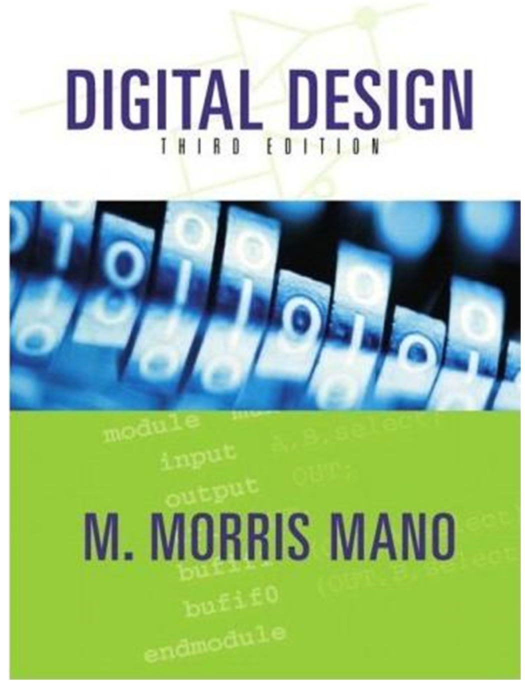


Digital Design 3e, Morris Mano

Chapter 2 – Boolean Algebra and Logic Gates

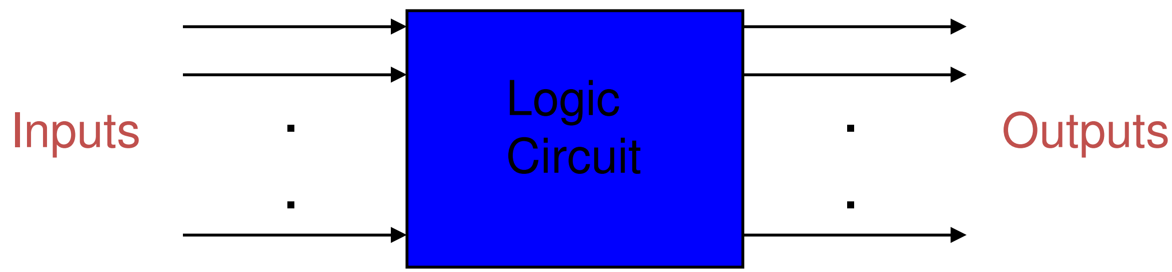
Dr. Eng. Kamalanath Samarakoon

Overview

- Logic functions with 1's and 0's
  - Building digital circuitry
- Truth tables
- Logic symbols and waveforms
- Boolean algebra
- Properties of Boolean Algebra
  - Reducing functions
  - Transforming functions

Digital Systems

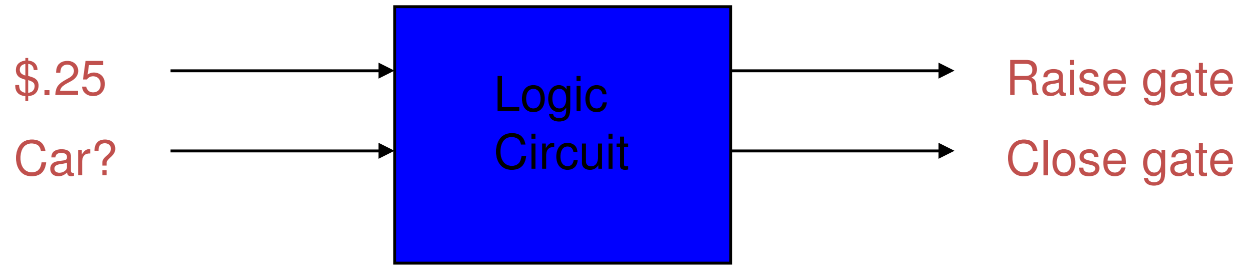
- Analysis problem:



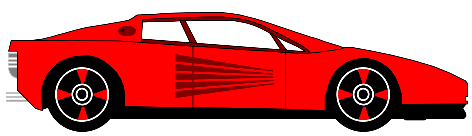
  - Determine binary outputs for each combination of inputs
- Design problem: given a task, develop a circuit that accomplishes the task
  - Many possible implementation
  - Try to develop “best” circuit based on some criterion (size, power, performance, etc.)

Toll Booth Controller

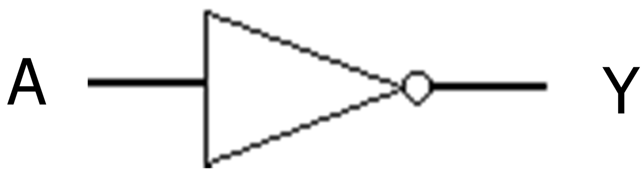
- Consider the design of a toll booth controller
- Inputs: quarter, car sensor
- Outputs: gate lift signal, gate close signal



- If driver pitches in quarter, raise gate.
- When car has cleared gate, close gate.



Describing Circuit Functionality: Inverter



Symbol

Truth Table

A	Y
N	**Y**
Y	**N**

Input Output

A	Y
F	**T**
T	**F**

- Basic logic functions have symbols.
- The same functionality can be represented with truth tables.
  - Truth table completely specifies outputs for all input combinations.
- Why it is called truth table?
- The above circuit is an inverter.
  - An input of 0 is inverted to a 1.
  - An input of 1 is inverted to a 0.

A	Y
0	**1**
1	**0**

Positive logic and Negative Logic

- Positive logic
- 0 = 0V
- 1 = 5V

- Negative logic
- 0=5V
- 1=0V

The AND Gate

- This is an AND gate.
- So, if the two inputs signals are asserted (high) the output will also be asserted. Otherwise, the output will be deasserted (low).

Truth Table

A	B	Y
0	**0**	**0**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

The OR Gate

- This is an OR gate.
- So, if either of the two input signals are asserted, or both of them are, the output will be asserted.

A	B	Y
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**1**

Describing Circuit Functionality: Waveforms

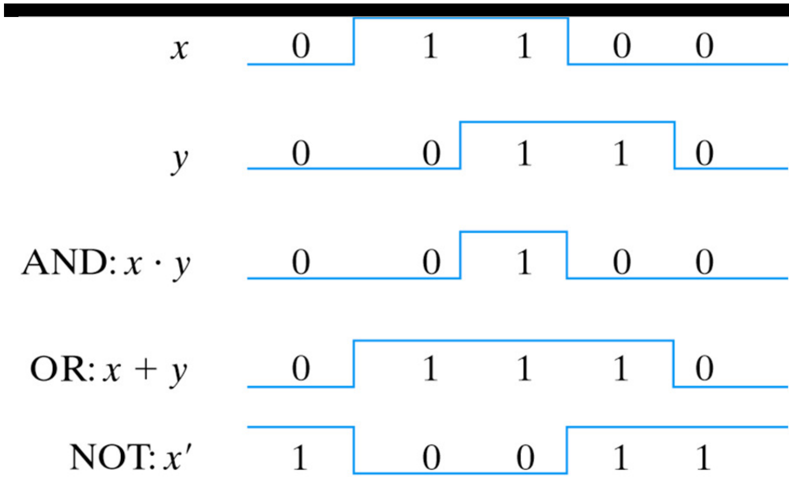


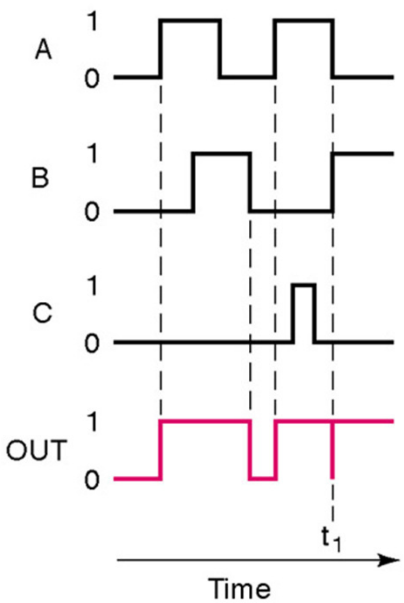
Fig. 1-5 Input-output signals for gates

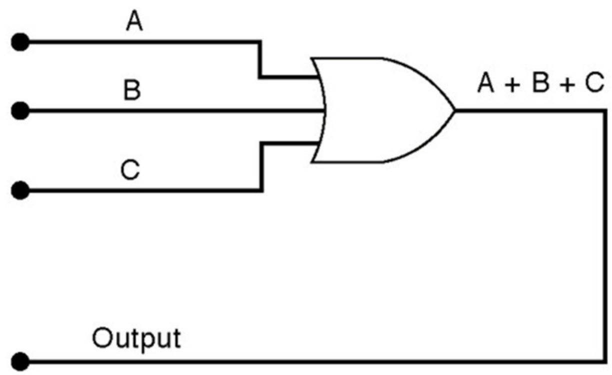
AND Gate

A	B	Y
0	**0**	**0**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

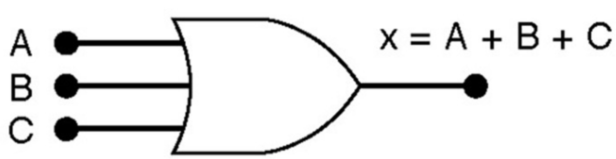
- Waveforms provide another approach for representing functionality.
- Values are either high (logic 1) or low (logic 0).
- Can you create a truth table from the waveforms?

Consider three-input gates





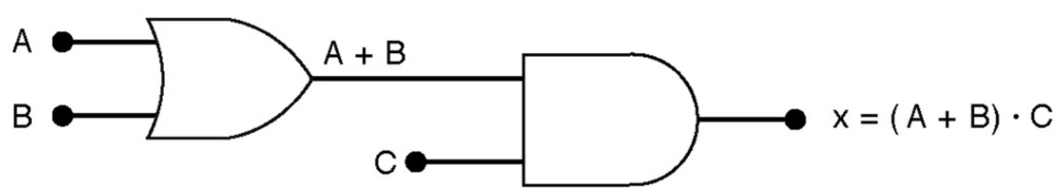
3 Input OR Gate



0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Ordering Boolean Functions

- How to interpret A•B+C?
  - Is it A•B ORed with C ?
  - Is it A ANDed with B+C ?
- Order of precedence for Boolean algebra: AND before OR.
- Note that parentheses are needed here :



Boolean Algebra

- A *Boolean algebra* is defined as a closed algebraic system containing a set K or two or more elements and the two operators, . and +.
- Useful for identifying and *minimizing* circuit functionality
- Identity elements
  - a + 0 = a
  - a . 1 = a
- 0 is the identity element for the + operation.
- 1 is the identity element for the . operation.

Commutativity and Associativity of the Operators

- The Commutative Property:

  For every a and b in K,

  - $a + b = b + a$
  - $a \cdot b = b \cdot a$

- The Associative Property:

  For every a, b, and c in K,

  - $a + (b + c) = (a + b) + c$
  - $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Distributivity of the Operators and Complements

- The Distributive Property:
  For every a, b, and c in K,
  - $a + (b . c) = (a + b) . (a + c)$
  - $a . (b + c) = (a . b) + (a . c)$
- The Existence of the Complement:
  For every a in K there exists a unique element called a’ (*complement of a*) such that,
  - $a + a' = 1$
  - $a . a' = 0$
- To simplify notation, the . operator is frequently omitted. When two elements are written next to each other, the AND (.) operator is implied…
  - $a + b . c = (a + b) . (a + c)$
  - $a + bc = (a + b)(a + c)$

Duality

- The principle of *duality* is an important concept. This says that if an expression is valid in Boolean algebra, the dual of that expression is also valid.
- To form the dual of an expression, replace all + operators with . operators, all . operators with + operators, all ones with zeros, and all zeros with ones.
- Form the dual of the expression

  $a + (bc) = (a + b)(a + c)$
- Following the replacement rules…

  $a(b + c) = ab + ac$
- Take care not to alter the location of the parentheses if they are present.

Involution

- This theorem states:

  $(a')' = a$

- Taking the double inverse of a value will give the initial value.

Absorption

- This theorem states:

  $a + ab = a$ $\qquad\qquad$ $a(a+b) = a$

- To prove the first half of this theorem:

$$
\begin{aligned}
a + ab &= a \,.\, 1 + ab \\
&= a\,(1 + b) \\
&= a\,(b + 1) \\
&= a\,(1) \\
a + ab &= a
\end{aligned}
$$

DeMorgan's Theorem

- A key theorem in simplifying Boolean algebra expression is DeMorgan's Theorem. It states:

  $(a + b)' = a'b'$ $\qquad\qquad$ $(ab)' = a' + b'$

- Complement the expression $a(b + z(x + a'))$ and simplify.

$$
\begin{aligned}
(a(b+z(x+a')))' &= a' + (b + z(x + a'))' \\
&= a' + b'(z(x + a'))' \\
&= a' + b'(z' + (x + a')') \\
&= a' + b'(z' + x'a'') \\
&= a' + b'(z' + x'a)
\end{aligned}
$$

Summary

- Basic logic functions can be made from AND, OR, and NOT (invert) functions
- The behavior of digital circuits can be represented with waveforms, truth tables, or symbols
- Primitive gates can be combined to form larger circuits
- Boolean algebra defines how binary variables can be combined
- Rules for associativity, commutativity, and distribution are similar to algebra
- DeMorgan’s rules are important.
  - Will allow us to reduce circuit sizes.

Acknowledgement

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