FIR Filter Design Report

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1 Filter Details (Bandpass)

1.1 Un-normalized Discrete Time Filter Specifications

Filter number = 104
Since filter number > 80, m = 104 - 80 = 24 and passband will be equiripple.
q(m) = greatest integer strictly less than 0.1*m = 2
r(m) = m - 10*q(m) = 4
B_L(m) = 10 + 5*q(m) + 13*r(m) = 10 + 5*2 + 13*4 = 72KHz
B_H(m) = B_L(m) + 75 = 147KHz

This filter is given to be a Bandpass Filter with passband from BL(m) kHz to BH(m) kHz. Therefore the specifications are:

- Passband : **72 - 147 KHz**
- Transition band : **5KHz** on either side of stopband
- Stopband : **0 - 67** and **152 - 300 KHz** (As **sampling rate** is **600KHz**)
- Tolerance : **0.15** in **magnitude** for both passband and stopband

1.2 Normalized Digital Filter Specifications

Sampling rate = 600KHz
In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Passband : (**0.24 - 0.49**) π
- Transition band : **0.017** π on either side of stopband
- Stopband : (**0 - 0.223**) π and (**0.507 - 1**) π
- Tolerance : **0.15** in **magnitude** for both passband and stopband

1.3 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be:-

$$A = -20 \log(0.15) = 16.4782\text{dB}$$

Since A ¡ 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 8}{2.285 * \Delta w_T}$$

Here Δw_T is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta w_T = \frac{5KHz * 2\pi}{600KHz} = 0.017\pi$$

$$\text{Hence, N} \geq 71$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **99** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters as done in class.

The impulse response sequence that we get from MATLAB is: (Note that the values are from n = -49 to 49)

```
 Columns 1 through 9

-0.0121   -0.0008    0.0089    0.0044    0.0000    0.0058   0.0064   -0.0073   -0.0153

  Columns 10 through 18
```

```
-0.0042    0.0071    0.0031   -0.0001    0.0095    0.0120   -0.0052   -0.0181   -0.0079

  Columns 19 through 27

0.0046   -0.0001   -0.0024    0.0131    0.0202   -0.0008    -0.0203   -0.0113    0.0018

  Columns 28 through 36

-0.0061   -0.0089    0.0162    0.0326    0.0073   -0.0222   -0.0141   -0.0000   -0.0175

  Columns 37 through 45

-0.0254    0.0186    0.0575    0.0250   -0.0260   -0.0160    0.0033   -0.0516    -0.0940

  Columns 46 through 54

0.0198    0.1929    0.1564   -0.1065   -0.2666   -0.1065    0.1564    0.1929    0.0198

  Columns 55 through 63

-0.0940   -0.0516    0.0033   -0.0160   -0.0260    0.0250    0.0575    0.0186   -0.0254

  Columns 64 through 72

-0.0175   -0.0000   -0.0141   -0.0222    0.0073    0.0326    0.0162   -0.0089   -0.0061

  Columns 73 through 81

0.0018   -0.0113   -0.0203   -0.0008    0.0202    0.0131    -0.0024   -0.0001    0.0046

  Columns 82 through 90

-0.0079   -0.0181   -0.0052    0.0120    0.0095   -0.0001    0.0031    0.0071   -0.0042

  Columns 91 through 99

-0.0153   -0.0073    0.0064    0.0058    0.0000    0.0044    0.0089   -0.0008   -0.0121
```

The z-transform can simply be read off from the sequence values since its finite sequence.

2 Filter Details (Bandstop)

2.1 Un-normalized Discrete Time Filter Specifications

Filter number = 104
Since filter number > 80, m = 104 - 80 = 24 and passband will be monotonic.
q(m) = greatest integer strictly less than 0.1*m = 2
r(m) = m - 10*q(m) = 4
B_L(m) = 20 + 3*q(m) + 11*r(m) = 20 + 3*2 + 11*4 = 70KHz
B_H(m) = B_L(m) + 40 = 110KHz

This filter is given to be a Bandstop Filter with stopband from BL(m) kHz to BH(m) kHz. Therefore the specifications are:

- Stopband : **70 - 110 KHz**
- Transition band : **5KHz** on either side of stopband
- Passband : **0 - 65** and **115 - 212.5 KHz** (As **sampling rate** is **425KHz**)
- Tolerance : **0.15** in **magnitude** for both passband and stopband

2.2 Normalized Digital Filter Specifications

Sampling rate = 425KHz
In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Stopband : (**0.329 - 0.518**) π
- Transition band : **0.024** π on either side of stopband
- Passband : (**0 - 0.306**) π and (**0.541 - 1**) π
- Tolerance : **0.15** in **magnitude** for both passband and stopband

2.3 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be:-

A = -20 log(0.15) = 16.4782dB

Since A ¡ 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A-8}{2.285 * \Delta w_T}$$

Here Δw_T is the minimum transition width. In our case, the transition width is the same on either side of the stopband.

$$\Delta w_T = \frac{5KHz * 2\pi}{425KHz} = 0.024\pi$$

Hence, N $\geq$ 51

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **77** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument.The bandstop impulse response samples were generated as the difference between three low-pass filters(all-pass - bandpass) as done in class.

The impulse response sequence that we get from MATLAB is: (Note that the values are from n = -38 to 38)

```
 Columns 1 through 9

-0.0014    0.0021   -0.0069   -0.0122    0.0055    0.0192    0.0031   -0.0148   -0.0068

 Columns 9 through 18
```

```
0.0032   -0.0021   0.0020    0.0170    0.0062   -0.0229   -0.0186    0.0136    0.0187

  Columns 19 through 27

-0.0011   -0.0015    0.0038   -0.0177   -0.0248    0.0182    0.0443    0.0008   -0.0387

  Columns 28 through 36

-0.0138    0.0088   -0.0085    0.0126    0.0655    0.0124   -0.1182   -0.0885    0.1178

  Columns 37 through 45

0.1744   -0.0495    0.7880   -0.0495    0.1744    0.1178   -0.0885   -0.1182    0.0124

  Columns 46 through 54

0.0655    0.0126   -0.0085    0.0088   -0.0138   -0.0387    0.0008    0.0443    0.0182

  Columns 55 through 63

-0.0248    -0.0177    0.0038   -0.0015   -0.0011    0.0187    0.0136   -0.0186   -0.0229

  Columns 63 through 72

0.0062    0.0170    0.0020    -0.0021    0.0032   -0.0068   -0.0148    0.0031    0.0192

  Columns 72 through 77

0.0055   -0.0122   -0.0069    0.0021   -0.0014
```

The z-transform can simply be read off from the sequence values since its finite sequence.

3 Matlab Plots

3.1 BandPass Filter

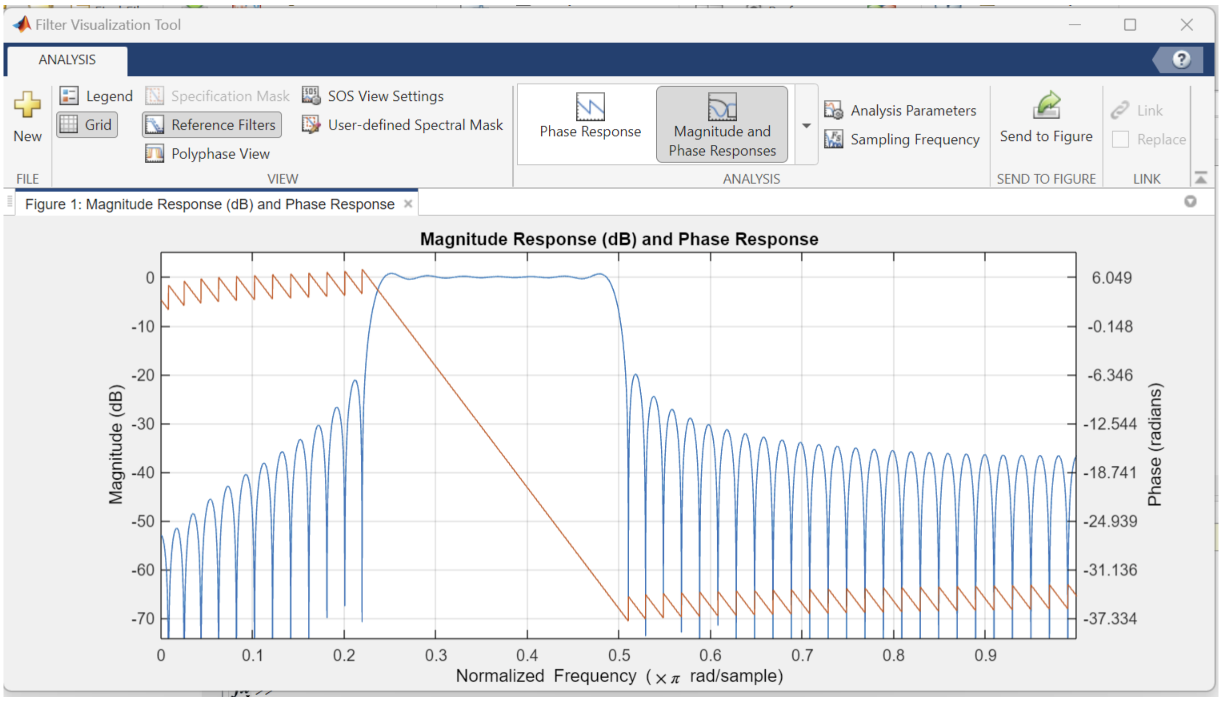

Figure 1: Frequency Response

It can be seen that the FIR Filter is indeed giving us a Linear Phase response which is desired.

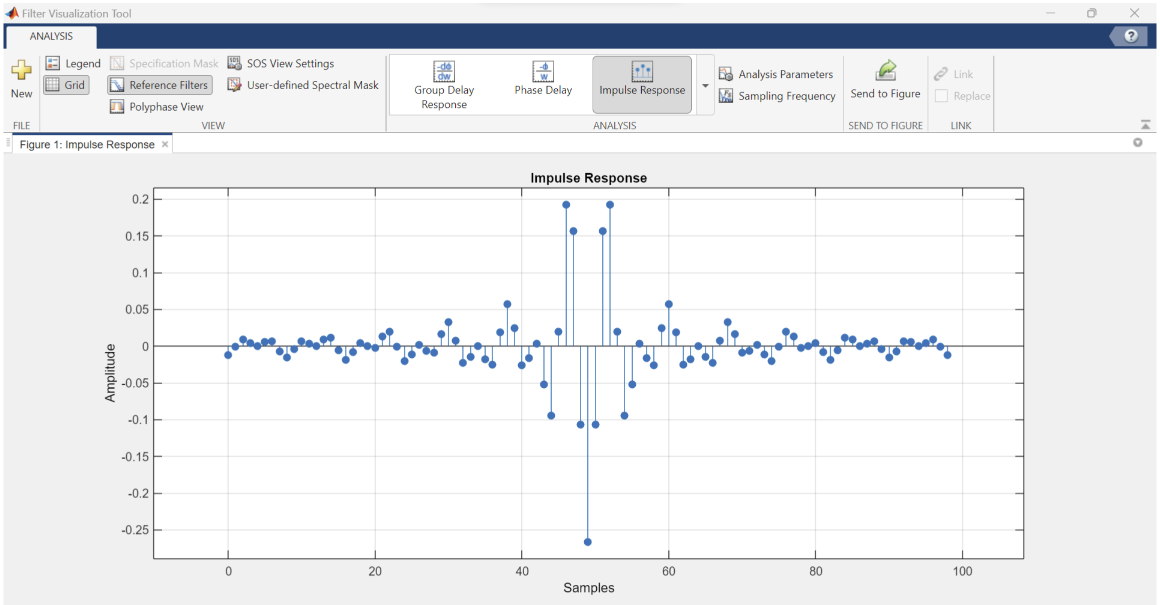

Figure 2: Impulse Response

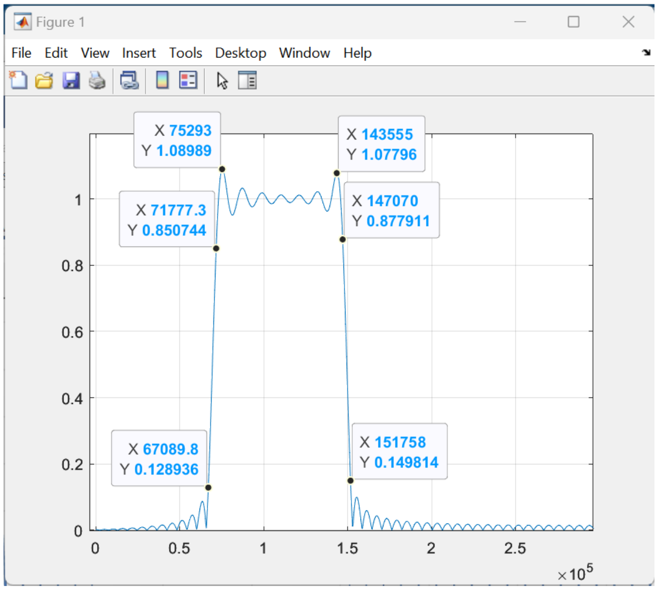

Figure 3: Magnitude plot

From the above figure, it can be seen that all the specifications have been satisfied.

3.2 BandStop Filter

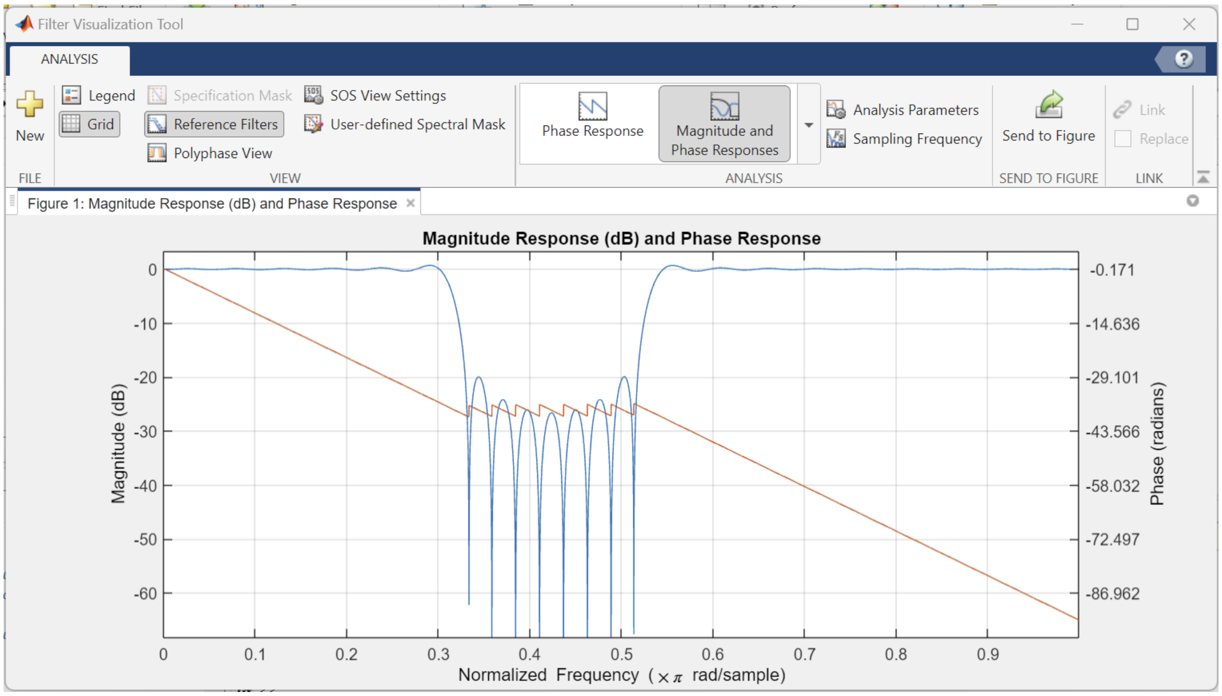

Figure 4: Frequency Response

It can be seen that the FIR Filter is indeed giving us a Linear Phase response which is desired.

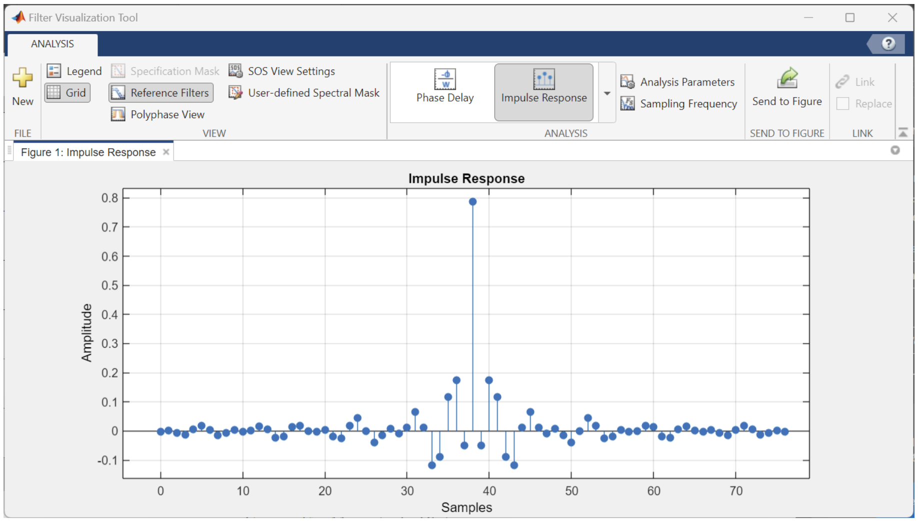

Figure 5: Impulse Response

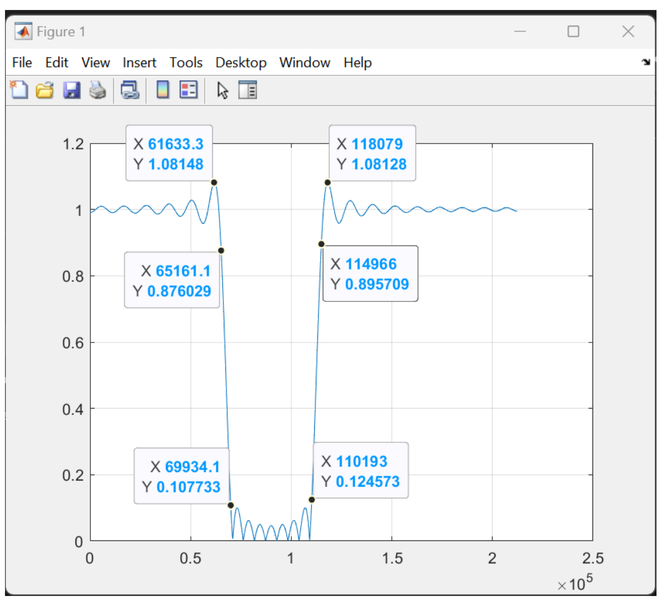

Figure 6: Magnitude plot

From the above figure, it can be seen that all the specifications have been satisfied.

4 Comparison Between FIR and IIR Realizations

Advantages of FIR Filters over IIR filters:

- We get a linear phase response in FIR filters, which we don't get using IIR filters.
- The effort required to implement FIR Filters is also easier as we have to just truncate the ideal impulse response using the window function, whereas for IIR, we need to find the analog filter specs, then design low pass filter, and then convert it into the required filter.

Advantages of IIR Filters over FIR filters:

- We don't have any control over the nature of passband and stopband in FIR filters, whereas in IIR filters, we can control that.
- We can't control the tolerances of passband and stopband independently in FIR filters. In IIR filters, we can do that.
- The Passband for IIR filters is more specific as it keeps the value of transfer function always less than 1, but in FIR, the value of transfer function can be more than, or less than 1.
- We usually need a lot more resources for FIR filters as compared to IIR filters, as we can see that the value of N for FIR filters is considerably large.

So, each type of filter has its own advantage, we can choose the one we want according to our needs.

5 Peer Review of Assignment

I have reviewed the report of my peer:

Annirudh K P
(Roll no: 210070009, Filter number: 96)

and certify it to be correct.