Problem 1a	EE 417	HW1

$\vec{y}(t) = \vec{h}_1 x_1(t) + \vec{h}_2 x_2(t) + \vec{n}(t)$ $\qquad ||\vec{h}_1|| = ||\vec{h}_2|| = 1 = ||\vec{h}_1^H|| = ||\vec{h}_2^H||$

Matched Filter:

$\langle \vec{y}, \vec{w} \rangle = \vec{w}^H \vec{h}_1 x_1(t) + \vec{w}^H \vec{h}_2 x_2(t) + \vec{w}^H \vec{n}(t)$

x_1 recovery:

pick $\vec{w}$ s.t. $\langle \vec{h}_1, \vec{w} \rangle$ is maximized

so $\vec{w} = \vec{h}_1$ makes $\langle \vec{y}, \vec{h}_1 \rangle = \vec{h}_1^H \vec{h}_1 x_1(t) + \vec{h}_1^H h_2 x_2(t) + \vec{h}_1^H \vec{n}(t)$

$= ||\vec{h}_1|| x_1(t) + \vec{h}_1^H h_2 x_2(t) + ||\vec{h}_1^H|| n(t)$

$= x_1(t) + \vec{h}_1^H h_2 x_2(t) + \vec{n}(t)$

if $\vec{h}_1 \perp \vec{h}_2$, then $\boxed{\langle \vec{y}, \vec{h}_1 \rangle = x_1(t) + 0 + \vec{n}(t)}$

x_2 recovery:

pick $\vec{w}$ s.t. $\langle \vec{h}_2, \vec{w} \rangle$ is maximized.

So $\vec{w} = \vec{h}_2$ makes

$\langle \vec{y}, \vec{h}_2 \rangle = \vec{h}_2^H \vec{h}_1 x_1(t) + \vec{h}_2^H h_2 x_2(t) + \vec{h}_2^H \vec{n}(t)$

$= \vec{h}_2^H \vec{h}_1 x_1(t) + ||\vec{h}_2|| x_2(t) + ||\vec{h}_2^H|| n(t)$

$= \vec{h}_2^H \vec{h}_1 x_1(t) + x_2(t) + n(t)$

if $\vec{h}_1 \perp \vec{h}_2$, then $\boxed{\langle \vec{y}, \vec{h}_2 \rangle = 0 + x_2(t) + n(t)}$

if $\vec{h}_1$ and $\vec{h}_2$ are not orthoginal, some of the jammed signal will remain. if $\vec{h}_1$ is in the same direction as $\vec{h}_2$ the signal will remain unchanged. i.e., still fully jammed.

if $x_2(t)$ increases in amplitude, then more of the jammed signal remains. Unless $\vec{h}_1 \perp \vec{h}_2$, it will still be fully removed.

Problem 1b | EE 417 | HW1

$$\vec{h}_1 = \begin{pmatrix} 0.7 \\ -0.9 \\ 0.3 \end{pmatrix} \qquad \vec{h}_2 = \begin{pmatrix} -0.4 \\ 0.8 \\ 0.2 \end{pmatrix} \qquad \|\vec{h}_2\| = \|\vec{h}_1\| = \|\vec{h}_2^T\| = \|\vec{h}_1^T\| = 1$$

$$\vec{y}(t) = \vec{h}_1 x_1(t) + \vec{h}_2 x_2(t) + \vec{n}(t) \qquad z_1(t) = \vec{\omega}^T \cdot \vec{y}(t)$$

$$\vec{\omega}^T \cdot \vec{y}(t) = \vec{\omega}^T \cdot \vec{h}_1 x_1(t) + \vec{\omega}^T \vec{h}_2 x_2(t) + \vec{\omega}^T \vec{n}(t)$$

$$\vec{\omega}^T \cdot \vec{h}_2 = 0 \qquad \boxed{\vec{\omega} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sqrt{4/5} \\ \sqrt{1/5} \\ 0 \end{pmatrix}}$$

$$(a \quad b \quad c) \cdot \begin{pmatrix} -.4 \\ .8 \\ .2 \end{pmatrix} = 0 = -0.4a + .8b + .2c$$

$$a^2 + b^2 + c^2 = 1$$

$$c = 0$$

$$0 = -.4a + .8b = -1a + 2b = 0$$

$$a^2 + b^2 = 1$$

$$(2b)^2 + b^2 = 1$$

$$4b^2 + b^2 = 1$$

$$5b^2 = 1$$

$$b = \pm\sqrt{\tfrac{1}{5}}$$

$$a = \pm\sqrt{\tfrac{4}{5}}$$

$$\vec{\omega}^T \cdot \vec{h}_2 = 0 = (\sqrt{4/5} \quad \sqrt{1/5} \quad 0) \cdot \begin{pmatrix} -.4 \\ .8 \\ .2 \end{pmatrix}$$

$$-0.4(\sqrt{4/5}) + .8\sqrt{1/5}$$

$$-\sqrt{4/5} + \sqrt{4/5} = 0 \checkmark$$

$$\vec{\omega}^T \cdot \vec{h}_1 = (\sqrt{4/5} \quad \sqrt{1/5} \quad 0) \cdot \begin{pmatrix} 0.7 \\ -0.9 \\ 0.3 \end{pmatrix}$$

$$= 0.7\sqrt{4/5} - 0.9\sqrt{1/5}$$

$$\vec{\omega}^T \cdot \vec{y}(t) = (0.7\sqrt{4/5} - 0.9\sqrt{1/5})\, x_1(t) + 0 \cdot x_2(t) + \vec{\omega}^T \cdot \vec{n}(t)$$

$$\boxed{\vec{\omega}^T \cdot \vec{y}(t) = (0.7\sqrt{4/5} - 0.9\sqrt{1/5})\, x_1(t) + \vec{\omega}^T \cdot \vec{n}(t)}$$

Problem 1c/d | EE 417 | HW1

c) $\vec{h}_1 = \begin{pmatrix} .7 \\ -.9 \end{pmatrix}$ $\vec{h}_2 = \begin{pmatrix} -0.4 \\ 0.8 \end{pmatrix}$

$\vec{y}(t) = \vec{h}_1 x_1(t) + \vec{h}_2 x_2(t) + \vec{n}(t)$

$\vec{y}(t) = \underbrace{(\vec{h}_1\ \vec{h}_2)}_{H}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \vec{n}(t)$

$z(t) = H^{-1}\vec{y}(t) = H^{-1}(\vec{h}_1\ \vec{h}_2)\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + H^{-1}\vec{n}(t)$

$H^{-1}(\vec{h}_1\ \vec{h}_2) = \vec{\vec{I}} \longrightarrow H = (\vec{h}_1\ \vec{h}_2)$ $H^{-1} = \begin{pmatrix} .7 & -0.4 \\ -.9 & 0.8 \end{pmatrix}^{-1} = \frac{1}{.7 \cdot .8 + .4 \cdot -.9}\begin{pmatrix} .8 & .4 \\ .9 & .7 \end{pmatrix}$

$\boxed{H^{-1} = \begin{pmatrix} 4 & 2 \\ 4.5 & 3.5 \end{pmatrix}}$

$H^{-1}\begin{pmatrix} .7 & -.4 \\ -.9 & .8 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 4.5 & 3.5 \end{pmatrix}\begin{pmatrix} .7 & -.4 \\ -.9 & .8 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

There is only one solution to H^{-1} because there are now three unknowns $(x_1(t), x_2(t), \vec{n}(t))$. So if we keep $\vec{n}(t)$, solving for $x_1(t)$ or $x_2(t)$ is only possible with one solution given two equations (given the two antennae).

Noise enhancement is the increase in the amplitude of noise from ZFE. Noise enhancement is greater in part c than part b. Assuming $\vec{n}(t)$ is the same, part b is lower because the ~~vector~~ $\vec{\omega}^T$ vector has negatives, while H^{-1} does not, so $\vec{\omega}^T \cdot \vec{n}(t)$ is lower in amplitude than $H^{-1}\vec{n}(t)$.

d)
- Wikipedia
- Class notes (matrix review)
- Khan academy

Problem 2 | EE 417 | HW1

$N = 25 \quad T = 400$

a)

$$\boxed{P(\underset{\sim}{K} = k) = \left(\frac{24}{25}\right)^{400-k} \cdot \left(\frac{1}{25}\right)^{k} \cdot \left(\frac{400!}{k!(400-k)!}\right)}$$

b) $\boxed{E[\underset{\sim}{K}] = \frac{1}{25} \cdot 400 = 16}$ also $E[\underset{\sim}{K}] = \sum_{k=0}^{400} k\, P(\underset{\sim}{K} = k) = \sum_{k=1}^{400} k\, P(\underset{\sim}{K} = k)$

c) $E[\underset{\sim}{K}] = E\left[\sum_{k=1}^{400} \underset{\sim}{1}_F(k)\right] = \boxed{\sum_{k=1}^{400} E[1_F(k)] = 400 \cdot \frac{1}{25} = 16}$

d) $E[\underset{\sim}{K}]$ would go down by a factor of p^2.

I.e., the new $E[\underset{\sim}{K}] = \text{old}\, E[\underset{\sim}{K}] \cdot p^2$. Because likelyhood of collision is now multiplied by p^2.

e) An FDMA system does not have collisions. Frequency bands are uniquely allocated. To make the FHSS system more reliable, you could increase the number of frequency bands, reduce the number of time slots, or decrease the number of users.

f)
- Wikipedia
- 416 notes

Problem 3 (p.1)	EE 417	HW1

a)
Because squaring in the time domain causes convolution with self in the frequency domain, the frequency band of $y(t)$ is double $x(t)$.

$x(t)$ should be such that doubling the frequency band does not create frequencies in the 535-1605 kHz range.
I.e. 0-267.5 kHz

b)
Adding a constant 1 does not impact the frequencies of the signal; however, the square root will decrease the frequency bands to half. Square root will deconvolve the signal, so frequency bands are halved. $x(t)$ should be such that the deconvolution with itself is not in the AM frequency range.
So $x(t)$ could have frequencies from 0-802.5 kHz

c) $p(t)$ should be designed in a way that its frequencies only span from 0-535 kHz. Convolution in time is multiplication, so it could be as simple as a pulse with amplitude 1 from 0-535 kHz, and 0 elsewhere.
$x(t) \rightarrow y(t)$ is not linear. $p(t)$ is non-linear and $x^2(t)$ is non-linear.

d) 1) Car accidents per day
2) Electricity usage per year
3) number of steps walked per day.

Band width = 10 kHz = 10^4 Hz Avg. Sample Rate $\frac{10^7}{10^2} = 10^5$

Even though the average sampling rate is sufficiently high, it is possible to sample some sections more, and others not enough. So statistically, there is a chance aliasing could occur at some parts of the signal that aren't sampled close enough.

Problem 3 (p.2)	EE 417	HW 1

e) i) $X(f) = \frac{e^{j2\pi f}}{f^2}$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(f)\, e^{jft}\, df$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{jf(2\pi + t)} \cdot f^{-2}\, df$

complex: $e^{jf(2\pi+t)}$ will become $\cos(f(2\pi+t)) + i\sin(f(2\pi+t))$ which has an imaginary component, and f^{-2} will ensure it converges.

ii) $X(f) = \frac{e^{j2\pi(f+1/6)}}{f^2} = \frac{e^{j\pi/3} \cdot e^{j2\pi f}}{f^2} = \frac{(0.5 + \frac{\sqrt{3}}{2}i)}{f^2} e^{j2\pi f}$

complex: $e^{j\pi/3}$ multiplies the transform by a real and imaginary component.

iii) $X(f) = \bar{X}(-f)$

real: if $X(f) = \bar{X}(-f)$, that means $X(f)$ is real and even, meaning $x(t)$ is also real and even.

iv) $X(f) = \begin{cases} G(f), & f \geq 0 \\ 0, & f < 0 \end{cases}$

if $G(f)$ is real, $x(t)$ will also be real. if $G(f)$ is complex (i.e., phase) $x(t)$ will be complex.

FM radios only show positive, because negative frequencies are the same, just shifted by π.

f) · Signals and Systems – Alan Oppenheim

· Wikipedia