

$\overbrace{\hspace{1cm}}^{\text{Es}_2}$ $\overbrace{\hspace{1cm}}^{\text{Es}_1}$

ii) loc(a[3][4]) = ? $100 + [(4-1) * 4 + (3-1)] * 2 = 126$

$\overbrace{\hspace{1cm}}^{\text{Es}_1}$ $\overbrace{\hspace{1cm}}^{\text{Es}_2}$

Question:- $a[-29 \dots +647][-3 \dots +799]$, BA = 3500, S = 8, $\text{loc}(a[611][738]) = ?$

$$\begin{aligned}\text{loc}(a[\underbrace{611}_{\uparrow}][\underbrace{738}_{\uparrow}]) &= 3000 + [(738 - 3) * (647 - -29 + 1) + (611 - -29)] * 8 \\ &= \underline{\underline{4021876}}\end{aligned}$$

Question: $a[-311 \dots +617][-428 \dots +618]$, $BA = 6800$, $SOE = 7$, $loc(a[599][577]) = ?$

$$loc(a[\underline{\underline{599}}][\underline{\underline{577}}]) = 6800 + [(577 - \cancel{428}) * (617 - \cancel{-311} + 1) + (599 - \cancel{-311})] * 7$$
$$= \underline{\underline{6548685}}$$

$\cancel{lb_1} - \cancel{lb_1} + \cancel{lb_1}$

NOTE:- $A[\underbrace{lb_1 \dots ub_1}_{\text{Row}}][\underbrace{lb_2 \dots ub_2}_{\text{Col}}]$, BA, SOE = S,

$$n_r = ub_1 - lb_1 + 1$$

$$\text{loc}(A[i][j]) = \underline{\text{Base}} + [(j-lb_2) * n_r + (i-lb_1)] * s$$

$$n_c = ub_2 - lb_2 + 1$$

Ex:- A[1...10][1...15] of integers. Assume that each integer take one byte, Base(A)=100,

find loc[A[i][j]]

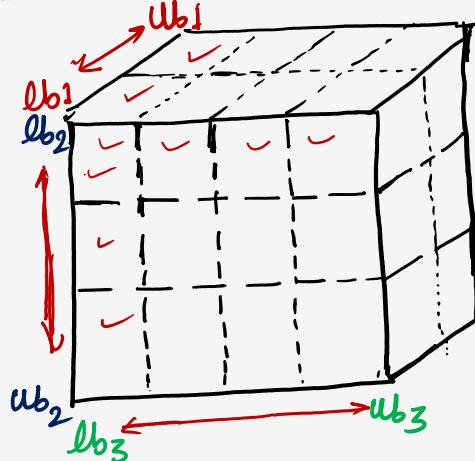
- a) $15i+j+84$
- b) $15j+i+84$
- c) $10i+j+89$
- d) $10j+i+89$
- e) ~~All are wrong~~

$$\begin{aligned} &= 100 + [(j-1)*10 + (i-1)] * 1 \\ &= 100 + 10j - 10 + i - 1 \\ &\Rightarrow \frac{i + 10j + 89}{10j + i + 89} \end{aligned}$$

3D-Array:-

$$a[\underbrace{lb_1 \dots ub_1}_{\text{Planes or Surfaces or layers}}][\underbrace{lb_2 \dots ub_2}_{\text{Row}}][\underbrace{lb_3 \dots ub_3}_{\text{Col}}]$$

$\text{int}[2][3][4]; []_{3 \times 4}$
↓



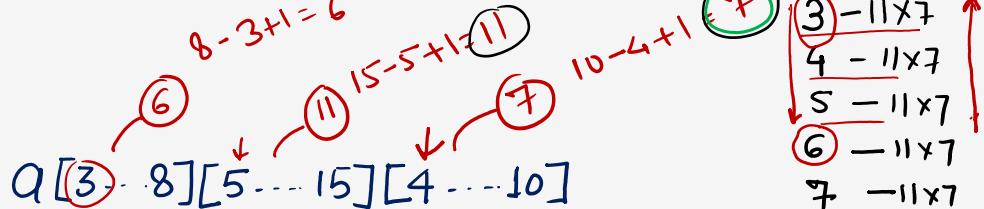
$$\text{No. of layers } (n_a) = ub_1 - lb_1 + 1 \quad \checkmark$$

$$\text{No. of rows } (n_r) = ub_2 - lb_2 + 1 \quad \checkmark$$

$$\text{No. of Columns } (n_c) = ub_3 - lb_3 + 1 \quad \checkmark$$

3D-Array stored into multiple 2D-Arrays

Row major order (RMO) :-



6 - 2D Arrays each of size 11×7

11 - 1D Arrays each of size 7

Ex: let $BA = 100$, $S = 2$, $\text{loc}(a[6][8][7]) = ?$

$$= 100 + [(6-3)*11*7 + (8-5)*7 + (7-4)] * 2$$

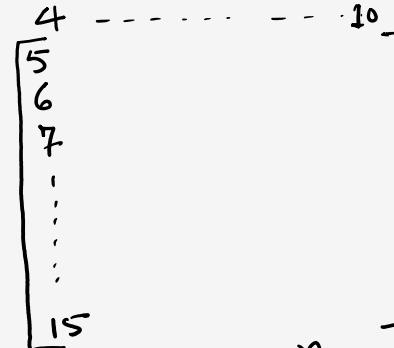
$$= 610$$

Ex:

$$\text{loc}(a[7][14][9]) = 100 + [(7-3)*11*7 + (14-5)*7 + (9-4)] * 2$$

$$= 852$$

Extra Add



Extra Add

$$n_c$$

$$lb_3$$

$$lb_1$$

$$n_r$$

$$lb_2$$

$$[(6-3)*11*7 + (8-5)*7 + (7-4)] * 2$$

NOTE:- $a[lb_1 \dots ub_1][lb_2 \dots ub_2][lb_3 \dots ub_3]$, BA, so E = S

Layers
Row
Column

RCMO:

$$loc(a[i][j][k]) = BA + [(i-lb_1)*n_r*n_c + (j-lb_2)*n_c + (k-lb_3)] * S$$

R₁₄₀
↓
Extraded
C₁₄₀

CMO:

$$loc(a[e][j][k]) = BA + [(i-lb_1)*n_r*n_c + (k-lb_3)*n_r + (j-lb_2)] * S$$

$$n_r = ub_2 - lb_2 + 1$$

$$n_c = ub_3 - lb_3 + 1$$

