Discrete Mathematics

- Propositional and first order logic
- Sets, relations, functions, partial orders and lattices, Monoids, Groups
- **Combinatorics:** counting, recurrence relations, generating functions
- **Graphs:** connectivity, matching, coloring

Propositional Logic

Proposition

A proposition is the basic building block of logic (science of reasoning).

It is defined as a declarative sentence that is either True or False, but not both.

The **Truth Value** of a proposition is True(denoted as T) if it is a true statement, and False(denoted as F) if it is a false statement.

2 is an even no → True

3 is an even no. → false

Truth values

What time it is?

The time is 9:17am

F

Example,

1. The sun rises in the East and sets in the West. True
2. 1 + 1 = 2 True
3. 'b' is a vowel. false

All of the above sentences are propositions, where the first two are Valid(True) and the third one is Invalid(False).

$x+1=2$??

Some sentences that do not have a truth value or may have more than one truth value are not propositions.

For Example,

1. What time is it?
2. Go out and play.

~~T/F~~

3. x + 1 = 2.

1+2 3 4 5

Compound propositions → operations

Propositions constructed using one or more propositions are called **compound propositions**.

The propositions are combined together using **Logical Connectives** or **Logical Operators.**

connectives

True → connective

Example : 1+1=2 and "All birds can fly".

“b is a vowel” or “e is a vowel”.

Connectives

The words and phrases(or symbols) used to form compound propositions are called connectives.

There are five basic connectives called Negation, Conjunction, Disjunction, Conditional and Biconditional.

1. Negation – If p is a proposition, then the negation of p is denoted by $\neg p$ which when translated to simple English means-
"It is not the case that "p" or simply "not p".

P : proposition

$\neg p \rightarrow$ negate

Example

The negation of "It is raining today", is "It is not the case that is raining today" or simply "It is not raining today".

2 is an even no $\rightarrow$ T

Neg. 2 is not an even no.

$\Downarrow$

F

p	$\neg p$
T	F
F	T

2. Conjunction – For any two propositions p and q, their conjunction is denoted by $p \wedge q$ which means “p and q”.

$p \wedge q$

The conjunction $p \wedge q$ is true when both p and q are true, otherwise it is false.

Example

The conjunction of the propositions

p – “Today is Friday” and

q – “It is raining today”,

is “Today is Friday and it is raining today”.

This proposition is true only on rainy Fridays and is false on any other rainy day or on Fridays when it does not rain.

T&F

T&F

The truth table of $p \wedge q$ is

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \wedge q \rightarrow$ and

$p \vee q \rightarrow$ or

3. Disjunction – For any two propositions p and q, their disjunction is denoted by $p \vee q$ which means "p or q".
The conjunction $p \vee q$ is true when either p or q are true, otherwise it is false.

Example

The disjunction of the propositions
p – "Today is Friday" and
q – "It is raining today",
is "Today is Friday or it is raining today". This proposition is true on any day that is a Friday or a rainy day(including rainy Fridays) and is false on any day other than Friday when it also does not rain.

The truth table of $p \vee q$ is

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

5. Implication – For any two propositions p and q, the statement "if p then q" is called an implication and it is denoted by $p \rightarrow q$.

$p \rightarrow q$ F

Hypothesis

If p then q
If p, q
q if p $\rightarrow$ $p \rightarrow q$
q when p
q whenever p
p only if q $p \rightarrow q$
q unless $\neg p$

p	q	$p \rightarrow q$
T	F	F
F	T	T
F	F	T
T	T	T

In the implication $p \rightarrow q$, p is called the **hypothesis** or **antecedent** or **premise** and q is called the **conclusion** or **consequence**.

The statement $p \rightarrow q$ is also called a **conditional statement**.
The implication is false when p is true and q is false otherwise it is true.

Example

"If it is Friday then it is raining today" is a proposition which is of the form $p \rightarrow q$.
The above proposition is true if it is not Friday(premise is false) or if it is Friday and it is raining, and it is false when it is Friday, but it is not raining.

$p \rightarrow q$

If is friday only if it is raining

$p \rightarrow q$ → If p happens then q happen

p only if q

$\neg q \rightarrow \neg p$

p iff

p

The truth table of $p \rightarrow q$ is

p	q	p→ q	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$p \rightarrow q$ have same truth values as $\neg p \vee q$.

$\Downarrow$ same truth values in every case.

$p \rightarrow q$ $\quad \neg p \vee q$

propositional equivalent

$p \rightarrow q \equiv \neg p \vee q$.

6. Biconditional or Double Implication – For any two propositions the statement "p if and only if (iff) q" is called biconditional and it is denoted by $p \leftrightarrow q$.

p iff q

$p \leftrightarrow q$

The statement $p \leftrightarrow q$ is also called bi-implication.
This statement is true when p and q have same truth values.

$(p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
F	F	T
T	F	F
F	T	F

$p \rightarrow q$

$q \rightarrow p$

The truth table of $p \leftrightarrow q$ is

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

and $\to \wedge$
or $\to \vee$
neg $\to \neg$

imp $\to \rightarrow$
bicond $\to \leftrightarrow$

$p \to q$	$q \to p$	$p \to q \wedge q \to p$
T	T	T
F	T	F
T	F	F
T	T	T

$p \leftrightarrow q$ have same truth values as $(p \to q) \wedge (q \to p)$.

$+ - \times \div$ % of

B
O
D
M
A
S

Translating English Sentences

A statement may have multiple interpretations. Therefore, it is important to convert these sentences into mathematical expressions involving propositional variables and logical connectives.

Precedence of Logical Operators

Precedence of operators helps us to decide which operator will get evaluated first in a complicated looking propositions.

$\neg p \vee q$

p

Operators	Names	Precedence
¬	Negation	1
∧	Conjunction	2
∨	Disjunction	3
→	Implication	4
↔	Biconditional	5

Once the sentences are translated into logical expressions they can be analyzed further to determine their truth values.

Example :

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

$(p \to (q \vee (\neg r)))$

p : You can access the internet

q : you are CS major

r : you are freshman

p only if q

$\Rightarrow$ p would occur if q has already occured

$\Rightarrow$ whenever I have p, q would be there

$\Rightarrow p \to q$.

$q \to F$

$p \to T$

p	- "You can access the Internet from campus."
q	- "You are a computer science major."
r	- "You are a freshman."

Using logical connectives, we can join the above-mentioned propositions to get a logical expression of the given statement.

Therefore, the logical expression would be –

$$p \rightarrow (q \lor \neg r)$$

GATE-CS-2014-(Set-3)

Consider the following statements:

P: Good mobile phones are not cheap $\Rightarrow$ If mobile phones are good then they are not cheap

Q: Cheap mobile phones are not good

$\Downarrow$ If mobile phones are cheap then they are not good

L: P implies Q $\rightarrow$

M: Q implies P

N: P is equivalent to Q $\longleftrightarrow$

Which one of the following about L, M, and N is CORRECT?

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

P: $p \rightarrow \neg q$

Q: $q \rightarrow \neg p$

L: $(p \rightarrow \neg q) \rightarrow (q \rightarrow \neg p)$

M: $(q \rightarrow \neg p) \rightarrow (p \rightarrow \neg q)$

N: $(p \rightarrow \neg q) \Longleftrightarrow (q \rightarrow \neg p)$

Let a and b be two proposition

a: Good Mobile phones.

b: Cheap Mobile Phones.

P and Q can be written in logic as

P: $a \rightarrow \neg b$

Q: $b \rightarrow \neg a$

Truth Table

a	b	¬a	¬b	P	Q
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

It clearly shows P and Q are equivalent.

$\neg p \quad \neg q \quad p \rightarrow \neg q \quad q \rightarrow \neg p \quad (p \rightarrow \neg q) \rightarrow (q \rightarrow \neg p)$

Special Conditional Statements

As we know that we can form new propositions using existing propositions and logical connectives.
New conditional statements can be formed starting with a conditional statement

In particular, there are three related conditional statements that occur so often that $p \to q$
they have special names.

Converse : The converse of the proposition $p \to q$ is $q \to p$

Contrapositive : The contrapositive of the proposition $p \to q$ is $\neg q \to \neg p$ $\equiv (p \to q)$

Inverse : The inverse of the proposition $p \to q$ is $\neg p \to \neg q$ $\equiv q \to p$

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg q \to \neg p$	$\neg p \to \neg q$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T
		Implic	Converse			Cont	Inverse

To summarize,

Statement	If p, then q
Converse	If q, then p
Contrapositive	If not q, then not p
Inverse	If not p, then not q

Example :

Implication : If today is Friday, then it is raining.

The given proposition is of the form $p \rightarrow q$ where 'p' is "Today is Friday" and 'q' is "It is raining today".

Contrapositive, Converse, and Inverse of the given proposition are-

- **Converse :** If it is raining, then today is Friday.

- **Contrapositive :**If it is not raining, then today is not Friday

- **Inverse :** If today is not Friday, then it is not raining

Truth table for converse, contrapositive and inverse are as follows:

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$	$q \to p$	$\neg p \to \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

Note : It is interesting to note that the truth value of the conditional statement $p \to q$ is the same as it's contrapositive, and the truth value of the Converse of $p \to q$ is the same as the truth value of its Inverse.

When two compound propositions always have the same truth value, they are said to be equivalent.

Therefore,

- $p \to q \equiv \neg q \to \neg p$
- $q \to p \equiv \neg p \to \neg q$

GATE CS 1998

What is the converse of the following assertion?

I stay only if you go

(A) I stay if you go

(B) If I stay then you go

(C) If you do not go then I do not stay

(D) If I do not stay then you go

I stay: p

you go : q

Imp: $p \to q$

Converse $q \to p$

If you go then I stay

Inverse ≡ Converse

q if p $\quad p \to q$.

Solution:

Let p : I stay

q : you go

I stay only if you go: $p \to q$

converse of the proposition $p \to q \; is \; q \to p$

$q \to p$: You go then I stay

Which is same as option (a)

There are three types of propositions when classified according to their truth values

1. **Tautology –** A proposition which is always true, is called a tautology.

2. **Contradiction –** A proposition which is always false, is called a contradiction.

3. **Contingency –** A proposition that is neither a tautology nor a contradiction is called a contingency.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Example,

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$p \vee q$ is contingency.

ISRO CS 2017

The proposition $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ is a

(A) tautology

(B) contradiction

(C) contingency

(D) absurdity

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Answer: (C)

Propositional Equivalence

$L \equiv M$

$L \Leftrightarrow M$

$pq \rightarrow 4$ $\quad$ $pqr \rightarrow 8$ $\quad$ $pqrs \rightarrow 16$

Logical Equivalence

Two propositions p and q are said to be logically equivalent if $p \leftrightarrow q$ is a **Tautology**.

$(p \vee q \wedge \neg r) \rightarrow (p \vee q) \rightarrow r$

tautology

One way of proving that two propositions are logically equivalent is to use a truth table.

Better way is to construct a mathematical proof which uses already established logical equivalences to construct additional more useful logical equivalences.

Some basic established logical equivalences are tabulated below-

Logical Equivalences using conjunction, disjunction and negation.

Identity laws

$p \wedge T \equiv p$

$p \vee F \equiv p$

p	T	$p \wedge T$	$p \vee F$	$p \wedge F$	$p \vee T$
T	T	T	T	F	T
F	T	F	F	F	T

Domination laws

$p \wedge \mathrm{F} \equiv F$

$p \vee \mathrm{T} \equiv T$

Idempotent laws

$p \wedge \mathrm{p} \equiv p$

$p \vee \mathrm{p} \equiv p$

Double Negation law

$$\neg(\neg p) \equiv p$$

$-(-1) = 1$

Commutative laws

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

not i~~mp~~

Associative laws

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee p)$$

$\wedge$ & $\vee$

Distributive laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Absorption laws

$$p \wedge (p \vee q) \equiv p$$

$$p \vee (p \wedge q) \equiv p$$

Negation laws

$$p \wedge \neg p \equiv F$$

$$p \vee \neg p \equiv T$$

Implication

$$\boxed{(p \rightarrow q) \equiv (\neg p \vee q)}$$

10. De Morgan's Law :

$(A \cup B)^c = A^c \cap B^c$

- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Proof by Truth Table –

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \vee \neg q$	$p \vee q$	$\neg p \wedge \neg q$
T	T	F	F	T	F	T	F
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
F	F	T	T	F	T	F	T

Negation of implication

Let p and q be two propositions

$$\neg(p \to q)$$
$$\equiv \neg(\neg p \vee q)$$
$$\equiv \neg(\neg p) \wedge \neg q$$
$$\equiv p \wedge \neg q$$

$\neg(p \to q) \equiv \neg(\neg p \vee q)$
$\equiv \neg\neg p \wedge \neg q$
$\equiv p \wedge \neg q$

Negation of biconditional

$$\neg(p \leftrightarrow q) \equiv (\neg p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$$

$\neg(p \leftrightarrow q) \equiv \neg[(p \to q) \wedge (q \to p)]$
$\equiv (\neg(p \to q)) \vee (\neg(q \to p))$
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

GATE CS 2008

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(A) Only I and II
(B) Only I, II and III
(C) Only I, II and IV
(D) All of I, II, III and IV

II

$\neg(\neg P \wedge Q)$

$\neg\neg P \vee \neg Q$

$P \vee \neg Q$

III IV

$\underline{(P \wedge Q)} \vee \underline{(P \wedge \neg Q)} \vee (\neg P \wedge \cancel{\neg} Q)$

$P \wedge (Q \vee \neg Q) \vee (\neg P \wedge \cancel{\neg} Q)$

$(P \wedge T) \vee (\neg P \wedge \cancel{\neg} Q)$

$P \vee (\neg P \wedge \cancel{\neg} Q)$

$(P \vee \neg P) \wedge (P \vee \cancel{\neg} Q)$

$T \wedge (P \vee \cancel{\neg} Q)$

$P \vee \cancel{\neg} Q$

$p \to q$
$\equiv \neg p \vee q$

GATE-CS-2006

Consider the following propositional statements:

P1 : $((A \wedge B) \to C)) \equiv ((A \to C) \wedge (B \to C))$

P2 : $((A \vee B) \to C)) \equiv ((A \to C) \vee (B \to C))$

Which one of the following is true?

(A) P1 is a tautology, but not P2

(B) P2 is a tautology, but not P1

(C) P1 and P2 are both tautologies

(D) Both P1 and P2 are not tautologies

P1: $(A \wedge B) \to C$

$\equiv \neg(A \wedge B) \vee C$

$\equiv (\neg A \vee \neg B) \vee C$

$\equiv (\neg A \vee C) \vee (\neg B \vee C)$

$\equiv \underline{(A \to C)} \vee (B \to C)$

Answer: (D)

$p \to q \equiv \neg p \vee q$

GATE-CS-2015 (Set 1)

Which one of the following is not equivalent to $p \leftrightarrow q$

(A) $(\neg p \vee q) \wedge (p \vee \neg q)$ (B) $(\neg p \vee q) \wedge (q \to p)$

(C) $(\neg p \wedge q) \vee (p \wedge \neg q)$ (D) $(\neg p \wedge \neg q) \vee (p \wedge q)$

$(p \leftrightarrow q)$

$\equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (q \to p)$

$= (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

$= F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$

$= (\neg p \wedge \neg q) \vee (p \wedge q)$

GATE-CS-2016 (Set 2)

Consider the following expressions:

(i) false

(ii) Q

(iii) true

(iv) P V Q

(v) ¬Q V P

The number of expressions given above that are logically implied by P ∧ (P ⇒ Q) is

A ________ 4

$(p \wedge q) \rightarrow Q$

$(p \wedge q) \rightarrow F \rightarrow F.$

$(p \wedge q)$

$F \vee (p \wedge q)$

$(p \wedge \neg p) \vee (p \wedge q)$

$p \wedge (\neg p \vee q)$

A!

$p \wedge q$	p	q	$A \rightarrow$ (i) $A \rightarrow F$	$A \rightarrow Q$	$A \rightarrow T$	$p \vee q$	$A \rightarrow (p \vee Q)$	$\neg Q$	$\neg Q \vee p$	$A \rightarrow (\neg Q \vee p)$
T	T	T	F	T	T	T	T	F	T	T
F	T	F	T	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T	F	~~T~~ F	T
F	F	F	T	T	T	F	T	T	T	T

GATE CS 1997

Which of the following propositions is a tautology?

a. ~~(p V q) → p~~

b. p V (q → p)

c. p V (p → q)

d. p → (p → q)

(A) a

(B) b

(C) c

(D) d

$p \vee q \to p$

T F

$\boxed{p \to F \quad q \to T}$

$p \vee (q \to p)$

F F

$q \to T$
$p \to F$

$p \vee (p \to q)$

F F

$p \to T$
$q \to F$

$p \to (p \to q)$

T T F F

$p \to T$
$q \to F$

Rules of Inference (Basic Terminology)

$P \to Q$

Premise: It is a proposition on the basis of which we would able to draw conclusion.
Basically, premise is an evidence or assumption.

$(P_1 \wedge P_2 \ldots \wedge P_n) \to Q$

Conclusion: It is a proposition that is reached from the given set of premises.
Basically, conclusion is a result of assumption that made in an argument.

If Premise then conclusion

Important Definitions :

1. **Argument –** A sequence of statements, **premises**, that end with a conclusion.

$(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow Q$

T F

T

2. Validity – A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.

3. Fallacy – An incorrect reasoning or mistake which leads to invalid arguments.

Structure of an Argument :

As defined, an argument is a sequence of statements called premises which end with a conclusion.

$(P_1 \wedge P_2 \ldots P_n) \rightarrow Q$

$$Premises \rightarrow Conclusion$$

P_1

P_2

$\vdots$

$\underline{P_n}$

Q

Alternate way

<u>Premises</u>

∴ Conclusion

Premise	Conclusion	Premise → Conclusion
T	T	T
F	F	T
F	T	T
T	F	F

If we have n number of premises,
An argument can be presented symbolically as: –
$(P_1 \land P_2 \land \ldots \land P_{\mathrm{n}}) \rightarrow$ Q
Where P_1 , P_2 , ..., P_{n} represent the premises and Q represents the conclusion.

An argument is valid if whenever the premises are all true, the conclusion must also be true.
(OR) $(P_1 \land P_2 \land \ldots \land P_{\mathrm{n}}) \rightarrow$ Q is a tautology.

P_1 : x is divisible by 2 or x is divisible by 3

P_2 : x is not divisible by 2

C : x is divisible by 3

Let p: x is divisible by 2

q: x is divisible by 3

Then argument will be $((p \vee q) \wedge \neg p) \rightarrow q$

p	q	p ∨ q	$(p \vee q) \wedge \neg p$	$((p \vee q) \wedge \neg p) \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	F	T

$P_1 : p \vee q$

$P_2 : \neg p$

$C : q$

$\rightarrow q$

$((p \vee q) \wedge \neg p) \rightarrow q$

valid

Rule of Inference	Tautology	Name
$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$	$(p \wedge (p \to q)) \to q$	Modus Ponens

Example:

Premises : If I work hard , then I get A. $\to p \to q$

I work hard. p

p : I work hard

q : I get A

$$\begin{array}{c} p \\ \underline{p \to q} \\ \therefore q \end{array}$$

q

Conclusion: I get A

p: I work hard

q: I get A

$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus Tollens

Example:

Premises : If I work hard , then I get A.

I do not get A.

p : I work hard

q : I get A

$$\neg q$$
$$\underline{p \to q}$$
$$\therefore \neg p$$

Conclusion: I ~~get A~~ I do not work hard

$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism

Example:

Premises : If it rains today, then I will be at home.

If I am at home, then I will see a movie.

p : It rains today

q : I am at home

r : I will see movie

$$\begin{array}{c} p \rightarrow q \\ \underline{q \rightarrow r} \\ p \rightarrow r \end{array}$$

Conclusion: If it rains today, then I will see a movie.

$\neg p$ $p \vee q$ $\therefore q$	$(\neg p \wedge (p \vee q)) \rightarrow q$	Disjunctive Syllogism

Example:

Premises : Either my pet is a dog, or my pet is a cat.

My pet is not a cat.

Conclusion: Therefore, my pet is a dog.

$\dfrac{p}{\therefore (p \vee q)}$	$p \rightarrow (p \vee q)$	Addition

$\dfrac{(p \wedge q) \rightarrow r}{\therefore p \rightarrow (q \rightarrow r)}$	$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	Exportation

$\dfrac{\begin{matrix} p \vee q \\ \neg p \vee r \end{matrix}}{\therefore q \vee r}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow q \vee r$	Resolution

Example : Show that the hypotheses
"It is not sunny this afternoon and it is colder than yesterday",
"We will go swimming only if it is sunny",
"If we do not go swimming, then we will take a canoe trip", and
"If we take a canoe trip, then we will be home by sunset"
lead to the conclusion
"We will be home by sunset".

p : It is sunny this afternoon
q : It is colder than yesterday
r : we will go swimming
s : we will take canoe trip
t : we will be home by sunset

$$\begin{array}{c} \neg p \wedge q \\ r \rightarrow p \\ \neg r \rightarrow s \\ s \rightarrow t \\ \hline t \end{array}$$

1.	$\neg p \wedge q$	
2.	$r \to p$	
3.	$\neg r \to s$	
4.	$s \to t$	$/ \therefore t$
~~5.~~	~~$\neg r \to t$~~	~~3, 4 H.S.~~
6.	$\neg p$	1, Simp
7.	$\neg r$	3, 6 M.T.
8.	T	5,7 M.P.

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5.	$\neg p$	1, Simplify
6.	$\neg r$.	2, 5, Modus Tolles
7	s	3, 6 Modus Ponen
8	t	4, 7 Modus Ponen

$p \to$ corrupt
$q \to$ elected
$r \to$ kind

Consider the following two statements.

S1: If a candidate is known to be corrupt, then he will not be elected.

S2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

$S_1: p \to \neg q$

$S_2: r \to q$

$\neg q \to \neg r$

$p \to q = \neg q \to \neg p$ contrapositive

$p \to \neg r$

$\neg \neg r \to \neg p$

$r \to \neg p$

p : candidate is corrupt

q : he will be elected

r : he is kind

$p \to \neg q$
$r \to q$

The truth value of the conditional statement r $\rightarrow q$ is the same as it's contrapositive $\neg q \rightarrow \neg r$.

So, $p \rightarrow \neg q$ and $\neg q \rightarrow \neg r$
gives $p \rightarrow \neg\, r$ (By Modus Ponens)

Again by Contrapositive

$$\neg \neg\, r \rightarrow \neg\, p$$

$$\mathrm{r} \rightarrow \neg\mathrm{p}$$

Answer: (C)

p : ~~cricket~~ rains

q : match played

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Consider the following logical inferences.

I1: If it rains then the cricket match will not be played. The cricket match was played.

Inference: there was no rain.

I2: If it rains then the cricket match will not be played. It did not rain

Inference : the cricket match was played. which of the following is TRUE?

A. Both I1 and I2 are correct inferences

B. I1 is correct but I2 is not a correct inference

C. I1 is not correct but I2 is a correct inference

D. Both I1 and I2 are not correct inferences

I1

$p \to \neg q$

$q \to \neg(\neg q)$

$\neg p$

$\neg p$ M.T

I2

$p \to \neg q$

$\neg p$

q

Answer: (B)

Thank you