Optimal LAP Altitude for Maximum Coverage

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Introduction

1. Broadband wireless networks are increasingly adopted by users of mission critical communications, such as public safety agencies and first responders.
2. Like every cellular network, the communication is largely dependent on fixed infrastructure (base stations) that could be severally disrupted in the case of natural disasters such as floods, earthquakes or tsunamis.
3. One of the temporary recovery solution which is rapid and cost-effective for realizing wireless recovery networks is by utilizing airborne base stations.
4. Airborne network recovery solutions mainly focuses on Low Altitude Platforms (LAPs).

LAP

What is LAP ?

1. Low Altitude Platform (LAP) is a quasi-stationary aerial platform usually characterized with an altitude laying within the troposphere.
2. It is used as an alternative solution to emergency communication system.
3. Examples: quad-copters, balloons and helicopter.

Why LAP ?

1. LAPs are much easier to deploy, and are inline with the broadband cellular concept.
2. Low altitude combines both coverage superiority and confined cell radius.

Motivation

1. Due to technical limitations, the number of deployable LAPs could be very limited.
2. So in order to provide the best possible coverage,we need full exploitation of each of the deployed LAPs by optimizing its altitude.
3. Let's discuss about a mathematical model capable of predicting the optimum altitude of a LAP based on the statistical parameters of the underlying urban environment.

Preliminaries required for radio propagation model

Classification of propagation groups

Air-To-Ground (ATG) communication occurs in accordance to two main propagation groups. where,

1. First group : Receivers favoring a Line-of-Sight (LoS) condition or near-Line-of-Sight condition.
2. Second group: Receivers with no LAP Line-of-Sight but still receiving coverage via strong reflections and diffractions.

What is Line of sight(LoS)?

A straight line between a transmitting antenna and receiving antenna when unobstructed by horizon.

Low Altitude Platforms radio propagation in urban environment

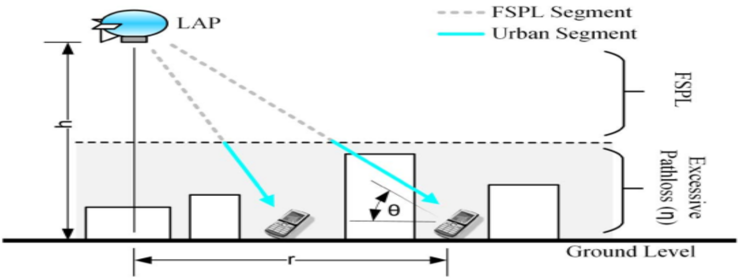

Figure 1: Low Altitude Platforms radio propagation in urban environment

Radio signals emitted by a LAP base station propagate in free space until reaching the urban environment where they incur shadowing and scattering caused by the man-made structures, creating an additional loss in the ATG link.

Let's understand some terms

PL

PathLoss is the reduction in power density (attenuation) of an electromagnetic wave as it propagates through space

FSPL

Free Space Path Loss (FSPL) represents the pathloss occured as it travels through free space.

Excessive pathloss

1. Excessive pathloss is the additive loss incurred on top of the free space pathloss
2. Excessive pathloss has a Gaussian distribution.
3. However in this study we deal with its mean value (expectation) rather than with its random behavior.

Mean Pathloss

1. ATG mean pathloss (expressed in dB) can be modeled as:

$$PL_{\xi} = FSPL + \eta_{\xi} \tag{1}$$

   where
   1. PL refers to Path Loss
   2. $FSPL$ refers to Free Space Path Loss between the LAP and a ground receiver
   3. η refers to the mean value of the excessive pathloss
   4. ξ refers to the propagation group

   We ignore the effect of small-scale fluctuations caused by the rapid changes in the propagation environment.
2. η affecting the ATG link depends largely on the propagation group rather than the elevation angle which is depicted θ in Figure 1

Spatial expectation of the pathloss

It is expectation of the pathloss between a LAP and all ground receivers having a common elevation angle θ

$$\Lambda = \sum_{\xi} PL_{\xi} P(\xi, \theta)_{\xi} \tag{2}$$

where

1. PL refers Path Loss
2. $P(\xi, \theta)_{\xi}$ represents the probability of occurrence of a certain propagation group which is strongly dependent on the elevation angle.

We assume that there are only two dominant propagation groups that strictly correspond to the LoS condition. So, $\xi \in \{LoS, NLoS\}$. Those groups' probability are linked as the following:

$$P(NLoS, \theta) = 1 - P(LoS, \theta). \tag{3}$$

Modeling Line of Sight Probability

1. The probability of geometrical LoS between a terrestrial transmitter at elevation h_{TX} and a receiver at elevation h_{RX} in an urban environment is independent of the system frequency but dependent on three statistical parameters related to the urban environment.
    1. Parameter α : Represents the ratio of built-up land area to the total land area .
    2. Parameter β : Represents the mean number of buildings per unit area .
    3. Parameter γ: A scale parameter that describes the buildings' heights distribution according to Rayleigh probability density function :

$$f(H) = \frac{H}{\gamma^2} exp(-\frac{H^2}{2\gamma^2})$$

where H is the building height in meters.

Probability of LoS

1. The LoS probability is:

$$P(\text{LoS}) = \prod_{n=0}^{m}\left[1 - exp\left(-\frac{\left[h_{TX} - \frac{\left(n+\frac{1}{2}\right)(h_{TX}-h_{RX})}{m+1}\right]^2}{2\gamma^2}\right)\right] \qquad (4)$$

where

1. m = floor($r\sqrt{\alpha\beta}$ - 1)
2. r is the ground distance between the transmitter and the receiver, as depicted in Figure 1

Observation

1. Geometrical LoS is independent of the system frequency,
2. And as (4) is generic, it can be used for any h_{TX} and h_{RX} heights.
3. In case of a LAP we can disregard h_{RX} since it is much lower than the average buildings heights and the LAP altitude.
4. Also, the ground distance becomes $r = \frac{h}{tan(\theta)}$, where h is the LAP altitude.

1. The resulting plot of the series in (4) will smooth our for large values of h, accordingly $P(LoS)$ can be considered as a continuous function of θ and the environment parameters.

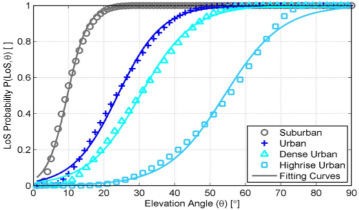

Figure 2: The calculated LoS probabilities, with their related S-curve fitting for different urban environments.

2. We can notice that the trend can be closely approximated to a simple modified Sigmoid function (S-curve) of the following form:

$$P(LoS, \theta) = \frac{1}{1 + aexp(-b\,[\theta - a])} \tag{5}$$

where a and b are called here the S-curve parameters.

1. To generalize the solution we have linked the S-curve parameters a and b directly to the environment variables α, β and γ. This linking was performed using two variables surface fitting where
   1. $(\alpha \times \beta)$ is assumed as the first variable
   2. (γ) as the second.
2. The surface equation yields a two-variables polynomial having the following form:

$$z = \sum_{j=0}^{3} \sum_{i=0}^{3-j} C_{ij} (\alpha\beta)^i \gamma^j \tag{6}$$

where

1. z represents the fitting parameter a or b
2. C_{ij} are the polynomial coefficients

1. To analyze the effect of the LAP's altitude on the provided service, firstly we define the service threshold in terms of the maximum allowable pathloss PL_{max}.
2. If the total pathloss between the LAP and a receiver exceeds this threshold then the link is deemed as failed.
3. For ground receivers, this threshold translates into a coverage disk (zone) of radius R, since all receivers within this disk have a pathloss that is less than or equal PL_{max}

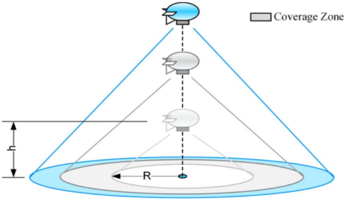

Figure 3: The coverage zone by a low altitude platform.

1. Mathematically, the cell radius of the coverage zone can be written as:

$$R = r|_{\Lambda = PL_{max}} \tag{7}$$

2. Optimization problem is to find the best altitude that will maximize R.

Relation between the LAP altitude h and the cell radius R

By rewriting Equation (1) we have:

$$PL_{LoS} = 20 \log d + 20 \log f + 20 \log \left(\frac{4\pi}{c}\right) + \eta_{LoS} \tag{8}$$

$$PL_{NLoS} = \underbrace{20 \log d + 20 \log f + 20 \log \left(\frac{4\pi}{c}\right)}_{\text{FSPL}} + \underbrace{\eta_{NLoS}}_{\eta_\xi} \tag{9}$$

where

1. The FSPL is according to Friis equation with the assumption of isotropic transmitter and receiver antennas.
2. $d = \sqrt{h^2 + r^2}$, is the distance between the LAP and a receiver at circle of radius r
3. f is the system frequency.

1. By solving equations (3), (5), (7), (8), (9) we get

$$PL_{max} = \frac{A}{1 + aexp\left(-b\left[arctan(\frac{h}{R} - a\right]\right)} + 10log(h^2 + R^2) + B \quad (10)$$

where

$$A = \eta_{LoS} - \eta_{NLoS}$$
$$B = 20logf + 20log(\frac{4\pi}{c}) + \eta_{NLoS}$$

2. In order to obtain the optimum point of the LAP altitude h_{OPT} that yields the best coverage, we need to search for the value of h that satisfies the equation of the critical point:

$$\frac{\partial R}{\partial h} = 0 \quad (11)$$

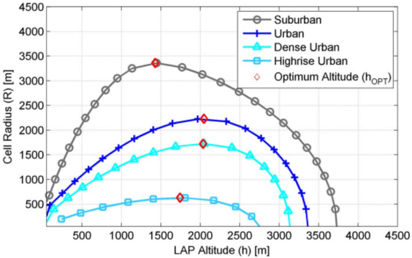

Figure 4: Cell radius vs. LAP altitude curve for four different urban environments.

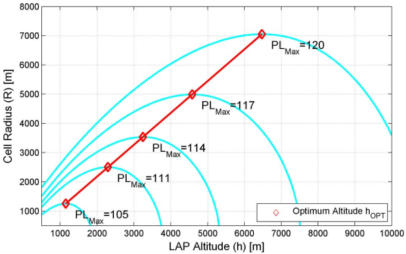

Figure 5: Cell radius vs. LAP altitude curve for different maximum pathloss, in an urban environment.

This shows that the optimum altitude of a LAP is strongly dependent on the specific urban environment condition.

We can notice that there is a certain elevation angle that always satisfies a constant ratio of $\frac{h_{opt}}{R}$, we call it here the optimum elevation angle or $\theta_{OPT} = arctan(\frac{h_{OPT}}{R})$

Optimum Elevation angle

On rewriting the expression in (10) in terms of θ and R as the following:

$$PL_{max} = \frac{A}{1 + a\exp(-b[\theta - a])} + 20\log(R\sec\theta) + B \tag{12}$$

The optimum point can then be found by solving the equation $\frac{\partial R}{\partial \theta} = 0$, which yields the following:

$$\frac{\pi}{9\ln(10)}\tan(\theta_{OPT}) + \frac{abA\exp(-b[\theta_{OPT} - a])}{[a\exp(-b[\theta_{OPT} - a]) + 1]^2} = 0 \tag{13}$$

Observation

1. θ_{OPT} is independent of the maximum allowed pathloss
2. It is also unique for a certain set of parameters (a, b, A)

Conclusion

1. We understood how Low-altitude aerial platforms (LAPs) have recently gained significant popularity as key enablers for rapid deployable relief networks where coverage is provided by onboard radio heads.
2. We learnt a method for optimizing the altitude of such platforms to provide maximum radio coverage on the ground.
3. Our analysis shows that the optimal altitude is a function of the maximum allowed pathloss and of the statistical parameters of the urban environment,