CBSE math

QUESTIONS

January 31, 2024

1 Matrices

1. For the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Find $(A + A')$ and verify that it is a symmetric matrix.
2. A is a square matrix with $\left|A\right| = 4$. Then find the value of $\left|A.\,(\text{adj}A)\right|$.
3. Using properties of determinants, find the value of x for which $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$.
4. Using elementary row transformations, find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$.
5. Using matrices, solve the following system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

2 Vectors

6. Find the coordinates of the foot of the perpendicular Q drawn from $P\,(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.
7. Find the acute angle between the planes $\overrightarrow{r}.\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$ and $\overrightarrow{r}.\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0$.
8. X and Y are two points with position vectors $3\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - 3\overrightarrow{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio $2 : 1$ externally.
9. Let $\overrightarrow{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $\left(\overrightarrow{a} + \overrightarrow{b}\right)$ and $\left(\overrightarrow{a} - \overrightarrow{b}\right)$ and perpendicular to each other.
10. Find the vector equation of the plane which contains the line of intersection of the planes $\overrightarrow{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\overrightarrow{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\overrightarrow{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.
11. Find the value of x such that the four points with position vectors,$A\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$, $B\left(4\hat{i} + x\hat{j} + 5\hat{k}\right)$, $C\left(4\hat{i} + 2\hat{j} - 2\hat{k}\right)$, $D\left(6\hat{i} + 5\hat{j} - \hat{k}\right)$ are coplanar.
12. Find the vector equation of a line passing through the point$(2, 3, 2)$ and parallel to the line $\overrightarrow{r} = \left(-2\hat{i} + 3\hat{j}\right)\lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance between these two lines.

3 Probability

13. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

14. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

15. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

16. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are $0 \cdot 3$, $0 \cdot 05$ and $0 \cdot 02$ respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist ?

4 Integration

17. Find :

$$\int x.tan^{-1}xdx$$

18. Find :

$$\int \frac{dx}{\sqrt{5-4x-2x^2}}$$

19. Find :

$$\int_{\frac{-\pi}{4}}^{0} \frac{1+\tan x}{1-\tan x} dx$$

20. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ w.r.t. x.

21. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, and hence evaluate $\int_0^1 x^2(1-x)^n\,dx$.

5 Differentiation

22. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of $2cm/sec$. How fast is the height on the wall decreasing when the foot of the ladder is $5m$ away from the wall?

23. Form the differential equation representing the family of curves $y = A\sin x$, by eliminating the arbitrary constant A.

24. Solve the following differential equation :

$$\frac{dy}{dx} + y = \cos x - \sin x$$

25. If $y = \log(\cos e^x)$, then find $\dfrac{dy}{dx}$.

26. If $x = \sin t, y = \sin pt$, prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

27. Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

with respect to $\cos^{-1} x^2$.

28. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

29. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

30. If $y = (\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.

6 Algebra

31. Prove that :

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

7 Linear forms

32. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

33. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the $X-$axis.

34. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

35. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

8 Functions

36. Let $*$ be an operation defined as $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $a * b = 2a + b, a, \in \mathbb{R}$.Check if $*$ is a binary operation. If yes, find if it is associative too.

37. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

38. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 3 $\}$ is an equivalence relation.

9 Optimization

39. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.