Problem sheet (2)

(6) a) $x\frac{dy}{dx} + y^2 - 1 = 0$ — ODE, First order, Non-linear

b) $y''' \sin\theta - \cos\theta y' = 2$ — ODE, 3rd order, Linear

c) $y' + P(x)y = Q(x)$ — ODE, 1st order, Linear

d) $x^3y'' - xy' = 2x$ — ODE, 2nd order, Linear

e) $\sin(y'') + e^{y'} = 1$ — ODE, 2nd order, non-linear

f) $y''' = (y')^2 + y'$ — ODE, 3rd order, non-linear.

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(7) a) $y' = e^x \quad y'' = e^x$

$\therefore$ L.H.S. $= y'' - y = e^x - e^x = 0 =$ R.H.S.

b) Error in question.

c) $y = \sin(e^x) \quad y' = \cos(e^x).e^x \quad y'' = -\sin(e^x).e^{2x} + \cos(e^x)e^x$

L.H.S. $= y'' - y' + e^{2x}y = -\sin(e^x).e^{2x} + \cos(e^x).e^x - \cos(e^x)e^x + \sin(e^x).e^{2x} = 0$

$=$ R.H.S.

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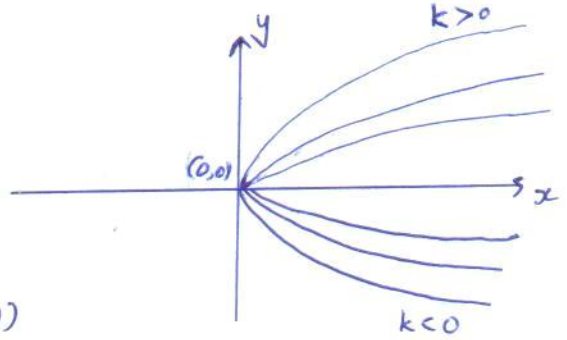
(8) a) i) $\frac{dy}{dx} = \frac{y}{2x} \iff \int \frac{1}{y}\,dy = \int \frac{1}{2x}\,dx \iff \ln y = \frac{1}{2}\ln x + C$ where C is an arbitrary constant

let $e = \ln k$

$\updownarrow$

$y = kx^{1/2} \iff \ln y = \ln x^{1/2} + \ln k = \ln kx^{1/2}$

$\therefore$ $y = kx^{1/2}$ satisfies $\frac{dy}{dx} = \frac{y}{2x}$



integral curves

ii) through (2, 1)

$1 = k\sqrt{2}$

$k = \frac{1}{\sqrt{2}}$ $\therefore$ $y = \frac{1}{\sqrt{2}}x^{1/2}$ passes through (2, 1)

b) $y = e^{\lambda x} \quad y' = \lambda e^{\lambda x} = \lambda y \quad y'' = \lambda^2 y$

$y'' - 5y' + 6y = 0 \iff (\lambda^2 - 5\lambda + 6)y = 0 \iff (\lambda - 3)(\lambda - 2)y = 0$

$\therefore \lambda = 3$ & $\lambda = 2$.

$\therefore$ general solution $y = C_1e^{3x} + C_2e^{2x}$ where C_1 & C_2 are arbitrary constants.

⑨ (i) $y = 0$

(a) $xy' = 2y$

(ii) $y = 2$

(c) $y'' + 9y = 18$, (d) $y' = 2y - 4$

(iii) $y = 2x$

(b) $y' = 2$

(iv) $y = 2x^2$

(a) $xy' = 2y$

(a) $xy' = 2y$

i) $y = 0$ (iv) $y = 2x^2$

(b) $y' = 2$

(iii) $y = 2x$

(c) $y'' + 9y = 18$

(ii) $y = 2$

(d) $y' = 2y - 4$

(ii) $y = 2$

[by substitution]

eg- (i) $y = 0$

a) $xy' = 2y$

L.H.S. $= x \times 0$

R.H.S $= 2 \times 0 =$ L.H.S. $\therefore$ $y = 0$ is a sol^{n} for this.

b) $y' = 2$

L.H.S $= 0$

R.H.S $= 2$

$\therefore$ $y = 0$ isn't a sol^{n} for this.