

AI1103: Assignment 8

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Download all latex codes from

<https://github.com/Geetha495/Assignment8/blob/main/Assignment8.tex>

Option 3 :

$$\begin{aligned}\phi_X(-t) &= E(e^{i(-t)X}) \\ &= E(e^{it(-X)}) \\ &= E(e^{itY}) \\ &= \phi_Y(t)\end{aligned}$$

1 PROBLEM

Let $\phi(t)$ be a characteristic function of some random variable. Then, which of the following is also a characteristic function ?

- 1) $f(t) = [\phi(t)]^2$ for all $t \in \mathbb{R}$
- 2) $f(t) = |\phi(t)|^2$ for all $t \in \mathbb{R}$
- 3) $f(t) = \phi(-t)$ for all $t \in \mathbb{R}$
- 4) $f(t) = \phi(t+1)$ for all $t \in \mathbb{R}$

Thus, $\phi(-t)$ is a characteristic function of random variable Y .

Option 4 :

$$\begin{aligned}\phi_X(t+1) &= E(e^{i(t+1)X}) \\ &= E(e^{itX} \times e^{iX})\end{aligned}$$

Thus, $\phi(t+1)$ is not a characteristic function.

2 SOLUTION

Properties of a Characteristic function

- 1) $\phi_X(t) = E(e^{itX})$
- 2) $\phi_X(t) \times \phi_Y(t) = \phi_{X+Y}(t)$
- 3) If Y and $-X$ have the same distribution then,
 $\phi_Y(t) = \overline{\phi_X(t)}$

Let X be the given random variable and let Y and $-X$ have the same distribution.

Option 1 :

$$\begin{aligned}[\phi_X(t)]^2 &= \phi_X(t) \times \phi_X(t) \\ &= \phi_{2X}(t)\end{aligned}$$

Thus, $[\phi(t)]^2$ is a characteristic function of random variable $2X$.

Option 2 :

$$\begin{aligned}|\phi_X(t)|^2 &= \phi_X(t) \times \overline{\phi_X(t)} \\ &= \phi_X(t) \times \phi_Y(t) \\ &= \phi_{X+Y}(t)\end{aligned}$$

Thus, $|\phi(t)|^2$ is a characteristic function of random variable $(X + Y)$.

Hence, correct options are 1, 2, 3.