

AI1103: Assignment 7

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Download all latex codes from

<https://github.com/Geetha495/Assignment7/blob/main/Assignment7.tex>

Download all python codes from

<https://github.com/Geetha495/Assignment7/blob/main/Assignment7.py>

1 PROBLEM

Suppose X is a positive random variable with the following probability density function,

$$f(x) = (\alpha x^{\alpha-1} + \beta x^{\beta-1})e^{-x^\alpha - x^\beta}; x > 0$$

for $\alpha > 0, \beta > 0$. Then the hazard function of X for some choices of α and β can be

- 1) an increasing function.
- 2) a decreasing function.
- 3) a constant function.
- 4) a non monotonic function

2 SOLUTION

CDF of X ,

$$F(x) = \int_{-\infty}^x f(t)dt \quad (2.0.1)$$

$$= \int_0^x f(t)dt \quad \text{as } x > 0 \quad (2.0.2)$$

$$= \int_{-\infty}^x ((\alpha t^{\alpha-1} + \beta t^{\beta-1}) \times e^{-t^\alpha - t^\beta}) dt \quad (2.0.3)$$

$$= -e^{-t^\alpha - t^\beta} \Big|_0^x \quad (2.0.4)$$

$$= 1 - e^{-x^\alpha - x^\beta} \quad (2.0.5)$$

Hazard function,

$$h(x) = \frac{f(x)}{1 - F(x)} \quad (2.0.6)$$

$$= \alpha x^{\alpha-1} + \beta x^{\beta-1} \quad (2.0.7)$$

As α, β are positive, $h(x)$ is an increasing function.
Correct option is 1.