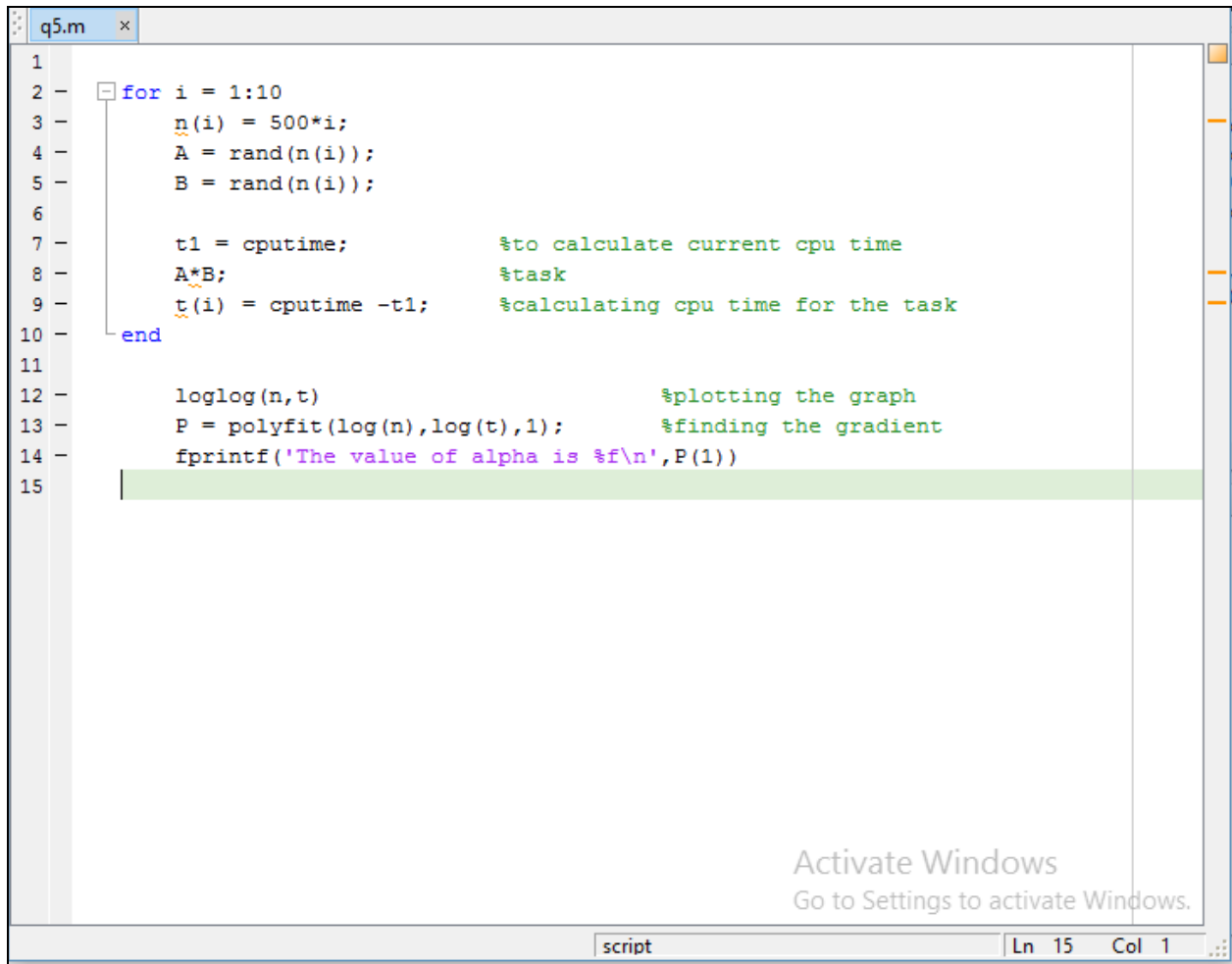


Computer Experiments

(5)



```
1
2 - for i = 1:10
3 -     n(i) = 500*i;
4 -     A = rand(n(i));
5 -     B = rand(n(i));
6
7 -     t1 = cputime;           %to calculate current cpu time
8 -     A*B;                   %task
9 -     t(i) = cputime -t1;     %calculating cpu time for the task
10 - end
11
12 -     loglog(n,t)            %plotting the graph
13 -     P = polyfit(log(n),log(t),1); %finding the gradient
14 -     fprintf('The value of alpha is %f\n',P(1))
15
```

Activate Windows
Go to Settings to activate Windows.

script Ln 15 Col 1

Figure 1: Script screenshot of q5

Code for question (5)

```
for i = 1:10
    n(i) = 500*i;
    A = rand(n(i));
    B = rand(n(i));

    t1 = cputime;           %to calculate current cpu time
    A*B;                   %task
    t(i) = cputime -t1;     %calculating cpu time for the task
end

loglog(n,t)            %plotting the graph
P = polyfit(log(n),log(t),1); %finding the gradient
fprintf('The value of alpha is %f\n',P(1))
```

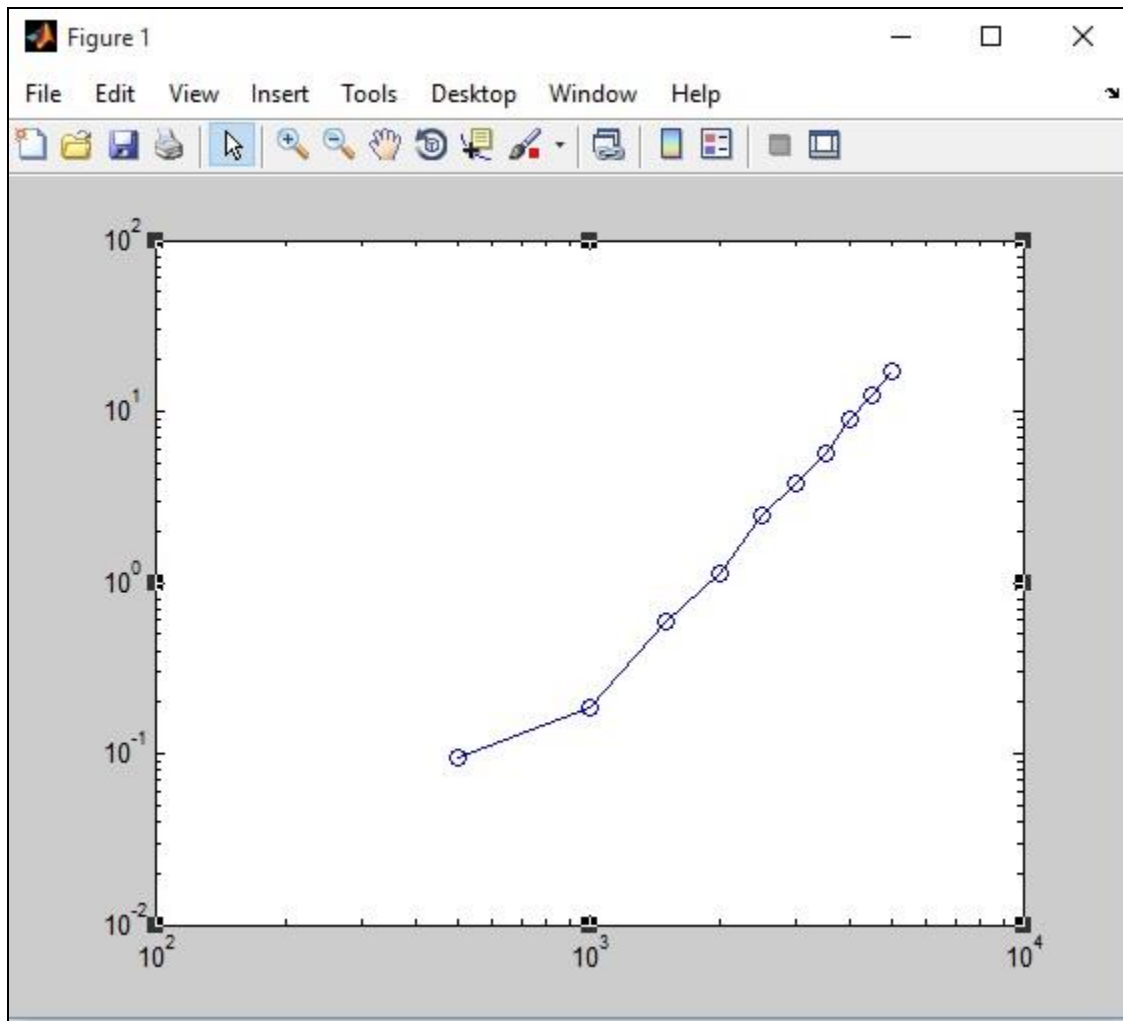


Figure 2: The variation of $\log(n)$ against $\log(t)$

Calculations

$$\begin{aligned} t &= Cn^\alpha \\ \log(t) &= \log(C) + \log(n^\alpha) \\ \log(t) &= k + \alpha \log(n) \end{aligned}$$

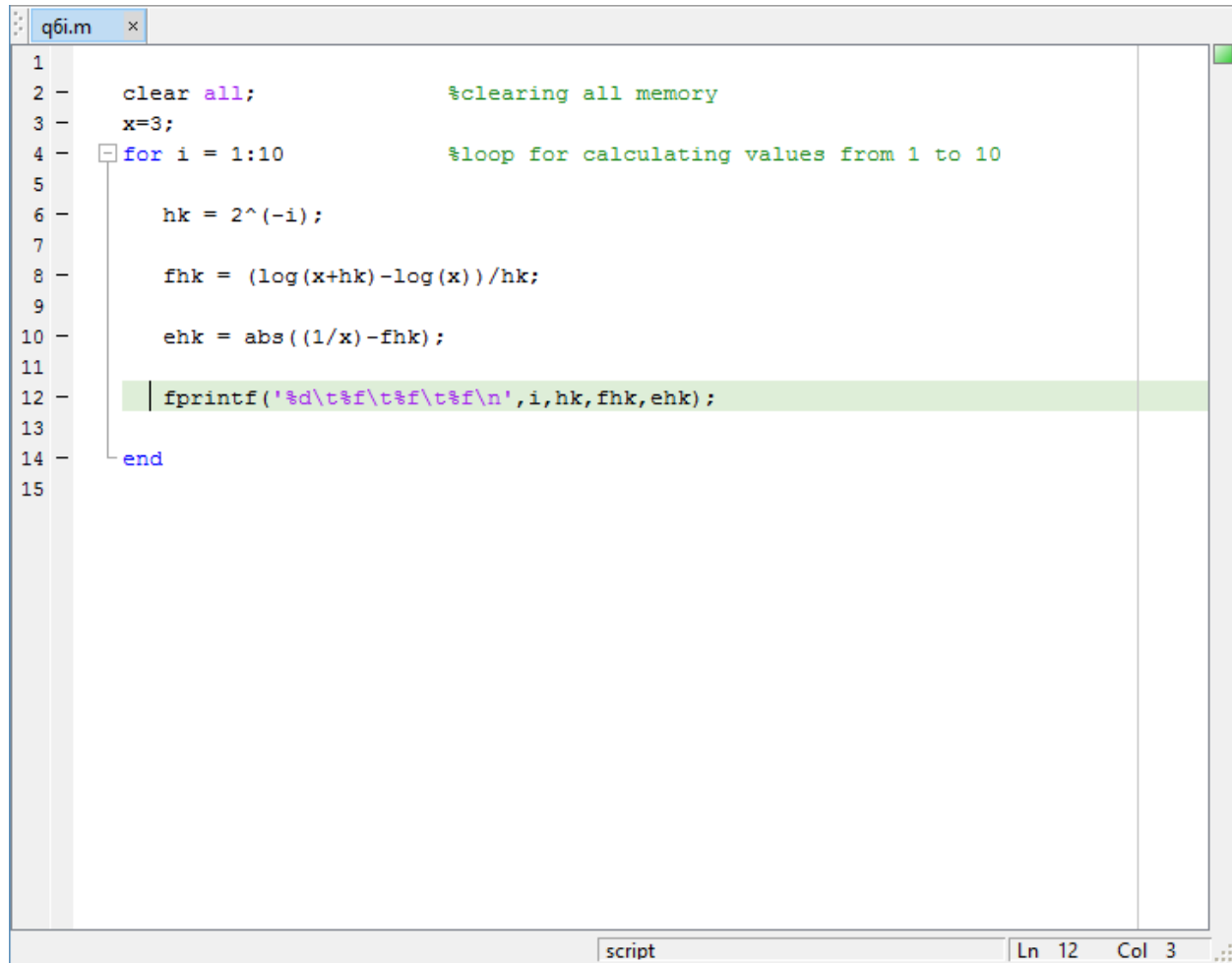
α can be estimate by 'polyfit' function

According to the theoretical problem value of alpha is 3

Output

- The value of alpha is 2.475217 (according to the polyfit of graph)

6) (a)

A screenshot of a MATLAB script editor window. The title bar shows the file name 'q6i.m' and a close button. The editor contains a script with 15 lines of code. Line 12 is highlighted in green. The status bar at the bottom indicates 'script', 'Ln 12', and 'Col 3'.

```
1  
2 - clear all; %clearing all memory  
3 - x=3;  
4 - for i = 1:10 %loop for calculating values from 1 to 10  
5  
6 - hk = 2^(-i);  
7  
8 - fhk = (log(x+hk)-log(x))/hk;  
9  
10 - ehk = abs((1/x)-fhk);  
11  
12 - fprintf('%d\t%f\t%f\t%f\n',i,hk,fhk,ehk);  
13  
14 - end  
15
```

Figure 3: Script screenshot for q6i

Code for question (6) (a)

```
clear all; %clearing all memory  
x=3;  
for i = 1:10 %loop for calculating values from 1 to 10  
  
    hk = 2^(-i);  
    fhk = (log(x+hk)-log(x))/hk;  
    ehk = abs((1/x)-fhk);  
    fprintf('%d\t%f\t%f\t%f\n',i,hk,fhk,ehk);  
  
end
```

```

Command Window
>> q6i
1  0.500000  0.308301  0.025032
2  0.250000  0.320171  0.013163
3  0.125000  0.326576  0.006757
4  0.062500  0.329909  0.003425
5  0.031250  0.331609  0.001724
6  0.015625  0.332468  0.000865
7  0.007813  0.332900  0.000433
8  0.003906  0.333117  0.000217
9  0.001953  0.333225  0.000108
10 0.000977  0.333279  0.000054
fx >> |

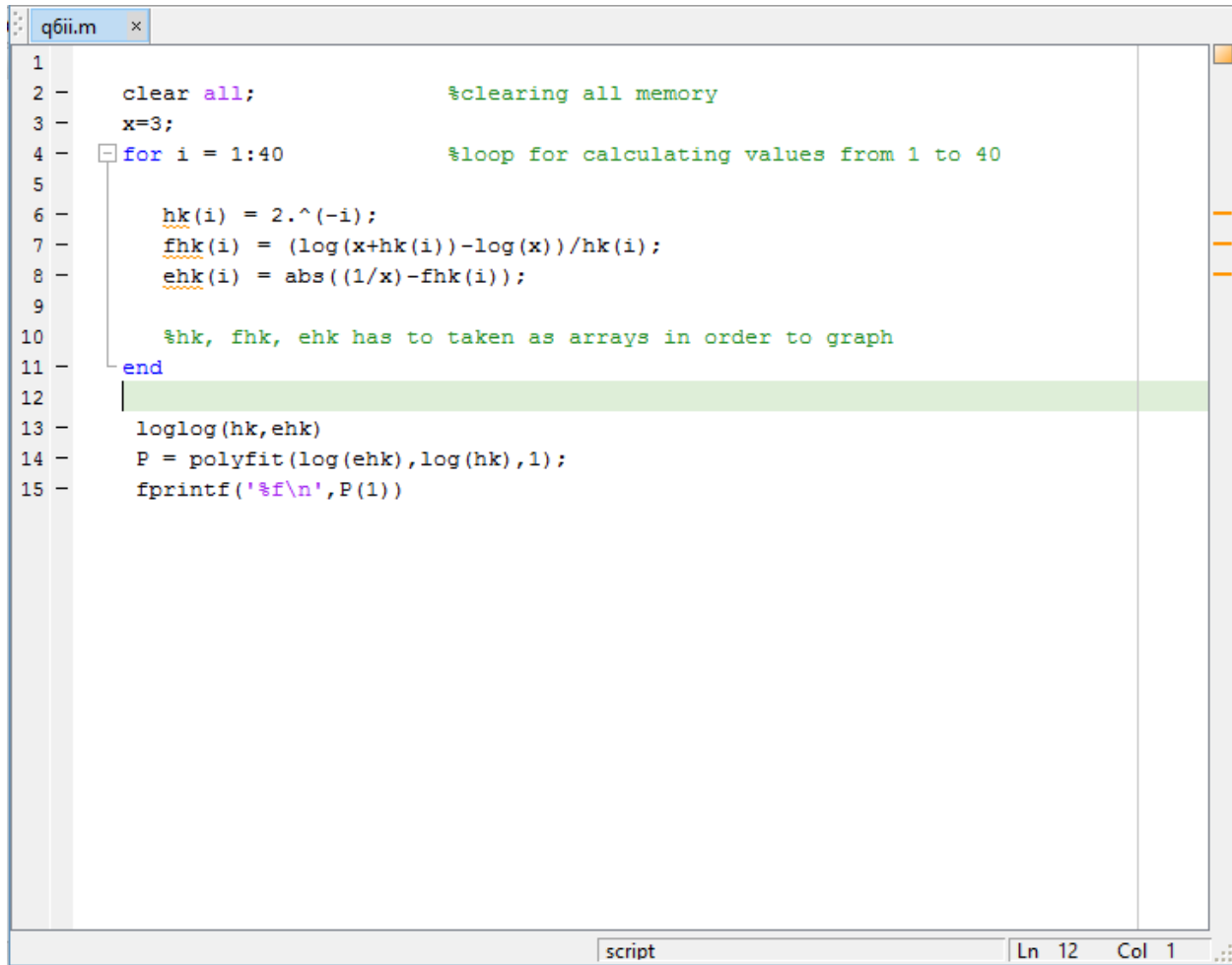
```

Figure 4: The resulting output screenshot for q6i

k	h_k	$f_{hk}(x)$	E_{hk}
1	0.500000	0.308301	0.025032
2	0.250000	0.320171	0.013163
3	0.125000	0.326576	0.006757
4	0.062500	0.329909	0.003425
5	0.031250	0.331609	0.001724
6	0.015625	0.332468	0.000865
7	0.007813	0.332900	0.000433
8	0.003906	0.333117	0.000217
9	0.001953	0.333225	0.000108
10	0.000977	0.333279	0.000054

Table 1: The resulting output for q6i

(b)

A screenshot of a MATLAB script editor window titled 'q6ii.m'. The script contains 15 lines of code. Line 2: 'clear all;' with a comment '%clearing all memory'. Line 3: 'x=3;'. Line 4: 'for i = 1:40' with a comment '%loop for calculating values from 1 to 40'. Line 6: 'hk(i) = 2.^(-i);'. Line 7: 'fhk(i) = (log(x+hk(i))-log(x))/hk(i);'. Line 8: 'ehk(i) = abs((1/x)-fhk(i));'. Line 10: A comment '%hk, fhk, ehk has to taken as arrays in order to graph'. Line 11: 'end'. Line 13: 'loglog(hk,ehk)'. Line 14: 'P = polyfit(log(ehk),log(hk),1);'. Line 15: 'fprintf('%f\n',P(1))'. The status bar at the bottom shows 'script', 'Ln 12', and 'Col 1'.

```
1  
2 - clear all; %clearing all memory  
3 - x=3;  
4 - for i = 1:40 %loop for calculating values from 1 to 40  
5  
6 - hk(i) = 2.^(-i);  
7 - fhk(i) = (log(x+hk(i))-log(x))/hk(i);  
8 - ehk(i) = abs((1/x)-fhk(i));  
9  
10 %hk, fhk, ehk has to taken as arrays in order to graph  
11 - end  
12  
13 - loglog(hk,ehk)  
14 - P = polyfit(log(ehk),log(hk),1);  
15 - fprintf('%f\n',P(1))
```

Figure 5: Script screenshot for q6ii

Code for question (6) (b)

```
clear all; %clearing all memory  
x=3;  
for i = 1:40 %loop for calculating values from 1 to 40  
  
hk(i) = 2.^(-i);  
fhk(i) = (log(x+hk(i))-log(x))/hk(i);  
ehk(i) = abs((1/x)-fhk(i));  
  
%hk, fhk, ehk has to taken as arrays in order to graph  
end  
  
loglog(hk,ehk)  
P = polyfit(log(ehk),log(hk),1);  
fprintf('%f\n',P(1))
```

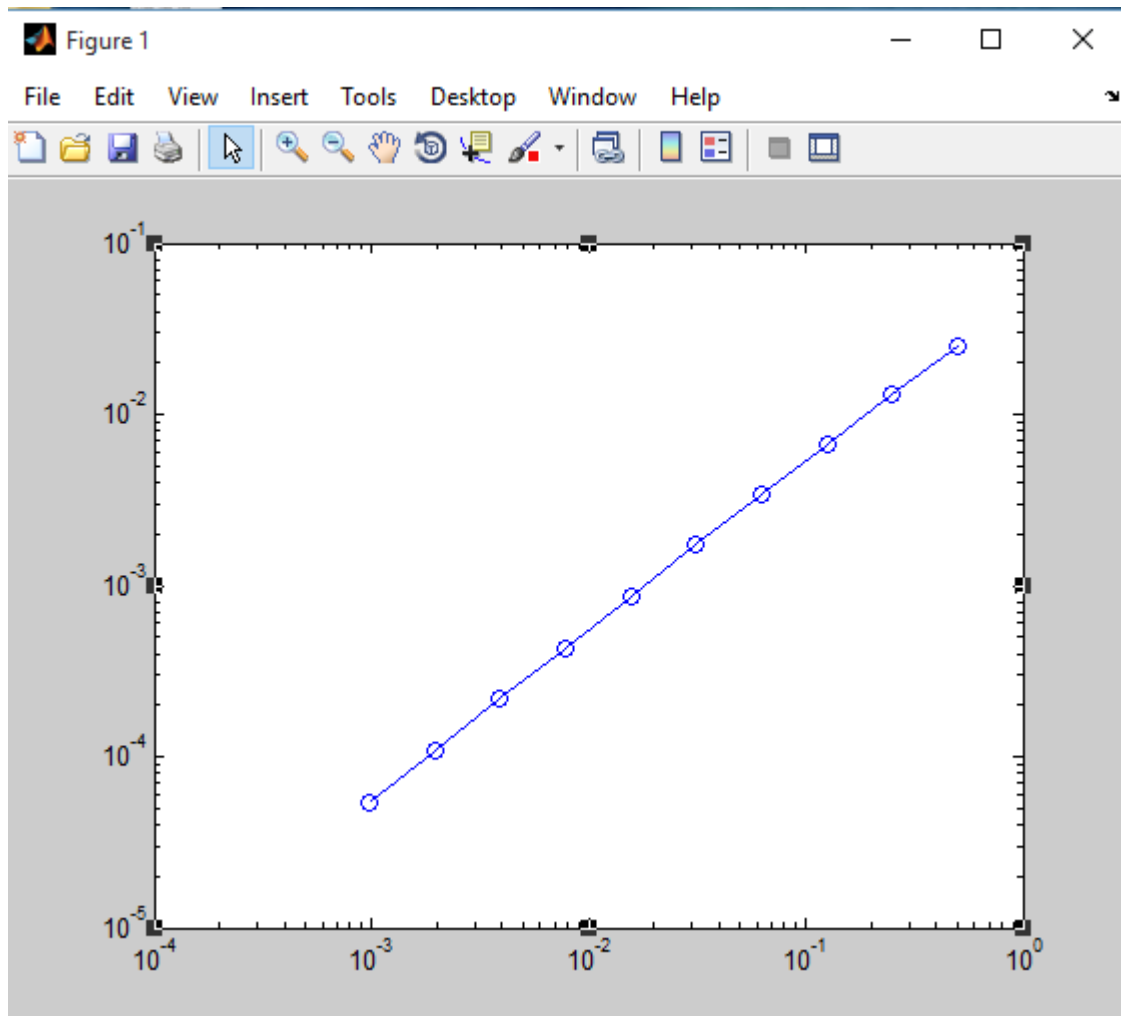


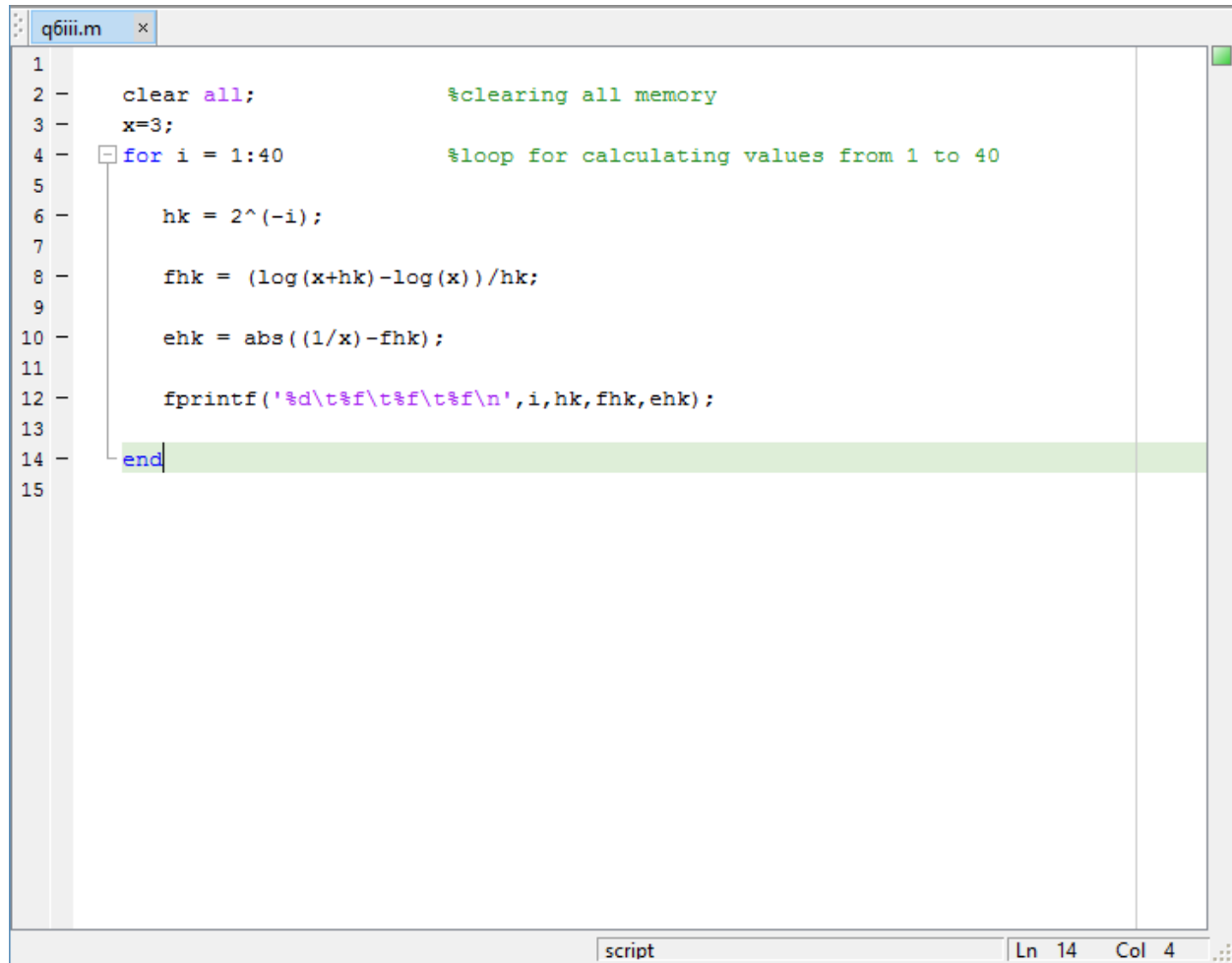
Figure 6: The variation of $\log(E_h)$ against $\log(h)$ ($N = 10$)

Let's assume that ,

$$\begin{aligned}
 E_h &\propto h^\gamma \\
 E_h &= C h^\gamma \\
 \log(E_h) &= k + \log(h^\gamma) \\
 \log(E_h) &= k + \gamma \log(h)
 \end{aligned}$$

- γ can be estimate by 'polyfit' function
- The value of gamma according to the polyfit 1.013012
- If $E_h = O(h)$ value of gamma is 1
- These values above are so close. Therefore, $E_h = O(h)$ is a valid expression

(c)

A screenshot of the MATLAB script editor window. The title bar shows 'q6iii.m' with a close button. The script is as follows:

```
1  
2 - clear all;           %clearing all memory  
3 - x=3;  
4 - for i = 1:40         %loop for calculating values from 1 to 40  
5  
6 -     hk = 2^(-i);  
7  
8 -     fhk = (log(x+hk)-log(x))/hk;  
9  
10 -    ehk = abs((1/x)-fhk);  
11  
12 -    fprintf('%d\t%f\t%f\t%f\n',i,hk,fhk,ehk);  
13  
14 - end  
15
```

The line numbers 1 through 15 are on the left. The script content is on the right. The 'end' statement on line 14 is highlighted with a green background. At the bottom, the status bar shows 'script', 'Ln 14', and 'Col 4'.

Figure 5: Script screenshot for q6iii

Code for question (6) (c)

```
clear all;           %clearing all memory  
x=3;  
for i = 1:40         %loop for calculating values from 1 to 40  
  
    hk = 2^(-i);  
    fhk = (log(x+hk)-log(x))/hk;  
    ehk = abs((1/x)-fhk);  
    fprintf('%d\t%f\t%f\t%f\n',i,hk,fhk,ehk);  
  
end
```

Command Window			
>> q6iii			
1	0.500000	0.308301	0.025032
2	0.250000	0.320171	0.013163
3	0.125000	0.326576	0.006757
4	0.062500	0.329909	0.003425
5	0.031250	0.331609	0.001724
6	0.015625	0.332468	0.000865
7	0.007813	0.332900	0.000433
8	0.003906	0.333117	0.000217
9	0.001953	0.333225	0.000108
10	0.000977	0.333279	0.000054
11	0.000488	0.333306	0.000027
12	0.000244	0.333320	0.000014
13	0.000122	0.333327	0.000007
14	0.000061	0.333330	0.000003
15	0.000031	0.333332	0.000002
16	0.000015	0.333332	0.000001
17	0.000008	0.333333	0.000000
18	0.000004	0.333333	0.000000
19	0.000002	0.333333	0.000000
20	0.000001	0.333333	0.000000
21	0.000000	0.333333	0.000000
22	0.000000	0.333333	0.000000
23	0.000000	0.333333	0.000000
24	0.000000	0.333333	0.000000
25	0.000000	0.333333	0.000000
26	0.000000	0.333333	0.000000
27	0.000000	0.333333	0.000000

Figure 6.1: The resulting output screenshot for q6iii

Command Window			
14	0.000061	0.333330	0.000003
15	0.000031	0.333332	0.000002
16	0.000015	0.333332	0.000001
17	0.000008	0.333333	0.000000
18	0.000004	0.333333	0.000000
19	0.000002	0.333333	0.000000
20	0.000001	0.333333	0.000000
21	0.000000	0.333333	0.000000
22	0.000000	0.333333	0.000000
23	0.000000	0.333333	0.000000
24	0.000000	0.333333	0.000000
25	0.000000	0.333333	0.000000
26	0.000000	0.333333	0.000000
27	0.000000	0.333333	0.000000
28	0.000000	0.333333	0.000000
29	0.000000	0.333333	0.000000
30	0.000000	0.333333	0.000000
31	0.000000	0.333333	0.000000
32	0.000000	0.333334	0.000001
33	0.000000	0.333334	0.000001
34	0.000000	0.333336	0.000003
35	0.000000	0.333336	0.000003
36	0.000000	0.333344	0.000010
37	0.000000	0.333344	0.000010
38	0.000000	0.333374	0.000041
39	0.000000	0.333374	0.000041
40	0.000000	0.333496	0.000163
fx >>			

Figure 6.2: The resulting output screenshot for q6iii (continued)

k	h_k	$f_{hk}(x)$	E_{hk}
1	0.500000	0.308301	0.025032
2	0.250000	0.320171	0.013163
3	0.125000	0.326576	0.006757
4	0.062500	0.329909	0.003425
5	0.031250	0.331609	0.001724
6	0.015625	0.332468	0.000865
7	0.007813	0.332900	0.000433
8	0.003906	0.333117	0.000217
9	0.001953	0.333225	0.000108
10	0.000977	0.333279	0.000054
11	0.000488	0.333306	0.000027
12	0.000244	0.333320	0.000014
13	0.000122	0.333327	0.000007
14	0.000061	0.333330	0.000003
15	0.000031	0.333332	0.000002
16	0.000015	0.333332	0.000001
17	0.000008	0.333333	0.000000
18	0.000004	0.333333	0.000000
19	0.000002	0.333333	0.000000
20	0.000001	0.333333	0.000000
21	0.000000	0.333333	0.000000
22	0.000000	0.333333	0.000000
23	0.000000	0.333333	0.000000
24	0.000000	0.333333	0.000000
25	0.000000	0.333333	0.000000
26	0.000000	0.333333	0.000000
27	0.000000	0.333333	0.000000
28	0.000000	0.333333	0.000000
29	0.000000	0.333333	0.000000
30	0.000000	0.333333	0.000000
31	0.000000	0.333333	0.000000
32	0.000000	0.333334	0.000001
33	0.000000	0.333334	0.000001
34	0.000000	0.333336	0.000003
35	0.000000	0.333336	0.000003
36	0.000000	0.333344	0.000010
37	0.000000	0.333344	0.000010
38	0.000000	0.333374	0.000041
39	0.000000	0.333374	0.000041
40	0.000000	0.333496	0.000163

Table 2: The resulting output for q6iii

- (d) According to the values in the table 2, E_{hk} decreases at the beginning. But after the 32nd iteration, again E_{hk} starts rising.

E_{hk} is the absolute difference of $f'(x)$ and $f'_{hk}(x)$. As it appears up to 32nd iteration E_{hk} reaches to zero and conceptually it should be happened afterwards too. For MATLAB the machine epsilon (the lowest number that can be represented) is about 10^{-16} . As E_{hk} value decreases below this eps value, particular value can't be represent properly and therefore, E_{hk} value varies in an unusual manner.

- (e)

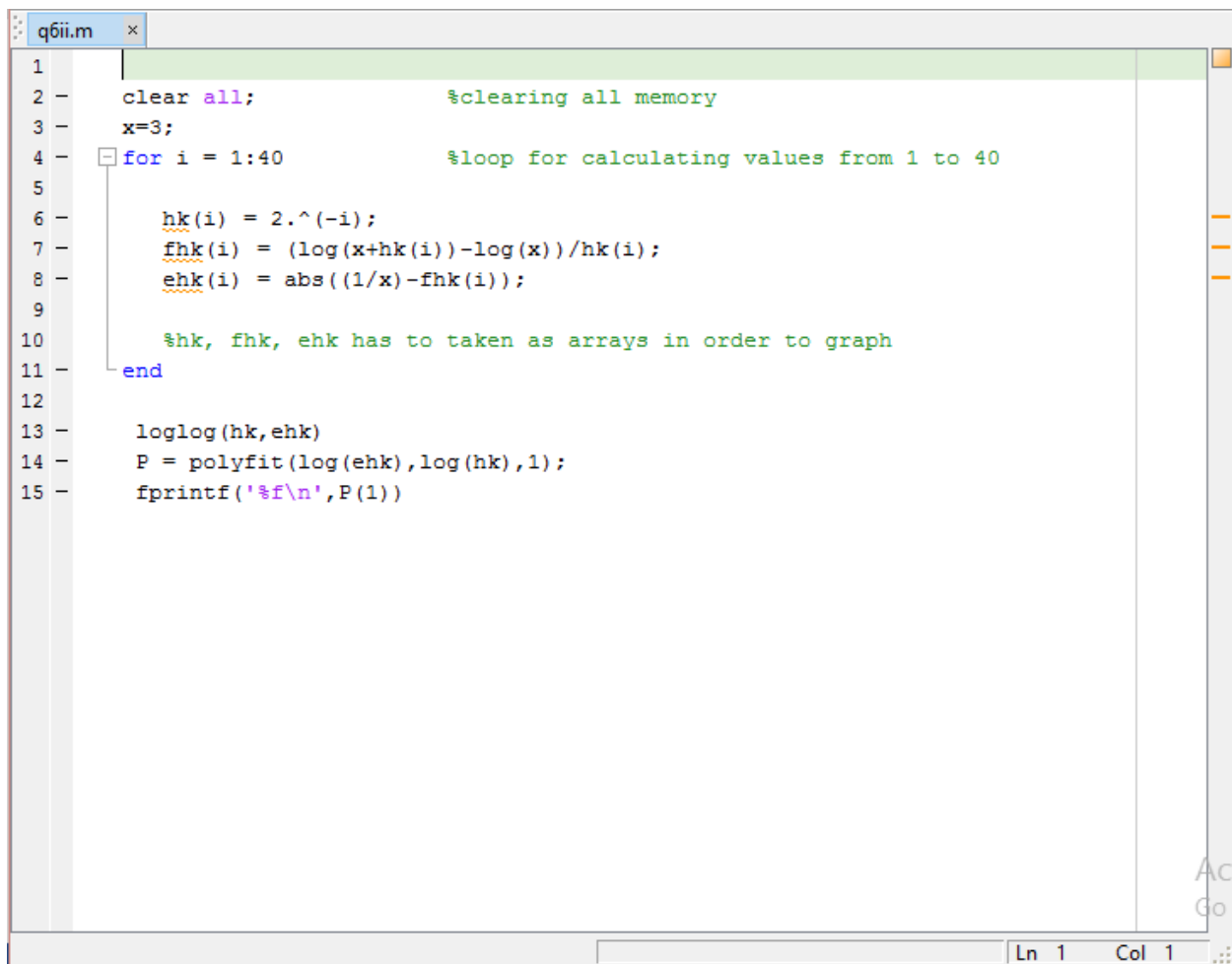
A screenshot of a MATLAB script editor window titled 'q6ii.m'. The script contains 15 lines of code. Line 1 is a blank line. Line 2 is 'clear all;' with a comment '%clearing all memory'. Line 3 is 'x=3;'. Line 4 is 'for i = 1:40' with a comment '%loop for calculating values from 1 to 40'. Line 5 is a blank line. Line 6 is 'hk(i) = 2.^(-i);'. Line 7 is 'fhk(i) = (log(x+hk(i))-log(x))/hk(i);'. Line 8 is 'ehk(i) = abs((1/x)-fhk(i));'. Line 9 is a blank line. Line 10 is a comment '%hk, fhk, ehk has to taken as arrays in order to graph'. Line 11 is 'end'. Line 12 is a blank line. Line 13 is 'loglog(hk,ehk)'. Line 14 is 'P = polyfit(log(ehk),log(hk),1);'. Line 15 is 'fprintf('%f\n',P(1))'. The script is displayed in a monospaced font with syntax highlighting. The window has a standard MATLAB interface with a title bar, a menu bar, and a status bar at the bottom showing 'Ln 1 Col 1'.

Figure 7: Script screenshot for q6iv

```

clear all; %clearing all memory
x=3;
for i = 1:40 %loop for calculating values from 1 to 40

    hk(i) = 2.^(-i);
    fhk(i) = (log(x+hk(i))-log(x))/hk(i);
    ehk(i) = abs((1/x)-fhk(i));

    %hk, fhk, ehk has to taken as arrays in order to graph
end

loglog(hk,ehk)
P = polyfit(log(ehk),log(hk),1);
fprintf('%f\n',P(1))

```

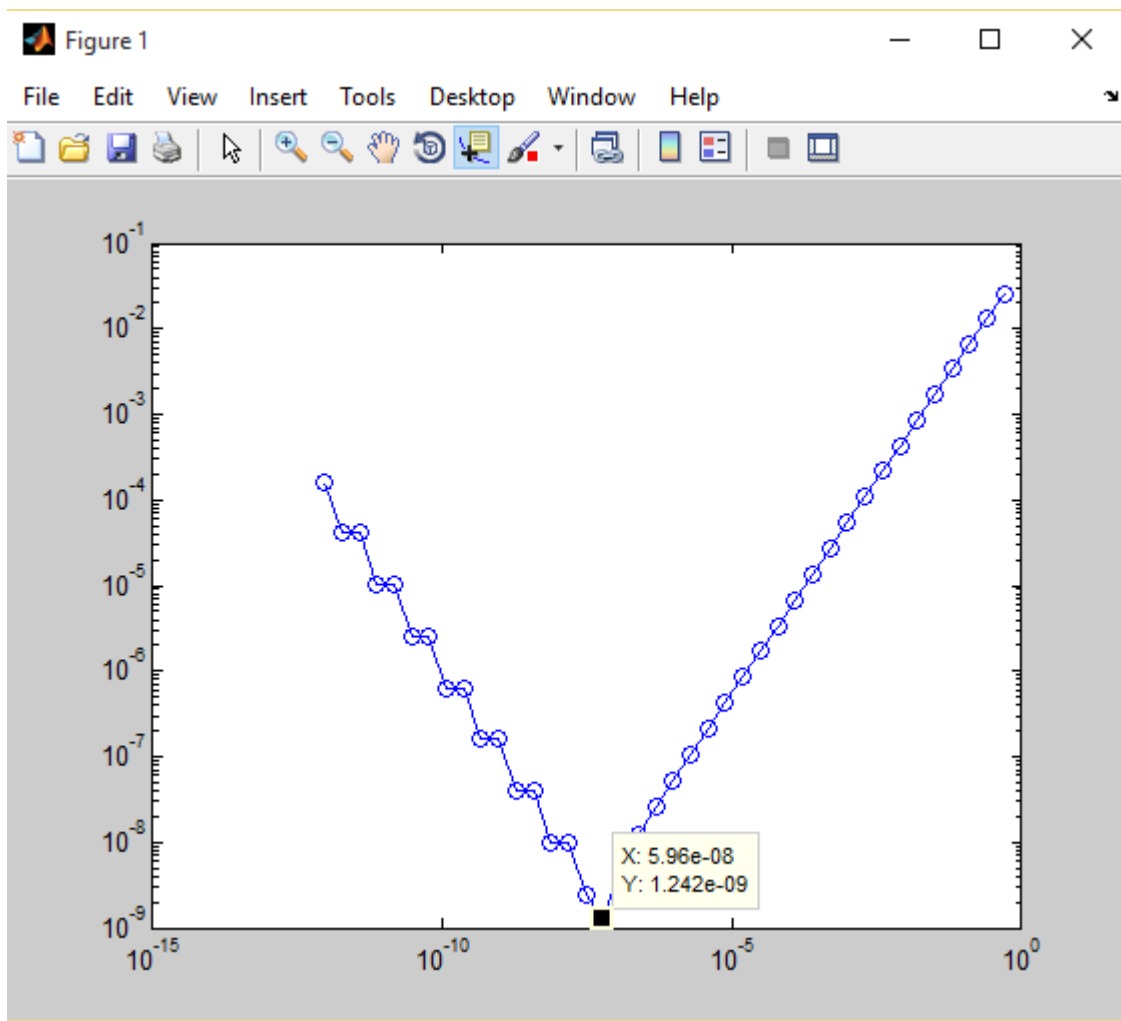


Figure 8: The variation of $\log(E_h)$ against $\log(h)$ ($N = 40$)

$$\log(h_{\min}) = 5.96 \times 10^{-8}$$

$$h_{\min} = 1.0000$$