Filter Design Report

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1 Filter Details (Bandstop)

1.1 Un-normalized Discrete Time Filter Specifications

Filter number = 104
Since filter number > 80, m = 104 - 80 = 24 and passband will be monotonic.
q(m) = greatest integer strictly less than 0.1*m = 2
r(m) = m - 10*q(m) = 4
B_L(m) = 20 + 3*q(m) + 11*r(m) = 20 + 3*2 + 11*4 = 70KHz
B_H(m) = B_L(m) + 40 = 110KHz

This filter is given to be a Bandstop Filter with stopband from BL(m) kHz to BH(m) kHz. Therefore the specifications are:

- Stopband : **70 - 110 KHz**
- Transition band : **5KHz** on either side of stopband
- Passband : **0 - 65** and **115 - 212.5 KHz** (As **sampling rate** is **425KHz**)
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Nature : Both passband and stopband are **monotonic**

1.2 Normalized Digital Filter Specifications

Sampling rate = 425KHz
In the normalized frequency axis, sampling rate corresponds to 2π
Therefore, any frequency can be normalized as follows :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s}$$

where Ω_s is the Sampling Rate.

For the normalized discrete filter specifications, the nature and tolerances being the dependent variables remain the same while the passband and stopband frequencies change as per the above transformations.

- Stopband : (**0.329 - 0.518**) π
- Transition band : **0.024** π on either side of stopband
- Passband : (**0 - 0.306**) π and (**0.541 - 1**) π
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Monotonic**
- Stopband nature: **Monotonic**

1.3 Band-stop analog Filter Specifications using Bilinear Transformation

To convert to analog domain, we use the following transformation :

$$\Omega = \tan(\frac{w}{2})$$

Applying the transformation at Band Edges we get :

ω	Ω
0	0
0.329 π	0.568
0.518 π	1.058
0.306 π	0.521
0.541 π	1.137
π	∞

Hence, the specifications for corresponding bandstop analog filter are:

- Stopband : (**0.568 - 1.058**)
- Transition band : **0.521 to 0.568** and **1.058 to 1.137**
- Passband : (**0 - 0.521**) and (**1.137 -** ∞)
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Monotonic**
- Stopband nature: **Monotonic**

1.4 Low pass analog Filter Specifications using Frequency Transformation

We need to transform the bandstop filter to a low pass filter. Hence, we can use the bandstop transformation:

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

Where Ω_0 and B can be determined by: We want Ω_{P1} to map to +1 and Ω_{P2} to map to -1.
Using this, we get:

$$\Omega_0 = \sqrt{\Omega_{P1}\Omega_{P2}} = \sqrt{0.521 \times 1.137} = 0.76966$$

$$B = \Omega_{P2} - \Omega_{P1} = 1.137 - 0.521 = 0.616$$

Ω	Ω_L
0^+	0^+
$0.521(\Omega_{P1})$	$+1(\Omega_{L_{P1}})$
$0.568(\Omega_{S1})$	$1.297\Omega_{L_{S1}})$
$0.76966(\Omega_0^-)$	∞
$0.76966(\Omega_0^+)$	$- \infty$
$1.058(\Omega_{S2})$	$-1.236\Omega_{L_{S2}})$
$1.137(\Omega_{P2})$	$-1(\Omega_{L_{P2}})$
∞	0^-

1.5 Low pass Analog filter specifications

- Passband Edge : 1 (Ω_{L_P})
- Stopband Edge : min($\Omega_{L_{S1}}$, -$\Omega_{L_{S2}}$) = min(1.297, 1.236) = 1.236 (Ω_{L_S})
- Tolerance : **0.15** in **magnitude** for both passband and stopband
- Passband nature: **Monotonic**
- Stopband nature: **Monotonic**

1.6 Analog Lowpass Transfer Function

The analog filter has both passband and stopband monotonic, hence, we need a **Butterworth** filter. Now, using the given tolerance as 0.15, we can define 2 new quantities D_1 and D_2:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Hence, the order of the filter N satisfies the inequality:

$$N \geq \frac{log(\frac{D_2}{D_1})}{2(log(\frac{\Omega_S}{\Omega_P}))} = 11.158$$

And we want minimum resources to design this filter, hence we use the smallest possible integer value of n that satisfies the above equation, that is **N = 12**

Also, the cutoff frequency(Ω_C) should satisfy the following constraint:

$$\frac{\Omega_P}{D_1^{\frac{1}{2N}}} \leq \Omega_C \leq \frac{\Omega_S}{D_2^{\frac{1}{2N}}}$$

$$1.0407 \leq \Omega_C \leq 1.0562$$

Thus we can choose the value of Ω_C to be equal to 1.05
Now, the poles of the Transfer function are the roots of the equation:

$$1 + (\frac{s}{j\Omega_C})^{2N} = 1 + (\frac{s}{1.05j})^{2N} = 0$$

Solving for the roots(using Wolfram) we get:

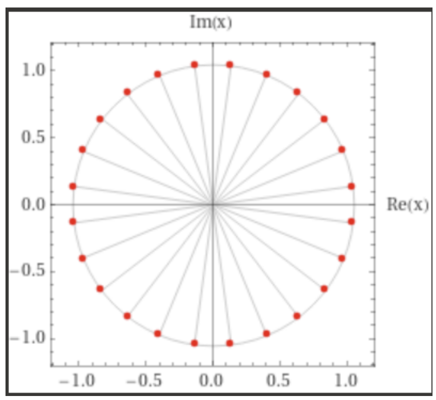

Figure 1: Poles of Magnitude Plot of Analog LPF

For a stable Filter, we must include the poles lying in the Left Half Plane in the Transfer Function.
p_1 = -1.04102 - 0.137053 i
p_2 = -1.04102 + 0.137053 i
p_3 = -0.970074 + 0.401818 i
p_4 = -0.970074 - 0.401818 i
p_5 = -0.833021 - 0.6392 i
p_6 = -0.833021 + 0.6392 i
p_7 = -0.6392 - 0.833021 i
p_8 = -0.6392 + 0.833021 i
p_9 = -0.401818 - 0.970074 i
p_{10} = -0.401818 + 0.970074 i
p_{11} = -0.137053 - 1.04102 i
p_{12} = -0.137053 + 1.04102 i

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as:

$H_{analog,LPF}(s_L) = \frac{(\Omega_C)^N}{\Pi_{i=1}^{12}(s_L - p_i)}$

$= \frac{1.796}{(s_L^2+1.94s_L+1.1025)(s_L^2+1.666s_L+1.1025)(s_L^2+1.2784s_L+1.1025)(s_L^2+0.8036s_L+1.1025)(s_L^2+0.274s_L+1.1025)(s_L^2+2.082s_L+1.1025)}$

Note that the scaling of the numerator is done in order to obtain a DC gain of 1.

1.7 Analog Bandstop Transfer Function

The transformation is given by:

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Substituting $\Omega_0 = 0.76966$ and B = 0.616, we get:

$$s_L = \frac{0.616s}{0.5924 + s^2}$$

Substituting this in the low pass transfer function gives us the analog band-stop transfer function. The denominator and numerator are represented by D(s) and N(s) respectively.

The coefficients of D(s) are:

Degree	s^{24}	s^{23}	s^{22}	s^{21}	s^{20}	s^{19}	s^{18}	s^{17}	s^{16}
Coefficient	1	4.5041	17.2642	44.4521	99.7834	181.2948	293.3468	407.6367	511.0157

Degree	s^{15}	s^{14}	s^{13}	s^{12}	s^{11}	s^{10}	s^{9}	s^{8}
Coefficient	564.2816	566.1763	506.0797	412.3625	300.3152	199.3739	117.9155	63.3677

Degree	s^{7}	s^{6}	s^{5}	s^{4}	s^{3}	s^{2}	s^{1}	s^{0}
Coefficient	29.9961	12.8095	4.6978	1.5344	0.4056	0.0935	0.0145	0.0019

Coefficients of N(s) are:

Degree	s^{24}	s^{22}	s^{20}	s^{18}	s^{16}	s^{14}	s^{12}	s^{10}	s^{8}
Coefficient	1	7.1210	23.2413	45.9725	61.3817	58.2796	40.3480	20.5227	7.6115

Degree	s^6	s^4	s^2	s^0
Coefficient	2.0075	0.3574	0.0386	0.0019

Coefficients of all odd powers of s in N(s) are all 0.

1.8 Discrete Time Filter Transfer Function

To get the Discrete time transfer function, we need to perform the bilinear transform on the analog bandstop transfer function. The bilinear transform is given by:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Substituting this in the analog bandstop transfer function, we get a rational function of z given by N(z)/D(z).

The coefficients of N(z) are:

Degree	z^{-24}	z^{-23}	z^{-22}	z^{-21}	z^{-20}	z^{-19}	z^{-18}	z^{-17}	z^{-16}
Coefficient	0.0617	-0.378	1.8018	-5.9632	16.7570	-38.7281	78.915	-139.6387	222.0975

Degree	z^{-15}	z^{-14}	z^{-13}	z^{-12}	z^{-11}	z^{-10}	z^{-9}	z^{-8}
Coefficient	-314.1368	403.4502	-465.8035	490.5015	-465.8035	403.4502	-314.1368	222.0975

Degree	z^{-7}	z^{-6}	z^{-5}	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^{0}
Coefficient	-139.6387	78.915	-38.7281	16.7570	-5.9632	1.8018	-0.378	0.0617

The coefficients of D(z) are:

Degree	z^{-24}	z^{-23}	z^{-22}	z^{-21}	z^{-20}	z^{-19}	z^{-18}	z^{-17}	z^{-16}
Coefficient	0.0038	-0.0286	0.1651	-0.6673	2.2861	-6.4758	16.1872	-35.3039	69.344

Degree	z^{-15}	z^{-14}	z^{-13}	z^{-12}	z^{-11}	z^{-10}	z^{-9}	z^{-8}
Coefficient	-121.6909	194.4124	-280.5670	370.4089	-443.3166	485.5183	-480.7720	433.7621

Degree	z^{-7}	z^{-6}	z^{-5}	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^{0}
Coefficient	-350.3115	255.0995	-162.5943	91.5297	-42.8253	16.9508	-4.7432	1

Hence, this, N(z)/D(z) is the final discrete time transfer function of our butterworth bandstop filter.

1.9 Matlab plots

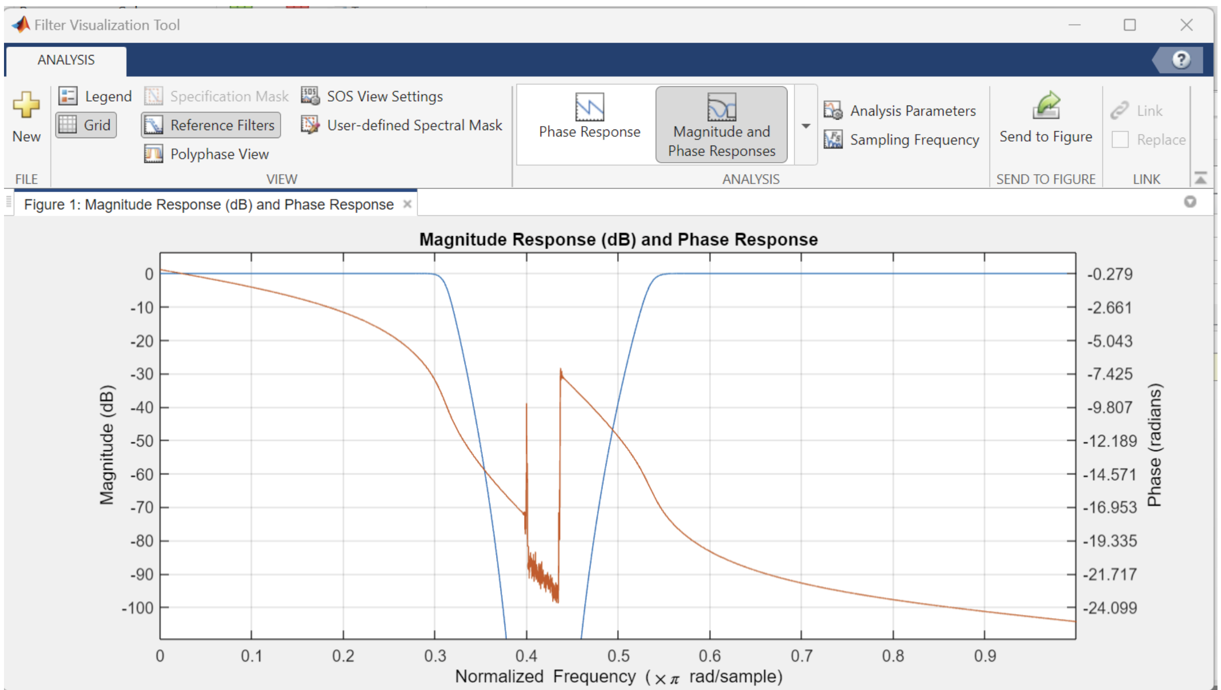

Figure 2: Frequency Response

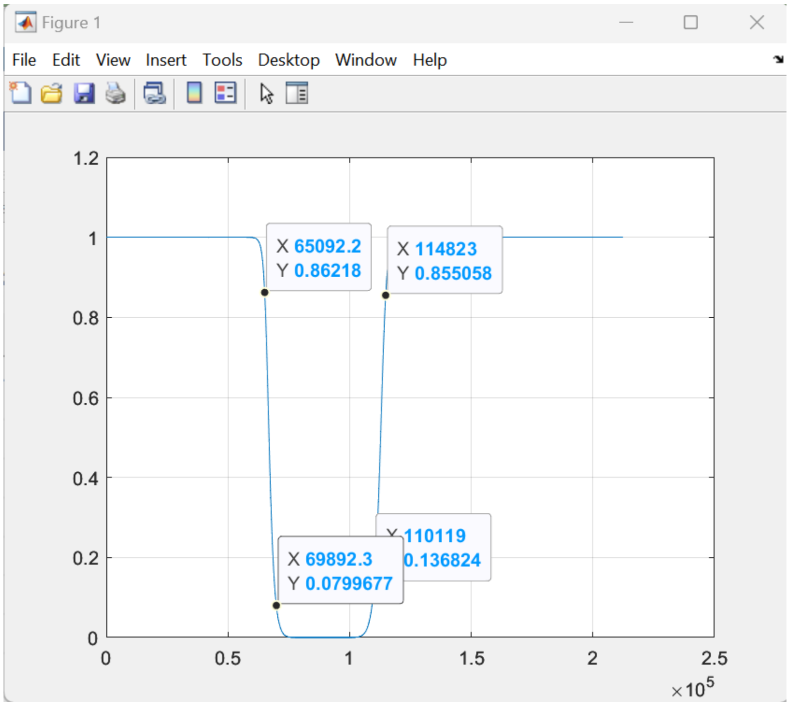

Figure 3: Magnitude plot

From the above figure, it can be seen that the required specifications are satisfied.

2 Peer Review of Assignment

I have reviewed the report of my peer:

Annirudh K P
(Roll no: 210070009, Filter number: 96)

and certify it to be correct.