Push Down Automata(PDA):

FA + 1 STACK

$$PDA = \underbrace{(Q, \Sigma, \delta, q_0, F)}\ \cup\ \underbrace{(Z_0, \Gamma)}$$

I/p Alphabet

set of pushdown symbol

$$\boxed{PDA = (Q, \Sigma, \delta, \Gamma, q_0, Z_0, F)}$$

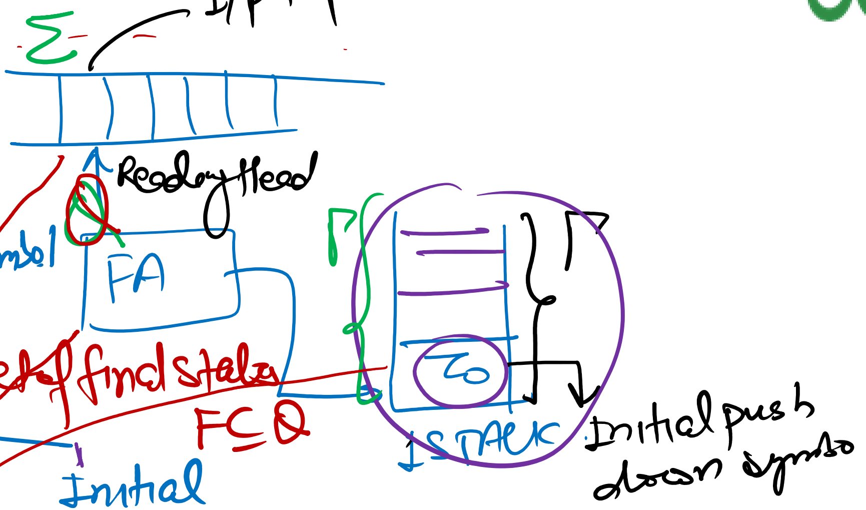
Finite Non empty set of states

Transition function

Initial state $q_0 \in Q$

set of final states $F \subseteq Q$

Initial pushdown symbol $\boxed{Z_0 \in \Gamma}$



$$\boxed{\delta : Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*}$$

DFA: $Q \times \Sigma \rightarrow Q$

NFA: $Q \times \Sigma \rightarrow 2^Q$

$$\Gamma^* = \{\epsilon, \Gamma, \Gamma\Gamma, \Gamma\Gamma\Gamma, \ldots\}$$

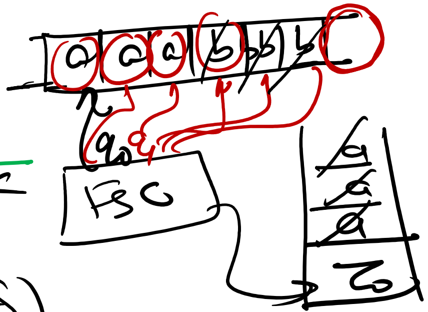
Instantaneous Description (ID):- Instantaneous Description (ID) is an informal notation of how a PDA "computes" a input string and make a Decision that string is accepted or rejected.

A - ID is a triple $\boxed{(q, \omega, \alpha)}$, when

$L = \{a^n b^n \mid n > 0\} = \{ab, a^2b^2, a^3b^3, a^4b^4 \ldots\}$

a^3b^3

1. q is the current state
2. ω is the remaining input
3. α is the stack contents, top at the Left.



Turnstile Notation: ⊢ Sign is Called a "turnstile notation" and represents one move

$\vdash^*$ Sign represents a sequence of Moves

$\vdash^*$ $\vdash$

i.e $(p, b, T) \vdash (q, \omega, \alpha)$

This implies that while taking a transition from state p to state q, the input symbol b is consumed and the top of the STACK 'T' is replaced by a new string 'α'

Acceptance by PDA:

$\delta(q_0, \omega, z_0) \vdash^* (q_f, \epsilon, \alpha)$, $q_f \in F$, $\alpha \in \Gamma^*$

1. **Acceptance by final:** A PDA accepts a string when, after reading the entire string, The PDA is in a final state

$PDA = (Q, \Sigma, \delta, \Gamma, q_0, z_0, F) \Rightarrow$ 7 tuples

Let $P = (Q, \Sigma, \delta, \Gamma, q_0, z_0, F)$ be a PDA the language accepted by the final state can be defined as

$L(PDA) = \{\omega \mid (q_0, \omega, z_0) \vdash^* (P, \varepsilon, X), P \in F, X \in \Gamma^*\}$

$PDA = (Q, \Sigma, \delta, \Gamma, q_0, z_0, \phi)$

2. **Acceptance by Empty stack:** Here a PDA accepts a string when, after reading the entire string, The PDA has emptied its stack.

STACK Empty

$N(PDA) = \{\omega \mid (q_0, \omega, z_0) \vdash^* (P, \varepsilon, \varepsilon), P \in Q\}$

$\delta(q_0, \omega, z_0) \vdash^* (q', \epsilon, \epsilon)$

Initial ID

$q' \in Q$

NOTE: - if acceptance is defined by empty stack, then there is no meaning of final state and it is represented by ϕ

Null Moves: The PDA can make a transition without taking input symbol such moves are called null moves

$\delta(Q \times \Sigma \cup \epsilon \times \Gamma)$

$\delta(q_0, \epsilon, z_0) \vdash (q_1, az_0)$

$\delta(q_1, a, b, z_1 z_0)$

$\delta : Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*$

$\delta(q_1, \epsilon, z_1) \vdash (q_1, az_1)$ $\quad$ $\delta(q_1, a, z_1) \vdash (q_2, b)$

① $\delta(q_1, \epsilon, z_1) \neq \phi$ $\quad$ ② $\delta(q_1, a, z_1) = \phi$

Deterministic PDA(DPDA): A PDA is said to be deterministic if it satisfying the following 2-Conditions

$\delta(q, a, z)$

1) Every $\delta(q, a, z)$ is either empty or has a single move

2) if $\delta(q, \epsilon, z) \neq \phi$ then $\delta(q, a, z) = \phi$, $\forall a \in \Sigma$

Null moves

① $\delta(q, \epsilon, z) \neq \phi$

② $\delta(q, a, z) \neq \phi$

Non Deterministic PDA (NPDA):

$$\boxed{\delta : Q \times \Sigma \cup \epsilon \times \Gamma \rightarrow 2^{Q \times \Gamma^*}}$$

$$\delta(q, a, z) \xrightarrow{NPDA} (q_1, az), (q_2, z), (q_3, bz), (q, z)$$

DPDA: $Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow \boxed{Q \times \Gamma^*}$

NPDA: $Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow \boxed{2^{Q \times \Gamma^*}}$

CFG

↓

CFL

DCFL | NDCFL (CF)

↓ | ↓

Imp

$$\boxed{P(\text{DPDA}) < P(\text{NPDA})(\text{PDA})}$$

Difference b/w DPDA & NPDA:

Deterministic PDA (DPDA)	Non Deterministic PDA (NPDA)
1. Choice is not Allowed in DPDA	1. Choice Allowed in NPDA
2. Dead Configuration is Allowed in DPDA	2. Dead configuration is allowed in NPDA
3. Null moves Allowed Conditionally	3. Null Moves is allowed unconditionally.
4. $\boxed{\delta: Q \times (\Sigma \cup \varepsilon) \times \Gamma \to Q \times \Gamma^*}$	4. $\boxed{\delta: Q \times (\Sigma \cup \varepsilon) \times \Gamma \to 2^{Q \times \Gamma^*}}$
5. it is less powerful than NPDA	5. It is more powerful than DPDA
6. Language Accepted by DPDA is called DCFL	6. Language Accepted by NDPA is called NDCFL
7. It is Possible to Convert every DPDA to a corresponding NPDA	7. It is not possible to convert NDPA to a corresponding DPDA.

Representation of PDA:

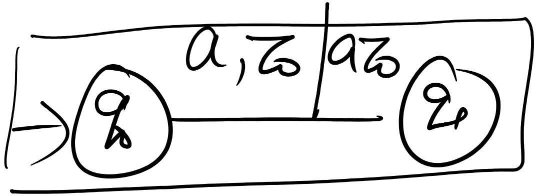
$\delta : Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*$

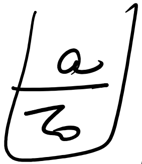
1) Push:

$\delta(q_0, a, Z_0) \vdash (q_1, aZ_0)$

(or)

Graphical:

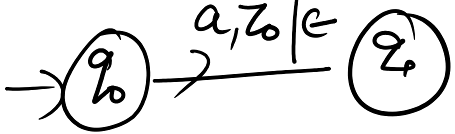


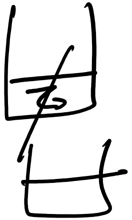


2) Pop:

$\delta(q_0, a, Z_0) \vdash (q_1, \epsilon)$

or

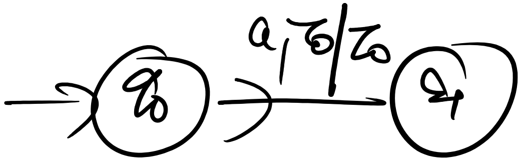


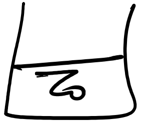


3) Read:

$\delta(q_0, a, Z_0) \vdash (q_1, Z_0)$

or



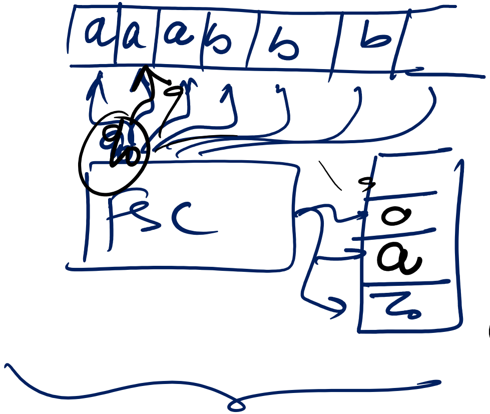


PDA Designing:

Ex:- Design a PDA for the Language $L = \{a^n b^n \mid n > 0\}$

$L = \{ab, a^2b^2, a^3b^3, a^4b^4 \ldots\}$, ~~ba, aaba~~, aabbaabb, ϵ

aaabbb



PDA

$\delta(q_0, \epsilon, Z_0) \vdash (q_f, Z_0)$: Acceptance by final State

or

$\delta(q_0, \epsilon, Z_0) \vdash (q_0, \epsilon, \epsilon)$] Empty stack Acceptance

$L = \{a^n b^n \mid n > 0\}$

$\delta(q_0, a, Z_0) \vdash (q_0, aZ_0)$

$\delta(q_0, a, a) \vdash (q_0, aa)$

$\delta(q_0, b, a) \vdash (q_1, \epsilon)$

$\boxed{\delta(q_0, \epsilon, Z_0) \vdash (q_0, \epsilon)}$

Wrong Design b/c of Accept $a^n b^n \mid n > 0$

~~not perfect~~

and some Non valid string ϵ, a^2b^2ab,

$a^4b^4 a^2b^2 a^6b^6$ – $(a^i b^i)^*$

a	a	b	b	a	b	ϵ

q_0

FSC

$\delta(q_0, a, Z_0)$

Analysis:

a^2b^3, ~~a^3~~ b^2

$\delta(q_0, b, Z_0) \vdash \delta(q_0, b, \boxed{\epsilon})$

reject

$\delta(q_0, \epsilon, a)$

Reject

$$\left\{\begin{array}{l}\delta(q_0, a, Z_0) \vdash (q_0, aZ_0)\\ \delta(q_0, a, a) \vdash (q_0, aa)\\ \delta(q_0, b, a) \vdash (q_1, \epsilon)\\ \delta(q_1, b, a) \vdash (q_1, \epsilon)\\ \boxed{\delta(q_1, \epsilon, Z_0) \vdash (q_1, \epsilon)}\end{array}\right.$$

or (q_f, Z_0)

Acceptance by Empty STACK

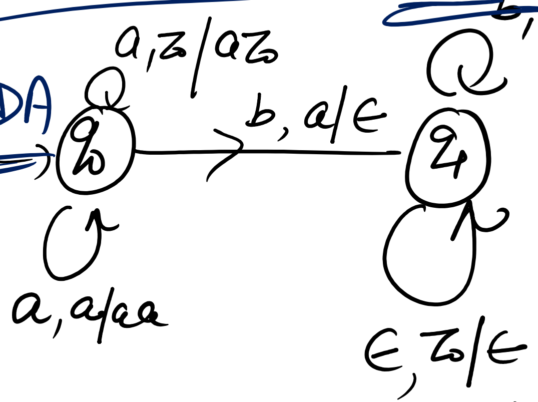
Acceptance by final State

$\delta(q_1, a, Z_0) = \phi$

$\delta(q_1, b, Z_0) = \phi$

$L = \{a^n b^n \mid n > 0\}$ Graphical → DCFL

DPDA



$\delta(q_0, \epsilon, Z_0) =$

DPDA

$L_{DCFL} = \{a^n b^n \mid n > 0\}$

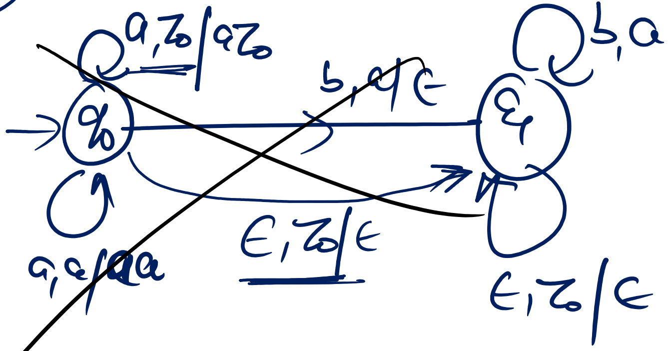
Acceptance by Empty STACK

Ex:- Design DPDA for $\boxed{L = \{a^n b^n \mid n \geq 0\}}$ ⇒ DCFL

$L = \{\epsilon, ab, a^2b^2, a^3b^3, a^4b^4, \ldots\ldots\}$

DPDA.

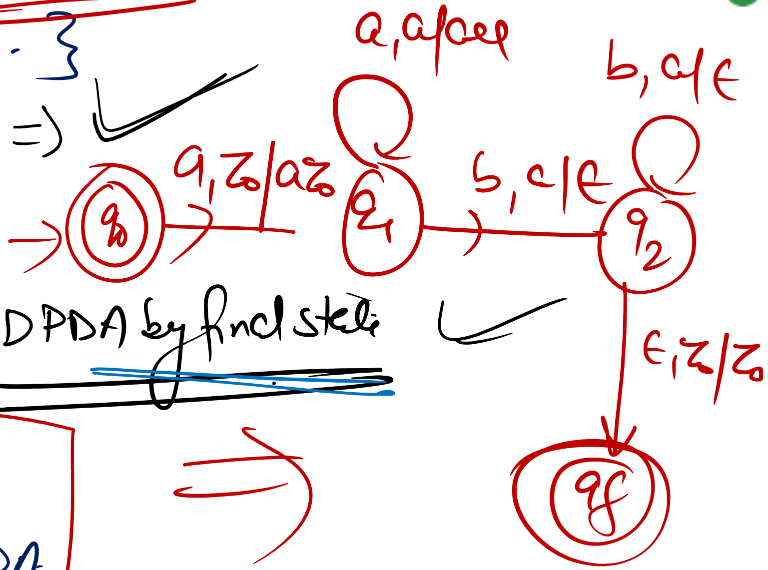
by Empty STACK



$\delta(q_0, \epsilon, Z_0) \vdash (q_1, \epsilon)$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

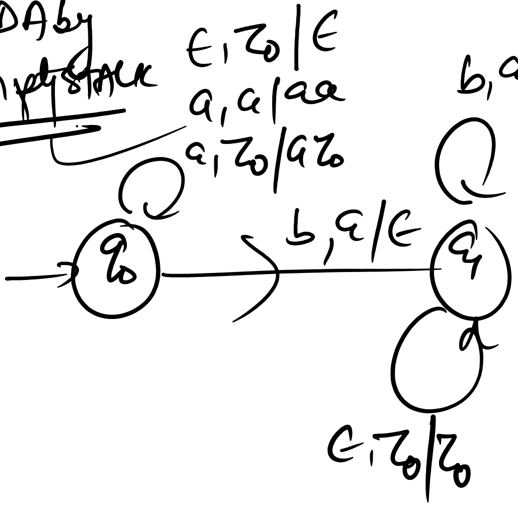
NPDA



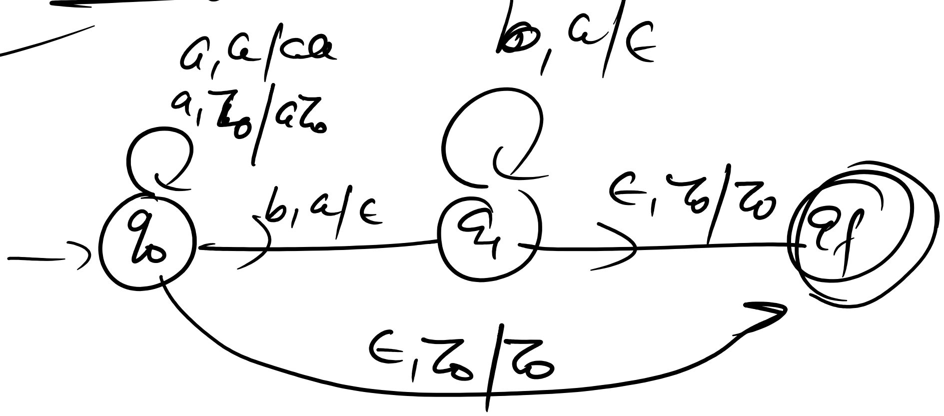
DPDA by final state

$L = \{a^n b^n \mid n \geq 0\}$

NPDA by Empty Stack



NPDA by final State

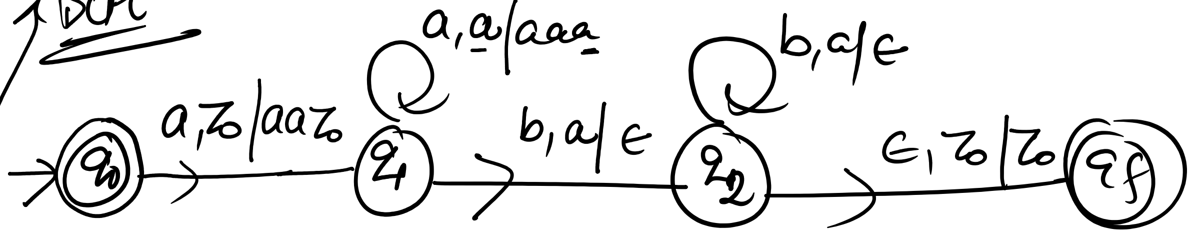


(Theory of Computation)

Ex:- $L = \{a^n b^{2n} \mid n \geq 0\}$ Construct DPDA

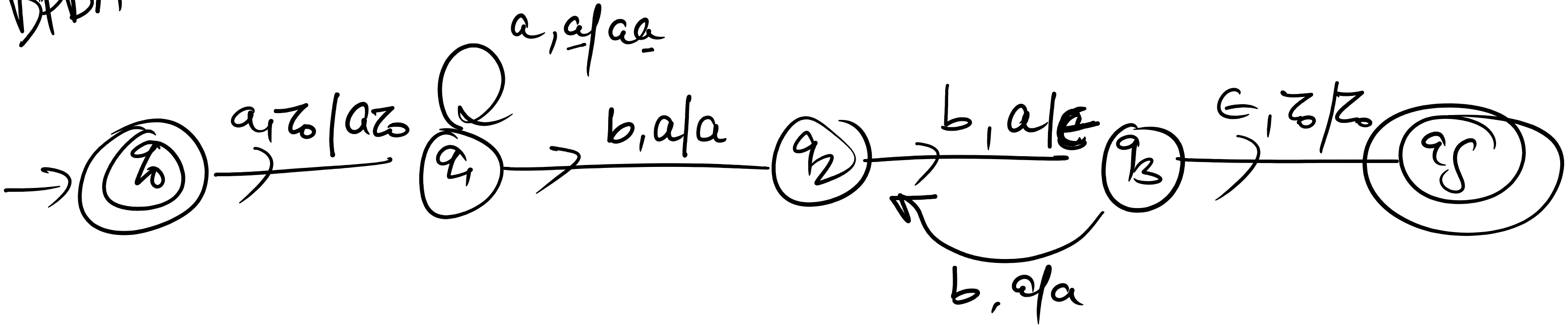
$L = \{\epsilon, ab^2, a^2b^4, a^3b^6, a^4b^8 \ldots\ldots \}$

DCFL



(or)

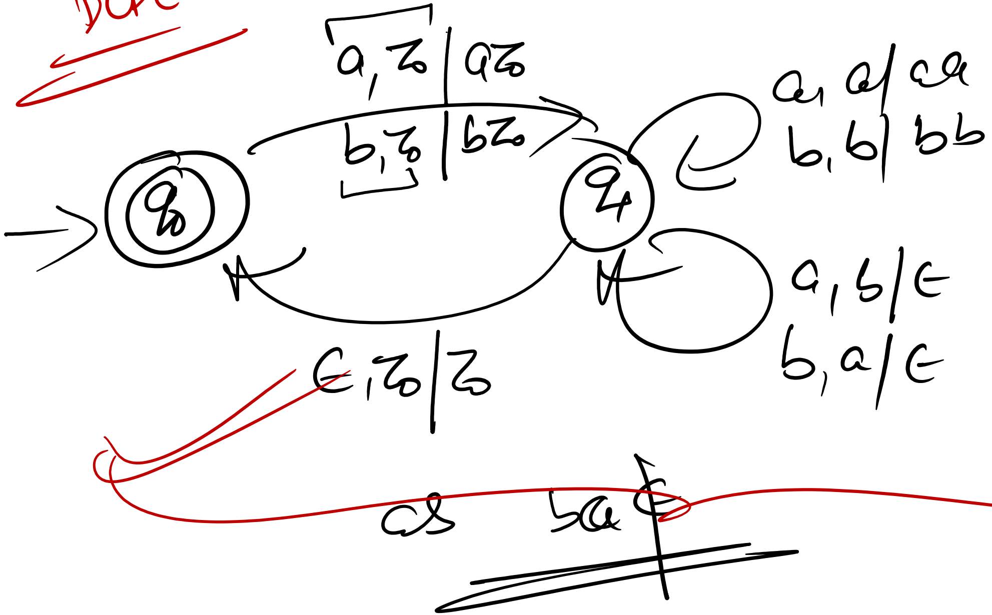
DPDA



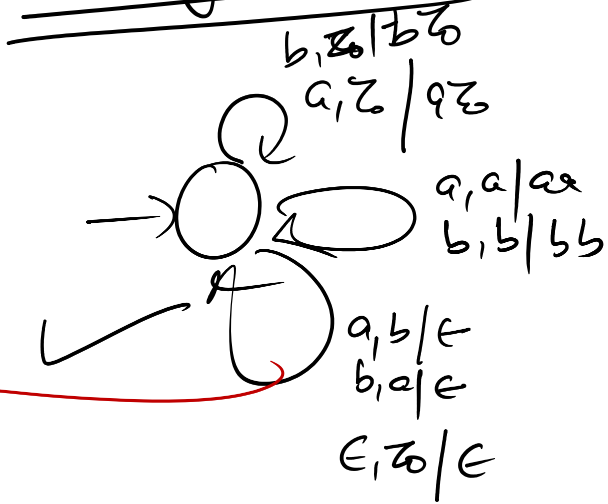
Ex:- $L = \{ w \in (a,b)^*,\ N_a(w) = N_b(w) \}$ is DCFL $\quad a^n b^n$

$L = \{ \epsilon, ab, ba, abab, baba, abba, aabbbb, \ldots \}$

DCFL



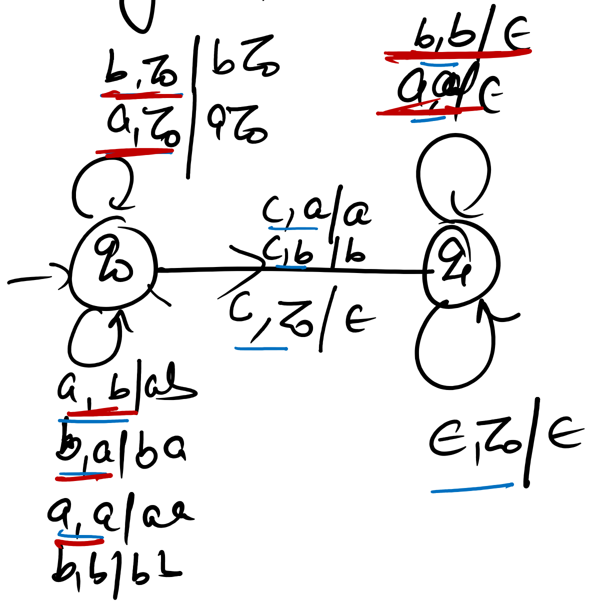
NPDA by Empty STACK

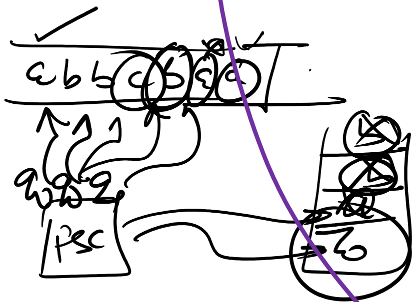


Ex:- $L = \{wcw^T \mid w \in (a,b)^*\}$ = odd length palindrome → DCFL

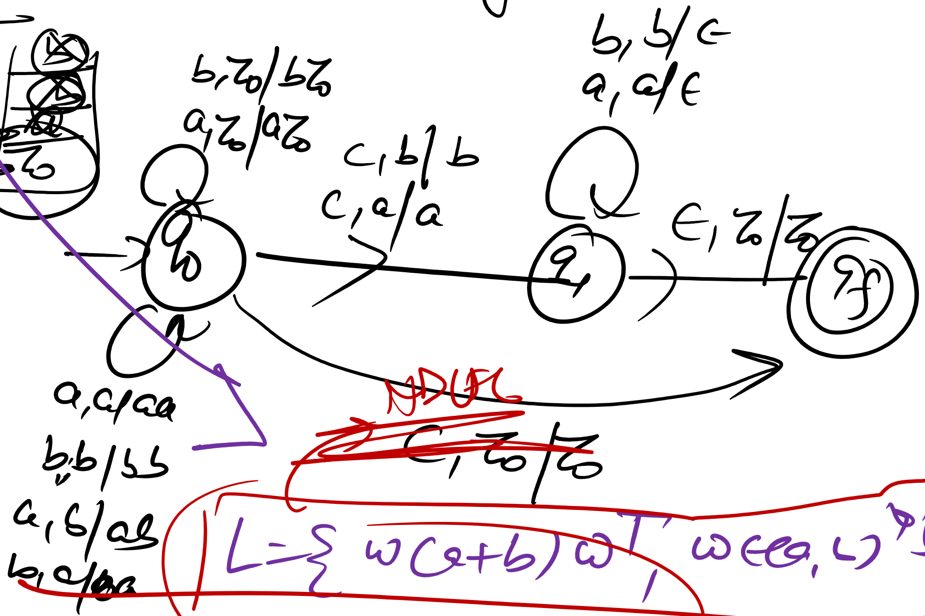
$= \{c, aca, bcb, cbcbc, bacab, \ldots\}$

DPDA by Empty STACK





DPDA by final state

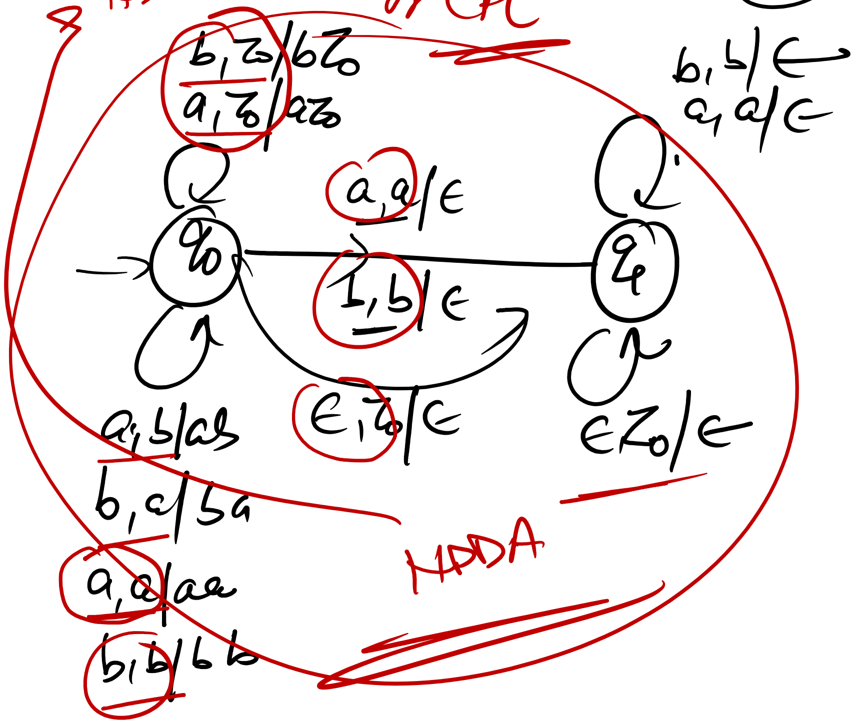


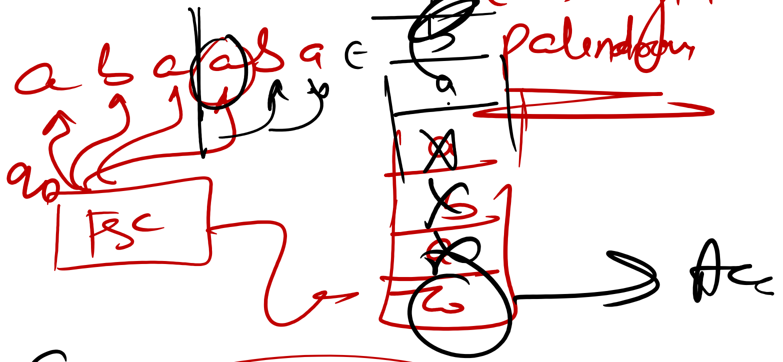
$L = \{w(a+b)w^T, w \in (a,b)^*\}$

Ex:- $L = \{W \cdot W^T \mid W \in (a,b)^*\} = \{\epsilon, aa, bb, abba, aaaa, bbbb \ldots\}$

NPDA or CFL

Even length palindrome





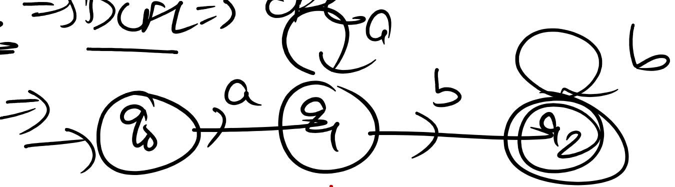
ε,

30 minits → 1:35

Lunch break

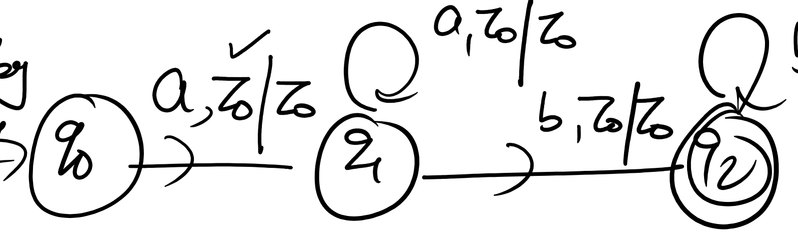
Ex:- $L=\{a^m b^n \mid m,n>0\}$ ⇒ PDA ⇒ RL ⇒ DCFL ⇒ CFL

$L=\{ab, a^2b, ab^2, a^2b^2, \ldots\}$ ⇒



~~DPD~~

DPDA by final state



aaabb\$

DPDA

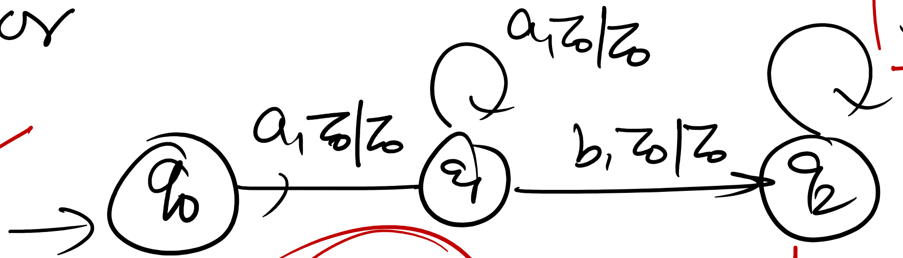
$\$, Z_0/\epsilon$

$b, Z_0/Z_0$

or

DPDA by Empty stack

\$



$\delta(q_2, \epsilon, Z_0) = (q_2, \epsilon)$

$\delta(q_2, b, Z_0) = (q_2, Z_0)$

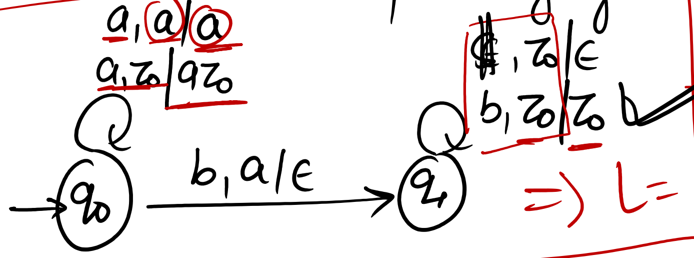
$\$, Z_0/Z_0$

$b, Z_0/Z_0$

Not allow

in DPDA

Ex:- Given PDA accept Language?



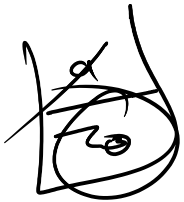
⇒ L = ?

HPDA by Emply STACK

$

$$L = \{ a^n b^m \mid m, n > 0 \}$$

aaabb$

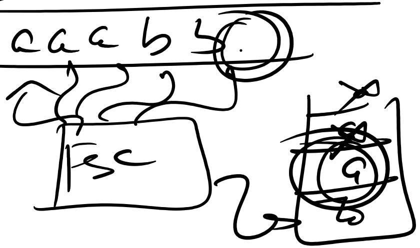


Ex:- $L = \{a^m b^n \mid m > n > 0\}$ is DCFL = ?

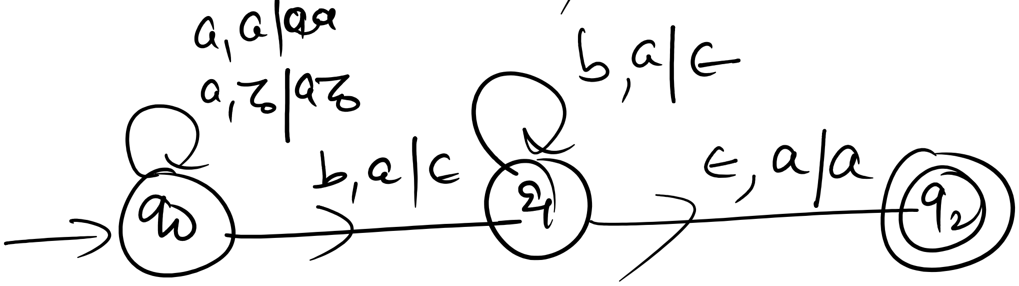
DCFL

$L = \{a^2 b, a^3 b^2, a^3 b, a^4 b^2, a^4 b^3 \text{ ---- } \}$

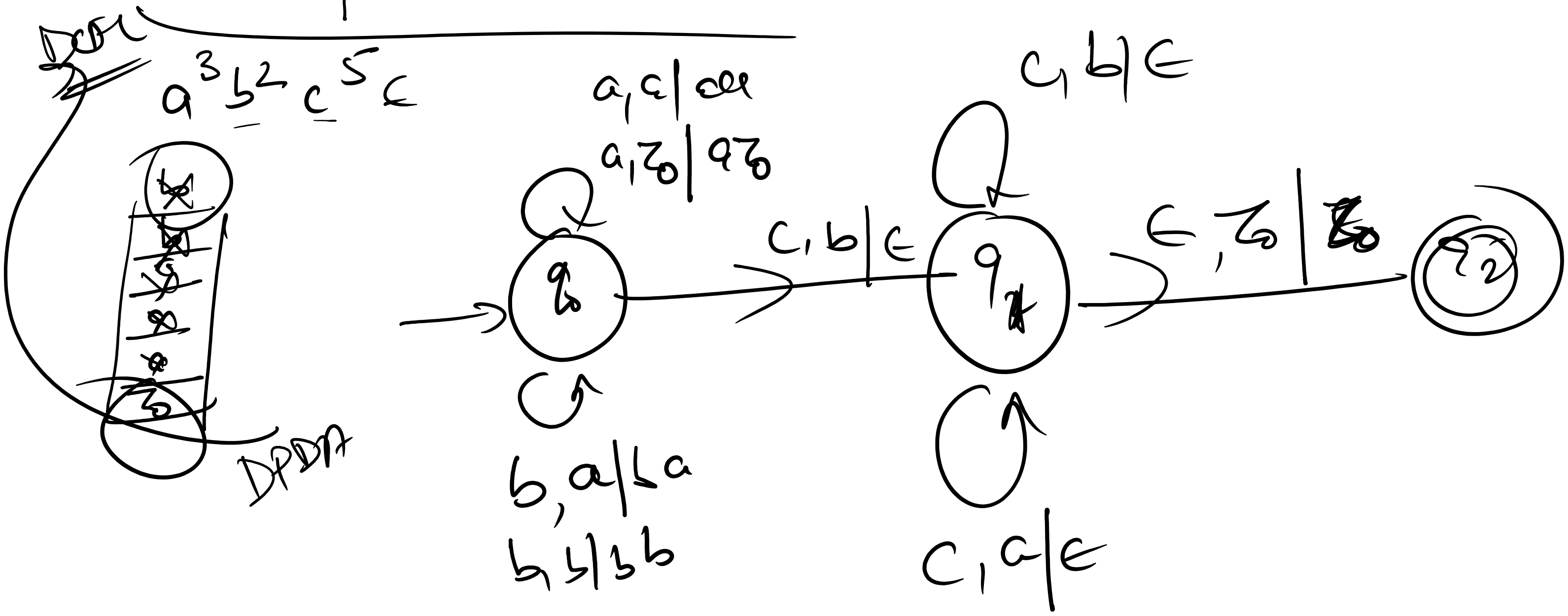
$m > n$



DPDA



Ex:- $L = \{a^m b^n c^{m+n} \mid m,n > 0\}$ is DCFL ?



Ex:- $L = \{a^n b^n c^n \mid n > 0\}$ ⇒ not CFL ⇒ CSL

aabbcc ∈

aabccc

aacc

$a^n b^n c^n$ ⇒ CSL

2 STACK PDA

LBTM

aabbcccc

⇒ TM

$L = \{a^n b^n c^n, 1 \leq n \leq 10^{10}\}$

⇒

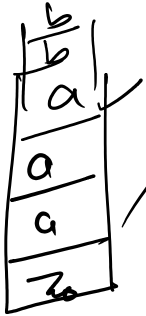
(Theory of Computation)

Ex:- $L = \{a^m b^n c^m d^n \mid m,n \geq 1\}$ CSL

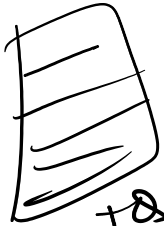
$L = \{a^m b^m c^n d^n \mid m,n \geq 1\}$

$a^3 b^2 c^3 d^2$

DCFL



$a^m b^n c^m d^n$



$\Rightarrow$ 2STACK

1 Queue

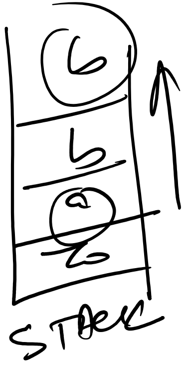
Ex:

$L = \{ w w \mid w \in (a,b)^* \} \rightarrow$ CSL

abb | abb

$|w| = 10$

$\underbrace{a\,b\,b}_{w} \mid \underbrace{a\,b\,b}_{w}$



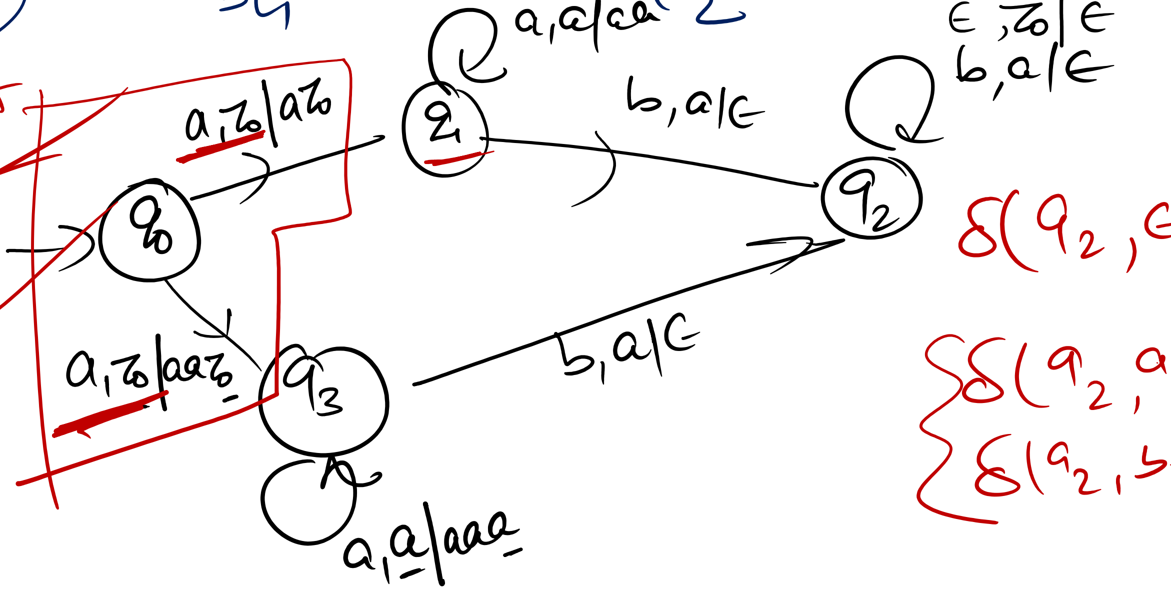
2 stack + FA $\Rightarrow$ TM

Ex: Design a PDA for the following language

$$L = \{a^n b^n \mid n>0\} \cup \{a^n b^{2n} \mid n>0\}$$

~~NDCFL~~ DCFL DCFL

NPDA



$$\delta(q_0, a, Z_0) = \{(q_1, aZ_0), (q_3, aaZ_0)\}$$

$$\delta(q_2, \epsilon, Z_0) = (q_2, \epsilon)$$

$$\delta(q_2, a, Z_0) = \phi$$

$$\delta(q_2, b, Z_0) = \phi$$

Ex: Design PDA for the following Languages

(i) $L = \{a^m b^n c^k \mid m = n \text{ or } n = k, \; m, n, k \geq 1\} \Rightarrow$ NDCFL

NDCFL

(or)

L_1 = DCFL L_2 = DCFL

$L = \{ \underbrace{a^n b^n c^k}_{\text{DCFL} \cdot n, k \geq 1} \cup a^m b^k c^k \quad m, k \geq 1 \}$

(ii) $L = \{a^m b^n c^k \mid m = n \text{ AND } n = k, \; m, n, k \geq 1\} \Rightarrow$ NotCFL

(or)

$L = \{a^m b^m c^m \mid m \geq 1\}$ → CSL