AI1103: Assignment 7

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Download all latex codes from

https://github.com/Geetha495/Assignment7/blob/main/Assignment7.tex

Download all python codes from

https://github.com/Geetha495/Assignment7/blob/main/Assignment7.py

1 Problem

Suppose X is a positive random variable with the following probability density function,

$$f(x) = (\alpha x^{\alpha-1} + \beta x^{\beta-1})e^{-x^{\alpha}-x^{\beta}}; x > 0$$

for $\alpha > 0, \beta > 0$. Then the hazard function of X for some choices of α and β can be

1) an increasing function.
2) a decreasing function.
3) a constant function.
4) a non monotonic function

2 Solution

CDF of X,

$$F(x) = \int_{-\infty}^{x} f(t)dt \tag{2.0.1}$$

$$= \int_{0}^{x} f(t)dt \qquad \text{as } x > 0 \tag{2.0.2}$$

$$= \int_{-\infty}^{t} \left((\alpha t^{\alpha-1} + \beta t^{\beta-1}) \times e^{-t^{\alpha}-t^{\beta}}\right) dt \tag{2.0.3}$$

$$= -e^{-t^{\alpha}-t^{\beta}}\Big|_{0}^{x} \tag{2.0.4}$$

$$= 1 - e^{-x^{\alpha}-x^{\beta}} \tag{2.0.5}$$

Hazard function,

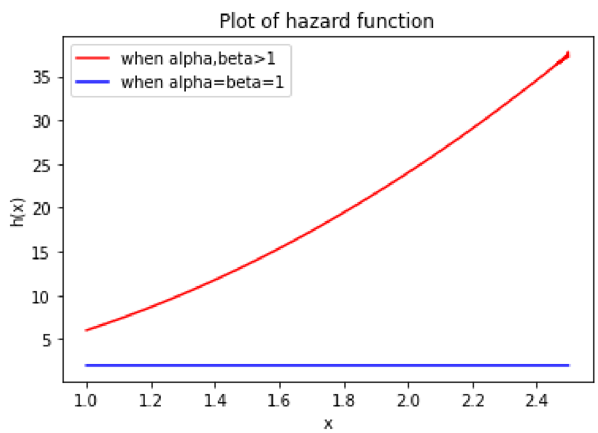
$$h(x) = \frac{f(x)}{1 - F(x)} \tag{2.0.6}$$

$$= \alpha x^{\alpha-1} + \beta x^{\beta-1} \tag{2.0.7}$$

$$h'(x) = \alpha(\alpha - 1)x^{\alpha-2} + \beta(\beta - 1)x^{\beta-2} \tag{2.0.8}$$

$$h'(x) = \begin{cases} 0 & \alpha = \beta = 1 \\ > 0 & \text{otherwise} \end{cases} \tag{2.0.9}$$

Thus $h(x)$ can be either constant function or an increasing function.



From the above figure, it is verified that $h(x)$ can be either constant function or an increasing function. Correct options are 1,3.