Chebyschev Filter Design Report

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1 Filter Details (Bandpass)

1.1 Un-normalized Discrete Time Filter Specifications

Filter number = 104
Since filter number > 80, m = 104 - 80 = 24 and passband will be equiripple.
q(m) = greatest integer strictly less than 0.1*m = 2
r(m) = m - 10*q(m) = 4
B_L(m) = 10 + 5*q(m) + 13*r(m) = 10 + 5*2 + 13*4 = 72KHz
B_H(m) = B_L(m) + 75 = 147KHz

This filter is given to be a Bandpass Filter with passband from BL(m) kHz to BH(m) kHz. Therefore the specifications are:

- Passband : **72 - 147 KHz**
- Transition band : **5KHz** on either side of stopband
- Stopband : **0 - 67** and **152 - 300 KHz** (As **sampling rate** is **600KHz**)
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Passband is **equiripple** and stopband is **monotonic**

1.2 Normalized Digital Filter Specifications

Sampling rate = 600KHz
In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Passband : (**0.24 - 0.49**) π
- Transition band : **0.017** π on either side of stopband
- Stopband : (**0 - 0.223**) π and (**0.507 - 1**) π
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Equiripple**
- Stopband nature: **Monotonic**

1.3 Band-pass analog Filter Specifications using Bilinear Transformation

To convert to analog domain, we use the following transformation :

$$\Omega = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.24 π	0.396
0.49 π	0.969
0.223 π	0.365
0.507 π	1.022
π	∞

Hence, the specifications for corresponding bandpass analog filter are:

- Passband : (**0.396 - 0.969**)
- Transition band : **0.365 to 0.396** and **0.969 to 1.022**
- Stopband : (**0 - 0.365**) and (**1.022 -** ∞)
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Equiripple**
- Stopband nature: **Monotonic**

1.4 Low pass analog Filter Specifications using Frequency Transformation

We need to transform the bandpass filter to a low pass filter. Hence, we can use the bandpass transformation:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

Where Ω_0 and B can be determined by: We want Ω_{P1} to map to -1 and Ω_{P2} to map to +1.
Using this, we get:

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = \sqrt{0.396 \times 0.969} = 0.61945$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.969 - 0.396 = 0.573$$

Ω	Ω_L
0^+	$-\infty$
$0.365(\Omega_{S1})$	$-1.198\Omega_{L_{S1}})$
$0.396(\Omega_{P1})$	$-1(\Omega_{L_{P1}})$
$0.61945(\Omega_0)$	0
$0.969(\Omega_{P2})$	$+1(\Omega_{L_{P2}})$
$1.022(\Omega_{S2})$	$1.128\Omega_{L_{S2}})$
∞	∞

1.5 Low pass Analog filter specifications

- Passband Edge : 1 (Ω_{L_P})
- Stopband Edge : min($-\Omega_{L_{S1}}$, $\Omega_{L_{S2}}$) = min(1.198, 1.128) = 1.128 (Ω_{L_S})
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Equiripple**
- Stopband nature: **Monotonic**

1.6 Analog Lowpass Transfer Function

The analog filter has Equiripple passband and Monotonic stopband, hence, we need a **Chebyschev** filter. Now, using the given tolerance as 0.15, we can define 2 new quantities D_1 and D_2:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Hence, the order of the filter N satisfies the inequality:

$$N \geq \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_S}{\Omega_P})} = 6.101$$

And we want minimum resources to design this filter, hence we use the smallest possible integer value of n that satisfies the above equation, that is $\mathbf{N = 7}$. Also, we choose ϵ to be $\sqrt{D_1}$.

Now, the poles are the roots of the equation:

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 cosh^2(4 cosh^{-1}(\frac{s}{j})) = 0$$

Solving for roots, using Wolfram, we get:

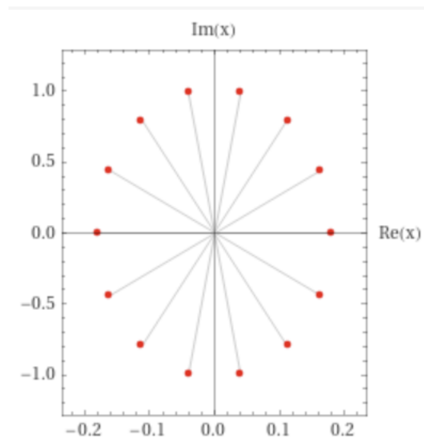

Figure 1: Poles of Magnitude Plot of Analog LPF

For a stable filter, we need to take only the LHP poles, so:

p_1 = -0.18041
p_2 = -0.11249 + 0.79445 i
p_3 = -0.11249 - 0.79445 i
p_4 = -0.04015 + 0.99067 i
p_5 = -0.04015 - 0.99067 i
p_6 = -0.16255 + 0.44089 i
p_7 = -0.16255 - 0.44089 i

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as:

$H_{analog,LPF}(s_L) = \frac{(-1)^N \Pi_{i=1}^N p_i}{\Pi_{i=1}^N (s_L - p_i)}$

$= \frac{0.0252}{(s_L+0.18)(s_L^2+0.225s_L+0.644)(s_L^2+0.08s_L+0.983)(s_L^2+0.325s_L+0.221)}$

Note that since it is odd order we take the DC Gain to be 1.

1.7 Analog Bandpass Transfer Function

The transformation is given by:

$$s_L = \frac{\Omega_0^2 + s^2}{Bs}$$

Substituting $\Omega_0 = 0.61945$ and B = 0.573, we get:

$$s_L = \frac{0.3837 + s^2}{0.573s}$$

Substituting this in the low pass transfer function gives us the analog bandpass transfer function. The denominator and numerator are represented by D(s) and N(s) respectively.

The coefficients of D(s) are:

Degree	s^{14}	s^{13}	s^{12}	s^{11}	s^{10}	s^9	s^8
Coefficient	1	0.4641	3.3685	1.2997	4.5353	1.4083	3.1431

Degree	s^7	s^6	s^5	s^4	s^3	s^2	s^1	s^0
Coefficient	0.7510	1.2061	0.2074	0.2562	0.0282	0.0280	0.0015	0.0012

And $N(s) = 5.1107 \times 10^{-4} s^7$
Coefficients of all other powers of s in N(s) are all 0.

1.8 Discrete Time Filter Transfer Function

To get the Discrete time transfer function, we need to perform the bilinear transform on the analog bandpass transfer function. The bilinear transform is given by:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this in the analog bandpass transfer function, we get a rational function of z given by N(z)/D(z).
The coefficients of N(z) are:

Degree	z^{-14}	z^{-12}	z^{-10}	z^{-8}	z^{-6}	z^{-4}
Coefficient	-2.8877×10^{-5}	2.0214×10^{-4}	-6.0641×10^{-4}	0.0010	-0.0010	6.0641×10^{-4}

Degree	z^{-2}	z^0
Coefficient	-2.0214×10^{-4}	2.8877×10^{-5}

Coefficients of all odd powers of s in N(s) are all 0.

The coefficients of D(z) are:

Degree	z^{-14}	z^{-13}	z^{-12}	z^{-11}	z^{-10}	z^{-9}	z^{-8}
Coefficient	0.5299	-3.1872	11.7694	-30.4846	61.8841	-101.2390	137.8179

Degree	z^{-7}	z^{-6}	z^{-5}	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^{0}
Coefficient	-156.8791	150.7805	-121.2137	81.1190	-43.7856	18.5293	-5.5069	1

Hence, this, N(z)/D(z) is the final discrete time transfer function of our Chebyschev bandpass filter.

1.9 Matlab plots

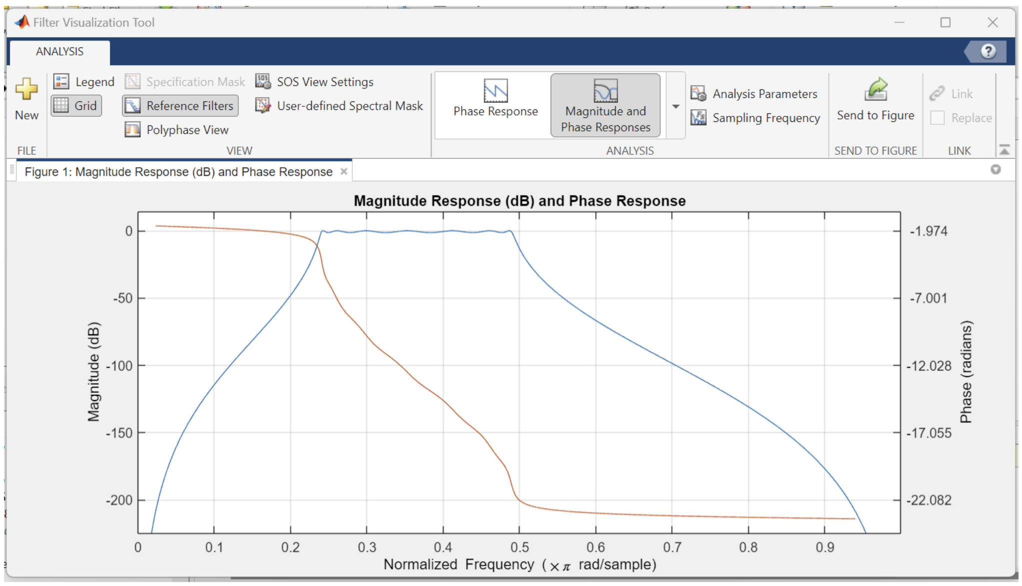

Figure 2: Frequency Response

From the above figure, it can be seen that the phase response is not linear.

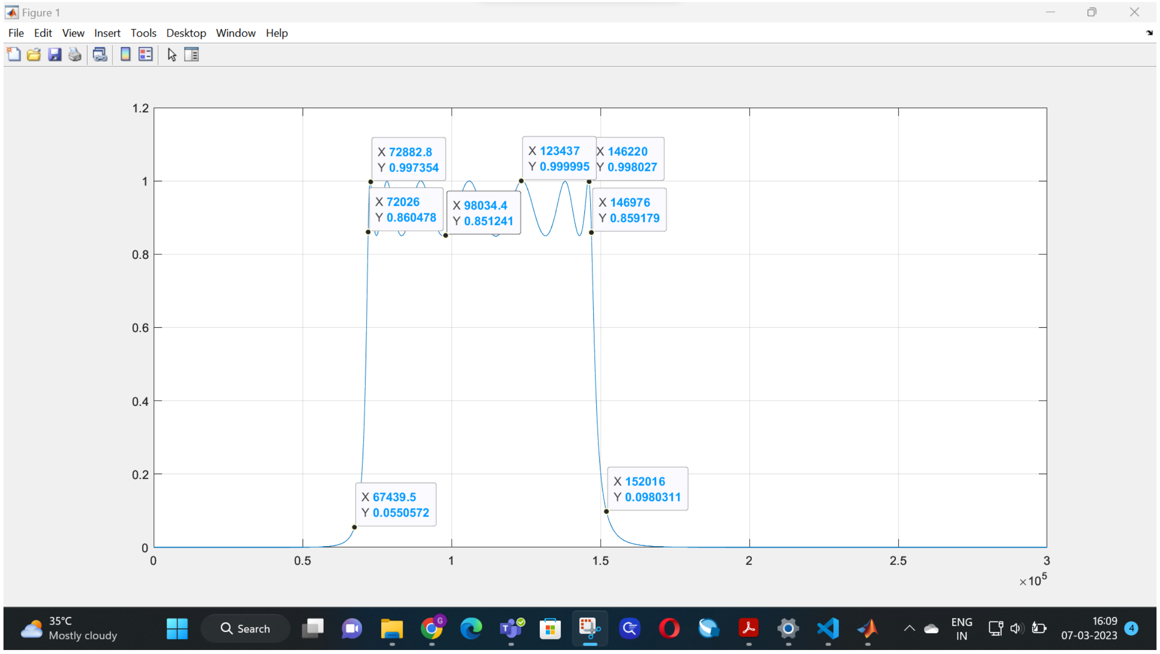

Figure 3: Magnitude plot

From the above figure, it can be seen that the required specifications are satisfied.

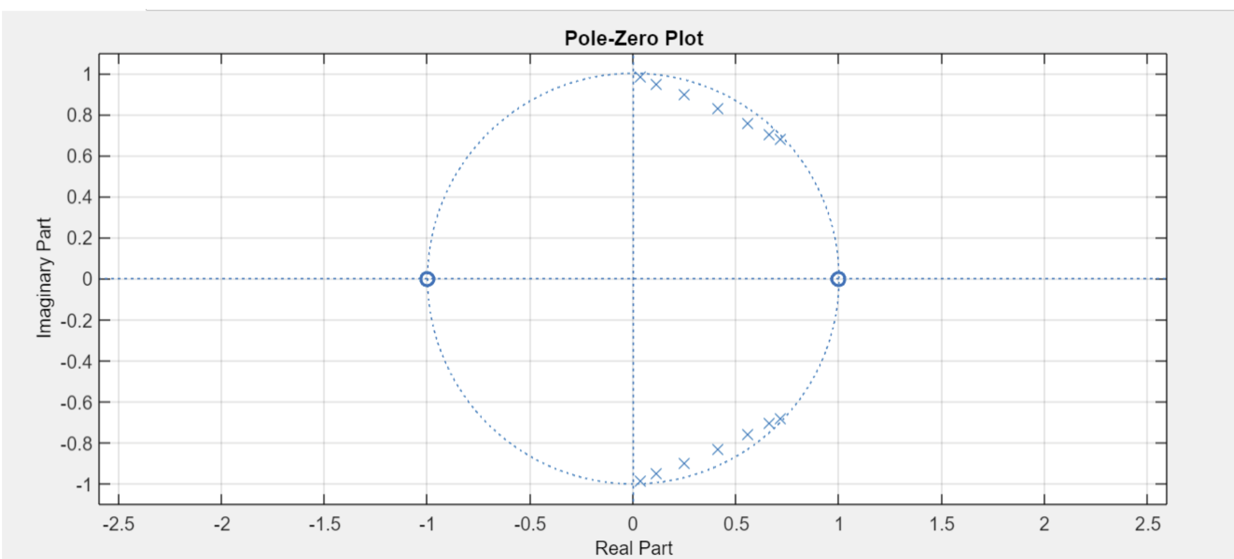

Figure 4: Pole zero plot

From the above fig, we can see that all the poles are in the unit circle. Hence the transfer function is stable.

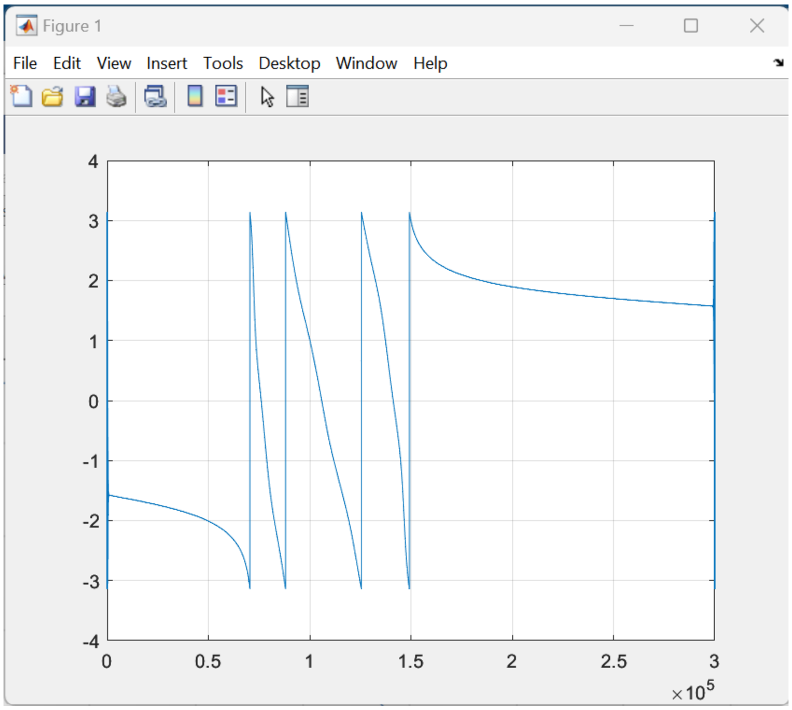

Figure 5: Unnormalised Phase response

2 Peer Review of Assignment

I have reviewed the report of my peer:

Annirudh K P
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and certify it to be correct.