
PyREx Documentation

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ABOUT PYREX

PyREx (**P**ython package for an IceCube **R**adio **E**xtension) is, as its name suggests, a Python package designed to simulate the measurement of Askaryan pulses via a radio antenna array around the IceCube South Pole Neutrino Observatory. The code is designed to be modular so that it can also be applied to other askaryan radio antennas (e.g. the ARA and ARIANA collaborations).

1.1 Installation

The easiest way to get the PyREx package is using `pip` as follows:

```
pip install git+https://github.com/bhokansonfasig/pyrex#egg=pyrex
```

PyREx requires python version 3.6+ as well as numpy version 1.13+ and scipy version 0.19+, which should be automatically installed when installing via `pip`.

Alternatively, you can download the code from <https://github.com/bhokansonfasig/pyrex> and then either include the `pyrex` directory (the one containing the python modules) in your `PYTHON_PATH`, or just copy the `pyrex` directory into your working directory. PyREx is not currently available on PyPI, so a simple `pip install pyrex` will not have the intended effect.

1.2 Quick Code Example

The most basic simulation can be produced as follows:

First, import the package:

```
import pyrex
```

Then, create a particle generator object that will produce random particles in a cube of 1 km on each side with a fixed energy of 100 PeV:

```
particle_generator = pyrex.ShadowGenerator(dx=1000, dy=1000, dz=1000,  
                                           energy_generator=lambda: 1e8)
```

An array of antennas that represent the detector is also needed. The base `Antenna` class provides a basic antenna with a flat frequency response and no trigger condition. Here we make a single vertical “string” of four antennas with no noise:

```
antenna_array = []  
for z in [-100, -150, -200, -250]:  
    antenna_array.append(
```

```
pyrex.Antenna(position=(0,0,z), noisy=False)
)
```

Finally, we want to pass these into the EventKernel and produce an event:

```
kernel = pyrex.EventKernel(generator=particle_generator,
                           ice_model=pyrex.IceModel, antennas=antenna_array)
kernel.event()
```

Now the signals received by each antenna can be accessed by their waveforms parameter:

```
import matplotlib.pyplot as plt
for ant in kernel.ant_array:
    for wave in ant.waveforms:
        plt.figure()
        plt.plot(wave.times, wave.values)
        plt.show()
```

1.3 Units

For ease of use, PyREx tries to use consistent units in all classes and functions. The units used are mostly SI with a few exceptions listed in bold below:

Metric	Unit
time	seconds (s)
frequency	hertz (Hz)
distance	meters (m)
density	grams per cubic centimeter (g/cm³)
material thickness	grams per square centimeter (g/cm²)
temperature	kelvin (K)
energy	gigaelectronvolts (GeV)
resistance	ohms (Ω)
voltage	volts (V)
electric field	volts per meter (V/m)

CODE EXAMPLES

The following code examples assume these imports:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.fftpack
import pyrex
```

All of the following examples can also be found (and quickly run) in the Code Examples python notebook.

2.1 Working with Signal Objects

The base `Signal` class is simply an array of times and an array of signal values, and is instantiated with these two arrays. The `times` array is assumed to be in units of seconds, but there are no general units for the `values` array. It is worth noting that the `Signal` object stores shallow copies of the passed arrays, so changing the original arrays will not affect the `Signal` object.

```
time_array = np.linspace(0, 10)
value_array = np.sin(time_array)
my_signal = pyrex.Signal(times=time_array, values=value_array)
```

Plotting the `Signal` object is as simple as plotting the times vs the values:

```
plt.plot(my_signal.times, my_signal.values)
plt.show()
```

While there are no specified units for a `Signal.values`, there is the option to specify the `value_type` of the values. This is done using the `Signal.ValueTypes` enum. By default, a `Signal` object has `value_type=ValueTypes.unknown`. However, if the signal represents a voltage, electric field, or electric power; `value_type` can be set to `Signal.ValueTypes.voltage`, `Signal.ValueTypes.field`, or `Signal.ValueTypes.power` respectively:

```
my_voltage_signal = pyrex.Signal(times=time_array, values=value_array,
                                value_type=pyrex.Signal.ValueTypes.voltage)
```

`Signal` objects can be added as long as they have the same time array and `value_type`. `Signal` objects also support the python `sum` function:

```
time_array = np.linspace(0, 10)
values1 = np.sin(time_array)
values2 = np.cos(time_array)
signal1 = pyrex.Signal(time_array, values1)
```

```
plt.plot(signal1.times, signal1.values, label="signal1 = sin(t)")
signal2 = pyrex.Signal(time_array, values2)
plt.plot(signal2.times, signal2.values, label="signal2 = cos(t)")
signal3 = signal1 + signal2
plt.plot(signal3.times, signal3.values, label="signal3 = sin(t)+cos(t)")
all_signals = [signal1, signal2, signal3]
signal4 = sum(all_signals)
plt.plot(signal4.times, signal4.values, label="signal4 = 2*(sin(t)+cos(t))")
plt.legend()
plt.show()
```

The `Signal` class provides many convenience attributes for dealing with signals:

```
my_signal.dt == my_signal.times[1] - my_signal.times[0]
my_signal.spectrum == scipy.fftpack.fft(my_signal.values)
my_signal.frequencies == scipy.fftpack.fftfreq(n=len(my_signal.values),
                                              d=my_signal.dt)
my_signal.envelope == np.abs(scipy.signal.hilbert(my_signal.values))
```

The `Signal` class also provides functions for manipulating the signal. The `resample` function will resample the times and values arrays to the given number of points (with the same endpoints):

```
my_signal.resample(1001)
len(my_signal.times) == len(my_signal.values) == 1001
my_signal.times[0] == 0
my_signal.times[-1] == 10
plt.plot(my_signal.times, my_signal.values)
plt.show()
```

The `with_times` function will interpolate/extrapolate the signal's values onto a new times array:

```
new_times = np.linspace(-5, 15)
new_signal = my_signal.with_times(new_times)
plt.plot(new_signal.times, new_signal.values, label="new signal")
plt.plot(my_signal.times, my_signal.values, label="original signal")
plt.legend()
plt.show()
```

The `filter_frequencies` function will apply a frequency-domain filter to the values array based on the passed frequency response function:

```
def lowpass_filter(frequency):
    if frequency < 1:
        return 1
    else:
        return 0

time_array = np.linspace(0, 10, 1001)
value_array = np.sin(0.1*2*np.pi*time_array) + np.sin(2*2*np.pi*time_array)
my_signal = pyrex.Signal(times=time_array, values=value_array)

plt.plot(my_signal.times, my_signal.values)
my_signal.filter_frequencies(lowpass_filter)
plt.plot(my_signal.times, my_signal.values)
plt.show()
```

A number of classes which inherit from the `Signal` class are included in PyREx: `EmptySignal`,

FunctionSignal, AskaryanSignal, and ThermalNoise. EmptySignal is simply a signal whose values are all zero:

```
time_array = np.linspace(0,10)
empty = pyrex.EmptySignal(times=time_array)
plt.plot(empty.times, empty.values)
plt.show()
```

FunctionSignal takes a function of time and creates a signal based on that function:

```
time_array = np.linspace(0, 10, num=101)
def square_wave(time):
    if int(time)%2==0:
        return 1
    else:
        return -1
square_signal = pyrex.FunctionSignal(times=time_array, function=square_wave)
plt.plot(square_signal.times, square_signal.values)
plt.show()
```

Additionally, FunctionSignal leverages its knowledge of the function to more accurately interpolate and extrapolate values for the with_times function:

```
new_times = np.linspace(0, 20, num=201)
long_square_signal = square_signal.with_times(new_times)
plt.plot(long_square_signal.times, long_square_signal.values, label="new signal")
plt.plot(square_signal.times, square_signal.values, label="original signal")
plt.legend()
plt.show()
```

AskaryanSignal produces an Askaryan pulse (in V/m) on a time array due to a neutrino of given energy observed at a given angle from the shower axis:

```
time_array = np.linspace(-10e-9, 40e-9, 1001)
neutrino_energy = 1e8 # GeV
observation_angle = 45 * np.pi/180 # radians
askaryan = pyrex.AskaryanSignal(times=time_array, energy=neutrino_energy,
                                theta=observation_angle)
print(askaryan.value_type)
plt.plot(askaryan.times, askaryan.values)
plt.show()
```

ThermalNoise produces Rayleigh noise (in V) at a given temperature and resistance which has been passed through a bandpass filter of the given frequency range:

```
time_array = np.linspace(-10e-9, 40e-9, 1001)
noise_temp = 300 # K
system_resistance = 1000 # ohm
frequency_range = (550e6, 750e6) # Hz
noise = pyrex.ThermalNoise(times=time_array, temperature=noise_temp,
                            resistance=system_resistance,
                            f_band=frequency_range)
print(noise.value_type)
plt.plot(noise.times, noise.values)
plt.show()
```

Note that since ThermalNoise inherits from FunctionSignal, it can be extrapolated nicely to new times. It may be highly periodic outside of its original time range however, unless a large number of frequencies is requested

on initialization.

```
short_noise = pyrex.ThermalNoise(times=time_array, temperature=noise_temp,
                                resistance=system_resistance,
                                f_band=(100e6, 400e6))
long_noise = short_noise.with_times(np.linspace(-10e-9, 90e-9, 2001))

plt.plot(short_noise.times, short_noise.values)
plt.show()
plt.plot(long_noise.times, long_noise.values)
plt.show()
```

2.2 Antenna Class and Subclasses

The base Antenna class provided by PyREx is designed to be inherited from to match the needs of each project. At its core, an Antenna object is initialized with a position, a temperature, and a frequency range, as well as optionally a resistance for noise calculations and a boolean dictating whether or not noise should be added to the antenna's signals (note that if noise is to be added, a resistance must be specified).

```
# Please note that some values are unrealistic in order to simplify demonstration
position = (0, 0, -100) # m
temperature = 300 # K
resistance = 1e17 # ohm
frequency_range = (0, 5) # Hz
basic_antenna = pyrex.Antenna(position=position, temperature=temperature,
                              resistance=resistance,
                              freq_range=frequency_range)
noiseless_antenna = pyrex.Antenna(position=position, noisy=False)
```

The basic properties of an Antenna object are `is_hit` and `waveforms`. `is_hit` specifies whether or not the antenna has been triggered by an event. `waveforms` is a list of all the waveforms which have triggered the antenna. The antenna also defines `signals`, which is a list of all signals the antenna has received, and `all_waveforms` which is a list of all waveforms (signal plus noise) the antenna has received including those which didn't trigger.

```
basic_antenna.is_hit == False
basic_antenna.waveforms == []
```

The Antenna class contains two attributes and three methods which represent characteristics of the antenna as they relate to signal processing. The attributes are `efficiency` and `antenna_factor`, and the methods are `response`, `directional_gain`, and `polarization_gain`. The attributes are to be set and the methods overwritten in order to customize the way the antenna responds to incoming signals. `efficiency` is simply a scalar which multiplies the signal the antenna receives (default value is 1). `antenna_factor` is a factor used in converting received electric fields into voltages ($\text{antenna_factor} = E / V$; default value is 1). `response` takes a frequency or list of frequencies (in Hz) and returns the frequency response of the antenna at each frequency given (default always returns 1). `directional_gain` takes angles `theta` and `phi` in the antenna's coordinates and returns the antenna's gain for a signal coming from that direction (default always returns 1). `directional_gain` is dependent on the antenna's orientation, which is defined by its `z_axis` and `x_axis` attributes. To change the antenna's orientation, use the `set_orientation` method which takes `z_axis` and `x_axis` arguments. Finally, `polarization_gain` takes a polarization vector and returns the antenna's gain for a signal with that polarization (default always returns 1).

```
basic_antenna.efficiency == 1
basic_antenna.antenna_factor == 1
freqs = [1, 2, 3, 4, 5]
basic_antenna.response(freqs) == [1, 1, 1, 1, 1]
```

```
basic_antenna.directional_gain(theta=np.pi/2, phi=0) == 1
basic_antenna.polarization_gain([0,0,1]) == 1
```

The Antenna class defines a `trigger` method which is also expected to be overwritten. `trigger` takes a `Signal` object as an argument and returns a boolean of whether or not the antenna would trigger on that signal (default always returns `True`).

```
basic_antenna.trigger(pyrex.Signal([0],[0])) == True
```

The Antenna class also defines a `receive` method which takes a `Signal` object and processes the signal according to the antenna's attributes (`efficiency`, `antenna_factor`, `response`, `directional_gain`, and `polarization_gain` as described above). To use the `receive` function, simply pass it the `Signal` object the antenna sees, and the Antenna class will handle the rest. You can also optionally specify the origin point of the signal (used in `directional_gain` calculation) and the polarization direction of the signal (used in `polarization_gain` calculation). If either of these is unspecified, the corresponding gain will simply be set to 1.

```
incoming_signal_1 = pyrex.FunctionSignal(np.linspace(0,2*np.pi), np.sin,
                                         value_type=pyrex.Signal.ValueTypes.voltage)
incoming_signal_2 = pyrex.FunctionSignal(np.linspace(4*np.pi,6*np.pi), np.sin,
                                         value_type=pyrex.Signal.ValueTypes.voltage)
basic_antenna.receive(incoming_signal_1)
basic_antenna.receive(incoming_signal_2, origin=[0,0,-300], polarization=[1,0,0])
basic_antenna.is_hit == True
for waveform, pure_signal in zip(basic_antenna.waveforms, basic_antenna.signals):
    plt.figure()
    plt.plot(waveform.times, waveform.values, label="Waveform")
    plt.plot(pure_signal.times, pure_signal.values, label="Pure Signal")
    plt.legend()
    plt.show()
```

Beyond `Antenna.waveforms`, the Antenna object also provides methods for checking the waveform and trigger status for arbitrary times: `full_waveform` and `is_hit_during`. Both of these methods take a time array as an argument and return the waveform `Signal` object for those times and whether said waveform triggered the antenna, respectively.

```
total_waveform = basic_antenna.full_waveform(np.linspace(0,20))
plt.plot(total_waveform.times, total_waveform.values, label="Total Waveform")
plt.plot(incoming_signal_1.times, incoming_signal_1.values, label="Pure Signals")
plt.plot(incoming_signal_2.times, incoming_signal_2.values, color="C1")
plt.legend()
plt.show()

basic_antenna.is_hit_during(np.linspace(0, 200e-9)) == True
```

Finally, the Antenna class defines a `clear` method which will reset the antenna to a state of having received no signals:

```
basic_antenna.clear()
basic_antenna.is_hit == False
len(basic_antenna.waveforms) == 0
```

To create a custom antenna, simply inherit from the Antenna class:

```
class NoiselessThresholdAntenna(pyrex.Antenna):
    def __init__(self, position, threshold):
        super().__init__(position=position, noisy=False)
```

```

        self.threshold = threshold

    def trigger(self, signal):
        if max(np.abs(signal.values)) > self.threshold:
            return True
        else:
            return False

```

Our custom NoiselessThresholdAntenna should only trigger when the amplitude of a signal exceeds its threshold value:

```

my_antenna = NoiselessThresholdAntenna(position=(0, 0, 0), threshold=2)

incoming_signal = pyrex.FunctionSignal(np.linspace(0,10), np.sin,
                                       value_type=pyrex.Signal.ValueTypes.voltage)

my_antenna.receive(incoming_signal)
my_antenna.is_hit == False
len(my_antenna.waveforms) == 0
len(my_antenna.all_waveforms) == 1

incoming_signal = pyrex.Signal(incoming_signal.times,
                               5*incoming_signal.values,
                               incoming_signal.value_type)
my_antenna.receive(incoming_signal)
my_antenna.is_hit == True
len(my_antenna.waveforms) == 1
len(my_antenna.all_waveforms) == 2

for wave in my_antenna.waveforms:
    plt.figure()
    plt.plot(wave.times, wave.values)
    plt.show()

```

For more on customizing PyREx, see the [Custom Sub-Package](#) section.

PyREx defines `DipoleAntenna` which as a subclass of `Antenna`, which provides a basic threshold trigger, a basic bandpass filter frequency response, a sine-function directional gain, and a typical dot-product polarization effect. A `DipoleAntenna` object is created as follows:

```

antenna_identifier = "antenna 1"
position = (0, 0, -100)
center_frequency = 250e6 # Hz
bandwidth = 300e6 # Hz
resistance = 100 # ohm
antenna_length = 3e8/center_frequency/2 # m
polarization_direction = (0, 0, 1)
trigger_threshold = 1e-5 # V
dipole = pyrex.DipoleAntenna(name=antenna_identifier, position=position,
                             center_frequency=center_frequency,
                             bandwidth=bandwidth, resistance=resistance,
                             effective_height=antenna_length,
                             orientation=polarization_direction,
                             trigger_threshold=trigger_threshold)

```

2.3 AntennaSystem and Detector Classes

The `AntennaSystem` class is designed to bridge the gap between the basic antenna classes and realistic antenna systems including front-end processing of the antenna's signals. It is designed to be subclassed, but by default it takes as an argument the `Antenna` class or subclass it is extending, or an object of that class. It provides an interface nearly identical to that of the `Antenna` class, but where a `front_end` method (which by default does nothing) is applied to the extended antenna's signals.

To extend an `Antenna` class or subclass into a full antenna system, subclass the `AntennaSystem` class and define the `front_end` method. Optionally a trigger can be defined for the antenna system (by default it uses the antenna's trigger):

```
class PowerAntennaSystem(pyrex.AntennaSystem):
    """Antenna system whose signals and waveforms are powers instead of
    voltages."""
    def __init__(self, position, temperature, resistance, frequency_range):
        super().__init__(pyrex.Antenna)
        # The setup_antenna method simply passes all arguments on to the
        # antenna class passed to super.__init__() and stores the resulting
        # antenna to self.antenna
        self.setup_antenna(position=position, temperature=temperature,
                           resistance=resistance,
                           freq_range=frequency_range)

    def front_end(self, signal):
        return pyrex.Signal(signal.times, signal.values**2,
                             value_type=pyrex.Signal.ValueTypes.power)
```

Objects of this class can then, for the most part, be interacted with as though they were regular antenna objects:

```
position = (0, 0, -100) # m
temperature = 300 # K
resistance = 1e17 # ohm
frequency_range = (0, 5) # Hz

basic_antenna_system = PowerAntennaSystem(position=position,
                                           temperature=temperature,
                                           resistance=resistance,
                                           frequency_range=frequency_range)

basic_antenna_system.trigger(pyrex.Signal([0],[0])) == True

incoming_signal_1 = pyrex.FunctionSignal(np.linspace(0,2*np.pi), np.sin,
                                          value_type=pyrex.Signal.ValueTypes.voltage)
incoming_signal_2 = pyrex.FunctionSignal(np.linspace(4*np.pi,6*np.pi), np.sin,
                                          value_type=pyrex.Signal.ValueTypes.voltage)
basic_antenna_system.receive(incoming_signal_1)
basic_antenna_system.receive(incoming_signal_2, origin=[0,0,-300],
                             polarization=[1,0,0])
basic_antenna_system.is_hit == True
for waveform, pure_signal in zip(basic_antenna_system.waveforms,
                                basic_antenna_system.signals):
    plt.figure()
    plt.plot(waveform.times, waveform.values, label="Waveform")
    plt.plot(pure_signal.times, pure_signal.values, label="Pure Signal")
    plt.legend()
    plt.show()
```

```

total_waveform = basic_antenna_system.full_waveform(np.linspace(0,20))
plt.plot(total_waveform.times, total_waveform.values, label="Total Waveform")
plt.plot(incoming_signal_1.times, incoming_signal_1.values, label="Pure Signals")
plt.plot(incoming_signal_2.times, incoming_signal_2.values, color="C1")
plt.legend()
plt.show()

basic_antenna_system.is_hit_during(np.linspace(0, 200e-9)) == True

basic_antenna_system.clear()
basic_antenna_system.is_hit == False
len(basic_antenna_system.waveforms) == 0

```

The `Detector` class is another convenience class meant to be subclassed. It is useful for automatically generating many antennas (as would be used to build a detector). Subclasses must define a `set_positions` method to assign vector positions to the `self.antenna_positions` attribute. By default `set_positions` will raise a `NotImplementedError`. Additionally subclasses may extend the default `build_antennas` method which by default simply builds antennas of a passed antenna class using any keyword arguments passed to the method. In addition to simply generating many antennas at desired positions, another convenience of the `Detector` class is that once the `build_antennas` method is run, it can be iterated directly as though the object were a list of the antennas it generated. An example of subclassing the `Detector` class is shown below:

```

class AntennaGrid(pyrex.Detector):
    """A detector composed of a plane of antennas in a rectangular grid layout
    some distance below the ice."""
    def set_positions(self, number, separation=10, depth=-50):
        self.antenna_positions = []
        n_x = int(np.sqrt(number))
        n_y = int(number/n_x)
        dx = separation
        dy = separation
        for i in range(n_x):
            x = -dx*n_x/2 + dx/2 + dx*i
            for j in range(n_y):
                y = -dy*n_y/2 + dy/2 + dy*j
                self.antenna_positions.append((x, y, depth))

grid_detector = AntennaGrid(9)

# Build the antennas
temperature = 300 # K
resistance = 1e17 # ohm
frequency_range = (0, 5) # Hz
grid_detector.build_antennas(pyrex.Antenna, temperature=temperature,
                             resistance=resistance,
                             freq_range=frequency_range)

plt.figure(figsize=(6,6))
for antenna in grid_detector:
    x = antenna.position[0]
    y = antenna.position[1]
    plt.plot(x, y, "kD")
plt.ylim(plt.xlim())
plt.show()

```

Due to the parallels between `Antenna` and `AntennaSystem`, an antenna system may also be used in the custom detector class. Note however, that the antenna positions must be accessed as `antenna.antenna.position` since

we didn't define a position attribute for the `PowerAntennaSystem`:

```
grid_detector = AntennaGrid(12)

# Build the antennas
temperature = 300 # K
resistance = 1e17 # ohm
frequency_range = (0, 5) # Hz
grid_detector.build_antennas(PowerAntennaSystem, temperature=temperature,
                             resistance=resistance,
                             frequency_range=frequency_range)

for antenna in grid_detector:
    x = antenna.antenna.position[0]
    y = antenna.antenna.position[1]
    plt.plot(x, y, "kD")
plt.show()
```

2.4 Ice and Earth Models

PyREx provides a class `IceModel`, which is an alias for whichever south pole ice model class is the preferred (currently just the basic `AntarcticIce`). The `IceModel` class provides class methods for calculating characteristics of the ice at different depths and frequencies outlined below:

```
depth = -1000 # m
pyrex.IceModel.temperature(depth)
pyrex.IceModel.index(depth)
pyrex.IceModel.gradient(depth)
frequency = 1e8 # Hz
pyrex.IceModel.attenuation_length(depth, frequency)
```

PyREx also provides two functions related to its earth model: `prem_density` and `slant_depth`. `prem_density` calculates the density in grams per cubic centimeter of the earth at a given radius:

```
radius = 6360000 # m
pyrex.prem_density(radius)
```

`slant_depth` calculates the material thickness in grams per square centimeter of a chord cutting through the earth at a given nadir angle, starting from a given depth:

```
nadir_angle = 60 * np.pi/180 # radians
depth = 1000 # m
pyrex.slant_depth(nadir_angle, depth)
```

2.5 Particle Generation

PyREx includes `Particle` as a container for information about neutrinos which are generated to produce Askaryan pulses. `Particle` contains three attributes: `vertex`, `direction`, and `energy`:

```
initial_position = (0,0,0) # m
direction_vector = (0,0,-1)
particle_energy = 1e8 # GeV
```

```
pyrex.Particle(vertex=initial_position, direction=direction_vector,
               energy=particle_energy)
```

PyREx also includes a `ShadowGenerator` class for generating random neutrinos, taking into account some Earth shadowing. The neutrinos are generated in a box of given size, and with an energy given by an energy generation function:

```
box_width = 1000 # m
box_depth = 500 # m
const_energy_generator = lambda: 1e8 # GeV
my_generator = pyrex.ShadowGenerator(dx=box_width, dy=box_width,
                                     dz=box_depth,
                                     energy_generator=const_energy_generator)
my_generator.create_particle()
```

2.6 Ray Tracing

PyREx provides ray tracing in the `RayTracer` and `RayTracerPath` classes. `RayTracer` takes a launch point and receiving point as arguments (and optionally an ice model and z-step), and will solve for the paths between the points (as `RayTracerPath` objects).

```
start = (0, 0, -250) # m
finish = (100, 0, -100) # m
my_ray_tracer = pyrex.RayTracer(from_point=start, to_point=finish)
```

The two most useful properties of `RayTracer` are `RayTracer.exists` and `RayTracer.solutions`. `RayTracer.exists` is a boolean value of whether or not path solutions exist between the launch and receiving points. `RayTracer.solutions` is the list of (zero or two) `RayTracerPath` objects which exist between the launch and receiving points. There are many other properties available in `RayTracer`, outlined in the [PyREx API](#) section, which are mostly used internally and maybe not interesting otherwise.

```
my_ray_tracer.exists
my_ray_tracer.solutions
```

The `RayTracerPath` class contains the attributes of the paths between points. The most useful properties of `RayTracerPath` are `RayTracerPath.tof`, `RayTracerPath.path_length`, `RayTracerPath.emitted_direction`, and `RayTracerPath.received_direction`. These properties provide the time of flight, path length, and direction of rays at the launch and receiving points respectively.

```
my_path = my_ray_tracer.solutions[0]
my_path.tof
my_path.path_length
my_path.emitted_direction
my_path.received_direction
```

`RayTracePath` also provides the `RayTracePath.attenuation()` method which gives the attenuation of the signal at a given frequency (or frequencies), and the `RayTracePath.coordinates` property which gives the x, y, and z coordinates of the path (useful mostly for plotting, and are not guaranteed to be accurate for other purposes).

```
frequency = 500e6 # Hz
my_path.attenuation(100e6)
my_path.attenuation(np.linspace(1e8, 1e9, 11))
plt.plot(my_path.coordinates[0], my_path.coordinates[2])
plt.show()
```

Finally, `RayTracePath.propagate()` propagates a `Signal` object from the launch point to the receiving point by applying the frequency-dependent attenuation of `RayTracePath.attenuation()`, and shifting the signal times by `RayTracePath.tof`. Note that it does not apply a $1/R$ effect based on the path length. If needed, this effect should be added in manually.

```
time_array = np.linspace(0, 5e-9, 1001)
my_signal = (pyrex.FunctionSignal(time_array, lambda t: np.sin(1e9*2*np.pi*t))
             + pyrex.FunctionSignal(time_array, lambda t: np.sin(1e10*2*np.pi*t)))
plt.plot(my_signal.times, my_signal.values)
plt.show()

my_path.propagate(my_signal)
my_signal.values /= my_path.path_length
plt.plot(my_signal.times, my_signal.values)
plt.show()
```

2.7 Full Simulation

PyREx provides the `EventKernel` class to control a basic simulation including the creation of neutrinos, the propagation of their pulses to the antennas, and the triggering of the antennas:

```
particle_generator = pyrex.ShadowGenerator(dx=1000, dy=1000, dz=500,
                                           energy_generator=lambda: 1e8)

detector = []
for i, z in enumerate([-100, -150, -200, -250]):
    detector.append(
        pyrex.DipoleAntenna(name="antenna_"+str(i), position=(0, 0, z),
                           center_frequency=250e6, bandwidth=300e6,
                           resistance=0, effective_height=0.6,
                           trigger_threshold=0, noisy=False)
    )

kernel = pyrex.EventKernel(generator=particle_generator,
                          ice_model=pyrex.IceModel,
                          antennas=detector)

triggered = False
while not triggered:
    kernel.event()
    for antenna in detector:
        if antenna.is_hit:
            triggered = True
            break

for antenna in detector:
    for i, wave in enumerate(antenna.waveforms):
        plt.plot(wave.times * 1e9, wave.values)
        plt.xlabel("Time (ns)")
        plt.ylabel("Voltage (V)")
        plt.title(antenna.name + " - waveform " + str(i))
```

2.8 More Examples

For more code examples, see the [PyREx Demo python notebook](#).

CUSTOM SUB-PACKAGE

While the PyREx package provides a basis for simulation, the real benefits come in customizing the analysis for different purposes. To this end the custom sub-package allows for plug-in style modules to be distributed for different collaborations.

By default PyREx comes with a custom module for IREX (IceCube Radio Extension) accessible at `pyrex.custom.irex`. This module includes a more thorough `IREXAntennaSystem` class inheriting from the `AntennaSystem` class which adds a front-end for amplifying the signal, processing signal envelopes, and downsampling the result. It also includes an `IREXDetector` class designed to easily produce different geometries of `IREXAntennaSystem` objects.

Other institutions and research groups are encouraged to create their own custom modules to integrate with PyREx. These modules have full access to PyREx as if they were a native part of the package. When PyREx is loaded it automatically scans for these custom modules in certain parts of the filesystem and includes any modules that it can find. The first place searched is the `custom` directory in the PyREx package itself. Next, if a `.pyrex-custom` directory exists in the user's home directory (note the leading `.`), its subdirectories are searched for `custom` directories and any modules in these directories are included. Finally, if a `pyrex-custom` directory exists in the current working directory (this time without the leading `.`), its subdirectories are similarly scanned for modules inside `custom` directories. Note that if any name-clashing occurs, the first result found takes precedence (without warning). Additionally, none of these `custom` directories should contain an `__init__.py` file, or else the plug-in system may not work (For more information on the implementation, see PEP 420 and/or David Beazley's 2015 PyCon talk on Modules and Packages at <https://youtu.be/0oTh1CXRaQ0?t=1h25m45s>).

As an example, in the following filesystem layout available custom modules are `pyrex.custom.pyspice`, `pyrex.custom.irex`, `pyrex.custom.ara`, `pyrex.custom.ariana`, and `pyrex.custom.my_analysis`:

```
/path/to/site-packages/pyrex/
|-- __init__.py
|-- signals.py
|-- antenna.py
|-- ...
|-- custom/
|   |-- pypspice.py
|   |-- irex/
|       |-- __init__.py
|       |-- antenna.py
|       |-- ...

/path/to/home_dir/.pyrex-custom/
|-- ara/
|   |-- custom/
|       |-- ara/
|           |-- __init__.py
|           |-- antenna.py
|           |-- ...
|-- ariana/
|   |-- custom/
|       |-- ariana.py

/path/to/cwd/pyrex-custom/
|-- my_analysis_module/
|   |-- custom/
|       |-- my_analysis.py
```

PYREX API

The API documentation here is split into three sections. First, the *Package contents* section documents all classes and functions that are imported by PyREx under a `from pyrex import *` command. Next, the *Submodules* section is a full documentation of all the modules which make up the base PyREx package. And finally, the *PyREx Custom Subpackage* section documents the custom subpackages contained in PyREx by default.

4.1 Package contents

class `pyrex.Signal` (*times, values, value_type=<ValueTypes.undefined: 0>*)

Base class for signals. Takes arrays of times and values (values array forced to size of times array by zero padding or slicing). Supports adding between signals with the same time values, resampling the signal, and calculating the signal's envelope.

class `ValueTypes`

Enum containing possible types (units) for signal values.

`undefined = 0`

`voltage = 1`

`field = 2`

`power = 3`

dt

Returns the spacing of the time array, or None if invalid.

envelope

Calculates envelope of the signal by Hilbert transform.

resample (*n*)

Resamples the signal into *n* points in the same time range.

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Uses `numpy.interp` on values by default.

spectrum

Returns the FFT spectrum of the signal.

frequencies

Returns the FFT frequencies of the signal.

filter_frequencies (*freq_response*)

Applies the given frequency response function to the signal.

class `pyrex.EmptySignal` (*times*, *value_type*=<ValueTypes.undefined: 0>)

Bases: `pyrex.signals.Signal`

Class for signal with no amplitude (all values = 0)

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Returns EmptySignal for new times.

class `pyrex.FunctionSignal` (*times*, *function*, *value_type*=<ValueTypes.undefined: 0>)

Bases: `pyrex.signals.Signal`

Class for signals generated by a function

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Leverages knowledge of the function to properly interpolate and extrapolate.

`pyrex.AskaryanSignal`

alias of FastAskaryanSignal

class `pyrex.signals.FastAskaryanSignal` (*times*, *energy*, *theta*, *n*=1.78, *t0*=0)

Bases: `pyrex.signals.Signal`

Askaryan pulse binned to times from a particle shower with given energy (GeV) observed at angle theta (radians) from the shower axis. Optional parameters are the index of refraction n, and pulse offset to start time t0 (s). Returned signal values are electric fields (V/m).

Note that the amplitude of the pulse goes as 1/R, where R is the distance from source to observer. R is assumed to be 1 meter so that dividing by a different value produces the proper result.

vector_potential

Recover the vector_potential from the electric field. Mostly just for testing purposes.

RAC (*time*)

Calculates $R * \text{vector potential (A)}$ at the Cherenkov angle in Vs at the given time (s).

charge_profile (*z*, *density*=0.92, *crit_energy*=0.0786, *rad_length*=36.08)

Calculates the longitudinal charge profile in the EM shower at distance z (m) with parameters for the density (g/cm³), critical energy (GeV), and electron radiation length (g/cm²) in ice.

max_length (*density*=0.92, *crit_energy*=0.0786, *rad_length*=36.08)

Calculates the maximum length (m) of an EM shower with parameters for the density (g/cm³), critical energy (GeV), and electron radiation length (g/cm²) in ice.

class `pyrex.ThermalNoise` (*times*, *f_band*, *f_amplitude*=1, *rms_voltage*=None, *temperature*=None, *resistance*=None, *n_freqs*=0)

Bases: `pyrex.signals.FunctionSignal`

Thermal Rayleigh noise in the frequency band *f_band*=[*f_min*,*f_max*] (Hz) at a given temperature (K) and resistance (ohms) or with a given RMS voltage (V). Optional parameters are *f_amplitude* (default 1) which can be a number or a function designating the amplitudes at each frequency, and *n_freqs* which is the number of frequencies to use (in *f_band*) for the calculation (default is based on the FFT bin size of the given times array). Returned signal values are voltages (V).

class `pyrex.Antenna` (*position*, *z_axis*=[0, 0, 1], *x_axis*=[1, 0, 0], *antenna_factor*=1, *efficiency*=1, *freq_range*=None, *noise_rms*=None, *temperature*=None, *resistance*=None, *noisy*=True)

Base class for an antenna with a given position (m), temperature (K), allowable frequency range (Hz), total resistance (ohm) used for Johnson noise, and whether or not to include noise in the antenna's waveforms. Defines default trigger, frequency response, and signal reception functions that can be overwritten in base classes to customize the antenna.

set_orientation (*z_axis*=[0, 0, 1], *x_axis*=[1, 0, 0])

is_hit
Test for whether the antenna has been triggered.

is_hit_during (*times*)
Test for whether the antenna has been triggered during the given times array.

clear ()
Reset the antenna to a state of having received no signals.

waveforms
Signal + noise (if noisy) at each triggered antenna hit.

all_waveforms
Signal + noise (if noisy) at all antenna hits, even those that didn't trigger.

full_waveform (*times*)
Signal + noise (if noisy) for the given times array.

make_noise (*times*)
Returns the noise signal generated by the antenna over the given array of times. Used to add noise to signal for production of the antenna's waveforms.

trigger (*signal*)
Function to determine whether or not the antenna is triggered by the given Signal object.

directional_gain (*theta*, *phi*)
Function to calculate the directive electric field gain of the antenna at given angles theta (polar) and phi (azimuthal) relative to the antenna's orientation.

polarization_gain (*polarization*)
Function to calculate the electric field gain due to polarization for a given polarization direction.

response (*frequencies*)
Function to return the frequency response of the antenna at the given frequencies (Hz). This function should return the response as imaginary numbers, where the real part is the amplitude response and the imaginary part is the phase response.

receive (*signal*, *origin*=None, *polarization*=None)
Process incoming signal according to the filter function and store it to the signals list. Subclasses may extend this function, but should likely end with super().receive(signal).

class pyrex.DipoleAntenna (*name*, *position*, *center_frequency*, *bandwidth*, *resistance*, *orientation*=[0, 0, 1], *trigger_threshold*=0, *effective_height*=None, *noisy*=True)
Bases: `pyrex.antenna.Antenna`

Antenna with a given name, position (m), center frequency (Hz), bandwidth (Hz), resistance (ohm), effective height (m), polarization direction, and trigger threshold (V).

trigger (*signal*)
Trigger on the signal if the maximum signal value is above the given threshold.

response (*frequencies*)
Butterworth filter response for the antenna's frequency range.

directional_gain (*theta*, *phi*)
Power gain of dipole antenna goes as $\sin(\theta)^2$, so electric field gain goes as $\sin(\theta)$.

polarization_gain (*polarization*)
Polarization gain is simply the dot product of the polarization with the antenna's z-axis.

class pyrex.AntennaSystem (*antenna*)
Base class for an antenna system consisting of an antenna and some front-end processes.

setup_antenna (*args, **kwargs)

Setup the antenna by passing along its init arguments. This function can be overwritten if desired, just make sure to assign the self.antenna attribute in the function.

front_end (signal)

This function should take the signal passed (from the antenna) and return the resulting signal after all processing by the antenna system's front-end. By default it just returns the given signal.

is_hit

is_hit_during (times)

signals

waveforms

all_waveforms

full_waveform (times)

receive (signal, origin=None, polarization=None)

clear ()

Reset the antenna system to a state of having received no signals.

trigger (signal)

Antenna system trigger. Should return True or False for whether the passed signal triggers the antenna system. By default just matches the antenna's trigger.

class pyrex.Detector (*args, **kwargs)

Class for automatically generating antenna positions based on geometry criteria. The set_positions method creates a list of antenna positions and the build_antennas method is responsible for actually placing antennas at the generated positions. Once antennas are placed, the class can be directly iterated over to iterate over the antennas (as if it were just a list of antennas itself).

set_positions (*args, **kwargs)

Not implemented. Should generate positions for the antennas based on the given arguments and assign those positions to the antenna_positions attribute.

build_antennas (antenna_class, **kwargs)

Sets up antenna objects at the positions stored in the class. By default takes an antenna class and passes a position to the 'position' argument, followed by the keyword arguments passed to this function.

pyrex.IceModel

alias of ArasimIce

class pyrex.ice_model.ArasimIce

Bases: [pyrex.ice_model.AntarcticIce](#)

Class containing characteristics of ice at the south pole. In all cases, depth z is given with negative values in the ice and positive values above the ice. Ice model index is the same as used in the ARA collaboration's arasim package.

k = 0.43

a = 0.0132

n0 = 1.78

pyrex.prem_density (r)

Returns the earth's density (g/cm³) for a given radius r (m). Calculated by the Preliminary Earth Model (PREM).

`pyrex.slant_depth` (*angle, depth, step=5000*)

Returns the material thickness (g/cm²) for a chord cutting through earth at Nadir angle and starting at depth (m).

class `pyrex.Particle` (*vertex, direction, energy*)

Class for storing particle attributes. Consists of a 3-D vertex (m), 3-D direction vector (automatically normalized), and an energy (GeV).

class `pyrex.ShadowGenerator` (*dx, dy, dz, energy_generator*)

Class to generate UHE neutrino vertices in (relatively) shallow detectors. Takes into account Earth shadowing (sort of). `energy_generator` should be a function that returns a particle energy in GeV. Note that the x and y ranges are (-dx/2, dx/2) and (-dy/2, dy/2) while the z range is (-dz, 0).

create_particle ()

Creates a particle with random vertex in cube with a random direction.

`pyrex.RayTracer`

alias of `SpecializedRayTracer`

class `pyrex.ray_tracing.SpecializedRayTracer` (*from_point, to_point, ice_model=<class 'pyrex.ice_model.ArasimIce'>, dz=1*)

Bases: `pyrex.ray_tracing.BasicRayTracer`

Ray tracer specifically for ice model with index of refraction $n(z) = n_0 - k \cdot \exp(a \cdot z)$. Calculations performed using true integral evaluation. Ice model must use methods inherited from `pyrex.AntarcticIce`

solution_class

alias of `SpecializedRayTracePath`

valid_ice_model

z_uniform

direct_r_max

peak_angle

`pyrex.RayTracePath`

alias of `SpecializedRayTracePath`

class `pyrex.ray_tracing.SpecializedRayTracePath` (*parent_tracer, launch_angle, direct*)

Bases: `pyrex.ray_tracing.BasicRayTracePath`

Class for storing a single ray-trace solution between points, specifically for ice model with index of refraction $n(z) = n_0 - k \cdot \exp(a \cdot z)$. Calculations performed using true integral evaluation (except attenuation). Ice model must use methods inherited from `pyrex.AntarcticIce`

uniformity_factor = 0.99999

beta_tolerance = 0.005

valid_ice_model

z_uniform

z_integral (*integrand, numerical=False, x_func=<function SpecializedRayTracePath.<lambda>>*)

Function for integrating a given integrand along the depths of the path.

path_length

tof

attenuation (*f*)

Returns the attenuation factor for a signal of frequency *f* (Hz) traveling along the path. Supports passing a list of frequencies.

coordinates

class `pyrex.EventKernel` (*generator, ice_model, antennas*)

Kernel for generation of events with a given particle generator, ice model, and list of antennas.

event ()

Generate particle, propagate signal through ice to antennas, process signal at antennas, and return the original particle.

4.2 Submodules

4.2.1 `pyrex.signals` module

Module containing classes for digital signal processing

class `pyrex.signals.Signal` (*times, values, value_type=<ValueTypes.undefined: 0>*)

Bases: `object`

Base class for signals. Takes arrays of times and values (values array forced to size of times array by zero padding or slicing). Supports adding between signals with the same time values, resampling the signal, and calculating the signal's envelope.

class `ValueTypes`

Bases: `enum.Enum`

Enum containing possible types (units) for signal values.

undefined = 0

voltage = 1

field = 2

power = 3

dt

Returns the spacing of the time array, or None if invalid.

envelope

Calculates envelope of the signal by Hilbert transform.

resample (*n*)

Resamples the signal into n points in the same time range.

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Uses `numpy.interp` on values by default.

spectrum

Returns the FFT spectrum of the signal.

frequencies

Returns the FFT frequencies of the signal.

filter_frequencies (*freq_response*)

Applies the given frequency response function to the signal.

class `pyrex.signals.EmptySignal` (*times, value_type=<ValueTypes.undefined: 0>*)

Bases: `pyrex.signals.Signal`

Class for signal with no amplitude (all values = 0)

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Returns EmptySignal for new times.

class `pyrex.signals.FunctionSignal` (*times, function, value_type=<ValueTypes.undefind: 0>*)

Bases: `pyrex.signals.Signal`

Class for signals generated by a function

with_times (*new_times*)

Returns a signal object representing this signal with a different times array. Leverages knowledge of the function to properly interpolate and extrapolate.

class `pyrex.signals.SlowAskaryanSignal` (*times, energy, theta, n=1.78, t0=0*)

Bases: `pyrex.signals.Signal`

Askaryan pulse binned to times from a particle shower with given energy (GeV) observed at angle theta (radians) from the shower axis. Optional parameters are the index of refraction n, and pulse offset to start time t0 (s). Returned signal values are electric fields (V/m).

Note that the amplitude of the pulse goes as $1/R$, where R is the distance from source to observer. R is assumed to be 1 meter so that dividing by a different value produces the proper result.

RAC (*time*)

Calculates $R * \text{vector potential}$ at the Cherenkov angle in Vs at the given time (s).

charge_profile (*z, density=0.92, crit_energy=0.0786, rad_length=36.08*)

Calculates the longitudinal charge profile in the EM shower at distance z (m) with parameters for the density (g/cm^3), critical energy (GeV), and electron radiation length (g/cm^2) in ice.

max_length (*density=0.92, crit_energy=0.0786, rad_length=36.08*)

Calculates the maximum length (m) of an EM shower with parameters for the density (g/cm^3), critical energy (GeV), and electron radiation length (g/cm^2) in ice.

class `pyrex.signals.FastAskaryanSignal` (*times, energy, theta, n=1.78, t0=0*)

Bases: `pyrex.signals.Signal`

Askaryan pulse binned to times from a particle shower with given energy (GeV) observed at angle theta (radians) from the shower axis. Optional parameters are the index of refraction n, and pulse offset to start time t0 (s). Returned signal values are electric fields (V/m).

Note that the amplitude of the pulse goes as $1/R$, where R is the distance from source to observer. R is assumed to be 1 meter so that dividing by a different value produces the proper result.

vector_potential

Recover the vector_potential from the electric field. Mostly just for testing purposes.

RAC (*time*)

Calculates $R * \text{vector potential}$ (A) at the Cherenkov angle in Vs at the given time (s).

charge_profile (*z, density=0.92, crit_energy=0.0786, rad_length=36.08*)

Calculates the longitudinal charge profile in the EM shower at distance z (m) with parameters for the density (g/cm^3), critical energy (GeV), and electron radiation length (g/cm^2) in ice.

max_length (*density=0.92, crit_energy=0.0786, rad_length=36.08*)

Calculates the maximum length (m) of an EM shower with parameters for the density (g/cm^3), critical energy (GeV), and electron radiation length (g/cm^2) in ice.

`pyrex.signals.AskaryanSignal`

alias of `FastAskaryanSignal`

class `pyrex.signals.GaussianNoise` (*times, sigma*)

Bases: `pyrex.signals.Signal`

Gaussian noise signal with standard deviation sigma

```
class pyrex.signals.ThermalNoise (times, f_band, f_amplitude=1, rms_voltage=None, temperature=None, resistance=None, n_freqs=0)
```

Bases: `pyrex.signals.FunctionSignal`

Thermal Rayleigh noise in the frequency band `f_band=[f_min,f_max]` (Hz) at a given temperature (K) and resistance (ohms) or with a given RMS voltage (V). Optional parameters are `f_amplitude` (default 1) which can be a number or a function designating the amplitudes at each frequency, and `n_freqs` which is the number of frequencies to use (in `f_band`) for the calculation (default is based on the FFT bin size of the given times array). Returned signal values are voltages (V).

4.2.2 pyrex.antenna module

Module containing antenna class capable of receiving signals

```
class pyrex.antenna.Antenna (position, z_axis=[0, 0, 1], x_axis=[1, 0, 0], antenna_factor=1, efficiency=1, freq_range=None, noise_rms=None, temperature=None, resistance=None, noisy=True)
```

Bases: `object`

Base class for an antenna with a given position (m), temperature (K), allowable frequency range (Hz), total resistance (ohm) used for Johnson noise, and whether or not to include noise in the antenna's waveforms. Defines default trigger, frequency response, and signal reception functions that can be overwritten in base classes to customize the antenna.

```
set_orientation (z_axis=[0, 0, 1], x_axis=[1, 0, 0])
```

```
is_hit
```

Test for whether the antenna has been triggered.

```
is_hit_during (times)
```

Test for whether the antenna has been triggered during the given times array.

```
clear ()
```

Reset the antenna to a state of having received no signals.

```
waveforms
```

Signal + noise (if noisy) at each triggered antenna hit.

```
all_waveforms
```

Signal + noise (if noisy) at all antenna hits, even those that didn't trigger.

```
full_waveform (times)
```

Signal + noise (if noisy) for the given times array.

```
make_noise (times)
```

Returns the noise signal generated by the antenna over the given array of times. Used to add noise to signal for production of the antenna's waveforms.

```
trigger (signal)
```

Function to determine whether or not the antenna is triggered by the given Signal object.

```
directional_gain (theta, phi)
```

Function to calculate the directive electric field gain of the antenna at given angles theta (polar) and phi (azimuthal) relative to the antenna's orientation.

```
polarization_gain (polarization)
```

Function to calculate the electric field gain due to polarization for a given polarization direction.

response (*frequencies*)

Function to return the frequency response of the antenna at the given frequencies (Hz). This function should return the response as imaginary numbers, where the real part is the amplitude response and the imaginary part is the phase response.

receive (*signal, origin=None, polarization=None*)

Process incoming signal according to the filter function and store it to the signals list. Subclasses may extend this function, but should likely end with `super().receive(signal)`.

class `pyrex.antenna.DipoleAntenna` (*name, position, center_frequency, bandwidth, resistance, orientation=[0, 0, 1], trigger_threshold=0, effective_height=None, noisy=True*)

Bases: `pyrex.antenna.Antenna`

Antenna with a given name, position (m), center frequency (Hz), bandwidth (Hz), resistance (ohm), effective height (m), polarization direction, and trigger threshold (V).

trigger (*signal*)

Trigger on the signal if the maximum signal value is above the given threshold.

response (*frequencies*)

Butterworth filter response for the antenna's frequency range.

directional_gain (*theta, phi*)

Power gain of dipole antenna goes as $\sin(\theta)^2$, so electric field gain goes as $\sin(\theta)$.

polarization_gain (*polarization*)

Polarization gain is simply the dot product of the polarization with the antenna's z-axis.

4.2.3 pyrex.detector module

Module containing higher-level AntennaSystem and Detector classes

class `pyrex.detector.AntennaSystem` (*antenna*)

Bases: `object`

Base class for an antenna system consisting of an antenna and some front-end processes.

setup_antenna (**args, **kwargs*)

Setup the antenna by passing along its init arguments. This function can be overwritten if desired, just make sure to assign the `self.antenna` attribute in the function.

front_end (*signal*)

This function should take the signal passed (from the antenna) and return the resulting signal after all processing by the antenna system's front-end. By default it just returns the given signal.

is_hit**is_hit_during** (*times*)**signals****waveforms****all_waveforms****full_waveform** (*times*)**receive** (*signal, origin=None, polarization=None*)**clear** ()

Reset the antenna system to a state of having received no signals.

trigger (*signal*)

Antenna system trigger. Should return True or False for whether the passed signal triggers the antenna system. By default just matches the antenna's trigger.

class `pyrex.detector.Detector` (*args, **kwargs)

Bases: `object`

Class for automatically generating antenna positions based on geometry criteria. The `set_positions` method creates a list of antenna positions and the `build_antennas` method is responsible for actually placing antennas at the generated positions. Once antennas are placed, the class can be directly iterated over to iterate over the antennas (as if it were just a list of antennas itself).

set_positions (*args, **kwargs)

Not implemented. Should generate positions for the antennas based on the given arguments and assign those positions to the `antenna_positions` attribute.

build_antennas (*antenna_class*, **kwargs)

Sets up antenna objects at the positions stored in the class. By default takes an antenna class and passes a position to the 'position' argument, followed by the keyword arguments passed to this function.

4.2.4 pyrex.ice_model module

Module containing ice models. Ice model classes contain static and class methods for convenience. `IceModel` class is set to the preferred ice model.

class `pyrex.ice_model.AntarcticIce`

Bases: `object`

Class containing characteristics of ice at the south pole. In all cases, depth z is given with negative values in the ice and positive values above the ice. Index of refraction goes as $n(z)=n_0-k*\exp(az)$.

k = 0.438

a = 0.0132

n0 = 1.758

thickness = 2850

classmethod `gradient` (z)

Returns the gradient of the index of refraction at depth z (m).

classmethod `index` (z)

Returns the medium's index of refraction, n , at depth z (m). Supports passing a numpy array of depths.

classmethod `depth_with_index` (n)

Returns the depth z (m) at which the medium has the given index of refraction (inverse of index function, assumes index function is monotonic so only one solution exists). Supports passing a numpy array of indices.

static `temperature` (z)

Returns the temperature (K) of the ice at depth z (m). Supports passing a numpy array of depths.

classmethod `attenuation_length` (z, f)

Returns the attenuation length at depth z (m) and frequency f (Hz). Supports passing a numpy array of depths and/or frequencies. If both are passed as arrays, a 2-D array is returned where each row is a single depth and each column is a single frequency.

class `pyrex.ice_model.NewcombIce`

Bases: `pyrex.ice_model.AntarcticIce`

Class inheriting from `AntarcticIce`, with new `attenuation_length` function based on Matt Newcomb's fit (DOESN'T CURRENTLY WORK).

classmethod `attenuation_length` (*z, f*)

Returns the attenuation length at depth *z* (m) and frequency *f* (MHz) by Matt Newcomb's fit (DOESN'T CURRENTLY WORK - USE BOGORODSKY).

class `pyrex.ice_model.ArasimIce`

Bases: `pyrex.ice_model.AntarcticIce`

Class containing characteristics of ice at the south pole. In all cases, depth *z* is given with negative values in the ice and positive values above the ice. Ice model index is the same as used in the ARA collaboration's arasim package.

k = 0.43

a = 0.0132

n0 = 1.78

`pyrex.ice_model.IceModel`

alias of `ArasimIce`

4.2.5 `pyrex.earth_model` module

Module containing earth model. Uses PREM for density as a function of radius and a simple integrator for calculation of the slant depth as a function of nadir angle.

`pyrex.earth_model.prem_density` (*r*)

Returns the earth's density (g/cm³) for a given radius *r* (m). Calculated by the Preliminary Earth Model (PREM).

`pyrex.earth_model.slant_depth` (*angle, depth, step=5000*)

Returns the material thickness (g/cm²) for a chord cutting through earth at Nadir angle and starting at depth (m).

4.2.6 `pyrex.particle` module

Module for particles (namely neutrinos) and neutrino interactions in the ice. Interactions include Earth shadowing (absorption) effect.

class `pyrex.particle.NeutrinoInteraction` (*c, p*)

Bases: `object`

Class for neutrino interaction attributes.

cross_section (*E*)

Return the cross section (cm²) at a given energy *E* (GeV).

interaction_length (*E*)

Return the interaction length (cm) in water equivalent at a given energy *E* (GeV).

class `pyrex.particle.Particle` (*vertex, direction, energy*)

Bases: `object`

Class for storing particle attributes. Consists of a 3-D vertex (m), 3-D direction vector (automatically normalized), and an energy (GeV).

`pyrex.particle.random_direction` ()

Generate an arbitrary 3D unit vector.

class `pyrex.particle.ShadowGenerator(dx, dy, dz, energy_generator)`

Bases: `object`

Class to generate UHE neutrino vertices in (relatively) shallow detectors. Takes into account Earth shadowing (sort of). `energy_generator` should be a function that returns a particle energy in GeV. Note that the x and y ranges are $(-dx/2, dx/2)$ and $(-dy/2, dy/2)$ while the z range is $(-dz, 0)$.

create_particle()

Creates a particle with random vertex in cube with a random direction.

4.2.7 pyrex.ray_tracing module

Module containing class for ray tracing through the ice.

class `pyrex.ray_tracing.BasicRayTracePath(parent_tracer, launch_angle, direct)`

Bases: `pyrex.internal_functions.LazyMutableClass`

Class for storing a single ray-trace solution between points. Calculations preformed by integrating z-steps of size `dz`. Most properties lazily evaluated to save on re-computation time.

z_turn_proximity

Parameter for how closely path approaches `z_turn`. Necessary to avoid diverging integrals.

z0

Depth of the launching point.

z1

Depth of the receiving point.

n0

rho

phi

beta

z_turn

emitted_direction

received_direction

theta(z)

Polar angle of the ray at given depth or array of depths.

z_integral(integrand)

Returns the integral of the integrand (a function of `z`) along the path.

path_length

tof

fresnel

attenuation(f)

Returns the attenuation factor for a signal of frequency `f` (Hz) traveling along the path. Supports passing a list of frequencies.

propagate(signal)

Applies attenuation to the signal along the path.

coordinates

class `pyrex.ray_tracing.SpecializedRayTracePath` (*parent_tracer, launch_angle, direct*)

Bases: `pyrex.ray_tracing.BasicRayTracePath`

Class for storing a single ray-trace solution between points, specifically for ice model with index of refraction $n(z) = n_0 - k \cdot \exp(a \cdot z)$. Calculations performed using true integral evaluation (except attenuation). Ice model must use methods inherited from `pyrex.AntarcticIce`

uniformity_factor = 0.99999

beta_tolerance = 0.005

valid_ice_model

z_uniform

z_integral (*integrand, numerical=False, x_func=<function SpecializedRayTracePath.<lambda>>*)

Function for integrating a given integrand along the depths of the path.

path_length

tof

attenuation (*f*)

Returns the attenuation factor for a signal of frequency *f* (Hz) traveling along the path. Supports passing a list of frequencies.

coordinates

class `pyrex.ray_tracing.BasicRayTracer` (*from_point, to_point, ice_model=<class 'pyrex.ice_model.ArasimIce'>, dz=1*)

Bases: `pyrex.internal_functions.LazyMutableClass`

Class for proper ray tracing. Calculations performed by integrating z-steps with size *dz*. Most properties lazily evaluated to save on re-computation time.

solution_class

alias of `BasicRayTracePath`

z_turn_proximity

Parameter for how closely path approaches *z_turn*. Necessary to avoid diverging integrals.

z0

Depth of lower point. Ray tracing performed as if launching from lower point to higher point.

z1

Depth of higher point. Ray tracing performed as if launching from lower point to higher point.

n0

rho

max_angle

peak_angle

direct_r_max

indirect_r_max

exists

expected_solutions

solutions

direct_angle

indirect_angle_1

indirect_angle_2

static angle_search (*true_r*, *r_function*, *min_angle*, *max_angle*, *tolerance=1e-12*,
max_iterations=100)
 Root-finding algorithm.

class `pyrex.ray_tracing.SpecializedRayTracer` (*from_point*, *to_point*, *ice_model=<class*
'pyrex.ice_model.ArasimIce'>, *dz=1*)

Bases: `pyrex.ray_tracing.BasicRayTracer`

Ray tracer specifically for ice model with index of refraction $n(z) = n_0 - k \cdot \exp(a \cdot z)$. Calculations performed using true integral evaluation. Ice model must use methods inherited from `pyrex.AntarcticIce`

solution_class

alias of `SpecializedRayTracePath`

valid_ice_model

z_uniform

direct_r_max

peak_angle

`pyrex.ray_tracing.RayTracer`

alias of `SpecializedRayTracer`

`pyrex.ray_tracing.RayTracePath`

alias of `SpecializedRayTracePath`

class `pyrex.ray_tracing.PathFinder` (*ice_model*, *from_point*, *to_point*)

Bases: `object`

Class for pseudo ray tracing. Just uses straight-line paths.

exists

Boolean of whether path exists based on basic total internal reflection calculation.

emitted_ray

Direction in which ray is emitted.

received_ray

Direction from which ray is received.

path_length

Length of the path (m).

tof

Time of flight (s) for a particle along the path. Calculated using default values of `self.time_of_flight()`

time_of_flight (*n_steps=100*)

Time of flight (s) for a particle along the path.

attenuation (*f*, *n_steps=100*)

Returns the attenuation factor for a signal of frequency *f* (Hz) traveling along the path. Supports passing a list of frequencies.

propagate (*signal*)

Applies attenuation to the signal along the path.

class `pyrex.ray_tracing.ReflectedPathFinder` (*ice_model*, *from_point*, *to_point*, *reflec-*
tion_depth=0)

Bases: `object`

Class for pseudo ray tracing of ray reflected off ice surface. Just uses straight-line paths.

get_bounce_point (*reflection_depth=0*)

Calculation of point at which signal is reflected by the ice surface ($z=0$).

exists

Boolean of whether path exists based on whether its sub-paths exist and whether it could reflect off the ice surface.

emitted_ray

Direction in which ray is emitted.

received_ray

Direction from which ray is received.

path_length

Length of the path (m).

tof

Time of flight (s) for a particle along the path. Calculated using default values of `self.time_of_flight()`

time_of_flight (*n_steps=100*)

Time of flight (s) for a particle along the path.

attenuation (*f, n_steps=100*)

Returns the attenuation factor for a signal of frequency f (Hz) traveling along the path. Supports passing a list of frequencies.

propagate (*signal*)

Applies attenuation to the signal along the path.

4.2.8 pyrex.kernel module

Module for the simulation kernel. Includes neutrino generation, ray tracking (no raytracing yet), and hit generation.

class `pyrex.kernel.EventKernel` (*generator, ice_model, antennas*)

Bases: `object`

Kernel for generation of events with a given particle generator, ice model, and list of antennas.

event ()

Generate particle, propagate signal through ice to antennas, process signal at antennas, and return the original particle.

4.2.9 pyrex.internal_functions module

Helper functions for use in PyREx modules.

`pyrex.internal_functions.normalize` (*vector*)

Returns the normalized form of the given vector.

`pyrex.internal_functions.lazy_property` (*fn*)

Decorator that makes a property lazily evaluated.

class `pyrex.internal_functions.LazyMutableClass` (*static_attributes=None*)

Bases: `object`

Class whose properties can be lazily evaluated by using `lazy_property` decorator, but will re-evaluate lazy properties if any of its specified `static_attributes` change. By default, `static_attributes` is set to all attributes of the class at the time of the init call.

4.3 PyREx Custom Subpackage

Note that more modules are available as plug-ins, see *Custom Sub-Package*.

4.3.1 pyrex.custom.pyspice module

Module containing setup and wrappers for PySpice module into PyREx

```
class pyrex.custom.pyspice.NgSpiceSharedSignal (**kwargs)
    Bases: PySpice.Spice.NgSpice.Shared.NgSpiceShared
    Helper class for bridging gap between PyREx Signal and PySpice NgSpiceShared classes
    get_vsrc_data (voltage, time, node, ngspice_id)

class pyrex.custom.pyspice.SpiceSignal (signal, shared=<pyrex.custom.pyspice.NgSpiceSharedSignal
    object>)
    Bases: object
    Class for passing PyREx Signal object into PySpice
```

4.3.2 pyrex.custom.irex package

Customizations of pyrex package specific to IREX (IceCube Radio Extension)

pyrex.custom.irex.antenna module

Module containing customized antenna classes for IREX

```
class pyrex.custom.irex.antenna.IREXAntenna (position, center_frequency, bandwidth,
    resistance, orientation=(0, 0, 1), effective_height=None, noisy=True)
    Bases: pyrex.antenna.Antenna
    Antenna to be used in IREX. Has a position (m), center frequency (Hz), bandwidth (Hz), resistance (ohm),
    effective height (m), and polarization direction.
    response (frequencies)
        Butterworth filter response for the antenna's frequency range.
    directional_gain (theta, phi)
        Power gain of dipole antenna goes as sin(theta)^2, so electric field gain goes as sin(theta).
    polarization_gain (polarization)
        Polarization gain is simply the dot product of the polarization with the antenna's z-axis.

class pyrex.custom.irex.antenna.IREXAntennaSystem (name, position, trigger_threshold,
    time_over_threshold=0, orientation=(0, 0, 1), amplification=1,
    amplifier_clipping=3, noisy=True, envelope_method='analytic')
    Bases: pyrex.detector.AntennaSystem
    IREX antenna system consisting of dipole antenna, low-noise amplifier, optional bandpass filter, and envelope
    circuit.
```

setup_antenna (*center_frequency=250000000.0, bandwidth=300000000.0, resistance=100, orientation=(0, 0, 1), effective_height=None, noisy=True*)

Sets attributes of the antenna including center frequency (Hz), bandwidth (Hz), resistance (ohms), orientation, and effective height (m).

make_envelope (*signal*)

Return the signal envelope based on the antenna's `envelope_method`.

front_end (*signal*)

Apply the front-end processing of the antenna signal, including amplification, clipping, and envelope processing.

all_waveforms

full_waveform (*times*)

trigger (*signal*)

pyrex.custom.irex.detector module

Module containing customized detector geometry classes for IREX

class `pyrex.custom.irex.detector.IREXDetector` (**args, **kwargs*)

Bases: `pyrex.detector.Detector`

Base class for IREX detector classes which implements the `build_antennas` method, but not `set_positions`.

build_antennas (*trigger_threshold, time_over_threshold=0, amplification=1, naming_scheme=<function IREXDetector.<lambda>>, orientation_scheme=<function IREXDetector.<lambda>>, noisy=True, envelope_method='analytic'*)

Sets up IREXAntennas at the positions stored in the class. Takes as arguments the trigger threshold, optional time over threshold, and whether to add noise to the waveforms. Other optional arguments include a naming scheme and orientation scheme which are functions taking the antenna index *i* and the antenna object. The naming scheme should return the name and the orientation scheme should return the orientation z-axis and x-axis of the antenna.

class `pyrex.custom.irex.detector.IREXGrid` (**args, **kwargs*)

Bases: `pyrex.custom.irex.detector.IREXDetector`

Class for (semi)automatically generating a rectangular grid of strings of antennas, which can then be iterated over.

set_positions (*number_of_strings=1, string_separation=500, antennas_per_string=2, antenna_separation=40, lowest_antenna=-200*)

Generates antenna positions in a grid of strings. Takes as arguments the number of strings, the distance between strings, the number of antennas per string, the separation (in z) of the antennas on the string, and the position of the lowest antenna.

class `pyrex.custom.irex.detector.IREXClusteredGrid` (**args, **kwargs*)

Bases: `pyrex.custom.irex.detector.IREXDetector`

Class for (semi)automatically generating a rectangular grid of clusters of strings of antennas, which can then be iterated over.

set_positions (*number_of_stations=1, station_separation=500, antennas_per_string=2, antenna_separation=40, lowest_antenna=-200, strings_per_station=4, string_separation=50*)

Generates antenna positions in a grid of strings. Takes as arguments the number of stations, the distance between stations, the number of antennas per string, the separation (in z) of the antennas on the string, the

position of the lowest antenna, and the name of the geometry to use. Optional parameters (depending on the geometry) are the number of strings per station and the distance from station to string.

```
class pyrex.custom.irex.detector.IREXCoxeterClusters(*args, **kwargs)
```

Bases: `pyrex.custom.irex.detector.IREXDetector`

Class for (semi)automatically generating a rectangular grid of Coxeter-plane-like clusters (one string at center) of strings of antennas, which can then be iterated over.

```
set_positions(number_of_stations=1, station_separation=500, antennas_per_string=2,
              antenna_separation=40, lowest_antenna=-200, strings_per_station=4,
              string_separation=25)
```

Generates antenna positions in a grid of strings. Takes as arguments the number of stations, the distance between stations, the number of antennas per string, the separation (in z) of the antennas on the string, the position of the lowest antenna, and the name of the geometry to use. Optional parameters (depending on the geometry) are the number of strings per station and the distance from station to string.

```
class pyrex.custom.irex.detector.IREXPairedGrid(*args, **kwargs)
```

Bases: `pyrex.custom.irex.detector.IREXDetector`

Class for (semi)automatically generating a rectangular grid of strings of antennas, which can then be iterated over.

```
set_positions(number_of_strings=1, string_separation=500, antennas_per_string=16,
              antenna_separation=10, lowest_antenna=-95, antennas_per_clump=2,
              clump_separation=1)
```

Generates antenna positions in a grid of strings. Takes as arguments the number of strings, the distance between strings, the number of antennas per string, the separation (in z) of the antennas on the string, and the position of the lowest antenna.

pyrex.custom.irex.frontends module

Module containing IREX front-end circuit models

```
pyrex.custom.irex.frontends.basic_envelope_model(signal, cap=2e-11, res=500)
```

Model of a basic diode-capacitor-resistor envelope circuit. Takes a signal object as the input voltage and returns the output voltage signal object.

```
pyrex.custom.irex.frontends.bridge_rectifier_envelope_model(signal, cap=2e-11,
                                                            res=500)
```

Model of a diode bridge rectifier envelope circuit. Takes a signal object as the input voltage and returns the output voltage signal object.

pyrex.custom.irex.reconstruction module

Module containing reconstruction methods for IREX

```
pyrex.custom.irex.reconstruction.quick_vertex_reconstruction(detector, thresh-
                                                            old=None,
                                                            get_waveform=<function
                                                            <lambda>>)
```

```
pyrex.custom.irex.reconstruction.full_vertex_reconstruction(detector, thresh-
                                                            old=None,
                                                            get_waveform=<function
                                                            <lambda>>)
```

```
pyrex.custom.irex.reconstruction.get_xcorr_times(waveforms)
```

```
pyrex.custom.irex.reconstruction.minimizer_vertex_reconstruction(positions,  
                                                                    times,  
                                                                    guess=None)  
pyrex.custom.irex.reconstruction.least_squares(vertex,           positions,           times,  
                                                  method='trace')  
pyrex.custom.irex.reconstruction.bancroft_vertex(positions,       times,           veloc-  
                                                  ity=170940170.94017094)  
pyrex.custom.irex.reconstruction.bancroft_scan_vertex(positions,   times,           veloc-  
                                                  ity=170940170.94017094)
```

VERSION HISTORY

5.1 Version 1.4.1

Changes

- Improved ray tracing and defaulted to the almost completely analytical `SpecializedRayTracer` and `SpecializedRayTracePath` classes as `RayTracer` and `RayTracePath`
- Added ray tracer into `EventKernel` to replace `PathFinder` completely

5.2 Version 1.4.0

New Features

- Implemented full ray tracing in the `RayTracer` and `RayTracePath` classes.

5.3 Version 1.3.1

New Features

- Added diode bridge rectifier envelope circuit analytic model to `irex.frontends` and made it the default analytic envelope model in `IREXAntennaSystem`.
- Added `allow_reflection` attribute to `EventKernel` class to determine whether `ReflectedPathFinder` solutions should be allowed.

Changes

- Changed neutrino interaction model to include all neutrino and anti-neutrino interactions rather than only charged-current neutrino (relevant for `ShadowGenerator` class).

5.4 Version 1.3.0

New Features

- Added and implemented `ReflectedPathFinder` class for rays which undergo total internal reflection and subsequently reach an antenna.

Changes

- Change `AksaryanSignal` angle to always be positive and remove < 90 degree restriction (Alvarez-Muniz, Romero-Wolf, & Zas paper suggests the algorithm should work for all angles).

Performance Improvements

- Improve performance of ice index calculated at many depths.

5.5 Version 1.2.1

New Features

- Added `set_orientation` function to `Antenna` class for setting the `z_axis` and `x_axis` attributes appropriately.

Bug Fixes

- Fixed bug where `Antenna._convert_to_antenna_coordinates` function was returning coordinates relative to (0,0,0) rather than the antenna's position.

5.6 Version 1.2.0

Changes

- Changed `custom` module to a package containing `irex` module.
- `custom` package leverages “Implicit Namespace Package” structure to allow plug-in style additions to the package in either the user's `~/pyrex-custom/` directory or the `./pyrex-custom` directory.

5.7 Version 1.1.2

New Features

- Added `with_times` method to `Signal` class for interpolation/extrapolation of signals to different times.
- Added `full_waveform` and `is_hit_during` methods to `Antenna` class for calculation of waveform over arbitrary time array and whether said waveform triggers the antenna, respectively.

- Added `front_end_processing` method to `IREXAntenna` for processing envelope, amplifying signal, and downsampling result (downsampling currently inactive).

Changes

- Change `Antenna.make_noise` to use a single master noise object and use `with_times` to calculate noise at different times.
 - To ensure noise is not obviously periodic (for <100 signals), uses 100 times the recommended number of frequencies, which results in longer computation time for noise waveforms.

5.8 Version 1.1.1

Changes

- Moved `ValueTypes` inside `Signal` class. Now access as `Signal.ValueTypes.voltage`, etc.
- Changed signal envelope calculation in custom `IREXAntenna` from hilbert transform to a basic model. Spice model also available, but slower.

5.9 Version 1.1.0

New Features

- Added `directional_gain` and `polarization_gain` methods to base `Antenna`.
 - `receive` method should no longer be overwritten in most cases.
 - `Antenna` now has orientation defined by `z_axis` and `x_axis`.
 - `antenna_factor` and `efficiency` attributes added to `Antenna` for more flexibility.
- Added `value_type` attribute to `Signal` class and derived classes.
 - Current value types are `ValueTypes.undefined`, `ValueTypes.voltage`, `ValueTypes.field`, and `ValueTypes.power`.
 - `Signal` objects now must have the same `value_type` to be added (though those with `ValueTypes.undefined` can be coerced).

Changes

- Made units consistent across PyREx.
- Added ability to define `Antenna` noise by RMS voltage rather than temperature and resistance if desired.
- Allow `DipoleAntenna` to guess at `effective_height` if not specified.

Performance Improvements

- Increase speed of `IceModel.__atten_coeffs` method, resulting in increased speed of attenuation length calculations.

5.10 Version 1.0.3

New Features

- Added `custom` module to contain classes and functions specific to the IREX project.

5.11 Version 1.0.2

New Features

- Added `Antenna.make_noise()` method so custom antennas can use their own noise functions.

Changes

- Allow passing of numpy arrays of depths and frequencies into most `IceModel` methods.
 - `IceModel.gradient()` must still be calculated at individual depths.
- Added ability to specify RMS voltage of `ThermalNoise` without providing temperature and resistance.
- Removed (deprecated) `Antenna.isHit()`.

Performance Improvements

- Allowing for `IceModel` to calculate many attenuation lengths at once improves speed of `PathFinder.propagate()`.
- Improved speed of `PathFinder.time_of_flight()` and `PathFinder.attenuation()` (and improved accuracy to boot).

5.12 Version 1.0.1

Changes

- Changed `Antenna` not require a temperature and frequency range if no noise is produced.

Bug Fixes

- Fixed bugs in `AskaryanSignal` that caused the convolution to fail.
- Fixed bugs resulting from converting `IceModel.temperature()` from Celsius to Kelvin.

5.13 Version 1.0.0

- Created PyREx package based on original notebook.
- Added all signal classes to produce full-waveform Askaryan pulses and thermal noise.

- Changed `Antenna` class to `DipoleAntenna` to allow `Antenna` to be a base class.
- Changed `Antenna.isHit()` method to `Antenna.is_hit` property.
- Introduced `IceModel` alias for `AntarcticIce` (or any future preferred ice model).
- Moved `AntarcticIce.attenuationLengthMN` to its own `NewcombIce` class inheriting from `AntarcticIce`.
- Added `PathFinder.propagate()` to propagate a `Signal` object in a customizable way.
- Changed naming conventions to be more consistent, verbose, and “pythonic”:
 - `AntarcticIce.attenuationLength()` becomes `AntarcticIce.attenuation_length()`.
 - In `pyrex.earth_model`, `RE` becomes `EARTH_RADIUS`.
 - In `pyrex.particle`, `neutrino_interaction` becomes `NeutrinoInteraction`.
 - In `pyrex.particle`, `NA` becomes `AVOGADRO_NUMBER`.
 - `particle` class becomes `Particle` namedtuple.
 - * `Particle.vtx` becomes `Particle.vertex`.
 - * `Particle.dir` becomes `Particle.direction`.
 - * `Particle.E` becomes `Particle.energy`.
 - In `pyrex.particle`, `next_direction()` becomes `random_direction()`.
 - `shadow_generator` becomes `ShadowGenerator`.
 - `PathFinder` methods become properties where reasonable:
 - * `PathFinder.exists()` becomes `PathFinder.exists`.
 - * `PathFinder.getEmittedRay()` becomes `PathFinder.emitted_ray`.
 - * `PathFinder.getPathLength()` becomes `PathFinder.path_length`.
 - `PathFinder.propagateRay()` split into `PathFinder.time_of_flight()` (with corresponding `PathFinder.tof` property) and `PathFinder.attenuation()`.

5.14 Version 0.0.0

Original PyREx python notebook written by Kael Hanson:

<https://gist.github.com/physkael/898a64e6fbf5f0917584c6d31edf7940>

GITHUB README

6.1 PyREx - (Python package for an IceCube Radio Extension)

PyREx (**P**ython **p**ackage for an **I**ceCube **R**adio **E**xtension) is, as its name suggests, a Python package designed to simulate the measurement of Askaryan pulses via a radio antenna array around the IceCube South Pole Neutrino Observatory. The code is designed to be modular so that it can also be applied to other askaryan radio antennas (e.g. the ARA and ARIANA collaborations).

6.1.1 Useful Links

- Source (GitHub): <https://github.com/bhokansonfasig/pyrex>
- Documentation: <https://bhokansonfasig.github.io/pyrex/>
- Release notes: <https://bhokansonfasig.github.io/pyrex/build/html/versions.html>

6.1.2 Getting Started

Requirements

PyREx requires python version 3.6+ as well as numpy version 1.13+ and scipy version 0.19+. After installing python from <https://www.python.org/downloads/>, numpy and scipy can be installed with `pip` as follows, or by simply installing pyrex as specified in the next section.

```
pip install numpy>=1.14
pip install scipy>=0.19
```

Installing

The easiest way to get the PyREx package is using `pip` as follows:

```
pip install git+https://github.com/bhokansonfasig/pyrex#egg=pyrex
```

Note that since PyREx is not currently available on PyPI, a simple `pip install pyrex` will not have the intended effect.

6.1.3 Examples

For examples of how to use PyREx, see the [examples](#) page in the documentation, or the PyREx [Demo notebook](#).

6.1.4 Authors

- Ben Hokanson-Fasig

6.1.5 License

MIT License

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