Risk Management and Value in Banking and Insurance: Evaluating market risk models during the recent crises

Agenda

1. Calculating VAR using the parametric approach
2. Backtesting our model using the Kupiec test
3. Calculating VAR using a hybrid approach

First, we estimated log returns and volatility of the individual stocks using a 52-week standard MA and an EWMA with a decay factor of 0.94

Assumptions underlying our volatility estimations

- Historical sample for volatility estimation: **24/02/2020 – 14/03/2022**
- Large historical sample with high information content

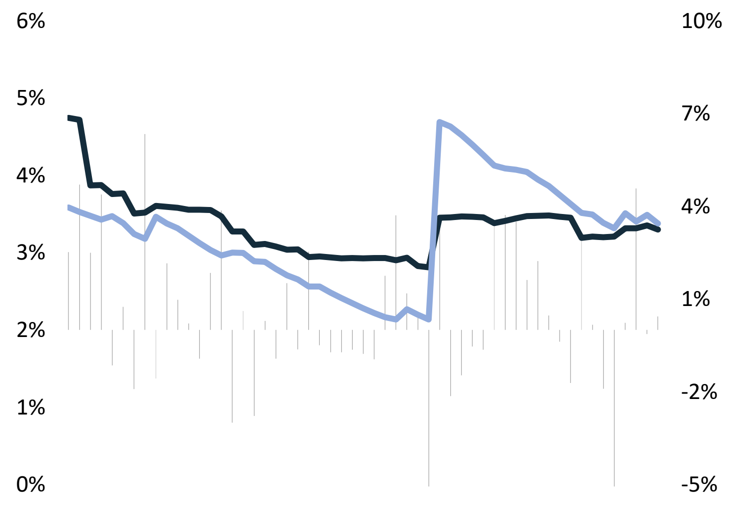
- 52-week standard moving average
- 52-week EWMA with a decay factor of 0.94 $\frac{\lambda^0 x_{t-1}+\lambda x_{t-2}+\lambda^2 x_{t-3}+\lambda^3 x_{t-4}+...+\lambda^{n-1}x_{t-n}}{1+\lambda+\lambda^2+\lambda^3+...+\lambda^{n-1}} \Big| 0<\lambda<1$
- EWMA attributing more weight to recent data in the large sample

52-week EWMA

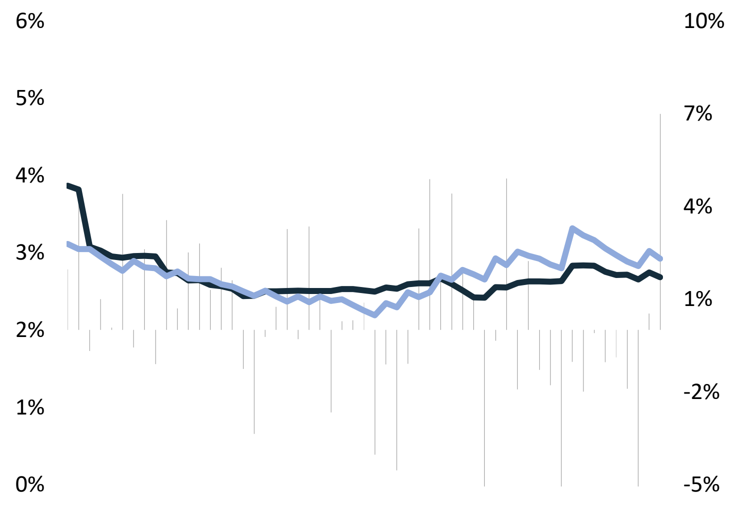
52-week MA

Weekly log returns

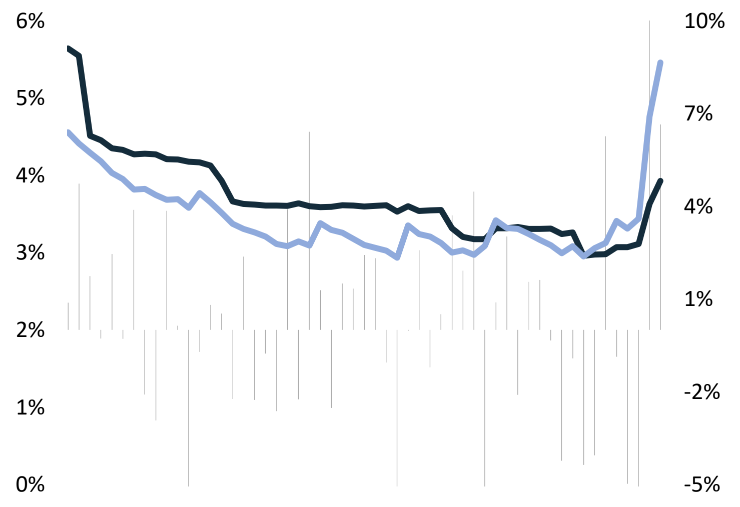
IBM (IBM)



L'Oréal (OR)



Siemens (SIE)



Parametric VaR | Backtesting | Hybrid VaR

We then computed the weekly VaR for each individual stock and the portfolio VaR using our EMWA volatility estimation of the previous step

Assumptions underlying our VaR estimation

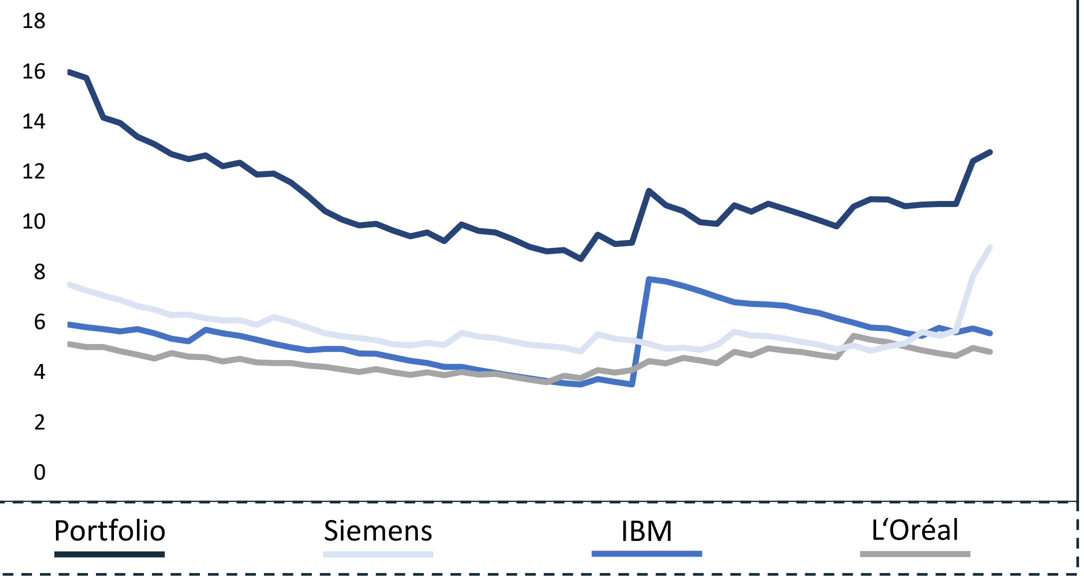
- Computation of the weekly VaR from **03/2021 to 03/2022**
- Use of our 52-week EMWA volatility estimation (which leads to a lower level of VaR)
- Use of rolling correlation between the individual daily log returns of the three stocks in our portfolio to compute portfolio VaR
- Investment of 100€ into each of the three stocks and weekly rebalancing
- Confidence interval of 95%

Course formulas used in our VaR calculations

$$VaR_{P,99\%} = \sqrt{VaR_A^2 + VaR_B^2 + VaR_C^2 + 2VaR_A VaR_B \rho_{A,B} + 2VaR_A VaR_C \rho_{A,C} + 2VaR_B VaR_C \rho_{B,C}}$$

$$VaR_i = MV_i \cdot \beta_i \cdot \sigma_j \cdot \alpha$$

Individual stock & portfolio VaR



Date	Individual stocks VaR			Correlation (rolling)			Portfolio VaR
	IBM	LOREAL	SIEMENS	IBM/ L'OREAL	IBM/ SIEMENS	L'OREAL/ SIEMENS	
31.01.22	5,75	5,20	5,03	-0,04	0,08	0,59	10,89
07.02.22	5,57	5,03	5,15	-0,05	0,07	0,56	10,63
14.02.22	5,46	4,88	5,61	-0,03	0,05	0,51	10,69
21.02.22	5,78	4,75	5,45	-0,02	0,06	0,51	10,71
28.02.22	5,60	4,66	5,66	-0,01	0,06	0,50	10,71
07.03.22	5,74	4,97	7,84	-0,07	-0,04	0,55	12,42
14.03.22	5,56	4,81	8,99	-0,11	-0,08	0,49	12,78

We performed the Kupiec test (unconditional coverage likelihood ratio) in order to evaluate the performance of our parametric VaR model

Method

- Test based on the consistency between the actual exception rate recorded by backtesting (π) and the theoretical exception rate if the model is correct (α)

- Calculate the two likelihood functions, one with probability of obtaining an error set equal to the error rate observed in the sample, the other in which the probability of obtaining an exception is set equal to α:

$$L(x|\pi) = \pi^x(1-\pi)^{N-x} \qquad L(x|\pi=\alpha) = \alpha^x(1-\alpha)^{N-x}$$

- To test the null hypothesis of consistency between the frequency of empirical exceptions and the desired empirical one, we compute the likelihood ratio:

$$LR_{uc}(\alpha) = -2\ln\left[\frac{\alpha^x(1-\alpha)^{N-x}}{\pi^x(1-\pi)^{N-x}}\right]$$

- Finally, we compute the test's p-value through a chi-square distribution with 1 degree of freedom, in order to quantify the reliability of the model

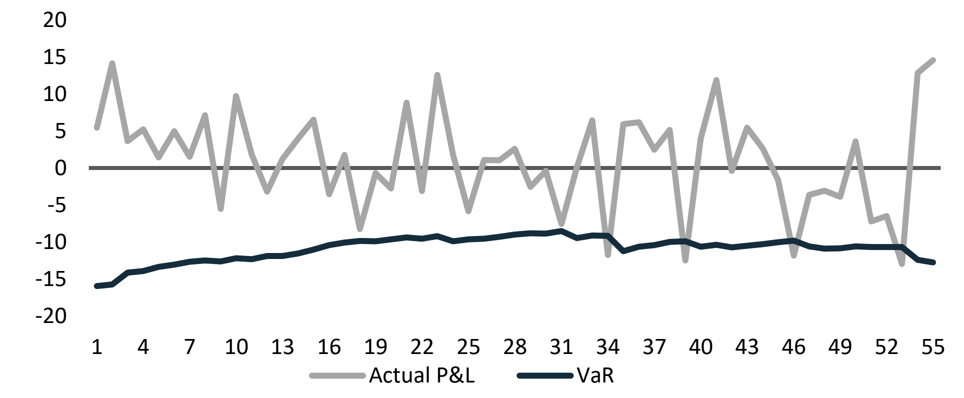
Results

- A LR_{uc} of 0.5277 shows a value of π not too far from α, and jointly with a p-value of 47%, a relatively high type I error and small type II error.
- Such a result leads us to consider the model as a good one since the most costly error to be avoided is to consider an incorrect model to be correct.

N. of exceptions	4
Exception rate (π)	7.3%
Runs (N)	55
VaR confidence level	95%
Expected exception rate (α)	5%

Non-constrained LF	5.95E-07
Constrained LF	4.57E-07
LR(unconditional)	**0.5277**
P-value	**47%**

Portfolio P&L and portfolio VaR



Parametric VaR | **Backtesting** | Hybrid VaR

We attempted to conduct the independence test but a zero N_{11} caused problems in calculating the likelihood ratio

Method

- Compare actual weekly Portfolio P&L with predicted Parametric VaR based on our model
- Calculate the number and size of exceptions, classify the result into four categories, and compute conditional probabilities
- To test the null hypothesis of independence, we calculate the likelihood ratio based on the likelihood function
- $$L(\pi_{0,0},\pi_{1,0},\pi_{0,1},\pi_{1,1}) = (1-\pi_{0,1})^{N_{0,0}}\pi_{0,1}^{N_{0,1}}(1-\pi_{1,1})^{N_{1,0}}\pi_{1,1}^{N_{1,1}}$$
  $$L(\pi) = (1-\pi)^{N_{0,0}+N_{1,0}}\pi^{N_{0,1}+N_{1,1}} \quad LR_{ind} = -2\ln\left[\frac{L(\pi)}{L(\pi_{0,0},\pi_{1,0},\pi_{0,1},\pi_{1,1})}\right]$$

N. of exceptions	4
Exception rate	7.3%
Runs (N)	55
VaR confidence level	95%
Expected exception rate (alpha)	5%

N_{11}	0	π_{11}	0.0%
N_{10}	4	π_{10}	100.0%
N_{01}	4	π_{01}	7.8%
N_{00}	47	π_{00}	92.2%
x	4	π	7.3%

- There is no exception followed by another exception, therefore the number and frequency of N_{11} are both equal to zero, indicating that we are not able to get the likelihood ratio using listed methodologies. Here we will only consider the result from the unconditional test.
- Potential reason: Sample size too small, only 55 runs are used in backtesting
- Possible solution: Extend the backtesting period, use weekly data for 2 to 3 years

Based on the Hybrid Model we used the cumulative weight from the first two rows to derive the VaR at 5% significance level with a decay factor of 0.94

Assumptions underlying our VaR estimation

- Choose one year of lookback period (use data from 2021/3/8 to compute VaR at 2022/3/7)
- Decay factor of 0.94, Confidence Level 95%
- Calculate and sort portfolio weekly returns assuming that we invest €100 in each of the 3 stocks

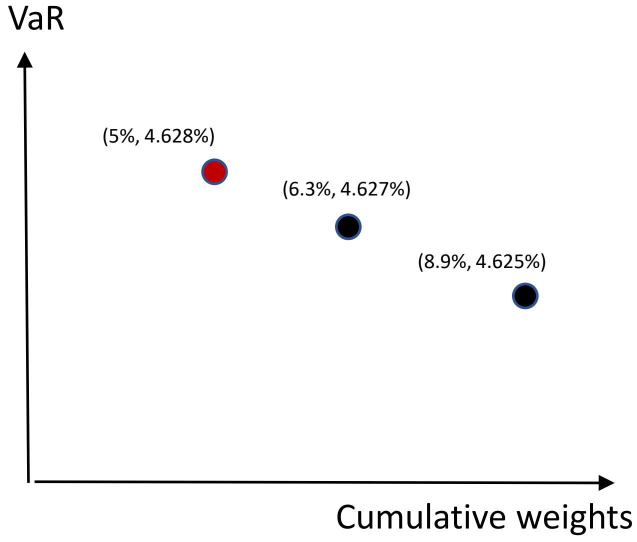
Cumulative weights

The cumulative weight in the first row with portfolio log return of –4.627% has already exceeded the significance level required

Portfolio log returns	Weeks ago	Decay factor weights	Cumulative weights
-4.627%	1	6.3%	6.3%
-4.625%	15	2.6%	8.9%
-4.252%	8	4.1%	12.9%
-3.860%	20	1.9%	14.9%
-2.826%	23	1.6%	16.5%

Portfolio VaR

Using linear interpolation on the first two rows (-4.627%, 6.3%) and (-4.625%, 8.9%), we can derive the VaR at a 5% significance level.



Conclusion

Results

The hybrid portfolio VaR is 4.628% with an absolute value of 13.88.

Portfolio 95% VaR first week of March 2022	
In % of value	In absolute value
4.628%	13.88

Potential issues

Stability: The first negative return happens to be 1 week ago and is assigned much weight (6.3%)

The decay factor and lookback period may require adjustment to reduce overweight on recent data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