	Old Computer	New Computer
Data	1000000 elements in array	"
	<u>Linear Search:</u> W.C: Element does not exist in the array	"
	Time: 10 sec	Time: 1 sec

$\boxed{\text{Time Complexity} \neq \text{Time taken}}$

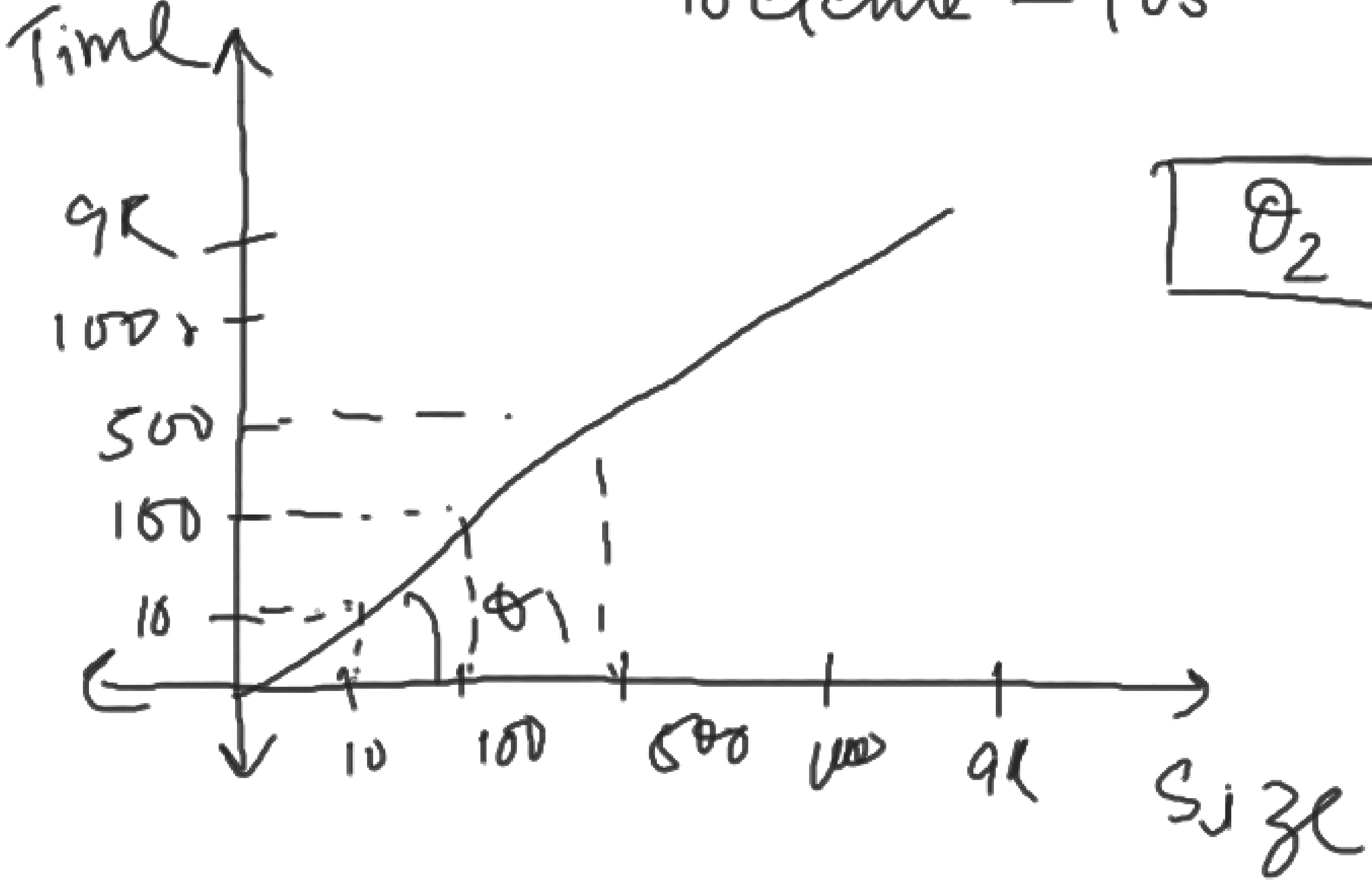
W.C → O(N)

Old Comp.

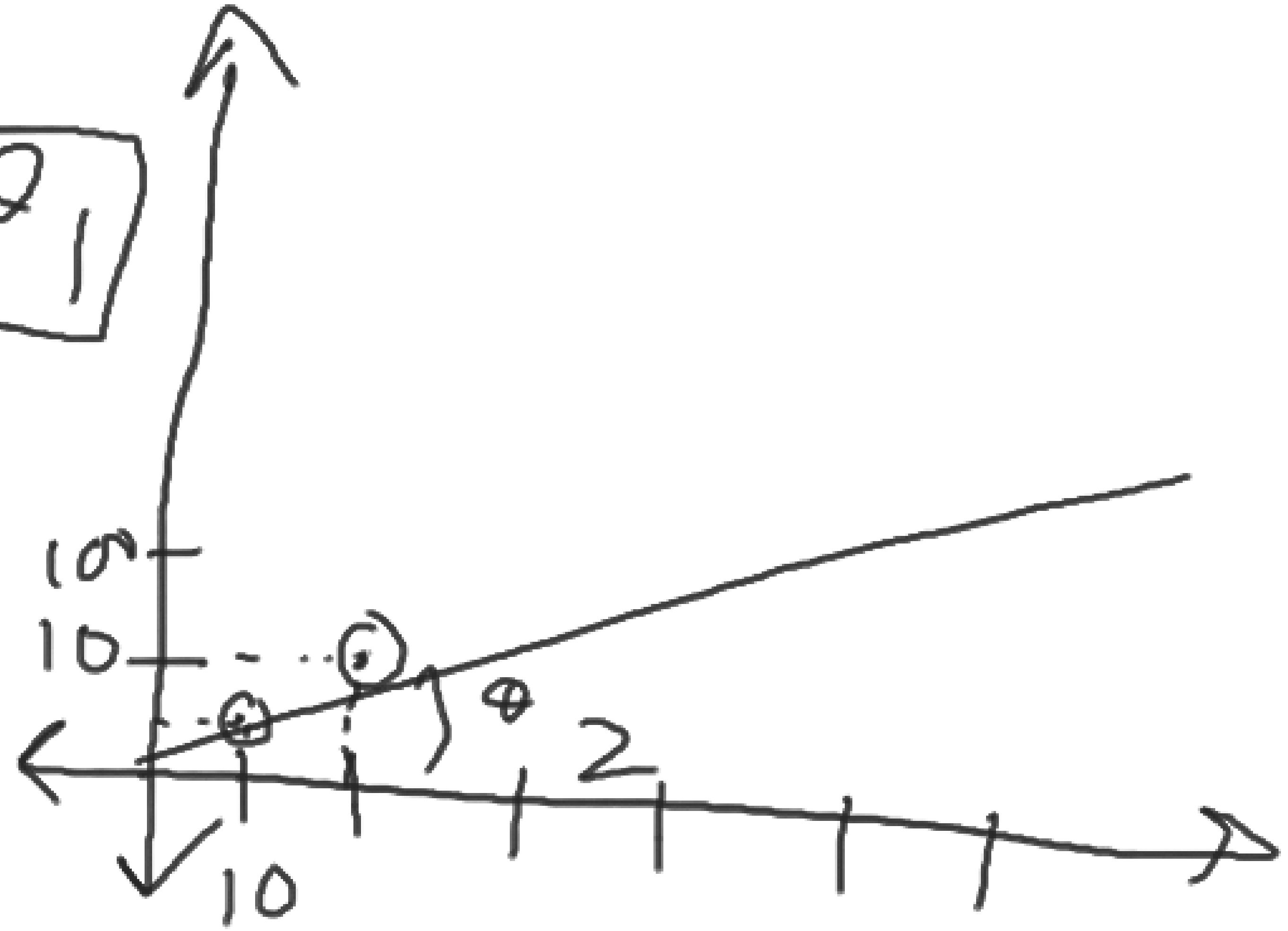
New Comp

10 eleme — 10s

10 elm — 1s



$\boxed{\theta_2 < \theta_1}$



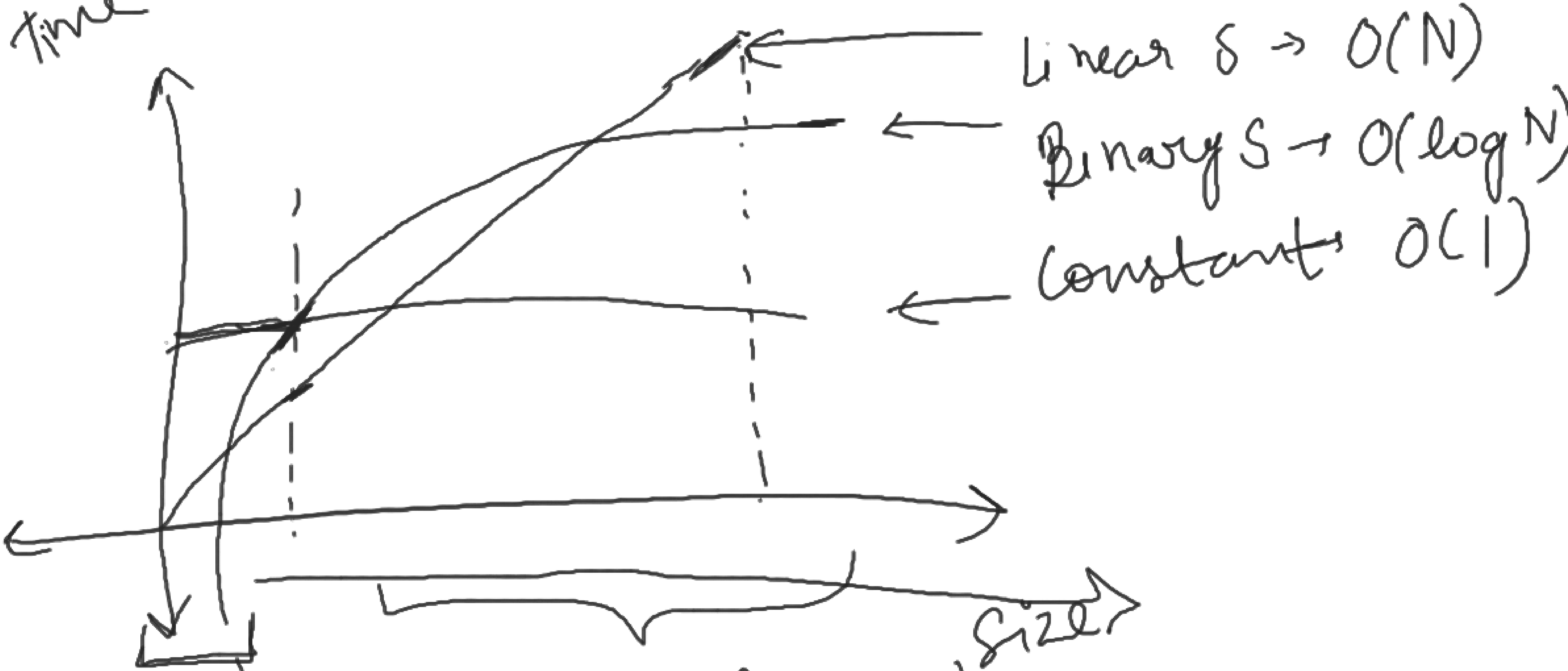
Time Complexity is the Mathematical function which tells us how the time is growing as the input grows.

T.C

1

2

$N^3 + \cancel{N} + \cancel{1}$



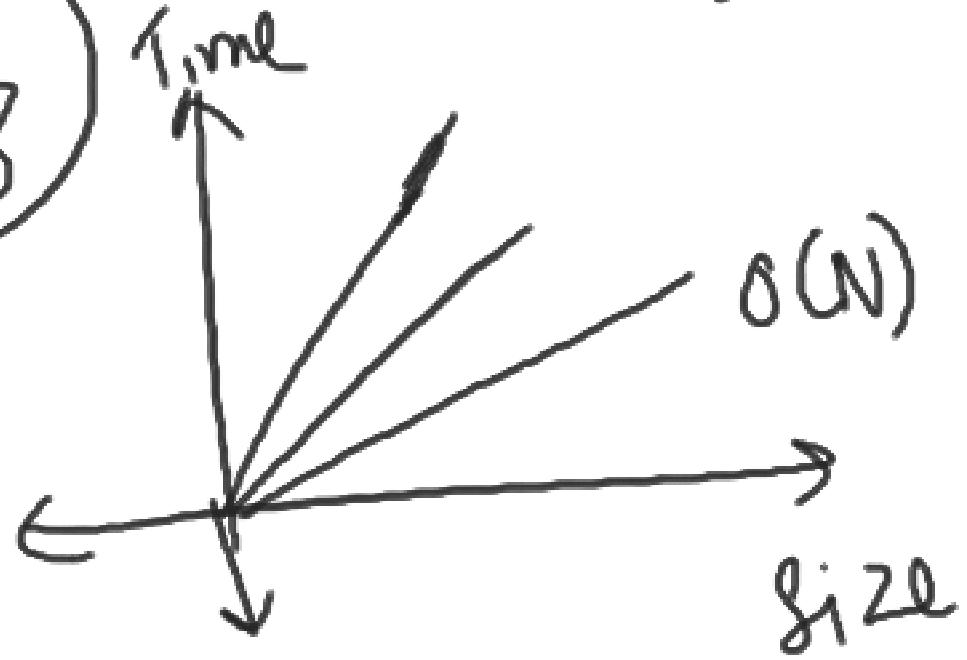
$O(1) > O(\log N) > O(N)$

$\boxed{O(1) < O(\log N) < O(N)}$

TC Analysis

1) We always look for Worst Case Complexity

2) Always look at complexity with large / ∞ data.

3) 

* Even though time taken in each case is different, all are growing linearly ∴ O(N)

* We don't care about actual time taken

ex

size
$= 1000000$

$$N^3 + \cancel{N^2} + \cancel{\log N} + \cancel{1}$$

$N =$ size of array

$$= 1\,\text{mil}^3 + 1\,\text{mil}^2 + \log(1\,\text{mil}) + 1$$

$$= 1\,\text{mil}^3 + 1\,\text{mil}^2 + \cancel{6} + \cancel{1}^{\,0}$$

$$= \boxed{1000000^3} + \cancel{1000000^2}$$

$$= O(N^3)$$

We ignore all constants and less dominating terms

Big O Notation :

word define

ex $\boxed{O}(N^3)$

Upper Bound

Why we consider Big O while talking about Time Complexity

Mathematical definition

$f(N) = O\,g(N)$

$$\boxed{\lim_{n \to \infty} \frac{f(N)}{g(N)} < \infty}$$

Big Omega : Opposite of Big O.

word definition   Lower Bound

en   T.C = $\omega(N^3)$

at least T.C

10 S

Mathematical definition

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0}$$

Recursion

ex $Fibo(n) = Fibo(n-1) + Fibo(n-2)$

Recurrence reln

Linear

ex Fibo

ex $F(N) = F(N-1) + F(N-2)$

Divide & Conquer

ex Binary Search

$F(N) = F\left(\frac{N}{2}\right) + O(1)$

Divide & Conquer Recurrences:

$$T(x) = \left[a_1 T(b_1 x + \varepsilon_1(x)) + a_2 T(b_2 x + \varepsilon_2 x) \right.$$

$$\left. - - - - \; a_k T(b_k x + \varepsilon_k x) + g(x) \right]$$

for $x \geq x_0$

$x = N$

Q $\left[T(N) = 1 \cdot T\left(\frac{N}{2}\right) + C \right.$ $\qquad \varepsilon_1 x = 0$

$a_1 = 1 \quad b_1 = \frac{1}{2} \quad g(n) = C$

$$T(N) = 9T\left(\frac{N}{3}\right) + \frac{4}{3}T\left(\frac{5}{6}N\right) + 4N^3$$

$$a_1 = 9 \qquad b_1 = \frac{1}{3} \qquad a_2 = \frac{4}{3} \qquad b_2 = \frac{5}{6}$$

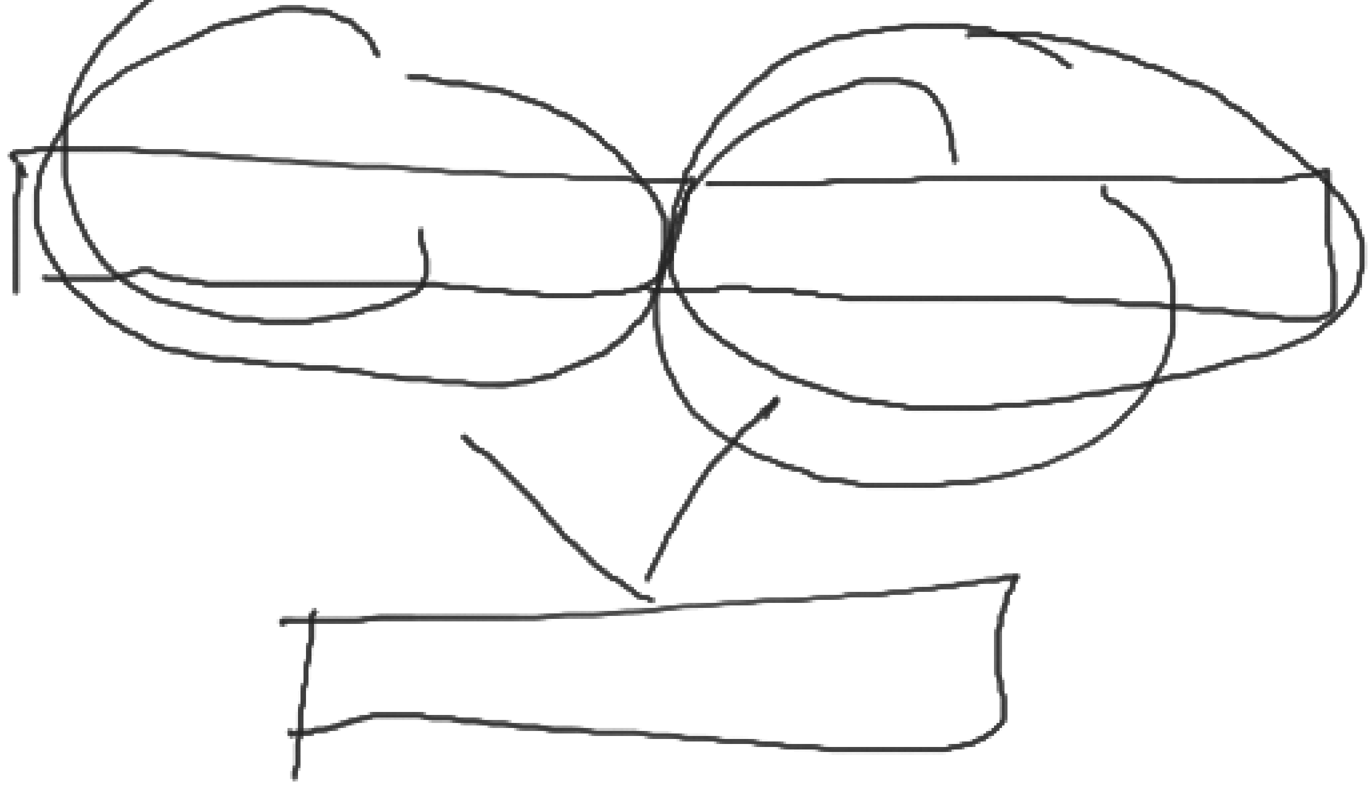
$$g(x) = 4N^3$$

Merge Sort

$$T(N) = \boxed{2T\left(\frac{N}{2}\right)} + \underline{N-1}$$

$a_1 = 2$ $\quad b_1 = \frac{1}{2}$ $\quad g(n) = N-1$

$$T\left(\frac{N}{2}\right) + T\left(\frac{N}{2}\right) +$$



when you get ans

+

what you do with the answer

How do we actually get the T.C?

1. Plug & Chug
2. Master's Theorem
3. Akra Bazzi Theorem — MIT

Akra Bazzi

$$\cancel{T}(x) = \Theta\left(x^p + x^p \int_1^x \underbrace{\frac{g(u)}{u^{p+1}}}_{\text{constant}} du\right)$$

constant

$$\boxed{\int x^n = \frac{x^{n+1}}{n+1}\, dx}$$

What is p?

$$a_1 b_1^p + a_2 b_2^p + \cdots = 1$$

$$\boxed{\sum_{i=1}^{k} a_i b_i^p = 1}$$

Q $T(N) = 2T\left(\frac{N}{2}\right) + N - 1$ $\boxed{MS = O(N\log N)}$

$a_1 = 2$ $b_1 = \frac{1}{2}$ $g(x) = N - 1$

$$\boxed{a_1 b_1^p + \text{---} = 1}$$

Let $p = 1$ $2 \times \left(\frac{1}{2}\right)^p = 1 \Rightarrow \cancel{2} \times \left(\frac{1}{\cancel{2}}\right)^1 = 1$

$$\boxed{p = 1}$$

$$= \Theta\left(x^1 + x^1 \int_1^x \frac{u-1}{u^2}\,du\right)$$

$$= \Theta\left(x + x \int_1^x \left(\frac{u}{u^{\cancel{2}}} - \frac{1}{u^2}\right) du\right)$$

$$= \Theta\left(x + x \left[\left[\log u\right]_1^x + \left[\frac{1}{u}\right]_1^x\right]\right)$$

$$= \Theta\left(x + x \left[\log x - \cancel{\log 1}^{\,0} + \frac{1}{x} - \frac{1}{1}\right]\right)$$

$$= \Theta\left(\cancel{x} + x\log x + 1 - \cancel{x}\right)$$

$$u^{-2}$$

$$\frac{u^{-2+1}}{-2+1}$$

$$\frac{u^{-1}}{-1}$$

$$= -\frac{1}{u}$$

$$= \theta(x \log x + \cancel{1})$$

$$T(x) = \theta(x \log x$$

$$\boxed{T(N) = \theta(N \log N)}$$

Q $\boxed{T(N) = 2T\left(\frac{N}{2}\right) + \frac{8}{9} T\left(\frac{3}{4}N\right) + N^2}$

T.C

$a_1 = 2 \quad b_1 = \frac{1}{2} \quad a_2 = \frac{8}{9} \quad b_2 = \frac{3}{4} \quad g(x) = N^2$

$$a_1 b_1^p + a_2 b_2^p = 1$$

Let $p = 2$

$$\Rightarrow 2 \times \left(\frac{1}{2}\right)^p + \frac{8}{9} \times \left(\frac{3}{4}\right)^p = 1$$

$$\Rightarrow 2 \times \left(\frac{1}{2}\right)^2 + \frac{8}{9}\left(\frac{3}{4}\right)^2$$

$$\Rightarrow \cancel{2} \times \frac{1}{\cancel{4}_{2}} + \frac{\cancel{8}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{16}_{2}} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow p = 2$$

$$= \Theta\left(x^2 + x^2 \int_1^x \frac{u^2}{u^3} \, du \right)$$

$$= \Theta\left(x^2 + x^2 \left[\log u \right]_1^x \right)$$

$$= \Theta\left(\cancel{x^2} + x^2 \log x - \cancelto{0}{x^2 \log 1} \right)$$

$$= \Theta\left(x^2 \log x \right)$$

$$\boxed{T(N) = \Theta\left(N^2 \log N \right)}$$

If you can't find value of p

$$T(n) = 3T\left(\frac{x}{3}\right) + 4T\left(\frac{x}{4}\right) + \underbrace{x^2}_{=gx}$$

Let $p = 1$ $\quad a_1 = 3 \quad b_1 = \frac{1}{3} \quad a_2 = 4 \quad b_2 = \frac{1}{4}$

$$3 \times \left(\frac{1}{3}\right)^1 + 4 \times \left(\frac{1}{4}\right)^1 = 1$$

$$1 + 1 = 1$$

$$\boxed{2 \neq 1}$$

let $p = 2$ $\quad \cancel{3} \times \frac{1}{\cancel{9}_{3}} + \cancel{4} \times \frac{1}{\cancel{16}_{4}} = 1 \Rightarrow \boxed{\frac{7}{12} \neq 1}$