

# AI1103: Assignment 8

Chitneedi Geetha Sowmya  
CS20BTECH11011

Download all latex codes from

<https://github.com/Geetha495/Assignment8/blob/main/Assignment8.tex>

Download all python codes from

<https://github.com/Geetha495/Assignment8/blob/main/Assignment8.py>

Option 3 :

$$\begin{aligned}\phi_X(-t) &= E(e^{i(-t)X}) \\ &= E(e^{it(-X)}) \\ &= E(e^{itY}) \\ &= \phi_Y(t)\end{aligned}$$

Thus,  $\phi(-t)$  is a characteristic function of random variable  $Y$ .

Option 4 :

$$\begin{aligned}\phi_X(t+1) &= E(e^{i(t+1)X}) \\ &= E(e^{itX} \times e^{iX})\end{aligned}$$

Thus,  $\phi(t+1)$  is not a characteristic function.

Hence, correct options are 1, 2, 3.

## 1 PROBLEM

Let  $\phi(t)$  be a characteristic function of some random variable. Then, which of the following is also a characteristic function ?

- 1)  $f(t) = [\phi(t)]^2$  for all  $t \in \mathbb{R}$
- 2)  $f(t) = |\phi(t)|^2$  for all  $t \in \mathbb{R}$
- 3)  $f(t) = \phi(-t)$  for all  $t \in \mathbb{R}$
- 4)  $f(t) = \phi(t+1)$  for all  $t \in \mathbb{R}$

## 2 SOLUTION

Let  $X$  be the random variable.

Option 1 :

$$\begin{aligned}[\phi_X(t)]^2 &= \phi_X(t) \times \phi_X(t) \\ &= \phi_{(X+X)}(t) \\ &= \phi_{2X}(t)\end{aligned}$$

Thus,  $[\phi(t)]^2$  is a characteristic function of random variable  $2X$ .

Let  $Y$  and  $-X$  have the same distribution then

$$\phi_Y(t) = \overline{\phi_X(t)}$$

Option 2 :

$$\begin{aligned}|\phi_X(t)|^2 &= \phi_X(t) \times \overline{\phi_X(t)} \\ &= \phi_X(t) \times \phi_Y(t) \\ &= \phi_{X+Y}(t)\end{aligned}$$

Thus,  $|\phi(t)|^2$  is a characteristic function of random variable  $(X+Y)$ .