AI1103: Challenging Problem 2

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Download all latex codes from

https://github.com/Geetha495/AI1103/blob/main/Challenging_Problems/Challenging_Problem2/Challenging_Problem2.pdf

1 Problem

Suppose X is a random variable such that $E(X) = 0, E(X^2) = 2, E(X^4) = 4$. Then

1) $E(X^3) = 0$
2) $\Pr(X \geq 0) = \frac{1}{2}$
3) $X \sim N(0, 2)$
4) X is bounded with Probability 1.

2 Solution

Let $Y = X^2$ be a random variable, Then

$$\begin{aligned}\sigma(Y) &= E(Y^2) - (E(Y))^2 \\ &= E(X^4) - (E(X^2))^2 \\ &= 0\end{aligned}$$

So, Y is a constant random variable.
Thus, for all $x \in X$, $x^2 = c$, where c is constant.

1)

$$\begin{aligned}E(X^3) &= \sum_{x\in X} x^3 p_X(x) \\ &= c \times \sum_{x\in X} x p_X(x) \\ &= c \times E(X) \\ &= 0\end{aligned}$$

Option 1 is correct.

2)

$$\begin{aligned}E(X) &= \sum_{x\in X} x p_X(x) \\ 0 &= \sqrt{c}\Pr(X \geq 0) + \left(-\sqrt{c}\Pr(X < 0)\right) \\ \Pr(X \geq 0) &= \Pr(X < 0)\end{aligned}$$

$$\text{As } \sum_{x\in X} p_X(x) = 1$$

$$\begin{aligned}\Pr(X \geq 0) + \Pr(X < 0) &= 1 \\ \Pr(X \geq 0) &= \frac{1}{2}\end{aligned}$$

Option 2 is correct.

3)

$$p_X(x) = \begin{cases} \frac{1}{2} & x = \pm\sqrt{c} \\ 0 & \text{otherwise} \end{cases}$$

Hence X forms a discrete probability distribution. So, it can't be normal distribution
Option 3 is wrong .

4) As $p_X(x)$ takes only values $\frac{1}{2}$ and 0 , X is bounded.
Option 4 is correct.

Thus, correct options are 1,2,4.