

EE 387 - Signal Processing

Assignment 1

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Problem 1. Determine the fundamental period of the following signals:

- (a) $x(t) = 3\cos(10t + 1) - \sin(4t - 1)$
- (b) $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$

Problem 2. Determine whether the following signals are periodic or aperiodic? If periodic, also find the period.

- (a) $x(t) = 2\cos(4t + \frac{\pi}{3})$
- (b) $x(t) = [\sin(2t - \frac{\pi}{4})]^2$
- (c) $x[n] = \sin(6\frac{\pi}{7}n + 1)$
- (d) $x[n] = \cos(\frac{\pi}{8}n^2)$
- (e) $x(t) = \sin(\frac{\pi}{8}t^2)$
- (f) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

Problem 3. Classify each of the signals below as a power signal or an energy signal. In addition, find the power or the energy of the signal.

(a)

$$x[n] = \begin{cases} \cos(\pi n) & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$x(t) = \begin{cases} \frac{1}{2}(\cos(\omega t) + 1) & -\pi/\omega \leq t \leq \pi/\omega \\ 0 & \text{otherwise} \end{cases}$$

Problem 4. Determine the Fourier series representation for the following signal, $x(t)$, with period 4:

$$x(t) = \begin{cases} \sin(\pi t) & 0 \leq t \leq 2 \\ 0 & 2 < t \leq 4 \end{cases}$$

Problem 5. Consider the following three continuous-time signals with a fundamental period of $T=1/2$:

$$\begin{aligned} x(t) &= \cos(4\pi t), \\ y(t) &= \sin(4\pi t), \\ z(t) &= x(t)y(t). \end{aligned}$$

- (a) Determine the Fourier series coefficients of $x(t)$.
- (b) Determine the Fourier series coefficients of $y(t)$.
- (c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

- (d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

Problem 6. Consider the following periodic square wave pulse (symmetric) with a period of 4,

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < 2 \end{cases}$$

- (a) Determine the Fourier series coefficients of $x(t)$.
 (b) $x(t)$ can be approximated by a linear combination of a finite number of harmonically related complex exponentials- that is, by a finite series of the form,

$$x_N(t) = \sum_{k=-N}^N a_k e^{jk\omega_0 t}$$

Plot $x_N(t)$ for $N = 1, 3, 7, 19, 43$, and 79 over the fundamental period (Hint: you may use any programming language).

- (c) Determine the percentage overshoot of the approximated square pulse for each of the above partial sums.

Problem 7. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may use the Fourier transform properties.

- (a) $x_1(t) = x(1-t) + x(-1-t)$
 (b) $x_2(t) = x(3t-6)$
 (c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

Problem 8. Consider the Fourier transform pair

$$e^{-|t|} \xleftrightarrow{\mathcal{F}} \frac{2}{1+\omega^2}$$

- (a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.
 (b) Use the above result, along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}$$

Problem 9. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure 1.

- (a) Find $\angle X(j\omega)$.
 (b) Find $X(j0)$.
 (c) Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$.
 (d) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.
 (e) Sketch the inverse Fourier transform of $\Re\{X(j\omega)\}$.

Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

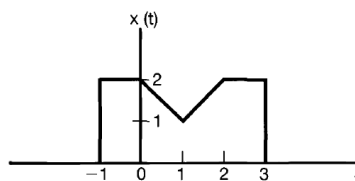


Figure 1