Regular Expression:

Regular Expression is a way of describing regular Language in algebraic form.

Rules of Regular Expression:

Example:	Regular Language	Regular Expression
	$L = \{\}$	$RE = \Phi$
	$L = \{\varepsilon\}$	$RE = \varepsilon$
	$L = \{a\}$	$RE = a$
	$L = \{a, b, c\}$	$RE = a + b + c$
	$L = \{\varepsilon, a, aa, aaa, \ldots\}$	$RE = a^*$
	$L = \{a, aa, aaa, \ldots\}$	$RE = a^+$
	$L = \{\varepsilon, a, b, aa, ab, ba, bb, \ldots\}$	$RE = (a + b)^*$

Precedence of operators:

Example: $\Phi \cap \varepsilon \cup a^* = ?$

Sol. $\Phi \cap \varepsilon \cup a^*$

$((\Phi \cap \varepsilon) \cup a^*)$ $\quad$ $(\Phi \cap (\varepsilon \cup a^*))$

$\Phi \cup a^*$ $\quad$ $\Phi \cap a^*$

a^* $\quad$ Φ

Regular Expression for the following languages:

$$L_1 = \{w \mid w \in (a, b)^*, \sum = \{a, b\}\}$$

i. $|w| = 2 \Rightarrow$ $RE = (a + b)(a + b)$

ii. $|w| \geq 2 \Rightarrow$ $RE = (a + b)(a + b)(a + b)^*$

iii. $|w| \leq 2 \Rightarrow$ $RE = (a + b)(a + b) + (a + b) + \varepsilon = (a + b + \varepsilon)(a + b + \varepsilon)$

iv. $|w| = \text{Even} \Rightarrow$ $RE = [(a + b)(a + b)]^*$

v. $|w| = \text{Odd} \Rightarrow$ $RE = [(a + b)(a + b)]^*(a + b)$

$L_2 = \{w \mid |w| \bmod 3 = 0, w \in (a, b)^*$

L_3 = Set of all strings start with a, over $\sum = \{a, b\} \Rightarrow$

L_4 = Set of all strings end with a, over $\sum = \{a, b\} \Rightarrow$

L_5 = Set of all strings contain a as a sub string $\Rightarrow$

L_6 = Set of all strings start with a & end with a $\Rightarrow$

L_7 = Start & end with different symbol $\Rightarrow$

L_8 = Start & end with same symbol $\Rightarrow$

L_9 = Set of all strings contain at least two a $\Rightarrow$

L_{10} = Contain exact two a $\Rightarrow$

L_{11} = Contain at most two a $\Rightarrow$

L_{12} = Set of all strings contain aa as a sub string over $\sum = \{a, b\} \Rightarrow$

L_{13} = Set of all strings does not contain aa as a sub string over $\sum = \{a, b\} \Rightarrow$

L_{14} = Set of all strings contain even no. of a, over $\sum = \{a, b\} \Rightarrow$

L_{15} = Set of all strings contain at least one pair of a $\Rightarrow$

(Theory of Computation)

L_{16} = Every 0 immediately followed by at least two 1 over $\sum = \{0, 1\} \Rightarrow$

$L_{17} = \{a^m b^n \mid m, n \geq 0\} \Longrightarrow$

$L_{18} = \{a^m b^n \mid m + n = \text{Even} \mid m, n \geq 0\} \Longrightarrow$

$L_{19} = \{(ab)^n \mid n \geq 0\} \Longrightarrow$

$L_{20} = \{a^{2n} \mid n \geq 0\} \Longrightarrow$

$L_{21} = \{0, 1, 10, 100, 1000, \ldots\} \Longrightarrow$

Q23: Write a RE for the set of strings of 0's and 1's whose 10th symbol from right end is 1.

Q24: $L = \{a^{2n} b^{2m+1} \mid n \geq 0, m \geq 0\}$

Q25: Write RE for $L = \{a^n b^m \mid n \geq a, m \leq 3\}$

Q26: $L = \{w \mid |w| \bmod 3 = 0\}, w \in (a, b)^*$

Q27: $L = \{w \mid w \in (a, b)^*, n_a(w) \bmod 3 = 0\}$

Identities Related to Regular Expression:

There are many identities for the regular expression. Let p, q and r are regular expressions.

1. $\Phi + R = R$
2. $\Phi . R = R . \Phi = \Phi$
3. $\varepsilon . R = R . \varepsilon = R$
4. $\varepsilon^* = \varepsilon, \Phi^* = \varepsilon$
5. $R + R = R$
6. $R^* . R^* = R^*$
7. $R . R^* = R^* . R = R^+$
8. $(R^*)^* = R^*$
9. $\varepsilon + RR^* = \varepsilon + R^*R = R^*$
10. $(PQ)^* P = P (QP)^*$
11. $\boxed{(P + Q)^* = (P^* Q^*)^* = (P^* + Q^*)^* = P^* (QP^*)^*}$ Imp
12. $(P + Q) R = PR + QR$ and $R (P + Q) = RP + RQ$

Note: Two regular expression P and Q are equivalent (P = Q) if and P and Q represent the same set of strings.

Example: Simplify RE $(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1)$

Example: The regular expression $0^*(10^*)^*$ denotes the same set as

a) $(1^*0)^*1^*$
b) $0 + (0 + 10)^*$
c) $(0 + 1)^*10\,(0 + 1)^*$
d) None

Example: Let $A = (m + n^*)^*$, $B = (m + n)^*$

a) $A = B$
b) $A \subset B$
c) $B \subset A$
d) $A \cap B = \Phi$

Example: Let $A = [(01)^*1^*]$, $B = (01 + 1)^*$

a) $A = B$
b) $A \subset B$
c) $B \subset A$
d) None

Regular Expression to NFA:

1. $\Phi =$
2. $\varepsilon =$
3. $a =$
4. $b =$
5. $a + b =$
6. $a.b =$
7. $a^* =$
8. $(a + b)^* =$

9. $(ab)^*$

10. ab^*

11. $(ab + ba)^*$

12. $aa + b$

13. $(a + b)a^*$

14. $(aa + b)^*$

15. a^*b

16. $(a + b)^*abb$,

17. (a + b + (a) (bab + (a + b))*(ab)*

18. [a + ba(a + b)]*a(ba)*b*

19. a*b(a + b)*

20. (ab)*ab*

21. $(ab)^* + (a + ab)^*b^*(a + b)^*$

22. $(aa + aaa)^*$

23. $(a + aaaa)^*$

24. $(a + b)^*(aa + bb)(a + b)$

Peg 21.5

Finite Automata to Regular Expression:

Arden's Theorem: Let P, Q and R be three regular expressions if P does not contain ε, then $R = Q + RP$ has a unique solution, that is $R = QP^*$, if P contain ε then $R = Q + RP$ has more than one solution.

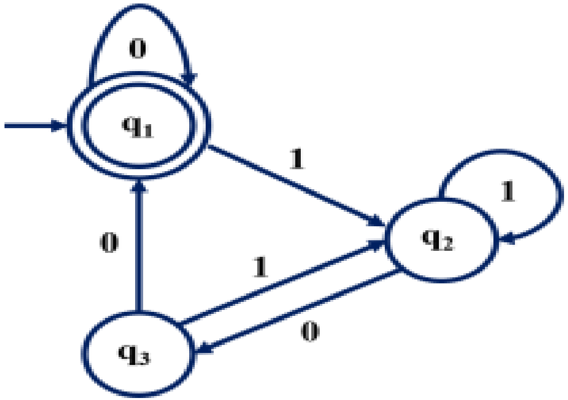
That means, whenever we get any equation in the form of $R = Q + RP$.

Then we can directly replaced by $R = QP^*$.

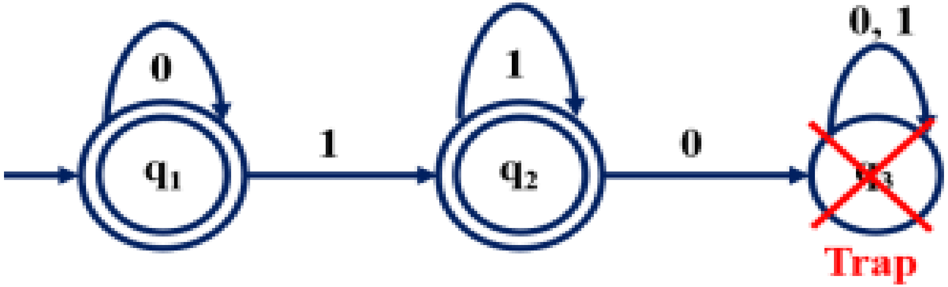
Assumption for applying Arden's Theorem:

1. The transition diagram must not have ε – transition.
2. It must have only one initial state.
3. Arden's Theorem used only for DFA, NFA but not for ε – NFA.

Example: Find regular expression:



Example: Find regular expression.



Peg 24.5

FA to Regular Expression Direct Method:

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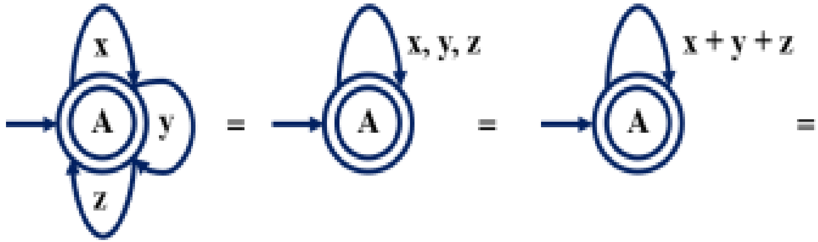

$= (x + y + z)^*$

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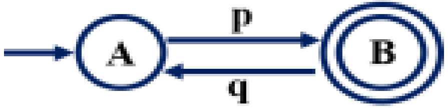

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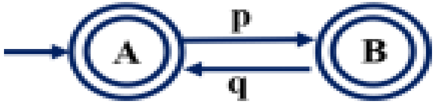

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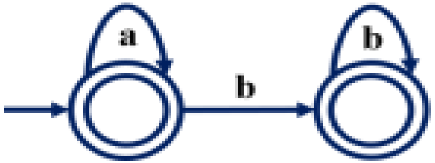

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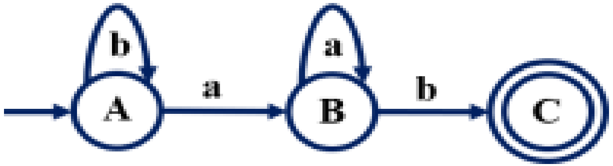

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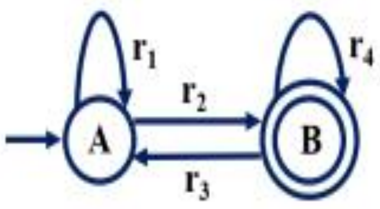

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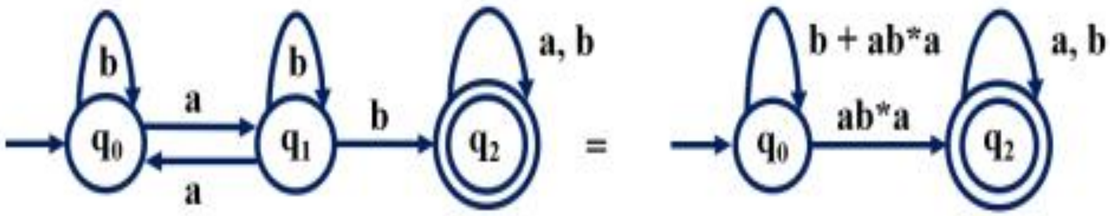

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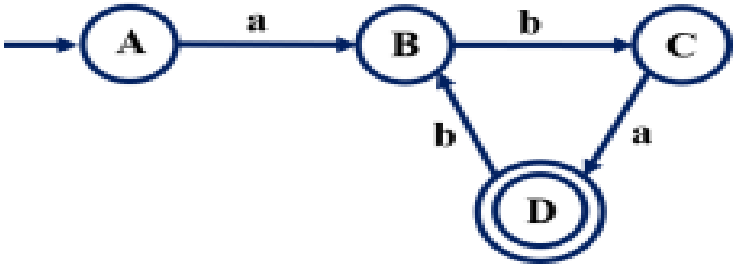

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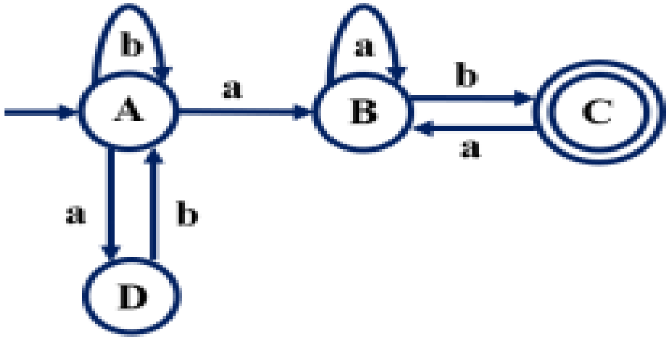

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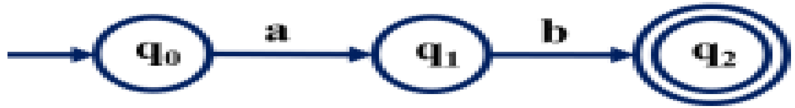

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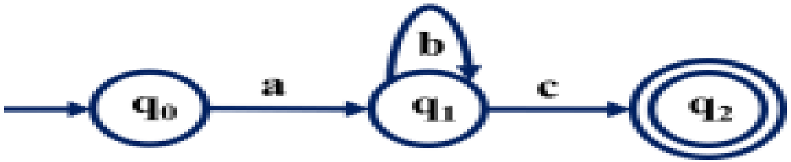

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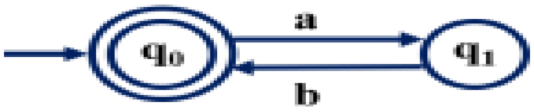

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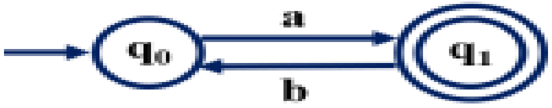

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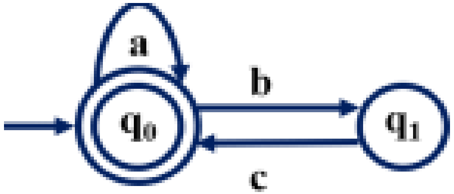

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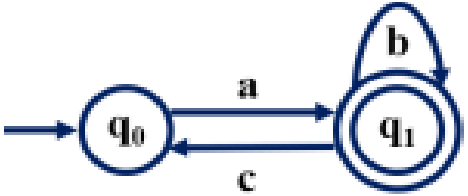

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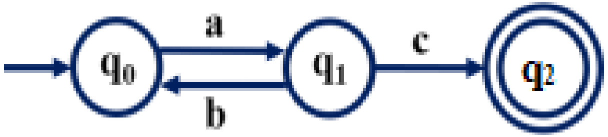

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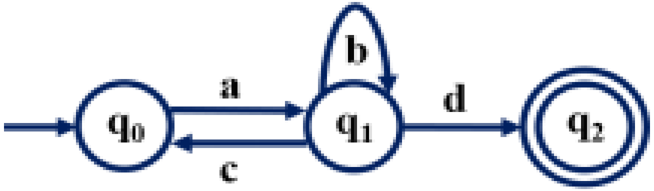

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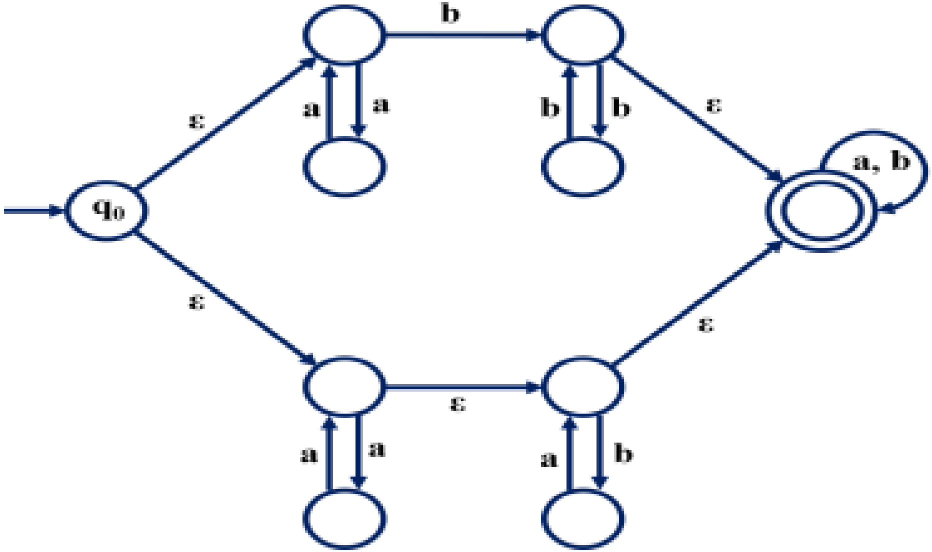

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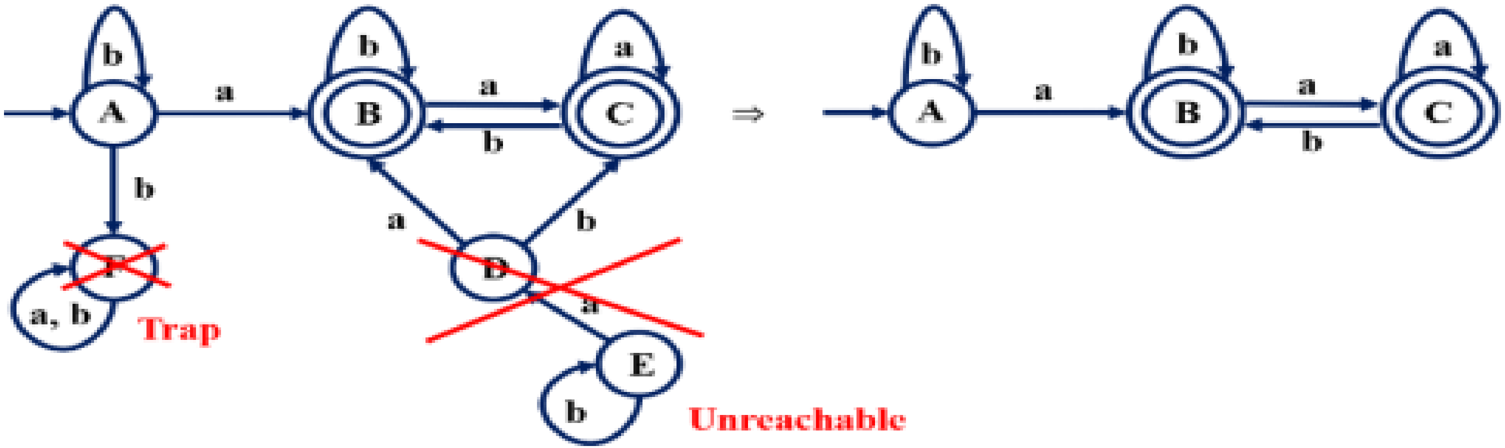

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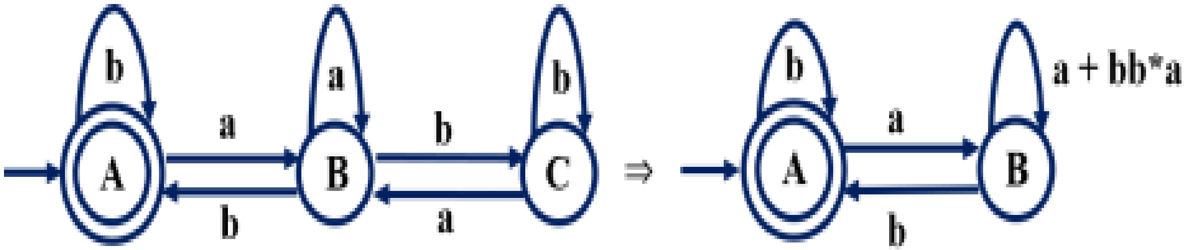

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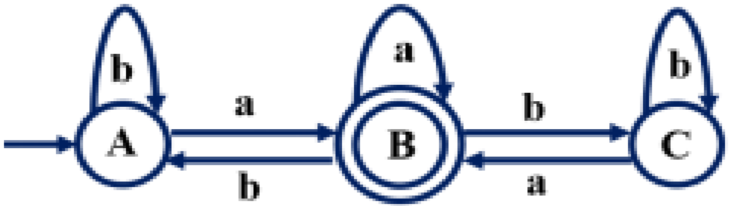

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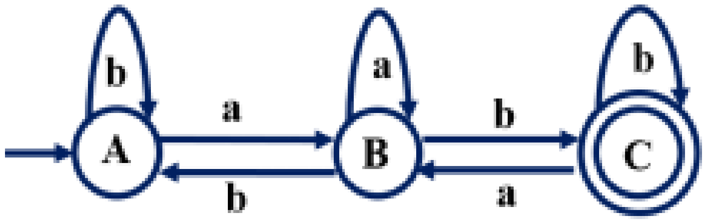

24.