

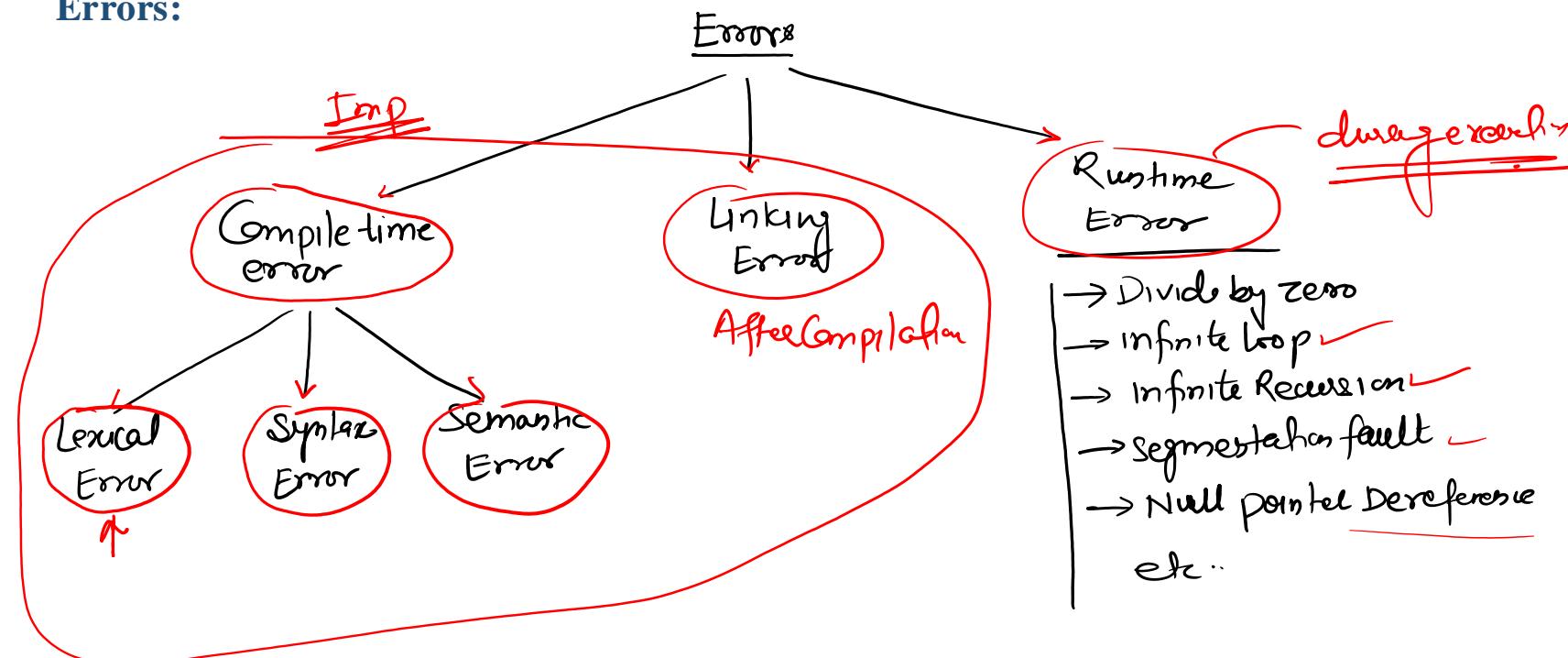
Today Class Topics

- Errors ✓
- Context free grammar(CFG) ✓
- Parse tree or derivation tree ✓
- Ambiguous and Unambiguous grammar ✓
- Left recursion and how to remove left recursion ✓
- Left factoring and left factored grammar ✓

Syntax Analysis

(Compiler Design)

Errors:



(Compiler Design)

Lexical Error: if the scanned group of characters are not matching with any of rules (pattern) of the Tokens 36



Lexical Analyzer produce an error.

- Unterminated Comment:

Ex: ~~/* Comment~~, ~~/* Comment */~~ ~~lexer~~ ~~lexer~~ ✓

$$\underline{L} \underline{(L+D)}^*$$

- Unmatched string: ~~“eabcd”~~, ~~“abcd”~~, ~~“eascl”~~

$$\underline{(L,-)} \underline{(L+D+-)}^*$$

- Invalid Identifier: Ex: int ~~1a~~, ~~2a~~; lexer error

- Appearance of illegal characters:

Ex: Smt a= ~~\$10; 2~~ lexer voidmain() { int a, b, c; c=a+b; @ }

- Exceeding the length of identifier: 32 character

(Compiler Design)

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Ex: `printf("Delhi is a beautiful city);` LAE

Ex: `int a=10.26;`

`a=a+5;`

`b=b*a; /* Multiply b with a */`

`printf("%d",b);` \Rightarrow Lexical error

Ex3 `int a=09; // octal no. (0 - 7) Lexical error.`

Ex4 `int a=0b12 // (0 or 1) allows Lexical error`

Ex5 `int a=0xgAGH; // not allowed lexical error`

Hexadecimal No. $(0 - 9, A - F)$

$0 - 9, 10, 11, 12, 13, 14, 15$

$A - F$

(Compiler Design)

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Syntax Error: (Pattern)

- 1) missing (, &, ; etc
- 2) missing operator $ab+c$
- 3) misspelled keyword Ex! - for

$ab + c$

for ($i=0$; $i < n$; $i++$)
or
for ($i=0$, $i < n$, $i++$)

int ab ;
int a, b ; op1 @ op2

Syntax error

Syntax errors

$a = b + cd *$

Ex! - While(1 Syntax error
 { prompt("Hello");
 }

Ex! - int x, y ; Syntax error

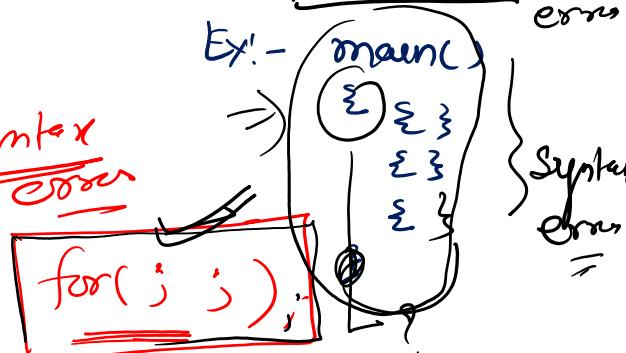
Ex! - void main()
 {

Ex! - ef(); Syntax error

Ex! - main() Syntax error

Ex! - int x, y ;
 $x = 2;$
 $y = 3;$

Ex! - while();



Ex! - for(, ,);

Ex! - $a = b + cd *$;

(Compiler Design)

(Semantic Analysis)

Semantic Error: Type mismatch

- 1) Incompatible type of operands
- 2) Undeclared variable
- 3) Unmatch Actual & formal parameter

int a, b, c;

C = a + b

Symbol Table

SN	Vname	Type	Value
1	a	int	
2	b	int	
3	c	int	

Ex 1

void main()

{ int a, c;
float b;
C = a + b;

 | |
 int float

Semantic error

Ex 2

int sum;

Sum = 15;

SN	Vname	Type
1	Sum	int

③

void main()

{ int x;
x = y;

 |
 y

④

whiel(1);

Whiel(1);

linking error

function calling

Ex' int Sum;
Sum = 15;

(Compiler Design)

Ex:

Consider line number 3 of the following C-program.

```
int main() {  
    int I, N;  
    fro (I=0, I<N, I++);  
}
```

/*Line 1 */
/*Line 2 */
/*Line 3 */

$$\bullet n = 2$$

$$0 < 2$$

fro (l=0, l < n, l++);

fro (0, 1, 0);

lenkey

Identify the compiler's response about this line while creating the object-module:

- A. No compilation error
- B. Only a lexical error
- C. Only syntactic errors
- D. Both lexical and syntactic errors

$\Rightarrow \underline{\text{fro}}(\underline{l=0}, \underline{l < n}, \underline{l++});$

↓ notmatch ↓ notmatch

fro(l=0; l < n; l++); // Syntax error

for(l=0, l < n, l++); // Syntax error

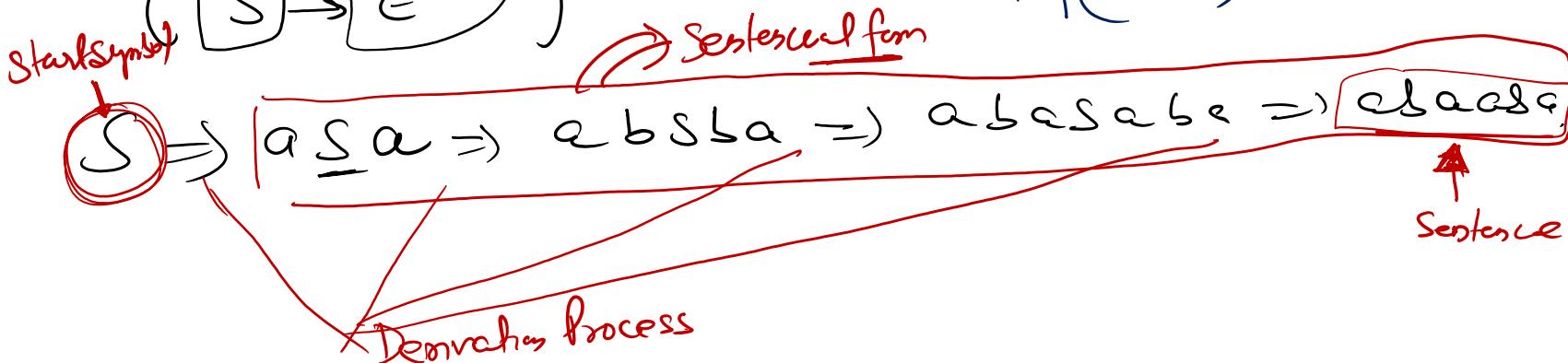
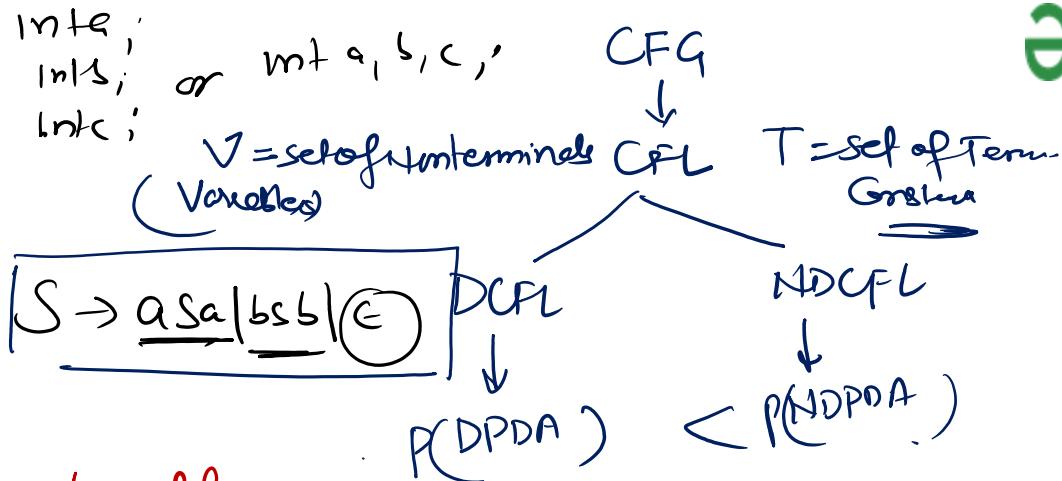
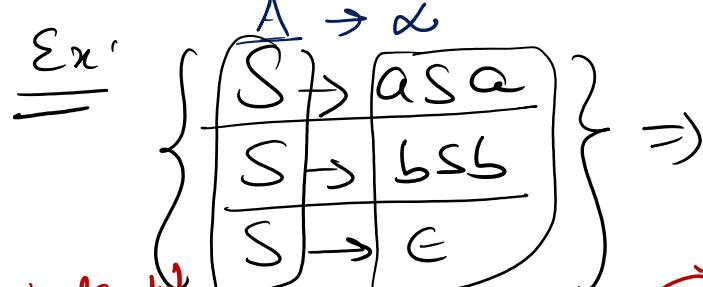
for(l=0; l < n; l++); // No Syntax error

(Compiler Design)

Context Free Grammar (CFG)

$$A \rightarrow \alpha, \text{ where } A \in V$$

$\alpha \in (V \cup T)^*$



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Derivation Tree or Parse Tree: Graphical Representation of Derivation Process

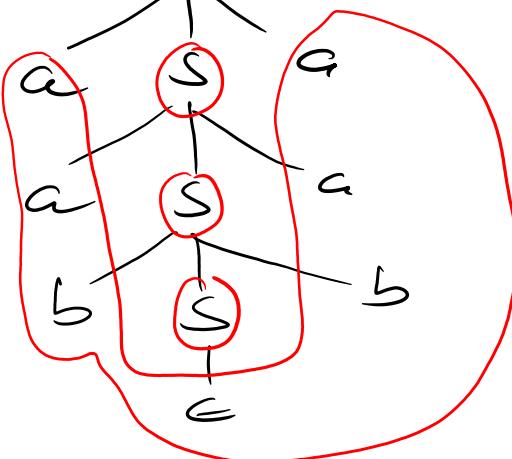
$$S \rightarrow aSa / bSb / c$$

Derivation Process:

$$S \Rightarrow aSa \Rightarrow \underline{a} \underline{S} \underline{a} \Rightarrow a \underline{a} \underline{b} \underline{S} \underline{b} \underline{c} \Rightarrow \underline{\underline{a}} \underline{\underline{a}} \underline{\underline{b}} \underline{\underline{c}}$$

$\xrightarrow{\text{RSI}}$ Starts^s NonTerm Internalnode = Nonterminal Sentence.

leafnode = Terminal symb/

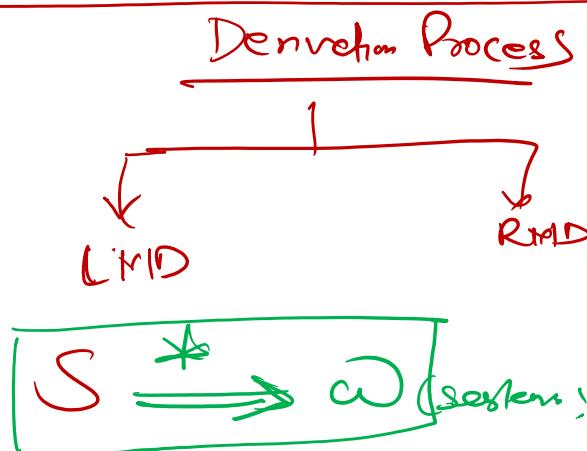
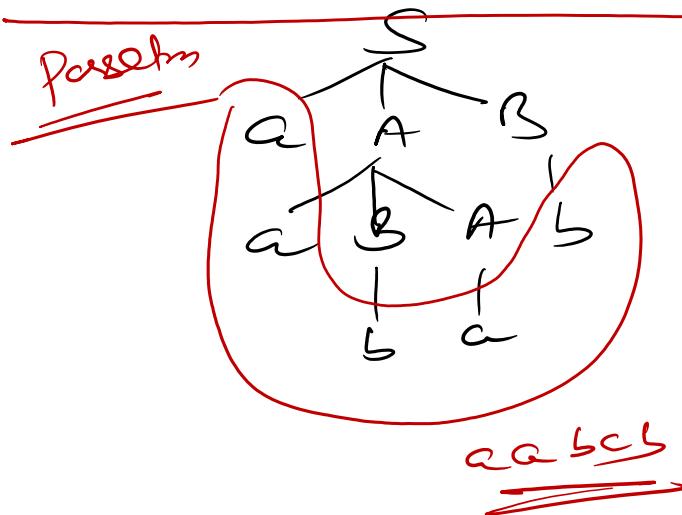
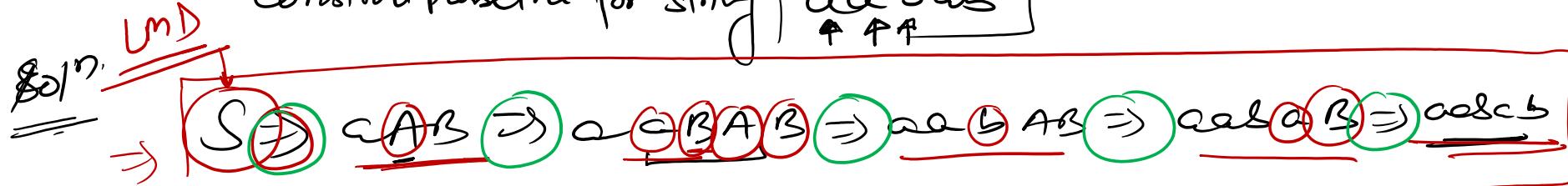


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Ex:- $G: S \rightarrow aAB|a, A \rightarrow aba|a, B \rightarrow b$



Construct Parse tree for string aabab



Derivation process



There are two types of derivation process,

1. Left most derivation (LMD) ✓
2. Right most derivation (RMD) ✓

Left most derivation (LMD):

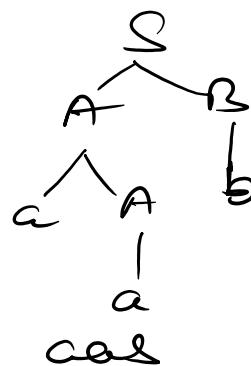
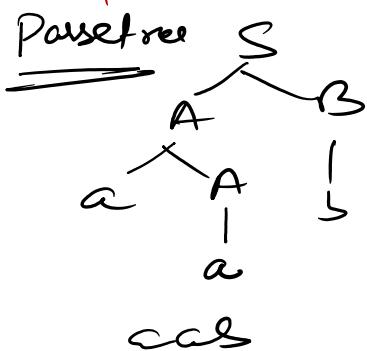
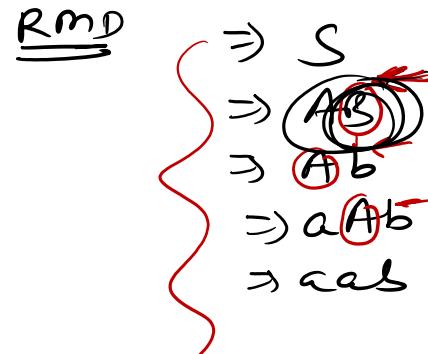
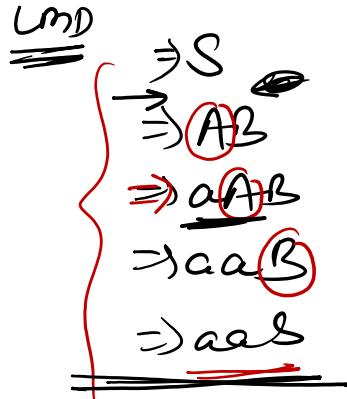
A derivation $\boxed{S \xrightarrow{*} w}$ is called a leftmost derivation if we apply a production only to the leftmost variable at every step.

Right most derivation (RMD):

A derivation $\boxed{S \xrightarrow{*} w}$ is called a rightmost derivation if we apply a production to the right most variable at every step.

(Compiler Design)

Ex: $G = \{ S \xrightarrow{\cdot} AB, A \xrightarrow{\cdot} aA/a, B \xrightarrow{\cdot} bB/b \}$



Note: LMD & RMD are different but their derivation tree need not be different

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Example: Consider a grammar G for the language, $L = \{a^{2n}b^m, m, n \geq 0\}$

$G: S \rightarrow AB, A \rightarrow aaA \mid \epsilon, B \rightarrow bB \mid \epsilon$, find the LMD and RMD for string $w = aab$

$w = aab$

LMD

$S \Rightarrow AB$
 $\Rightarrow aa\overline{AB}$
 $\Rightarrow a\overline{aB}$

$\Rightarrow a\overline{aB}$

$\Rightarrow a\overline{ab}$

RMD

Image

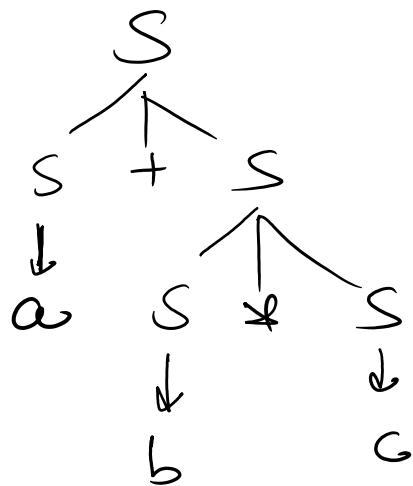
$S \Rightarrow AB$
 $\Rightarrow A b\overline{B}$
 $\Rightarrow \underline{A}\overline{b}\epsilon$
 $\Rightarrow aa\overline{AB}$
 $\Rightarrow aa\overline{b}$

(Compiler Design)

Example: $G: S \rightarrow S+S \mid S*S \quad a \mid b \mid c$, find the LMDT and RMDT for string $w = a + b * c$

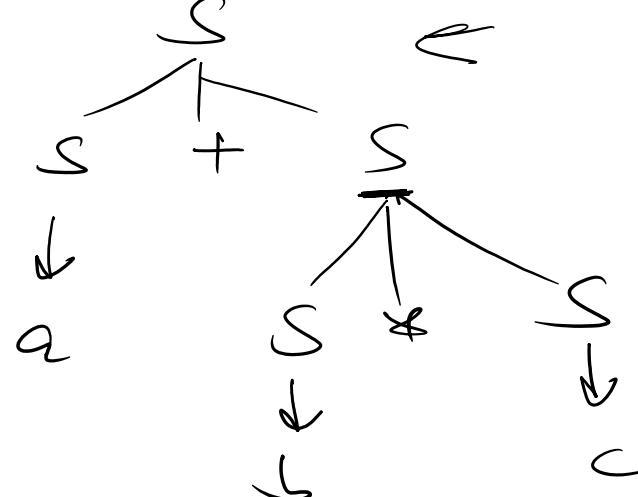


LMDT



$\Rightarrow a + b * c$

RMDT



$a + b * c$

(Compiler Design)

Slide 8



Ex:

$$S \rightarrow a$$

$$A \rightarrow Ab$$

$$B \rightarrow a$$

Starting Symbol is S

Note: ① LMD and RMD need not be same

② No. of LMD = No. of RMD = No. of parse trees.

$$\omega = a$$

No. of derivations = No. of LMD = No. of RMD = No. of parse trees

LMD

$$① S \Rightarrow a$$

$$② S \Rightarrow AB \Rightarrow B \Rightarrow a$$

RMD

$$① S \Rightarrow a$$

$$② S \Rightarrow AB \Rightarrow Aa \Rightarrow a$$

Parse tree

$$① S \\ | \\ a$$

②



(Compiler Design)

~~Ambiguous Grammar~~



A grammar G , is ambiguous grammar if $\exists w \in L(G)$, such that w has > 1 Parse tree either using 2 LMD or 2 RMD i.e., a grammar G is ambiguous if there is more than one Parse Tree or LMD / RMD for a string $w \in L(G)$.

Example: $G: S \rightarrow S+S \mid S^*S \mid a \mid b \mid c$, Grammar G is ambiguous because there exist two different LMD for a string $w = a + b * c$

In Ambiguous

G is Ambiguous if $\exists w \in L(G)$

$$G \Rightarrow L(G) = \{ \quad \}$$

$\exists w \in L(G) - , w = \text{SPT}$
 $= \text{SLMD}$
 $= \text{IRB}$

$w = \text{SPT}$
or
 SLDT
or
 SLMD
or
 IRMD

(Compiler Design)



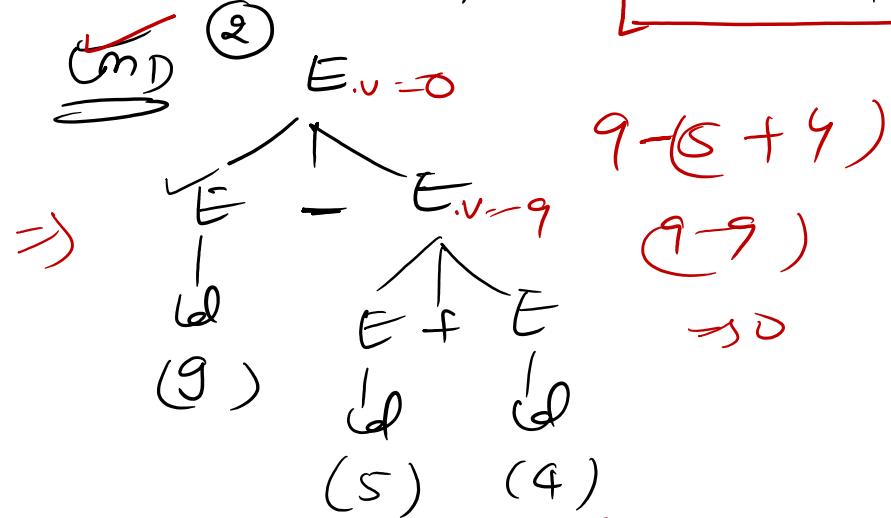
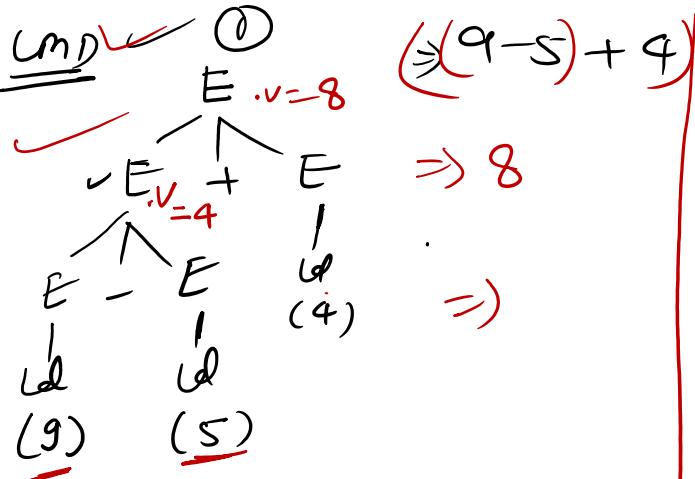
NOTE:

1- A grammar G is unambiguous if there exist exactly one parse tree or LMD / RMD for all the string, $w \in L(G)$

2- If a grammar G is ambiguous, it doesn't mean language (L) is ambiguous.

Ambig^{ous}

~~Example:~~ State whether given grammar is ambiguous or not. $G: E \rightarrow E+E \mid E-E \mid id$, $w = 9-5+4$



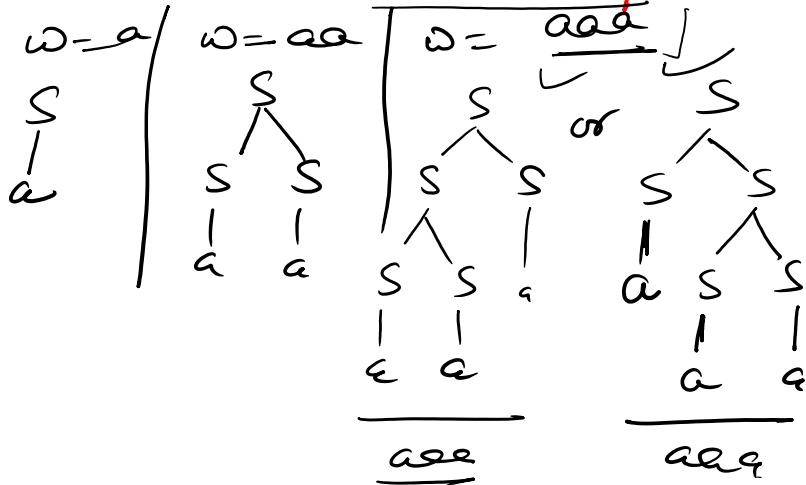
(Compiler Design)

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Ex 1)

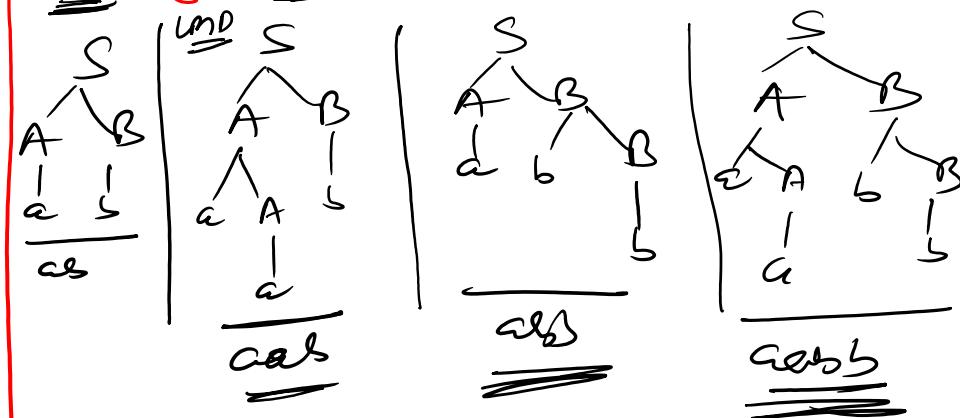
$$G = \{ S \rightarrow SS | a \}$$



Ambiguous Grammars

Ex 2

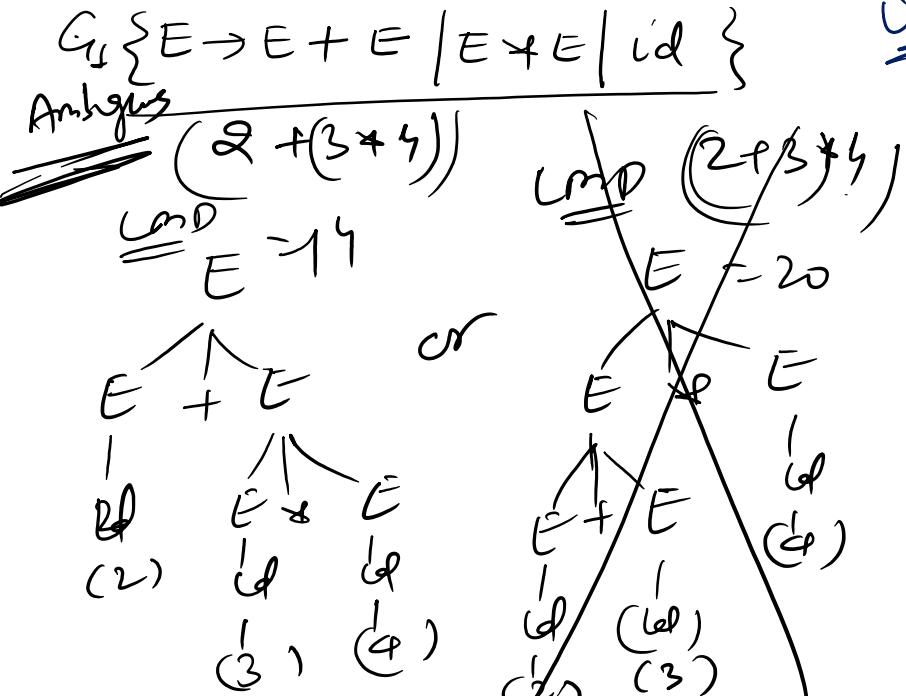
$$G = \{ S \rightarrow AB, A \rightarrow aA | a, B \rightarrow bB | b \}$$



On Ambiguity

(Compiler Design)

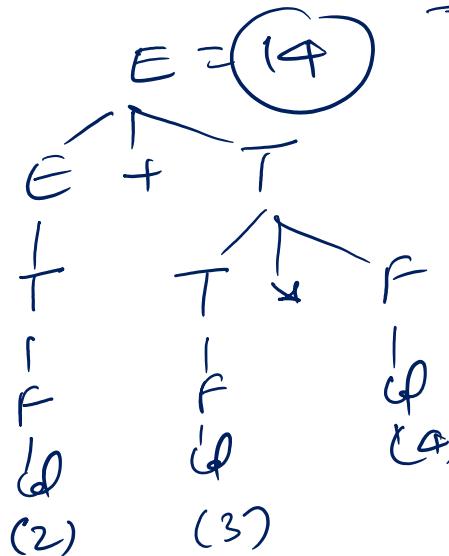
Slide 12



~~G₂ { E → E + T | T }~~

T → T * F | ·F
F → id

2 + 3 * 4



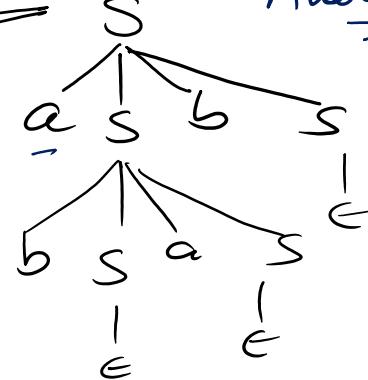
(Compiler Design)

Slide 13

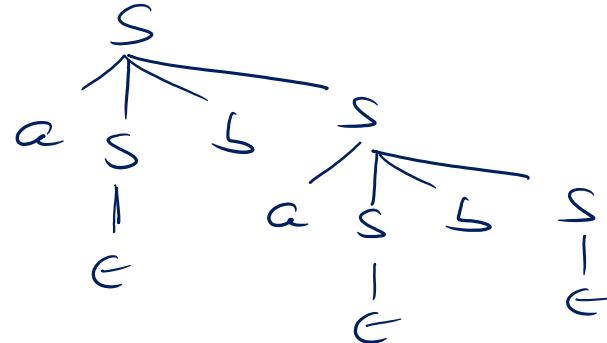


$G = \{ S \rightarrow aSbS \mid bSaS \mid \epsilon \}$ Check Ambiguous?

Soln: LMD:



Always Ambiguous LMD



$$\frac{a \cdot \cancel{b} \cdot a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{e}}{\Rightarrow abaa}$$

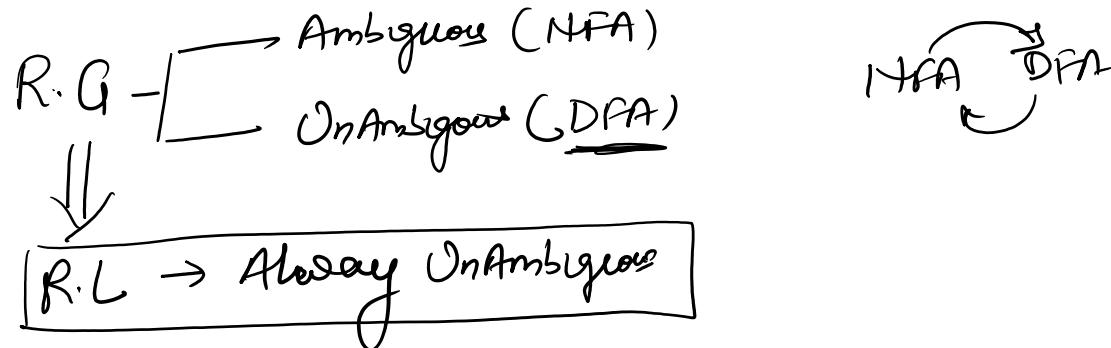
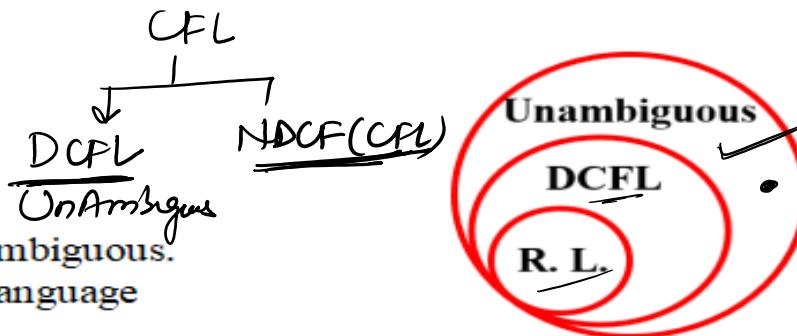
$\Rightarrow \boxed{abaa}$

(Compiler Design)



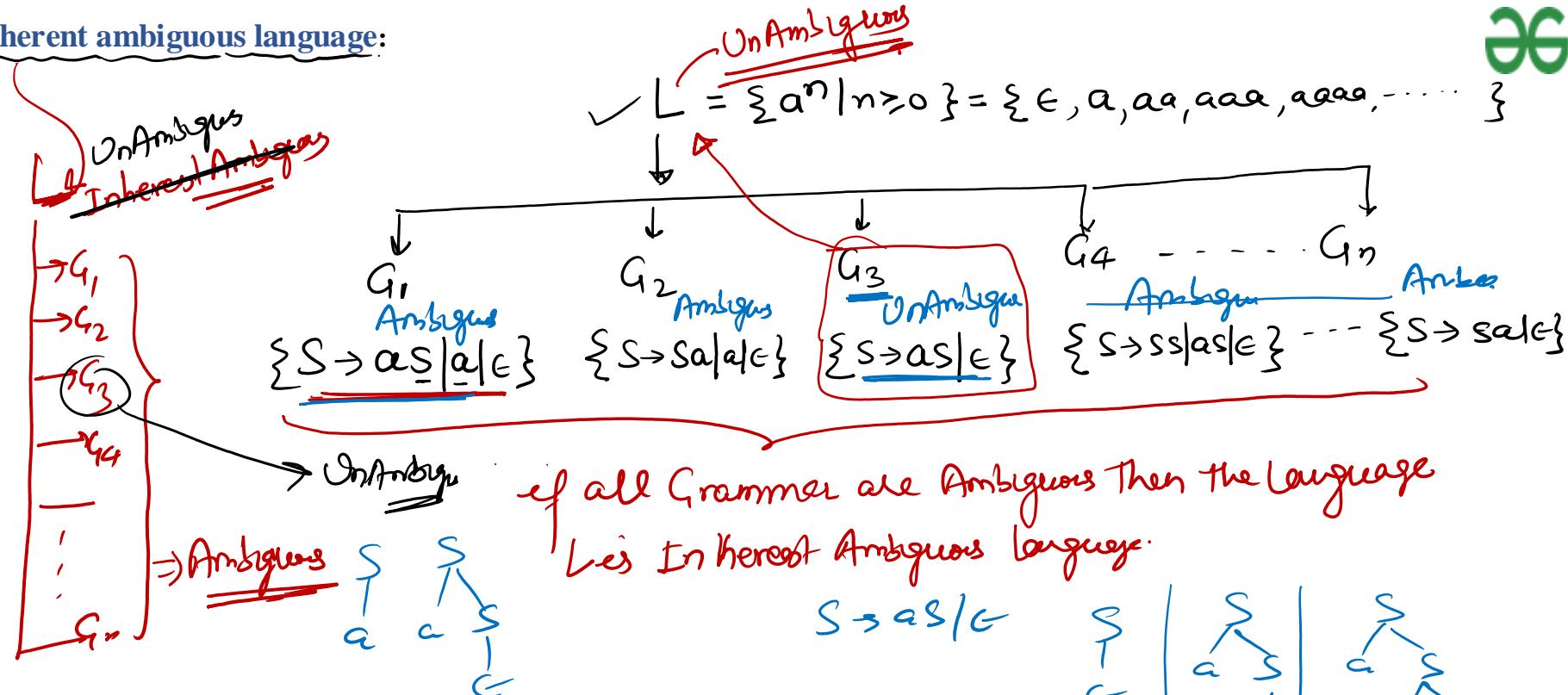
NOTE:

- Regular Grammar:
 - Ambiguous (NFA) ✓
 - Unambiguous (DFA) ✓
- Regular language is always unambiguous.
- DCFL is always unambiguous language
- Ambiguity starts from CFL



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Inherent ambiguous language:

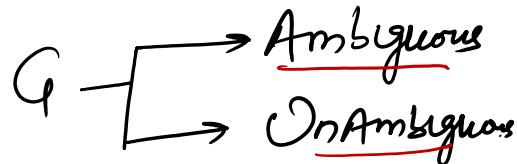


(Compiler Design)

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Example: Check whether the given language $L = \{a^n \mid n \geq 0\}$ is an inherently ambiguous or not

UnAmbiguous language means not Inherently Ambiguous



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Example: Given language $L = \{a^m b^n c^k \mid m, n, k \geq 1, \text{ either } m = n \text{ or } n = k\}$ is Inherent Ambiguous?

$$\Rightarrow L = \left\{ \overbrace{a^m b^m c^k}^{\substack{L_1 \\ A \cdot B}} \mid m, k \geq 1 \right\} \cup \left\{ \overbrace{a^m b^n c^n}^{\substack{L_2 \\ C \cdot D}} \mid m, n \geq 1 \right\}$$

Inherent Ambiguity



G:

$$\left\{ \begin{array}{l} S \rightarrow \underline{AB} \mid \underline{CD} \\ A \rightarrow aAb \mid ab \\ B \rightarrow cB \mid c \\ C \rightarrow aC \mid a \\ D \rightarrow bDc \mid bc \end{array} \right\}$$

$$L_1 = \left\{ \underline{a^m b^m c^k} \mid m, k \geq 1 \right\} = \left\{ \underline{a^n b^n c^n} \mid n \geq 1 \right\}$$

$$L_2 = \left\{ \underline{a^m b^n c^n} \mid m, n \geq 1 \right\}$$

for this type strings
 $\boxed{\underline{a^n b^n c^n}}$

$$L_1 \cap L_2 = \left\{ \underline{a^i b^i c^i} \mid i \geq 1 \right\}$$

$$\left\{ \text{no of } a = \text{no of } b \right\} \text{ AND } \left\{ \text{no of } b = \text{no of } c \right\}$$

(Compiler Design)

Note: If grammar is ambiguous then it is not suitable for any kind of parsing technique except backtracking and operator precedence parsing

(Compiler Design)

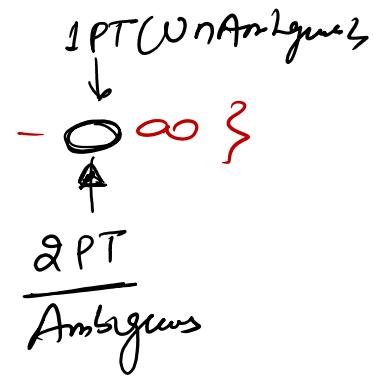


Remove Ambiguity:

It is not always possible to convert an ambiguous grammar into an unambiguous grammar because
ambiguity finding & removal both are undecidable

$$G \Rightarrow L(G) = \sum \underline{\omega_1, \omega_2, \omega_3, \omega_4} \dots \dots \dots$$

\uparrow \overline{T} $\overline{1}$ $\overline{1}$ \uparrow
 $\downarrow \text{PT}$ $\downarrow \text{PT}$ $\downarrow \text{PT}$ $\downarrow \text{PT}$



Id + Id * Id

(Compiler Design)



Remove Ambiguity by precedence & associativity rules:

An ambiguous grammar may be converted into an unambiguous grammar by implementing:

- Precedence Constraints ✓
- Associativity Constraints ✓

These constraints are implemented using the following rules:

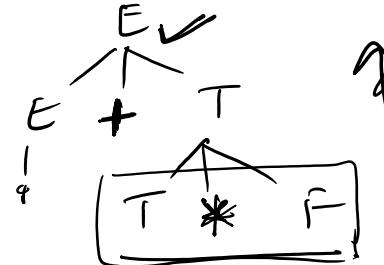
Rule 1: The precedence constraint is implemented using the following rules:

- The level at which the production is present defines the priority of the operator contained in it.
- The top level of the production, the lower the priority of the operator.
- The bottom level of production, the higher the priority of the operator.

Rule 2: The associativity constraint is implemented using the following rule:

- If the operator is left associative, induce left recursion in its production.

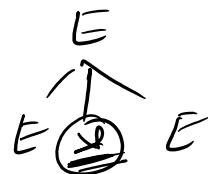
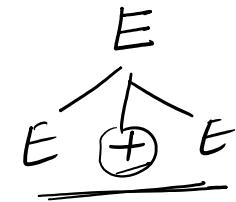
If the operator is right associative, induce right recursion in its production



Left Assoc → Left Recur
Right Assoc → Right Recur

(Compiler Design)

Example: $G: E \rightarrow E+E \mid E^*E \mid \text{id}$. Ambiguous grammar converts into unambiguous grammar



~~Id + Id * Id~~

$E \rightarrow E+E \mid T/T$
 $T \rightarrow T+F \mid F$
 $F \rightarrow \text{id}$

On Ambiguity

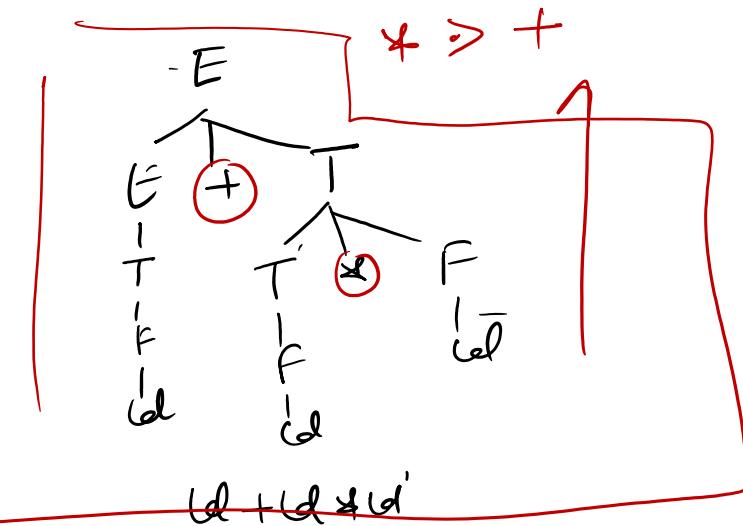
X

$* > +$

$[+ > +]$ left Assoc.



$[* > *]$ left Assoc.



(Compiler Design)

Example: Convert the following ambiguous grammar into unambiguous grammar.

$$G: E \rightarrow E+E \mid E^*E \mid E^\wedge E \mid id$$

$$E \rightarrow E+E$$

$$E \rightarrow E^*E$$

$$E \rightarrow E^\wedge E$$

$$E \rightarrow id$$

The priority order: $id > ^\wedge > ^* > . > +$

Associativity: $+$ and $*$ are left associative and $^\wedge$ operator is right associative

$$\begin{array}{c} \uparrow > * > + \\ + > + \\ * > * \\ \uparrow < \cdot \uparrow \end{array}$$

Unambiguous Grammar

Apply Rule 1 & 2

Starting Symbol left Assoc

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow G \uparrow F \mid G$$

$$G \rightarrow id$$

(+) left associative

(*) left associative

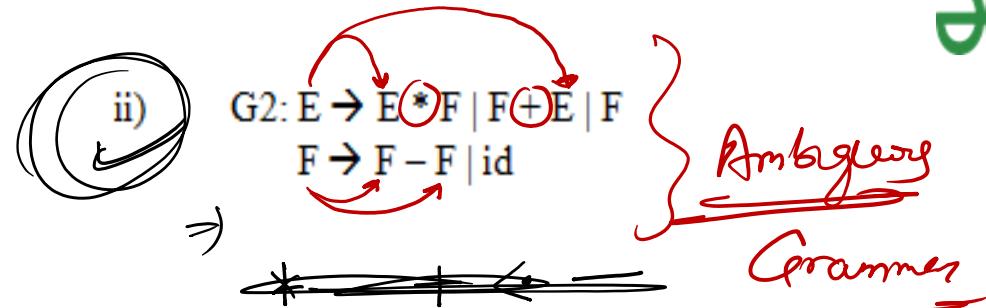
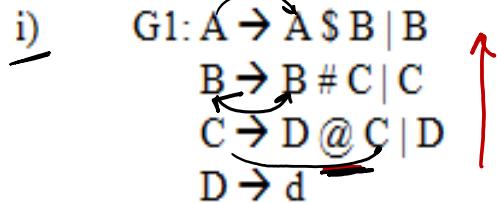
(^) Right associative

$$\uparrow > * > > +$$

(Compiler Design)



Example: Find the precedence and associativity



$$@ > \# > \$$$

$$@ < @$$

$$\# > \#$$

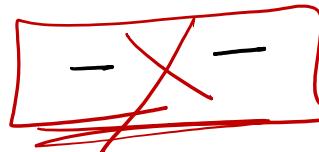
$$\$ > \$$$

$$\textcircled{1} \quad - > [* \doteq +]$$

$$\textcircled{2} \quad * > *$$

$$\textcircled{3} \quad + < +$$

\textcircled{4}



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Q1

Consider the grammar defined by the following production rules, with two operators * and +

```
S --> T * P  
T --> U | T * U  
P --> Q + P | Q  
Q --> Id  
U --> Id
```



+ > *

+ = Right Associatw

* = Left Assocat.

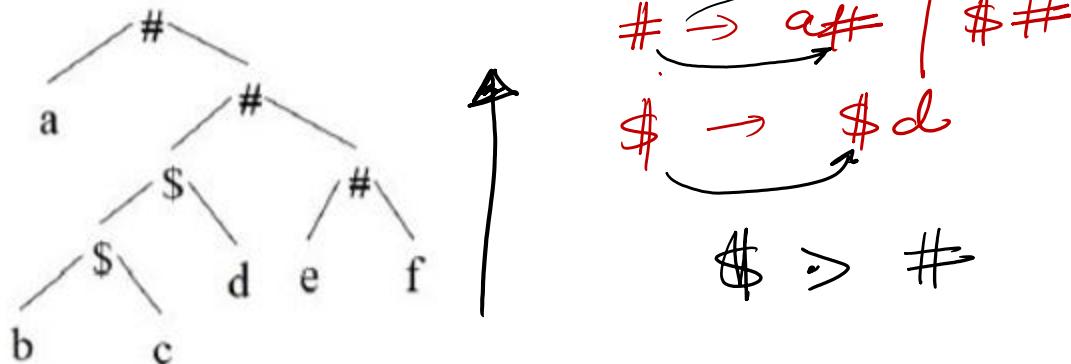
Which one of the following is TRUE?

- (A) + is left associative, while * is right associative
- (B) + is right associative, while * is left associative
- (C) Both + and * are right associative
- (D) Both + and * are left associative

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Ques. Consider the following parse tree for the expression $a\#b\$c\$d\#e\#f$, involving two binary operators $\$$ and $\#$



Which one of the following is correct for the given

parse tree?

(A) $\$$ has higher precedence and is left associative; $\#$ is right associative

(B) $\#$ has higher precedence and is left associative; $\$$ is right associative

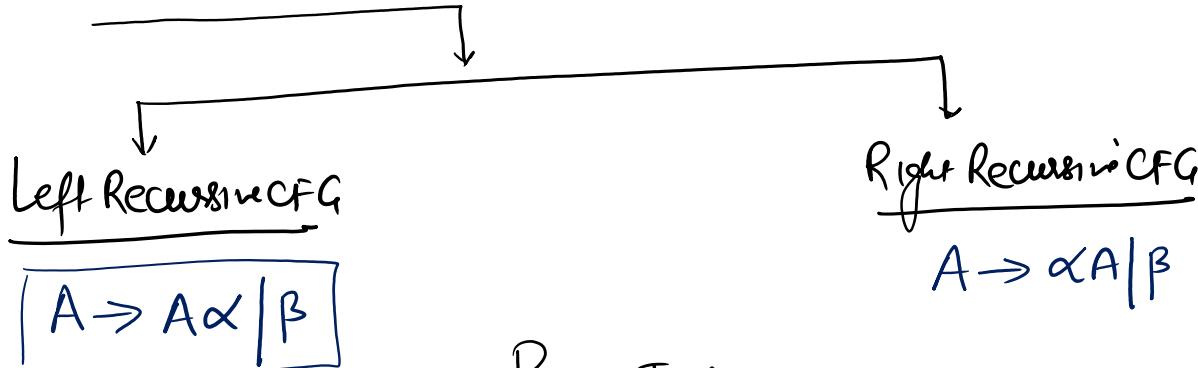
(C) $\$$ has higher precedence and is left associative; $\#$ is left associative

(D) $\#$ has higher precedence and is right associative; $\$$ is left associative

(Compiler Design)



Recursive CFG:



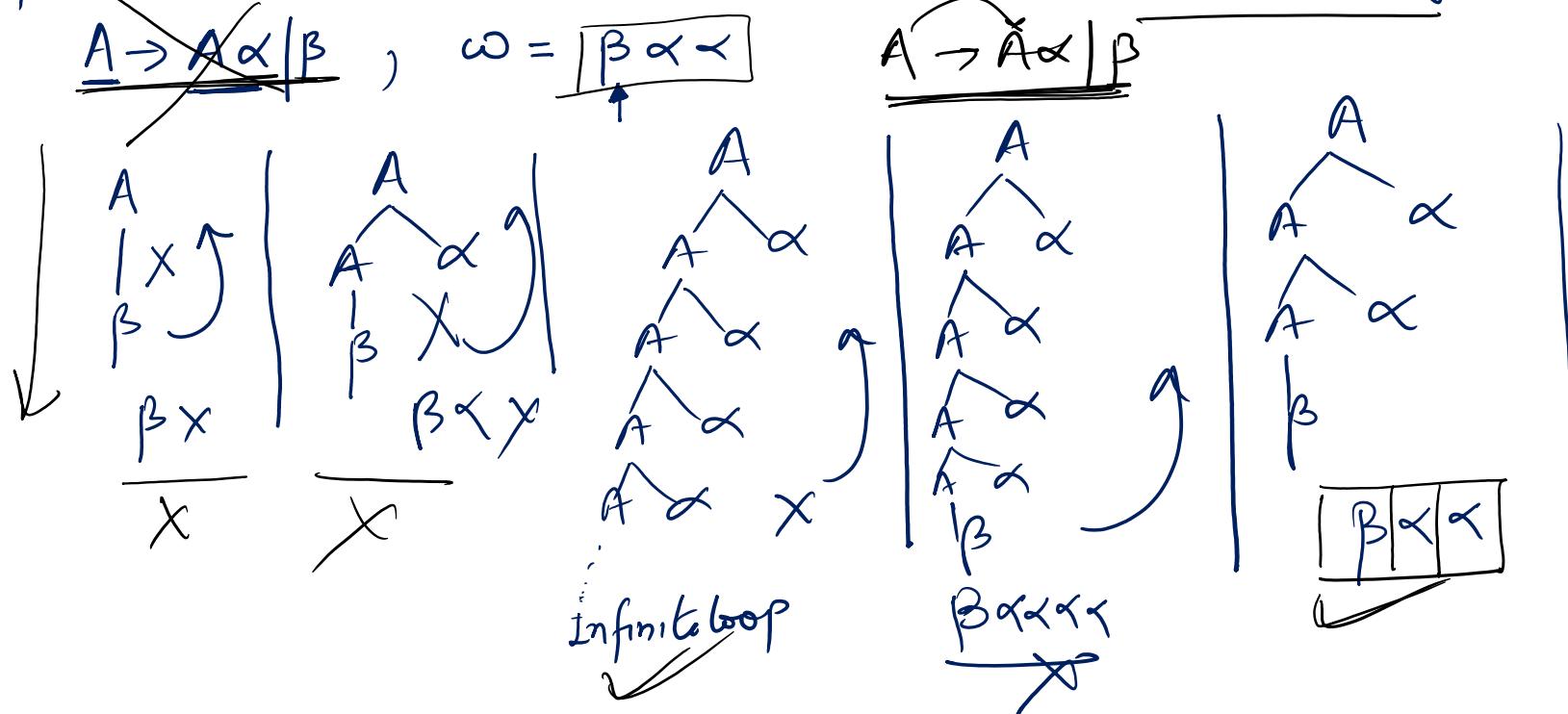
Parsing Technique

	<u>Topdown Parsing</u>	<u>Bottom up parsing</u>
<u>Left Recursive CFG</u>	X	
<u>Right Recursive CFG</u>	✓	✓

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→ if Grammar is Left Recursiv then it is not suitable for topdown Parsing.



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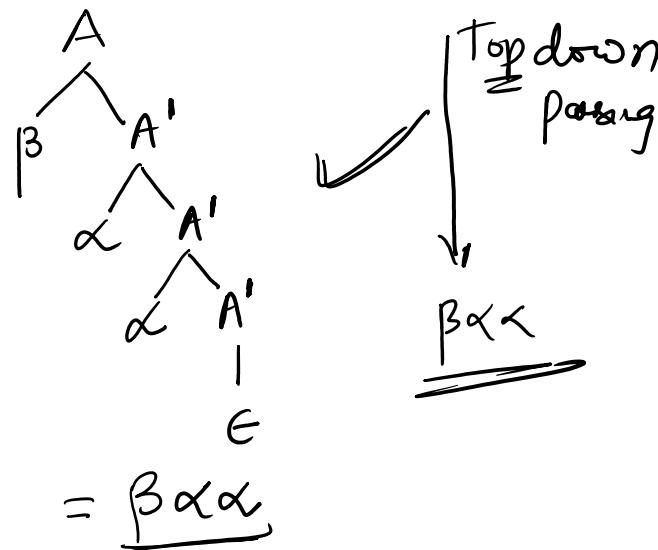
How to Eliminate Direct left Recursion:

$$G \{ A \rightarrow A\alpha | \underline{\beta} \} \xrightarrow[\text{Left Recursion}]{\text{Remove}} G'$$

$$\omega = \underline{\beta} \alpha \alpha \#$$

non left Recursive Grammar:

$$G' \left\{ \begin{array}{l} A \rightarrow \beta A' \\ A' \rightarrow \alpha A' | \underline{\epsilon} \end{array} \right\}$$



(Compiler Design)

Eliminate Direct Left Recursion:

$$1) G = \{ A \rightarrow A\alpha | \underline{\beta} \}$$

Non left Recursion

$$G' \left\{ \begin{array}{l} A \rightarrow \beta A' \\ A' \rightarrow \alpha A' | \epsilon \end{array} \right.$$

$$2) G = \{ A \rightarrow A\alpha_1 | A\alpha_2 | A\alpha_3 | \dots | A\alpha_r | \beta_1 | \beta_2 | \beta_3 | \dots | \beta_s \}$$

↓ Non Left Recursive Grammatical

✓ $G = \left\{ \begin{array}{l} A \rightarrow \beta_1 A' | \beta_2 A' | \beta_3 A' | \dots | \beta_s A' \\ A' \rightarrow \alpha_1 A' | \alpha_2 A' | \alpha_3 A' | \dots | \alpha_r A' \end{array} \right. \epsilon \right\}$

(Compiler Design)

$$1) G = \{ S \rightarrow \underline{s} @ | \underline{b} | \underline{aa} \}$$

\Downarrow Eliminate Left Recursion

$$G' = \left\{ \begin{array}{l} S \rightarrow bS' | aaS' \\ S' \rightarrow aS' | \epsilon \end{array} \right\}$$

$$\begin{aligned} A &\rightarrow A\alpha | \beta \\ &\Downarrow \\ A &\rightarrow \underline{B}A' \\ A' &\rightarrow \underline{\alpha A'} | \epsilon \end{aligned}$$

$$2) S \rightarrow \underline{s} \underline{b} | \underline{S} \underline{ab} | \underline{a} \underline{S} \underline{b} | \underline{b} | \epsilon$$

\Downarrow

$$\begin{aligned} S &\rightarrow asbs' | bs' | s' \\ S' &\rightarrow bs' | abs' | \epsilon \end{aligned}$$

$$\epsilon \cdot s' = s'$$

(Compiler Design)

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Ex:- G:

$$\left\{ \begin{array}{l} A \rightarrow A \alpha | \beta \Rightarrow \\ \cancel{\begin{array}{l} E \rightarrow E + T | I \\ T \rightarrow T * F | E \\ F \rightarrow (E) | id \end{array}} \\ \downarrow \end{array} \right. \quad \begin{array}{l} A \rightarrow \beta A' \\ A' \rightarrow \underline{\alpha A'} | \epsilon \end{array}$$

Non Left Recursive Grammar

$$\left\{ \begin{array}{l} E \rightarrow TE' \\ E' \rightarrow +TE' | \epsilon \\ T \rightarrow FT' \\ T' \rightarrow *FT' | \epsilon \\ F \rightarrow (E) | id \end{array} \right.$$

Top down
parsing

(Compiler Design)

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How to Eliminate indirect Left Recursion:

$$\begin{array}{l} S \rightarrow A\alpha \\ \cancel{A \rightarrow S\beta | \gamma} \end{array}$$

$$S \rightarrow A$$

Step1: Choose order of production

$$\overbrace{S, A, B, C}^{\longrightarrow}$$

Step2: Choose S , Apply direct left Recursion elimination method if left Recursion exist

Step3: ~~Choose A~~ Substitute S production in A production
 \Rightarrow Apply Direct left Recursion eliminate

Step4: ~~Choose S~~
 \Rightarrow Substitute S, A
 \Rightarrow Apply Direct left Recursion

(Compiler Design)

Ex:- $G = \{ S \rightarrow \underline{AaS} \mid bS \mid a, A \rightarrow \underline{Ab} \mid \underline{SbA} \mid b \}$

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Indirect
left Recursion Direct Indirect
left Recursion left Recursion

Solⁿ $\overrightarrow{S, A}$

Step 1: $\overrightarrow{S, A}$

Step 2: Choose $\boxed{S \rightarrow \underline{AaS} \mid bS \mid a}$, No Direct left Recursion

Step 3: Choose A:

$$Q' = \left\{ \begin{array}{l} S \rightarrow AaS \mid bS \mid a \\ A \rightarrow bSbAA' \mid abAA' \mid bA' \\ A' \rightarrow ba' \mid aSbAA' \mid c \end{array} \right\}$$

$$A \rightarrow Ab \mid \overset{\circlearrowleft}{SbA} \mid b$$

$$A \rightarrow A(b) \mid A(aSbA) \mid b\underline{SbA} \mid abA \mid b$$

↓ Eliminate Direct left Recursion

$$\rightarrow \boxed{A \rightarrow bSbAA' \mid abAA' \mid bA' \\ A' \rightarrow ba' \mid aSbAA' \mid c}$$

(Compiler Design)

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Ex:- $G = \{ S \xrightarrow{\text{ }} \underbrace{S(b)}_{\text{Direct}} | \underbrace{Aa}_{\text{Indirect}} | b, A \xrightarrow{\text{ }} \underbrace{Ab}_{\text{Direct}} | \underbrace{Sa}_{\text{Indirect}} | a \}$

Solⁿ: $G' = \left\{ \begin{array}{l} S \xrightarrow{\text{ }} \underbrace{Aas'}_{\text{ }} | \underbrace{bs'}_{\text{ }} , A \xrightarrow{\text{ }} Ab | \underbrace{Sa}_{\text{ }} | a \\ S' \xrightarrow{\text{ }} bs' | \epsilon \end{array} \right\}$

$$G'' = \left\{ \begin{array}{l} S \xrightarrow{\text{ }} Aas' | bs' , A \xrightarrow{\text{ }} \underbrace{A(b)}_{\text{ }} | \underbrace{A(as'a)}_{\text{ }} | \underbrace{bs'a}_{\text{ }} | a \\ S' \xrightarrow{\text{ }} bs' | \epsilon \end{array} \right\}$$

Non left Recursive Grammar

$$G''' = \left\{ \begin{array}{l} S \xrightarrow{\text{ }} Aas' | bs' , A \xrightarrow{\text{ }} bs'aA' | aA' \\ S' \xrightarrow{\text{ }} bs' | \epsilon \quad A' \xrightarrow{\text{ }} bA' | as'aA' | \epsilon \end{array} \right\}$$

(Compiler Design)

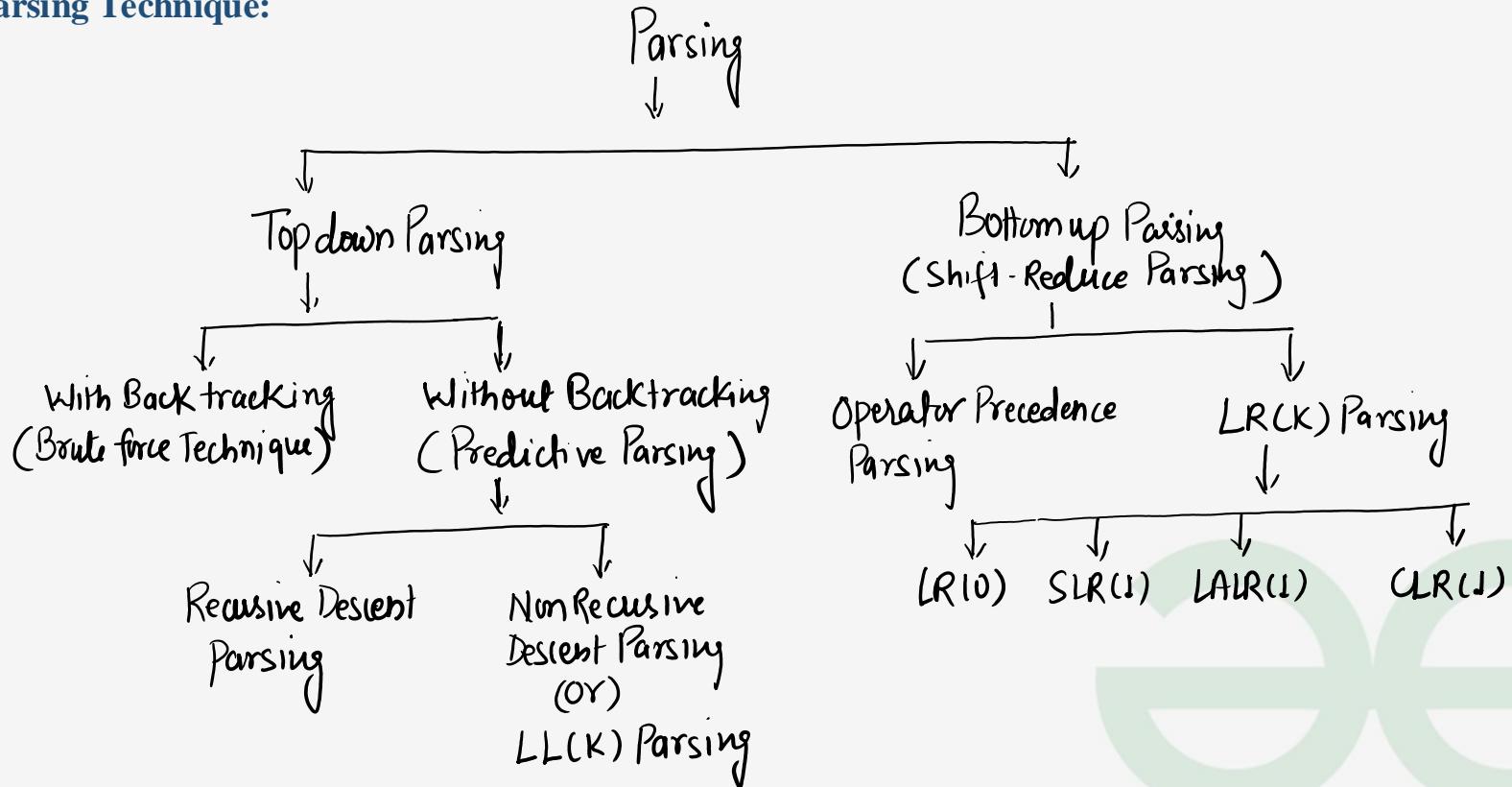
Left Factoring:



(Compiler Design)

Compiler Design

Parsing Technique:



Thank You !