AI1103: Assignment 8

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Download all latex codes from

https://github.com/Geetha495/Assignment8/blob/main/Assignment8.tex

1 Problem

Let $\phi(t)$ be a characteristic function of some random variable. Then, which of the following is also a characteristic function ?

1) $f(t) = [\phi(t)]^2$ for all $t \in \mathbb{R}$
2) $f(t) = |\phi(t)|^2$ for all $t \in \mathbb{R}$
3) $f(t) = \phi(-t)$ for all $t \in \mathbb{R}$
4) $f(t) = \phi(t+1)$ for all $t \in \mathbb{R}$

2 Solution

Let X be the random variable.
Option 1 :

$$\begin{aligned}[\phi_X(t)]^2 &= \phi_X(t) \times \phi_X(t) \\ &= \phi_{(X+X)}(t) \\ &= \phi_{2X}(t)\end{aligned}$$

Thus, $[\phi(t)]^2$ is a characteristic function of random variable $2X$.
Let Y and $-X$ have the same distribution then

$$\phi_Y(t) = \overline{\phi_X(t)}$$

Option 2 :

$$\begin{aligned}|\phi_X(t)|^2 &= \phi_X(t) \times \overline{\phi_X(t)} \\ &= \phi_X(t) \times \phi_Y(t) \\ &= \phi_{X+Y}(t)\end{aligned}$$

Thus, $|\phi(t)|^2$ is a characteristic function of random variable $(X+Y)$.

Option 3 :

$$\begin{aligned}\phi_X(-t) &= E(e^{i(-t)X}) \\ &= E(e^{it(-X)}) \\ &= E(e^{itY}) \\ &= \phi_Y(t)\end{aligned}$$

Thus, $\phi(-t)$ is a characteristic function of random variable Y.
Option 4 :

$$\begin{aligned}\phi_X(t+1) &= E(e^{i(t+1)X}) \\ &= E(e^{itX} \times e^{iX})\end{aligned}$$

Thus, $\phi(t+1)$ is a not a characteristic function.

Hence, correct options are 1, 2, 3.