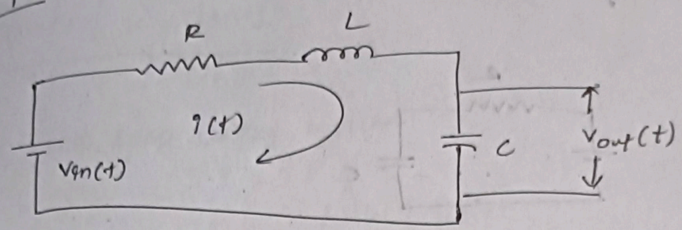
15/02/24



$$V_{in}(t) = R\,i(t) + L\cdot\frac{di(t)}{dt} + V_{out}(t)$$

$$V_{out}(t) = \cancel{C\cdot\frac{d}{dt}V_{out}(t)}\ \frac{1}{C}\int i(t)\,dt$$

$$i(t) = C\cdot\frac{d}{dt}V_{out}(t).$$

$$V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + LC\cdot\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t).$$

$$LC\cdot\frac{d^2}{dt^2}V_{out}(t) + RC\cdot\frac{d}{dt}V_{out}(t) + V_{out}(t) = V_{in}(t)$$

$$LCs^2V_{out}(s) + RCsV_{out}(s) + V_{out}(s) = V_{in}(s)$$

Voltage divider ckt.

$$V_{out}(t) = \frac{R_2}{R_1+R_2}V_{in}(t).$$

$$V_{out}(s) = \frac{R_2}{R_1+R_2}V_{in}(s).$$

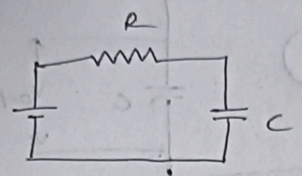
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+sRC+s^2LC}$$

$$K = \frac{V_{out}(s)}{V_{in}(s)}$$

$$G(s) = \frac{1}{s^2LC+sRC+1}$$

$$G(s) = K.$$

low pass ckt.



$$V_{in}(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$$

$$V_{out}(t) = \frac{1}{C}\int i(t)\,dt$$

$$i(t) = C\frac{d}{dt}V_{out}(t)$$

$$V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + \frac{1}{\cancel{C}}V_{out}(t)\,\cancel{dt}$$

$$V_{in}(S) = RCS\,V_{out}(S) + V_{out}(S)$$

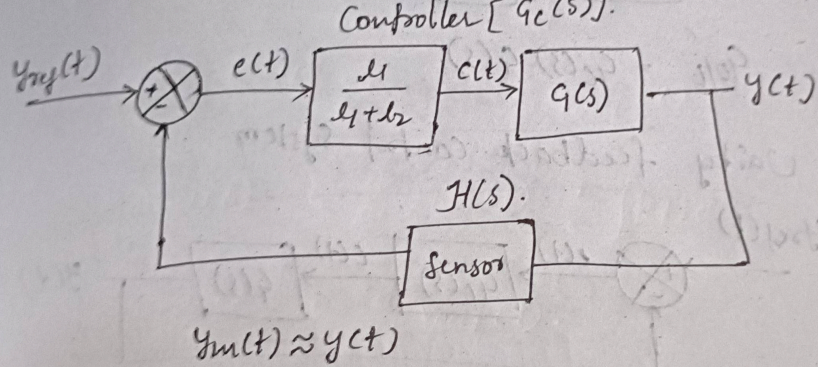
$$\frac{V_{out}(S)}{V_{in}(S)} = \frac{1}{1+RCS}$$

$$G(S) = \frac{1}{SRC+1}$$

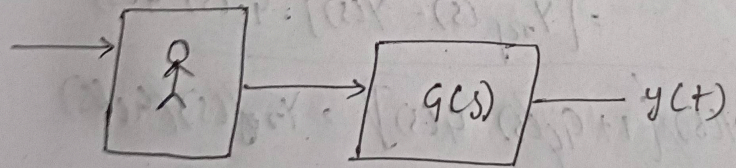
2

Control Systems

closed loop control system.



Open loop control system.



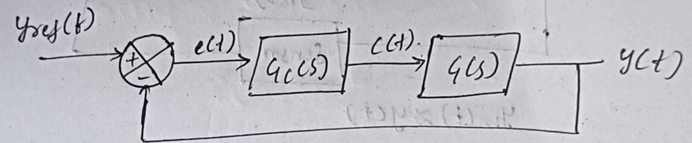
16/02/24

Open loop vs. Closed loop system

$$c(t) \longrightarrow \boxed{G_c(s)} \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$$G_{O/L} = G_c(s) \cdot G(s)$$

Unity feedback control system



$$Y(s) = E(s) \cdot G_c(s) \cdot G(s)$$

$$= [Y_{ref}(s) - Y(s)] \cdot G_c(s) \cdot G(s)$$

$$Y(s)[1 + G_c(s) \cdot G(s)] = Y_{ref}(s) G_c(s)$$

$$G_c(s) = \frac{Y(s)}{Y_{ref}} (1 + G_c(s) G(s))$$

$$G_c(s)[Y(s) - G_c(s)]$$

$$G(s) = Y(s) / [Y_{ref}(s) - Y(s)] G_c(s)$$

$$G(s) = \frac{Y(s)}{Y_{ref}(s) G_c(s) - Y(s) G_c(s)}$$

$$G(s) = \frac{[Y_{ref}(s) - Y(s)] G_c(s) \cdot G(s)}{[Y_{ref}(s) - Y(s)] G_c(s)}$$

$$Y(s) = [Y_{ref}(s) - Y(s)] \cdot G_c(s) \cdot G(s).$$

$$Y(s) = Y_{ref}(s) \cdot G_c(s)\, G(s) - Y(s) \cdot G_c(s) \cdot G(s).$$

$$Y(s)\,[1 + G_c(s)\, G(s)] = Y_{ref}\,[G_c(s)\, G(s)].$$

$$\frac{Y_{ref}(s)}{Y(s)} = \frac{G_c(s)\, G(s)}{1 + G_c(s)\, G(s)}$$

$$\boxed{G_{c/L}(s) = \frac{G_c(s)\, G(s)}{1 + G_c(s)\, G(s)}}$$

$$S^{G_{o/L}(s)}_{G(s)} = \frac{\%\ \text{change in } G_{o/L}(s)}{\%\ \text{change in } G(s).}$$

$$S^{G_{c/L}(s)}_{G(s)} = \frac{\%\ \text{change in } G_{c/L}(s).}{\%\ \text{change in } G(s)}$$

$$S^{G_{o/L}(s)}_{G(s)} = \frac{\dfrac{\partial G_{o/L}(s)}{G_{o/L}(s)} \times 100}{\dfrac{\partial G(s)}{G(s)} \times 100} = \frac{\partial G_{o/L}(s)}{\partial G(s)} \times \frac{G(s)}{G_{o/L}(s)}$$

$$= G_c(s) \cdot \frac{1}{G_c(s)} = 1 \,//$$

$$S^{G_{C/L}(s)}_{G(s)} = \frac{\dfrac{\partial G_{C/L}(s)}{G_{C/L}(s)} \times 100}{\dfrac{\partial G(s)}{G(s)} \times 100}$$

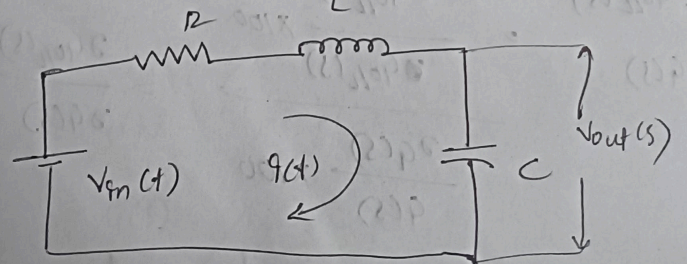
$$= \frac{\partial G_{C/L}(s)}{\partial G(s)} \times \frac{G(s)}{G_{C/L}(s)}$$

$$= \frac{G_c(s)}{[1+G_c(s)G(s)]^2} \times \frac{[1+G_c(s)G(s)]}{G_c(s)}$$

$$= \frac{1}{1+G_c(s)G(s)}$$

open loop system is more sensitive than closed loop system

RLC Ckt



$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC} I(s)}{R I(s) + Ls I(s) + \frac{1}{sC} I(s)}$$

$$= \frac{\frac{1}{sC}}{R + Ls + \frac{1}{sC}}$$

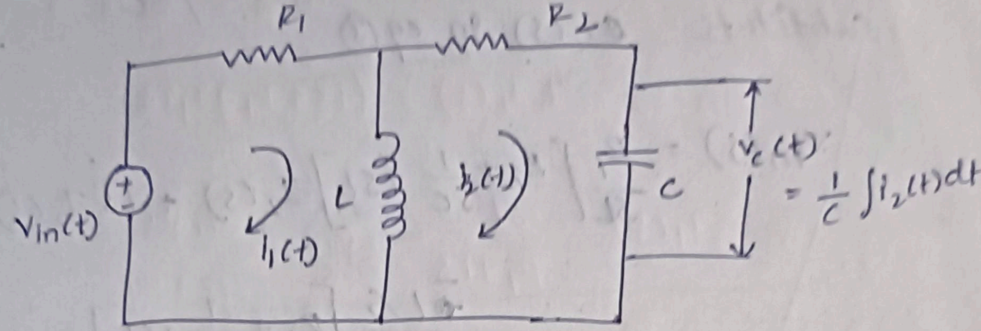
$$= \frac{1}{sRC + s^2LC + 1}$$

$$G(s) = \frac{1}{s^2LC + sRC + 1}$$

Transfor fn model of series RLC ckt

19/02

Control Systems



input $= V_{in}(t)$.

output $= i_2(t)$.

$$G(s) = \left.\frac{I_2(s)}{V_{in}(s)}\right|_{I.C.=0}$$

Transfer function model.

KVL for loop 1.

$$V_{in}(t) = i_1(t)R_1 + L\cdot\frac{d}{dt}(i_1 - i_2).$$

$$= i_1(t)R_1 + L\cdot\left[\frac{di_1(t)}{dt} - \frac{d}{dt}i_2(t)\right] \rightarrow ①$$

KVL for loop 2

$$i_2(t)R_2 + V_c(t) + L\cdot\left[\frac{di_2(t)}{dt} - \frac{di_1(t)}{dt}\right] = 0$$

$$R_2 i_2(t) + \frac{1}{C}\int i_2(t)dt + L\cdot\left[\frac{di_2(t)}{dt} - \frac{di_1(t)}{dt}\right] = 0 \quad — ②.$$

Apply Laplace Transform to eq ① & ②

$$V_{in}(s) = R_1 I_1(s) + L[sI_1(s) - sI_2(s)] \quad — ③$$

$$R_2 I_2(s) + \frac{1}{sC}I_2(s) + L[sI_2(s) - sI_1(s)] = 0. \quad — ④$$

G(s)

(3) $LS\,I_1(S) - SL\,I_2(S) = V_{in}(S) - R_1 I_1(S)$

$$V_{in}(S) + SL\,I_2(S) = (LS + R_1)\,I_1(S)$$

$$I_1(S) = \frac{SL}{R_1 + LS} I_2(S) + \frac{V_{in}(S)}{LS + R_1}$$

(4) $R_2 I_2(S) + \frac{1}{SC} I_2(S) + LS\,I_2(S) - LS\,I_1(S) = 0$

$$I_2(S)\left[R_2 + \frac{1}{SC} + LS\right] - LS\left[\frac{SL}{R_1 + SL} I_2(S) + \frac{V_{in}(S)}{LS + R_1}\right] = 0$$

$$I_2(S)\left[R_2 + \frac{1}{SC} + LS - \frac{S^2L^2}{R_1 + SL}\right] - \frac{LS}{LS + R_1} V_{in}(S) = 0$$

$$\frac{I_2(S)}{V_{in}(S)} = \frac{\frac{LS}{LS + R_1}}{R_2 + \frac{1}{SC} + LS - \frac{S^2L^2}{R_1 + SL}}$$

$$= \frac{\frac{LS}{\cancel{LS + R_1}}}{\frac{R_1R_2 + R_2SL + \frac{R_1 + SL}{SC} + R_1LS + L^2S^2 - L^2S^2}{\cancel{LS + R_1}}}$$

$$= \cancel{\frac{LS}{R_1R_2/LS + R_2 + \frac{R_1}{S}}}$$

$R_1 I_1(S)$

$I_1(S)$

$\frac{\ldots(S)}{\ldots + R_1}$

$(S) = 0$

$\frac{V_{in}(S)}{LS+R_1} \Big] = 0$

0

$\frac{\cancel{2}L^2 S^2}{}$

$$G(S) = \frac{LS}{R_1R_2 + SR_2L + \frac{R_1+SL}{SC} + R_1LS + \cancel{2L^2S^2}}$$

$$= \frac{S^2LC}{SR_1R_2C + S^2R_2LC + R_1 + SL + S^2R_1LC + \cancel{2L^2S^3C}}$$

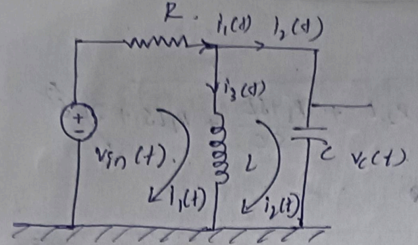
$$= \frac{S^2LC}{\cancel{2S^3L^2C} + S^2(R_1LC + R_2LC) + S(R_1R_2C + L) + R_1}$$

$$= \frac{S^2LC}{\cancel{2S^3L^2C} + S^2LC[R_1+R_2] + S[R_1R_2C+L] + R_1}$$

$$G(S) = \frac{S^2LC}{S^2LC[R_1+R_2] + S[R_1R_2C+L] + R_1}$$

$$L\frac{d}{dt} i_1(t) = V_{in}(t) - R_1 i_1(t) + L\cdot\frac{d}{dt} i_2(t).$$

$$\frac{d}{dt} i_2(t) = R_2 i_2(t) + V_C(t) + L\cdot\frac{d}{dt} i_1(t).$$



$$G(s) = \frac{V_c(s)}{V_{in}(s)}$$

$$V_{in}(t) = i_1(t)R + L\left[\frac{d}{dt}i_1(t) - \frac{d}{dt}i_2(t)\right]$$

$$V_{in}(s) = I_1(s)R + LSI_1(s) - LSI_2(s)$$

$$I_1(s)[R+LS] = V_{in}(s) + LSI_2(s)$$

$$I_1(s) = \frac{V_{in}(s)}{R+LS} + \frac{LS}{R+LS}I_2(s)$$

$$i_1(t) = i_2(t) + i_3(t)$$

$$i_1(t) = \frac{V_{in}(t) - V_c(t)}{R}$$

$$i_2(t) = C\cdot\frac{d}{dt}V_c(t) \qquad i_3(t) = \frac{1}{L}\int V_c(t)\,dt$$

$$\frac{V_{in}(t) - V_c(t)}{R} = C\cdot\frac{d}{dt}V_c(t) + \frac{1}{L}\int V_c(t)\,dt$$

$$\frac{V_{in}(s) - V_c(s)}{R} = C\cdot SV_c(s) + \frac{1}{LS}V_c(s)$$

$$V_{in}(s) - V_c(s) = \left(SCR + \frac{R}{LS}\right)V_c(s)$$

$$V_{in}(s) = \left(SCR + \frac{R}{LS} + 1\right)V_c(s).$$

$$\frac{V_c(s)}{V_{in}(s)} = \frac{1}{sCR + \frac{R}{Ls} + 1}$$

$$= \frac{sL}{s^2RLC + R + Ls}$$

$$G(s) = \frac{V_c(s)}{V_{in}(s)} = \frac{sL}{s^2RLC + Ls + R}$$

$$\frac{1}{C}\int i_2(t)\,dt + L\frac{d}{dt}i_2(t) - L\frac{d}{dt}i_1(t) = 0$$

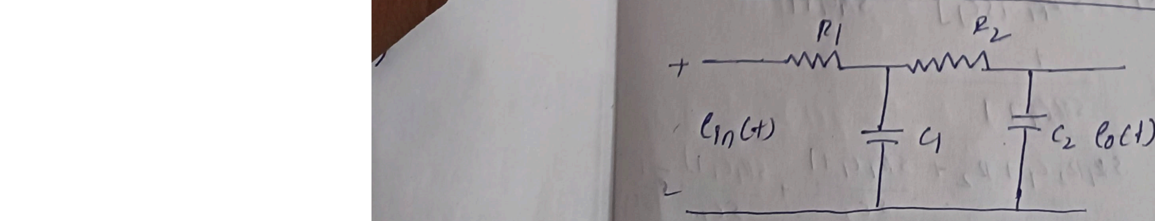
$$\frac{1}{Cs}I_2(s) + Ls\,I_2(s) - Ls\,I_1(s) = 0$$

$$\left(\frac{1}{Cs} + Ls\right)I_2(s) - Ls\left[\frac{Ls}{R+Ls}I_2(s) + \frac{V_{in}(s)}{R+Ls}\right] = 0$$

$$\left(\frac{1}{Cs} + Ls - \frac{L^2s^2}{R+Ls}\right)I_2(s) - \frac{Ls}{R+Ls}V_{in}(s) = 0$$

$$\frac{I_2(s)}{V_{in}(s)} = \frac{\frac{Ls}{\cancel{R+Ls}}}{\frac{\frac{R+Ls}{Cs} + RLs + \cancel{s^2L^2} - \cancel{L^2s^2}}{\cancel{R+Ls}}}$$

$$G(s) = \frac{Ls}{s^2RLC + Ls + R}$$



$$G(S) = \frac{E_o(S)}{E_{in}(S)}.$$

$$e_{in}(t) = i_1(t)R_1 + \frac{1}{C_1}\left[\int i_1(t)dt - \int i_2(t)dt\right]$$

$$E_{in}(S) = I_1(S)R_1 + \frac{1}{C_1}\left[\frac{1}{S}I_1(S) - \frac{1}{S}I_2(S)\right]$$

$$E_{in}(S) = I_1(S)R_1 + \frac{1}{SC_1}I_1(S) - \frac{1}{SC_1}I_2(S).$$

$$E_{in}(S) + \frac{1}{SC_1}I_2(S) = I_1(S)\left[R_1 + \frac{1}{SC_1}\right]$$

$$I_1(S) = \frac{1}{R_1 + \frac{1}{SC_1}}E_{in}(S) + \frac{1}{SC_1}\cdot\frac{1}{R_1+\frac{1}{SC_1}}I_2(S).$$

$$I_1(S) = \frac{SC_1}{SR_1C_1+1}E_{in}(S) + \frac{1}{SR_1C_1+1}I_2(S).$$

$$i_2(t)R_2 + \frac{1}{C_2}\int i_2(t)dt + \frac{1}{C_1}\left[\int i_2(t)dt - \int i_1(t)dt\right] = 0$$

$$I_2(S)R_2 + \frac{1}{C_2}\frac{1}{S}I_2(S) + \frac{1}{C_1}\frac{1}{S}I_2(S) - \frac{1}{C_1}\frac{1}{S}I_1(S) = 0$$

$$I_2(S)\left[R_2 + \frac{1}{SC_2} + \frac{1}{SC_1}\right] - \frac{1}{SC_1}\left(\frac{SC_1}{SR_1C_1+1}E_{in}(S) + \frac{1}{SR_1C_1+1}I_2(S)\right) = 0.$$

$$I_2(S)\left[R_2 + \frac{1}{SC_2} + \frac{1}{SC_1} - \frac{1}{S^2R_1C_1^2+SC_1}\right] = \frac{1}{SR_1C_1+1}E_{in}(S)$$

$$\frac{I_2(S)}{E_{in}(S)} = \frac{\frac{1}{SR_1C_1+1}}{\frac{SR_1R_2C_1 + R_2 + \frac{SR_1C_1+1}{SC_2} + \frac{SR_1C_1+1}{SC_1} - \frac{1}{SC_1}}{SR_1C_1+1}}$$

$$\frac{I_2(S)}{E_{in}(S)} = \frac{1}{SR_1R_2C_1 + R_2 + \frac{SR_1C_1+1}{SC_2} + \frac{SR_1C_1}{SC_1}}$$

$$= \frac{1}{SR_1R_2C_1 + R_2 + \frac{1}{S}\left[\frac{SR_1C_1^2 + SC_1 + SR_1C_1C_2}{C_1C_2}\right]}$$

$$= \frac{SC_1C_2}{S^2R_1R_2C_1^2C_2 + SR_2C_1C_2 + SR_1C_1^2 + SC_1 + SR_1C_1C_2}$$

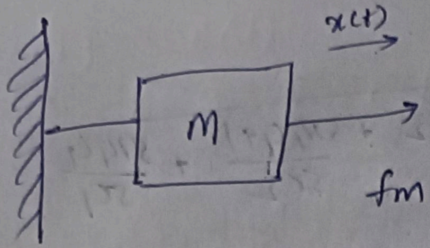
$$= \frac{SC_1C_2}{S^2R_1R_2C_1^2C_2 + S(R_2C_1C_2 + R_1C_1^2 + C_1) + SR_1C_1C_2}$$

$$G(S) = \frac{SC_1C_2}{S^2R_1R_2C_1^2C_2 + SC_1(R_2C_2 + R_1C_1 + 1) + SR_1C_1C_2}$$

$$= \frac{SC_1C_2}{S^2R_1R_2C_1^2C_2 + SC_1(R_2C_2 + R_1C_1 + 1 + R_1C_2)}$$

$$= \frac{SC_1C_2}{S^2R_1R_2C_1^2C_2 + SC_1(R_1C_2 + R_2C_2 + R_1C_1 + 1)}$$

$$G(S) = \frac{C_2}{SR_1R_2C_1C_2 + (R_1+R_2)C_2 + R_1C_1 + 1}$$

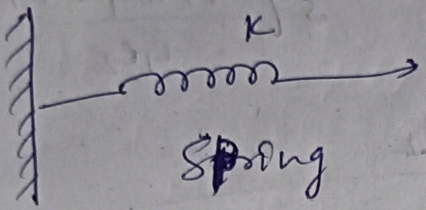


Mass

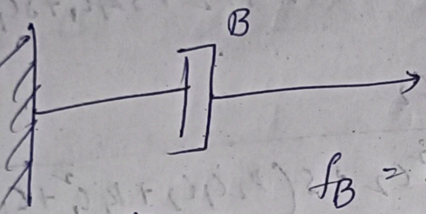
$f_m = M \cdot a(t)$

$= M \cdot \frac{dv(t)}{dt}$

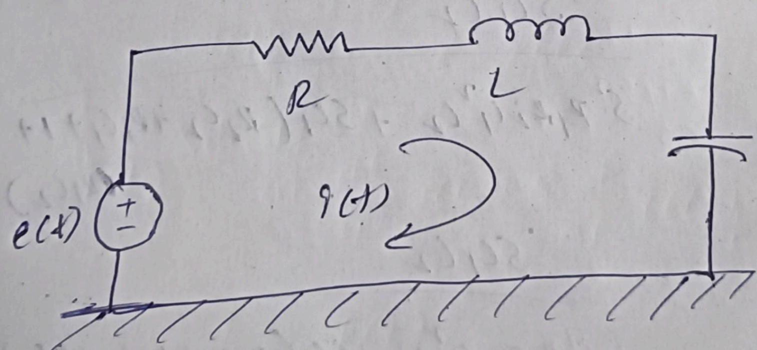
$f_M = M \cdot \frac{d^2 x(t)}{dt^2}$



Spring $\quad f_K = K \cdot x(t)$ ✓



Damper $\quad f_B = B \cdot \frac{d}{dt} x(t)$



Charge

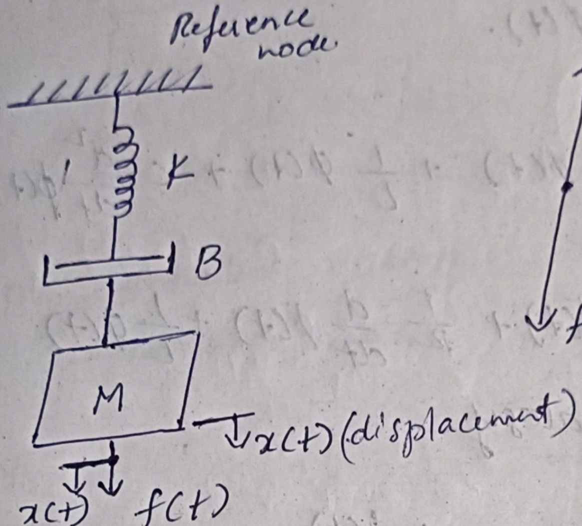
$i(t) = \frac{d}{dt} q(t)$

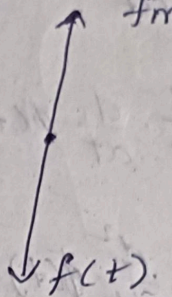
$$e(t) = R\,i(t) + L \cdot \frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

$$e(t) = L \cdot \frac{d^2}{dt^2} q(t) + R\frac{d}{dt} q(t) + \frac{1}{C} q(t)$$

21/02

Mechanical Systems





$$f(t) = M \cdot \frac{d^2}{dt^2} x(t) + B \cdot \frac{d}{dt} x(t) + K x(t).$$

Force-voltage Analysis (Analogy)

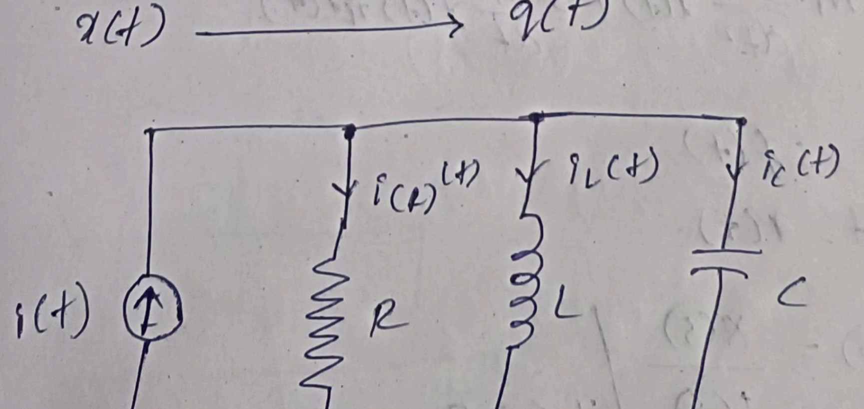
$f(t) \longleftrightarrow e(t)$.

$M \longrightarrow L$

$B \longrightarrow R$

$K \longrightarrow \frac{1}{C}$

$x(t) \longrightarrow q(t)$



$$i(t) = \frac{e(t)}{R} + \frac{1}{L}\int e(t)\,dt + C \cdot \frac{de(t)}{dt}$$

$$e(t) = \frac{d}{dt}\phi(t).$$

$$i(t) = \frac{1}{R} \cdot \frac{d}{dt}\phi(t) + \frac{1}{L}\phi(t) + C \cdot \frac{d^2}{dt}\phi(t)$$

$$i(t) = C \cdot \frac{d^2}{dt}\phi(t) + \frac{1}{R}\frac{d}{dt}\phi(t) + \frac{1}{L}\phi(t)$$

force-current analogy

$f(t) \longrightarrow i(t)$

$M \longrightarrow C.$

$B \longrightarrow \frac{1}{R}$

$K \longrightarrow \frac{1}{L}$

$x(t) \longrightarrow \phi(t).$

$\frac{d}{dt}x(t) = \text{velocity}.$

$$f(t) = M \cdot \frac{d^2}{dt^2}x(t) + B \cdot \frac{d}{dt}x(t) + Kx(t).$$

Input : $f(t)$.

output : $x(t)$.

$$G(S) = \frac{X(S)}{F(S)}\Bigg|_{I.C = 0}$$

$$F(S) = M \cdot S^2 X(S) + B \cdot S X(S) + KX(S)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2M + sB + K}$$

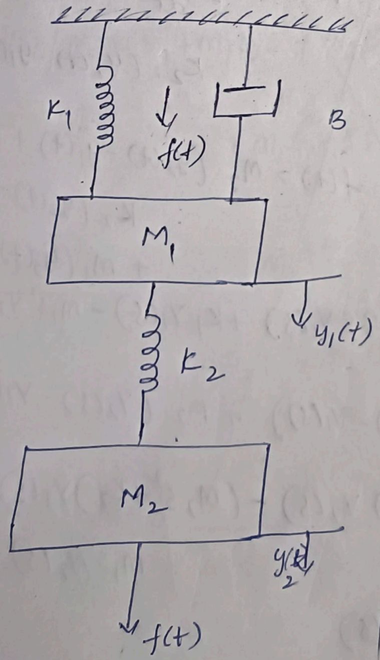
$$\boxed{G(s) = \frac{1}{Ms^2 + Bs + K}}$$

Sample Controlled system

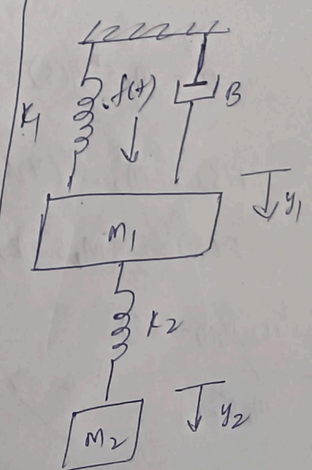
Event controlled systems

22/02/24

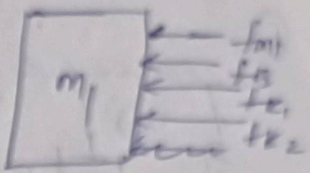
Modeling of Mechanical Translational Systems



$$G(s) = \frac{Y_2(s)}{F(s)}$$



→ The opposing forces are m_1 FBD



$f_{m1}, f_B, f_{k1}, f_{k2}$.

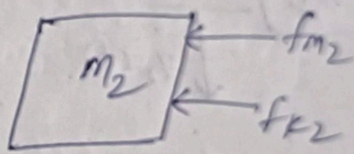
$$f_{m1} + f_B + f_{k1} + f_{k2} = f(t).$$

$$m_1\ddot{y}_1(t) + B.\dot{y}_1(t) + K_1y_1 + K_2(y_1 - y_2) = f(t)$$

$$m_1 s^2 Y_1(s) + BsY_1(s) + (K_1+K_2)Y_1(s) - K_2Y_2(s) = F(s)$$

M_2

f_{m2}, f_{k2}



$$f_{m2} + f_{k2} = 0$$

$$M_2\ddot{y}_2(t) + K_2(y_2(t) - y_1(t)) = 0$$

$$M_2s^2Y_2(s) + K_2Y_2(s) - K_2Y_1(s) = 0$$

$$Y_1(s) = Y_2(s)\,\frac{M_2s^2 + K_2}{K_2}$$

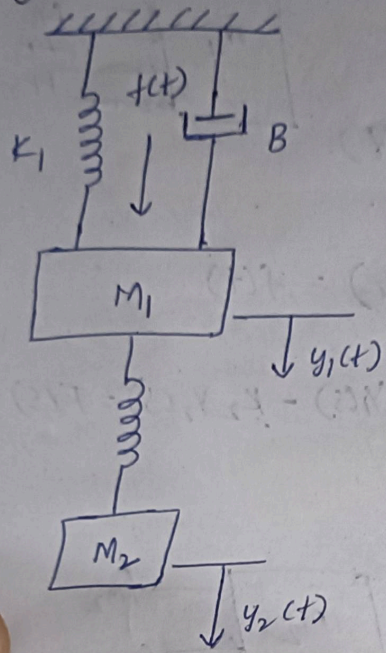
$$(M_1s^2 + Bs + (K_1+K_2))\left(\frac{M_2s^2+K_2}{K_2}\right)Y_2(s) - K_2Y_2(s) = F(s)$$

$$\frac{F(s)}{Y_2(s)} = \frac{[M_1s^2 + Bs + (K_1+K_2)][M_2s^2+K_2]}{K_2} - K_2$$

$$G(s) = \frac{Y_2(s)}{F(s)} = \frac{K_2}{\quad}$$

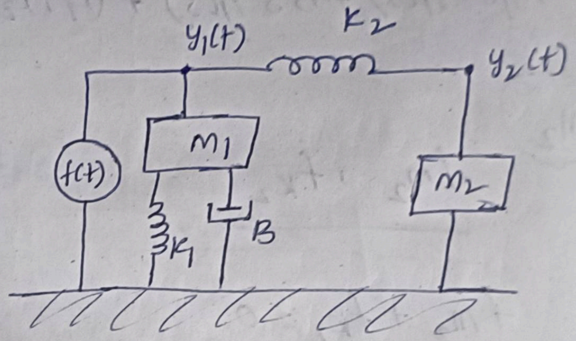
23|02|24

①



$$G(s) = \frac{Y_2(s)}{F(s)}$$

Transfer function mode



Node 1 : $f(t) = f_{K_1}(t) + f_B(t) + f_{M_1}(t) + f_{K_2}(t)$

$$f(t) = M_1 \ddot{y}_1(t) + B\dot{y}_1(t) + K_1 y_1(t) + K_2(y_1(t) - y_2(t))$$

$$F(s) = M_1 s^2 Y_1(s) + BsY_1(s) + K_1 Y_1(s) + K_2 Y_1(s) - K_2 Y_2(s)$$

$$F(s) = [M_1 s^2 + Bs + (K_1 + K_2)] Y_1(s) - K_2 Y_2(s)$$

Node 2 :

$$0 = M_2 \ddot{y}_2(t) + K_2(y_2(t) - y_1(t))$$

$$0 = M_2 s^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s)$$

$$0 = (M_2 s^2 + K_2) Y_2(s) - K_2 Y_1(s)$$

$$Y_1(s) = \frac{M_2 s^2 + K_2}{K_2} Y_2(s)$$

$$F(s) = [M_1 s^2 + Bs + (K_1+K_2)] \frac{M_2 s^2 + K_2}{K_2} Y_2(s) - K_2 Y_2(s)$$

$$\frac{F(s)}{Y_2(s)} = \frac{[M_1 s^2 + Bs + (K_1+K_2)][M_2 s^2 + K_2]}{K_2} - K_2$$

$$\frac{F(s)}{Y_2(s)} = \frac{[M_1 s^2 + Bs + (K_1+K_2)][M_2 s^2 + K_2] - K_2^2}{K_2}$$

$$G(s) = \frac{Y_2(s)}{F(s)} = \frac{K_2}{[M_1 s^2 + Bs + (K_1+K_2)][M_2 s^2 + K_2] - K_2^2}$$

$$\mathcal{L}\{M_1 \ddot{y}_1(t)\} = M_1 \cdot s^2 Y_1(s)$$

$$\mathcal{L}\{B \cdot \dot{y}_1(t)\} = B \cdot s Y_1(s)$$

$$\mathcal{L}\{K_1 y_1(t)\} = K_1 Y_1(s)$$

$$\mathcal{L}\{K_2 y_2(t)\} = K_2 Y_2(s)$$

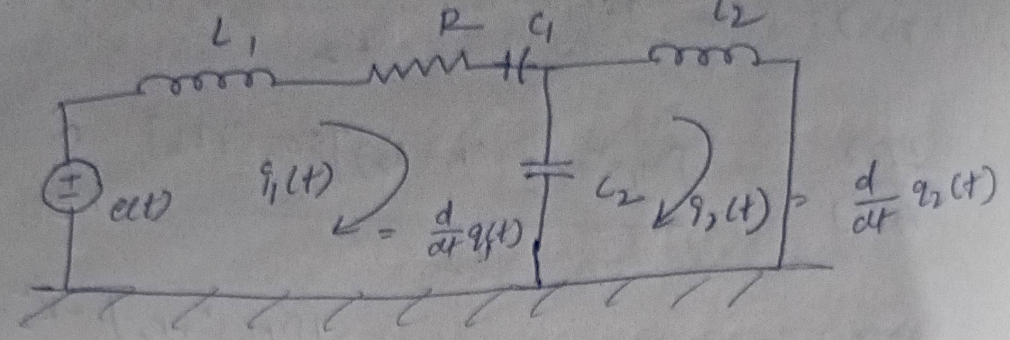
Use force-voltage analogy :

$$e(t) = L_1 \ddot{q}_1(t) + R \dot{q}_1(t) + \frac{1}{C_2}(q_1(t) - q_2(t)) + \frac{1}{C_1} q_1(t)$$

$$0 = L_2 \ddot{q}_2(t) + \frac{1}{C_2}[q_2(t) - q_1(t)]$$

$$e(t) = L_1 \frac{d}{dt} i_1(t) + R \cdot i_1(t) + \frac{1}{C_1}\int i_1(t)\,dt + \frac{1}{C_2}\int (i_1(t) - i_2(t))\,dt$$

$$0 = L_2 \frac{d}{dt} i_2(t) + \frac{1}{C_2}\left(\int (i_2(t) - i_1(t))\,dt\right)$$



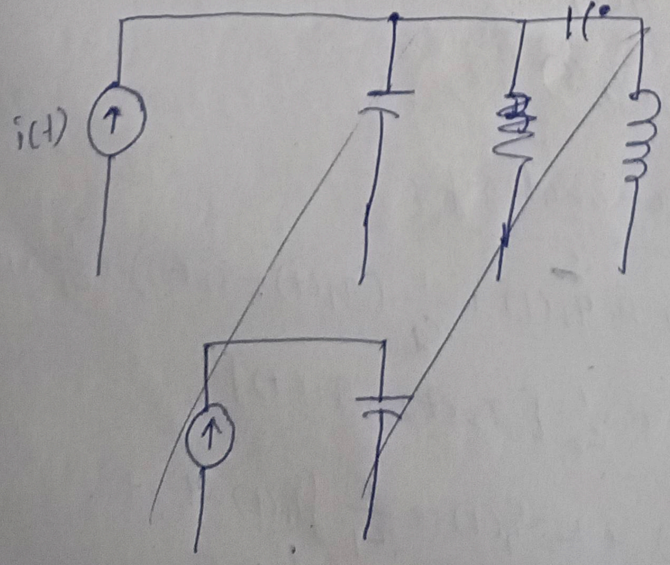
force-current Analogy

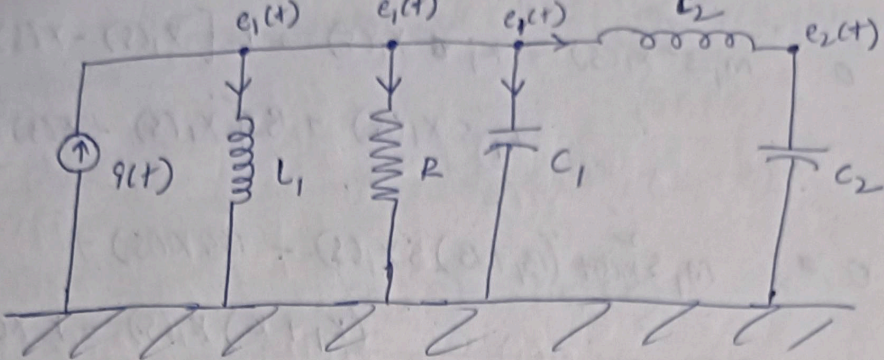
$$i(t) = C_1 \ddot{\phi}_1(t) + \frac{1}{R}\dot{\phi}_1(t) + \frac{1}{L_1}\phi_1(t) + \frac{1}{L_2}[\phi_1(t) - \phi_2(t)]$$

$$0 = C_2 \ddot{\phi}_2(t) + \frac{1}{L_2}(\phi_2(t) - \phi_1(t))$$

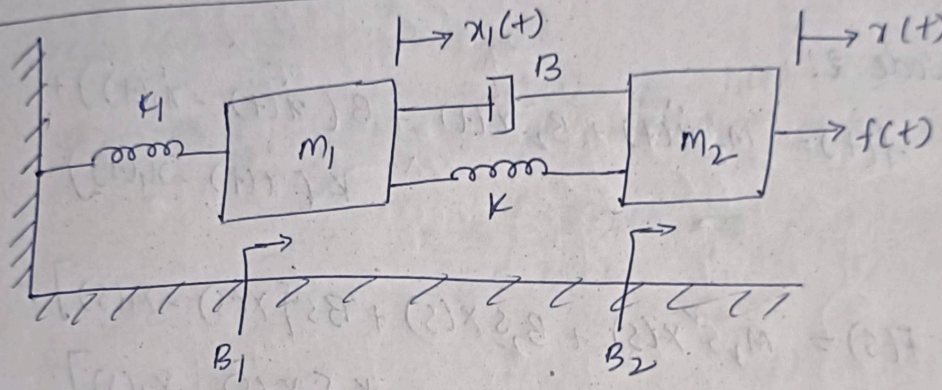
$$i(t) = C_1 \frac{d}{dt} e_1(t) + \frac{1}{R} e_1(t) + \frac{1}{L_1}\int e_1(t)\,dt + \frac{1}{L_2}\left[\int (e_1(t) - e_2(t))\,dt\right]$$

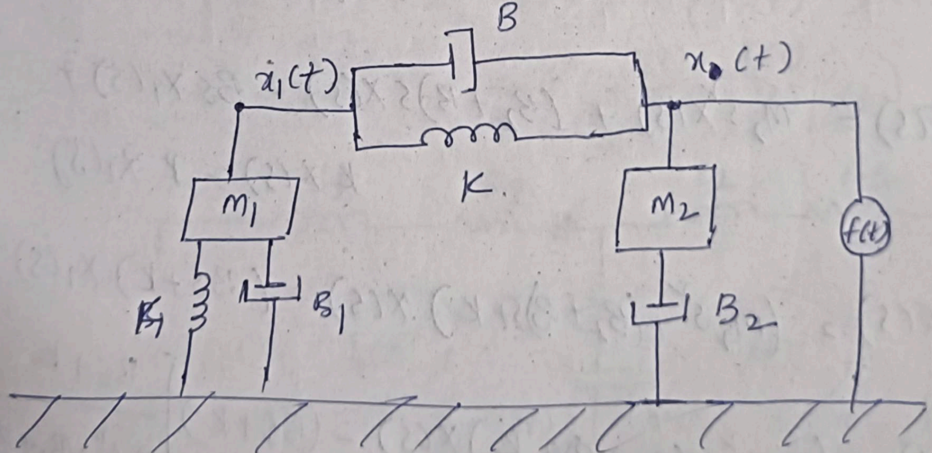
$$0 = C_2 \frac{d}{dt} e_2(t) + \frac{1}{L_2}\left(\int (e_2(t) - e_1(t))\,dt\right)$$





$$i(t) = i_C(t) + i_R(t) + i_{L_1}(t) + i_{L_2}(t).$$

② 



Node 1:

$$0 = f_{m_1}(t) + f_{K_1}(t) + f_{B_1}(t) + f_B(t) + f_K(t)$$

$$0 = M_1 \ddot{x}_1(t) + K_1 x_1(t) + K(x_1(t) - x_2(t)) + B_1 \dot{x}_1(t) + B(\dot{x}_1(t) - \dot{x}_2(t))$$

$$0 = M_1 s^2 X_1(s) + K_1 X_1(s) + K(X_1(s) - X(s)) \longrightarrow$$

$$0 = M_1 s^2 X_1(S) + K_1 X_1(S) + K[X_1(S) - X(S)] + B_1 s X_1(S) + Bs[X_1(S) - X(S)]$$

$$0 = M_1 s^2 X_1(S) + (B_1 + B) s X_1(S) - Bs X(S) + (K_1 + K) X_1(S) - K X(S)$$

$$0 = [M_1 s^2 + (B_1 + B)s + (K_1 + K)] X_1(S) - (Bs + K) X(S)$$

Node 2:

$$f(t) = M_2 \ddot{x}(t) + B_2 \dot{x}(t) + B(\dot{x}(t) - \dot{x}_1(t)) + K(x(t) - x_1(t)).$$

$$F(S) = M_2 s^2 X(S) + B_2 s X(S) + Bs[X(S) - X_1(S)] + K[X(S) - X_1(S)]$$

$$F(S) = M_2 s^2 X(S) + (B_2 + B) s X(S) - Bs X_1(S) + K X(S) - K X_1(S)$$

$$F(S) = (M_2 s^2 + (B_2 + B)s + K) X(S) - (Bs + K) X_1(S)$$

$$F(S) = (M_2 s^2 + (B_2 + B)s + K) X(S) - (Bs + K)\left[\frac{Bs + K}{M_1 s^2 + (B_1 + B)s + K_1 + K}\right] X(S)$$

$$\frac{F(S)}{X(S)} = (M_2 s^2 + (B_2 + B)s + K) - \frac{(Bs + K)^2}{M_1 s^2 + (B_1 + B)s + K_1 + K}$$

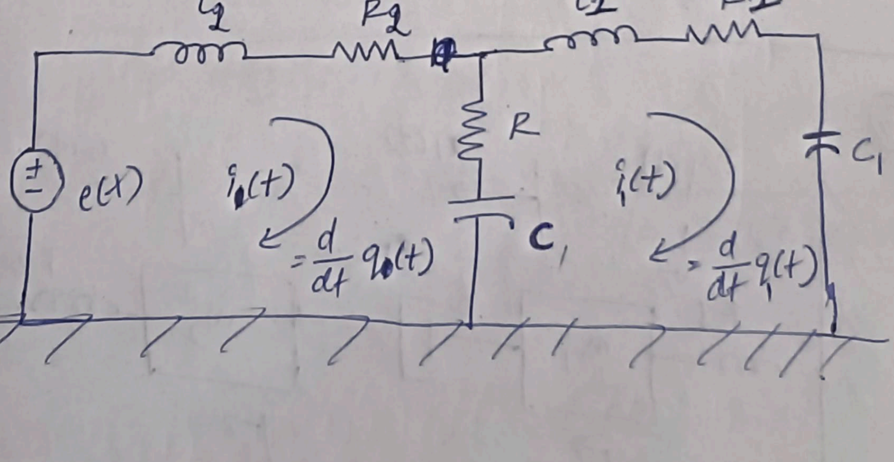
$$\frac{F(s)}{X(s)} = \frac{[M_1 s^2+(B_1+B)s+(K_1+K)][M_2 s^2+(B_2+B)s+K] - (Bs+K)^2}{M_1 s^2+(B_1+B)s+(K_1+K)}$$

$$\frac{X(s)}{F(s)}\, G(s) = \frac{M_1 s^2+(B_1+B)s+(K_1+K)}{[M_1 s^2+(B_1+B)s+(K_1+K)][M_2 s^2+(B_2+B)s+K] - (Bs+K)^2}$$

Force - Voltage Analogy

$$0 = \underbrace{L_1}_{L_1} \frac{d}{dt}\underbrace{\dot{q}_1(t)}_{i_1(t)} + \overset{R_1}{R_1}\,\overset{i_1(t)}{\dot{q}_1(t)} + \overset{R}{R}\,\overset{i_1(t)-i(t)}{(\dot{q}_1(t)-\dot{q}_2(t))} + \frac{1}{C_1}\int \underbrace{q_1(t)}_{i_1(t)} dt + \frac{1}{C}\left[\int \left(\underbrace{q_1(t)}_{i_1(t)} - \underbrace{q(t)}_{i(t)}\right) dt\right.$$

$$e(t) = L_2 \frac{d}{dt}\underbrace{\overset{i(t)}{\ddot{q}(t)}}_{i(t)} + R_2\,\dot{q}(t) + R\,(\overset{i(t)}{\dot{q}(t)} - \overset{i_1(t)}{\dot{q}_1(t)}) + \frac{1}{C}\left(\int [\overset{q(t)}{q(t)} - \overset{q_1(t)}{q_1(t)}]\, dt\right)$$



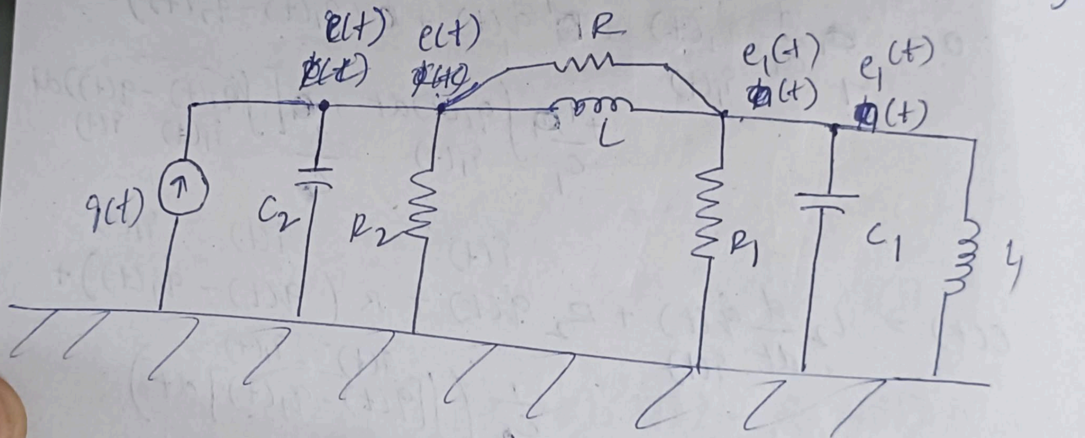
force current Analogy

$$i(t) = C_2 \frac{d}{dt} \underbrace{\dot{\phi}(t)}_{e(t)} + \frac{1}{R_2} \overset{e(t)}{\dot{\phi}(t)} + \frac{1}{R}\overset{e(t)\,-\,e_1(t)}{\left(\dot{\phi}(t) - \dot{\phi}_1(t)\right)} +$$

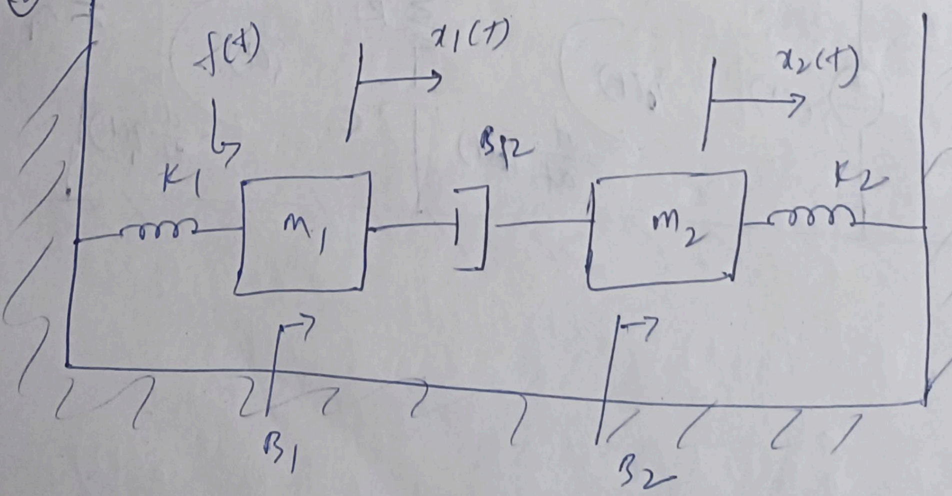
$$\frac{1}{L}\left[\int \underbrace{(\phi(t) - \phi_1(t))}_{e(t)\,-\,e_1(t)}\, dt\right]$$

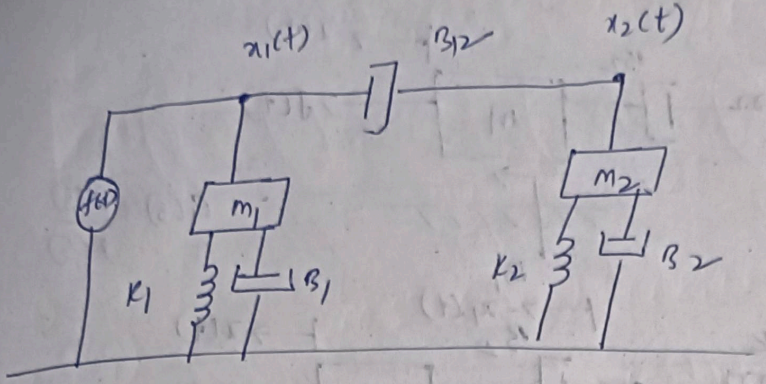
$$0 = C_1 \frac{d}{dt} \overset{e_1(t)}{\dot{\phi}_1(t)} + \frac{1}{R_1} \overset{e_1(t)}{\dot{\phi}_1(t)} + \frac{1}{R}\overset{e_1(t)\,-\,e(t)}{\left(\dot{\phi}_1(t) - \dot{\phi}(t)\right)}$$

$$+ \frac{1}{L_1}\left(\cancel{\frac{d\phi}{dt}} \int \underbrace{\phi_1(t)}_{e_1(t)}\right) + \frac{1}{L}\int \underbrace{(\phi_1(t) - \phi(t))}_{e_1(t)\,-\,e(t)}\, dt$$



(3)





$$G(s) = \frac{X_1(s)}{F(s)}$$

Node 1

$$f(t) = M_1 \ddot{x}_1(t) + B_1 \dot{x}_1(t) + B_{12}[\dot{x}_1(t) - \dot{x}_2(t)] + K_1 x_1(t).$$

$$F(s) = m_1 s^2 X_1(s) + B_1 s X_1(s) + B_{12} s [X_1(s) - X_2(s)] + K_1 X_1(s)$$

$$F(s) = (m_1 s^2 + B_1 s + B_{12} s + K_1) X_1(s) - B_{12} s X_2(s)$$

Node 2

$$0 = m_2 \ddot{x}_2(t) + B_2 \dot{x}_2(t) + B_{12}[\dot{x}_2(t) - \dot{x}_1(t)] + K_2 x_2(t)$$

$$0 = m_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s [X_2(s) - X_1(s)] + K_2 X_2(s)$$

$$0 = (m_2 s^2 + B_2 s + B_{12} s + K_2) X_2(s) - B_{12} s X_1(s)$$

$$X_2(s) = \frac{B_{12} s X_1(s)}{m_2 s^2 + (B_2 + B_{12}) s + K_2}$$

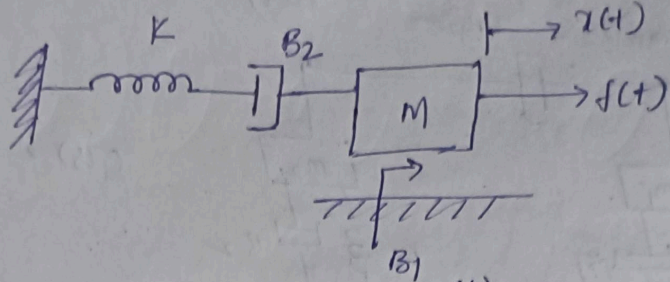
$$F(s) = (m_1 s^2 + (B_1 + B_{12}) s + K_1) X_1(s) - B_{12} s \left[\frac{B_{12} s}{m_2 s^2 + (B_2 + B_{12}) s + K_2}\right] X_1(s)$$

$$\frac{F(s)}{X_1(s)} = \frac{(m_1 s^2 + (B_1 + B_{12}) s + K_1)(m_2 s^2 + (B_2 + B_{12}) s + K_2) - B_{12}^2 s^2}{m_2 s^2 + (B_2 + B_{12}) s + K_2}$$

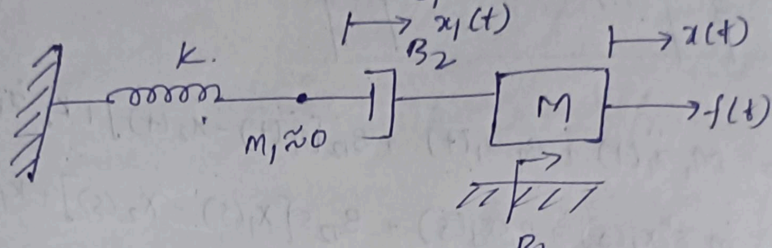
$$G(s) = \frac{X_1(s)}{F(s)} = \frac{m_2 s^2 + (B_2 + B_{12}) s + K_2}{(m_1 s^2 + (B_1 + B_{12}) s + K_1)(m_2 s^2 + (B_2 + B_{12}) s + K_2) - B_{12}^2 s^2}$$

24|02|24

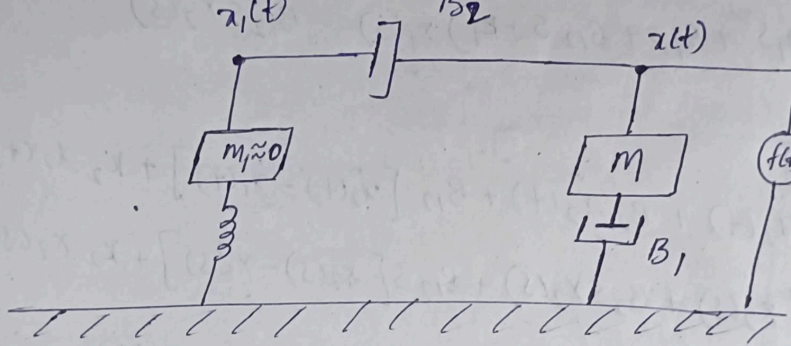
(9)



$$G(S) = \frac{X(S)}{F(S)}$$



FBD.



At node 1

$$0 = 0 + K\,x_1(t) + B_2\left[\dot{x}_1(t) - \dot{x}(t)\right]$$

$$0 = K\,X_1(S) + B_2 S\left[X_1(S) - X(S)\right]$$

$$0 = \left[B_2 S + K\right] X_1(S) - B_2 S\, X(S).$$

$$X_1(S) = \left[\frac{B_2 S}{B_2 S + K}\right] X(S)$$

At node 2

$$f(t) = M\ddot{x}(t) + B_1\dot{x}(t) + B_2\left[\dot{x}(t) - \dot{x}_1(t)\right]$$

$$F(S) = Ms^2 X(S) + B_1 S X(S) + B_2 S\left[X(S) - X_1(S)\right]$$

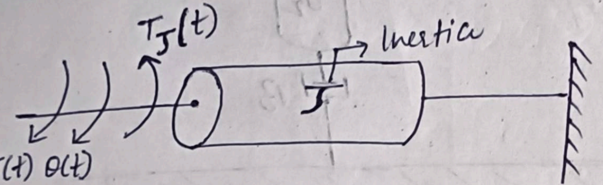
$$F(S) = \left[Ms^2 + B_1 S + B_2 S\right] X(S) - B_2 S\, X_1(S)$$

$$F(s) = [ms^2 + (B_1 + B_2)s]X(s) - B_2 s\left[\frac{B_2 s}{B_2 s + K}\right]X(s)$$

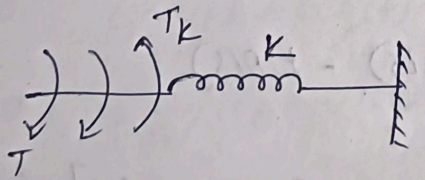
$$\frac{F(s)}{X(s)} = \frac{[ms^2 + (B_1 + B_2)s][B_2 s + K] - (B_2 s)^2}{B_2 s + K}$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{B_2 s + K}{[ms^2 + (B_1 + B_2)s][B_2 s + K] - B_2^2 s^2}$$

Mechanical Rotational Systems:

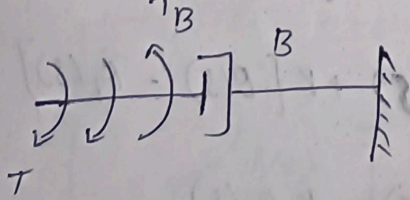


Functionally, it is same as mechanical translational system, the difference is displacement is angular, the excitation is torque and [J = Inertia].



Spring and damper are same.

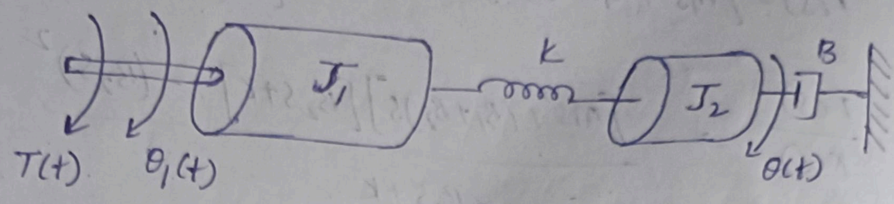
$$\boxed{\frac{d}{dt}\theta(t) = \omega(t)}$$ → Angular velocity



$$\boxed{\frac{d}{dt}\omega(t) = \alpha(t)}$$

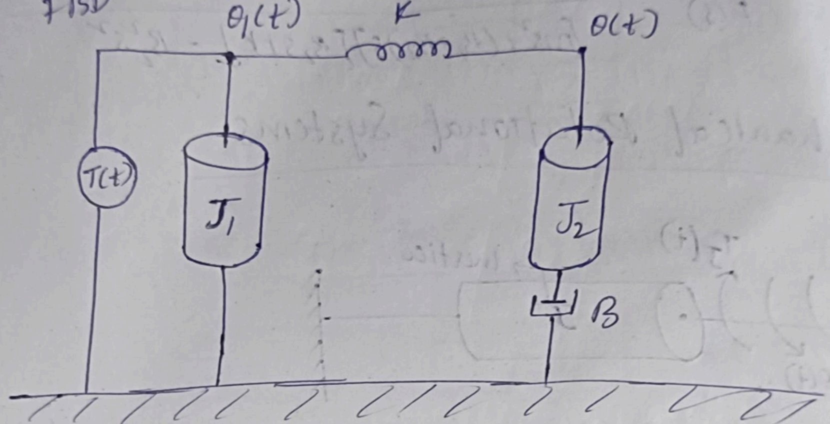
∴ Inertia (J), spring (K), damper (B) are the building blocks

①



$$G(s) = \frac{\theta(s)}{T(s)}$$

FBD



Node ①

$$T(t) = J_1\, \ddot{\theta}_1(t) + K[\theta_1(t) - \theta(t)]$$

$$T(s) = J_1 s^2 \theta_1(s) + K[\theta_1(s) - \theta(s)]$$

$$T(s) = [J_1 s^2 + K]\, \theta_1(s) - K\theta(s)$$

Node ②

$$0 = J_2\, \ddot{\theta}(t) + B\, \dot{\theta}(t) + K[\theta(t) - \theta_1(t)]$$

$$0 = J_2 s^2 \theta(s) + Bs\, \theta(s) + K[\theta(s) - \theta_1(s)]$$

$$0 = [J_2 s^2 + Bs + K]\, \theta(s) - K\theta_1(s)$$

$$\theta_1(s) = \frac{J_2 s^2 + Bs + K}{K}\, \theta(s)$$

$$T(s) = [J_1 s^2 + K]\left[\frac{J_2 s^2 + Bs + K}{K}\right]\theta(s) - K\theta(s)$$

$$\frac{T(s)}{\theta(s)} = \frac{[J_1 s^2 + K][J_2 s^2 + Bs + K] - K^2}{K}$$

$$G(s) = \frac{\theta(s)}{T(s)} = \frac{K}{[J_1 s^2 + K][J_2 s^2 + Bs + K] - K^2}$$

Torque - Voltage Analogy

$T(t) \rightarrow e(t)$

$J \rightarrow L$

$B \rightarrow R$

$K \rightarrow \frac{1}{C}$

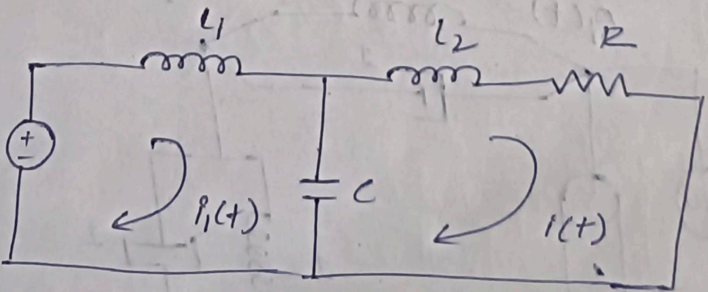
$\theta(t) \rightarrow q(t)$

$$e(t) = L_1 \ddot{q}_1(t) + \frac{1}{C}[q_1(t) - q(t)]$$

$$e(t) = L_1 \frac{d}{dt} i(t) + \frac{1}{C}\left[\int (i_1(t) - i(t))\,dt\right].$$

$$0 = L_2 \ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}[q(t) - q_1(t)]$$

$$0 = L_2 \frac{d}{dt} i(t) + R\,i(t) + \frac{1}{C}\left[\int (i(t) - i_1(t))\,dt\right]$$



Torque - current Analogy

$T(t) \rightarrow i(t)$

$J \rightarrow C$

$B \rightarrow \frac{1}{R}$

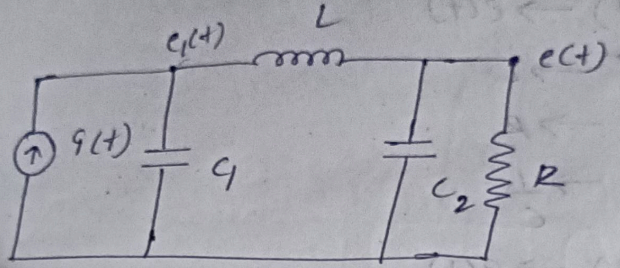
$\theta(t) \rightarrow \phi(t)$

$$q(t) = C_1 \ddot{\phi}_1(t) + \frac{1}{L}[\phi_1(t) - \phi(t)]$$

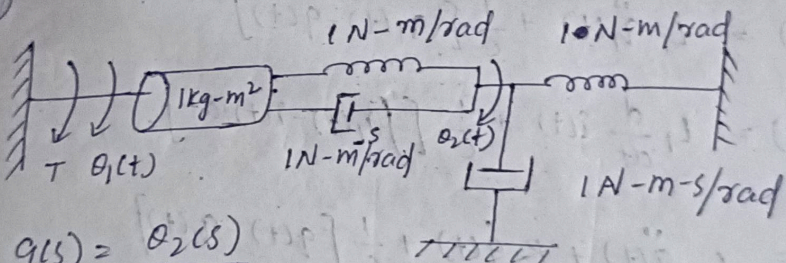
$$q(t) = C_1 \cdot \frac{d}{dt} e_1(t) + \frac{1}{L}\left[\int e_1(t) - e(t)\, dt\right]$$

$$0 = C_2 \ddot{\phi}(t) + \frac{1}{R}\dot{\phi}(t) + \frac{1}{L}[\phi(t) - \phi_1(t)]$$

$$0 = C_2 \frac{d}{dt} e(t) + \frac{1}{R} e(t) + \frac{1}{L}\left[\int e(t) - e_1(t)\, dt\right]$$

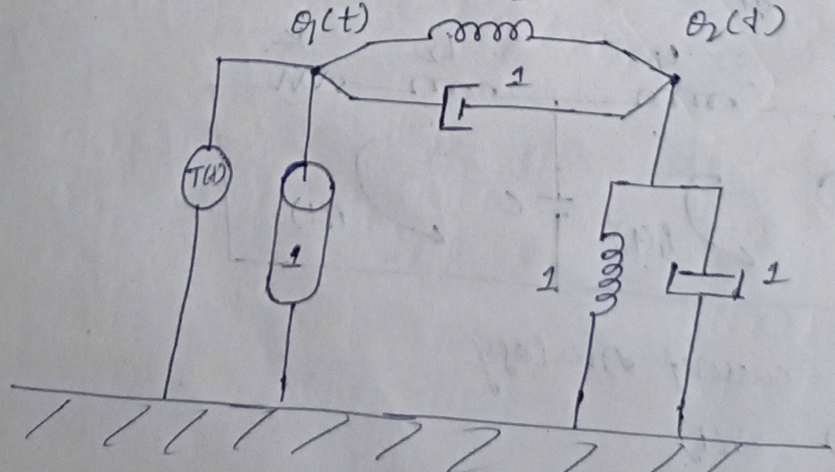


②



$$G(s) = \frac{\theta_2(s)}{T(s)}$$

FBD



Node ①

$$T(t) = 1.\ddot{\theta}_1(t) + 1[\theta_1(t) - \theta_2(t)] + 1[\dot{\theta}_1(t) - \dot{\theta}_2(t)]$$

$$T(S) = S^2\theta_1(S) + \theta_1(S) - \theta_2(S) + S[\theta_1(S) - \theta_2(S)]$$

$$T(S) = (S^2+1+S)\theta_1(S) - (1+S)\theta_2(S)$$

Node ②

$$0 = \theta_2(t) + \dot{\theta}_2(t) + (\theta_2(t) - \theta_1(t)) + (\dot{\theta}_2(t) - \dot{\theta}_1(t))$$

$$0 = \theta_2(S) + S\theta_2(S) + \theta_2(S) - \theta_1(S) + S(\theta_2(S) - \theta_1(S))$$

$$0 = (2+2S)\theta_2(S) - (1+S)\theta_1(S)$$

$$\theta_1(S) = \frac{2(1+S)}{(1+S)}\theta_2(S)$$

$$\theta_1(S) = \frac{2\cancel{(1+S)}}{\cancel{(1+S)}}\theta_2(S) \Rightarrow \theta_1(S) = 2\theta_2(S)$$

$$T(S) = (S^2+S+1)\,2\,\theta_2(S) - (1+S)\theta_2(S)$$

$$\frac{T(S)}{\theta_2(S)} = 2(S^2+S+1) - (1+S)$$

$$G(S)\ \frac{\theta_2(S)}{T(S)} = \frac{1}{2S^2+2S+2-1-S}$$

$$G(S) = \frac{1}{2S^2+S+1}$$

$$\Rightarrow \frac{dy_1}{dt} = c(t) + 3 \text{ (linear}$$

$$\frac{dy_1}{dt} = \frac{y_{i+1} - y_i}{\Delta t}$$

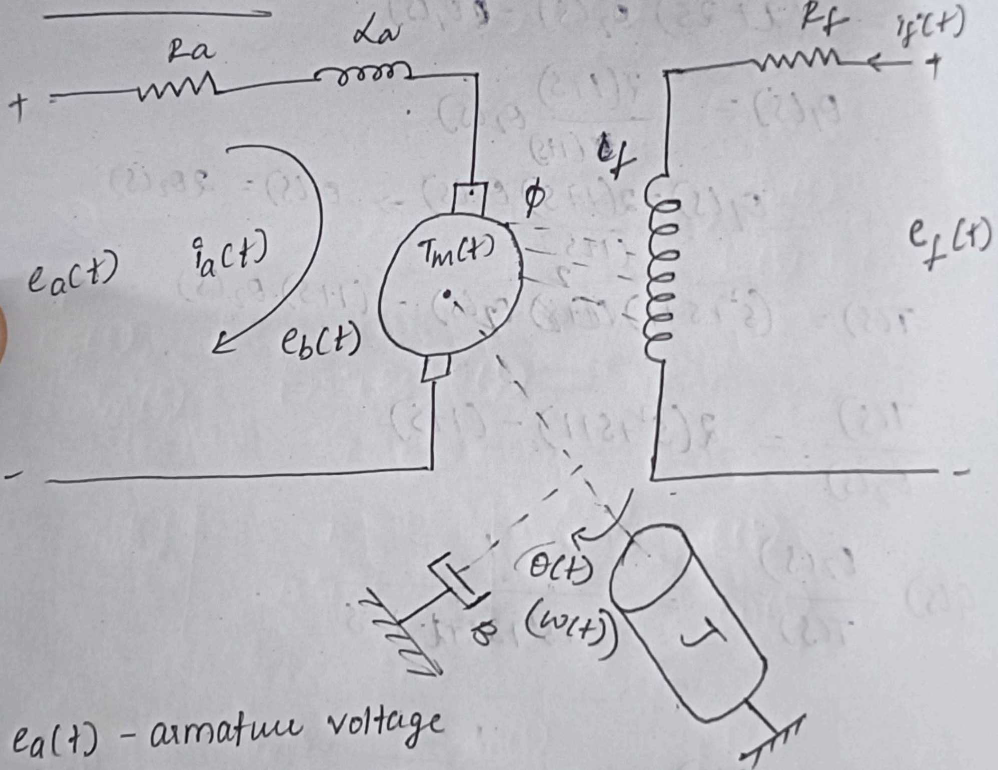
$$\frac{y_{i+1}(t) - y_i(t)}{\Delta t} = c(t)+3$$

$$y_{i+1}(t) = y_i(t) + \Delta t\,[c(t)+3]$$

$$\frac{d}{dt} y(t) = c^2(t) + 5 \quad \text{[non-linear]}$$

28/2/24

DC motor



$e_a(t)$ - armature voltage
R_a - Armature resistance
L_a - Armature inductance
$e_b(t)$ - Back emf
$i_a(t)$ - Armature current

$e_f(t)$ field voltage
$i_f(t)$ - field current
R_f - field resistance
L_f - field inductance

$T_m(t)$ - Torque
$\theta(t)/\omega(t)$ - angular displacement / velocity
B - Damper
J - Inertia

modeling of

Armature - controlled DC motor

e_f = constant

$\phi \propto i_f$

$\phi = K_f i_f$

$T_m(t) \propto \phi \, i_a(t)$

$T_m(t) \propto K_f i_f i_a(t)$

$T_m(t) = K_T K_f i_f i_a(t)$

$$\boxed{T_m(t) = K_T i_a(t)} \quad \text{—— ①}$$

K_T = (motor) torque constant

$e_b(t) \propto \frac{d}{dt} \theta(t)$

$$\boxed{e_b(t) = K_b \frac{d}{dt} \theta(t)} \quad \text{—— ②}$$

K_b = back emf constant

KVL to i/p loop $\quad e_a(t) = R_a i_a(t) + L_a \frac{d}{dt} i_a(t) + e_b(t)$ —— ③

$T_m(t) = J \ddot{\theta}(t) + B \dot{\theta}(t)$

$T_m(t) = J \frac{d^2}{dt^2} \theta(t) + B \frac{d}{dt} \theta(t)$ —— ④

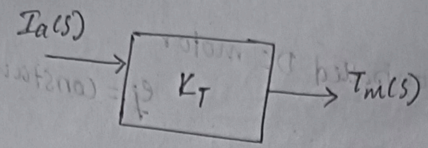
$G(s) = \frac{\theta(s)}{E_a(s)}$

$T_m(s) = K_T I_a(s)$

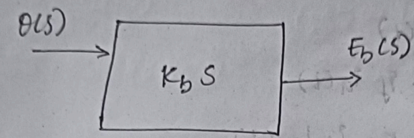
$E_b(s) = K_b \cdot s \, \theta(s)$

$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s)$

$T_m(s) = J s^2 \theta(s) + B s \theta(s)$

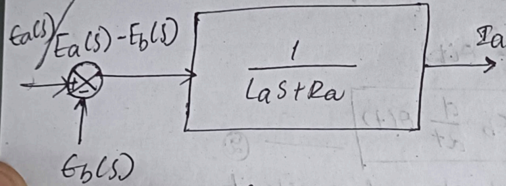


$\Rightarrow \dfrac{T_m(S)}{I_a(S)} = K_T$



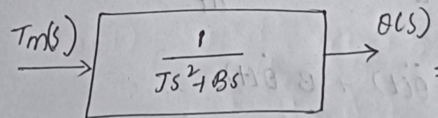
$\Rightarrow \dfrac{E_b(S)}{\theta(S)} = K_b S$

$E_a(S) - E_b(S) = (L_a S + R_a)\, I_a(S)$

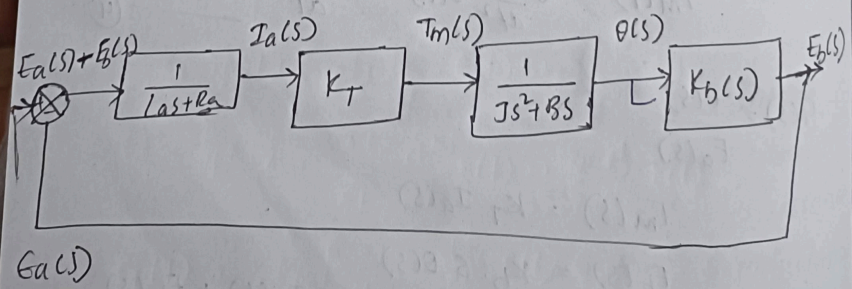


$\Rightarrow \dfrac{I_a(S)}{E_a(S) - E_b(S)} = \dfrac{1}{L_a S + R_a}$

$T_m(S) = (JS^2 + BS)\,\theta(S)$



$\Rightarrow \dfrac{\theta(S)}{T_m(S)} = \dfrac{1}{JS^2 + BS}$



$$\frac{T_m(s)}{Js^2+Bs} = \theta(s)$$

$$\frac{K_T\, I_a(s)}{Js^2 Bs} = \theta(s)$$

$$E_a(s) = (R_a + L_a s)\, I_a(s) + K_b\, s\, \theta(s)$$

$$\frac{E_a - K_b s\, \theta(s)}{R_a + L_a s} = I_a(s)$$

$$\frac{K_T}{Js^2+Bs}\left[\frac{E_a - K_b s\, \theta(s)}{R_a + L_a s}\right] = \theta(s)$$

$$\frac{K_T\, E_a(s)}{(Js^2+Bs)(R_a+L_a s)} = \underbrace{\cancel{\frac{(K_T K_b s)\, \theta(s)}{R_a + L_a(s)}}}_{} + \theta(s)$$

$$\cancel{\frac{K_T}{(Js^2+Bs)(R_a+L_a s)(1+K_T K_b s)}} = \frac{\theta(s)}{E_a(s)}$$

$K_T K_b s\, \theta(s)$

$$\frac{K_T E_a(s)}{(Js^2+Bs)(R_a+L_a s)} - \frac{K_b K_T s\theta(s)}{(Js^2+Bs)(R_a+L_a s)} = \theta(s)$$

$$\frac{K_T E_a(s)}{\cancel{(Js^2+Bs)(R_a+L_a s)}} = \frac{(Js^2+Bs)(R_a+L_a s) + K_b K_T s}{\cancel{(Js^2+Bs)(R_a+L_a s)}}\,\theta(s)$$

$$\frac{K_T}{(Js^2+Bs)(R_a+L_a s) + K_b K_T s} = \frac{\theta(s)}{E_a(s)} = G(s)$$

Field-controlled DC motor

$e_a(t) = \text{constant}$

$i_a = \text{constant}$

$$T_m(t) = K_T\, i_f(t)$$

$$T_m(t) = J.\frac{d^2}{dt}\theta(t) + B\frac{d}{dt}\theta(t)$$

$$e_f(t) = R_f\, i_f(t) + L_f\frac{d}{dt} i_f(t)$$

$$K_T\, i_f(t) = J.\frac{d^2}{dt}\theta(t) + B\frac{d}{dt}\theta(t)$$

$$i_f(t) = \frac{J}{K_T}\frac{d^2}{dt}\theta(t) + B\cdot\frac{d}{dt}\theta(t)$$

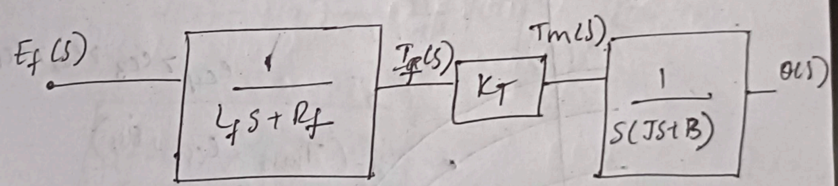
$$e_f(t) = R_f\left[\frac{J}{K_T}\frac{d^2}{dt}\theta(t) + B\cdot\frac{d}{dt}\theta(t)\right] + L_f\left[\frac{J}{K_T}\frac{d^3}{dt^3}\theta(t) + B\cdot\frac{d^2}{dt^2}\theta(t)\right]$$

$$E_f(s) = R_f\left[\frac{J}{K_T}s^2\theta(s) + B\cdot s\,\theta(s)\right] + L_f\left[\frac{J}{K_T}s^3\theta(s) + B\cdot s^2\theta(s)\right]$$

$$E_f(s) = \left(\frac{R_f J}{K_T}s^2 + Bs + \frac{L_f J}{K_T}s^3 + Bs^2\right)\theta(s)$$

$$\frac{E_f(s)}{\theta(s)} = \frac{L_f J s^3 + R_f J s^2 + K_T B s^2 + K_T B s}{K_T}$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K_T}{(L_f s + R_f)Js^2 + K_T B s\,[s+1]}$$

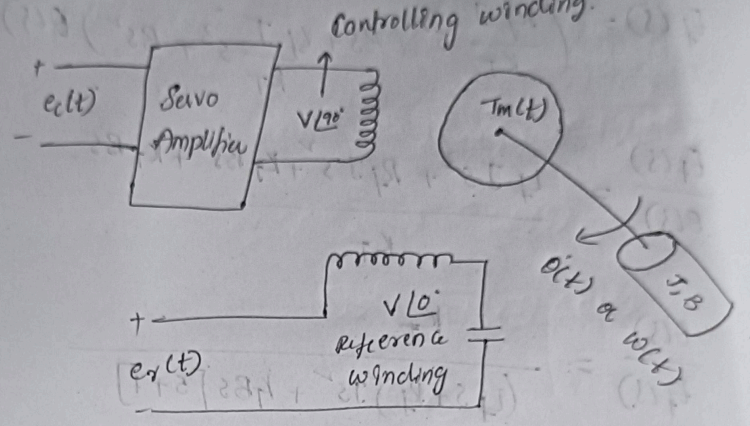


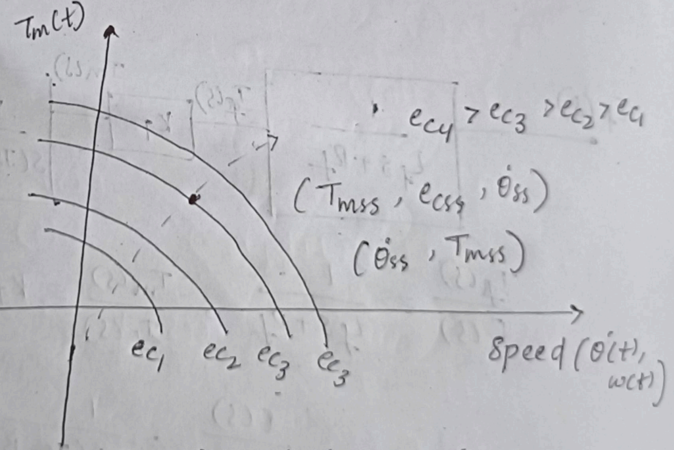
$$\frac{I_f(s)}{E_f(s)} = \frac{1}{L_f s + R_f}, \quad \frac{T_m(s)}{I_f(s)} = K_T$$

$$\frac{\theta(s)}{T_m(s)} = \frac{1}{s(Js+B)}$$

28/2/24

Modeling of AC Servo Motor.





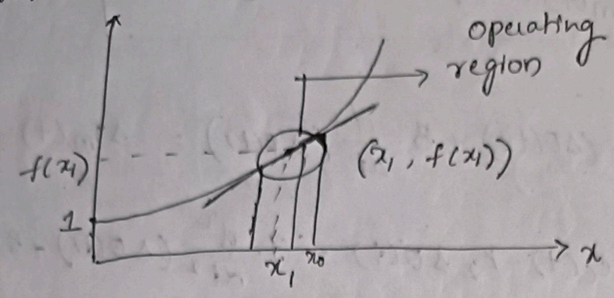
$$T_m(t) = f(e_c(t), \dot\theta(t))$$

$$T_m(t) = J \cdot \frac{d^2}{dt^2}\theta(t) + B \cdot \frac{d}{dt}\theta(t).$$

$$e^x = \sum_{i=0}^{\infty} \cancel{\frac{x^i}{i!}} \quad \frac{x^i}{i!}$$

$$(T_{mss}, e_{css}, \dot\theta_{css})$$

$f(x) = e^x$



$f'(x)\Big|_{x=x_1}$ tangent drawn at $x = x_1$

$f(x_0) \quad f'(x_0)$

$$f(x) = f(x_1) + \underbrace{\frac{d}{dx} f(x)\Big|_{x=x_1}}_{\text{slope}} \underbrace{(x_1 - x_0)}_{\text{distance}}$$

$$f(x) - f(x_1) = \frac{d}{dx} f(x)\Big|_{x=x_1} (x_1 - x_0)$$

$$\boxed{\Delta f(x) = f'(x) \Delta x}$$

Operating point, $(\dot{\theta}_{ss}, T_{mss})$

$$T_m(t) = f(e_{ss}, \dot{\theta}_{ss}) + \frac{\partial T_m(t)}{\partial e_c(t)}\Big|_{(e_{css}, \dot{\theta}_{ss})} (e_c(t) - e_{ss})$$

$$+ \frac{\partial T_m(t)}{\partial \dot{\theta}(t)}\Big|_{(e_{css}, \dot{\theta}_{ss})} (\dot{\theta}(t) - \dot{\theta}_{ss})$$

$T_m(t) - T_{mss} =$

$$\Delta T_m(t) = \frac{\partial T_m(t)}{\partial e_c(t)}\Big| (\Delta e_c(t)) + \frac{\partial T_m(t)}{\partial \dot{\theta}(t)}\Big| (\Delta \dot{\theta}(t))$$

$$\boxed{\Delta T_m(t) = K_1 \Delta e_c(t) - K_2 \Delta \dot{\theta}(t)}$$

$$J \cdot \frac{d^2}{dt^2}(\Delta\theta(t)) + B \cdot \frac{d}{dt}(\Delta\theta(t)) = \Delta T_m(t)$$

$$K_1 \Delta e_c(t) - K_2 \Delta\dot{\theta}(t) = J \cdot \frac{d^2}{dt^2}(\Delta\theta(t)) + B \cdot \frac{d}{dt}(\Delta\theta(t))$$

$$K_1 \Delta E_c(S) - K_2 S \Delta\theta(S) = J \cdot S^2 \Delta\theta(S) + B \cdot S\, \theta(S)$$

$$K_1 \cdot \Delta E_c(S) = (JS^2 + BS + K_2 S)\Delta\theta(S)$$

$$\frac{K_1}{JS^2 + BS + K_2 S} = \frac{\Delta\theta(S)}{\Delta E_c(S)}$$

$$\Delta G(S) = \frac{K_1}{S(JS + (B + K_2))}$$

$$= \frac{K_1/(B+K_2)}{\left(1 + \frac{JS}{B+K_2}\right)S}$$

motor constant

$$K_m = \frac{K_1}{B+K_2}$$

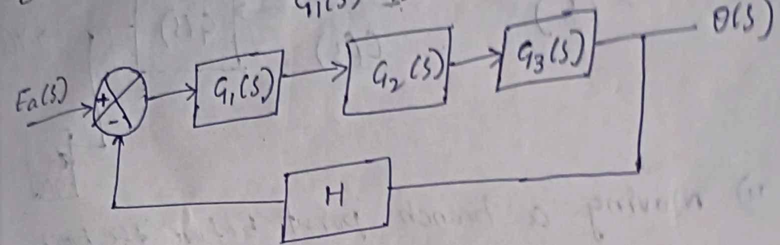
$$= \frac{K_m}{1 + (T_m S + 1)}$$

$$T = \frac{J}{B+K_2}$$

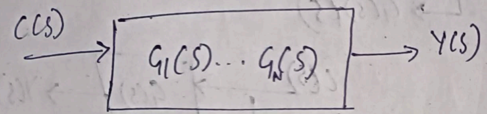
29/02/24

Block diagram of Armature controlled DC motor

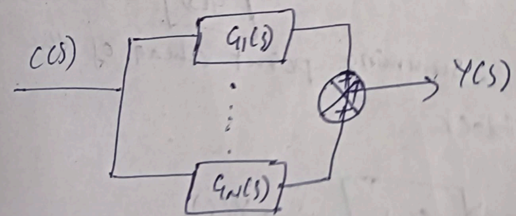
$G_1(S) \cdot G_2(S) \cdot G_3(S)$

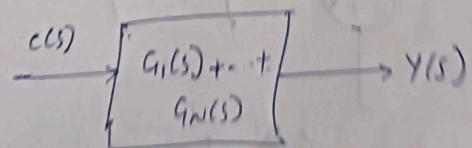


① ~~Series~~ Cascade

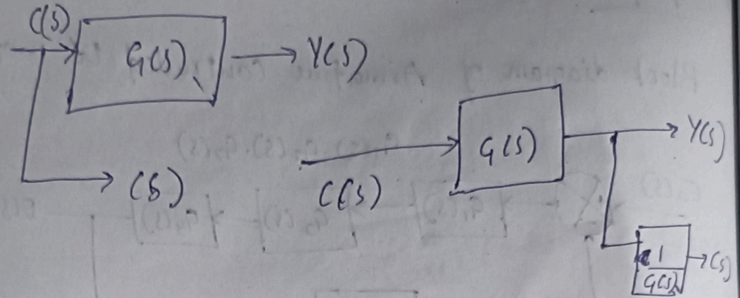


② Parallel

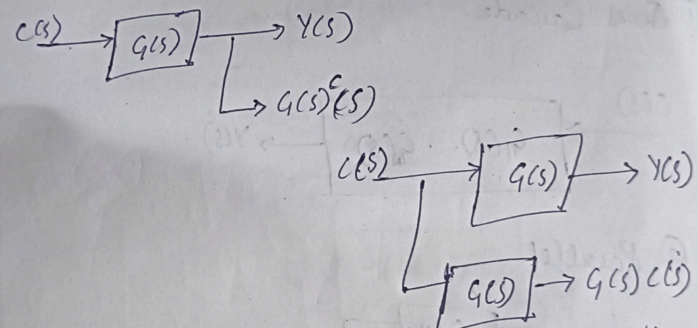




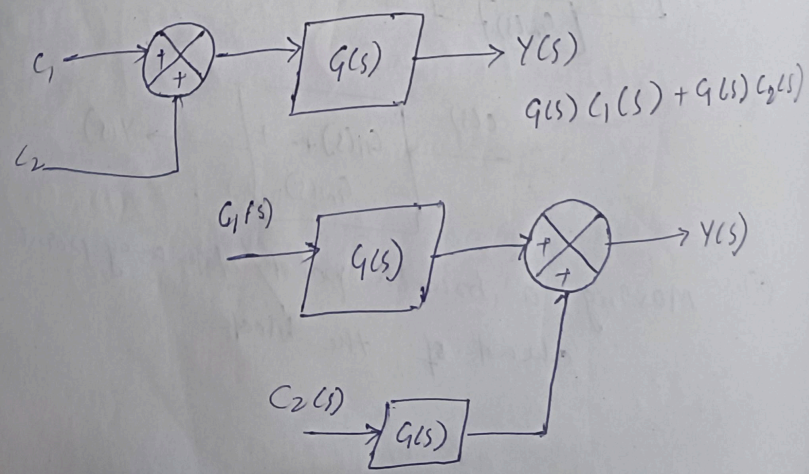
③ Moving a branch point/ take away point ahead of the block



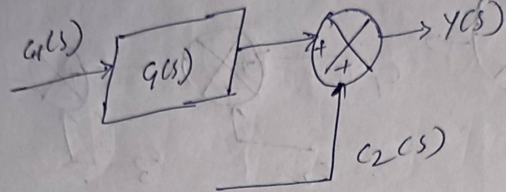
4) Moving a branch point before the block

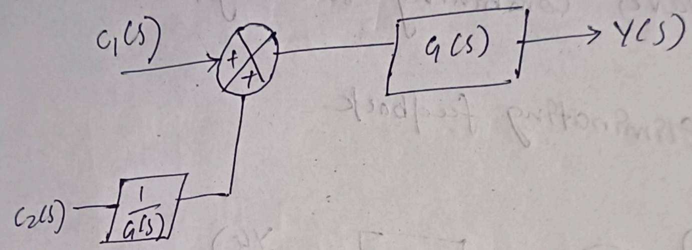


(5) Moving the summing point ahead of the block

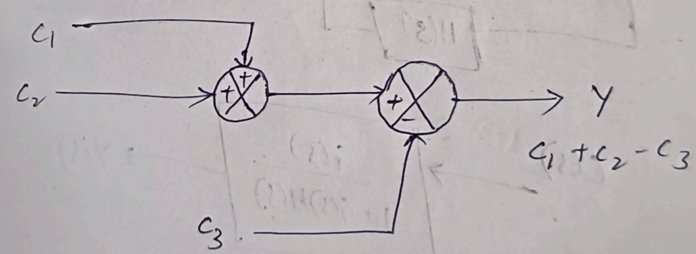


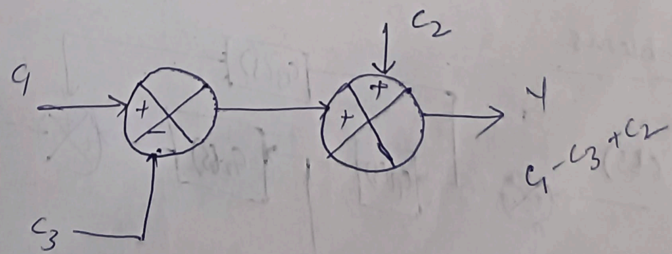
⑥ Moving the summing point before the block



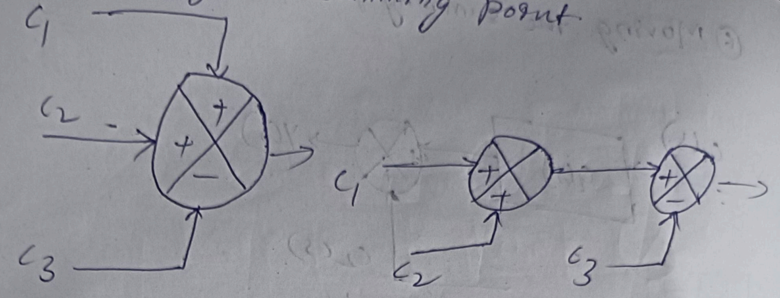


⑦ Interchanging the summing points.



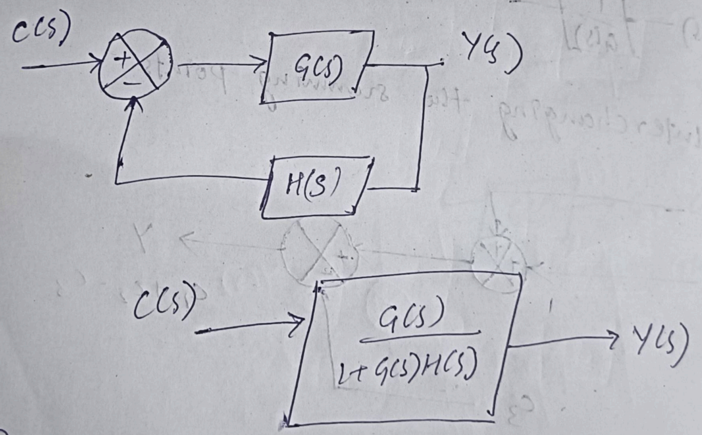


(8) Splitting the summing point.



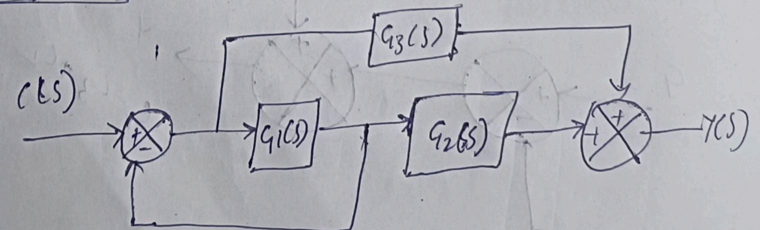
(9) (Rev) Combining summing point.

(10) Eliminating feedback



Problems

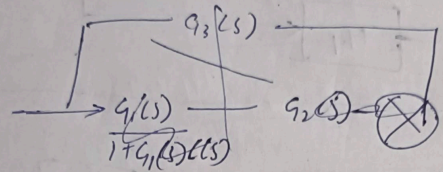
(1)



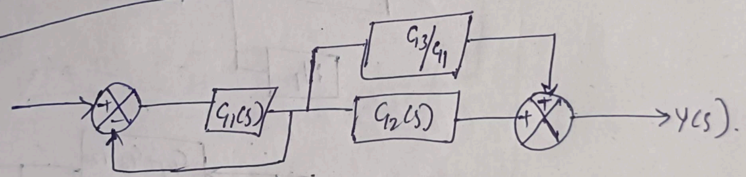
Y(s) =

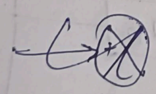
$G_1(s)\,C(s)$

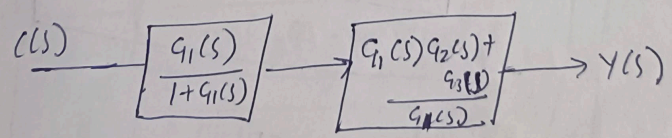
$\dfrac{\cancel{G_1(s)}}{1+G_1(s)C(s)}$

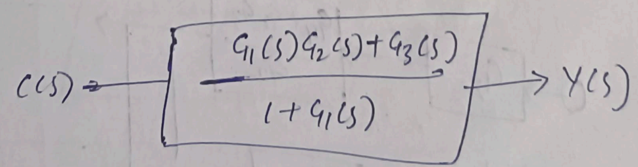


$C(s) \rightarrow \dfrac{G_1(s)}{1+G_1(s)C(s)} + G_2(s) \,||\, G_2(s) \rightarrow Y(s).$



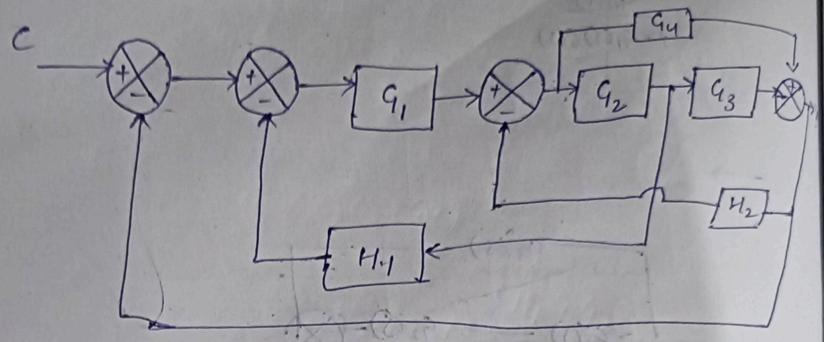


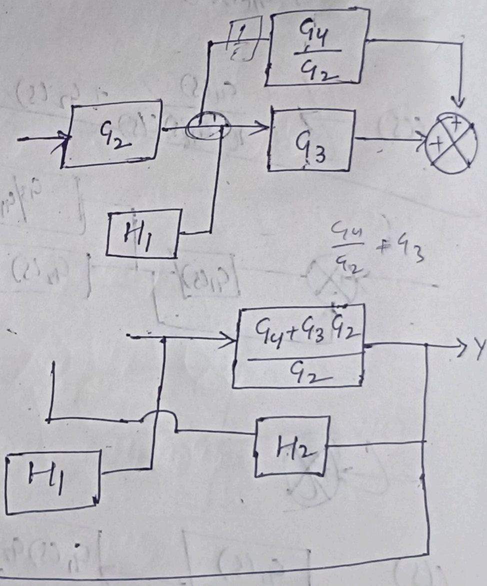


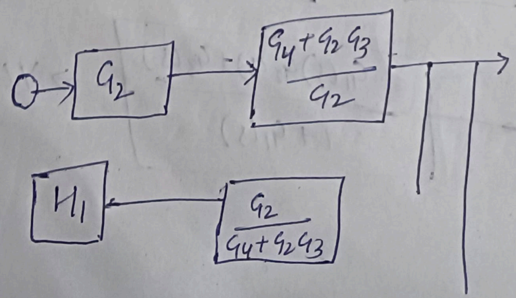


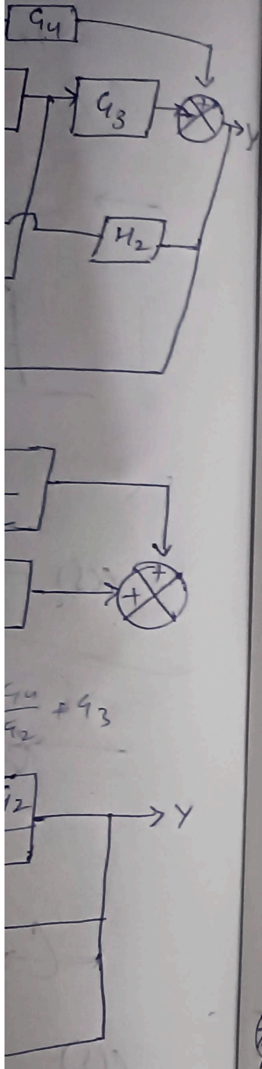
01/08/24

1

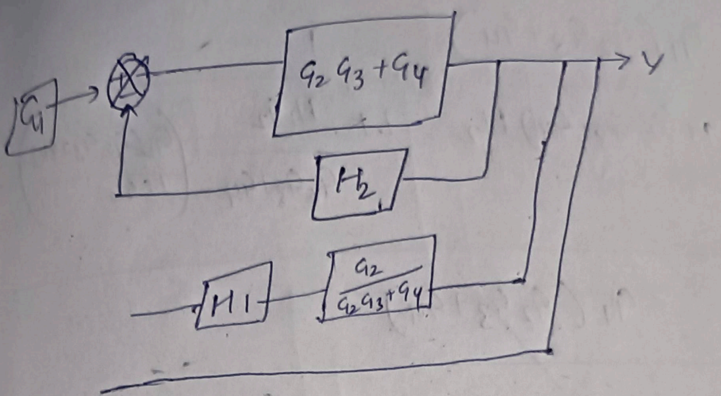

C
G1
G2
G3
G4
H1
H2


G4/G2
G2
G3
H1
G4/G2 + G3
(G4+G3G2)/G2
Y
H1
H2

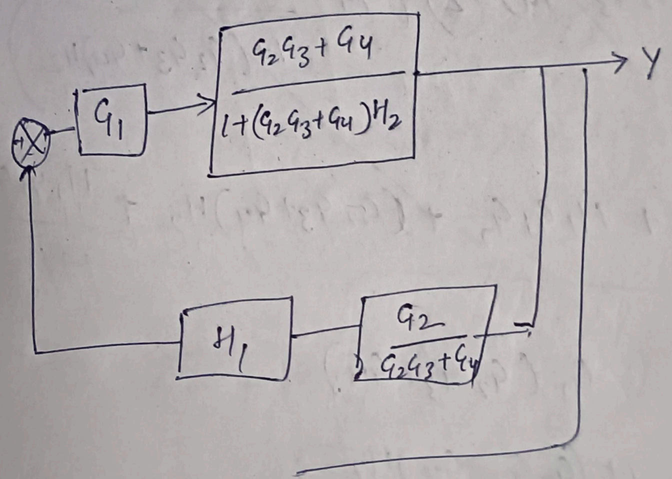

G2
(G4+G2G3)/G2
H1
G2/(G4+G2G3)



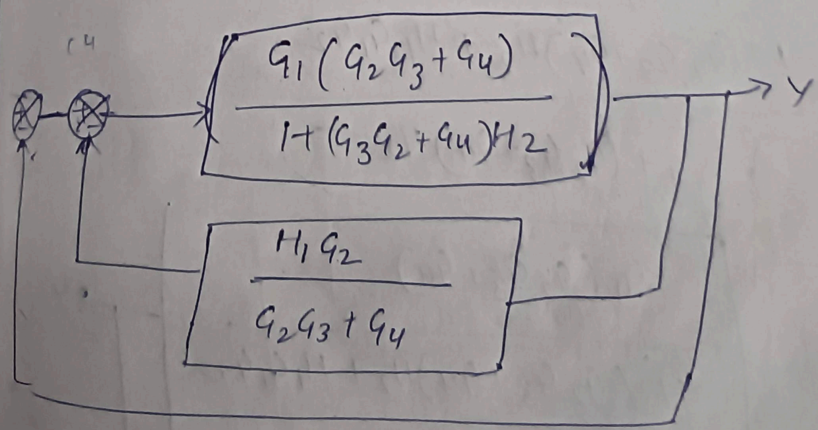
G4
G3
H2
Y
+
G4/G2 + G3
Y



G1
G2 G3 + G4
Y
H2
H1
G2 / (G2G3+G4)



G1
(G2G3 + G4) / (1+(G2G3+G4)H2)
Y
H1
G2 / (G2G3+G4)



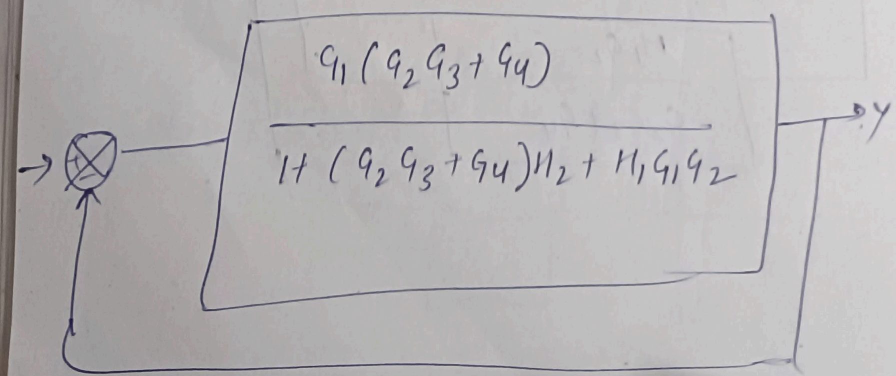
G1 (G2G3 + G4) / (1+ (G3G2 + G4)H2)
Y
H1G2 / (G2G3 + G4)

$$\frac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2} \times \frac{1}{1+\frac{H_1G_2}{G_2G_3+G_4}\left(\frac{G_1(G_2G_3+G_4)}{1+G_2}\right)}$$

$$\frac{G_1(G_2G_3+G_4)}{\left(1+(G_2G_3+G_4)H_2\right)\left(1+\frac{H_1G_1G_2}{1+(G_2G_3+G_4)H_2}\right)}$$

$$1+H_1G_1G_2+(G_2G_3+G_4)H_2+\frac{H_1G_1G_2}{1+1}$$

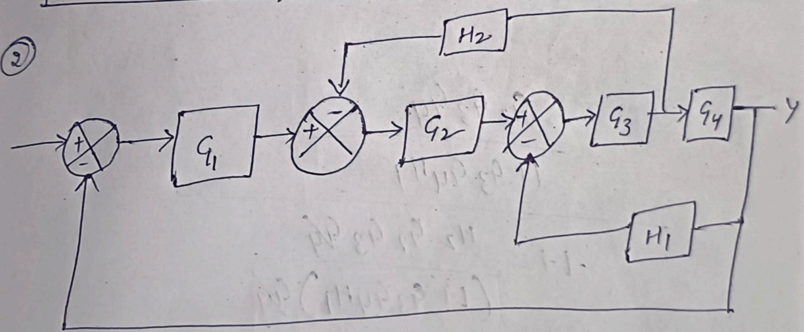
$$\frac{\frac{G_1(G_2G_3+G_4)}{\cancel{1+(G_2G_3+G_4)H_2}}}{\frac{1+(G_2G_3+G_4)H_2+H_1G_1G_2}{\cancel{1+(G_2G_3+G_4)H_2}}}$$

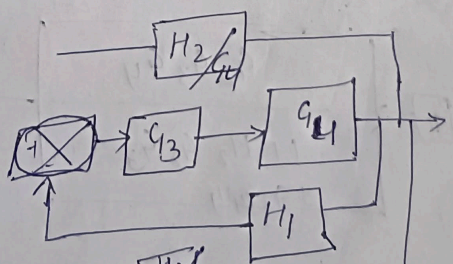


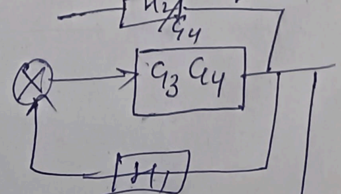
$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+H_1G_1G_2}}{\dfrac{1+(G_2G_3+G_4)H_2+H_1G_1G_2+G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+H_1G_1G_2}}$$

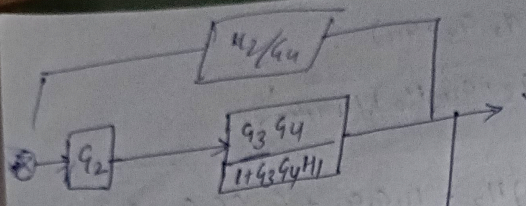
$$C \rightarrow \boxed{\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+(G_1+H_2)(G_2G_3+G_4)}} \rightarrow Y$$

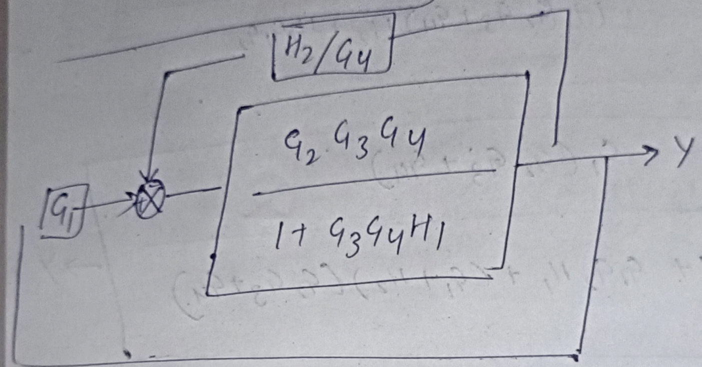
②



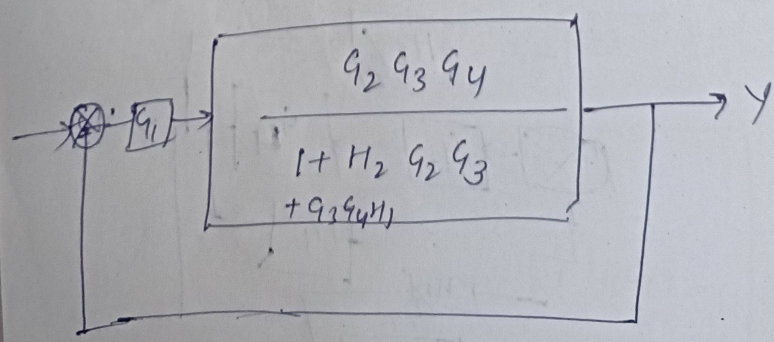


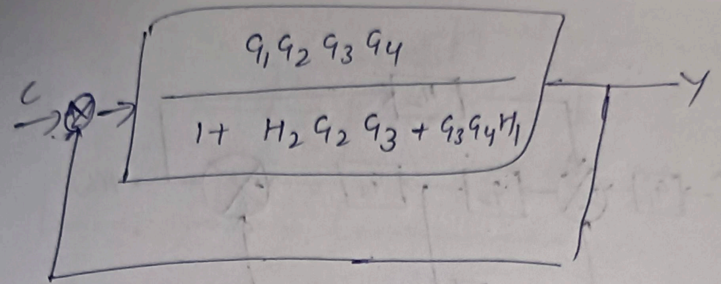




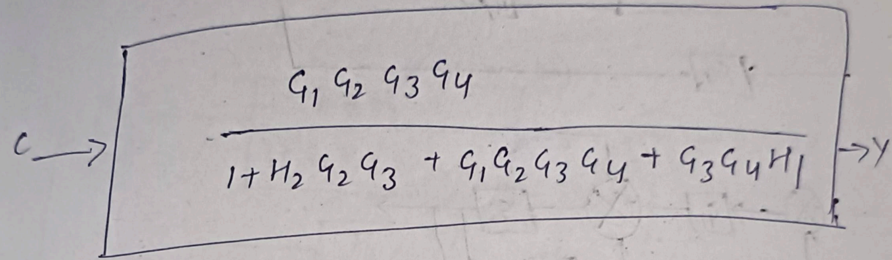


$$\frac{\dfrac{G_2 G_3 G_4}{1+G_3 G_4 H_1}}{1+\dfrac{H_2 G_2 G_3 G_4}{(1+G_3 G_4 H_1) G_4}}$$



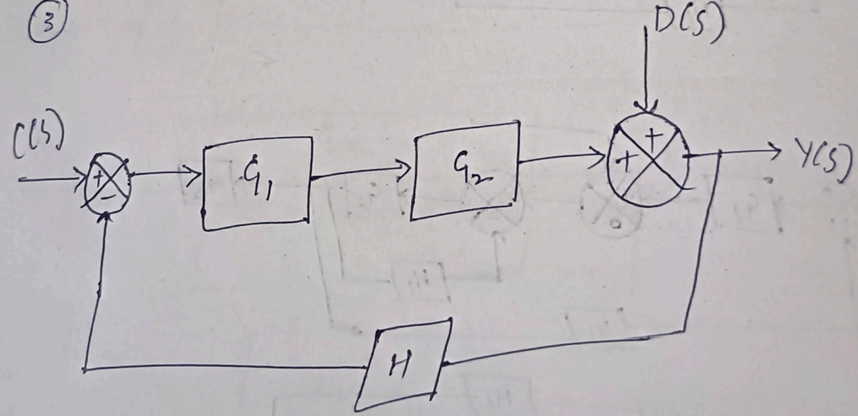


C
$\frac{G_1 G_2 G_3 G_4}{1+H_2 G_2 G_3 + G_3 G_4 H_1}$
Y



C
$\frac{G_1 G_2 G_3 G_4}{1+H_2 G_2 G_3 + G_1 G_2 G_3 G_4 + G_3 G_4 H_1}$
Y

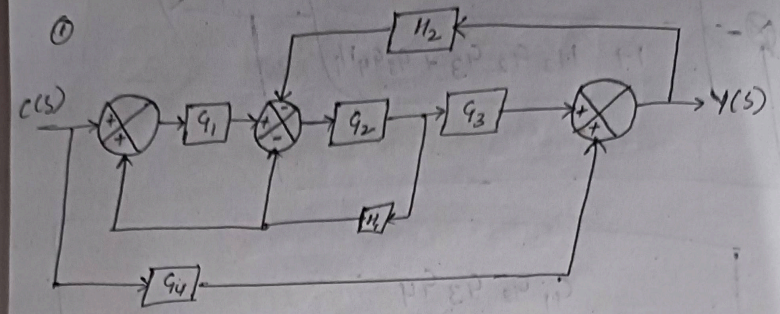
③



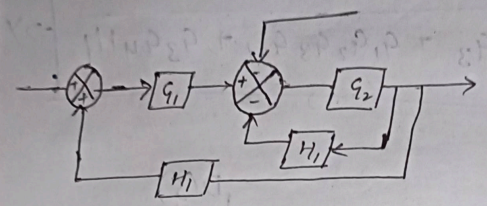
D(S)
C(S)
+
-
G_1
G_2
+
+
Y(S)
H

02/8/24

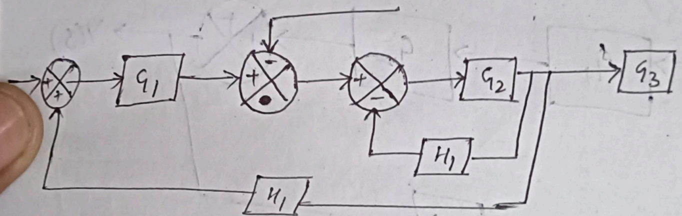
1



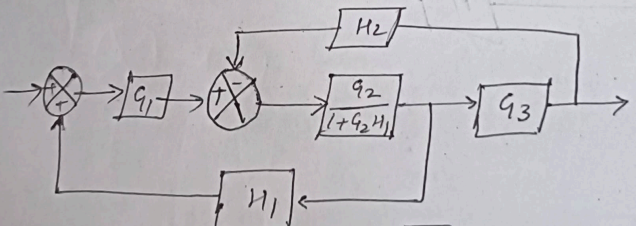
C(S)
G1
G2
G3
H2
H1
G4
Y(S)



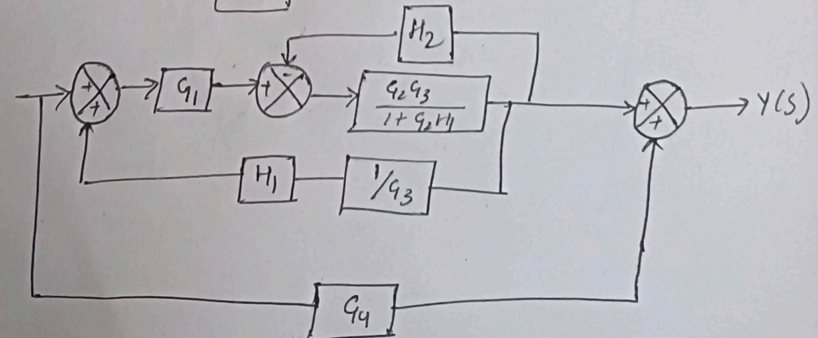
G1
G2
H1
H1



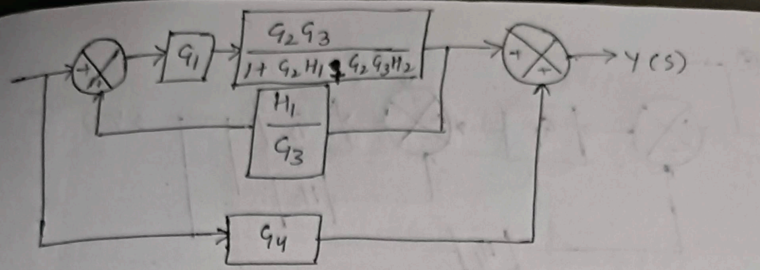
G1
G2
G3
H1
H1

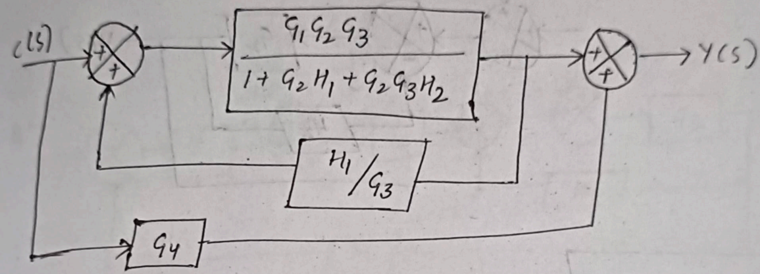


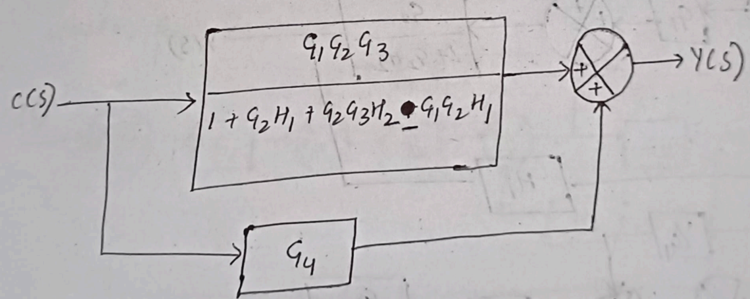
H2
G1
G2 / 1+G2H1
G3
H1

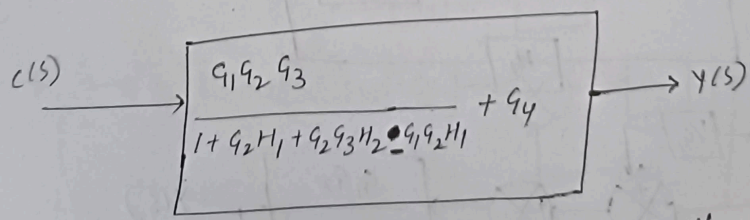


H2
G1
G2G3 / 1+G2H1
H1
1/G3
G4
Y(S)



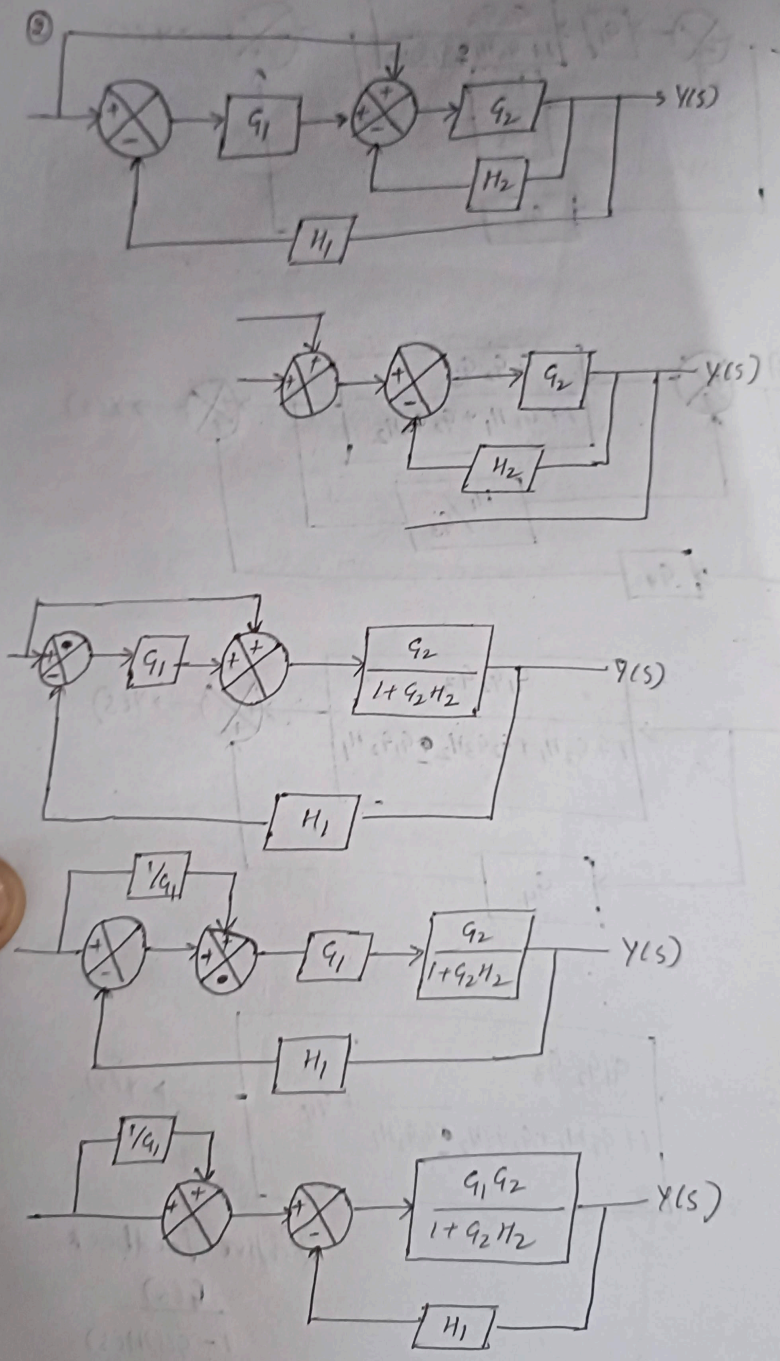




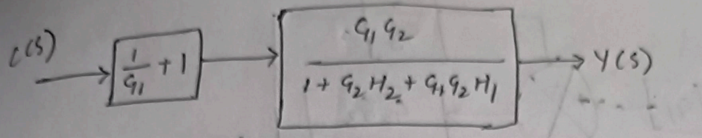


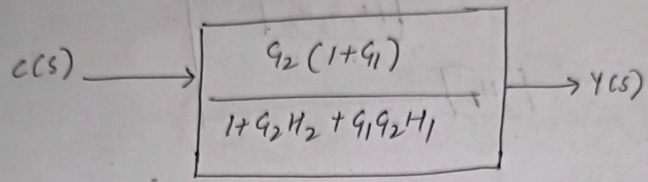
Positive feedback

$$\frac{G(S)}{1-G(S)H(S)}.$$

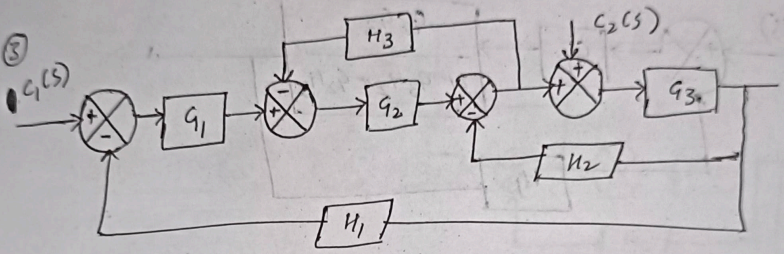


G_1
G_2
H_2
H_1
$Y(s)$
$\frac{G_2}{1+G_2H_2}$
$1/G_1$
$\frac{G_1G_2}{1+G_2H_2}$

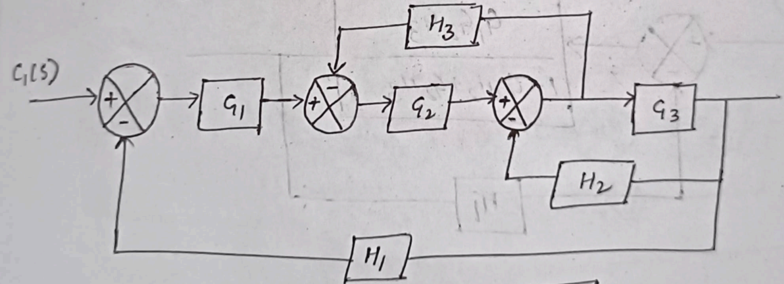

C(s)
1/G1 + 1
G1G2
1 + G2H2 + G1G2H1
Y(s)

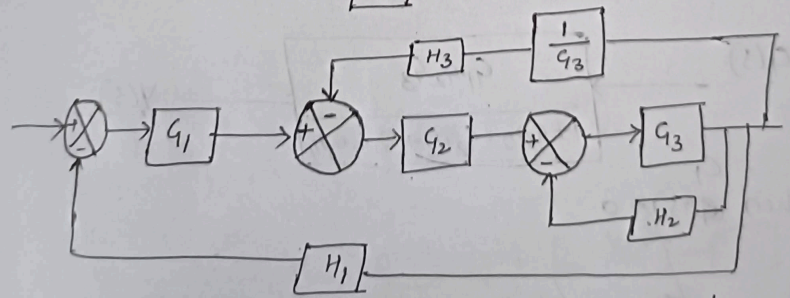

C(s)
G2 (1+G1)
1+G2H2 + G1G2H1
Y(s)

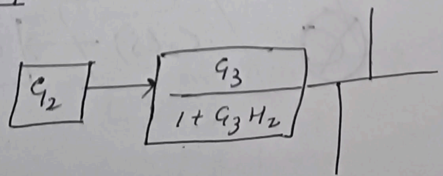
⑧

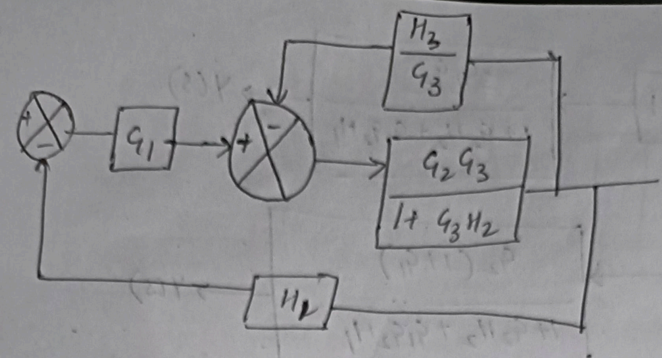

C1(s)
G1
H3
G2
C2(s)
G3
H2
H1

where, $C_2 = 0$

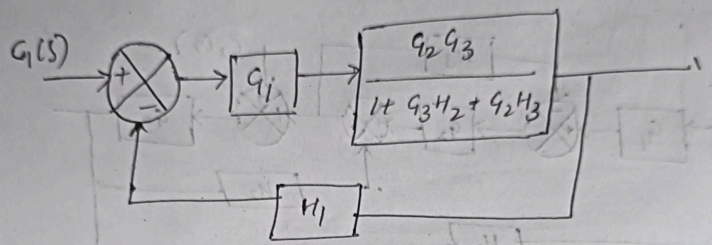

C1(s)
G1
H3
G2
G3
H2
H1


G1
H3
1/G3
G2
G3
H2
H1

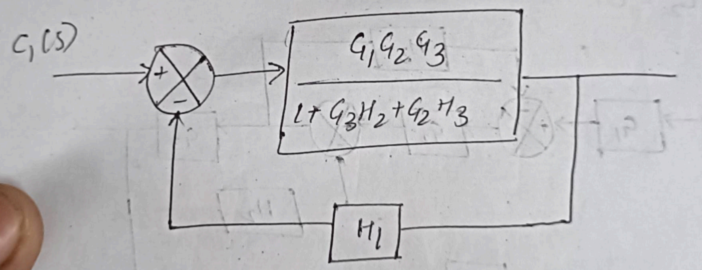

G2
G3
1 + G3H2



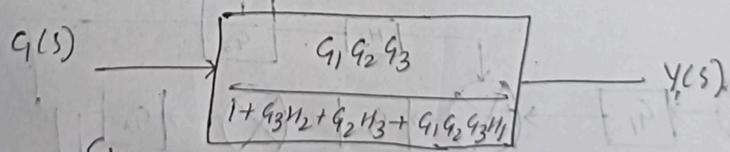
G_1
$\frac{H_3}{G_3}$
$\frac{G_2G_3}{1+G_3H_2}$
H_1



$G_1(S)$
G_1
$\frac{G_2G_3}{1+G_3H_2+G_2H_3}$
H_1

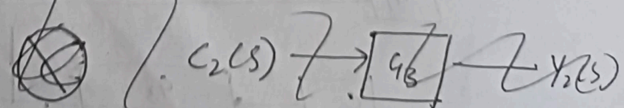


$C_1(S)$
$\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}$
H_1



$G_1(S)$
$\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$
$Y_1(S)$

when $C_1(S) = 0$



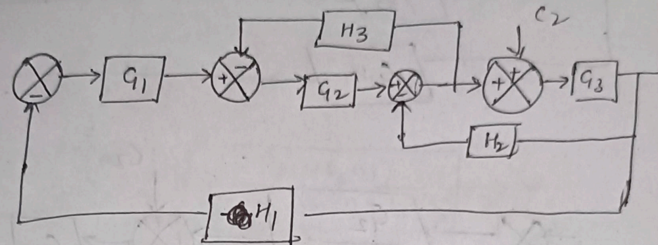
$C_2(S)$
G_3
$Y_2(S)$

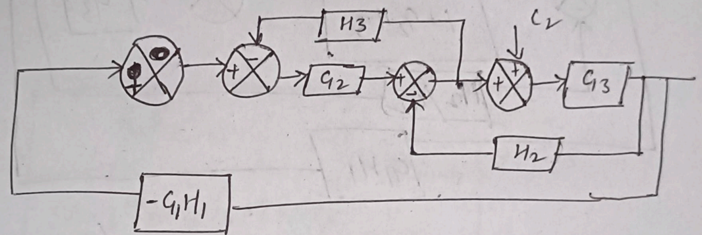
04/03

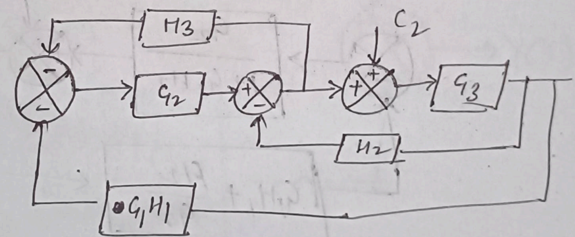
$$G_1 = \left.\frac{Y(s)}{C_1(s)}\right|_{C_2(s)=0} \qquad G_2 = \left.\frac{Y(s)}{C_2(s)}\right|_{C_1(s)=0}$$

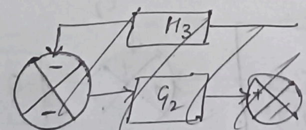
$$Y(s) = G_1(s)\,C_1(s) + G_2(s)\,C_2(s)$$

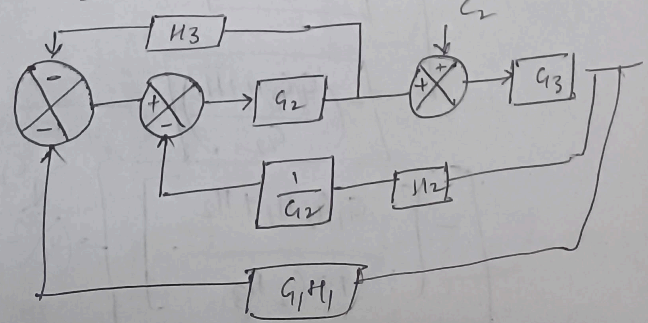
when $C_1 = 0$

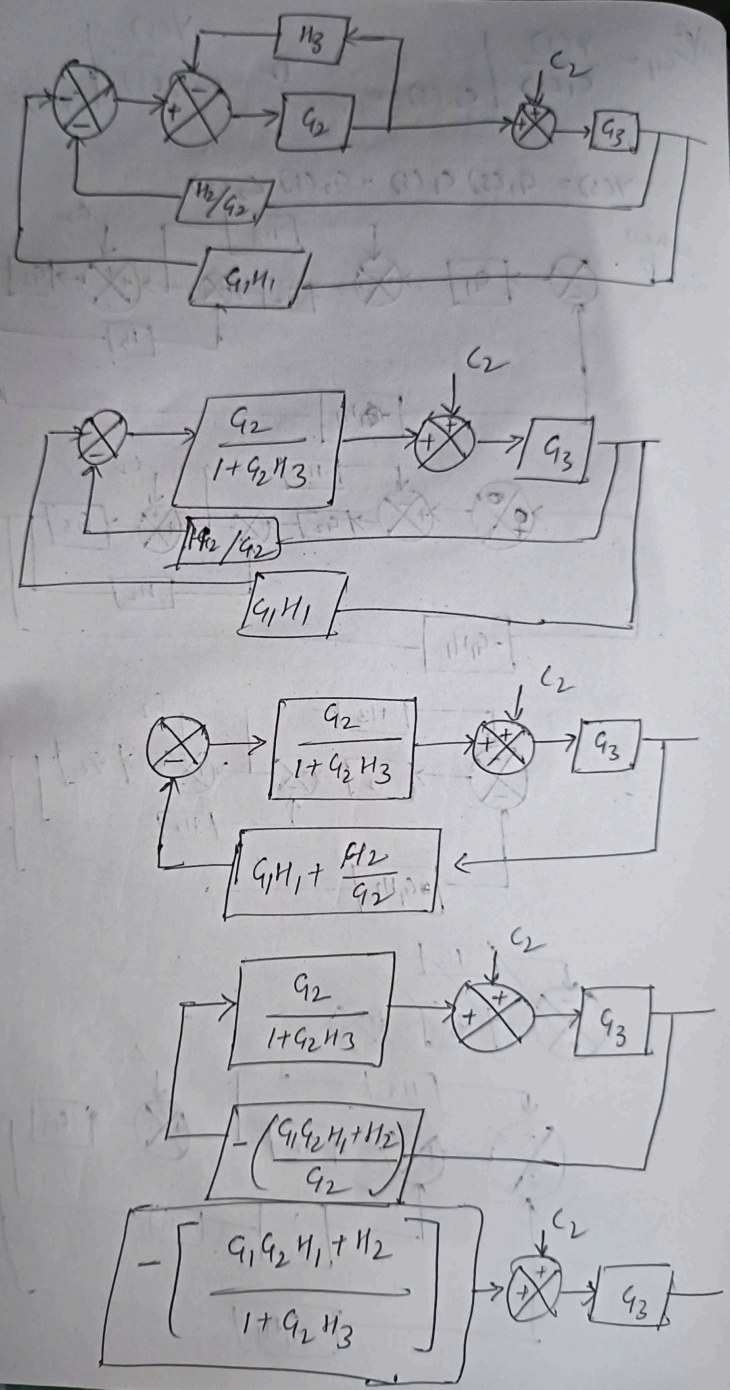


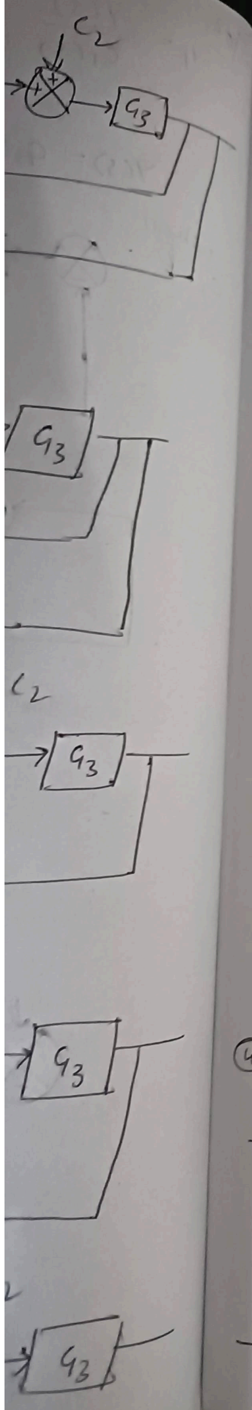




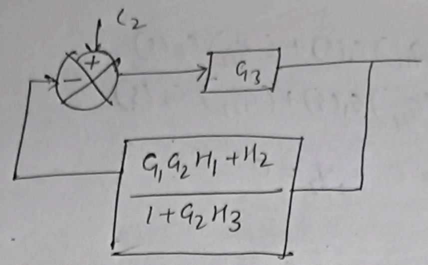




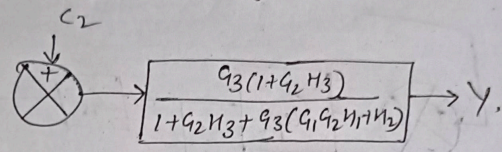

H_3
C_2
G_2
G_3
H_2/G_2
G_1H_1
$\frac{G_2}{1+G_2H_3}$
H_2/G_2
G_1H_1
G_3
C_2
$\frac{G_2}{1+G_2H_3}$
G_3
C_2
$G_1H_1 + \frac{H_2}{G_2}$
$\frac{G_2}{1+G_2H_3}$
G_3
C_2
$-\left(\frac{G_1G_2H_1+H_2}{G_2}\right)$
$-\left[\frac{G_1G_2H_1+H_2}{1+G_2H_3}\right]$
C_2
G_3



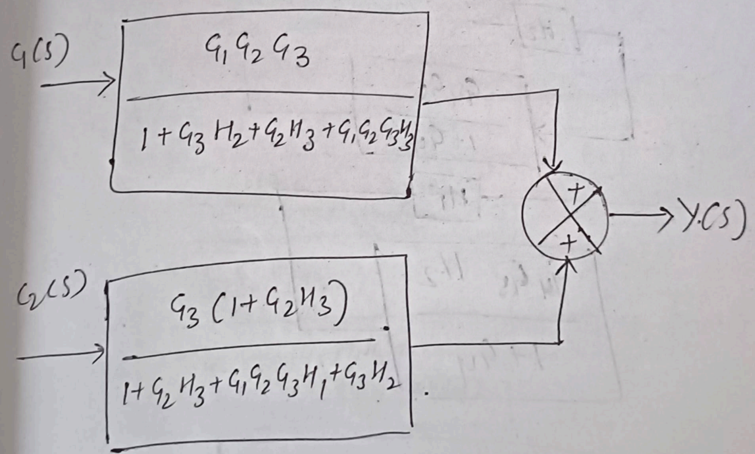
C_2
G_3



C_2
G_3
$\frac{G_1G_2H_1+H_2}{1+G_2H_3}$

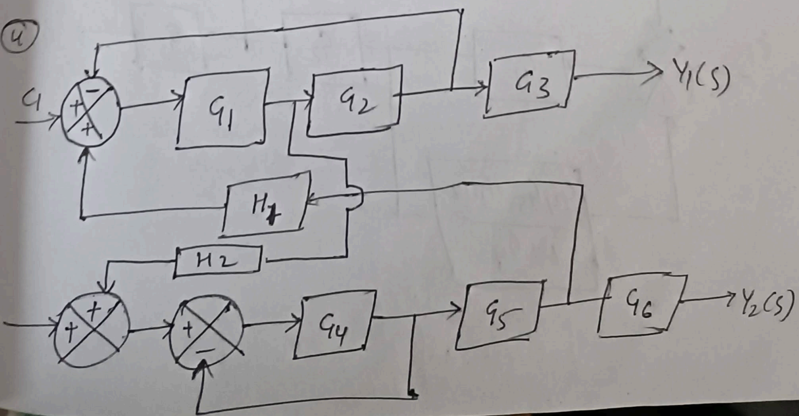


C_2
$\frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$
Y.



$C_1(s)$
$\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$
$C_2(s)$
$\frac{G_3(1+G_2H_3)}{1+G_2H_3+G_1G_2G_3H_1+G_3H_2}$
Y(S)

(4)

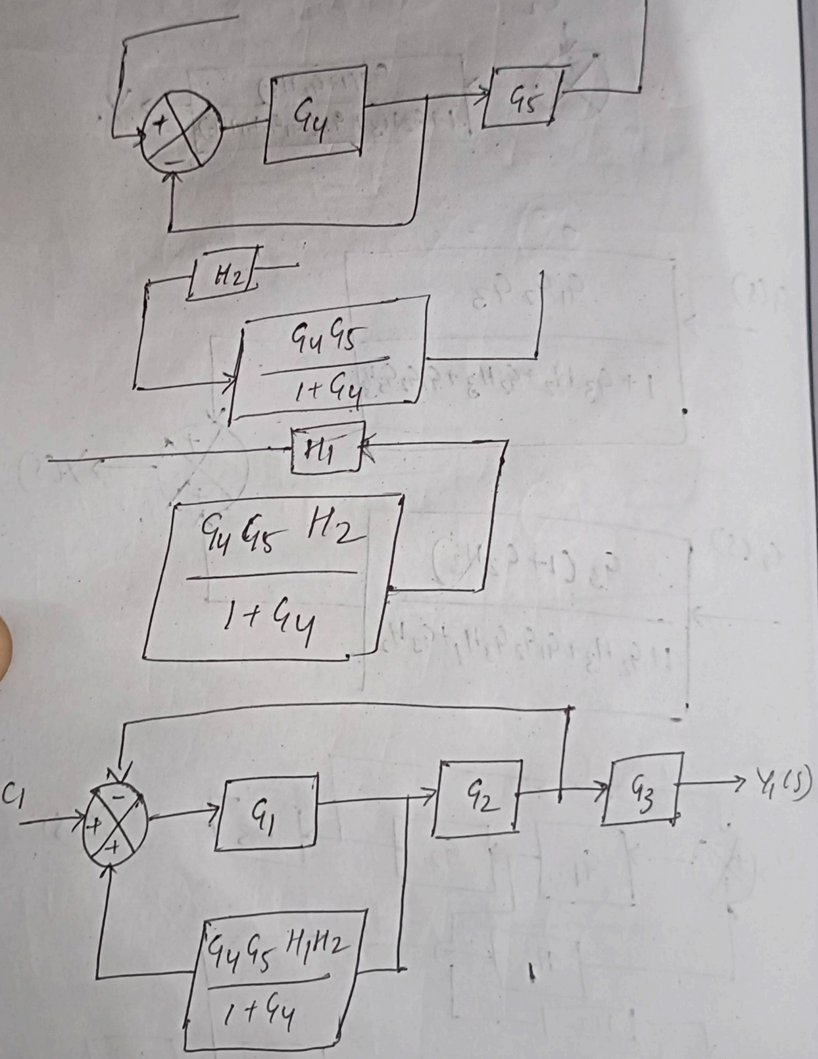


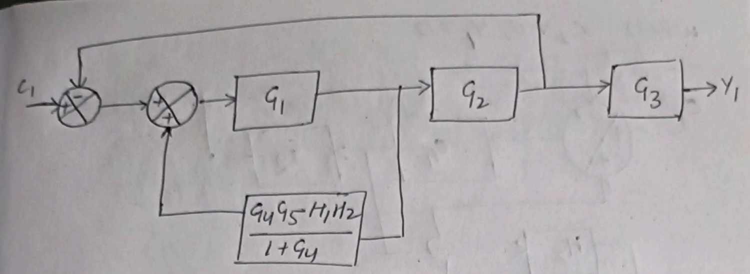
C_1
G_1
G_2
G_3
$Y_1(S)$
H_1
H_2
G_4
G_5
G_6
$Y_2(S)$

$Y_1(s) = (G_{11})C_1(s) + (G_{12})C_2(s)$

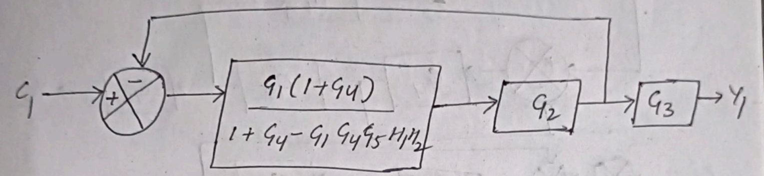
$Y_2(s) = (G_{21})C_1(s) + (G_{22})C_2(s)$

when $C_2 = 0$, $Y_2 = 0$

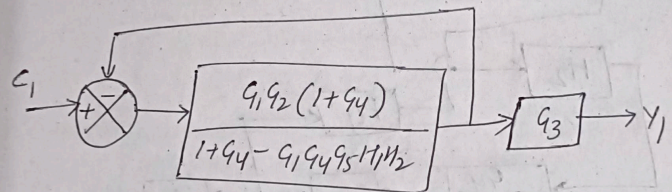




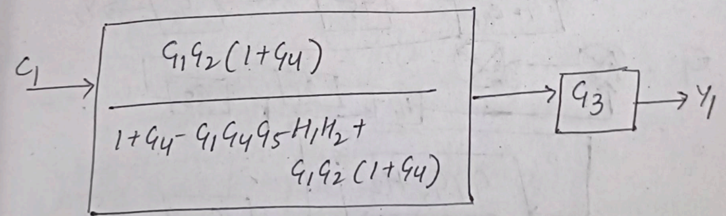
C1
G1
G2
G3
Y1
G4G5-H1H2
1+G4



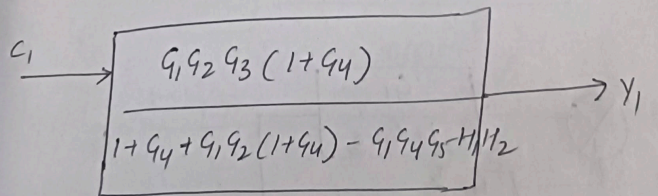
C1
G1(1+G4)
1+G4-G1G4G5-H1H2
G2
G3
Y1



C1
G1G2(1+G4)
1+G4-G1G4G5-H1H2
G3
Y1

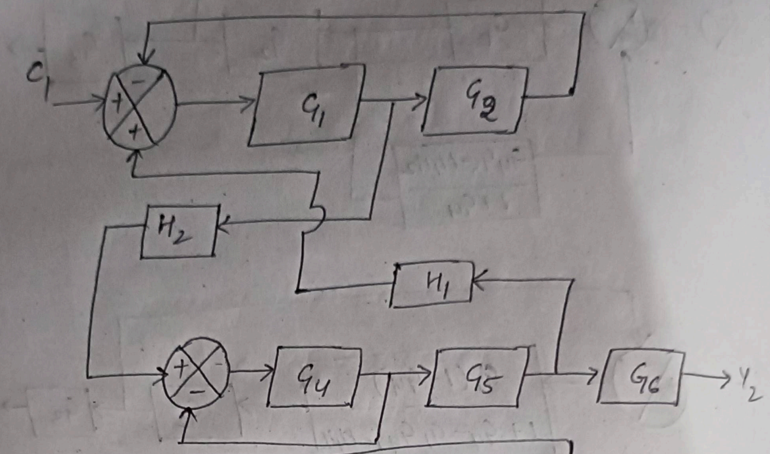


C1
G1G2(1+G4)
1+G4-G1G4G5-H1H2+
G1G2(1+G4)
G3
Y1

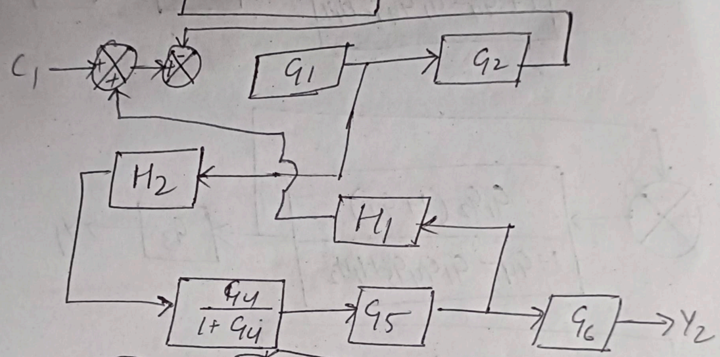


C1
G1G2G3(1+G4)
1+G4+G1G2(1+G4)-G1G4G5-H1H2
Y1

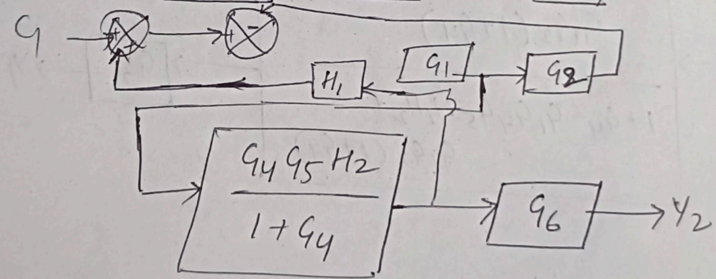
when $C_2 = 0,\ Y_1 = 0$



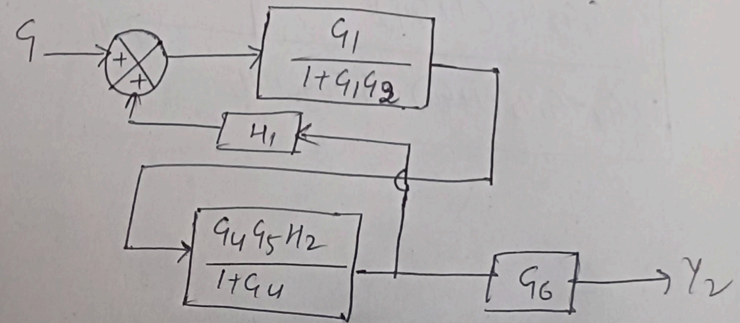
C1
G1
G2
H2
H1
G4
G5
G6
Y2



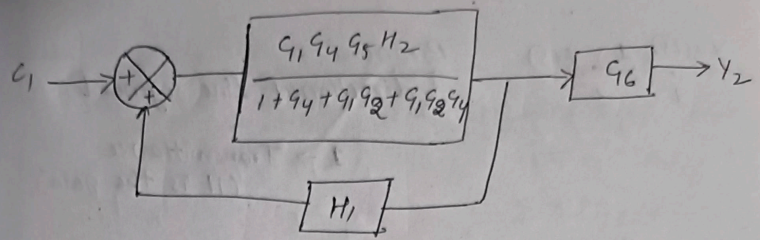
C1
G1
G2
H2
H1
G4 / 1+G4
G5
G6
Y2

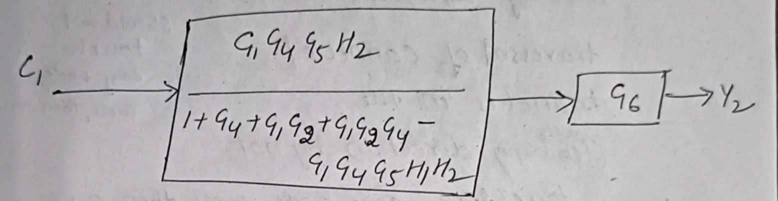


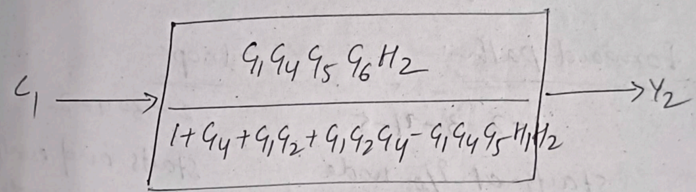
C1
H1
G1
G2
G4G5H2 / 1+G4
G6
Y2



C1
G1 / 1+G1G2
H1
G4G5H2 / 1+G4
G6
Y2

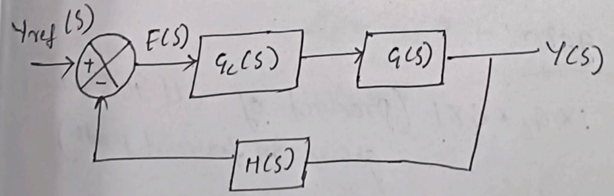


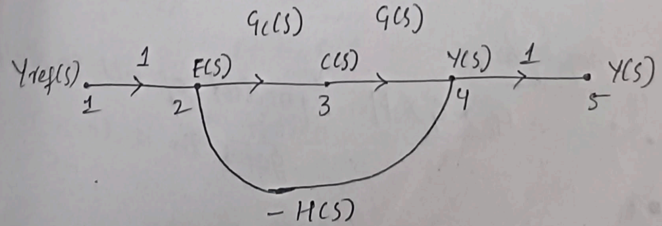




06/03/24

Signal Flow Graph





Input node (source)

Output node

mixed nodes

$Y_{ref}(s) \xrightarrow{1} E(s)$
(nodes 1, 2)

Branch :-
directional line segment

1 → Transmittance
(It is the gain)

Path:-

Journey between two branches

traversal of connected branches in the ~~flowing~~ allo direction by not touching any node more than once

should not touch any node more than once

Forward path :-

1-2-3-4-5

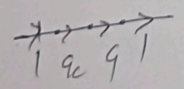
starts at i/p node
ends at o/p node

loop:-

2-3-4-2

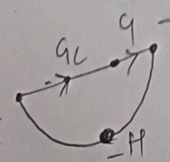
starts and ends at same node

Path gain:-



$1 \times G_c \times G \times 1$ (product of all individual gains in forward path)

loop gain:-



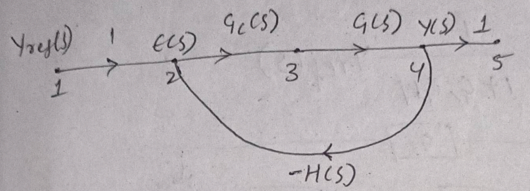
$-G_c \times G \times H$ (product of all individual gains in a loop)

non-touching loops

$$T/F = \frac{\sum P_K \Delta K}{\Delta}$$

, K = no. of forward paths
Δ = determinant of graph
$1 - \sum l_i + \sum \pi l_{ij} \pm \sum \pi l_{ijk} + \dots$

ΔK = determinent of k forward path



$$T/F = \frac{\sum P_K \Delta K}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$$, $P_1 = G_c G$

$l_1 = -G_c G H$

$\Delta = 1 - l_1$

$\Delta = 1 + G_c G H$

$\Delta_1 = 1$

$$T/F = \frac{G_c G \cdot 1}{1 + G_c G H}$$

$$= \frac{G_c G}{1 + G_c G H}$$

$$T/F = \frac{Y(s)}{Y_{ref}(s)} = \frac{G_c G}{1 + G_c G H}$$

$$\frac{E(s)}{Y_{ref}(s)} = \frac{P_1 \Delta_1}{\Delta} =$$

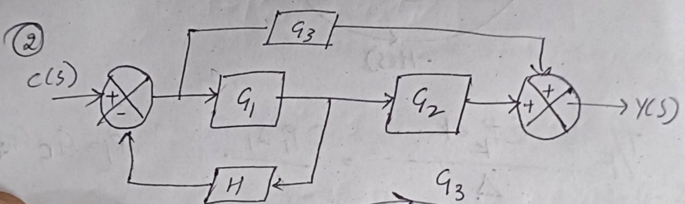
$\Delta = 1 + G_c G H$

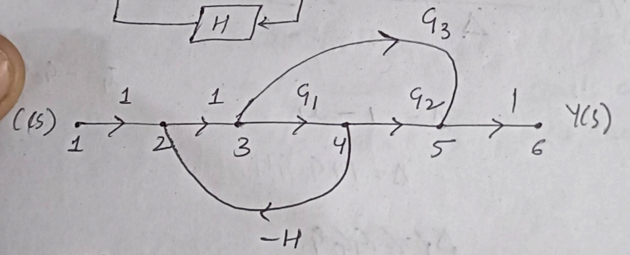
$\Delta_1 = 1$, $P_1 = 1$

$$= \frac{1}{1 + G_c G H}$$

$$E(s) = \frac{1}{1 + G_c G H} Y_{ref}(s)$$

(2)





$$\frac{Y(s)}{C(s)} = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_2 \Delta_2}{\Delta}$$

$P_1 = G_1 G_2 \quad {}^{P_2} G_3$

$\Delta = 1 - (-G_1 H)$

$\Delta_1 = 1$

$\Delta_2 = 1$

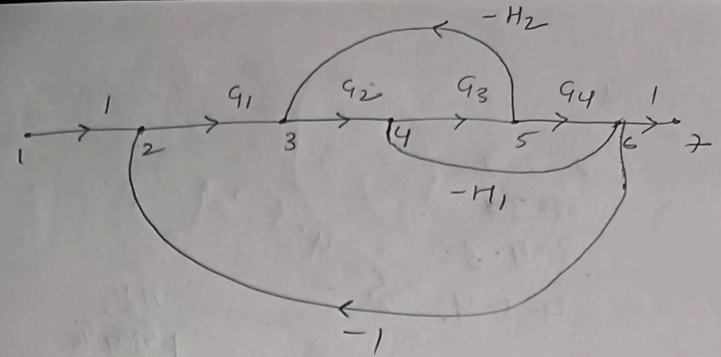
$$\frac{Y(s)}{C(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 + G_3}{1 + G_1 H}$$

(3)

(4)

(3)



$$\frac{Y(s)}{C(s)} = \frac{\Sigma P_k \Delta_k}{\Delta}$$

$K = 1$.

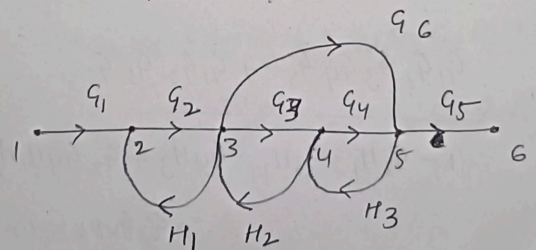
$P_1 = G_1 G_2 G_3 G_4$

$$\Delta = 1 - \left[-G_3 G_4 H_1 - G_1 G_2 G_3 G_4 - G_2 G_3 H_2 \right]$$

$\Delta_1 = 1$.

$$= \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 G_3 G_4 + G_3 G_4 H_1 + G_2 G_3 H_2}$$

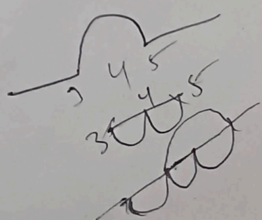
(4)



$K = 2$.

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_1 G_2 G_6 G_5$



$\Delta =$

2-3-2

~~4-3-4~~

3-4-3

4-5-4 5-4-3-5

$\Delta = 1 - [G_2H_1 + G_3H_2 + G_4H_3] + G_2G_4H_1H_3 + G_3G_4G_6H_2H_3$

$\Delta_1 = 1$

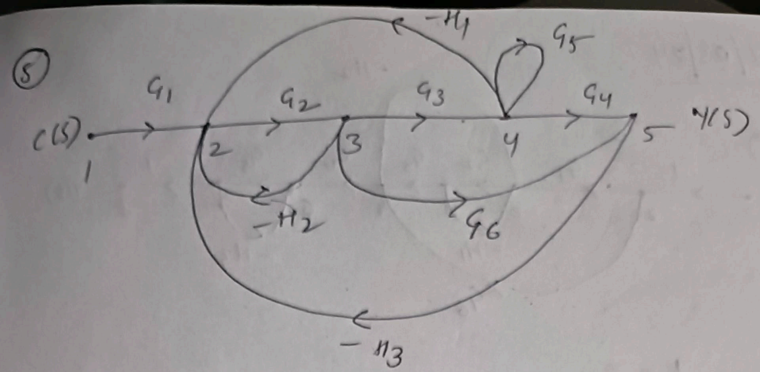
$\Delta_2 = 1 - [G_4H_3 + G_3H_2]$

$$\frac{Y(s)}{C(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_6G_5(}{1 - G_2H_1 - G_3H_2 - G_4H_3 + G_2G_4H_1H_3 - G_3G_4G_6H_2H_3}$$

$$= \frac{G_1G_2G_5(G_3G_4 + G_6)}{1 - G_2H_1 - G_3H_2 - G_4H_3 - G_3G_4G_6H_2H_3 + G_2G_4H_1H_3}$$

$$\frac{Y(s)}{C(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1 - G_2H_1 - G_3H_2 - G_4H_3 + G_2G_4H_1H_3 - G_6H_2H_3}$$

(5)



$k = 2.$

$\cancel{P_1 = G_1G_2G_3G_4}.$

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_1 G_2 G_6.$

2-3-2
~~3-4-5-3~~
2-3-4-5-2
2-3-4-2.
4 / 2-3-5-2

$$\Delta = 1 - [-G_2H_2 - G_2G_3H_1 - G_5 - G_2G_3G_4H_3 \cancel{+G_6} - G_2G_6H_3$$

$$+ (-G_5 \cdot G_2H_2 - G_2G_5G_6H_3)].$$

$$\Delta = 1 + G_2H_2 + G_2G_3H_1 + G_2G_3G_4H_3 + G_2G_6H_3 - G_2G_5H_2 - G_2G_5G_6H_3 + G_5$$

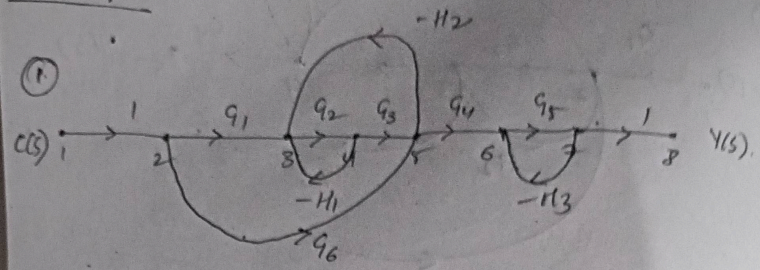
$\Delta_1 = 1$

$\Delta_2 = 1 - G_5$

$$\frac{Y(s)}{C(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_6(1 - G_5)}{1 + G_2H_2 + G_2G_3H_1 + G_2G_3G_4H_3 + G_2G_6H_3 - G_2G_5H_2 - G_2G_5G_6H_3 + G_5}$$

07/03/24

(1)



$k = 2$

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_6 G_4 G_5$

3-4-3

3-4-5-3

6-7-6

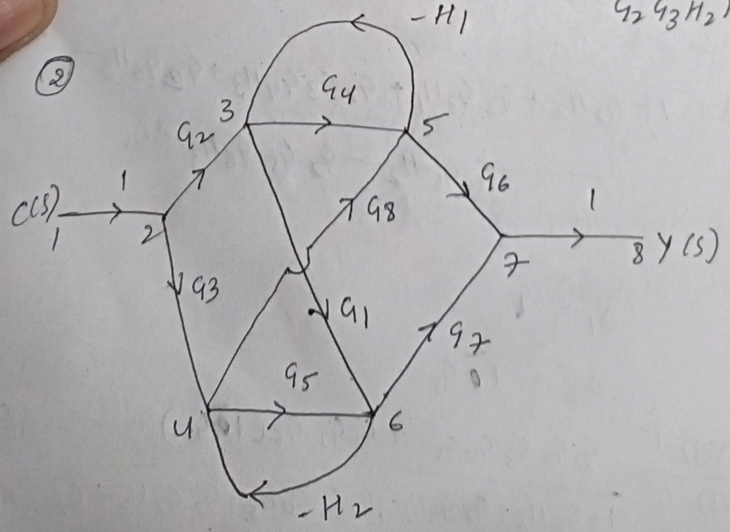
$\Delta = 1 - [-G_2 H_1 - G_2 G_3 H_2 - G_5 H_3] + (+G_2 G_5 H_1 H_3 + G_2 G_3 H_2 H_3)$

$\Delta_1 = 1$

$\Delta_2 = 1 + G_2 H_1$

$$\frac{Y(s)}{C(s)} = \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_5 G_6 (1 + G_2 H_1)}{1 + G_2 H_1 + G_5 H_3 + G_2 G_3 H_2 + G_2 G_5 H_1 H_3 + G_2 G_3 H_2 H_3}$$

(2)



$k = 6$

$P_1 = g_2 g_4 g_6$

$P_2 = g_3 g_5 g_7$

$P_3 = g_2 g_1 g_7$

$P_4 = g_3 g_8 g_6$

$P_5 = -g_2 g_1 H_2 g_8 g_6$

$P_6 = -g_3 g_8 H_1 g_1 g_7$

$-g_3 g_8 H_1 g_1 g_7$

1-2-3-5-6-4,7,5,7.

1-2-4-5-3-6

1-2-4-5-3-6-7-8

3-5-3

4-6-4

3-6-4-5-3

$$\Delta = 1 - [-g_4 H_1 - g_5 H_2 + g_1 g_8 H_1 H_2] + g_4 g_5 H_1 H_2$$

$\Delta_1 = 1 + g_5 H_2$

$\Delta_2 = 1 + g_4 H_1$

$\Delta_3 = 1$

$\Delta_4 = 1$

$\Delta_5 = 1$

$\Delta_6 = 1.$

$$\frac{Y(s)}{C(s)} = \frac{g_2 g_4 g_6 (1 + g_5 H_2) + g_3 g_5 g_7 (1 + g_4 H_1) + g_1 g_2 g_7 + g_3 g_8 g_6 - g_1 g_2 g_6 g_8 H_2 - g_1 g_3 g_7 g_8 H_1}{1 + g_4 H_1 + g_5 H_2 - g_1 g_8 H_1 H_2 + g_4 g_5 H_1 H_2}$$