

Making Concurrent Data Structures Recoverable

July 21, 2018

Abstract

Recent developments foreshadow the emergence of new systems, in which byte-addressable *non-volatile main memory (NVRAM)*, combining the performance benefits of conventional main memory with the durability of secondary storage, co-exists with (or eventually even replaces) traditional volatile memory. Consequently, there is increased interest in *recoverable* concurrent objects: objects that are made robust to crash-failures by allowing their operations to recover from such failures. This paper presents a principled approach to deriving recoverable versions of several widely-used concurrent data structures, in particular, a linked list and an elimination stack.

1 Introduction (based on our PODC)

Shared-memory multiprocessors are asynchronous in nature. Asynchrony is related to reliability, since algorithms that provide nonblocking progress properties (e.g., lock-freedom and wait-freedom [8]) in an asynchronous environment with reliable processes continue to provide the same progress properties in the presence of *crash failures*. This happens because a process that crashes permanently during the execution of the algorithm is indistinguishable to the other processes from one that is merely very slow. Owing to its simplicity and intimate relationship with asynchrony, the crash-failure model is almost ubiquitous in the treatment of concurrent algorithms.

The attention to the crash-failure model has so far mostly neglected the *crash-recovery* model, in which a failed process may be resurrected after it crashes. Recent developments foreshadow the emergence of new systems, in which byte-addressable *non-volatile main memory (NVRAM)*, combining the performance benefits of conventional main memory with the durability of secondary storage, co-exists with (or eventually even replaces) traditional volatile memory. Consequently, there is increased interest in *recoverable concurrent objects* (also called *persistent* [4, 5] or *durable* [11]): objects that are made robust to crash-failures by allowing their operations to recover from such failures.

This paper consider an abstract individual-process crash-recovery model for non-volatile memory [2]. Processes communicate via non-volatile shared-memory variables. Each process also has local variables stored in volatile processor registers. [[YF: What about program counter? HA: in this paper, it can be volatile.]] At any point, a process may incur a crash-failure, causing all its local variables to be reset to arbitrary values. Operation response values are returned via volatile processor registers, which may become inaccessible to the calling process if it fails just before persisting the response value. A data structure has an associated *recovery function* that is responsible for restoring it upon the recovery from a crash-failure. The recovery function completes the current outstanding operation on the data structure, if there was any, returning either its *response* or a *fail* indication, if it was unsuccessful. Both responses are consistent with the resulting state of the data structure, to which the operation was applied (in the former case) or not (in the latter case).

We present a principled approach to deriving recoverable versions and describe in detail in two widely-used concurrent data structures: a linked list [] and elimination stack [].

Several correctness conditions for the crash-recovery model were defined in recent years (see, e.g., [1, 3, 6, 7, 10]). The goal of these conditions is to maintain the state of concurrent objects consistent in the face of crash failures.

2 Model and Definitions (based on our PODC)

We consider a system where N asynchronous *processes* p_1, \dots, p_N communicate by accessing *concurrent objects*. The system provides *base objects* that support atomic read, write, and compare&swap (CAS). Base objects can be used for implementing more complex concurrent objects (e.g. lists, trees and stacks), by defining access procedures that simulate each operation on the implemented object using operations on base objects.

The state of each process consists of non-volatile *shared-memory variables*, as well as *local variables stored in volatile processor registers*. Each process can incur at any point during the execution a *crash-failure* (or simply a *crash*) that resets all its local variables to arbitrary values, but preserves the values of all its non-volatile variables. A process p *invokes an operation* Op on

an object by performing an *invocation step*; upon Op 's completion, a *response step* is executed.

Operation Op is *pending* if it was invoked but was not yet completed. For simplicity, we assume that, at all times, each process has at most a single pending operation on any one object.

[[HA: Define data structure D .] Each data structure has an associated *recovery function*, denoted $D.\text{Recover}$, which is responsible for restoring the data structure to a consistent state, upon recovery from a crash.

More formally, a *history* H is a sequence of *steps*. There are four types of steps:

1. an *invocation step*, denoted (INV, p, O, Op) , represents the invocation by process p of operation Op on object O ;
2. an operation Op can be completed either normally or when, following one or more crashes, the execution of $Op.\text{Recover}$ is completed. In either case, a *response step* s , denoted (RES, p, O, Op, ret) , represents the completion by process p of operation Op invoked on object O by some step s' of p , with response ret being written to a local variable of p . We say that s is the *response step that matches* s' ;
3. a *crash step* s , denoted $(CRASH, p)$, represents the crash of process p . We call the inner-most recoverable operation Op of p that was pending when the crash occurred the *crashed operation* of s . $(CRASH, p)$ may also occur while p is executing some recovery function $Op.\text{Recover}$ and we say that Op is the crashed operation of s also in this case;
4. a *recovery step* s for process p , denoted (REC, p) , is the only step by p that is allowed to follow a $(CRASH, p)$ step s' . It represents the resurrection of p by the system, in which it invokes $Op.\text{Recover}$,¹ where Op is the crashed operation of s' . We say that s is the *recovery step that matches* s' .

For a history H , we let $H|p$ denote the subhistory of H consisting of all the steps by process p in H . H is *crash-free* if it contains no crash steps (hence also no recover steps). We let $H|<p, O>$ denote the subhistory consisting of all the steps on O by p .

Given two operations op_1 and op_2 in a history H , we say that op_1 *happens before* op_2 , denoted $op_1 <_H op_2$, if op_1 's response step precedes the invocation step of op_2 in H . $H|O$ is a *sequential object history*, if it is an alternating series of invocations and the matching responses starting with an invocation (the history may end by a pending invocation). The *sequential specification* of an object O is the set of all possible (legal) sequential histories over O . A history H is *sequential* if $H|O$ is a sequential object history for all objects O .

Two histories H and H' are *equivalent*, if $H|<p, O> = H'|<p, O>$ for all processes p and objects O . Given a history H , a *completion* of H is a history H' constructed from H by selecting separately, for each object O that appears in H , a subset of the operations pending on O in H and appending matching responses to all these operations, and then removing all remaining pending operations on O (if any).

Definition 1 (Linearizability [9], rephrased) *A finite crash-free history H is linearizable if it has a completion H' and a legal sequential history S such that:*

L1. H' is equivalent to S ; and

¹A history does not contain invocation/response steps for recovery functions.

L2. $<_H \subseteq <_S$ (i.e., if $op_1 <_H op_2$ and both ops appear in S then $op_1 <_S op_2$).

Thus, a finite history is linearizable, if we can linearize the subhistory of each object that appears in it. Next, we define a more general notion of well-formedness that applies also to histories that contain crash/recovery steps. For a history H , we let $N(H)$ denote the history obtained from H by removing all crash and recovery steps.

Definition 2 (Recoverable Well-Formedness) *A history H is recoverable well-formed if the following holds.*

1. Every crash step in $H|p$ is either p 's last step in H or is followed in $H|p$ by a matching recover step of p .
2. $N(H)$ is well-formed.

We can now define the notion of nesting-safe recoverable linearizability.

Definition 3 (Nesting-safe Recoverable Linearizability (NRL)) *A finite history H satisfies nesting-safe recoverable linearizability (NRL) if it is recoverable well-formed and $N(H)$ is a linearizable history. An object implementation satisfies NRL if all of its finite histories satisfy NRL.*

2.1 General Overview

Designing a data structure which is persistent in the presence of a crash-recovery is not an easy task. Several different correctness conditions and implementations have been proposed for such data structures [cite...]. However, most of these conditions and implementations does not support detectability, that is, the ability to conclude upon recovery whether the crashed operation took effect or not.

Attiya et al [cite] presented a novel crash-recovery model together with a correctness condition. Roughly speaking, Nesting-safe Recoverable Linearizability (NRL) requires each recoverable operation to supply an recover function, such that invoking it after a crash inside the operation allows the process to complete the operation, as well as restore the response value if needed. Moreover, they presented recoverable implementations for read,write and CAS operations. As suggested by the name NRL, it allows nesting. Therefore, taking any algorithm which uses only read,write and CAS primitives, and replacing each primitive with its recoverable version yields an NRL implementation (some minor changes are still needed in order to use this transformation). In particular, this is the case for all the algorithms presented in this paper. However, this transformation is very costly, in terms of both time and space.

Cohen et al [cite.] presented a universal construction that implements durably any object in a lock-free manner, using at most one persistent fence per operation, which is optimal. Moreover, the algorithm uses only read, write and CAS operations. However, universal construction by its nature is not efficient. In particular, their implementation requires to keep the entire history of the object in a designated shared queue, as well as per-process persistent log, such that these logs as whole keeps the entire history, and different logs may have a big overlap. In addition, the response of an operation is determine by looking at the entire history up to the linearization point of the operation. Their universal construction can be made detectable in the following way: upon recovery from a

crash inside an *Update(op)* operation, and after the system completes its recovery routine, *op* was linearized if and only if *op* appears in the shared queue (assuming each *op* structure has a unique address). In such case, the process can conclude the response value, since the queue represents the object's history.

Clearly, although the construction has an optimal fence complexity, in practice it is not efficient. Moreover, the crash model they consider is a system-wide crash, in which all processes crash at once. In such case, upon recovery, a single recover function is being executed by the system in order to reconstruct the queue data-structure in a consistent way. The correctness condition they consider is durable linearizability [cite], which requires that after a full system crash, the state of the object must reect a consistent operation sub-history that includes all operations completed by the time of the crash. Therefore, some of the crashed operation may get lost.

Different concurrent implementation for specific data-structures are known. These implementations exploit the structure and requirements of the specific object in order to optimize the resulting implementation. For example, operations that effects different parts of the data structure can be done concurrently without interfering each other. A lot of work and effort has been put in order to develop these implementations, as well as proving their correctness, thus we would like to have a general approach to turn a given implementation to a recoverable one, while preserving its structure and complexity as much as possible. This way, we can enjoy both an efficient algorithm, as well as recoverability, while avoiding the need to design new algorithms.

Ben-David, Blelloch and Wei [ref] presented a transformation for implementations which uses only read, write and CAS operation, which results a recoverable and detectable implementation. The transformation splits the code into capsules, each contains a single CAS operation followed by reads, and then replacing each CAS with its recoverable version. Each capsule can be recovered in case of a crash inside the capsule. In addition, the paper presents an optimisation for normalized algorithms, such that only two capsules are required for any operation. However, not all implementation are given in a normalized form, and the process of converting an implementation into a normalized form may be costly by itself.

Since Ben-David at el transformation is general, it may be improved in certain cases. In more details, each CAS is replaced with a recoverable CAS. This requires each CAS operation to use different arguments (they implement it using an increasing sequence number), which will be stored in the CAS location. This in turn implies that the CAS uses an unbounded word length, even if the original implementation uses a bounded CAS (for example, the operations are over a finite domain). Furthermore, although two capsules are used for each normalized operation, these two capsules are executed repeatedly, in each attempt of performing the operation (the implementation is lock-free, hence a process can fail to complete its operation over and over). In many cases we can avoid it, and not paying the extra complexity of the capsules for a failed attempt. In addition, we would like the transformation to change as little as necessary from the original code, even in the price of having a costly recover function. Assuming crash is a rare event, this may result a more efficient implementation in practice.

Linked-List

Harris' Linked-List presented in details in Section [cite]. The algorithm has the following property, which makes it persistent to crashes - each operation is linearized at its first successful CAS (not including CAS which help other operation). However, as discussed before, this does not imply it is also detectable. The main issue is a crash right after such a CAS. Upon recovery the crashed

process needs to know whether the last CAS was successful or not, in order to conclude the right response value. Moreover, two or more processes can try and perform the same operation at the same time, for example, deleting the same node. Therefore, upon recovery p not only needs to know whether the node was deleted, but also to determine if it was performed by it, or by some other process. In order to solve these issues we use the structure of the Linked-List implementation.

First, we notice that, as in many algorithms, data is added to the Linked-List using nodes, and each node is unique (different nodes may have the same key, but must have different references). Therefore, if a new node nd is added to the list using a CAS, then from this point on nd is accessible as part of the list, until it is deleted. This implies a recovery function for the Insert operation, by testing the former two conditions p can conclude whether its last attempt to add nd was successful. For the Delete operation, we notice that once a node is logically delete (the marked bit is turned on), it remains so forever. Hence, if process p tries to delete node nd and crash, upon recovery p can know if the node was logically deleted by looking at the marked bit. However, this is not enough information, as there might be many processes trying to delete nd at the same time, and exactly one needs to succeed. For that, each node is equipped with an extra field named deleter which is initialised to \perp . This field is used by processes trying to delete the same node nd in order to reach a consensus regarding which process is the one to delete nd . After nd is logically delete, the first process which writes to deleter its id (using an atomic CAS) is the one to declare its Delete operation as successful, while all others return false. Notice that crashed operation are also trying to win the consensus, and that deleter is written to exactly once. If a crash occurs after deleter was written to, then a process can recover by reading deleter and compare it to its own id.

Although the given implementation is specific for Linked-List, the same technique can be used for other implementations which share some similarities with the Linked-List implementation. For example, an operation which takes effect using a single CAS, and needs to guarantee exactly one process is the one to perform it, can use a new field (such as the deleter) in order to determine which process executed it. This requires the ability to conclude whether some process successfully performed the CAS upon recovery. In addition, the Insert operation of data-structures in which a new and unique object is used each time a new key is added (such as a node), can be recovered by looking for the new object, or an indication for it being removed from the data-structure. For example, Michel-Scott queue is such an implementation.

Binary Search Tree

Ellen et al [cite] presented a lock-free implementation for BST using a single-word CAS (see Section [ref...] for the full algorithm). The main idea is that each operation saves its relevant data in a unique info record. A process first declare its operation by flagging and writing its record (using a single atomic CAS) to all relevant nodes, and then the operation is carried out. This way, processes can help each other - a flagged node contains an info record for the operation, such that any process can complete it.

Algorithms in which helping mechanism is used, in many cases implies that many processes can try and perform the same operation concurrently, where only the invoking process will consider it as completed. Moreover, multiple attempts of the same process p may be indistinguishable from a run in which many processes tries to help p complete its operation, as the helping is anonymous. This already implies that in case of a crash a process may recover by retry its own operation according to it last info record. Once a process marks a node, it remains so until it is unmarked because the operation either complete or failed (by the process or any helping process). Therefore, upon

recovery p checks to see if the node is still marked with its own operation. If so, it will try to complete the operation, starting from right after the marking CAS. Otherwise, it will restart the operation.

The recover scheme describe above may still result a scenario in which p crash right after completing its operation and unmarking the node. Thus, upon recovery p will restart the operation, causing it to take effect twice. In order to avoid this scenario we update a field in the info record which signifies the operation is done. A key point is to update this field before the cleaning phase in which the unmarking is done. This way, if upon recovery the node is not marked with the operation, then the new field allows p to conclude the reason, that is, if the operation has been completed or failed. Notice that if process crash after updating the done field and before cleaning, upon recovery it still observes a marked node (unless some other process performed the cleaning), thus it will try to complete the operation. Therefore, a process may return only after the cleaning phase is done.

One advantage our approach has is that it relies on several key properties the implementation have, and not necessarily on the type of base objects. Mainly, the fact that each attempt to perform an operation uses a unique info record, so to avoid the ABA problem. Therefore, the CAS operation can be replaced with the LL/SC instruction with some minor modifications. In such case, the recoverable version is still correct, as the new implementation delivers the same guarantees as the original.

The simple but efficient approach we demonstrate for BST can be used for other implementation in which a process first declare its operation by marking a node and adding an info record. Moreover, in some cases it is very natural to transform the implementation to such a form, that is, to perform operations by writing info records that contains the entire data required for the operation (as demonstrate in Section [ref] using the elimination stack). This way, we can both be able to recover, as well as be able to design an helping mechanism. Note that we require each attempt to create a new info record, to avoid the ABA problem.

Elimination Stack

Finally, we present a recoverable implementation for elimination stack (full description can be found in Section [ref]). This is used in order to demonstrate the two techniques presented for Linked-List and BST can be used in other data-structures as well. The elimination stack implementation is composed of two components - the central stack, and an elimination array.

The central stack is implemented in a way which resembles Harris' Linked-List. In order to push or pop the process tries to atomically swing the head pointer using a CAS. Therefore, the same approach taken in the Linked-List can be used for this case also. In more details, the Pop operation will use a new *popby* field in a way similar to the deleter field, in order to determine which process is the one to pop the node.

However, unlike the Linked-List where a node is first marked before removing it from the list, in the stack a Pop operation is done using a single CAS without the need to mark. Hence, if a process p crash right after successfully adding a new node (using a Push operation), in case the node has been removed from the list, p can not distinguish the resulting configuration from a one in which the Push operation has failed. We address this issue by adding a simple helping mechanism, where a process marks a node as part of the list before trying to pop it. Notice that this mark does not imply the node has been removed, as the Pop operation may fail. For efficiency, we use the *popby* field for this marking. A process first writes NULL to it, and in case of a successful Pop try to

write its id. This is all done using CAS, so to avoid overwriting.

In the elimination array each entry is an exchanger object which allows exactly two processes to exchange values. First, we transform the implementation such that processes are now try to trade info records instead of values. Each process creates an info record containing its value. The exchange is done using a CAS, the first process to perform CAS writes its info record to the exchanger, and the second replace it with its own record. In order to allow recovery, the info record contains a state, identifying whether p is the first or second process to write the exchanger. Also, in case it is the second, the record also contains a reference to the info record p is trying to collide with. In case p crash, then if the info record in the exchanger implies it is part of an ongoing exchange, then it tries to complete it. Otherwise, either its exchange attempt failed, or that it succeeded and completed by some other process. In the former case, its info record contains an indication for it, as well as the response value.

The use of info records also allows the design of a simple and efficient helping mechanism. In case a collision is done by some process writing its info record second to the exchanger, then any other process can read this record and help to complete the exchange by updating both records with the right response value. Follow that, it can set the exchanger back to EMPTY, that is, to release it. This way, other processes do not need to wait for the two colliding processes to take steps, but can rather complete the exchange on behalf of them, and then use the exchanger again.

3 Linked-List

In this section, we present a recoverable version of the linked-list set algorithm of Harris [?].² Harris' algorithm maintains a linked list of nodes sorted in increasing order of keys. The *next* field of each node consists of two components: a reference to the next node and a *marked* bit that is set when the node is logically deleted. Both components can be manipulated atomically, either together or individually, using a single-word CAS operation.³ For presentation simplicity, we assume in the pseudo-code that reading the *next* field returns the reference only, while invoking the `get_data()` function on *next* returns both the reference and the mark bit. We also assume that the list always contain a *head* and *tail* sentinel nodes, containing keys $-\infty, \infty$ respectively.

We now provide a brief description of the Harris algorithm. The linked-list set supports the FIND, INSERT and DELETE set operations. INSERT and DELETE use the SEARCH helper procedure in order to find the node with the smallest key greater than or equal to the input key (this node is denoted *curr*), as well as its predecessor on the list (denoted *pred*). While traversing the list, SEARCH attempts to physically remove from the list any marked node it encounters. To insert a key α , a process p first calls SEARCH in order to find the position in the list where α should be added. If α is already in the list, INSERT returns false, otherwise it tries to set *pred.next* to point to a new node containing α using CAS (which would fail if *pred* has been logically deleted in the interim). To delete a key α , p searches for it using SEARCH and returns false if it not in the set. Otherwise, p tries to logically delete it by marking the *next* field of its node using CAS. If marking is successful, p tries to physically remove the node. To find a key α , a process simply looks for a node in the list with key α which is unmarked.

The linearization point for the Harris algorithm are as follows:

²Some implementation details follow the algorithm's presentation by [?].

³In the Java implementation of the algorithm in [?], *next* fields are represented by AtomicMarkableReference objects, in whose implementation the marked bit uses the least significant bit of the reference.

INSERT: Either when Lookup finds α in the list, or upon a successful CAS inserting α to the list.

DELETE: Upon a successful CAS that marks the node (logical deletion).

FIND: Upon the last read of either *curr.key* or *curr.next*.

Following the given linearization points (omitting proof...), INSERT and DELETE operation are linearized at the point where they affect the system. That is, if an INSERT operation performed a successful CAS, then all processes will see the new node starting from this point, and if a node was logically delete, then all processes treat it as if it was removed. Therefore, once a process p recovers following a crash, the list data structure is consistent - if p has a pending operation, either the operation already had a linearization point which affected all other processes, or it did not affect the data structure at all, nor will in any future run.

However, even though the list data structure is consistent, the response of the pending operation is lost. Consider for example a scenario in which process p performs DELETE(α) and crash right after applying a successful CAS to mark a node. Upon recovery, p may be able to conclude α was removed, as the node is marked. Nevertheless, even if no other process takes steps, p is not able to determine whether it is the process to successfully delete α , or that it was done by some other process, and therefore it does not able to determine the right response. Moreover, in case the node was physically removed, p is not able to determine whether α has been deleted at all, as it is no longer part of the list.

3.0.1 Linked-List Recoverable Version

To solve the problems mention above, we present a modification for the algorithm such that in case of a process crash, upon recovery it is able to complete its last pending operation if needed, and also return the response value in such case. The algorithm presented in figure 1. Blue lines represents changes comparing to the original algorithm.

Each node is equipped with a new field named deleter. This field is used to determine which process is the one to delete the node. After the node was successfully marked (logical delete), process p tries to announce itself as the one to delete the node by writing its id to deleter using CAS. This way, if a process crash during a delete, it can use deleter in order to determine the response value. We assume deleter is initialised to null when creating a new node.

Each process p has a designated location in the memory, Backup[p]. Before trying to apply an operation, p writes to Backup[p] the entire data needed to complete the operation. Upon recovery, p can read Backup[p], and based on it to complete its pending operation, in case there is such. Formally, Backup[p] contains a pointer to a structure containing all the relevant data. For simplicity, process p creates new such structure for each of its operations, although a more efficient way will be to use two such structures in an alternating way.

Algorithm 5: boolean RECOVER ()

```
53 Data: Node *nd := Backup[pid].nd
54 if Backup[pid].result  $\neq \perp$  then                                     // operation was completed
55 |   return Backup[pid].result
56 if Backup[pid].optype = INSERT then
57 |    $\langle pred, curr \rangle := \text{SEARCH}(nd.key)$                                // search for nd in the list
58 |   if curr = nd || nd.next is marked then                         // nd is in the list or marked
59 |   |   Backup[pid] := true
60 |   |   return true
61 |   return FAIL
62 if Backup[pid].optype = DELETE then
63 |   if nd  $\neq \perp$  && nd.next is marked then                               // nd was logically deleted
64 |   |   nd.deleter.CAS( $\perp$ , pid)                                           // try to complete the deletion
65 |   |   if nd.deleter = pid then                                         // you are the deleter
66 |   |   |   Backup[pid].result := true
67 |   |   return true
68 |   return FAIL
```

Algorithm 1: $\langle \text{Node}, \text{Node} \rangle$ SEARCH (T key)

```
1  Data: Node *pred, *curr, *succ
   boolean mbit
2  retry: while true do
3  |   pred := head
4  |   curr := pred.next
5  |   while true do
6  |   |    $\langle succ, mbit \rangle := curr.next.get\_data()$ 
7  |   |   if mbit then                                                     // succ was logically deleted
8  |   |   |   if pred.next.CAS(unmarked curr, unmarked succ) = false then // help physical delete
9  |   |   |   |   go to retry                                             // help failed
10 |   |   |   curr := succ                                               // help succeed
11 |   |   else
12 |   |   |   if curr.key  $\geq$  key then                                     // curr is the first unmarked node with key  $\geq$  key
13 |   |   |   |   return  $\langle pred, curr \rangle$ 
14 |   |   |   pred := curr                                               // advance pred and curr
15 |   |   |   curr := succ
16 |   end
17 end
```

Shared variables: Node **head*

```
Type Info {
  {INSERT, DELETE} optype
  Node *nd
  boolean result
}
```

Code for process p:

Algorithm 2: boolean INSERT (T *key*)

```
18  Data:   Node *pred, *curr
19  Node newnd := new Node (key)
20  Backup[pid] := new Info (INSERT, newnd,  $\perp$ )
21  while true do
22     $\langle pred, curr \rangle$  := SEARCH(key)           // search for the right location for insertion
23    if curr.key = key then                     // key is already in the list
24      Backup[pid].result := false
25      return false
26    else
27      newnd.next := unmarked curr
28      if pred.next.CAS (unmarked curr, unmarked newnd) then           // try to add newnd
29        Backup[pid].result := true
30        return true
31  end
```

Algorithm 3: boolean DELETE (T *key*)

```
31  Data:   Node *pred, *curr, *succ
32  Backup[pid] := new Info (DELETE,  $\perp$ ,  $\perp$ )
33   $\langle pred, curr \rangle$  := SEARCH(key)           // search for key in the list
34  if curr.key  $\neq$  key then                     // key is not in the list
35    Backup[pid].result := false
36    return false
37  else
38    Backup[pid].nd := curr
39    while curr.next is unmarked do               // repeatedly attempt logical delete
40      succ := curr.next
41      curr.next.CAS (unmarked succ, marked succ)
42    end
43    succ := curr.next
44    pred.next.CAS (unmarked curr, unmarked succ)           // physical delete attempt
45    res := curr.deleter.CAS( $\perp$ , pid)           // try to announce yourself as deleter
46    Backup[pid].result := res
47    return res
```

Algorithm 4: boolean FIND (T *key*)

```
48  Data:   Node *curr := head
49  while curr.key < key do           // search for the first node with key greater or equal to key
50    curr = curr.next
51  end
52  return (curr.key = key && curr.next is unmarked)
```

Figure 1: Recoverable Non-Blocking Linked-List

Correctness Argument

In the following, we give an high-level proof for the correctness of the algorithm.

First, notice that quitting the Lookup procedure at any point, or repeating it, can not violate the list consistency. The Lookup procedure simply traverse the list, while trying to physically delete marked nodes. Once `curr.next` is marked, a single process can perform the physical delete. This follows from the fact that at any point there is a single node in the list which points to `curr`. Once `curr` is physically delete, no node in the list points to `curr`, and thus any CAS operation with `curr` as the first parameter will fail. This observation relies on the fact that any new allocated node has a different address than `curr`. As a result, repeating the attempt to physically delete a node does not affect the list.

Assume a process p performs an *insert(key)* operation. First, p writes to `Backup[p]`, updating it is about to perform an Insert. If a process p does not crash, then, as in the original algorithm, it repeatedly tries to find the right location for the new node, and insert it by performing a CAS changing `pred.next` to point to `newnd`. In addition, it is clear from the code that a crash after updating `Backup[p].result` is after the operation had its linearization point, and the Recover procedure will return the right response. Therefore, we need to consider a crash before an update to `Backup[p].result`. There are two scenarios to consider.

Assume p crash without performing a successful CAS in line 27. p is the only process to have a reference to `newnd`, and it is yet to update any node with this reference, and thus no node points to `newnd`. As a result, the operation did not affects any other process, nor it will be in the future. Hence, considering the operation as not having a linearization point does not violate the list consistency. Indeed, since no node points to `newnd`, upon recovery p will see that `newnd` is not in the list and also not marked, and thus will return FAIL. Notice this argument holds whether key is already in the tree, or not, as the operation in both cases did not affect the system.

Assume now p crash after performing a successful CAS in line 27. In such case, `newnd` is part of the list, as `pred.next` points to it. Also notice we did not delete any other node, since `pred.next` pointed to `curr`, and after the CAS it points to `newnd` which points to `curr`. As a result, when p executes the Recover procedure, either it will see `newnd` in the list, or that it is no longer part of the list, and it must be some other process deleted it, and hence `newnd.next` is marked. In any case, p will return true as required. The above argument relies on the fact a marked node can not be unmarked, and that an Insert and Delete can not mistakenly remove nodes from the list. We have claimed it for Insert, and we will prove the same holds for Delete. Therefore, if a node is no longer in the list, it must be marked.

Assume a process p performs a *delete(key)* operation. First, p writes to `Backup[p]`, updating it is about to perform a Delete. As before, a crash after writing to `Backup[p].result` will return the right response. Also, a crash before updating any of `Backup[p].result` or `Backup[p].nd` implies p is yet to try and mark any node, and thus the operation did not affect the system so far, nor it will be in the future. Therefore, we can consider the operation as not having a linearization point (even in case key is not the list), and indeed, the Recover procedure returns FAIL in such case.

Assume thus p writes to `Backup[p].nd`. It follows that p completed the lookup procedure and finds a node `curr` storing key. The lookup procedure guarantees there is a point in time (of the procedure execution) where `curr` is in the list and `curr.next` is not marked. If p crash and recovers, and observe that `curr` is unmarked, then it returns FAIL. Since a marked node can not be unmarked, as there is no CAS changing a marked node, it follows that p did not mark `curr`. Therefore, the operation did not affects any process, nor it will be, and we consider it as having no linearization

point. Otherwise, the Recover function observe curr as marked, and we can conclude the marking point of curr is along the delete operation. We now prove we can linearize the operation, according to its response.

Let q be the process to mark curr. Since once curr.next is marked it will never be changed, the reference of curr.next is fixed to succ (of q at the point of the marking). This also implies q is unique and well defined, and any future CAS on curr.next will fail. As a result, any process leaving the while loop in line 39 reads the same value in line 43, which is this succ. The attempt to physically delete curr in line 44 will succeed only if pred.next points to curr, and as we said, curr point to succ, and any other attempt will fail. Thus, if this attempt succeed, it deletes only curr, and can not delete additional nodes.

In line 45 process p tries to writes its id to curr.deleter. As it is initialised to null, only the first process to perform this CAS will succeed. Also, any p must go through line 45 in order to complete its operation, as the Recover procedure redirect the process to this line. Therefore, if there is a process to complete its delete operation while observing curr.next is marked, there must be a CAS to curr.deleter. Let q' be the first process to perform this CAS. As proved above, q' tries to delete curr, and the point in time where curr is marked must be contained in its operation interval. Moreover, q' is the only process to write to curr.deleter, and the first one to do so, thus q' is the only process to obtain true when testing (curr.deleter = q') in line 46 (and thus to also return true), while any other process will obtain false. We linearize the operation of q' at the point of the marking, and any other attempt to delete curr is linearized after it (in an arbitrary order).

A corollary of the analysis is that processes trying to delete the same node curr "helps" each other, in the sense that they all keep trying to mark curr. However, the marking process is not necessarily the one to return true. Also, in the original algorithm, if a process fails to mark a node, it starts the delete operation from the beginning. In our implementation, process can keep trying to mark the node without the need to perform a lookup again after each failed CAS. We guarantee that once curr is marked, exactly one process will return true, while the rest can consider curr as being deleted (in the course of their delete execution), and thus there is a point along their execution is which key is not in the tree, and they can return false.

4 Robust BST

The original BST algorithm does not support the crash-recovery model. It is clear from the code a process does not persist the operation's response in the non-volatile memory, and thus, once a process crash the response is lost. For example, assume a process q apply $\text{INSERT}(k)$, performs a successful CAS in line 101 and fails after completing the HELPINSERT routine. In this case, the INSERT operation took effect, that is, the new key appears as a leaf in the tree, and any $\text{FIND}(k)$ operation will return it. However, even though the operation must be linearized before the crash, upon recovery process q is unaware of it. Moreover, looking for the new leaf in the tree may be futile, as it might be k has been removed from the tree after the crash.

Furthermore, if no recover routine is supplied, it may result an execution which is not well-formed. Consider for example the following scenario. A process q invoke an $Op_1 = \text{INSERT}(k_1)$ operation. q performs a successful CAS in line 101 followed by a crash. After recovering, q invoke an $Op_2 = \text{INSERT}(k_2)$ operation. Assume k_1 and k_2 belongs to a different parts of the tree (do not share parent or grandparent). Then, q can complete the insertion of k_2 without having any affect on k_1 . Now, a process q' performs $\text{FIND}(k_1)$ which returns NULL, as the insertion of k_1 is not completed, followed by $\text{FIND}(k_2)$, which returns the leaf of k_2 . The $\text{INSERT}(k_1)$ operation will be completed later by any INSERT or DELETE operation which needs to make changes to the flagged node. We get that Op_2 must be before Op_1 in the linearization, although Op_1 invoked first.

The kind of anomaly described above can be addressed by having the first CAS of a successful attempt for INSERT or DELETE as the linearization point, as in the Linked-List. For that, the FIND routine should take into consideration future unavoidable changes, for example, a node flagged with IFlag ensures an insertion of some key. A simple solution is to change the FIND routine such that it also helps other operations, as described in figure 2. The FIND routine will search for key k in the tree. If the SEARCH routine returns a grandparent or a parent that is flagged, then it might be that an insert or delete of k is currently in progress, thus we first help the operation to complete, and then search for k again. Otherwise, if $gpupdate$ or $pupdate$ has been changed since the last read, it means some change already took affect, and there is a need to search for k again. If none of the above holds, there is a point in time where gp points to p which points to l , and there is no attempt to change this part of the tree. As a result, if k is in the tree at this point, it must be in l , and the find can return safely.

The approach described above is not efficient in terms of time. We would like a solution which maintain the desirable behaviour of the original FIND routine, where a single SEARCH is needed. A more refined solution is given in figure 3. The intuition for it is drawn from the Linked-List algorithm. In the Linked-List algorithm it was enough to consider a marked node as if it has been deleted, without the need to complete the deletion. Nonetheless, the complex BST implementation is more challenging, as the DELETE routine needs to successfully capture two nodes using CAS in order to complete the deletion. Therefore, if a process p executes $\text{FIND}(k)$ procedure, and observes a node flagged with DFlag attempting to delete the key k , it can not know whether in the future this delete attempt will succeed or fail, and thus does not know whether to consider the key k as part of the tree or not. To overcome this problem, in such case the process will first try and validate the delete operation by marking the relevant node. According to whether the marking attempt was successful, the process can conclude if the delete operation is successful or not. In order to easily implement the modified FIND routine there is a need to conclude from IInfo what is the new leaf (leaf *new* in the INSERT routine). For simplicity of presentation, we do not add this field, and abstractly refer to it in the code.

The correctness of the two suggested solutions relies on the following argument. Once a process flags a node during operation Op with input key k (either INSERT or DELETE), then if this attempt to complete the operation eventually succeed (i.e., the marking is also successful in the case of DELETE), then any FIND(k) operation invoked from this point consider Op as if it is completed.

The suggested modification, although being simple and local, only guarantee the implementation satisfy R-linearizability. However, the problem of response being lost in case of a crash is not addressed. Roughly speaking, the critical points in the code for recovery are the CAS primitives, as a crash right after applying CAS operation results the lost of the response, and in order to complete the operation the process needs to know the result of the CAS. In addition, because of the helping mechanism, a suspended DELETE operation which flagged a node and yet to mark one, may be completed by other process in the future, and may not. Upon recovery, the process needs to distinguish between the two cases, in order to obtain the right response.

To address this issue, we expend the helping mechanism so that it also update the info structure in case of a success. This is done by adding a boolean field, *done*, to the Flag structure. This way, if a process crash along an operation Op , upon recovery it can check to see if the operation was already completed. A crucial point is to update the *done* bit before performing the unflagging. Therefore, if a node is no longer flagged we can be sure *done* was already updated. If we switch the order, then it might be an operation and unflagging were completed, but the *done* bit is yet to be updated. Therefore, other processes can change the BST structure. However, if the process crash and recover at this point, the *done* bit is off, and the BST structure has been changed, so it will be harder for the process to conclude whether the operation took affect.

Before a process q attempt to perform an operation, as it creates the Flag structure op describing the operation and its affect on the data structure, the process stores op in a designated location in the shared memory (for simplicity, we use an array). As a result, upon recovery q has an access to this information. Now, q can check to see if the operation is still in progress, i.e., if the relevant node (parent or grandparent) is still flagged. If so, it first tries to complete the operation. Otherwise, it implies either the operation was completed, and therefore *done* bit is updated, or that the attempt was unsuccessful and there was no write to the *done* bit. Hence, the *done* bit can distinguish between the two scenarios. Notice that there is a scenario in which process q recovers and observes an operation Op as it is in a progress, but just before it retries it, some other process complete the operation. We need to prove that even in such case, the operation will affect the data structure exactly once, and the right response is returned.

The given implementation does not recover the FIND routine, since this routine does not make any changes to the BST, hence it is always safe to consider it as having no linearization point and reissue it. Also, for ease of presentation, we only write to $Announce[id]$ once we are about to capture a node using a CAS. However, writing to $Announce[id]$ at the beginning of the routine may be helpful in case of a crash early in the routine, so that the process will be able to use the data stored in $Announce[id]$ in such case also. The same is true with response value, $Announce[id] \rightarrow done$ is updated only if the routine made changes to the BST.

4.0.1 Correctness

In the following section we give a proof sketch for the algorithm correctness. We assume for simplicity nodes and Flag records are always allocated new memory locations, although it is enough to require no location is reallocated as long as there is a chain of pointers leading to it. The proof relies on the correctness of the original algorithm, which can be found on [...].

The proof relies on several key arguments given below.

[**Arg1**] The original algorithm is anonymous and uniform, i.e., any number of processes can use the BST, and there is no need to know the number of processes in the system in order to use the BST. Notice that all helping routines in the given implementation are completely anonymous, and an execution of such a routine by either the process which invoked op or any other helping process executes the exact same code. This observation allows the use of the following argument. If a process crash while executing some helping routine, we can consider it as an helping process which stop taking steps (more formally, there is an equivalent execution in which there is such a process, and it is indistinguishable to all process in the system). Since such process can not cause a wrong behaviour of the algorithm, so does the crash. A corollary of this argument is that repeating an helping routine multiple times by the same process can not violate the BST specification, as there is an equivalent executions in which multiple processes executes the different helping routines.

[**Arg2**] It is easy to verify the post-conditions of the SEARCH routine still holds, as they follow directly from the routine's code, and does not rely on the structure or correctness of the BST. Also, the SEARCH routine does not make any changes to the BST, but rather simply traverse it. Therefore FIND routine, which only uses SEARCH, does not affect any process, and in case of a failure along FIND execution, reissuing it satisfies NRL.

[**Arg3**] If an internal node nd_1 stops pointing to a node nd_2 at some point of the execution, it can not point to nd_2 again. This attributes to the fact an INSERT presents a node with two new children. Therefore, if nd_2 is a leaf, it can either be delete, or replaced by a new copy of an INSERT operation. Otherwise, nd_2 is an internal node, and as such, the pointer to it by nd_1 can not be replaced by an INSERT operation (which only allows to replacement a leaf), and therefore it can only be removed from the tree.

[**Arg4**] The field update of a node nd can have any value only once along an execution. Any attempt to perform an operation creates a new record in the memory. If $nd \rightarrow update$ is marked, it can not be unmarked or changed. Otherwise, any attempt to flag it uses a new created record op . If the attempt succeed, then eventually it will be unflagged while still referring to op . In order to replace the value again, there must be an operation reading $nd \rightarrow update$ after it was unflagged (as any operation first help a flagged node). This operation must create a new record, and thus we can use the same argument again. As a corollary, if a process successfully flag or mark a node, there was no change to the node since the last time it read the update field of the node.

Proof Sketch Assume a process q performs an operation Op (either INSERT or DELETE). If q does not crash, the algorithm is identical to the original algorithm, except for the additional write to $Announce[q]$ and $op \rightarrow done$, and thus the correctness of the original algorithm can be applied. Otherwise, q crash at some point, and upon recovery it reads op from $Announce[q]$. This record represent the last attempt of q to complete Op . We split the proof based on the type of operation.

$Op = \text{INSERT}$. Consider the read of $op \rightarrow p \rightarrow update$ upon recovery, and denote this value by $pupdate$. If $pupdate = \langle \text{IFlag}, op \rangle$, this implies the iflag CAS in line 101 was successful and the operation is yet to complete. It might be that INSERT already took affect, that is, the new key is part of the tree, but the unflagging is yet to happen. In such case, q calls $\text{HELPINSERT}(op)$ in order to try and complete the operation. Considering arg1, this call can not violate the BST correctness, even if it not the first time q executes it. Moreover, during HELPINSERT there is a write to $op \rightarrow done$, and thus after completing the routine q returns TRUE, as required.

Else $pupdate \neq \langle \text{IFlag}, op \rangle$. There are two scenarios to consider. Either the iflag CAS of q in

line 101 was successful or not. If it was successful, then $p \rightarrow update = \langle IFlag, op \rangle$ at this point. The only way to change it is to first unflag p . To do so, a process needs to complete an $HELPINSERT(op)$ routine, and in particular must write to $op \rightarrow done$. In such case, the INSERT operation was completed, and q returns TRUE. Otherwise, the CAS was not successful, either because it failed, or the crash was before the CAS. In both cases, the INSERT operation will not be completed, as op is not stored in $p \rightarrow update$, and thus no process has an access to it. Consequently, no process can update $op \rightarrow done$, and q returns FAIL.

$Op = DELETE$. Consider the read of $op \rightarrow gp \rightarrow update$ upon recovery, and denote this value by $gpupdate$. If $gpupdate = \langle DFlag, op \rangle$, this implies the dflag CAS of q in line 128 was successful, and the operation is yet to complete. As in the INSERT, it might be the operation already changed the tree. After reading $gpupdate$ q invokes $HELPDELETE(op)$ routine. Again, following $arg1$, executing this multiple times by q can not violate the BST correctness. The first process to try and mark $op \rightarrow p \rightarrow update$ during an $HELPDELETE(op)$ routine is the one to determine the outcome of it. If it is successful, then p is marked, and the $update$ field can not be changed. That is, any $HELPDELETE(op)$ execution will obtain true in line 139, and will call $HELPMARKED(op)$ routine. Otherwise, the CAS fails, and so $p \rightarrow update$ is no longer equal to $op \rightarrow pupdate$. By $arg4$ it will never get this value again, and thus any marking CAS during a $HELPDELETE(op)$ execution will fail, and there is no call to $HELPMARKED(op)$. In the first case, any $HELPDELETE(op)$ routine must first complete a $HELPMARKED(op)$, and thus must write to $op \rightarrow done$, while in the later case, there is no write to $op \rightarrow done$, as no $HELPMARKED(op)$ is ever invoked. Therefore, in both cases, when q completes $HELPMARKED(op)$ it reads $op \rightarrow done$ and returns the right response.

Otherwise $gpupdate \neq \langle DFlag, op \rangle$, and there are two scenarios to consider. If the dflag CAS of q in line 128 never took affect, because it either failed, or the crash preceded it, then op is never written to $gp \rightarrow update$, or to any update field. Thus, no process is aware of it, and $op \rightarrow done$ remains FALSE, resulting q returning FAIL as required. Else, the CAS was successful, and $gp \rightarrow update$ was flagged. The only way to change it is to first unflag it, and this in turn can be done only during an $HELPDELETE(op)$ routine. In this case, it can be unflagged in either the $HELPMARKED$ routine in line 154, or in line 145 of the $HELPDELETE$ routine. As mention before, the first CAS in line 138 of an $HELPDELETE(op)$ execution determines the outcome for all $HELPDELETE(op)$. If it is successful, $p \rightarrow update$ is forever marked, and all $HELPDELETE(op)$ must invoke $HELPMARKED(op)$. Therefore, the only option to unflag $gp \rightarrow update$ is at the end of $HELPMARKED(op)$ routine, and this done only after setting $op \rightarrow done$. In such case, the DELETE operation took affect, and q will return TRUE. On the other hand, if the CAS was not successful, then any $HELPDELETE(op)$ will fail to mark $p \rightarrow update$, and hence no $HELPMARKED(op)$ is ever invoked. As a result, there is no write to $op \rightarrow done$. In such case, the DELETE operation did not took affect, nor will be, and indeed q will return FAIL.

```

FIND(Key k) : Leaf* {
1      Internal *gp, *p
2      Leaf *l
3      Update pupdate, gpupdate

4      while (TRUE) {
5           $\langle gp, p, l, pupdate, gpupdate \rangle := \text{SEARCH}(k)$ 
6          if gpupdate.state  $\neq$  CLEAN then HELP(gpupdate)
7          else if pupdate.state  $\neq$  CLEAN then HELP(pupdate)
8          else if gp  $\rightarrow$  update = gpupdate and p  $\rightarrow$  update = pupdate then {
9              if l  $\rightarrow$  key = k then return l
10             else return NULL
11         }
12     }
13 }

```

Figure 2: Solution 1: R-linearizable FIND routine

```

FIND(Key k) : Leaf* {
14     Internal *gp, *p
15     Leaf *l
16     Update pupdate, gpupdate

17      $\langle gp, p, l, pupdate, gpupdate \rangle := \text{SEARCH}(k)$ 
18     if l  $\rightarrow$  key  $\neq$  k then {
19         if (pupdate.state = IFlag and pupdate.info attempt to add key k) then
20             return leaf with key k from pupdate.info
21         else return NULL
22     }
23     if (pupdate.state = MARK and pupdate.info  $\rightarrow$  l  $\rightarrow$  key = k) then return NULL
24     if (gpupdate.state = DFlag and gpupdate.info  $\rightarrow$  l  $\rightarrow$  key = k) then {
25         op := gpupdate.info
26         result := CAS(op  $\rightarrow$  p  $\rightarrow$  update, op  $\rightarrow$  pupdate,  $\langle \text{MARK}, op \rangle$ )  $\triangleright$  mark CAS
27         if (result = op  $\rightarrow$  pupdate or result =  $\langle \text{MARK}, op \rangle$ ) then return NULL  $\triangleright$  op  $\rightarrow$  p is successfully marked
28     }
29     return l
30 }

```

Figure 3: Solution 2: R-linearizable FIND routine

```

31 type Update {                                ▷ stored in one CAS word
32     {CLEAN, DFlag, IFlag, MARK} state
33     Flag *info
34 }
35 type Internal {                               ▷ subtype of Node
36      $Key \cup \{\infty_1, \infty_2\}$  key
37     Update update
38     Node *left, *right
39 }
40 type Leaf {                                   ▷ subtype of Node
41      $Key \cup \{\infty_1, \infty_2\}$  key
42 }
43 type IInfo {                                  ▷ subtype of Flag
44     Internal *p, *newInternal
45     Leaf *l
46     boolean done
47 }
48 type DInfo {                                  ▷ subtype of Flag
49     Internal *gp, *p
50     Leaf *l
51     Update pupdate
52     boolean done
53 }
    ▷ Initialization:
54 shared Internal *Root := pointer to new Internal node
    with key field  $\infty_2$ , update field  $\langle \text{CLEAN}, \text{NULL} \rangle$ , and
    pointers to new Leaf nodes with keys  $\infty_1$  and
     $\infty_2$ , respectively, as left and right fields.

```

Figure 4: Type definitions and initialization.

```

RECOVER() {
55     Flag *op = Announce[id]
56     if op of type IInfo then {
57         result := op → p → update
58         if result =  $\langle \text{IFlag}, op \rangle$  then HELPINSERT(op)
59     }
60     if op of type DInfo then {
61         result := op → gp → update
62         if result =  $\langle \text{DFlag}, op \rangle$  then HELPDELETE(op)
63     }
64     if op → done = TRUE then return TRUE
65     else return FAIL
66 }

```

▷ Finish the insertion

▷ Either finish deletion or unflag

Figure 5: RECOVER routine

```

67 SEARCH(Key k) : (Internal*, Internal*, Leaf*, Update, Update) {
    ▷ Used by INSERT, DELETE and FIND to traverse a branch of the BST; satisfies following postconditions:
    ▷ (1)  $l$  points to a Leaf node and  $p$  points to an Internal node
    ▷ (2) Either  $p \rightarrow left$  has contained  $l$  (if  $k < p \rightarrow key$ ) or  $p \rightarrow right$  has contained  $l$  (if  $k \geq p \rightarrow key$ )
    ▷ (3)  $p \rightarrow update$  has contained  $pupdate$ 
    ▷ (4) if  $l \rightarrow key \neq \infty_1$ , then the following three statements hold:
    ▷ (4a)  $gp$  points to an Internal node
    ▷ (4b) either  $gp \rightarrow left$  has contained  $p$  (if  $k < gp \rightarrow key$ ) or  $gp \rightarrow right$  has contained  $p$  (if  $k \geq gp \rightarrow key$ )
    ▷ (4c)  $gp \rightarrow update$  has contained  $gpupdate$ 
68     Internal *gp, *p
69     Node *l := Root
70     Update gpupdate, pupdate
    ▷ Each stores a copy of an update field
71     while  $l$  points to an internal node {
72         gp := p
73         p := l
74         gpupdate := pupdate
75         pupdate := p → update
76         if  $k < l \rightarrow key$  then  $l := p \rightarrow left$  else  $l := p \rightarrow right$ 
77     }
78     return (gp, p, l, pupdate, gpupdate)
79 }

80 FIND(Key k) : Leaf* {
81     Leaf *l
82     (−, −, l, −, −) := SEARCH(k)
83     if  $l \rightarrow key = k$  then return  $l$ 
84     else return NULL
85 }

86 INSERT(Key k) : boolean {
87     Internal *p, *newInternal
88     Leaf *l, *newSibling
89     Leaf *new := pointer to a new Leaf node whose key field is  $k$ 
90     Update pupdate, result
91     IInfo *op
92     while TRUE {
93         (−, p, l, pupdate, −) := SEARCH(k)
94         if  $l \rightarrow key = k$  then return FALSE
95         if pupdate.state ≠ CLEAN then HELP(pupdate)
96         else {
97             newSibling := pointer to a new Leaf whose key is  $l \rightarrow key$ 
98             newInternal := pointer to a new Internal node with key field  $\max(k, l \rightarrow key)$ ,
99                 update field (CLEAN, NULL), and with two child fields equal to new and newSibling
100                 (the one with the smaller key is the left child)
101             op := pointer to a new IInfo record containing (p, l, newInternal, FALSE)
102             Announce[id] := op
103             result := CAS(p → update, pupdate, (IFlag, op))
104             if result = pupdate then {
105                 HELPINSERT(op)
106                 return TRUE
107             }
108             else HELP(result)
109         }
110     }
    ▷ Cannot insert duplicate key
    ▷ Help the other operation
    ▷ The iflag CAS failed; help the operation that caused failure

110 HELPINSERT(IInfo *op) {
    ▷ Precondition: op points to an IInfo record (i.e., it is not NULL)
111     CAS-CHILD(op → p, op → l, op → newInternal)
112     op → done := TRUE
113     CAS(op → p → update, (IFlag, op), (CLEAN, op))
114 }
    ▷ ichild CAS
    ▷ announce the operation completed
    ▷ iunflag CAS

```

Figure 6: Pseudocode for SEARCH, FIND and INSERT.

```

115 DELETE(Key  $k$ ) : boolean {
116   Internal * $gp$ , * $p$ 
117   Leaf * $l$ 
118   Update  $pupdate$ ,  $gpupdate$ ,  $result$ 
119   DInfo * $op$ 

120   while TRUE {
121      $\langle gp, p, l, pupdate, gpupdate \rangle := \text{SEARCH}(k)$ 
122     if  $l \rightarrow key \neq k$  then return FALSE ▷ Key  $k$  is not in the tree
123     if  $gpupdate.state \neq \text{CLEAN}$  then HELP( $gpupdate$ )
124     else if  $pupdate.state \neq \text{CLEAN}$  then HELP( $pupdate$ )
125     else { ▷ Try to flag  $gp$ 
126        $op :=$  pointer to a new DInfo record containing  $\langle gp, p, l, pupdate, \text{FALSE} \rangle$ 
127        $\text{Announce}[id] := op$ 
128        $result := \text{CAS}(gp \rightarrow update, gpupdate, \langle \text{DFlag}, op \rangle)$  ▷ dflag CAS
129       if  $result = gpupdate$  then { ▷ CAS successful
130         if HELPDELETE( $op$ ) then return TRUE ▷ Either finish deletion or unflag
131       }
132       else HELP( $result$ ) ▷ The dflag CAS failed; help the operation that caused the failure
133     }
134   }
135 }

136 HELPDELETE(DInfo * $op$ ) : boolean {
137   ▷ Precondition:  $op$  points to a DInfo record (i.e., it is not NULL) ▷ Stores result of mark CAS
138   Update  $result$ 
139    $result := \text{CAS}(op \rightarrow p \rightarrow update, op \rightarrow pupdate, \langle \text{MARK}, op \rangle)$  ▷ mark CAS
140   if  $result = op \rightarrow pupdate$  or  $result = \langle \text{MARK}, op \rangle$  then { ▷  $op \rightarrow p$  is successfully marked
141     HELPMARKED( $op$ ) ▷ Complete the deletion
142     return TRUE ▷ Tell DELETE routine it is done
143   }
144   else { ▷ The mark CAS failed
145     HELP( $result$ ) ▷ Help operation that caused failure
146      $\text{CAS}(op \rightarrow gp \rightarrow update, \langle \text{DFlag}, op \rangle, \langle \text{CLEAN}, op \rangle)$  ▷ backtrack CAS
147     return FALSE ▷ Tell DELETE routine to try again
148   }

149 HELPMARKED(DInfo * $op$ ) {
150   ▷ Precondition:  $op$  points to a DInfo record (i.e., it is not NULL)
151   Node * $other$ 
152   ▷ Set  $other$  to point to the sibling of the node to which  $op \rightarrow l$  points
153   if  $op \rightarrow p \rightarrow right = op \rightarrow l$  then  $other := op \rightarrow p \rightarrow left$  else  $other := op \rightarrow p \rightarrow right$ 
154   ▷ Splice the node to which  $op \rightarrow p$  points out of the tree, replacing it by  $other$ 
155   CAS-CHILD( $op \rightarrow gp, op \rightarrow p, other$ ) ▷ dchild CAS
156    $op \rightarrow done := \text{TRUE}$  ▷ announce the operation completed
157    $\text{CAS}(op \rightarrow gp \rightarrow update, \langle \text{DFlag}, op \rangle, \langle \text{CLEAN}, op \rangle)$  ▷ dunflag CAS
158 }

159 HELP(Update  $u$ ) { ▷ General-purpose helping routine
160   ▷ Precondition:  $u$  has been stored in the  $update$  field of some internal node
161   if  $u.state = \text{IFlag}$  then HELPINSERT( $u.info$ )
162   else if  $u.state = \text{MARK}$  then HELPMARKED( $u.info$ )
163   else if  $u.state = \text{DFlag}$  then HELPDELETE( $u.info$ )
164 }

165 CAS-CHILD(Internal * $parent$ , Node * $old$ , Node * $new$ ) {
166   ▷ Precondition:  $parent$  points to an Internal node and  $new$  points to a Node (i.e., neither is NULL)
167   ▷ This routine tries to change one of the child fields of the node that  $parent$  points to from  $old$  to  $new$ .
168   if  $new \rightarrow key < parent \rightarrow key$  then
169      $\text{CAS}(parent \rightarrow left, old, new)$ 
170   else
171      $\text{CAS}(parent \rightarrow right, old, new)$ 
172 }

```

Figure 7: Pseudocode for DELETE and some auxiliary routines.

5 Elimination Stack

For simplicity, we assume a value \perp , which is different from NULL and any other value the stack can store. Since NULL is used as a legit return value, representing the value of POP operation (when exchanging values using the elimination array), NULL can not be used to represent an initialization value, different then any stack value. The same holds for a Node, since a NULL node represent an empty stack, the value \perp is used to distinguish between initialization value and empty stack.

For simplicity, we split the RECOVER routine into sub-routines, based on which operation (PUSH, POP, EXCHANGE) is pending, or needs to be recover. This can be concluded easily by the type of record stored in *Announce[pid]* (ExInfo or OpInfo), thus there is no need to explicitly know where exactly in the code the crash took place. Also, the RECOVER routine returns FAIL in case the last pending operation did not took affect (no linearization point), nor it will take in any future run. In such case, the user has the option to either re-invoke the operation, or to skip it, depends on the needs and circumstances of the specific use of the data structure.

The given implementation ignores the log of failures and successes of the exchange routine when recovering. That is, in case of a crash during an EXCHANGE, a process is able to recover the EXCHANGE routine, however, the log of successes and failures is not update, since it might be the process already updated it. In addition, in case of a FAIL response, we do not know whether the time limit (timeout) was reached, or that the process simply crashed earlier in the routine without completing it. The given implementation can be expanded to also consider the log. Nonetheless, for ease of presentation we do not handle the log in case of a crash. Assuming crash events are rare, the log still gives a roughly good approximation to the number of failures and successes, thus our approach might be useful in practice.

5.1 A Lock-Free Exchanger

An exchanger object supports the EXCHANGE procedure, which allows exactly two processes to exchange values. If process A calls the EXCHANGE with argument a , and B calls the EXCHANGE of the same object with argument b , then A's call will return value b and vice versa.

On the original algorithm [cite the book?!], processes race to win the exchanger using a **CAS** primitive. A process accessing the exchanger first reads its content, and act according to the state of it. The first process observe an EMPTY state, and tries to atomically writes its value and change the state to WAITING. In such case, it spins and wait for the second process to arrive. The second, observing the state is now WAITING, tries to write its value and change the state to BUSY. This way, it informs the first one a successful collision took place. Once the first process notice the collision, it reads the other process value and release the exchanger by setting it back to EMPTY. In order to avoid an unbounded waiting, if a second process does not show up, the call eventually timeout, and the process release the exchanger and return.

Assume a process p successfully capture the exchanger by setting its status to WAITING, followed by a crash. Now, some other process q complete the exchange by setting the exchanger to BUSY. Upon recovery, p can conclude some exchange was completed, but it can not tell whether its value is part of the exchange, and thus it can not complete the operation. Moreover, p and q must agree, otherwise q will return p 's value, and thus the operation of p must be linearized together with q operation.

In order to avoid the above problem, we take an approach resembling the BST implementation. Instead of writing a value to the exchanger, processes will use an info record, containing the relevant

information for the exchange. This way, processes use the exchanger in order to exchange info records (more precisely, pointers to such records), and not values. To overcome the problematic scenario described earlier, if a process q observe the exchanger state is **WAITING** with some record $yourop$, it first update its own record $myop$ it is about to try and collide with $yourop$, and only then performs the **CAS**. This way, if the collision is successful, the record $myop$ which now stored in the exchanger implies which two records collide. Also, the fact that different processes uses different records guarantee that at most one record can collide with $yourop$.

Using records instead of values, when using wisely, allows us to farther improve the algorithm. First, there is no need to store the exchanger's state in it (by using 2 bits of it to mark the state), but we can rather have this info in the record. Second, if there is a **BUSY** record in the exchanger, it contains the info of the two colliding records. Therefore, a third process, trying to also use the exchanger, can help the processes to complete the collision, and then can try and set the exchanger back to **EMPTY**, so it can use it again. In the original implementation, a process observing a **BUSY** exchanger, have to wait for the first process to read the value and release the exchanger. Therefore, if the first process crash after the collision, the exchanger will be hold by it forever. The helping mechanism avoids this scenario, making the exchange routine non-blocking.

Notice that no exchange record with **EMPTY** state is ever created, except for the *default* record. Therefore, reading **EMPTY** state is equivalent to the exchanger storing a pointer to *default*. A process p creates a new record $myop$ when accessing the exchanger, with a unique address. As long as p fails to perform a successful **CAS**, and thus fails to store $myop$ in *slot*, it is allowed to try again. However, once a process performs a successful **CAS** and stores $myop$ in *slot*, the only other **CAS** it is allowed to do are in order to try and store *default* in *slot*. Thus, $myop$ can be written exactly once to *slot*. It follows that a collision can occur between two processes exactly - once a **WAITING** record stored in *slot*, only a single **CAS** can replace it with a **BUSY** record. As the two records can not be written again to *slot*, no other process can collide with any of the records.

The **EXCHANGE-RECOVER** routine relies on the following argument. If a process p successfully wrote op_p to *slot* using the **CAS** in line 81, the only way to overwrite it by a different process q , is by a **CAS** in line 104 with a record op_q such that its state is **BUSY**, and $op_q.partner = op_p$. In addition, the only way to overwrite op_q is by a **CAS** replacing it with *default*, and this is done only after **SWITCHPAIR**(op_p, op_q) is completed, and thus both *result* fields are updated.

The correctness of the **EXCHANGE-RECOVER** routine is based on the above argument. There are few scenarios to consider. If p crash after a successful **CAS** in line 81, then op_p state is **WAITING**. Therefore, when reading *slot* in the **EXCHANGE-RECOVER** one of the following must hold. If *slot* contains op_p , then no process collide with p , and p continue to run as if the time limit has been reached. Otherwise, there was a collision. From the above argument, it must be that either op_q that collide with op_p is stored in *slot*, in this case $op_q.partner = op_p$, and p will try to complete the collision and release *slot*, or that op_q has been overwritten, and in this case the *result* field of op_p is updated. In both cases, p returns $op_p.result$. If p crash after a successful **CAS** in line 104, then op_p state is **BUSY**. It follows from the argument that the only way to overwrite op_p is only after completing the collision by **SWITCHPAIR**. Thus, either upon recovery p reads op_p from *slot*, and in this case it tries to complete the the operation, or that $op_p.result$ was already updated. In both cases, p returns it. If non of the above holds, then op_p was not involved in any collision, because either no successful **CAS** was done by p , or p reached the time limit while no process show up, and was able to set *slot* back to *default*. In any case, after the crash of p , op_p will never be written again to *slot*, nor any other op_q such that $op_q.partner = op_p$, as any such op_q tries to perform

CAS (op_p, op_q) that will fail. Also, as no process can collide with op_p , no SWITCHPAIR with op_p as parameter is ever invoked, and in particular $op_p.result = \perp$ for the rest of the execution. This in turn implies that upon recovery p will return FAIL, as required.

5.2 Lock-Free Stack

The stack implementation is due to [...]. The TRY PUSH routine tries to atomically have a new node pointing to the old top, and then updating the top to be the new node. The TRY POP routine tries to atomically read the top of the stack, and change the top to the next node of it. The two routines uses **CAS** in order to guarantee no change for the top was made between the read and write. PUSH (resp. POP) routine is alternating between a TRY PUSH (TRY POP) routine, which access the central stack, and the EXCHANGE, trying to collide with an opposite operation.

In order to make the implementation recoverable, we need a way to infer whether a POP or PUSH already took affect, in case of a crash. Moreover, in case of a POP, we also need to infer which process is the one to pop the node. For that, we use an approach similar to the Linked-List implementation. Each node contains a new field *popby* which is used to identify a PUSH of the node completed, as well as a POP of the node was completed, and who is the process to pop it. Consider the following scenario. Assume a process p performs a PUSH operation with node nd , and using a **CAS** succeed to update the stack top to point to nd , followed by a crash. Now, process q performing a POP operation performs a **CAS** causing the removal of nd from the stack (by changing top to the next node). In this case, once p recovers, nd is no longer part of the stack, and it is also not marked as deleted. This is indistinguishable from a configuration in which the PUSH of nd was yet to take affect (a crash before **CAS**), and thus p can not know what the right response is.

One way to solve this issue is by first marking a node for removal, and only then remove it. This way, if a node is no longer part of the stack it must be marked, and thus we can conclude it was in the stack, and the PUSH routine was successful. However, such an implementation, in addition for the need of to system to support a markable reference, also requires process to help each other. If a node is marked for delete, then a process trying to perform a different operation first needs to complete the deletion, before applying its own operation, otherwise the physical delete of the node may not take place, leaving the node forever in the stack. As the original algorithm avoids any marking, and simply tries to swing the *Top* pointer, we would like to maintain this property.

A field *popby* is initialised to \perp when a node nd is created. Once the node is successfully insert to the stack by a PUSH operation, the inserting process tries to mark it by changing *popby* to NULL using a **CAS**. Before a process tries to remove the node from the stack during a POP routine, it first mark it as part of the stack by doing the same thing, helping the inserting process conclude the node is in the stack. This replace the logic delete of the node, as we only need to know the node was part of the stack if it is removed. After a successful **CAS** to remove nd from the stack, another **CAS** is used in order to try and set *popby* to the identifier of the process who performed the **CAS**. The use of **CAS** to change *popby* from \perp to NULL, and from NULL to an identifier guarantee that only the first process to perform each of these **CAS** will succeed. Note that before writing an identifier to *popby* a process must try and set it to NULL, and thus it can not store two different identifiers along in any execution.

The correctness proof follows the same guidelines as of the proof for the Linked-List. If a PUSH operation did not introduce a new node nd into the stack, then no process but p is aware of nd . Thus, upon recovery the SEARCH routine will not find nd in the stack, nor its *popby* field has been

changed, and the PUSH-ROCEOVER returns FAIL. Otherwise, nd was successfully inserted to the stack. As discussed above, the only way to delete nd from the stack is by first changing its *popby* field to NULL. Thus, upon recovery p will either find nd in the stack, using the SEARCH routine, or that *popby* is different then \perp in case it was deleted, and in both cases it returns **true**. For the POP routine, if p tries to remove a node nd from the top of the stack and crash, then upon recovery it first check if nd is still in the stack using the SEARCH routine. If it is so, then clearly nd was yet to delete, and it returns FAIL. Otherwise, nd was deleted, either by p or by some other process. Only the first process of which to performs a **CAS**, writing its identifier to *popby* will return the value stored in nd , while the others return either \perp (in the TRYPOP routine) or FAIL (in the POP-RECOVER routine).

Notice that both PUSH-ROCEOVER and POP-RECOVER are wait-free. Due to the structure of stack, no *next* pointer of any node in the stack is ever changed. Therefore, once a process reads *Top* at the beginning of its RECOVER routine, the chain of pointers from this *Top* to the last node in the stack is fixed for the rest of the execution, and thus traversing it using the SEARCH routine is wait-free.

```

Type Node {
    T value
    int popby
    Node *next
}

Type PushInfo {                                ▷ subtype of Info
    Node *pushnd
}

Type PopInfo {                                  ▷ subtype of Info
    Node *popnd
}

Type ExInfo {                                   ▷ subtype of Info
    {EMPTY, WAITING, BUSY} state
    T value, result
    ExInfo *partner, *slot
}

```

Figure 8: Type definition

ExInfo *default* - global static ExInfo object with state = EMPTY

Algorithm 6: T EXCHANGE (ExInfo *slot, T *myitem*, long *timeout*)

```

69 long timeBound := getNanos() + timeout
70 ExInfo myop := new ExInfo(WAITING, myitem,  $\perp$ ,  $\perp$ , slot)
71 Announce[pid] := myop
72 while true do
73   if getNanos() > timeBound then
74     | myop.result := TIMEOUT // time limit reached
75     | return TIMEOUT
76   yourop := slot
77   switch yourop.state do
78     case EMPTY do
79       | myop.state := WAITING // attempt to replace default
80       | myop.partner :=  $\perp$ 
81       | if slot.CAS(yourop, myop) then // try to collide
82         | while getNanos() < timeBound do
83           | | yourop := slot
84           | | if yourop  $\neq$  myop then // a collision was done
85             | | | if youop.parnter = myop then // yourop collide with myop
86               | | | SWITCHPAIR(myop, yourop)
87               | | | slot.CAS(yourop, default) // release slot
88             | | | return myop.result
89           | | end
90           | | // time limit reached and no process collide with me
91           | | if slot.CAS(myop, default) then // try to release slot
92             | | | myop.result := TIMEOUT
93             | | | return TIMEOUT
94           | | else // some process show up
95             | | | yourop := slot
96             | | | if yourop.partner = myop then
97               | | | | SWITCHPAIR(myop, yourop) // complete the collision
98               | | | | slot.CAS(yourop, default) // release slot
99             | | | return myop.result
100          | end
101          break
102        case WAITING do // some process is waiting in slot
103          | | myop.partner := yourop // attempt to replace yourop
104          | | myop.state := BUSY
105          | | if slot.CAS(yourop, myop) then // try to collide
106            | | | SWITCHPAIR(myop, yourop) // complete the collision
107            | | | slot.CAS(myop, default) // release slot
108            | | | return myop.result
109          | | break
110        case BUSY do // a collision in progress
111          | | SWITCHPAIR(yourop, yourop.parnter) // help to complete the collision
112          | | slot.CAS(yourop, default) // release slot
113          | | break
114        end
115      end
116    end
117  end

```

Algorithm 7: void SWITCHPAIR(ExInfo <i>first</i> , ExInfo <i>second</i>)	
/* exchange the value of the two operations	*/
115 <i>first.result</i> := <i>second.value</i>	
116 <i>second.result</i> := <i>first.value</i>	
<hr/>	
Algorithm 8: T VISIT (T <i>value</i> , int <i>range</i> , long <i>duration</i>)	
/* invoke EXCHANGE on a random entry in the collision array	*/
117 int <i>cell</i> := randomNumber(<i>range</i>)	
118 return EXCHANGE(<i>exchanger</i> [<i>cell</i>], <i>value</i> , <i>duration</i>)	
<hr/>	
Algorithm 9: T EXCHANGE-RECOVER ()	
119 ExInfo * <i>myop</i> := Announce[<i>pid</i>]	// read your last operation record
120 ExInfo * <i>slot</i> := <i>myop.slot</i>	// and the slot on which it acts
121 if <i>myop.state</i> = WAITING then	
/* crash while trying to exchange <i>default</i> , or waiting for a collision	*/
122 <i>yourop</i> := <i>slot</i>	
123 if <i>yourop</i> = <i>myop</i> then	// still waiting for a collision
124 if <i>slot.CAS</i> (<i>myop</i> , <i>default</i>) then	// try to release <i>slot</i>
125 return FAIL	
126 else	// some process show up
127 <i>yourop</i> := <i>slot</i>	
128 if <i>yourop.partner</i> = <i>myop</i> then	
129 SWITCHPAIR(<i>myop</i> , <i>yourop</i>)	// complete the collision
130 <i>slot.CAS</i> (<i>yourop</i> , <i>default</i>)	// release <i>slot</i>
131 return <i>myop.result</i>	
132 else if <i>yourop.partner</i> = <i>myop</i> then	// <i>yourop</i> collide with <i>myop</i>
133 SWITCHPAIR(<i>myop</i> , <i>yourop</i>)	// complete the collision
134 <i>slot.CAS</i> (<i>yourop</i> , <i>default</i>)	// release <i>slot</i>
135 return <i>myop.result</i>	
136 if <i>myop.state</i> = BUSY then	
/* crash while trying to collide with <i>myop.partner</i>	*/
137 <i>yourop</i> := <i>slot</i>	
138 if <i>yourop</i> = <i>myop</i> then	// collide was successful and in progress
139 SWITCHPAIR(<i>myop</i> , <i>myop.partner</i>)	// complete the collision
140 <i>slot.CAS</i> (<i>myop</i> , <i>default</i>)	// release <i>slot</i>
141 return <i>myop.result</i>	
142 if <i>myop.result</i> ≠ ⊥ then	
143 return <i>myop.result</i>	// collide was successfully completed
144 else	
145 return FAIL	

Figure 9: Elimination Array routines

Algorithm 10: boolean TRY PUSH (Node **nd*)

```

    /* attempt to perform PUSH to the central stack */
146 Node *oldtop := Top
147 nd.next := oldtop
148 if Top.CAS(oldtop, nd) then           // try to declare nd as the new Head
149     nd.popby.CAS( $\perp$ , NULL)         // announce nd is in the stack
150     return true
151 return false

```

Algorithm 11: boolean PUSH (T *myitem*)

```

152 Node *nd = new Node (myitem)
153 nd.popby :=  $\perp$ 
154 PushInfo data := new PushInfo (nd)
155 while true do
156     Announce[pid] := data           // declare - trying to push node nd
157     if TRY PUSH(nd) then             // if central stack PUSH is successful
158         return true
159     range := CalculateRange()         // get parameters for collision array
160     duration := CalculateDuration()
161     othervalue := VISIT(myitem, range, duration) // try to collide
162     if othervalue = NULL then         // successfully collide with POP operation
163         RecordSuccess ()
164         return true
165     else if othervalue = TIMEOUT then // failed to collide
166         RecordFailure ()
167 end

```

Algorithm 12: boolean PUSH-ROCEOVER ()

```

168 Node *nd := Announce[pid].pushnd
169 if nd.popby  $\neq \perp$  then             // nd was announced to be in the stack
170     return true
171 if SEARCH(nd) || nd.popby  $\neq \perp$  then // nd in the stack, or was announced as such
172     nd.popby.CAS( $\perp$ , NULL)         // announce nd is in the stack
173     return true
174 return FAIL

```

Algorithm 13: boolean SEARCH (Node **nd*)

```

    /* search for node nd in the stack */
175 Node *iter := Top
176 while iter  $\neq \perp$  do
177     if iter = nd then
178         return true
179     iter := iter.next
180 end
181 return false

```

Figure 10: PUSH routine

Algorithm 14: T TRYPOP()

```
182 Node *oldtop := Top
183 Node *newtop
184 Announce[pid].popnd := oldnop           // declare - trying to pop node oldtop
185 if oldtop =  $\perp$  then                     // stack is empty
186   | return EMPTY
187 newtop := oldtop.next
188 oldtop.popby.CAS( $\perp$ , NULL)                // announce oldtop is in the stack
189 if Top.CAS(oldtop, newtop) then // try to pop oldtop by changing Top to newtop
190   | if newtop.popby.CAS(NULL, pid) then // try to announce yourself as winner
191     | return oldtop.value
192 else
193   | return  $\perp$ 
```

Algorithm 15: T POP ()

```
194 Node *result
195 PopInfo data := new PopInfo (Top)
196 while true do
197   | Announce[pid] := data           // declare - trying to perform POP
198   | result := TRYPOP()              // attempt to pop from central stack
199   | if result  $\neq \perp$  then           // if central stack POP is successful
200     | return result
201   | range := CalculateRange()        // get parameters for collision array
202   | duration := CalculateDuration()
203   | othervalue := VISIT(NULL, range, duration) // try to collide
204   | if othervalue = TIMEOUT then     // failed to collide
205     | RecordFailure ()
206   | else if othervalue  $\neq$  NULL then // successfully collide with PUSH operation
207     | RecordSuccess ()
208   | return othervalue
209 end
```

Algorithm 16: T POP-RECOVER()

```
210 Node *nd := Announce[pid].popnd // crash while trying to pop node nd
211 if nd =  $\perp$  then                  // pop from an empty stack
212   | return EMPTY
213 if SEARCH(nd) then                // nd was not removed from the stack
214   | return FAIL
215 nd.popby.CAS(NULL, pid)           // nd was removed. Try to complete the operation
216 if nd.popby = pid then            // you are the process to win the pop of nd
217   | return nd.value
218 return FAIL
```

Figure 11: POP routine

References

- [1] M. K. Aguilera and S. Frølund. Strict linearizability and the power of aborting. In *Tech. Rep. HPL-2003-241*, 2003.
- [2] H. Attiya, O. Ben-Baruch, and D. Hendler. Nesting-safe recoverable linearizability: Modular constructions for non-volatile memory. In *Proceedings of the Symposium on Principles of Distributed Computing*, PODC, 2018.
- [3] R. Berryhill, W. M. Golab, and M. Tripunitara. Robust shared objects for non-volatile main memory. In *19th International Conference on Principles of Distributed Systems, OPODIS*, pages 20:1–20:17, 2015.
- [4] S. Chen and Q. Jin. Persistent b+-trees in non-volatile main memory. *Proceedings of the VLDB Endowment*, 8(7):786–797, 2015.
- [5] J. Coburn, A. M. Caulfield, A. Akel, L. M. Grupp, R. K. Gupta, R. Jhala, and S. Swanson. NV-Heaps: Making persistent objects fast and safe with next-generation, non-volatile memories. In *Proceedings of the Sixteenth International Conference on Architectural Support for Programming Languages and Operating Systems, ASPLOS*, pages 105–118, 2011.
- [6] W. M. Golab and A. Ramaraju. Recoverable mutual exclusion: [extended abstract]. In *Proceedings of the 2016 ACM Symposium on Principles of Distributed Computing, PODC*, pages 65–74, 2016.
- [7] R. Guerraoui and R. R. Levy. Robust emulations of shared memory in a crash-recovery model. In *24th International Conference on Distributed Computing Systems (ICDCS)*, pages 400–407, 2004.
- [8] M. Herlihy. Wait-free synchronization. *ACM Trans. Program. Lang. Syst.*, 13(1):124–149, 1991.
- [9] M. Herlihy and J. M. Wing. Linearizability: A correctness condition for concurrent objects. *ACM Trans. Program. Lang. Syst.*, 12(3):463–492, 1990.
- [10] J. Izraelevitz, H. Mendes, and M. L. Scott. Linearizability of persistent memory objects under a full-system-crash failure model. In *Distributed Computing - 30th International Symposium, DISC*, pages 313–327, 2016.
- [11] S. Venkataraman, N. Tolia, P. Ranganathan, and R. H. Campbell. Consistent and durable data structures for non-volatile byte-addressable memory. In *FAST*, volume 11, pages 61–75, 2011.