

Compiler Design

Today's Class Topics

- Properties of LL(1) Grammar
- How to Test LL(1) Grammar :
- LL(1) PYQ discussion
- Bottom up parsing



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Properties of LL(1) Grammar:

1) A Left Recursive Grammar is not LL(1)

$$G = \{ S \rightarrow Sa \mid b \} \text{ not LL(1)}$$

$$\boxed{S \rightarrow \underline{Sa}}$$

$$\underline{S \rightarrow b}$$

2) A left factor Grammar is not LL(1)

$$G = \{ S \rightarrow aS \mid a \mid b \}$$

S	a	b	\$
		$S \rightarrow b$ $S \rightarrow Sa$	

Not LL(1)

S	c	b	\$
	$S \rightarrow aS$ $S \rightarrow a$		

not LL(1)

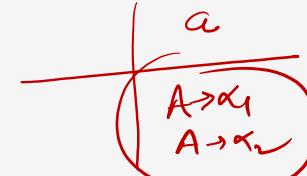
3) An Ambiguous Grammar is not LL(1)

4) Every LL(1) Grammar is Unambiguous But every Unambiguous Grammar is not LL(1)

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5) if Any Production are of the form $A \rightarrow \alpha_1 | \alpha_2$ Then.

$$\text{first}(\alpha_1) \cap \text{first}(\alpha_2) \neq \emptyset \Rightarrow \text{not LL(1)}$$



6) if Any Production are of the form $A \rightarrow \alpha_1 | \alpha_2 | \alpha_3 | \dots | \alpha_n$ Then only two productions having common elements in first set then it is not LL(1)

7) if any Production are of form $A \rightarrow \underline{\alpha} | \epsilon$, Then $A \rightarrow \underline{\alpha} \quad (A) \rightarrow \epsilon$

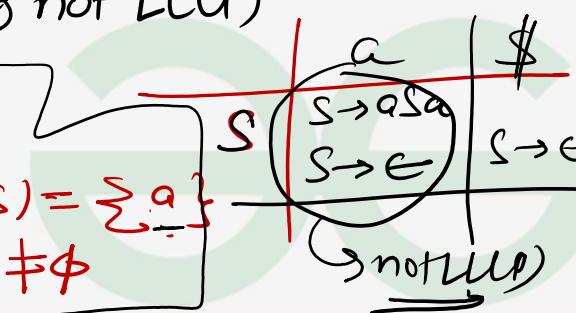
$$\text{if } \text{first}(\alpha) \cap \text{follow}(A) \neq \emptyset \Rightarrow \text{not LL(1)}$$

Ex:

$$G = \{ S \rightarrow \underline{as} \alpha | \epsilon \} \text{ not LL(1)}$$

$$\alpha \text{ first}(as) \cap \text{follow}(S) = \sum a \neq \emptyset$$

$$\neq \emptyset$$



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Not LL(1):

1- if any entry of table is multiple defined then G is not LL(1)

2- if G is Ambiguous

3- if G is Left Recursive

4- if G is Left factor

5- And Some other cases also

Ex: ① $S \rightarrow AB \mid ab$ ② $S \rightarrow Sa \mid b$

not LL(1)
 $A \rightarrow a$
 $B \rightarrow b$

Ambiguous
left factoring
 $\frac{S}{a}$ $\frac{S}{b}$

(Non left recursive \wedge Left factored \wedge Unambiguous) \Rightarrow LL(1)

$x \rightarrow y \cong \sim y \rightarrow \sim x$

LL(1) \rightarrow (Non left recursive) \wedge (Left factored) \wedge (Unambiguous)

(Left Recursive \vee Left factoring \vee Ambiguous) \rightarrow LL'(1)

Ex: ③ $S \rightarrow a \mid ab$ ④ $S \rightarrow a \underline{sa} \mid c$

not LL(1)
 $A \rightarrow a$
 $B \rightarrow b$

Ambiguous
left factoring
 $\frac{S}{a}$ $\frac{S}{ab}$

③ $S \rightarrow a \mid ab$

not LL(1)

because of
Contain common prefix

④ $S \rightarrow a \underline{sa} \mid c$

not LL(1)

1) it is Unambiguous
2) it is non left recursive
3) it is Left factored

$\text{First}(aS) \cap \text{Follow}(S)$

$= a$
 $\neq \emptyset$ not LL(1)

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Testing LL(1) Grammar:

Case 1: $A \rightarrow \alpha_1 \mid \alpha_2 \mid \alpha_3$

$\in \notin \text{first}(\alpha_1)$
 $\in \notin \text{first}(\alpha_2)$
 $\in \notin \text{first}(\alpha_3)$

$$\left. \begin{array}{l} \text{first}(\alpha_1) \cap \text{first}(\alpha_2) = \emptyset \\ \text{first}(\alpha_2) \cap \text{first}(\alpha_3) = \emptyset \\ \text{first}(\alpha_3) \cap \text{first}(\alpha_1) = \emptyset \end{array} \right\} \Rightarrow \underline{\underline{U}}(U)$$

Case 2: $A \rightarrow \alpha_1 \mid \alpha_2 \mid \alpha_3$

$\in \notin \text{first}(\alpha_1)$
 $\in \notin \text{first}(\alpha_2)$
 $\in \in \text{first}(\alpha_3)$

$$\left. \begin{array}{l} \text{first}(\alpha_1) \cap \text{first}(\alpha_2) = \emptyset \\ \text{first}(\alpha_1) \cap [(\text{first}(\alpha_3) - \in) \cup \text{follow}(A)] = \emptyset \\ \text{first}(\alpha_2) \cap [(\text{first}(\alpha_3) - \in) \cup \text{follow}(A)] = \emptyset \end{array} \right\} \Rightarrow \underline{\underline{U}}(U)$$

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Case 3: $A \rightarrow \alpha_1 | \underline{\alpha_2} | \underline{\alpha_3}$

$\in \notin \text{first}(\alpha_1)$

$\in \in \text{first}(\alpha_2)$

$\in \in \text{first}(\alpha_3)$

}

never LCA

$$\text{first}(\alpha_1) \cap [(\text{first}(\alpha_2) - \in) \cup \text{follow}(A)] = \emptyset$$

$$\text{first}(\alpha_1) \cap [(\text{first}(\alpha_3) - \in) \cup \text{follow}(A)] = \emptyset$$

$$[(\text{first}(\alpha_2) - \in) \cup \text{follow}(A)] \cap [(\text{first}(\alpha_3) - \in) \cup \text{follow}(A)] \neq \emptyset$$

LL(1) = ?

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Ex: $G = \{ S \rightarrow \underline{aS} | \underline{Ab}, A \rightarrow \underline{bA} | \underline{b} \}$

$$\text{first}(bA) \cap \text{first}(b) = b \neq \emptyset \quad \left\{ \text{not LL(1)} \right.$$

Ex: $G = \{ S \rightarrow \underline{Sa} | \underline{Ab} | \underline{bA}, A \rightarrow \underline{bA} | \underline{b} \}$

not LL(1)

Ex: $G = \{ S \rightarrow \underline{AaBb} | \underline{c}, A \rightarrow \underline{Bb} | \underline{b}, B \rightarrow \underline{aB} | \underline{c} \}$

$$\begin{aligned} & \text{first}(AaBb) \cap \text{Follow}(S) \\ &= \{\underline{a}, \underline{b}\} \cap \{\underline{c}\} \end{aligned}$$

$$= \emptyset \quad \checkmark$$

$$\begin{aligned} & \text{first}(Bb) \cap \text{first}(b) \\ &= \{\underline{a}, \underline{b}\} \cap \{\underline{b}\} \end{aligned}$$

$$\begin{aligned} &= b \\ &\neq \emptyset \quad \text{not LL(1)} \end{aligned}$$

$\text{first}(aB) \cap \text{Follow}(b)$

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Ex:- $G = \{ S \rightarrow \underline{a}S \mid \underline{Ab}S \mid C, A \rightarrow \underline{a}A \mid b \}$

$$\text{first}(aS) \cap \text{first}(AbS)$$

$$= \{a\} \cap \{a, b\}$$

$$\begin{matrix} = a \\ \neq \phi \end{matrix} \quad] \text{not LL(1)}$$



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$$G = \{ S \rightarrow e E t S | S' | a, S' \rightarrow e S | \epsilon, E \rightarrow a \}$$

$\text{First}(eS) \cap \text{Follow}(S')$

$$e \cap \{ e, \$ \} \xrightarrow{\exists} e \quad \{ \not\$ \} \text{ not } \underline{\text{LL(1)}}$$

$$\text{Follow}(S') = \text{Follow}(S) = \{ \$, \text{First}(S') \}$$

$$= \{ \$, e, \text{follow}(S) \} = \{ e, \$ \}$$

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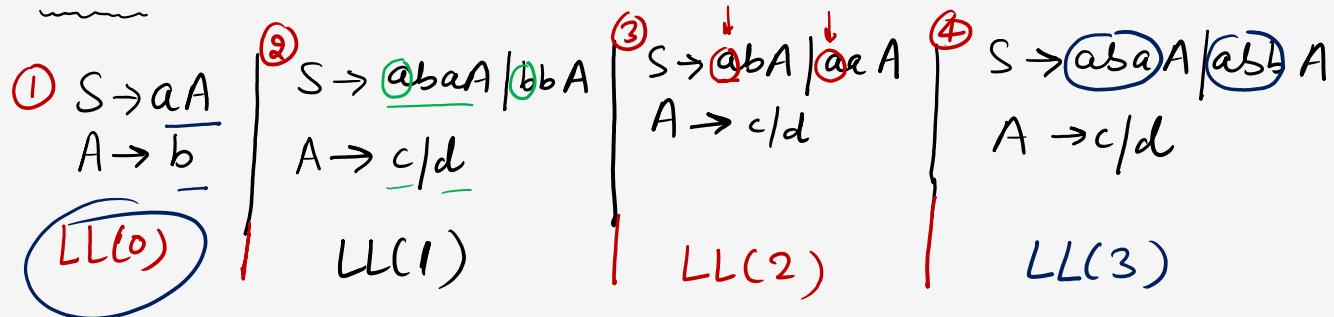
LL(1)

a ↑ b a



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LL(K) :-



Ex:

$$S \rightarrow \underline{abc} \mid \underline{def} \Rightarrow \text{LL}(0) \Rightarrow \text{LL}(1)$$

$\begin{array}{c} S \\ \downarrow \\ abc \end{array} \quad \begin{array}{c} def \\ \underline{\hspace{2cm}} \end{array}$



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$$\begin{array}{l} S \rightarrow Aa \\ A \rightarrow bc \end{array} \quad \left| \begin{array}{l} LL(0) \\ \cancel{LL(2)} \Rightarrow LL(1) \end{array} \right.$$

$$\boxed{S \rightarrow abc | ade} \Rightarrow LL(2)$$

$$LL(0) \Rightarrow LL(1) \Rightarrow LL(2) \Rightarrow LL(3) \Rightarrow LL(4) \Rightarrow \dots \Rightarrow LL(K)$$

$$\boxed{LL(K) \Rightarrow LL(K+1)} = \boxed{\cancel{LL(K+1) \Rightarrow LL(K)}}$$

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$LL(0) \Rightarrow LL(1) \Rightarrow LL(2) \Rightarrow LL(3) \Rightarrow \dots \Rightarrow LL(K)$

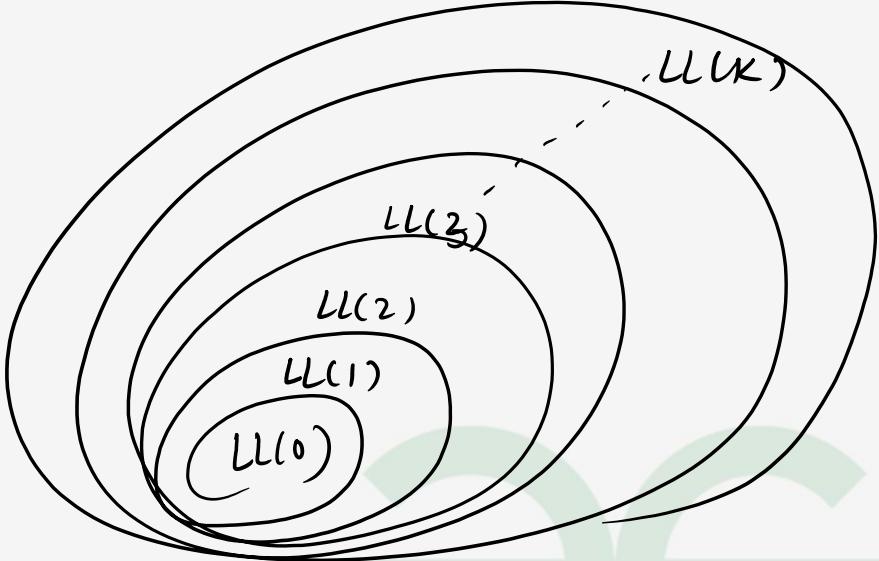
$LL(K) \Rightarrow LL(K+1)$

$LL(K+1) \Rightarrow \cancel{LL(K)}$

$LL(K+1) \Rightarrow LL(K)$

Ex: $\left\{ \begin{array}{l} S \rightarrow CC \times \\ C \rightarrow cC / d \end{array} \right\} \Rightarrow LL(1) = ?$

$$\text{first}(CC) \cap \text{first}(d) = c \cap d = \emptyset$$



Every $LL(1) \subset LL(2) \subset LL(3) \dots \subset LL(K)$

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Question:- Which of the following grammar is LL(3) but not LL(2) and not LL(1)

- a) $\{ S \rightarrow aS | b \} \text{ LL(1)}$
- b) $\{ S \rightarrow aaS | ab | b \} \text{ LL(2)}$
- c) $\{ S \rightarrow aaaS | aab | ab | b \} \text{ LL(3) but not LL(2) & not LL(1)}$
- d) None

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Question: The following Grammar $G = \{ S \rightarrow aA | b, A \rightarrow aB | a, B \rightarrow bB | a \}$

- a) LL(1) but not LL(2)
- ~~b) LL(2) but not LL(1) Ans~~
- c) Both LL(1) & LL(2)
- d) Neither LL(1) nor LL(2)

$$B \rightarrow bB | a \quad \checkmark$$

$\text{first}(bB) \cap \text{first}(a)$

$b \text{first}(B) \cap \text{first}(a)$

$b \{\underline{a}, \underline{b}\} \cap \{\underline{a}\} \Rightarrow$

$$\begin{aligned} &\stackrel{\text{LL(2)}}{=} S \rightarrow a\underline{A} | b \\ &\quad \swarrow \\ &\text{first}(aA) \cap \text{first}(b) \\ &a \text{first}(A) \cap \text{first}(b) \\ &a \{a\} \cap \{b\} \\ &\Rightarrow aa \cap b = \emptyset \end{aligned}$$

$\stackrel{\text{LL(2)}}{=}$

$\{ba, bb\} \cap \{a\} = \emptyset \Rightarrow \text{LL(2)}$

$$\begin{aligned} &A \rightarrow a\underline{B} | a \\ &\text{first}(aB) \cap \text{first}(a) \\ &a \{a\} \cap \text{first}(a) \\ &a \{a, b\} \cap \{a\} \\ &\{aa, ab\} \cap \{a\} \\ &= \emptyset \Rightarrow \text{LL(2)} \end{aligned}$$

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Question: $G = \{ S \rightarrow \underline{aA} / \underline{ab}, A \rightarrow aA / b \}$

- a) LL(2) but not LL(1)
- b) LL(1) but not LL(2)
- c) LL(1) & LL(2)
- d) None of these.

$\text{first}(aA) \cap \text{first}(ab)$

$a \text{first}(A) \cap \{ab\}$

$a\{a, b\} \cap \{ab\}$

$\{aa, ab\} \cap \{ab\}$

neither LL(1) nor LL(2) Ans

= ab
+ ab

not LL2

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Qustion: LL(1)

$S \rightarrow aBDh$, $LL(1) = ?$

$B \rightarrow cC$

$C \rightarrow bC \mid \epsilon$ $\rightarrow \text{first}(bC) \cap \text{follow}(S) = \emptyset$

$D \rightarrow EF$

$E \rightarrow g \mid \epsilon$ $\rightarrow \text{first}(g) \cap \text{follow}(E) = \emptyset$

$F \rightarrow f \mid \epsilon$ $\rightarrow \text{first}(f) \cap \text{follow}(F) = \emptyset$

Qustion:

$S \rightarrow \underline{AaAb} \mid \underline{BbBa}$

$A \rightarrow \epsilon$

$B \rightarrow \epsilon$

$\text{First}(\underline{AaAb}) \cap \text{First}(\underline{BbBa})$

$a \cap b = \emptyset$

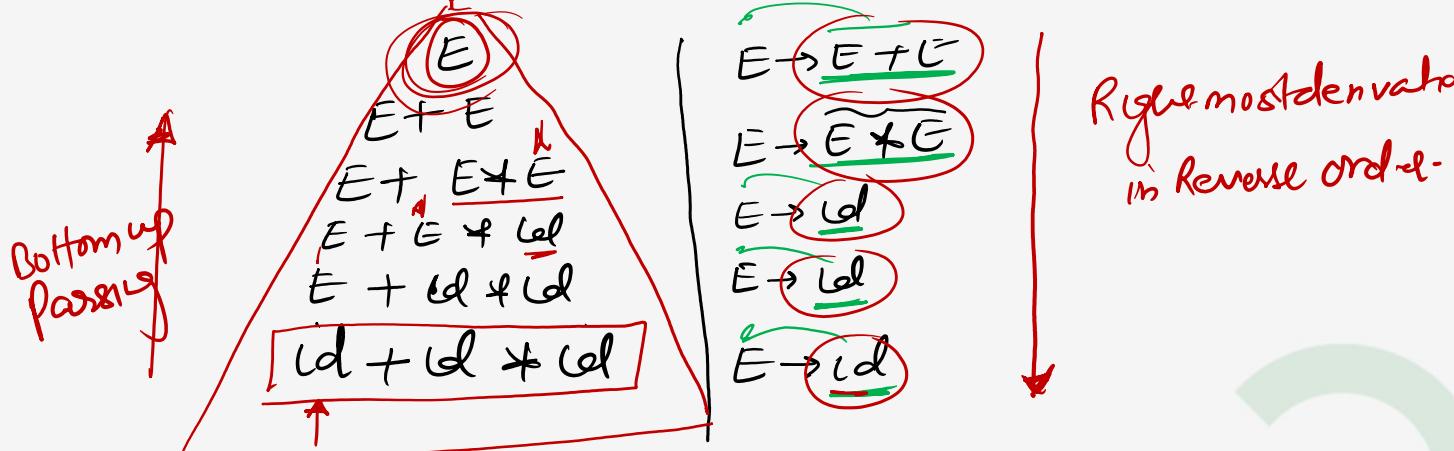
$\Rightarrow LL(1)$

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Bottom-up parsing: From leaves to root, uses right most derivation in reverse order. (uses reduction process)

Ex: ... $G = \sum E \rightarrow E + E \mid E * E \mid id \{ \}$



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Handle Pruning: Reducing the string to start symbol using the handles

Handle: A Handle is a substring of a string that matched with any of the right side of the productions then That handle will be reduced with left side of the production.

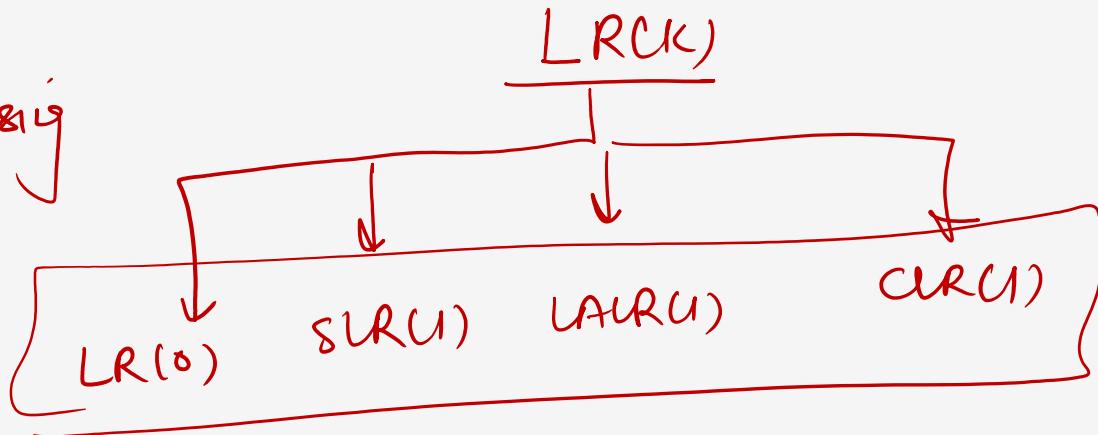
Bottom-up parsing is a process of finding the handles And using them in the reductions to get the start symbol, This entire process of reducing the string to the start symbol is called as handle pruning

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Shift-Reduce Parsing:

or

Bottom up Parsing



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Question:- Consider the following grammar and Parse the input string aab using Shift Reduce Parser

$$G_1 = \{ S \rightarrow AB, A \rightarrow aA \mid a, B \rightarrow bB \mid b \}$$

Always No. of Shift Action Will be length of string = 3

ReduceAction = 4

Accept = ⊥

Total Actions = 8

STACK	Input Buffer	Actions
\$	aab \$	Shift
\$ a	ab \$	Shift
\$ aa	b \$	Reduce A → a ①
\$ <u>aA</u>	b \$	Reduce A → aA ②
\$ A	b \$	Shift
\$ Ab	#	Reduce B → b ③
\$ AB	#	Reduce S → AB ④
\$ S	#	Accept

Viable prefixes

Starting Symbol for Rootnode

Handles

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Thank You !

