| Problem 1 (p.1) | EE 417 | HW 3 |
| --- | --- | --- |

a) Simulates sending bits over an AWGN channel with pulses and matched-filters

b) $T=1$

$p_1(t) = rect(t) \quad w_1(t) = rect(-t) = rect(t)$

$p_2(t) = sinc(t) \quad w_2(t) = sinc(-t) = sinc(t)$

$y_1(t) = x_1(t) + n(t) \qquad Z_1(t) = w_1(t) * y_1(t)$

$y_2(t) = x_2(t) + n(t) \qquad Z_2(t) = w_2(t) * y_2(t)$

$Z_{1k} = x_{2k} h_{2k} + \tilde{n} \qquad Z_{2k} = x_{2k} h_{2k} + \tilde{n}$

c) $T=1$, $\alpha = .2$

d) $y_1(t)$ shows the rectangular time pulsed signal that was sent accross an AWGN channel. $Z_1(t)$ shows the demodulated signal (matched filter).

$y_2(t)$ shows the sinc time pulsed signal that was sent across an AWGN channel. $Z_2(t)$ shows the demodulated signal (matched filter)

e) $Y_1(f)$ shows the frequencies of the rectangular signal that was sent over the AWGN channel. $Z_1(f)$ shows the demodulated signal's frequencies. $Y_1(f)$ shows the frequencies of the signals of the sinc functions sent over the AWGN channel. $Z_2(f)$ shows the demodulated frequencies of $Y_2(f)$.

f) Z is the demodulated signal, so the noise has been smoothed out with the convolution.

g) the Z signals have much less of a DC signal, and less noise. This also explains why the figure 1 y and z have more smoothing.

Problem 1 (p.2) | EE 417 | HW3

$$z_{1k} = \sum_{m=-\infty}^{\infty} x_m p(-t) * p(t) * \delta(t-mT) + p(-t) * n(t), \; p(t) = rect(t), \; T=1$$

$$x_{1k} = G(z_{1k})$$

$$z_{2k} = \sum_{m=-\infty}^{\infty} x_m p(-t) * p(t) * \delta(t-mT) + p(-t) * n(t), \; p(t) = sinc(t), \; T=1$$

$$x_{2k} = G(z_{2k})$$

There is No ISI because when the signal is sampled at T intervals, the value at $p(-t) * p(t)$ is zero for all pulses besides the one we are looking at.

i) The BER using matched filters is 0.

The BER using A/D is 0.1570.

Without the matched filter, there is significantly more noise, so more chances of bit errors for A/D.

j) The BER using SRRC is 0.1450

With $\alpha = .5$, BER = 0.002.

With alpha closer to 1, it behaves more like the rect pulse, which has 0 BER.

| Problem 1 (p.3) | EE 417 | HW3 |
|---|---|---|

$y(t) = x(t) + n(t) \qquad x(t) = \sum_{-\infty}^{\infty} x_k\, p(t-kT)$

a)

$z(t) = w(-t) * y(t)$

$= w(-t) * \left[\sum_{-\infty}^{\infty} x_k\, p(t-kT) + n(t)\right]$

$= \sum_{-\infty}^{\infty} x_k\, w(-t) * p(t-kT) + w(-t) * n(t)$

$= \sum_{-\infty}^{\infty} x_k\, w(-t) * p(t) * \delta(t-kT) + w(-t) * n(t)$ , $h(t) = w(-t) * p(t)$ , $\tilde{n}(t) = w(-t) * n(t)$

$= \sum_{-\infty}^{\infty} x_k\, h(t-kT) + \tilde{n}(t)$

$z_k = z(kT) = \sum_{m=-\infty}^{\infty} x_m h(kT-mT) + \tilde{n}(kT) = x_k * h_k + \tilde{n}(kT)$

$= \sum_{m=-\infty}^{\infty} x_k h_{k-m} + n_k = x_k h_0 + \underbrace{\sum_{m\neq 0} x_k h_{k-m}}_{\text{ISI}} + \tilde{n}_k$

$\sum_{m\neq 0} x_m h_{k-m} = \sum_{m\neq 0} x(mT) \cdot h((k-m)T)$

$w(-t) * p(t) = 0$ for $|t| \geq T$, so $\sum_{m\neq 0} x(mT)\cdot h((k-m)T) = 0$

So there is no ISI. $\qquad \downarrow$ $h_k = 0$ for $k \neq 0$

b) $\tilde{n}_k = \tilde{n}(kT) \qquad E[\tilde{n}_k \tilde{n}_{m-k}] = \frac{N_0}{2} h_k = 0$ for $k \neq 0$, so it's <u>iid</u>

| Problem 2 (p. 1) | EE 417 | HW3 |
| --- | --- | --- |

$y(t) = x(t) + n(t)$ $\quad x(t) = \sum_{-\infty}^{\infty} x_k p(t-kT)$

a)
$z(t) = w(-t) * y(t)$

$= w(-t) * \left[\sum_{-\infty}^{\infty} x_k p(t-kT) + n(t)\right]$

$= \sum_{-\infty}^{\infty} x_k w(-t) * p(t-kT) + w(-t) * n(t)$

$= \sum_{-\infty}^{\infty} x_k w(-t) * p(t) * \delta(t-kT) + w(-t) * n(t)$ , $h(t) = w(-t) * p(t)$ , $\tilde{n}(t) = w(-t) * n(t)$

$= \sum_{-\infty}^{\infty} x_k h(t-kT) + \tilde{n}(t)$

$z_k = z(kT) = \sum_{m=-\infty}^{\infty} x_m h(kT-mT) + \tilde{n}(kT) = x_k * h_k + \tilde{n}(kT)$

$= \sum_{m=-\infty}^{\infty} x_k h_{k-m} + n_k = x_k h_0 + \underbrace{\sum_{m\neq 0} x_k h_{k-m} + \tilde{n}_k}_{\text{ISI}}$

$\sum_{m\neq 0} x_m h_{k-m} = \sum_{m\neq 0} x(mT) \cdot h((k-m)T)$

$w(-t) * p(t) = 0$ for $|t| \geq T$, so $\sum_{m\neq 0} x(mT) \cdot h((k-m)T) = 0$

So there is no ISI. $\quad \downarrow$ $h_k = 0$ for $k \neq 0$

b) $\tilde{n}_k = \tilde{n}(kT)$ $\quad E[\tilde{n}_k \tilde{n}_{m-k}] = \frac{N_0}{2} h_k = 0$ for $k \neq 0$, so it's <u>iid</u>

Problem 2 (p.2) | EE 417 | HW3

c)

$$Z_k = z(kT) = \boxed{h_k * x_k + n_k = Z_k}$$ derived for part a

$$= (\omega(-kT) * p(kT)) * x_k + \omega(-kT) * n(kT)$$

d)

$$Z_k = x_k h_0 + n_k$$

for $\omega(t)$, $h_0 = \omega(-t) * p(t) = .5 \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$

for $p(t)$, $h_0 = p(-t) * p(t) = 1$

So $p(t)$ has a clearer signal

e)

$$y(t) = x(t) + \underset{\sim}{Y}$$

$$z(t) = \omega(-t) * y(t) = \omega(-t) * \left(\sum_{-\infty}^{\infty} x_k\, p(t-kT) + \underset{\sim}{Y}\right)$$

$$= \sum_{-\infty}^{\infty} x_k\, \omega(-t) * p(t-kT) + \omega(-t) * \underset{\sim}{Y}$$

$$= \sum_{-\infty}^{\infty} x_k\, \omega(-t) * p(t) * \delta(t-kT) + \omega(-t) * \underset{\sim}{Y},$$ $h(t) = \omega(-t) * p(t)$

$n(t) = \omega(-t) * \underset{\sim}{Y}$

$$= \sum_{-\infty}^{\infty} x_k h(t-kT) + \tilde{n}(t)$$

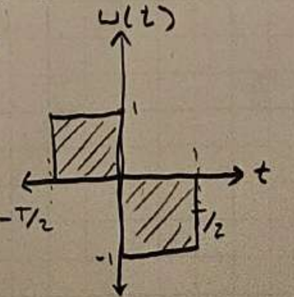
$Z_k = h_k * x_k + n_k$ $\qquad$ $n_k = 0 = \omega(-t) * \underset{\sim}{Y}$

$\omega(t)$ must be odd.

$h_k = \omega(-t) * p(t) \neq 0$

Ex. take $\omega(t)$

Ex:




$\omega(-t) * p(t) \neq 0$ for $t \neq 0$, $T < t < T$

So $Z_k = h_k * x_k$, noise is gone, signal is still there

| Problem 3 (p. 1) | EE 417 | HW3 |
|---|---|---|

$x(t) = \sqrt{2}\sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)\cos(2\pi f_c t)$, $p^b(t) = \frac{1}{\sqrt{T}}\mathrm{rect}\left(\frac{t}{T}\right)$

$y(t) = x(t)$ $\qquad x_n^Q = 0$

a)

$m^I(t) = \sqrt{2}x(t)\cos(2\pi f_c t) = \sqrt{2}\cdot\sqrt{2}\sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)\cos^2(2\pi f_c t)$

$= 2\sum_{m=-\infty}^{\infty} x_n^I p^b(t-mT)\left(\frac{1}{2}\right)(1+\cos(4\pi f_c t))$

$= \sum_{m=-\infty}^{\infty}\left[x_m^I p^b(t-mT) + x_m^I p^b(t-mT)\cos(2\pi f_c t)\right]$

$m^Q(t) = \sqrt{2}\,x(t)\sin(2\pi f_c t) = \sqrt{2}^2\sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)\cos(2\pi f_c t)\sin(2\pi f_c t)$

$= \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)\sin(4\pi f_c t)$

$y^I(t) = \omega^{LP}(t) * m^I(t) = \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)$

$y^Q(t) = \omega^{LP}(t) * m^Q(t) = 0$

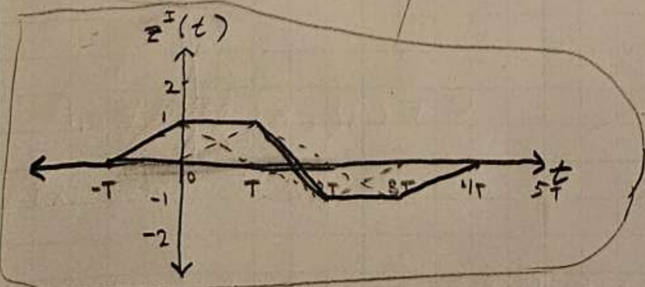
$z^I(t) = p^b(-t) * y^I(t) = \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) * p^b(-t)$

$= \sum_{m=-\infty}^{\infty} x_m^I p^b(t) * p^b(-t) * \delta(t-mT)$

$\boxed{z^Q(t) = p^b(-t) * y^Q(t) = p^b(-t) * 0 = 0}$ ✓

b) $x_n = \{1, 1, -1, -1\}$, 0 otherwise

$z^I(t) = \sum_{m=-\infty}^{\infty} x_m^I h^b(t) * \delta(t-mT)$, $h^b(t) = p^b(t) * p^b(-t)$



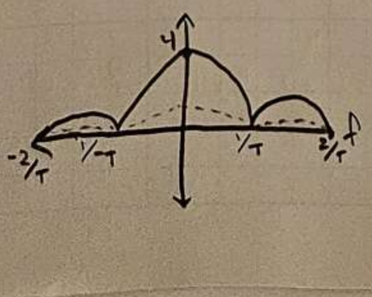

$h^b(t) =$ triangle of width 2T

$Z^I(f) = \sum_{m=-\infty}^{\infty} x_m^I H^b(f)e^{jfmT}$

$H^b(f) = \mathrm{sinc}(fT)$

$Z^I(f) = H^b(f) + H^b(f)e^{jfT} - H^b(f)e^{jf\cdot 2T} - H^b(f)e^{jf\cdot 3T}$

$= H^b(f)(1 + e^{jfT} + e^{jf\cdot 2T} + e^{jf\cdot 3T})$



| Problem 3 (p.2) | EE 417 | HW3 |
|---|---|---|

c)

$$m^I(t) = \sqrt{2}\, x(t) \cos(2\pi f_c t + \pi/4)$$

$$m^Q(t) = \sqrt{2}\, x(t) \sin(2\pi f_c t + \pi/4)$$

$$m^Q(t) = \sqrt{2}\cdot\sqrt{2} \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) \cos(2\pi f_c t) \sin(2\pi f_c t + \pi/4)$$

$$= 2 \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) \cos(2\pi f_c t)[\sin(2\pi f_c t)\cos(\pi/4) + \cos(2\pi f_c t)\sin(\pi/4)]$$

$$= 2 \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) \cos(2\pi f_c t)[\sin(2\pi f_c t)\cdot \sqrt{2}/2 + \cos(2\pi f_c t)\cdot \sqrt{2}/2]$$

$$= \sqrt{2} \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)[\sin(4\pi f_c t) + \cos^2(2\pi f_c t)]$$

$$= \sqrt{2} \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)[\sin(4\pi f_c t) + \tfrac{1}{2} + \frac{\cos(4\pi f_c t)}{2}]$$

LP-filter

$$y^Q(t) = \frac{\sqrt{2}}{2} \sum_{m=-\infty}^{\infty} x_m^I p(t-mT)$$

$$\boxed{z^Q(t) = \frac{\sqrt{2}}{2} \sum_{m=-\infty}^{\infty} x_m^I p(t) * p(-t) * \delta(t-mT)}$$

d)

$$y(t) = x(t) + \sqrt{2} A \cos(2\pi(f_c + \tfrac{1}{2T})t)$$

$$m^I(t) = \left[\sqrt{2} \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT)\cos(2\pi f_c t) + \sqrt{2} A \cos(2\pi(f_c + \tfrac{1}{2T})t)\right]\sqrt{2}\cdot\cos(2\pi f_c t)$$

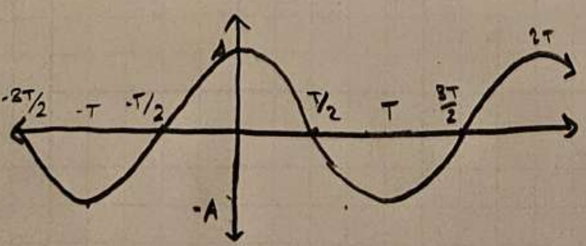
$$= 2 \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) \cos^2(2\pi f_c t) + A[\cos(4\pi f_c t + \tfrac{\pi t}{T}) + \cos(\tfrac{\pi t}{T})]$$

LP-filter

$$y^I(t) = 2 \sum_{m=-\infty}^{\infty} x_m^I p^b(t-mT) + A\cos(\tfrac{\pi}{T} t)$$

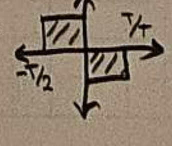
$$\boxed{z^I(t) = 2 \sum_{m=-\infty}^{\infty} x_m^I p(t) * p(-t) * \delta(t-mT) + A p(-t) * \cos(\tfrac{\pi}{T} t)}$$

e)




f) the receiver can just subtract $A p(-t) * \cos(\frac{\pi}{T} t)$ from $z^I(t)$

they could also use a pulse that looks as such:




Then when sampling at $mT$, $A p(-t) * \cos(\frac{\pi}{T} t)$ will be 0.

Problem 3 (p.3) | EE 417 | HW3

g) $T=1$

i) $p(t) = rect(\frac{t}{1.5})$

Not Nyquist: $p(-t) * p(t)$ has width 3, meaning there will be ISI at sampling intervals $mT$

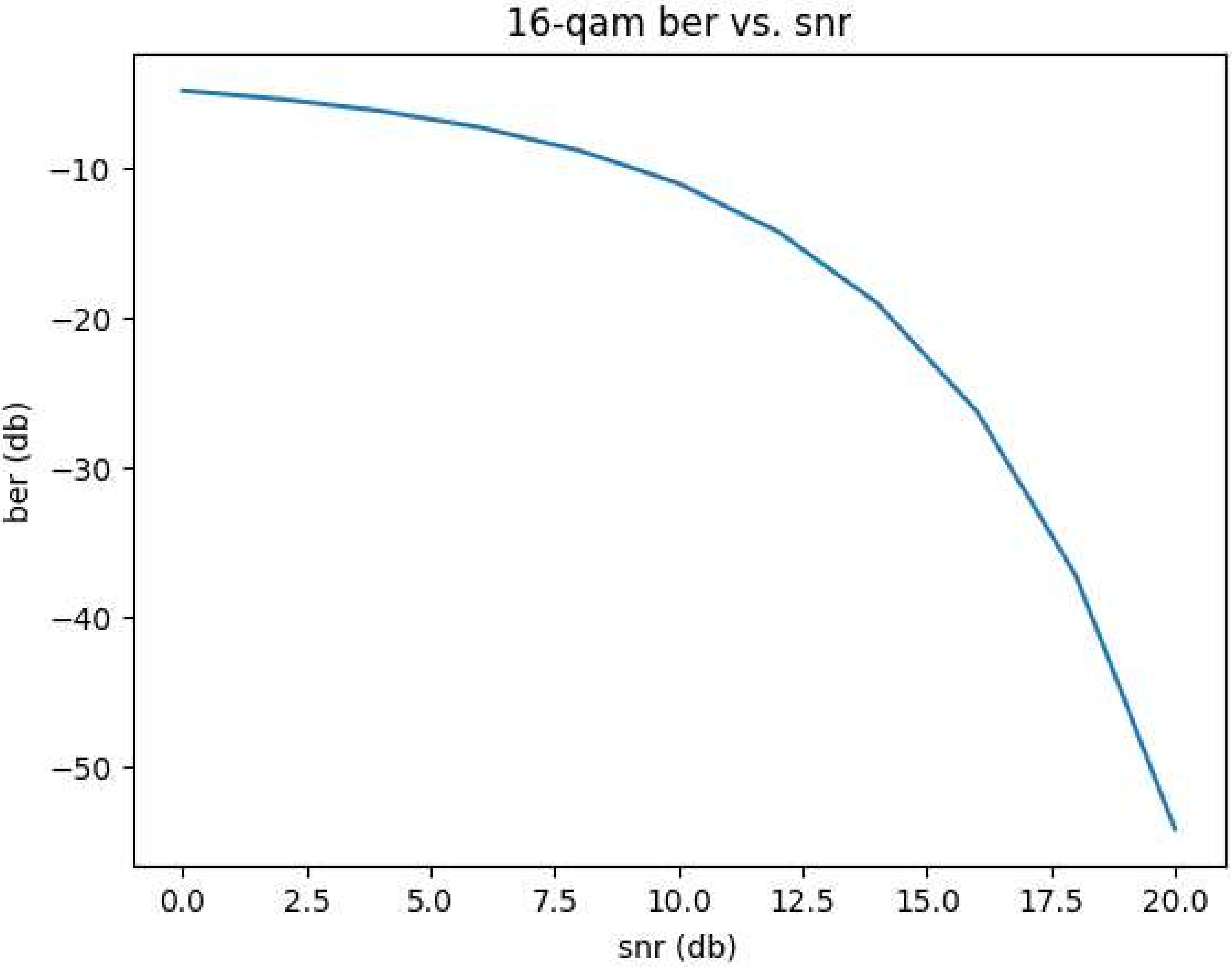
ii) $p(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

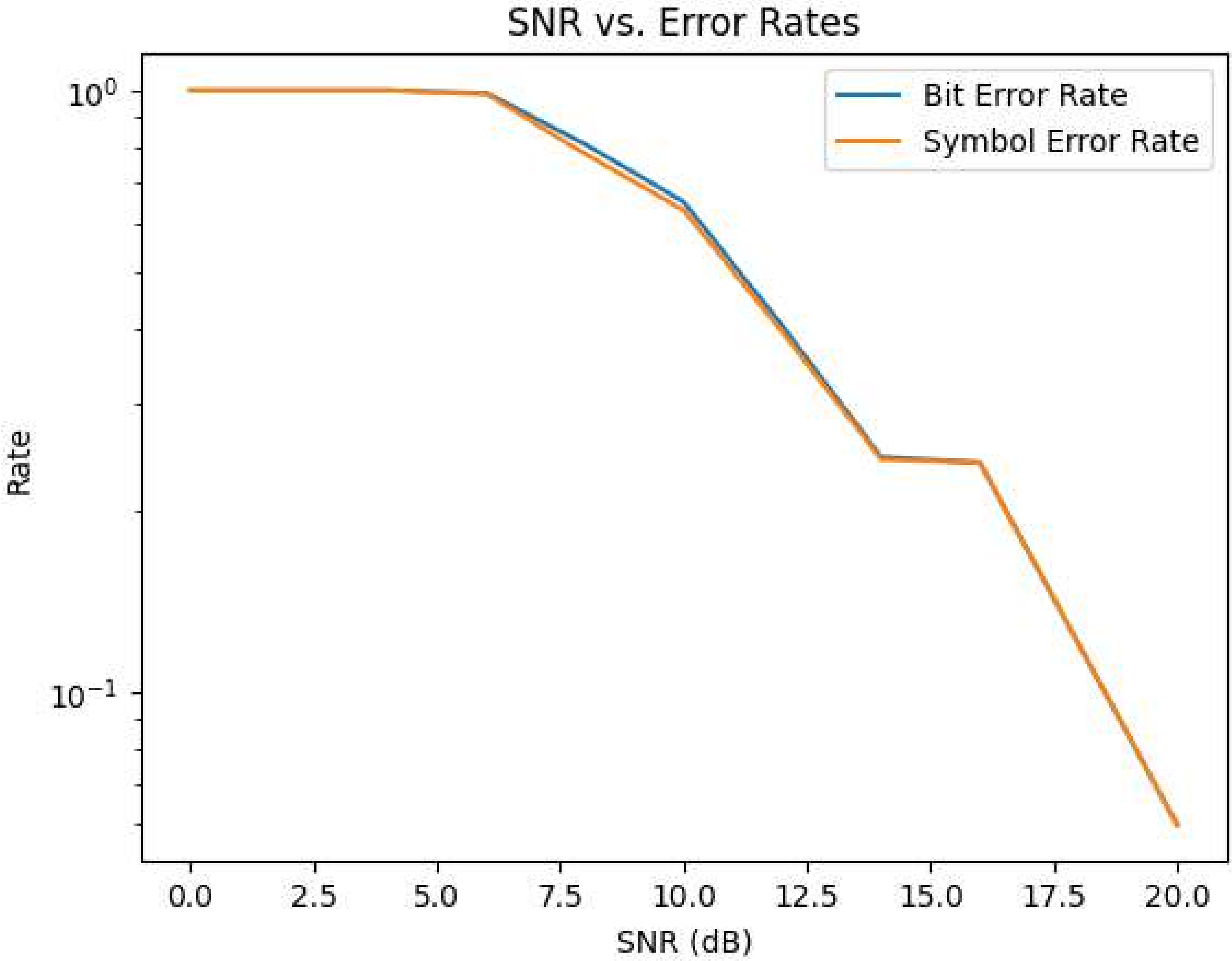
Not Nyquist: $p(-t) * p(t)$ has width $\infty$, meaning there will be ISI at sampling intervals $mT$

iii) $P(f) = \sqrt{|f|}$ $\quad [P(f)]^2 = |f|$

$$P(t) * P(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |f| e^{jft} df = \frac{-1}{2\pi^2 t^2}$$

Not Nyquist: $p(-t) * p(t)$ has infinite width.


16-qam ber vs. snr
ber (db)
snr (db)
−10
−20
−30
−40
−50
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



SNR vs. Error Rates
Bit Error Rate
Symbol Error Rate
$10^0$
$10^{-1}$
Rate
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
SNR (dB)