Stage 1

SAMPLE QUESTION PAPER 1

Mathematics (Basic)

Time : **3 hrs** Max. Marks : **80**

Instructions

1. This question paper has 5 Sections A, B, C, D and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) Type Questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) Type Questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) Type Questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All questions are compulsory. However, an internal choice in 2 questions of 2 marks, 2 questions of 3 marks and 2 questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section-A

Section A consists of 20 questions of 1 mark each

1. The probability of an impossible event is

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) None of these

Sol. (a) The probability of an impossible event is 0.

2. A die is thrown once. The probability of getting a number greater than 4 is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) 1

Sol. (c) On a die, there are six numbers 1, 2, 3, 4, 5 and 6.

$\therefore$ Total number of possible outcomes = 6

Number on dice which are greater than 4 = 5, 6

$\therefore$ Favourable number of elementary events = 2

$\therefore$ Required probability $= \frac{2}{6} = \frac{1}{3}$

3. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is

(a) $2\sin^2\theta$ (b) 1

(c) $\sin^2\theta - \cos^2\theta$ (d) 0

Sol. (b) We have, $\sin^2\theta + \dfrac{1}{1+\tan^2\theta}$

$$= \sin^2\theta + \frac{1}{\sec^2\theta} \quad [\because \sec^2 A = 1 + \tan^2 A]$$

$$= \sin^2\theta + \cos^2\theta \quad \left[\because \sec A = \frac{1}{\cos A}\right]$$

$$= 1 \quad [\because \sin^2 A + \cos^2 A = 1]$$

4. The area of a sector of a circle of radius 5 cm formed by an arc of length 3.5 cm, is

(a) $16.5\ \text{cm}^2$ (b) $17.5\ \text{cm}^2$
(c) $19.62\ \text{cm}^2$ (d) $8.75\ \text{cm}^2$

Sol. (d) We know that area of sector A of radius r and length of arc l is given by

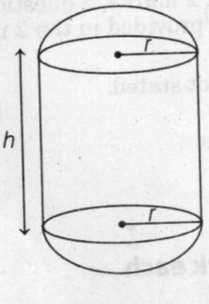
$$A = \frac{1}{2}lr$$

$$\therefore \quad A = \frac{1}{2} \times 3.5 \times 5 = 8.75\text{cm}^2$$

5. If a cylinder is covered by two hemispherical lid of equal shape, then the total curved surface area of the new object will be (where r is the radius and h is the height of cylinder)

(a) $4\pi rh + 2\pi r^2$ (b) $4\pi rh - 2\pi r^2$
(c) $2\pi rh + 4\pi r^2$ (d) $2\pi rh + 4\pi r$

Sol. (c)



Total curved surface area

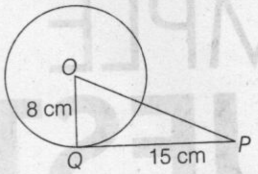
$= $ Curved surface area of cylinder $+ 2 \times$ Curved surface area of hemispheres

$$= 2\pi rh + 2 \times (2\pi r^2)$$
$$= 2\pi rh + 4\pi r^2$$

6. If radius of circle is 8 cm and tangent is drawn from an external point to the circle is 15 cm, the distance from centre of circle to the external point is

(a) $\sqrt{241}$ cm (b) 17 cm
(c) 10 cm (d) None of these

Sol. (b) Given, $OQ = 8$ cm and $PQ = 15$ cm.



In right angled ΔOPQ, using Pythagoras theorem

$$OP = \sqrt{OQ^2 + QP^2} = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64 + 225} = \sqrt{289} = 17 \text{ cm}$$

7. Suppose mean of 10 observations is 12.5 and each observation is multiplied by 5, then what is the new mean?

(a) 50 (b) 62.5 (c) 60 (d) 48.2

Sol. (b) Let 10 observations be $x_1, x_2, x_3, \ldots, x_{10}$

Given, $\dfrac{x_1 + x_2 + x_3 + \ldots + x_{10}}{10} = 12.5$ ...(i)

Now, if each observation multiplied by 5, then new

$$\text{mean} = \frac{5x_1 + 5x_2 + \ldots + 5x_{10}}{10}$$

$$= \frac{5(x_1 + x_2 + \ldots + x_{10})}{10} \quad \text{[using Eq. (i)]}$$

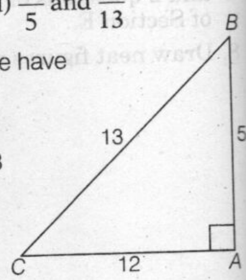
$$= 5 \times 12.5 = 62.5$$

8. In ΔABC, right angled at A and $AB = 5$, $AC = 12$ and $BC = 13$, then find the value of $\sin B$ and $\tan B$.

(a) $\dfrac{12}{5}$ and $\dfrac{5}{13}$ (b) $\dfrac{5}{13}$ and $\dfrac{5}{12}$
(c) $\dfrac{12}{13}$ and $\dfrac{12}{5}$ (d) $\dfrac{13}{5}$ and $\dfrac{12}{13}$

Sol. (c) With reference to $\angle B$, we have

Base $= AB = 5$,
perpendicular $= AC = 12$
and hypotenuse $= BC = 13$



$$\therefore \quad \sin B = \frac{AC}{BC} = \frac{12}{13}$$

$$\text{and } \tan B = \frac{AC}{AB} = \frac{12}{5}$$

9. Find the value of $3\sin 30° - 4\sin^3 60°$.

(a) $3 - \sqrt{3}$ (b) $\dfrac{3(1-\sqrt{3})}{2}$
(c) $\dfrac{3\sqrt{3}-1}{2}$ (d) $-\dfrac{3}{2}$

Sol. (b) $3\sin 30° - 4\sin^3 60°$

$$= 3 \times \frac{1}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3}{2} - 4 \times \frac{3\sqrt{3}}{8}$$

$$= \frac{3}{2} - \frac{3\sqrt{3}}{2} = \frac{3 - 3\sqrt{3}}{2} = \frac{3(1-\sqrt{3})}{2}$$

10. If median $=143$ and mean $=143.06$, then the mode is

(a) 143.18 (b) 142.94
(c) 142.88 (d) 143

Sol. (c) We know that

$$\text{Mode} = 3\,(\text{Median}) - 2\,(\text{Mean})$$
$$= 3(143) - 2(143.06)$$
$$= 429 - 286.12 = 142.88$$

11. If the lines given by $4x + ky = 1$ and $6x - 10y = 14$ has unique solution, then the value of k is

(a) $\frac{20}{3}$
(b) $-\frac{5}{7}$
(c) -15
(d) all real values except $-\frac{20}{3}$

Sol. (d) The given equations can be re-written as

$4x + ky - 1 = 0$ and $6x - 10y - 14 = 0$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 4,\ b_1 = k,\ c_1 = -1$$

and $a_2 = 6,\ b_2 = -10,\ c_2 = -14$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$
$$\Rightarrow \quad \frac{4}{6} \neq \frac{k}{-10}$$
$$\Rightarrow \quad k \neq -\frac{20}{3}$$

Thus, given lines have a unique solution for all real values of k, except $-\frac{20}{3}$.

12. Which term of the AP : 21, 18, 15, ... is -81?

(a) 34 (b) 36 (c) 35 (d) 33

Sol. (c) Given, AP is 21, 18, 15, ...

Here, $a = 21$ and $d = 18 - 21 = -3$

Let nth term of given AP be -81.

Then, $a_n = -81$

$\Rightarrow a + (n-1)d = -81 \quad [\because a_n = a + (n-1)d]$ ...(i)

On putting the values of a and d in Eq. (i), we get

$$21 + (n-1) \times (-3) = -81$$
$$\Rightarrow \quad 21 - 3n + 3 = -81$$
$$\Rightarrow \quad 24 - 3n = -81$$
$$\Rightarrow \quad -3n = -81 - 24$$
$$\therefore \quad n = \frac{-105}{-3} = 35$$

Hence, 35th term of given AP is -81.

13. The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

(a) $\sqrt{(x_1 + x_2) + (y_1 + y_2)}$
(b) $\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$
(c) $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
(d) $\sqrt{(x_1 - x_2) + (y_1 - y_2)}$

Sol. (c) $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

14. If $a = 2$ and 20th term $= 62$, then the sum of first 20 terms of an AP is

(a) 1200 (b) 640 (c) 600 (d) 1280

Sol. (b) Given, $a = 2$, $a_{20} = 62$ and $n = 20$

Now, sum of first 20 terms

$$S_{20} = \frac{20}{2}(2 + 62) \quad \left[\because S_n = \frac{n}{2}(a + a_n)\right]$$
$$= 10 \times 64 = 640$$

15. The product of the HCF and LCM of two prime numbers a and b is

(a) $\frac{a}{b}$ (b) $a - b$ (c) $a + b$ (d) $a \times b$

Sol. (d) HCF $(a, b) = 1$

LCM $(a, b) = ab$

$\therefore$ HCF $(a, b) \times$ LCM $(a, b) = 1 \times ab = ab$

16. If the product of the zeroes of the polynomial $mx^2 - 6x - 6$ is -3, then the value of m is

(a) 6 (b) 2 (c) -2 (d) -3

Sol. (b) Let α and β be the zeroes of $(mx^2 - 6x - 6)$.

Here, $a = m$, $b = -6$ and $c = -6$

Given, $\alpha\beta = -3$

$$\therefore \quad \frac{c}{a} = -3 \Rightarrow \frac{-6}{m} = -3 \Rightarrow m = 2$$

17. The solution of $2x^2 - 5x - 3 = 0$ is

(a) $-\frac{1}{2}, 3$ (b) $1, -3$
(c) $-1, 3$ (d) $-1, \frac{1}{2}$

Sol. (a) Given, $2x^2 - 5x - 3 = 0$

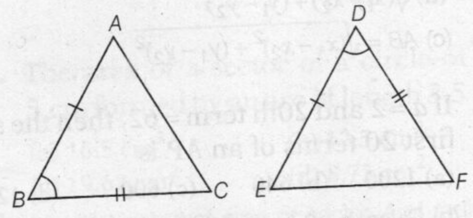
Splitting the middle term, we get

$$2x^2 - 6x + x - 3 = 0$$
$$\Rightarrow \quad 2x(x-3) + 1(x-3) = 0$$
$$\Rightarrow \quad (x-3)(2x+1) = 0$$
$$\Rightarrow \quad x = -\frac{1}{2}, 3$$

18. If in ΔABC and ΔDEF, $\frac{AB}{DE}=\frac{BC}{FD}$, then they will be similar, when

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

Sol. (c) Given, in ΔABC and ΔDEF, $\frac{AB}{DE}=\frac{BC}{FD}$



ΔABC and ΔEDF will be similar, if

$\angle B = \angle D$ [by SAS similarity criterion]

Directions *In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.*

19. Assertion (A) The value of $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$ is 1.

Reason (R) $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but Reason (R) is false
(d) Assertion (A) is false but Reason (R) is true

Sol. (b) **Assertion** $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$=\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{4}+\frac{1}{4}=\frac{4}{4}=1$$

So, Assertion is true.

Reason We know, $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$

So, Reason is true.

But Reason is not the correct explanation of Assertion.

20. Assertion (A) The system of equations $2x+3y+5=0$ and $4x+ky+7=0$ is inconsistent when $k=6$.

Reason (R) The system of equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is inconsistent when $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but Reason (R) is false
(d) Assertion (A) is false but Reason (R) is true

Sol. (a) **Assertion (A)**

Here, $a_1=2, b_1=3, c_1=5$

and $a_2=4, b_2=6, c_2=7$ $[\because k=6]$

So, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ $\left[\because \frac{2}{4}=\frac{3}{6}\neq\frac{5}{7}\right]$

So, the given system of equations has no solution (i.e. inconsistent).

So, the Assertion is true.

Reason (R) $a_1x+b_1y+c_1=0$

and $a_2x+b_2y+c_2=0$

We know, for the system of equations to be inconsistent,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

So, both Assertion and Reason are true and Reason is a correct explanation of Assertion.

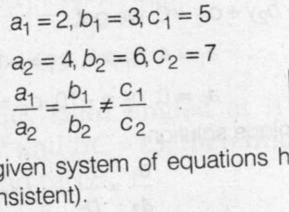
Section-B

Section B consists of 5 questions of 2 marks each

21. Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

Sol. **Given** AB and CD are two parallel tangents at the point P and Q of a circle with centre O.

To prove POQ is a diameter of the circle.



Construction Join OP and OQ and draw $OT \parallel AB$.

Proof $\angle APO + \angle TOP = 180°$ $[\because OT \parallel AB]$

$\Rightarrow$ $90° + \angle TOP = 180°$ $[\because OP \perp AB]$

$\Rightarrow \angle TOP = 180° - 90° = 90°$

Similarly, $\angle TOQ = 90°$

$\therefore$ $\angle TOP + \angle TOQ = 90° + 90° = 180°$

Since, POQ is a straight line.

Hence, POQ is a diameter of the circle with centre O. **Hence proved.** (1)

22. What is the probability that a number selected from the numbers 1, 2, 3, ..., 25 is a prime number, when each of the given numbers is equally likely to be selected?

Or

A letter is chosen at random from the letters of word 'ASSASSINATION'. Find the probability that the letter chosen is a (i) vowel (ii) consonant.

Sol. Out of 25 numbers, 1, 2, 3, ..., 25 one number can be chosen in 25 ways.

$\therefore$ Total number of elementary events = 25 (1)

The number selected will be a prime number, if it is chosen from the numbers

2, 3, 5, 7, 11, 13, 17, 19, 23.

$\therefore$ Favourable number of elementary events = 9

Hence, required probability $= \frac{9}{25}$ (1)

Or

There are 13 letters in the word 'ASSASSINATION' out of which one letter can be chosen in 13 ways.

$\therefore$ Total number of elementary events = 13

(i) There are 6 vowels in the word 'ASSASSINATION'. So, there are 6 ways of selecting a vowel.

$\therefore$ Probability of selecting a vowel $= \frac{6}{13}$ (1)

(ii) We have, probability of selecting a consonant

= 1 − Probability of selecting a vowel

$= 1 - \frac{6}{13} = \frac{7}{13}$ (1)

23. Find the sum of first 15 even natural numbers.

Sol. The sequence goes like this

2, 4, 6, 8, ...

Here, $4-2=6-4=8-6=2$

So, it is an AP with first term, $a=2$,

common difference, $d = 4-2=2$

and total number of terms, $n=15$ (1)

$\therefore$ Sum of first 15 even natural numbers

$$S_{15} = \frac{n}{2}[2a+(n-1)d] = \frac{15}{2}[2\times 2+(15-1)2]$$

$$\left[\because S_n = \frac{n}{2}\{2a+(n-1)d\}\right]$$

$$= \frac{15}{2}[4+28] = \frac{15}{2}\times 32 = 240$$ (1)

24. If one root of the quadratic equation $2x^2+kx-6=0$ is 2, find the value of k. Also, find the other root.

Sol. Since, $x=2$ is a root of the equation

$$2x^2+kx-6=0$$

$\therefore$ $2\times 2^2+2k-6=0$

$\Rightarrow$ $8+2k-6=0$

$\Rightarrow$ $2k+2=0 \Rightarrow k=-1$ (1)

On putting $k=-1$ in the equation $2x^2+kx-6=0$, we get

$$2x^2-x-6=0 \Rightarrow 2x^2-4x+3x-6=0$$

$\Rightarrow$ $2x(x-2)+3(x-2)=0 \Rightarrow (x-2)(2x+3)=0$

$\Rightarrow$ $x-2=0$ or $2x+3=0$

$\Rightarrow$ $x=2$ or $-\frac{3}{2}$

Hence, the other root is $-\frac{3}{2}$. (1)

25. Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$.

Or Evaluate $\cos^2 30° + \sin^2 45° - \frac{1}{3}\tan^2 60°$.

Sol. LHS $= \cot A + \tan A = \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos^2 A+\sin^2 A}{\sin A\cdot\cos A} = \frac{1}{\cos A\cdot\sin A}$$

$[\because \sin^2\theta+\cos^2\theta = 1]$ (1)

$$= \frac{1}{\sin A}\cdot\frac{1}{\cos A} = \operatorname{cosec} A \sec A$$

$$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta}\right]$$

= RHS **Hence proved.** (1)

Or

We have, $\cos^2 30° + \sin^2 45° - \frac{1}{3}\tan^2 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{3}(\sqrt{3})^2$$ (1)

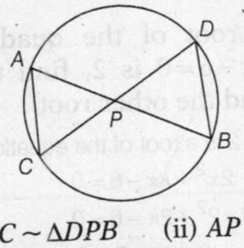
$$\left[\because \cos 30° = \frac{\sqrt{3}}{2}, \sin 45° = \frac{1}{\sqrt{2}} \text{ and } \tan 60° = \sqrt{3}\right]$$

$$= \frac{3}{4}+\frac{1}{2}-\frac{3}{3} = \frac{3+2}{4}-1 = \frac{5}{4}-1 = \frac{5-4}{4} = \frac{1}{4}$$ (1)

Section-C

Section C consists of 6 questions of 3 marks each

26. In the given figure, two chords AB and CD intersect each other at the point P. Prove that



(i) $\Delta APC \sim \Delta DPB$ (ii) $AP \cdot PB = CP \cdot DP$

Sol. **Given** In figure, two chords AB and CD intersect each other at point P.

To prove (i) $\Delta APC \sim \Delta DPB$

(ii) $AP \cdot PB = CP \cdot DP$

Proof

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [vertically opposite angles]

and $\angle CAP = \angle BDP$

[angles in the same segment]

$\therefore$ $\Delta APC \sim \Delta DPB$ [by AA similarity criterion] (1)

(ii) We have, $\Delta APC \sim \Delta DPB$ [proved in part (i)]

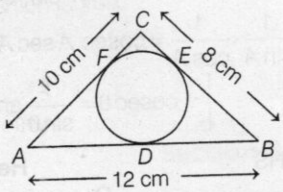
$\therefore$ $\dfrac{AP}{DP} = \dfrac{CP}{BP}$ (1)

[$\because$ if two triangles are similar, then the ratio of their corresponding sides is equal]

$\therefore$ $AP \cdot BP = CP \cdot DP$

or $AP \cdot PB = CP \cdot DP$ **Hence proved.** (1)

27. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF.



Sol. Given, a circle is inscribed in the triangle, whose sides are $BC = 8$ cm, $AC = 10$ cm and $AB = 12$ cm.

Let $AD = AF = x$, $BD = BE = y$

and $CE = CF = z$

[$\because$ the length of two tangents drawn from an external point to a circle are equal]

We have, $AB = 12$

$\Rightarrow$ $AD + DB = 12$

$\Rightarrow$ $x + y = 12$ ...(i)

$AC = 10$

$\Rightarrow$ $AF + FC = 10$

$\Rightarrow$ $x + z = 10$ ...(ii)

and $BC = 8$

$\Rightarrow$ $CE + EB = 8$

$\Rightarrow$ $z + y = 8$ ...(iii) (1)

On adding Eqs. (i), (ii) and (iii), we get

$2(x + y + z) = 12 + 10 + 8$

$\Rightarrow$ $x + y + z = \dfrac{30}{2} = 15$ ...(iv)

On putting $x + y = 12$ from Eq. (i) in Eq. (iv), we get

$12 + z = 15$

$\Rightarrow$ $z = 3$

On putting $z + y = 8$ from Eq. (iii) in Eq. (iv), we get

$x + 8 = 15$

$\Rightarrow$ $x = 7$ (1)

On putting $x + z = 10$ from Eq. (ii) in Eq. (iv), we get

$10 + y = 15$

$\Rightarrow$ $y = 5$

Hence, $AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm (1)

28. Prove that $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A} = \dfrac{2}{\sin^2 A - \cos^2 A}$

Or

Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

Sol. LHS $= \dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A}$

$= \dfrac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$ (1)

$= \dfrac{\left[\begin{array}{r}\sin^2 A + 2\sin A\cos A + \cos^2 A + \sin^2 A \\ - 2\sin A\cos A + \cos^2 A\end{array}\right]}{\sin^2 A - \cos^2 A}$

[$\because (a \pm b)^2 = a^2 + b^2 \pm 2ab$] (1)

$= \dfrac{2\sin^2 A + 2\cos^2 A}{\sin^2 A - \cos^2 A} = \dfrac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$

$= \dfrac{2}{\sin^2 A - \cos^2 A}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

= RHS **Hence proved.** (1)

Or

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

[dividing numerator and denominator by $\cos\theta$]

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{[(\tan\theta + \sec\theta) - 1][\tan\theta - \sec\theta]}{[(\tan\theta - \sec\theta) + 1][\tan\theta - \sec\theta]}$$

[multiplying and dividing by $(\tan\theta - \sec\theta)$] (1)

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

[$\because (a-b)(a+b) = a^2 - b^2$]

$$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

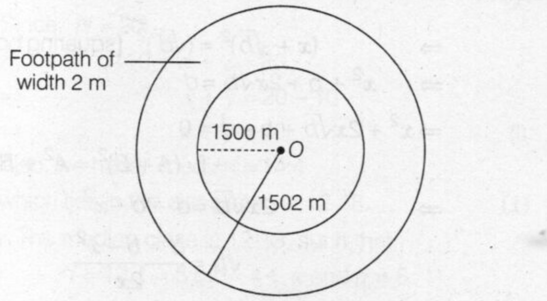
[$\because \tan^2 A - \sec^2 A = -1$] (1)

$$= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{RHS}$$ **Hence proved.** (1)

29. Circular footpath of width 2 m is constructed at the rate of ₹ 20 per m^2 around a circular park of radius 1500 m.



Contractor Ravi was assigned for constructing this footpath. Then, he proposed a estimate of ₹ 377051.2. Is his estimation right? Explain.

Sol. Yes, radius of the park, i.e. inner radius of the park $r = 1500$ m

Width of the footpath around the park = 2 m

Let R be the outer radius of the park including the footpath.

Then, $R = 1500 + 2 = 1502$ m (1)

Now, area of footpath $= \pi R^2 - \pi r^2$

$$= \pi(R^2 - r^2) = 3.14[(1502)^2 - (1500)^2] \quad (1)$$

$$= 3.14[(1502 + 1500)(1502 - 1500)]$$

$$= 3.14 \times 6004 = 18852.56 \text{ m}^2$$

$\therefore$ Total cost of construction of the footpath at the rate of ₹ 20 per m$^2 = 20 \times 18852.56$

$= ₹\, 377051.2$ (1)

30. Find the roots of the following equation

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7.$$

Or

Solve for x, $2\left(\dfrac{2x+3}{x-3}\right) - 25\left(\dfrac{x-3}{2x+3}\right) = 5.$

Sol. Given equation is $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30},\ x \neq -4, 7$

$$\Rightarrow \quad \frac{(x-7) - (x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{x - 7 - x - 4}{x^2 - 7x + 4x - 28} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{-1}{x^2 - 3x - 28} = \frac{1}{30}$$

$$\Rightarrow \quad -30 = x^2 - 3x - 28$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0 \quad (1)$$

On comparing with the standard quadratic equation $ax^2 + bx + c = 0$, we get

$a = 1,\ b = -3$ and $c = 2$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2 \times 1} \quad (1)$$

$$= \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$$

$$\Rightarrow \quad x = \frac{3+1}{2} \text{ or } x = \frac{3-1}{2} \Rightarrow x = \frac{4}{2} \text{ or } x = \frac{2}{2}$$

$\therefore \quad x = 2$ or $x = 1$

Hence, the roots of the given equation are 2 and 1. (1)

Or

Let $\dfrac{2x+3}{x-3} = y$ ...(i)

Then, $\dfrac{x-3}{2x+3} = \dfrac{1}{y}$ (1/2)

Therefore, the given equation reduces to

$$2y - 25\frac{1}{y} = 5$$

$$\Rightarrow \quad 2y^2 - 25 = 5y$$

$$\Rightarrow \quad 2y^2 - 5y - 25 = 0$$

$\Rightarrow 2y^2 - 10y + 5y - 25 = 0$ [by factorisation method]

$\Rightarrow 2y(y-5) + 5(y-5) = 0$

$\Rightarrow (y-5)(2y+5) = 0$

$\Rightarrow y = 5$ or $y = \frac{-5}{2}$ (1)

Now, putting $y = 5$ in Eq. (i), we get

$$\frac{2x+3}{x-3} = \frac{5}{1}$$

$\Rightarrow 5x - 15 = 2x + 3$

$\Rightarrow 3x = 18$

$\Rightarrow x = 6$ (1/2)

Again, putting $y = -\frac{5}{2}$ in Eq. (i), we get

$$\frac{2x+3}{x-3} = -\frac{5}{2}$$

$\Rightarrow -5x + 15 = 4x + 6$

$\therefore 9x = 9$

$\Rightarrow x = 1$

Hence, the values of x are 1 and 6. (1)

31. The Resident Welfare Association of a colony decided to build two straight paths in their neighbourhood park such that they do not cross each other, to plant trees along the boundary lines of each path. One of the members of association, Sarika suggested that the paths should be constructed represented by the two linear equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the two paths will cross each other or not.

Sol. Given, linear equations are

$x - 3y = 2$

$\Rightarrow x - 3y - 2 = 0$ ...(i)

and $-2x + 6y = 5$

$\Rightarrow -2x + 6y - 5 = 0$ ...(ii)

Here, $a_1 = 1, b_1 = -3, c_1 = -2$

and $a_2 = -2, b_2 = 6, c_2 = -5$ (1)

Now, $\frac{a_1}{a_2} = \frac{1}{-2} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (1)

So, the paths represented by the equations are parallel, i.e. not cross (intersect) each other. (1)

Section-D

Section D consists of 4 questions of 5 marks each

32. Prove that, if a, b, c and d are positive rationals such that, $a + \sqrt{b} = c + \sqrt{d}$, then either $a = c$ and $b = d$ or b and d are squares of rationals.

Or

In a seminar, the number of participants, in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the number of rooms required, if in each room the same number of participants are to be seated and all of them bring the same subject.

Sol. Given, $a + \sqrt{b} = c + \sqrt{d}$

If $a = c$, then $a + \sqrt{b} = a + \sqrt{d}$

$\Rightarrow \sqrt{b} = \sqrt{d}$

$\Rightarrow b = d$ [squaring both sides]

If $a \neq c$, then there exists a positive rational number x such that, $a = c + x$.

Now, $a + \sqrt{b} = c + \sqrt{d}$

$\Rightarrow c + x + \sqrt{b} = c + \sqrt{d}$ [$\because a = c + x$]

$\Rightarrow x + \sqrt{b} = \sqrt{d}$ ...(i) (1)

$\Rightarrow (x + \sqrt{b})^2 = (\sqrt{d})^2$ [squaring both sides]

$\Rightarrow x^2 + b + 2x\sqrt{b} = d$

$\Rightarrow x^2 + 2x\sqrt{b} + b - d = 0$

[$\because (A+B)^2 = A^2 + B^2 + 2AB$]

$\Rightarrow 2x\sqrt{b} = d - b - x^2$

$\therefore \sqrt{b} = \frac{d - b - x^2}{2x}$

Since, d, x and b are rational numbers and $x > 0$.

So, $\frac{d - b - x^2}{2x}$ is a rational.

Then, $\sqrt{b}$ is a rational number. (2)

Hence, b is the square of a rational number.

From Eq. (i), we get

$\sqrt{d} = x + \sqrt{b}$

Also, $\sqrt{d}$ is a rational.

So, d is the square of a rational number.

Hence, either $a = c$ and $b = d$ or b and d are the squares of rationals. **Hence proved.** (2)

Or

The number of participants in each room must be the HCF of 60, 84 and 108. (1)

Now, prime factors of numbers 60, 84 and 108 are

$$60 = 2^2 \times 3 \times 5,$$
$$84 = 2^2 \times 3 \times 7$$
and $$108 = 2^2 \times 3^3$$

HCF of (60, 84, 108) $= 2^2 \times 3 = 12$ (2)

Therefore, in each room maximum 12 participants can be seated.

$\therefore$ Total number of participants $= 60 + 84 + 108 = 252$

$\therefore$ Number of rooms required $= \dfrac{252}{12} = 21$ (2)

33. **The median of the distribution given below is 14.4. Find the values of x and y, if the total frequency is 20.**

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1

Sol. Table for cumulative frequency is given below

Class interval	Frequency	Cumulative frequency
0-6	4	$4 + 0 = 4$
6-12	x	$4 + x = (4 + x)\ (cf)$
12-18	$5\ (f)$	$5 + (4 + x) = 9 + x$
18-24	y	$y + (9 + x) = 9 + x + y$
24-30	1	$1 + (9 + x + y) = 10 + x + y$

(1)

Since, $N = 20$

$\therefore \quad 10 + x + y = 20$

$\Rightarrow \quad x + y = 20 - 10$

$\Rightarrow \quad x + y = 10$ ...(i)

Also, we have, median = 14.4

which lies in the class interval 12-18. (1)

$\therefore$ The median class is 12-18, such that

$l = 12, f = 5, cf = 4 + x$ and $h = 6$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$ (1)

$$\Rightarrow \quad 14.4 = 12 + \left[\frac{10 - (4 + x)}{5}\right] \times 6$$

$$\Rightarrow \quad 14.4 - 12 = \frac{6 - x}{5} \times 6$$

$$\Rightarrow \quad 2.4 = \frac{36 - 6x}{5}$$

$$\Rightarrow \quad 12 = 36 - 6x$$

$$\Rightarrow \quad 6x = 24 \Rightarrow x = 4$$ (1)

Now, put the value of x in Eq. (i), we get

$$4 + y = 10 \Rightarrow y = 10 - 4 = 6$$

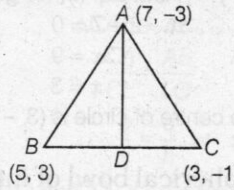
Thus, $x = 4$ and $y = 6$ (1)

34. **Find the length of the median drawn through A on BC of a ΔABC, whose vertices are $A(7, -3)$, $B(5, 3)$ and $C(3, -1)$ and also find the distance of the point $A(7, -3)$ from the origin.**

Or

Find the centre of a circle passing through the points $(6, -6)$, $(3, -7)$ and $(3, 3)$.

Sol. The median from a vertex of a triangle bisects the opposite side, to that vertex. So, let AD be the median through A, then D be the mid-point of the side BC.



Now, coordinates of $D = \left(\dfrac{5+3}{2}, \dfrac{3-1}{2}\right) = (4, 1)$ (2)

$\left[\because \text{coordinates of mid-point of line segment joining } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)\right]$

and length of median AD is given by

$$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[by distance formula] (1)

$$= \sqrt{(4 - 7)^2 + (1 + 3)^2} = \sqrt{(-3)^2 + (4)^2}$$

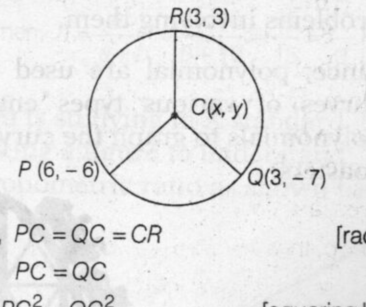
$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$ (1)

Also, $OA = \sqrt{(0 - 7)^2 + (0 + 3)^2}$

$$= \sqrt{49 + 9} = \sqrt{58} \text{ units}$$ (1)

Or

Let $C(x, y)$ be the centre of the circle passing through the points $P(6, -6)$, $Q(3, -7)$ and $R(3, 3)$.



(2)

Then, $PC = QC = CR$ [radii of circle]

Now, $PC = QC$

$\Rightarrow PC^2 = QC^2$ [squaring both sides]

$\Rightarrow (x - 6)^2 + (y + 6)^2 = (x - 3)^2 + (y + 7)^2$

$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$\Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36$

$= x^2 - 6x + 9 + y^2 + 14y + 49$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$$\Rightarrow -12x + 6x + 12y - 14y + 72 - 58 = 0$$
$$\Rightarrow \quad -6x - 2y + 14 = 0$$
$$\Rightarrow \quad 3x + y - 7 = 0 \quad \text{[dividing by } -2] \quad \ldots\text{(i)}$$

and $\quad QC = CR$

$$\Rightarrow \quad QC^2 = CR^2 \quad \text{[squaring both sides]}$$
$$\Rightarrow (x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$$
$$\Rightarrow \quad (y+7)^2 = (y-3)^2 \quad (1)$$
$$\Rightarrow \quad y^2 + 14y + 49 = y^2 - 6y + 9$$
$$\Rightarrow \quad 20y + 40 = 0$$
$$\Rightarrow \quad y = -\frac{40}{20} = -2 \quad \ldots\text{(ii)}$$

On putting $y = -2$ in Eq. (i), we get

$$3x - 2 - 7 = 0$$
$$\Rightarrow \quad 3x = 9$$
$$\Rightarrow \quad x = 3$$

Hence, the centre of circle is $(3, -2)$. (1)

35. **A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.**

Sol. Given, diameter of hemispherical bowl = 36 cm

$\Rightarrow$ Radius $(r) = 18$ cm

$$\text{Volume of liquid in the bowl} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3} \times \pi \times 18 \times 18 \times 18$$
$$= 3888\pi \text{ cm}^3 \quad (1)$$
$$\text{Amount of the liquid wasted} = 3888\pi \times \frac{10}{100}$$
$$= \frac{3888\,\pi}{10} \text{ cm}^3 \quad (1)$$

Liquid transferred into the bottles

$$= 3888\pi - \frac{3888\pi}{10} \text{ cm}^3$$
$$= \frac{34992\pi}{10} \text{ cm}^3 \quad \ldots\text{(i)}$$

Also, given diameter of bottle = 6 cm

$\Rightarrow$ Radius $(r) = 3$ cm (1)

Let h be the height of the bottle.

Volume of bottle $= \pi r^2 h = \pi \times 3 \times 3 \times h = 9\pi h \text{ cm}^3$

Volume of 72 such bottles $= 72 \times 9\pi h \text{ cm}^3$

$$= 648\pi h \text{ cm}^3 \quad (1)$$

Now, volume of 72 bottles

= Volume of liquid transferred into the bottles

$$\Rightarrow \quad 648\pi h = \frac{34992\pi}{10} \quad \text{[using Eq. (i)]}$$
$$\Rightarrow \quad h = 5.4 \text{ cm}$$

$\therefore$ Height of each bottle is 5.4 cm. (1)

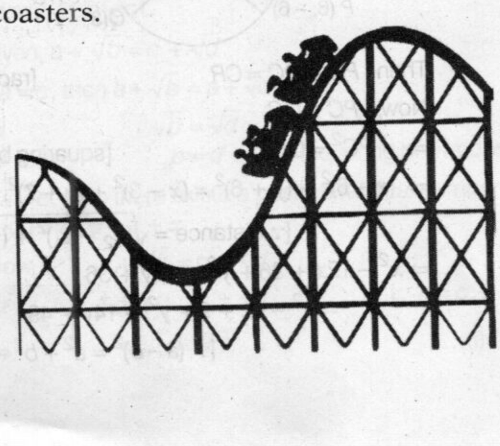
Section-E

Case study based questions are compulsory

36. **Roller Coaster Polynomials**

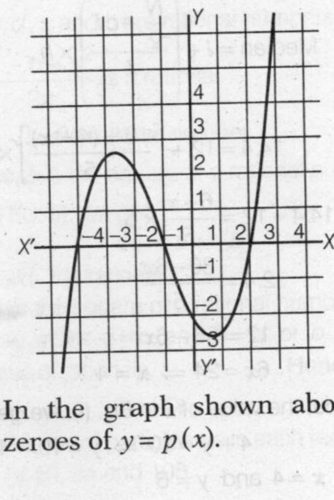
Polynomials are everywhere. They play a key role in the study of algebra in analysis and on the whole many mathematical problems involving them.

Since, polynomial are used to describe curves of various types engineers use polynomials to graph the curves on roller coasters.



On the basis of above information, answer the following questions.

(i) If the roller coaster is represented by the following graph $y = p(x)$, then name the type of the polynomial it traces. (1)



(ii) In the graph shown above, find the zeroes of $y = p(x)$. (1)

(iii) If $p(x) = 2x^3 - 5x^2 - 14x + 8$, then find the sum and product of zeroes. (2)

Or

Find a quadratic polynomial, the sum and product of whose roots are -3 and 2, respectively. (2)

Sol. (i) Since, graph of given polynomial intersect X-axis at 3 points. So, it has three zeroes. Hence, it is a cubic polynomial.

(ii) The graph of given polynomial intersect X-axis at points $-4, -1$ and 2. So, zeroes of given polynomial are $-4, -1$ and 2.

(iii) On comparing given polynomial $2x^3 - 5x^2 - 14x + 8$ with $ax^3 + bx^2 + cx + d$, we get

$$a = 2, b = -5, c = -14 \text{ and } d = 8$$

Sum of roots, $\alpha + \beta + \gamma = \frac{-b}{a}$

$$= \frac{-(-5)}{2} = \frac{5}{2}$$

Product of roots, $\alpha\beta\gamma = \frac{-d}{a} = \frac{-8}{2} = -4$

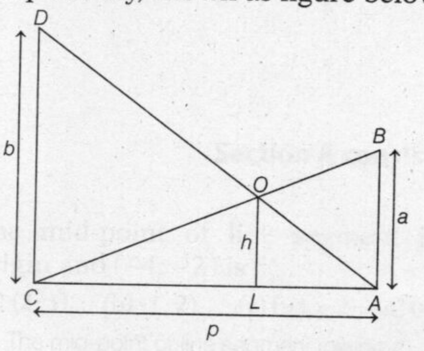
Or

Sum of roots, $\alpha + \beta = -3 = \frac{-b}{a}$

and product of roots, $\alpha\beta = 2 = \frac{c}{a}$

$\therefore$ Quadratic polynomial is $x^2 + 3x + 2$.

37. Ritu is studying in X standard. She observes two poles AB and DC and the heights of these poles are a m and b m respectively, shown as figure below.



These poles are p m apart and O is the point of intersection of the lines joining the top of each pole to the foot of opposite pole and the distance between point O and L is h. Few questions came to his mind while observing the poles. Give answer to his questions by looking at figure.

(i) If $CL = x$, then find x in terms of a, b and h. (1)

(ii) If $AL = y$, then find y in terms of a, b and h. (1)

(iii) Find h in terms of a and b. (2)

Or

If $a = 5$ m and $b = 10$ m, then find h. (2)

Sol. (i) In ΔABC and ΔLOC, we have

$$\angle CAB = \angle CLO = 90^\circ$$

$$\angle C = \angle C \quad \text{[common]}$$

$\therefore$ By AA-criterion of similarity,

$$\Delta CAB \sim \Delta CLO$$

$$\Rightarrow \quad \frac{CA}{CL} = \frac{AB}{LO}$$

$$\Rightarrow \quad \frac{p}{x} = \frac{a}{h} \Rightarrow x = \frac{ph}{a} \quad \text{...(i)}$$

(ii) In ΔALO and ΔACD, we have

$$\angle ALO = \angle ACD = 90^\circ$$

$$\angle A = \angle A \quad \text{[common]}$$

$\therefore \Delta ALO \sim \Delta ACD$ [by AA similarity criterion]

$$\Rightarrow \quad \frac{AL}{AC} = \frac{OL}{DC}$$

$$\Rightarrow \quad \frac{y}{p} = \frac{h}{b} \Rightarrow y = \frac{ph}{b} \quad \text{...(ii)}$$

(iii) From Eqs. (i) and (ii), we get

$$x + y = \frac{ph}{a} + \frac{ph}{b} = ph\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\Rightarrow \quad p = ph\left(\frac{a+b}{ab}\right) \quad [\because x + y = CL + LA = p]$$

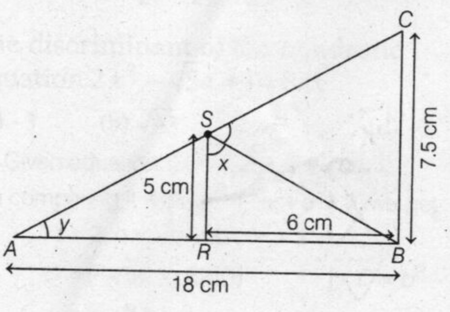
$$\Rightarrow \quad h = \frac{ab}{a+b} \text{ m}$$

Or

If $a = 5$ m and $b = 10$ m.

Then, $h = \frac{ab}{a+b} = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3}$ m

38. Sara is studying in X standard. She is making a figure to understand trigonometric ratio as shown below.



In $\Delta ABC, \angle B$ is right angled, ΔBSC is right angled at S and ΔBRS is right angled at R. $AB = 18$ cm, $BC = 7.5$ cm, $RS = 5$ cm, $RB = 6$ cm, $\angle BSR = x$ and $\angle SAB = y$.

Give answers to her questions by looking at the figure.

(i) What is the length of AS? (1)

(ii) Find $\tan x$. (1)

(iii) Find $\sec x$. (2)

Or Find $\operatorname{cosec} x$. (2)

Sol. (i) We have, $AR = AB - RB = 18 - 6 = 12$ cm

In ΔARS, by using Pythagoras theorem

$$AS^2 = AR^2 + RS^2$$

$$= (12)^2 + (5)^2 = 144 + 25 = 169$$

$$\Rightarrow \quad AS = \sqrt{169} = 13 \text{ cm}$$

(ii) In ΔBRS,

$$\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BR}{RS} = \frac{6}{5}$$

(iii) We know that

$$\sec x = \sqrt{1 + \tan^2 x}$$

$$= \sqrt{1 + \left(\frac{6}{5}\right)^2}$$

$$= \sqrt{1 + \frac{36}{25}}$$

$$= \sqrt{\frac{61}{25}} = \frac{\sqrt{61}}{5}$$

Or

We know that

$$\operatorname{cosec} x = \sqrt{1 + \cot^2 x}$$

$$= \sqrt{1 + \frac{1}{\tan^2 x}}$$

$$= \sqrt{1 + \left(\frac{5}{6}\right)^2}$$

$$= \sqrt{\frac{36 + 25}{36}} = \sqrt{\frac{61}{36}} = \frac{\sqrt{61}}{6}$$