Lower Triangular matrix (LTM) :- LTM is Possible only for square matrix

$$a[1 \cdots 4][1 \cdots 4]$$

$lb_1 \quad ub_1 \quad lb_2 \quad ub_2$

$\boxed{lb_1 = lb_2}$, $\boxed{ub_1 = ub_2}$

$$a_{ij} = \begin{cases} 0, & i<j \\ \text{Nonzero or Stored}, & \boxed{i \geq j} \end{cases}$$

$$= \begin{array}{c|cccc} & J & 2 & 3 & 4 \\ \hline 1 & a_{11} & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & 0 & 0 \\ 3 & a_{31} & a_{32} & a_{33} & 0 \\ 4 & a_{41} & a_{42} & a_{43} & a_{44} \end{array} \quad 4 \times 4$$

RMO:-

R_1	R_2		R_3			R_4			
a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{44}
100	102	104	106	108	110	112	114	116	118

Total No. of non zero elements = ? $= \frac{n(n+1)}{2}$

$1 + 2 + 3 + 4 + \cdots n$

No. of non zero elements upto a_{ij} = ?

$1 + 2 + 3 + \cdots (i-1) + j$

$\boxed{\frac{i(i-1)}{2} + j}$

$$\boxed{Loc(a[i][j]) = ? \quad BA + \left[\frac{i(i-1)}{2} + j - 1\right] * S}$$

Ex:- $loc(a[4][3]) = ?$ $\quad 100 + \left[\frac{4 \times 3}{2} + 2\right] * 2 \Rightarrow 116$

$$\begin{array}{c|cccc}
 & 0 & 1 & 2 & 3 \\ \hline
0 & a_{00} & 0 & 0 & 0 \\
1 & a_{10} & a_{11} & 0 & 0 \\
2 & a_{20} & a_{21} & a_{22} & 0 \\
3 & a_{30} & a_{31} & a_{32} & a_{33}
\end{array}$$

$n \times n$ ↓ $4 \times 4 \Rightarrow$

R_0	R_1		R_2			R_3			
0	0	1	0	1	2	0	1	2	3
a_{00}	a_{10}	a_{11}	a_{20}	a_{21}	a_{22}	a_{30}	a_{31}	a_{32}	a_{33}
100	102	104	106	108	110	112	114	116	118

i) Total ~~No.~~ of non zero elements = ? $\quad 1+2+3+4+ \cdots + n = \frac{n(n+1)}{2}$

ii) No. of non zero elements upto a_{ij} $= \underbrace{1+2+3+\cdots+i}+j+1$

$= \boxed{\frac{i(i+1)}{2} + j + 1}$

iii) $loc(a[i][j]) = ?$

$\Rightarrow \boxed{BA + \left[\frac{i(i+1)}{2} + j\right] * S}$ if index start (0, 0)

$\Rightarrow \boxed{BA + \left[\frac{i(i+1)}{2} + j - 1\right] * S}$

Ex:- $loc(a[3][2]) = ?$

$= 100 + \left[\frac{3 \times 4}{2} + 2\right] * 2 = 116$

NOTE:- if lower Bound (lb_1, lb_2), BA, SOE = S

b/c of Square matrix

$\Rightarrow lb_1 = lb_2$

$$Loc[A[i][J]] = Base(A) + \left[\frac{(i-lb_1)(i-lb_1+1)}{2} + J - \cancel{lb_2}\,lb_1\right] * S$$

LTM:

Question: $a[-25 \ldots +760][-25 \ldots +760]$, $BA = 6500$, $S = 9$

$loc(a[697][530]) = ?$

$$Loc[A[I][J]] = Base(A) + \left[\frac{(i-lb_1)(i-lb_1+1)}{2} + J - lb_2\right] * S$$

$$= 6500 + \left[\frac{(697 - -25)(697 - -25 + 1)}{2} + (530 - -25)\right] * 9$$

$$\Rightarrow 2360522$$

Ans

LTM

Question:- $a[-350 \cdots +650][-350 \cdots +650]$, $BA = 4600$, $SOE = 8$

$loc(a[599][610]) = ?$

$i \geq J$

$$Loc[A[i][J]] = Base(A) + \left[\frac{(i-lb_1)(i-lb_1+1)}{2} + J - lb_2\right] * S$$

Lunch Break

2:06 $\rightarrow$ 2:20

CMO:-

$$
\begin{array}{c|c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
a_{11} & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 \\
a_{31} & a_{32} & a_{32} & 0 \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}_{4\times 4}
\end{array}
$$

a_{ij} : $i \geq j$

C_1				C_2			C_3		C_4
a_{11}	a_{21}	a_{31}	a_{41}	a_{22}	a_{32}	a_{42}	a_{32}	a_{43}	a_{44}
100	102	104	106	108	110	112	114	116	118

i) Total No. of non zero elements = ? $\frac{n(n+1)}{2}$

$n + n-1 + n-2 + \cdots + 2 + 1$

ii) No. of non zero elements upto a_{ij} = ?

= Total elements in the first $(j-1)$ colom + no. of elements upto i^{th} row in the j^{th} colom

$= n + n-1 + n-2 + \cdots\cdots n-j+2 + i-j+1$

$\Rightarrow n*(j-1) - (1+2+3+4+ \cdots (j-2)+(j-1)) + i$

$\Rightarrow \boxed{n*(j-1) - \frac{j(j-1)}{2} + i} \Rightarrow$

iii) loc [a[i][j]] = ?

$$\boxed{BA + \left[n*(j-1) - \frac{j(j-1)}{2} + i-1\right] * S}$$

Ex:- $loc(a[3][2]) = ?$ ($i = 3$, $j = 2$)

$\Rightarrow 100 + \left[4*1 - \frac{2\times 1}{2} + 3-1\right]*2$

$100 + [4-1+2]*2$

$= 110$

NOTE:

$$\boxed{loc(a[i][J]) = Base(A) + \left[(J-\cancel{lb_2}^{\,lb_1}) * n - \frac{(J-\cancel{lb_2}^{\,lb_1})(J-\cancel{lb_2}^{\,lb_1}+1)}{2} + (i-lb_1)\right] * S}$$

$n = 712-30+1 =$

Question: $a[30 \ldots 712][30 \ldots 712]$, $BA = 6500$, $S = 7$

$loc(a[\overset{i}{699}][\overset{J}{599}]) = ?$

$$6500 + \left[(599-30) * (712-30+1) - \frac{(599-30)(599-30+1)}{2} + (699-30)\right] * 7$$

$= \underline{15{,}96{,}417}$

$\Rightarrow loc(a(i)(J)) = BA + \left[n * J - \frac{J(J+1)}{2} + i\right] * S$

$\cancel{\text{if } lb_1 = lb_1}$ $\quad lb_1 = lb_2 \Rightarrow$

Question:- $a[-119 \ldots\ldots +378][-119 \ldots\ldots +378]$, $BA = 7200$, $S = 9$

$loc(a[319][296]) = ?$

$$\boxed{loc(a[i][J]) = Base(A) + \left[(J - lb_2) * n - \frac{(J-lb_2)(J-lb_2+1)}{2} + (i - lb_1)\right] * S}$$

$i \geq J$

$\Rightarrow$ 10,94,292 Ans

Upper Triangular matrix:- Square matrix

$\boxed{lb_1 = lb_2}$

Total elements = ? $\frac{n(n+1)}{2}$

$a[i][j] = \{ i \le j \}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}_{4\times4}$$

RMO:

$$\boxed{loc(a[i][j]) = B.A + \left[n*(i-lb_1) - \frac{(i-lb_1)(i-lb_1+1)}{2} + (j-lb_2)\right]*S}$$

$\boxed{lb_1 = lb_2}$

CMO:

$$\boxed{loc(a[i][j]) = B.A + \left[\frac{(j-lb_2)(j-lb_2+1)}{2} + (i-lb_1)\right]*S}$$

Question:- Assume that an upper triangular matrix a[0 ... n-1, 0 ... n-1] is stored in a linear Array b[0 ... $\frac{n(n+1)}{2} - 1$] in Lexicographical order if a[0,0] is Stored in b[0] then where is a[800, 900] stored in Array b for n = 1000?

a) b[490500]

b) b[480501]

c) b[500499]

d) ~~None~~

$$\boxed{loc(a[i][j]) = B.A + \left[n*(i-\cancel{lb_1}) - \frac{(i-\cancel{lb_1})(i-\cancel{lb_1}+1)}{2} + (j-\cancel{lb_2})\right] * S}$$

RMO

$$b\left[n*i - \frac{i(i+1)}{2} + j\right]$$

$$b\left(1000 * 800 - \frac{800 \times 801}{2} + 900\right)$$

$$= b[480500]$$ Ans

a[0 ... n-1, 0 ... n-1]

$a[n, n]_{n \times n}$

Total element = $\frac{n(n+1)}{2}$

(0 ——— $\frac{n(n+1)}{2} - 1$)

b | | | | | | |

0 1 2 3 9

Question:- lower triangular matrix $a[0 \cdots n-1, 0 \cdots n-1]$ is stored in a linear Array $b[0 \cdots \frac{n(n+1)}{2}-1]$ in lexicographical order. if $a[0,0]$ is stored in $b[0]$ where is $a[i,j]$ stored in b Array?

RMO

$a[900, 800]$

LTM RMO

$$Loc[A[i][j]] = Base(A) + \left[\frac{(i-\cancel{lb_1})(i-\cancel{lb_1}+1)}{2} + j - \cancel{lb_2}\right] * S$$

$$b\left[\frac{i(i+1)}{2} + j\right]$$

$$a[900, 800] = b\left[\frac{900 \times 901}{2} + 800\right] \Rightarrow Ans$$

Recursion:-

1) Function Calling itself is called recursion
2) Recursion should definitely have termination Condition otherwise it leads to stack overflow
3) Recursion internally makes use of Stack
4) Every time when the recursive call is made then activation record will be created in the Stack

Activation Record:

1) The stack frame allocated to the recursive function call is called as activation record
2) The creation and deletion of activation records depends on actual function calling sequence

Recursion is categorized into 4 types:-

1) Tail Recursion ✓

2) Non tail Recursion ✓

3) Indirect Recursion ✓

4) Nested Recursion ✓

Tail Recursion:- In the Program the very last statement is only one recursive call and there is no other statement after that then it is called as tail recursion

Ex:-

```
TR(int n)
{
1 [ if(n==0)
      return;
  else
2 [ { Pf(n);
      return TR(n-1);
    }
}
```

TR(4)

=> 4 3 2 1

Equivalent non Recursive code

```
int n;
for(n=4; n>0, n--)
    Pf("%d", n)
```

4321

NOTE:

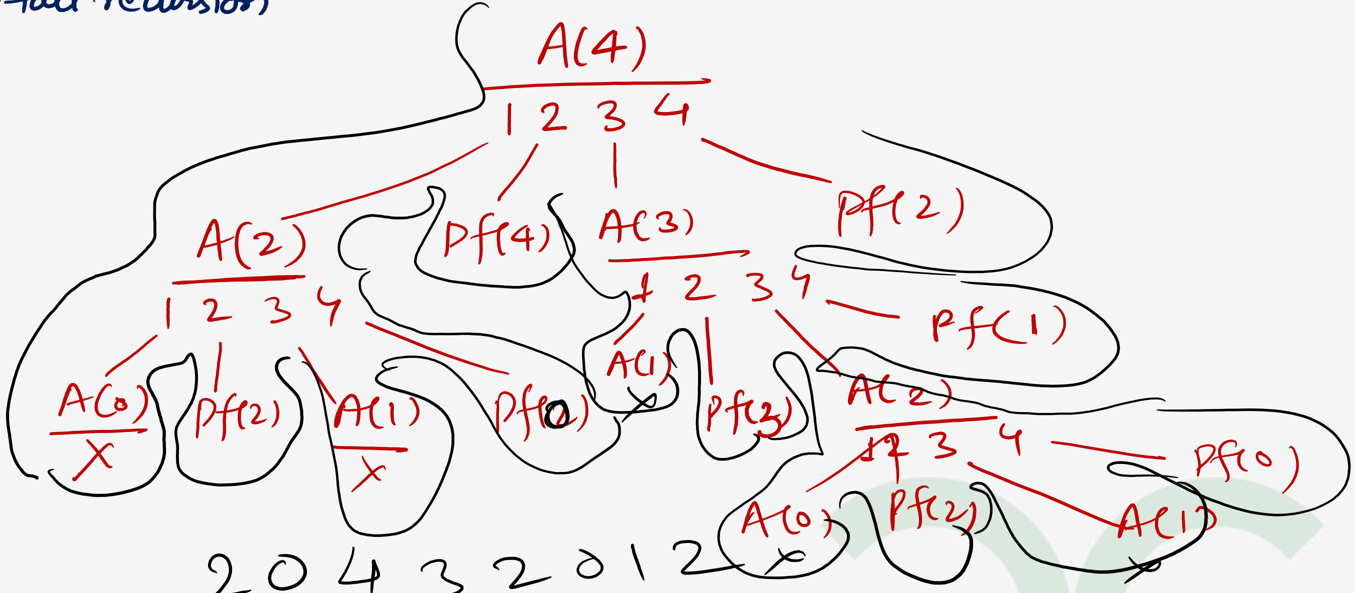
1) Tail recursion is unnecessary wasting lot of stack space and it is misusing the concept of Recursion

2) We can easily write equivalent non-recursive Program by using for loop or while loop

Non tail Recursion:- In the Program after the recursive call some statements to execute, Then it is called as non-tail recursion

Ex:-

```
=> A(int n)
   { if(n<=1)
       return;
     else
     {
1 -    A(n-2);
2 -    Pf(n);
3 -    A(n-1);
4 -    Pf(n-2);
     }
   }
```

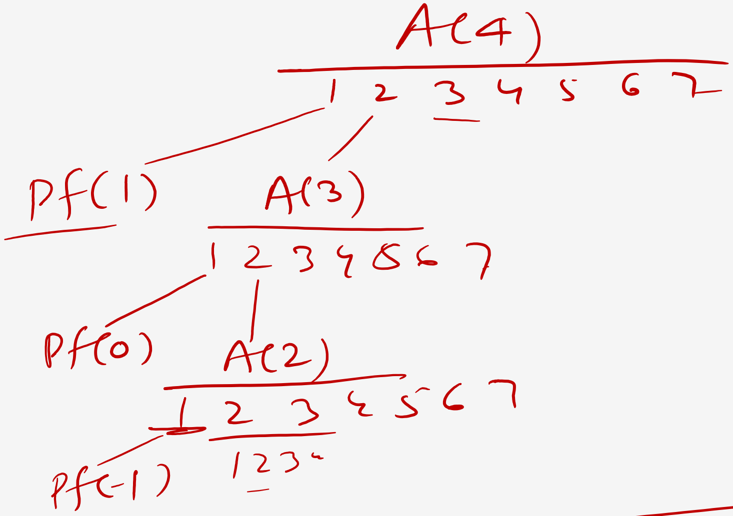


2 0 4 3 2 0 1 2

Ex:-

```
A(int n)
{
  if(n≤1)
    return;
  else
  {
    ① - Pf(n-3);
    ② - A(n-1);
    ③ - Pf(n-1);
    ④ - A(n-2);
    ⑤ - Pf(n);
    ⑥ - A(n-3);
    ⑦ - Pf(n-2);
  }
}
```

for n = 4



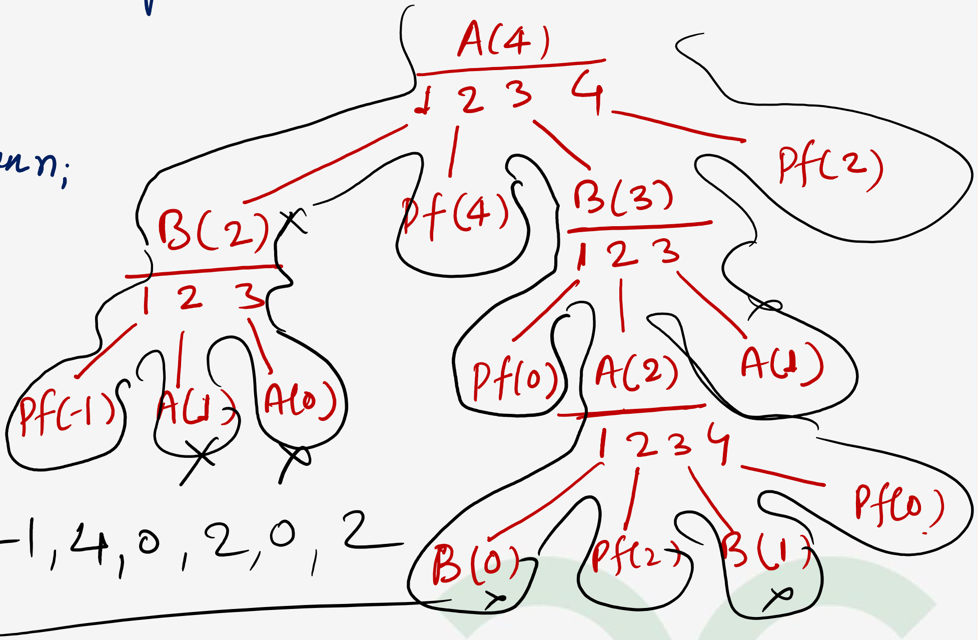
Ans: 1, 0, -1, 1, 2, 0, 2, 3, 1, 3, -1, 1, 2, 0, 4, 2

Indirect Recursion:- The two or more functions calling each other is called as indirect recursion

Ex:-

```
A(int n)
{
 if(n≤1) return;
 else
 {
1 - B(n-2);
2 - Pf(n);
3 - B(n-1);
4 - Pf(n-2);
 }
}
```

```
B(int n)
{
 if(n≤1) return n;
 else
 {
1 - Pf(n-3);
2 - A(n-1);
3 - A(n-2);
 }
}
```



-1, 4, 0, 2, 0, 2

Nested Recursion :- A recursive function which is passing itself as parameter to recursive call is called as a nested recursion

$$A(m,n) = \begin{cases} n+1, & \text{if } m=0 \\ A(m-1, 1), & \text{if } n=0 \\ A(m-1, A(m, n-1)), & \text{otherwise} \end{cases}$$

1) What is the output of $A(1,5) = ?$

$A(1,5) = 7$ Ans

$A(0, A(1,4)) = 6+1 = 7$

$A(0, A(1,3)) = 5+1 = 6$

$A(0, A(1,2)) = 4+1 = 5$

$A(0, A(1,1)) = 3+1 = 4$

$A(0, A(1,0)) = 2+1 = 3$

$A(0,1) = 1+1 = 2$

2) What is the output of $A(2,3) = ? \Rightarrow 9$ Ans

$A(1, \overset{7}{A(2,2)}) = 9$

$A(1, \overset{5}{A(2,1)}) = 7$

$A(1, \overset{3}{A(2,0)}) = 5$

$A(1,1) = 3$

$A(1,7) = 9$

$A(0, A(1,6)) = 8+1 = 9$

$A(0, A(1,5)) = 7+1 = 8$

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Thank You !