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- Q22. Construct a minimal DFA over {a,b} that accepts all strings having
- (i) At Least 2 a's and At Least 3 b's
- (ii) At Least 2 a's and At Most 3 b's
- (iii)At Most 2 a's and At Most 3 b's
- (iv)Exact 2 a's and Exact 3 b's

















**Q1:** The no. of DFA that can be drawn over  $\sum = \{a, b\}$  with 2 states  $q_0$  and  $q_1$  having  $q_0$  as the initial state is ?



Q2: How many will accept  $\sum^*$ 



Q3: How many will accept  $\Phi$ 



Q4: How many DFA neither accept  $\Phi$  nor  $\Sigma^*$ .



**Q5:** The no. of DFA over  $\{0,1\}$  with two states is ?



**Q6:** The no. of DFA over  $\{0,1\}$  with three states is ?



Formula count no. of DFA:



## **Operation on Finite Automata:**

- 1. Union
- 2. Cross Product
- 3. Subtraction



Example: 
$$M_1 = A$$

A

B

$$M_2 =$$



**Find:** 
$$M_1 \times M_2 = ?$$

$$M_1 + M_2 = ?$$

$$M_1 - M_2 = ?$$





NFA Designing: Simple Design as compared to DFA Q1. Construct NFA accepting a set of strings over {a, b} in which each string of the language start with abb.



Q2. Construct NFA accepting a set of strings over {a, b} in which each string of the language ends with 'abb'



Q3. Construction of NFA accepting a set of strings over {a, b} in which each string of the language containing 'abb' as the substring.

Q4. Design a NFA for 2<sup>nd</sup> symbol from LHS is a, over {a, b}



Q5. Design NFA for 2<sup>nd</sup> symbol from RHS is a, over {a, b}



Q6. Design NFA for 3rd symbol from RHS is a, over {a, b}

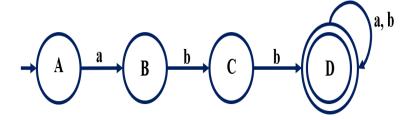




#### **Conversion from NFA to DFA:**

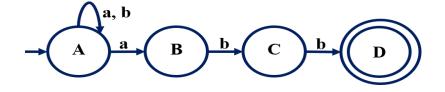
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Q1. Convert the following NFA to DFA



#### **Q2.** Convert the following NFA to DFA





Q3. Design NFA for  $3^{rd}$  symbol from RHS is 'a', over  $\Sigma = \{a, b\}$  and convert to DFA



Note: If n<sup>th</sup> symbol from RHS is a, over  $\Sigma = \{a, b\}$ . Then number of states in the corresponding DFA =  $2^n$ 

ε - **NFA**: NFA with ε - moves

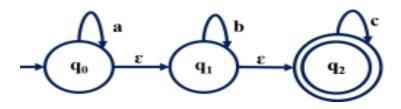


#### Conversion $\varepsilon$ - NFA to NFA (or) Removal of $\varepsilon$ - Move:



**1-**Find out all the  $\epsilon$  - transitions from each state from Q. That will be called as  $\epsilon$  - closur(q<sub>i</sub>), where q<sub>i</sub>  $\in$  Q

 $\epsilon$  - closure  $(q_i)$ : Set of all those states of the automata (NFA with  $\epsilon$  - transition) which can be reached from  $q_i$  on a path labeled by  $\epsilon$  i.e., without consuming any input symbol.

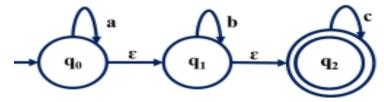


$$\varepsilon$$
 - closure  $(q_0) = \{q_0, q_1, q_2\}$   
 $\varepsilon$  - closure  $(q_1) = \{q_1, q_2\}$   
 $\varepsilon$  - closure  $(q_2) = \{q_2\}$ 

2-Then  $\delta$ ' transition can be obtained. The  $\delta$ ' transition means a  $\epsilon$  - closure on  $\delta$  moves



 $δ'(q_i, x) = ε - closure [δ(ε - closure(q_i), x)]$ 



$$\epsilon$$
 - closure  $(q_0) = \{q_0, q_1, q_2\}$ 

$$\varepsilon$$
 - closure  $(q_1) = \{q_1, q_2\}$ 

$$\varepsilon$$
 - closure  $(q_2) = \{q_2\}$ 

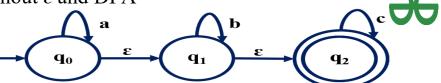
3. Repeat step 2 for each input symbol and each state of given NFA



4. Final State:

$$F' = \begin{cases} F \cup \{q\}, & \text{if } \epsilon \text{ - closure } (q) \text{ contains a state of } F \\ F & \text{otherwise} \end{cases}$$

**Example:** Convert the following NFA with  $\varepsilon$  to NFA without  $\varepsilon$  and DFA







### Minimization of DFA: $\epsilon$ -NFA $\rightarrow$ NFA $\rightarrow$ DFA $\rightarrow$ Minimize DFA

DFA minimization stands for converting a given DFA to its equivalent DFA with minimum number of states for minimum DFA.

- 1. Reduce Unreachable State
- 2. Reduce Equivalent State

```
Equivalent States: p and q are equivalent (p \approx q) state iff \delta(p, x) \in F and \delta(q, x) \in F (or) \delta(p, x) \notin F and \delta(q, x) \notin F
```

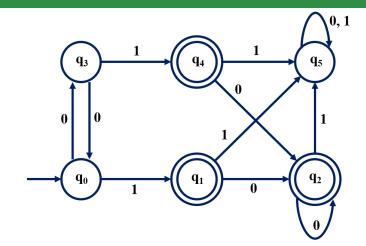
### Two Method for Minimization of DFA:



- 1. Set Partition Method
- 2. Myhill Nerode Theorem

### **Set Partition Method:**

**Example:** Minimize the following DFA

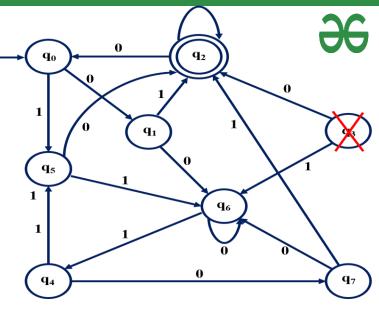






**Example:** Minimize the following DFA

<b>Q</b> /∑	0	1
$\rightarrow$ q <sub>0</sub>	$\mathbf{q_1}$	$\mathbf{q}_{5}$
$\mathbf{q_1}$	$\mathbf{q}_6$	$\mathbf{q}_2$
$*q_2$	$\mathbf{q}_0$	$\mathbf{q}_2$
$\mathbf{q_4}$	$\mathbf{q}_7$	$\mathbf{q}_{5}$
$\mathbf{q}_{5}$	$\mathbf{q_2}$	$\mathbf{q}_{6}$
$\mathbf{q}_{6}$	$\mathbf{q}_{6}$	$\mathbf{q_4}$
$\mathbf{q}_7$	$\mathbf{q}_{6}$	$\mathbf{q}_2$





#### Finite Automata with output:

Machine

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#### Moore Machine

"Output depends only present state"

Represented by 6 tuples

=  $(Q, \sum, \delta, q_0, \Delta, \lambda)$ , where

Q: Finite set of states

∑ : Input alphabet

 $\delta$ : transition function  $\delta: Q \times \sum \rightarrow Q$ 

 $q_0$ : initial state  $q_0 \in Q$ 

 $\Delta$ : Finite set of output

λ : Output function

 $\lambda : \mathbf{Q} \to \Delta$ 

#### Mealy Machine

"Output depends on the present state and input"

Represented by 6 tuples

=  $(Q, \sum, \delta, q_0, \Delta, \lambda)$ , where

Q: Finite set of states

 $\sum$ : Input alphabet

 $\delta$  : transition function  $\delta$  : Q x  $\sum \rightarrow Q$ 

 $q_0$ : initial state  $q_0 \in Q$ 

 $\Delta$ : Finite set of output

λ: Output function

 $\lambda: \mathbf{Q} \times \sum \rightarrow \Delta$ 

**Representation of Moore and Mealy Machine:** 



### **Design Moore and Mealy Machine:**

**Q1:** Design a mealy and moore m/c over  $\{0,1\}$  that produces output A if the no. of 1's in the input string is even otherwise produce output B

**Q2:** Construct a mealy and moore m/c that takes set of all strings over {0, 1} and produce 'A' as O/P if input ends with '10' or produce 'B' as O/P if input ends with '11' otherwise produces 'C'



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- Q1: Design mealy m/c for
  - (i) one's complement of binary no.
  - (ii) two's complement of binary no. (input read from LSB to MSB)

Design a mealy m/c which reads the input from  $(0 + 1)^*$  and produces the following outputs.



- (i) if input ends in 101, output is A
- (ii) if input ends in 110, output is B
- (iii) for other inputs, output is C

Q2:



Conversion Moore to Mealy & Mealy to Moore m/c:









