Eigen values and Eigen vectors

$X \neq 0$

$AX = \lambda X$

Eigen vector

eigen value

Let A be a square matrix of order n×n

Now to find a non-zero vector X and a constant λ such that the equation

$$AX = \lambda X$$

Then λ is an eigen value and X is an eigen vector.

Method to find eigen vectors and eigen values of any square matrix A

$A_{n\times n}$ $\quad X_{n\times 1}$ $\quad AX = \lambda X$

$n \times 1$

$AX - \lambda X = 0$

$|A - \lambda I| \rightarrow$ characteristic polynomial

$\rightarrow \boxed{(A - \lambda I)X = 0}$

$X \neq 0$

$BX = 0 \rightarrow$ non-trivial sol^n

$|B| = 0$

$|A - \lambda I| = 0$

characteristic eq^n

To find eigen values, solve ch. eq^n.

$\longrightarrow \left\{ ax^2+bx+c=0 \right. \Rightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

The roots of the characteristic equation $|A - \lambda I| = 0$ are the eigen values of the matrix A.

$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 5-\lambda \end{bmatrix}$

Now, to find the eigen vectors, we simply put each eigen value into (A – λI) X = 0 and solve it by Gaussian elimination,

that is, convert the augmented matrix (A – λI) X= 0 to row echelon form and solve the linear system of equations thus obtained.

$|A - \lambda I| = 0$

$(1-\lambda)(5-\lambda) - 8 = 0$

$5 - \lambda - 5\lambda + \lambda^2 - 8 = 0$

$\lambda^2 - 6\lambda - 3 = 0$

$\lambda = \dfrac{6 \pm \sqrt{36+12}}{2} = \dfrac{6 \pm \sqrt{48}}{2}$

$= 3 \pm 2\sqrt{3}$

Example: Consider given 2×2 matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

Step 1: Characteristic polynomial and Eigenvalues.

$$[A - \lambda I] = \begin{bmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(3-\lambda) - 2 = 0$$

$$12 - 4\lambda - 3\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 7\lambda + 10 = 0$$

$$\lambda^2 - 5\lambda - 2\lambda + 10 = 0$$

$$(\lambda - 5)(\lambda - 2) = 0$$

$$\downarrow$$

$$\lambda = 5, 2$$

Example: Consider given 2×2 matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ $12 - 2 = 10$

Step 1: Characteristic polynomial and Eigenvalues.

Step 2: Eigenvectors

$AX = \lambda X$

$(A - \lambda I)X = 0 \rightarrow$ eigen vector

$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

corr. to $\lambda = 5$

$(A - 5I)X = 0$

$\begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\sim \left[\begin{array}{cc|c} -1 & 2 & 0 \\ 1 & -2 & 0 \end{array}\right]$

$R_2 \rightarrow R_2 + R_1 \Rightarrow \left[\begin{array}{cc|c} -1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Algebraic mult $\rightarrow$ no. of eigen values

Geometric mult $\rightarrow$ no. of eigen vector

Algebraic mult = Geom. mult

$\Rightarrow$ Diagonalizable.

Eigen vector corr. to $\lambda = 5$

$x_2 \rightarrow$ free

$-x_1 + 2x_2 = 0$

$x_2 = 1$

$x_1 = 2$.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Step 2: Eigenvectors

corr. to $\lambda = 2$

$$(A - 2I)X = 0$$

$$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 1 & 1 & 0 \end{array}\right]$$

$$R_2 \to R_2 - \frac{1}{2} R_1 \qquad \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Diagonalizable

$$\boxed{A = PDP^{-1}}$$

$$= P \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} P^{-1}$$

$$P = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

x_2 = free

$2x_1 + 2x_2 = 0$

$x_2 = 1$

$x_1 = -1$

Eigen vector corr. to $\lambda = 2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)^2 = 0$

$\lambda = 1, 1$

eigen vector corr. to $\lambda = 1$

$[A - 1I]X = 0$

$\rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_2 = 0$

$x_1 = \text{free}$

$x_1 = 1$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$P = \begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Not diagonalizable

Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 = 0$$

$$\lambda = 1, 1$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Eigen vector corr. to $\lambda = 1$

$$[A - 1I]X = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 1$, $x_2 = 0$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ | $x_2 = 1$, $x_1 = 0$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$(1-\lambda)(2-\lambda)(3-\lambda) = 0$$

Some important properties of eigen values

$\boxed{A \to 1, 2, 3}$

$5A \to 5, 10, 15$

$A^{-1} \to \frac{1}{1}, \frac{1}{2}, \frac{1}{3}$

- Eigen values of real symmetric and ~~Hermitian matrices~~ are real.
- Eigen values of real skew symmetric and ~~skew Hermitian~~ matrices are either pure imaginary or zero.

$A^T A = I$ $\quad \lambda = \pm 1$

$A^2 \to 1, 4, 9$

$I_{3\times3} \to 1, 1, 1$

- Eigen values of ~~unitary~~ and orthogonal matrices are of unit modulus $|\lambda| = 1$.

$A^2 + 2A \to 1^2 + 2\times1,\ 2^2 + 2\times2,\ 3^2 + 2\times3$

- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $k\lambda_1, k\lambda_2 \ldots\ldots k\lambda_n$ are eigen values of kA.
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $1/\lambda_1, 1/\lambda_2 \ldots\ldots 1/\lambda_n$ are eigen values of A^{-1}
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $\lambda_1^k, \lambda_2^k \ldots\ldots \lambda_n^k$ are eigen values of A^k

$A^2 + 7A + 4I$

$1^2 + 7\times1 + 4\times1 = 12$

$2^2 + 7\times2 + 4\times1 = 22$

$3^2 + 7\times3 + 4\times1 = 34$

$AB \neq BA$

$A(BC) = (AB)C$

- Eigen values of A = Eigen Values of A^T (Transpose)

- Sum of Eigen Values = Trace of A (Sum of diagonal elements of A)

- Product of Eigen Values = $|A|$

- Maximum number of distinct eigen values of A = Size of A

- If A and B are two matrices of same order then, Eigen values of AB = Eigen values of BA

- The Eigen values of a triangular matrix or diagonal matrix are given by its diagonal entries.

GATE CS 2011

Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \text{U.T}$$

Which one of the following options provides the CORRECT values of the eigenvalues of the matrix?

(A) 1, 4, 3
(B) 3, 7, 3
(C) 7, 3, 2
(D) 1, 2, 3

GATE CS 2010

Consider the following matrix

$$\begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(A) x=4, y=10 (B) x=5, y=8

(C) x=-3, y=9 (D) x= -4, y=10

Det A = prod of e.v.

$2y - 3x = 32$

$20 - 3x = 32$

$-12 = 3x$

$x = -4$

Trace of A = sum of E.V.

$2 + y = 12$

$y = 10$

GATE-CS-2015

The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ **is**

(A) 5
(B) 6
(C) 7
(D) 8

$\lambda_1 \lambda_2 = -6$

$\lambda_1 + \lambda_2 = 5$

$6, -1$

$\rightarrow |A - \lambda I| = 0$ $\quad$ $|A - A I| = 0$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$4 - 4\lambda - \lambda + \lambda^2 - 10 = 0$

$\lambda^2 - 5\lambda - 6 = 0$ $\quad \Rightarrow A^2 - 5A - 6I = 0$

$(\lambda - 6)(\lambda + 1) = 0$

$\lambda = 6, -1$

$A \rightarrow 2 \times 2$

$\boxed{\lambda^2 - (\text{trace of } A)\lambda + \text{Det } A = 0}$

characteristic eqn

GATE-CS-2016 (Set 1)

Two eigenvalues of a 3 x 3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is ______

$x^2 = -1$

$x = \pm i$

$2 + i$

then $2 - i$

$$\text{Det} = (2+i)(2-i)\,3$$

$$= (2^2 - (i)^2) \cdot 3$$

$$= (4+1)\,3$$

$$= 15.$$

GATE-CS-2016 (Set 2)

Suppose that the eigenvalues of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is

____

$$A^{-1} \rightarrow \frac{1}{1}, \frac{1}{2}, \frac{1}{4}$$

$$(A^{-1})^T \Rightarrow 1, \frac{1}{2}, \frac{1}{4}$$

$$|(A^{-1})^T| = 1 \times \frac{1}{2} \times \frac{1}{4}$$

$$= \frac{1}{8}$$

GATE CS Mock 2018

$\rightarrow 3\times 3$

The matrix A has $(1, 2, 1)^T$ and $(1, 1, 0)^T$ as eigenvectors, both with eigenvalue 7, and its trace is 2. The determinant of A is ____________ .

(A) 84 **(B)** 588

(C) 49 **(D)** None of these

Det = Prod. of E.V.

$= 7 \times 7 \times (-12)$

$=$

7, 7

7

Sum of E.V = trace

$7 + 7 + \lambda_3 = 2$

$\lambda_3 = -12$

GATE CS 2018

Consider a matrix $A = uv^T$ *where* $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ Note that v^T denotes the transpose of v.

The largest eigenvalue of A is ____3____ .

$A = uv^T$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \rightarrow$ Det = 0

trace = 3

0, 3

GATE CS 2022

Which of the following is/are the eigen vector(s) for the matrix given below?

$$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$

$X \neq 0$ → eigen vector

$AX = \lambda X$

$9 - 6 + 0 - 4$

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ $= X_1$

(b) $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

Let M be a 2×2 matrix with the property that the sum of each of the rows and also the sum of each of the columns is the same constant c. Which (if any) any of the vectors must be an eigenvector of M

$\rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(a) U = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(b) V = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) W = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) None of these

$$\begin{bmatrix} a & b \\ e & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ e \end{bmatrix}$$

α

$$\begin{bmatrix} a & b \\ e & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ e+d \end{bmatrix} = \begin{bmatrix} c \\ c \end{bmatrix} = c\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ e & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$$

$Av = \lambda v$

Let the $n \times n$ matrix A have an eigenvalue λ with corresponding eigenvector v. Which of the following statements are true for matrix A.

A. $-v$ is an eigenvector of $-A$ with eigenvalue $-\lambda$.

B. If v is also an eigenvector of the $n \times n$ matrix B with eigenvalue μ, then $\lambda\mu$ is an eigenvalue of AB.

C. Let μ be an eigenvalue of the $n \times n$ matrix B corresponding to the eigenvector v, Then $\lambda + \mu$ is an eigenvalue of $A + B$.

D. eigenvalue of A^3 is λ^3 and the eigenvector is v^3 $\rightarrow v$

$$Av = \lambda v$$

$$A^2 v = A(Av)$$

$$= A(\lambda v)$$

$$= \lambda(Av)$$

$$= \lambda(\lambda v)$$

$$A^2 v = \lambda^2 v$$

$$A^3 v = \lambda^3 v.$$

Answer: (A), (B) & (C)

Which of the following statements is/are FALSE?

A. For $n \times n$ real-symmetric matrices A and B, AB and BA always have the same eigenvalues.
B. For $n \times n$ matrices A and B with B invertible, AB and BA always have the same eigenvalues.
C. For $n \times n$ matrices A and B with B invertible, AB and BA always have the same eigenvectors.
D. For $n \times n$ real-symmetric matrices A and B, AB and BA always have the same eigenvectors

Answer: (A), (B)

For matrix $H = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$, one of the eigenvalues is 5. Then, the other eigenvalue is

A. 12
B. 10
C. 8
D. 6

Cayley Hamilton Theorem

$|A - \lambda I| = 0$

$\downarrow$

Every matrix satisfies its own characteristic equation

Example : $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$

$12 - 4\lambda - 3\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$\boxed{A^2 - 7A + 10I = 0}$

GATE CS 1993

If $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{pmatrix}$ the matrix A^4, calculated by the use of Cayley-Hamilton theorem or otherwise, is ______

Answer I

$$(\lambda - 1)(\lambda + 1)(\lambda - i)(\lambda + i) = 0$$

$$(\lambda^2 - 1)(\lambda^2 + 1) = 0$$

$$(\lambda^4 - 1) = 0$$

$$A^4 - I = 0$$

$$A^4 = I$$

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ then (A – 2I) (A – 3I) is

1. A

2. I

3. 0

4. 5 I

$$\begin{vmatrix} 4-\lambda & 2 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(1-\lambda)+2=0$$

$$4-4\lambda-\lambda+\lambda^2+2=0$$

$$\rightarrow \lambda^2-5\lambda+6=0$$

$$\rightarrow (\lambda-2)(\lambda-3)=0$$

$$(A-2I)(A-3I)=0 \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A 3 × 3 matrix is such that, $P^3 = P$. Then the eigenvalues of P are

$\lambda^3 = \lambda$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = 0, 1, -1$

Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix. Then (c + d) is equal to

Answer : 5

$\frac{6}{\lambda} = \lambda^2 + c\lambda + d$

$c = -6, \quad d = 11$

5

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)\left[(4-\lambda)(1-\lambda)+2\right] = 0$

$(1-\lambda)\left[4-4\lambda-\lambda+\lambda^2+2\right] = 0$

$(1-\lambda)\left[\lambda^2-5\lambda+6\right] = 0$

$\lambda^2-5\lambda+6-\lambda^3+5\lambda^2-6\lambda = 0$

$-\lambda^3+6\lambda^2-11\lambda+6 = 0$

$\lambda^3-6\lambda^2+11\lambda-6 = 0$

$A^3 - 6A^2 + 11A - 6I = 0$

$A^2 - 6A + 11I - 6A^{-1} = 0$

$A^2 - 6A + 11I = 6A^{-1}$

GATE CS 2015

In the given matrix, one of the eigenvalues is 1. the eigenvectors corresponding to the eigenvalue 1 are

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = \alpha X$$

(A) $\{\alpha(4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$ (B) $\{\alpha(-4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(C) $\{\alpha(\sqrt{2},0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$ (D) $\{\alpha(-\sqrt{2},0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

$$(A - I)X = 0$$

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_2 = 0$

$-x_2 + 2x_3 = 0$

$x_2 = 2\alpha$

$x_3 = \alpha$

$x_1 = -4\alpha$

$$\begin{bmatrix} -4\alpha \\ 2\alpha \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} \alpha$$

GATE CSE 2014

Which one of the following statements is TRUE about every n×n matrix with only real eigenvalues?

A. If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

B. If the trace of the matrix is positive, all its eigenvalues are positive.

C. If the determinant of the matrix is positive, all its eigenvalues are positive.

D. If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

Answer: A

$4, -1, -1 = \underline{2}$

$\downarrow$

$\underline{4}$

trace × Det

$2 \times 4 = 8$

trace = +ve

→ det = -ve

GATE CSE 2012

Let A be the 2×2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$.
Then the eigenvalues of the matrix A^{19} are

A. 1024 and -1024
B. $1024\sqrt{2}$ and $-1024\sqrt{2}$
C. $4\sqrt{2}$ and $-4\sqrt{2}$
D. $512\sqrt{2}$ and $-512\sqrt{2}$

Answer: D

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \rightarrow \begin{matrix} 0 \\ -2 \end{matrix}$

$\lambda_1 + \lambda_2 = 0$

$\lambda_1 \cdot \lambda_2 = -2$

$\lambda_1^2 = 2$

$\boxed{\lambda = \pm\sqrt{2}}$

$(\pm\sqrt{2})^{19}$

$(\pm 2)^{19/2}$

$\pm (2)^9 \cdot \sqrt{2}$

$\pm 512\sqrt{2}$

Thank you