# Making Concurrent Data Structures Recoverable

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**Abstract**

Recent developments foreshadow the emergence of new systems, in which byte-addressable *non-volatile main memory* (*NVRAM*), combining the performance benefits of conventional main memory with the durability of secondary storage, co-exists with (or eventually even replaces) traditional volatile memory. Consequently, there is increased interest in *recoverable* concurrent objects: objects that are made robust to crash-failures by allowing their operations to recover from such failures. This paper presents a principled approach to deriving recoverable versions of several widely-used concurrent data structures, in particular, a linked list and an elimination stack.

## 1 Introduction (based on our PODC)

Shared-memory multiprocessors are asynchronous in nature. Asynchrony is related to reliability, since algorithms that provide nonblocking progress properties (e.g., lock-freedom and wait-freedom [8]) in an asynchronous environment with reliable processes continue to provide the same progress properties in the presence of *crash failures*. This happens because a process that crashes permanently during the execution of the algorithm is indistinguishable to the other processes from one that is merely very slow. Owing to its simplicity and intimate relationship with asynchrony, the crash-failure model is almost ubiquitous in the treatment of concurrent algorithms.

The attention to the crash-failure model has so far mostly neglected the *crash-recovery* model, in which a failed process may be resurrected after it crashes. Recent developments foreshadow the emergence of new systems, in which byte-addressable *non-volatile main memory* (*NVRAM*), combining the performance benefits of conventional main memory with the durability of secondary storage, co-exists with (or eventually even replaces) traditional volatile memory. Consequently, there is increased interest in *recoverable concurrent objects* (also called *persistent* [4, 5] or *durable* [11]): objects that are made robust to crash-failures by allowing their operations to recover from such failures.

This paper consider an abstract individual-process crash-recovery model for non-volatile memory [2]. Processes communicate via non-volatile shared-memory variables. Each process also has local variables stored in volatile processor registers. [[YF: What about program counter? HA: in this paper, it can be volatile.]] At any point, a process may incur a crash-failure, causing all its local variables to be reset to arbitrary values. Operation response values are returned via volatile processor registers, which may become inaccessible to the calling process if it fails just before persisting the response value. A data structure has an associated *recovery function* that is responsible for restoring it upon the recovery from a crash-failure. The recovery function completes the current outstanding operation on the data structure, if there was any, returning either its *response* or a *fail* indication, if it was unsuccessful. Both responses are consistent with the resulting state of the data structure, to which the operation was applied (in the former case) or not (in the latter case).

We present a principled approach to deriving recoverable versions and describe in in detail in two widely-used concurrent data structures: a linked list [] and elimination stack [].

Several correctness conditions for the crash-recovery model were defined in recent years (see, e.g., [1, 3, 6, 7, 10]). The goal of these conditions is to maintain the state of concurrent objects consistent in the face of crash failures.

## 2 Model and Definitions (based on our PODC)

We consider a system where $N$ asynchronous *processes* $p_1, \ldots, p_N$ communicate by accessing *concurrent objects*. The system provides *base objects* that support atomic read, write, and compare&swap (CAS). Base objects can be used for implementing more complex concurrent objects (e.g. lists, trees and stacks), by defining access procedures that simulate each operation on the implemented object using operations on base objects.

The state of each process consists of non-volatile *shared-memory variables*, as well as *local variables stored in volatile processor registers*. Each process can incur at any point during the execution a *crash-failure* (or simply a *crash*) that resets all its local variables to arbitrary values, but preserves the values of all its non-volatile variables. A process *p invokes an operation Op* on an object by performing an *invocation step*; upon *Op*'s completion, a *response step* is executed.

Operation *Op* is *pending* if it was invoked but was not yet completed. For simplicity, we assume that, at all times, each process has at most a single pending operation on any one object.

[[HA: Define data structure $D$.] Each data structure has an associated *recovery function*, denoted $D$.`Recover`, which is responsible for restoring the data structure to a consistent state, upon recovery from a crash.

More formally, a *history* $H$ is a sequence of *steps*. There are four types of steps:

1. an *invocation step*, denoted $(INV, p, O, Op)$, represents the invocation by process $p$ of operation $Op$ on object $O$;

2. an operation $Op$ can be completed either normally or when, following one or more crashes, the execution of $Op$.`Recover` is completed. In either case, a *response step* $s$, denoted $(RES, p, O, Op, ret)$, represents the completion by process $p$ of operation $Op$ invoked on object $O$ by some step $s'$ of $p$, with response $ret$ being written to a local variable of $p$. We say that *$s$ is the response step that matches s'*;

3. a *crash step* $s$, denoted $(CRASH, p)$, represents the crash of process $p$. We call the inner-most recoverable operation $Op$ of $p$ that was pending when the crash occurred the *crashed operation of $s$*. $(CRASH, p)$ may also occur while $p$ is executing some recovery function $Op$.`Recover` and we say that $Op$ is the crashed operation of $s$ also in this case;

4. a *recovery step $s$ for process $p$*, denoted $(REC, p)$, is the only step by $p$ that is allowed to follow a $(CRASH, p)$ step $s'$. It represents the resurrection of $p$ by the system, in which it invokes $Op$.`Recover`,[1] where $Op$ is the crashed operation of $s'$. We say that *$s$ is the recovery step that matches $s'$*.

For a history $H$, we let $H|p$ denote the subhistory of $H$ consisting of all the steps by process $p$ in $H$. $H$ is *crash-free* if it contains no crash steps (hence also no recover steps). We let $H|\langle p, O\rangle$ denote the subhistory consisting of all the steps on $O$ by $p$.

Given two operations $op_1$ and $op_2$ in a history $H$, we say that $op_1$ *happens before* $op_2$, denoted $op_1 <_H op_2$, if $op_1$'s response step precedes the invocation step of $op_2$ in $H$. $H|O$ is a *sequential object history*, if it is an alternating series of invocations and the matching responses starting with an invocation (the history may end by a pending invocation). The *sequential specification* of an object $O$ is the set of all possible (legal) sequential histories over $O$. A history $H$ is *sequential* if $H|O$ is a sequential object history for all objects $O$.

Two histories $H$ and $H'$ are *equivalent*, if $H|\langle p, O\rangle = H'|\langle p, O\rangle$ for all processes $p$ and objects $O$. Given a history $H$, a *completion* of $H$ is a history $H'$ constructed from $H$ by selecting separately, for each object $O$ that appears in $H$, a subset of the operations pending on $O$ in $H$ and appending matching responses to all these operations, and then removing all remaining pending operations on $O$ (if any).

**Definition 1 (Linearizability [9], rephrased)** *A finite crash-free history $H$ is* linearizable *if it has a completion $H'$ and a legal sequential history $S$ such that:*

*L1. $H'$ is equivalent to $S$; and*

[1]A history does not contain invocation/response steps for recovery functions.

*L2.* $<_H \subseteq <_S$ *(i.e., if* $op_1 <_H op_2$ *and both ops appear in* $S$ *then* $op_1 <_S op_2$*).*

Thus, a finite history is linearizable, if we can linearize the subhistory of each object that appears in it. Next, we define a more general notion of well-formedness that applies also to histories that contain crash/recovery steps. For a history $H$, we let $N(H)$ denote the history obtained from $H$ by removing all crash and recovery steps.

**Definition 2 (Recoverable Well-Formedness)** *A history $H$ is recoverable well-formed if the following holds.*

1. *Every crash step in $H|p$ is either $p$'s last step in $H$ or is followed in $H|p$ by a matching recover step of $p$.*
2. *$N(H)$ is well-formed.*

We can now define the notion of nesting-safe recoverable linearizability.

**Definition 3 (Nesting-safe Recoverable Linearizability (NRL))** *A finite history $H$ satisfies* nesting-safe recoverable linearizability *(NRL) if it is recoverable well-formed and $N(H)$ is a linearizable history. An object implementation satisfies NRL if all of its finite histories satisfy NRL.*

## 2.1 General Overview

Harris' Linked-List algorithm is presented in Section... The implementation uses CAS and read/write primitives only. Attiya et al [ref...] presented a recoverable implementations for read/write and CAS operations. As suggested by the name NRL, it allows nesting. Therefore, taking any algorithm which uses only read/write and CAS primitives, and replacing each primitive with its recoverable version yields an NRL implementation of the object (some minor changes are still needed in order to use this transformation). In particular, this is the case for the Linked-List algorithm.

However, such a generic construction, by its nature, is not efficient. Optimisations can be made in order to improve the complexity. For example, a simple observation is that changes to the data structure are done using CAS operations only, while all other instructions are either to local variables or a read of shared variables. Therefore, it is enough to replace only the CAS operations with its recoverable version. If a crash occurs outside of a CAS operation, upon recovery the process can go back to the last CAS operation and continue the execution from this point.

Ben-David, Blelloch and Wei [ref..] construction can be used in order to convert Harris algorithm to a recoverable one. Their approach is similar to ours - it split the code into capsules, each contains a single CAS operation, and then replacing each CAS with its recoverable version. Each capsule can be recovered in case of a crash inside the capsule. Moreover, an optimize transformation for normalized algorithms is presented. Applying this optimization to Harris algorithm results a recoverable Linked-List such that each attempt to perform an operation is encapsulate using 2 capsules only.

In this paper we present a new recoverable implementation for Harris' Linked-List. This implementation is efficient in terms of both time and space, and avoids few of the disadvantages the above general constructions have. In particular, our implementation does not use a recoverable CAS, but rather a primitive one. Also, the content of a CAS object is the same as the original algorithm, while in Ben-David et al implementation a recoverable CAS stores also an unbounded sequence number. Under the assumption that crash is a rare event, we have tried to make the regular operations as efficient as possible, while willing to pay with a more complex recover functions. In more details, the recovery function may access the data structure and perform operations that does not change its structure, such as search.

Designing a recoverable implementation for a specific data structure, even though may result an efficient algorithm, is a non trivial and time consuming task. However, our implementation exploit a certain structure that can be found on other data structures as well, and thus our technique can be used in those cases as well. More specific, many data structures uses a CAS primitive in order to sync the operations, such that each operation is linearized at the time of a single successful CAS. In the Linked-List, for example, Insert operation is linearized at the point of adding a pointer to the new node using a successful CAS, while Delete operation is linearized at the point of a successful logical delete. The key point in recovering is that if a process crash when about to perform a CAS operation, it is not trivial to discover upon recovery whether its operation took effect. Moreover, two or more process can try and perform the same operation at the same time, for example, deleting the same node. Upon recovery, a process can not tell whether it is the process to successfully perform the operation, or that it was done by some other process, hence it does not know what value to return. Therefore, a mechanism to determine the ID of the process which performed the operation is needed, such that out of all processes attempting to perform it, exactly one is defined to successfully performed it.

## 3 Linked-List

In this section, we present a recoverable version of the linked-list set algorithm of Harris [**?**].[2] Harris' algorithm maintains a linked list of nodes sorted in increasing order of keys. The *next* field of each node consists of two components: a reference to the next node and a *marked* bit that is set when the node is logically deleted. Both componenets can be manipulated atomically, either together or individually, using a singe-word CAS operation.[3] For presentation simplicity, we assume in the pseudo-code that reading the *next* field returns the reference only, while invoking the get_data() function on *next* returns both the reference and the mark bit. We also assume that the list always contain a *head* and *tail* sentinel nodes, containing keys $-\infty, \infty$ respectively.

We now provide a brief description of the Harris algorithm. The linked-list set supports the FIND, INSERT and DELETE set operations. INSERT and DELETE use the SEARCH helper procedure in order to find the node with the smallest key greater than or equal to the input key (this node is denoted *curr*), as well as its predecessor on the list (denoted *pred*). While traversing the list, SEARCH attempts to physically remove from the list any marked node it encounters. To insert a key $\alpha$, a process $p$ first calls SEARCH in order to find the position in the list where $\alpha$ should be added. If $\alpha$ is already in the list, INSERT returns false, otherwise it tries to set *pred.next* to point to a new node containing $\alpha$ using CAS (which would fail if *pred* has been logically deleted in the interim). To delete a key $\alpha$, $p$ searches for it using SEARCH and returns false if it not in the set. Otherwise, $p$ tries to logically delete it by marking the *next* field if its node using CAS. If marking is successful, $p$ tries to physically remove the node. To find a key $\alpha$, a process simply looks for a

[2]Some implementation details follow the algorithm's presentation by [**?**].

[3]In the Java implementation of the algorithm in [**?**], *next* fields are represented by AtomicMarkableReference objects, in whose implementation the marked bit uses the least significant bit of the reference.

node in the list with key $\alpha$ which is unmarked.

The linearization point for the Harris algorithm are as follows:

Insert: Either when Lookup finds $\alpha$ in the list, or upon a successful CAS inserting $\alpha$ to the list.

Delete: Upon a successful CAS that marks the node (logical deletion).

Find: Upon the last read of either *curr.key* or *curr.next*.

Following the given linearization points (omitting proof...), Insert and Delete operation are linearized at the point where they affect the system. That is, if an Insert operation performed a successful CAS, then all processes will see the new node starting from this point, and if a node was logically delete, then all processes treat it as if it was removed. Therefore, once a process $p$ recovers following a crash, the list data structure is consistent - if $p$ has a pending operation, either the operation already had a linearization point which affected all other processes, or it did not affect the data structure at all, nor will in any future run.

However, even though the list data structure is consistent, the response of the pending operation is lost. Consider for example a scenario in which process $p$ performs Delete($\alpha$) and crash right after applying a successful CAS to mark a node. Upon recovery, $p$ may be able to conclude $\alpha$ was removed, as the node is marked. Nevertheless, even if no other process takes steps, $p$ is not able to determine whether it is the process to successfully delete $\alpha$, or that it was done by some other process, and therefore it does not able to determine the right response. Moreover, in case the node was physically removed, $p$ is not able to determine whether $\alpha$ has been deleted at all, as it is no longer part of the list.

### 3.0.1 Linked-List Recoverable Version

To solve the problems mention above, we present a modification for the algorithm such that in case of a process crash, upon recovery it is able to complete its last pending operation if needed, and also return the response value in such case. The algorithm presented in figure 1. Blue lines represents changes comparing to the original algorithm.

Each node is equipped with a new field named deleter. This field is used to determine which process is the one to delete the node. After the node was successfully marked (logical delete), process $p$ tries to announce itself as the one to delete the node by writing its id to deleter using CAS. This way, if a process crash during a delete, it can use deleter in order to determine the response value. We assume deleter is initialised to null when creating a new node.

Each process $p$ has a designated location in the memory, Backup[p]. Before trying to apply an operation, $p$ writes to Backup[p] the entire data needed to complete the operation. Upon recovery, $p$ can read Backup[p], and based on it to complete its pending operation, in case there is such. Formally, Backup[p] contains a pointer to a structure containing all the relevant data. For simplicity, process $p$ creates new such structure for each of its operations, although a more efficient way will be to use two such structures in an alternating way.

**Algorithm 5:** boolean RECOVER ()

```
Data:   Node $*nd := Backup[pid].nd$
if $Backup[pid].result \neq \perp$ then                          // operation was completed
    return $Backup[pid].result$
if $Backup[pid].optype =$ INSERT then
    $\langle pred, curr\rangle :=$ SEARCH($nd.key$)                 // search for $nd$ in the list
    if $curr = nd \,||\, nd.next$ is marked then                  // $nd$ is in the list or marked
        $Backup[pid] :=$ true
        return true
    return FAIL
if $Backup[pid].optype =$ DELETE then
    if $nd \neq \perp$ && $nd.next$ is marked then                 // $nd$ was logically deleted
        $nd.deleter.$CAS$(\perp, pid)$                              // try to complete the deletation
        if $nd.deleter = pid$ then                                // you are the deleter
            $Backup[pid].result :=$ true
            return true
    return FAIL
```

**Algorithm 1:** $\langle$Node, Node$\rangle$ SEARCH (T *key*)

```
Data:   Node $*pred, *curr, *succ$
        boolean $mbit$
retry: while true do
    $pred := head$
    $curr := pred.next$
    while true do
        $\langle succ, mbit\rangle := curr.next.$get_data()
        if $mbit$ then                                           // succ was logically deleted
            if $pred.next.$CAS(unmarked $curr$,unmarked $succ$) = false then    // help physical delete
                go to retry                                      // help failed
            $curr := succ$                                       // help succeed
        else
            if $curr.key \geq key$ then          // $curr$ is the first unmarked node with key $\geq key$
                return $\langle pred, curr\rangle$
            $pred := curr$                                       // advance $pred$ and $curr$
            $curr := succ$
    end
end
```

**Shared variables:** Node $*head$

```
Type Info {
    {INSERT, DELETE} $optype$
    Node $*nd$
    boolean $result$
}
```

Code for process p:

**Algorithm 2:** boolean INSERT (T $key$)

```
Data: Node $*pred, *curr$
      Node $newnd$ := new Node ($key$)
$Backup[pid]$ := new Info (INSERT, $newnd$, $\bot$)
while true do
    $\langle pred, curr\rangle$ := SEARCH($key$)          // search for the right location for insertion
    if $curr.key = key$ then                             // $key$ is already in the list
        $Backup[pid].result$ := false
        return false
    else
        $newnd.next$ := unmarked $curr$
        if $pred.next$.CAS (unmarked $curr$, unmarked $newnd$) then     // try to add $newnd$
            $Backup[pid].result$ := true
            return true
end
```

**Algorithm 3:** boolean DELETE (T $key$)

```
Data: Node $*pred, *curr, *succ$
$Backup[pid]$ := new Info (DELETE, $\bot$, $\bot$)
$\langle pred, curr\rangle$ := SEARCH($key$)             // search for $key$ in the list
if $curr.key \neq key$ then                              // $key$ is not in the list
    $Backup[pid].result$ := false
    return false
else
    $Backup[pid].nd := curr$
    while $curr.next$ is unmarked do                     // repeatedly attempt logical delete
        $succ := curr.next$
        $curr.next$.CAS (unmarked $succ$, marked $succ$)
    end
    $succ := curr.next$
    $pred.next$.CAS (unmarked $curr$, unmarked $succ$)   // physical delete attempt
    $res := curr.deleter$.CAS($\bot, pid$)               // try to announce yourself as deleter
    $Backup[pid].result := res$
    return $res$
```

**Algorithm 4:** boolean FIND (T $key$)

```
Data: Node $*curr := head$
while $curr.key < key$ do          // search for the first node with key greater or equal to $key$
    $curr = curr.next$
end
return ($curr.key = key$ && $curr.next$ is unmarked)
```

Figure 1: Recoverable Non-Blocking Linked-List

**Correctness Argument**

In the following, we give an high-level proof for the correctness of the algorithm.

First, notice that quitting the Lookup procedure at any point, or repeating it, can not violet the list consistency. The Lookup procedure simply traverse the list, while trying to physically delete marked nodes. Once curr.next is marked, a single process can perform the physical delete. This follows from the fact that at any point there is a single node in the list which points to curr. Once curr is physically delete, no node in the list points to curr, and thus any CAS operation with curr as the first parameter will fail. This observation relays on the fact that any new allocated node has a different address then curr. As a result, repeating the attempt to physically delete a node does not affect the list.

Assume a process *p* performs an *insert*(*key*) operation. First, *p* writes to Backup[p], updating it is about to perform an Insert. If a process *p* does not crash, then, as in the original algorithm, it repeatedly tries to find the right location for the new node, and insert it by performing a CAS changing *pred.next* to point to newnd. In addition, it is clear from the code that a crash after updating $Backup[p].result$ is after the operation had its linearization point, and the Recover procedure will return the right response. Therefore, we need to consider a crash before an update to $Backup[p].result$. There are two scenarios to consider.

Assume *p* crash without performing a successful CAS in line 27. *p* is the only process to have a reference to newnd, and it is yet to update any node with this reference, and thus no node points to newnd. As a result, the operation did not affects any other process, nor it will be in the future. Hence, considering the operation as not having a linearization point does not violate the list consistency. Indeed, since no node points to newnd, upon recovery *p* will see that newnd is not in the list and also not marked, and thus will return FAIL. Notice this argument holds whether key is already in the tree, or not, as the operation in both cases did not affect the system.

Assume now *p* crash after performing a successful CAS in line 27. In such case, newnd is part of the list, as pred.next points to it. Also notice we did not delete any other node, since pred.next pointed to curr, and after the CAS it points to newnd which points to curr. As a result, when *p* executes the Recover procedure, either it will see newnd in the list, or that it is no longer part of the list, and it must be some other process deleted it, and hence newnd.next is marked. In any case, *p* will return true as required. The above argument relies on the fact a marked node can not be unmarked, and that an Insert and Delete can not mistakenly remove nodes from the list. We have claimed it for Insert, and we will prove the same holds for Delete. Therefore, if a node is no longer in the list, it must be marked.

Assume a process *p* performs a *delete*(*key*) operation. First, *p* writes to Backup[p], updating it is about to perform a Delete. As before, a crash after writing to Backup[p].result will return the right response. Also, a crush before updating any of Backup[p].result or Backup[p].nd implies *p* is yet to try and mark any node, and thus the operation did not affect the system so far, nor it will be in the future. Therefore, we can consider the operation as not having a linearization point (even in case key is not the list), and indeed, the Recover procedure returns FAIL in such case.

Assume thus *p* writes to Backup[p].nd. It follows that *p* completed the lookup procedure and finds a node curr storing key. The lookup procedure guarantees there is a point in time (of the procedure execution) where curr is in the list and curr.next is not marked. If *p* crash and recovers, and observe that curr is unmarked, then in returns FAIL. Since a marked node can not be unmarked, as there is no CAS changing a marked node, it follows that *p* did not marked curr. Therefore, the operation did not affects any process, nor it will be, and we consider it as having no linearization point. Otherwise, the Recover function observe curr as marked, and we can conclude the marking point of curr is along the delete operation. We now prove we can linearize the operation, according to its response.

Let $q$ be the process to mark curr. Since once curr.next is marked it will never be changed, the reference of curr.next is fixed to succ (of $q$ at the point of the marking). This also implies $q$ is unique and well defined, and any future CAS on curr.next will fail. As a result, any process leaving the while loop in line 39 reads the same value in line 43, which is this succ. The attempt to physically delete curr in line 44 will succeed only if pred.next points to curr, and as we said, curr point to succ, and any other attempt will fail. Thus, if this attempt succeed, it deletes only curr, and can not delete additional nodes.

In line 45 process $p$ tries to writes its id to curr.deleter. As it is initialised to null, only the first process to perform this CAS will succeed. Also, any $p$ must go through line 45 in order to complete its operation, as the Recover procedure redirect the process to this line. Therefore, if there is a process to complete its delete operation while observing curr.next is marked, there must be a CAS to curr.deleter. Let $q'$ be the first process to perform this CAS. As proved above, $q'$ tries to delete curr, and the point in time where curr is marked must be contained in its operation interval. Moreover, $q'$ is the only process to write to curr.deleter, and the first one to do so, thus $q'$ is the only process to obtain true when testing (curr.deleter $= q'$) in line 46 (and thus to also return true), while any other process will obtain false. We linearize the operation of $q'$ at the point of the marking, and any other attempt to delete curr is linearized after it (in an arbitrary order).

A corollary of the analysis is that processes trying to delete the same node curr "helps" each other, in the sense that they all keep trying to mark curr. However, the marking process is not necessarily the one to return true. Also, in the original algorithm, if a process fails to mark a node, it starts the delete operation from the beginning. In our implementation, process can keep trying to mark the node without the need to perform a lookup again after each failed CAS. We guarantee that once curr is marked, exactly one process will return true, while the rest can consider curr as being deleted (in the course of their delete execution), and thus there is a point along their execution is which key is not in the tree, and they can return false.

## 4 Robust BST

The original BST algorithm does not support the crash-recovery model. It is clear from the code a process does not persist the operation's response in the non-volatile memory, and thus, once a process crash the response is lost. For example, assume a process $q$ apply INSERT($k$), performs a successful CAS in line 101 and fails after completing the HELPINSERT routine. In this case, the INSERT operation took effect, that is, the new key appears as a leaf in the tree, and any FIND($k$) operation will return it. However, even though the operation must be linearized before the crash, upon recovery process $q$ is unaware of it. Moreover, looking for the new leaf in the tree may be futile, as it might be $k$ has been removed from the tree after the crash.

Furthermore, if no recover routine is supplied, it may result an execution which is not well-formed. Consider for example the following scenario. A process $q$ invoke an $Op_1$ = INSERT($k_1$) operation. $q$ performs a successful CAS in line 101 followed by a crush. After recovering, $q$ invoke an $Op_2$ = INSERT($k_2$) operation. Assume $k_1$ and $k_2$ belongs to a different parts of the tree (do not share parent or grandparent). Then, $q$ can complete the insertion of $k_2$ without having any affect on $k_1$. Now, a process $q'$ performs FIND($k_1$) which returns NULL, as the insertion of $k_1$ is not completed, followed by FIND($k_2$), which returns the leaf of $k_2$. The INSERT($k_1$) operation will be completed later by any INSERT or DELETE operation which needs to make changes to the flagged node. We get that $Op_2$ must be before $Op_1$ in the linearization, although $Op_1$ invoked first.

The kind of anomaly described above can be addressed by having the first CAS of a successful attempt for INSERT or DELETE as the linearization point, as in the Linked-List. For that, the FIND routine should take into consideration future unavoidable changes, for example, a node flagged with IFlag ensures an insertion of some key. A simple solution is to change the FIND routine such that it also helps other operations, as described in figure 2. The FIND routine will search for key $k$ in the tree. If the SEARCH routine returns a grandparent or a parent that is flagged, then it might be that an insert or delete of $k$ is currently in progress, thus we first help the operation to complete, and then search for $k$ again. Otherwise, if *gpupdate* or *pupdate* has been changed since the last read, it means some change already took affect, and there is a need to search for $k$ again. If none of the above holds, there is a point in time where $gp$ points to $p$ which points to $l$, and there is no attempt to change this part of the tree. As a result, if $k$ is in the tree at this point, it must be in $l$, and the find can return safely.

The approach described above is not efficient in terms of time. We would like a solution which maintain the desirable behaviour of the original FIND routine, where a single SEARCH is needed. A more refined solution is given in figure 3. The intuition for it is drown from the Linked-List algorithm. In the Linked-List algorithm it was enough to consider a marked node as if it has been deleted, without the need to complete the deletion. Nonetheless, the complex BST implementation is more challenging, as the DELETE routine needs to successfully capture two nodes using CAS in order to complete the deletion. Therefore, if a process $p$ executes FIND($k$) procedure, and observes a node flagged with DFlag attempting to delete the key $k$, it can not know whether in the future this delete attempt will succeed or fail, and thus does not know whether to consider the key $k$ as part of the tree or not. To overcome this problem, in such case the process will first try and validate the delete operation by marking the relevant node. According to whether the marking attempt was successful, the process can conclude if the delete operation is successful or not. In order to easily implement the modified FIND routine there is a need to conclude from IInfo what is the new leaf (leaf *new* in the INSERT routine). For simplicity of presentation, we do not add this field, and abstractly refer to it in the code.

The correctness of the two suggested solutions relies on the following argument. Once a process flags a node during operation *Op* with input key *k* (either Insert or Delete), then if this attempt to complete the operation eventually succeed (i.e., the marking is also successful in the case of Delete), then any Find(k) operation invoked from this point consider *Op* as if it is completed.

The suggested modification, although being simple and local, only guarantee the implementation satisfy R-linearzability. However, the problem of response being lost in case of a crash is not addressed. Roughly speaking, the critical points in the code for recovery are the CAS primitives, as a crash right after applying CAS operation results the lost of the response, and in order to complete the operation the process needs to know the result of the CAS. In addition, because of the helping mechanism, a suspended Delete operation which flagged a node and yet to mark one, may be completed by other process in the future, and may not. Upon recovery, the process needs to distinguish between the two cases, in order to obtain the right response.

To address this issue, we expend the helping mechanism so that it also update the info structure in case of a success. This is done by adding a boolean field, done, to the Flag structure. This way, if a process crash along an operation *Op*, upon recovery it can check to see if the operation was already completed. A crucial point is to update the *done* bit before performing the unflagging. Therefore, if a node is no longer flagged we can be sure done was already updated. If we switch the order, then it might be an operation and unflagging were completed, but the done bit is yet to be updated. Therefore, other processes can change the BST structure. However, if the process crash and recover at this point, the done bit is off, and the BST structure has been changed, so it will be harder for the process to conclude whether the operation took affect.

Before a process *q* attempt to perform an operation, as it creates the Flag structure *op* describing the operation and its affect on the data structure, the process stores *op* in a designated location in the shared memory (for simplicity, we use an array). As a result, upon recovery *q* has an access to this information. Now, *q* can check to see if the operation is still in progress, i.e., if the relevant node (parent or grandparent) is still flagged. If so, it first tries to complete the operation. Otherwise, it implies either the operation was completed, and therefore done bit is updated, or that the attempt was unsuccessful and there wa no write to the done bit. Hence, the done bit can distinguish between the two scenarios. Notice that there is a scenario in which process *q* recovers and observes an operation *Op* as it in a progress, but just before it retries it, some other process complete the operation. We need to prove that even in such case, the operation will affect the data structure exactly once, and the right response is returned.

The given implementation does not recover the Find routine, since this routine does not make any changes to the BST, hence it is always safe to consider it as having no linearization point and reissue it. Also, for ease of presentation, we only write to $Announce[id]$ once we are about to capture a node using a CAS. However, writing to $Announce[id]$ at the beginning of the routine may be helpful in case of a crash early in the routine, so that the process will be able to use the data stored in $Announce[id]$ in such case also. The same is true with response value, $Announce[id] \rightarrow done$ is updated only if the routine made changes to the BST.

### 4.0.2 Correctness

In the following section we give a proof sketch for the algorithm correctness. We assume for simplicity nodes and Flag records are always allocated new memory locations, although it is enough to require no location is reallocated as long as there is a chain of pointers leading to it. The proof relies on the correctness of the original algorithm, which can be found on [....].

The proof relies on several key arguments given below.

[**Arg1**] The original algorithm is anonymous and uniform, i.e., any number of processes can use the BST, and there is no need to know the number of processes in the system in order to use the BST. Notice that all helping routines in the given implementation are completely anonymous, and an execution of such a routine by either the process which invoked *op* or any other helping process executes the exact same code. This observation allows the use of the following argument. If a process crash while executing some helping routine, we can consider it as an helping process which stop taking steps (more formally, there is an equivalent execution in which there is such a process, and it is indistinguishable to all process in the system). Since such process can not cause a wrong behaviour of the algorithm, so does the crash. A corollary of this argument is that repeating an helping routine multiple times by the same process can not violate the BST specification, as there is an equivalent executions in which multiple processes executes the different helping routines.

[**Arg2**] It is easy to verify the post-conditions of the SEARCH routine still holds, as they follow directly from the routine's code, and does not rely on the structure or correctness of the BST. Also, the SEARCH routine does not make any changes to the BST, but rather simply traverse it. Therefore FIND routine, which only uses SEARCH, does not affect any process, and in case of a failure along FIND execution, reissuing it satisfies NRL.

[**Arg3**] If an internal node $nd_1$ stops pointing to a node $nd_2$ at some point of the execution, it can not point to $nd_2$ again. This attributes to the fact an INSERT presents a node with two new children. Therefore, if $nd_2$ is a leaf, it can either be delete, or replaced by a new copy of an INSERT operation. Otherwise, $nd_2$ is an internal node, and as such, the pointer to it by $nd_1$ can not be replaced by an INSERT operation (which only allows to replacement a leaf), and therefore it can only be removed from the tree.

[**Arg4**] The field update of a node *nd* can have any value only once along an execution. Any attempt to perform an operation creates a new record in the memory. If $nd \rightarrow update$ is marked, it can not be unmarked or changed. Otherwise, any attempt to flag it uses a new created record *op*. If the attempt succeed, then eventually it will be unflagged while still referring to *op*. In order to replace the value again, there must be an operation reading $nd \rightarrow update$ after it was unflagged (as any operation first help a flagged node). This operation must create a new record, and thus we can use the same argument again. As a corollary, if a process successfully flag or mark a node, there was no change to the node since the last time it read the update field of the node.

**Proof Sketch** Assume a process $q$ performs an operation $Op$ (either INSERT or DELETE). If $q$ does not crash, the algorithm is identical to the original algorithm, except for the additional write to $Announce[q]$ and $op \rightarrow done$, and thus the correctness of the original algorithm can be applied. Otherwise, $q$ crash at some point, and upon recovery it reads $op$ from $Announce[q]$. This record represent the last attempt of $q$ to complete $Op$. We split the proof based on the type of operation.

$Op$ = INSERT. Consider the read of $op \rightarrow p \rightarrow update$ upon recovery, and denote this value by *pupdate*. If $pupdate = \langle \text{IFlag}, op \rangle$, this implies the iflag CAS in line 101 was successful and the operation is yet to complete. It might be that INSERT already took affect, that is, the new key is part of the tree, but the unflagging is yet to happen. In such case, $q$ calls HELPINSERT($op$) in order to try and complete the operation. Considering arg1, this call can not violate the BST correctness, even if it not the first time $q$ executes it. Moreover, during HELPINSERT there is a write to $op \rightarrow done$, and thus after completing the routine $q$ returns TRUE, as required.

Else $pupdate \neq \langle \text{IFlag}, op \rangle$. There are two scenarios to consider. Either the iflag CAS of $q$ in line 101 was successful or not. If it was successful, then $p \rightarrow \textit{update} = \langle \text{IFlag}, op \rangle$ at this point. The only way to change it is to first unflag $p$. To do so, a process needs to complete an HelpInsert($op$) routine, and in particular must write to $op \rightarrow \textit{done}$. In such case, the Insert operation was completed, and $q$ returns True. Otherwise, the CAS was not successful, either because it failed, or the crash was before the CAS. In both cases, the Insert operation will not be completed, as $op$ is not stored in $p \rightarrow \textit{update}$, and thus no process has an access to it. Consequently, no process can update $op \rightarrow \textit{done}$, and $q$ returns Fail.

$Op =$ Delete. Consider the read of $op \rightarrow gp \rightarrow \textit{update}$ upon recovery, and denote this value by *gpupdate*. If $\textit{gpupdate} = \langle \text{DFlag}, op \rangle$, this implies the dflag CAS of $q$ in line 128 was successful, and the operation is yet to complete. As in the Insert, it might be the operation already changed the tree. After reading *gpupdate* $q$ invokes HelpDelete($op$) routine. Again, following arg1, executing this multiple times by $q$ can not violate the BST correctness. The first process to try and mark $op \rightarrow p \rightarrow \textit{update}$ during an HelpDelete($op$) routine is the one to determine the outcome of it. If it is successful, then $p$ is marked, and the *update* field can not be changed. That is, any HelpDelete($op$) execution will obtain true in line 139, and will call HelpMarked($op$) routine. Otherwise, the CAS fails, and so $p \rightarrow \textit{update}$ is no longer equal to $op \rightarrow \textit{pupdate}$. By arg4 it will never get this value again, and thus any marking CAS during a HelpDelete($op$) execution will fail, and there is no call to HelpMarked($op$). In the first case, any HelpDelete($op$) routine must first complete a HelpMarked($op$), and thus must write to $op \rightarrow \textit{done}$, while in the later case, there is no write to $op \rightarrow \textit{done}$, as no HelpMarked($op$) is ever invoked. Therefore, in both cases, when $q$ completes HelpMarked($op$) it reads $op \rightarrow \textit{done}$ and returns the right response.

Otherwise $\textit{gpupdate} \neq \langle \text{DFlag}, op \rangle$, and there are two scenarios to consider. If the dflag CAS of $q$ in line 128 never took affect, because it either failed, or the crash preceded it, then $op$ is never written to $gp \rightarrow \textit{update}$, or to any update field. Thus, no process is aware of it, and $op \rightarrow \textit{done}$ remains False, resulting $q$ returning Fail as required. Else, the CAS was successful, and $gp \rightarrow \textit{update}$ was flagged. The only way to change it is to first unflag it, and this in turn can be done only during an HelpDelete($op$) routine. In this case, it can be unflagged in either the HelpMarked routine in line 154, or in line 145 of the HelpDelete routine. As mention before, the first CAS in line 138 of an HelpDelete($op$) execution determines the outcome for all HelpDelete($op$). If it is successful, $p \rightarrow \textit{update}$ is forever marked, and all HelpDelete($op$) must invoke HelpMarked($op$). Therefore, the only option to unflag $gp \rightarrow \textit{update}$ is at the end of HelpMarked($op$) routine, and this done only after setting $op \rightarrow \textit{done}$. In such case, the Delete operation took affect, and $q$ will return True. On the other hand, if the CAS was not successful, then any HelpDelete($op$) will fail to mark $p \rightarrow \textit{update}$, and hence no HelpMarked($op$) is ever invoked. As a result, there is no write to $op \rightarrow \textit{done}$. In such case, the Delete operation did not took affect, nor will be, and indeed $q$ will return Fail.

```
FIND($Key\ k$) : Leaf* {
1       Internal *$gp$, *$p$
2       Leaf *$l$
3       Update $pupdate, gpupdate$
4       while (TRUE) {
5           $\langle gp, p, l, pupdate, gpupdate \rangle$ := SEARCH($k$)
6           if $gpupdate.state \neq$ CLEAN then HELP($gpupdate$)
7           else if $pupdate.state \neq$ CLEAN then HELP($pupdate$)
8           else if $gp \rightarrow update = gpupdate$ and $p \rightarrow update = pupdate$ then {
9               if $l \rightarrow key = k$ then return $l$
10              else return NULL
11          }
12      }
13  }
```

Figure 2: Solution 1: R-linearizable FIND routine

```
FIND($Key\ k$) : Leaf* {
14      Internal *$gp$, *$p$
15      Leaf *$l$
16      Update $pupdate, gpupdate$
17      $\langle gp, p, l, pupdate, gpupdate \rangle$ := SEARCH($k$)
18      if $l \rightarrow key \neq k$ then {
19          if ($pupdate.state =$ IFlag and $pupdate.info$ attempt to add key $k$) then
20              return leaf with key $k$ from $pupdate.info$
21          else return NULL
22      }
23      if ($pupdate.state =$ MARK and $pupdate.info \rightarrow l \rightarrow key = k$) then return NULL
24      if ($gpupdate.state =$ DFlag and $gpupdate.info \rightarrow l \rightarrow key = k$) then {
25          $op := gpupdate.info$
26          $result :=$ CAS($op \rightarrow p \rightarrow update, op \rightarrow pupdate, \langle$MARK$, op\rangle$)          ▷ **mark** CAS
27          if ($result = op \rightarrow pupdate$ or $result = \langle$MARK$, op\rangle$) then return NULL  ▷ $op \rightarrow p$ is successfully marked
28      }
29      return $l$
30  }
```

Figure 3: Solution 2: R-linearizable FIND routine

```
type Update {                      ▷ stored in one CAS word
    {CLEAN, DFlag, IFlag, MARK} $state$
    Flag *$info$
}
type Internal {                    ▷ subtype of Node
    $Key \cup \{\infty_1, \infty_2\}$ $key$
    Update $update$
    Node *$left$, *$right$
}
type Leaf {                        ▷ subtype of Node
    $Key \cup \{\infty_1, \infty_2\}$ $key$
}
type IInfo {                       ▷ subtype of Flag
    Internal *$p$, *$newInternal$
    Leaf *$l$
    boolean $done$
}
type DInfo {                       ▷ subtype of Flag
    Internal *$gp$, *$p$
    Leaf *$l$
    Update $pupdate$
    boolean $done$
}
▷ Initialization:
shared Internal *$Root$ := pointer to new Internal node
    with $key$ field $\infty_2$, $update$ field $\langle$CLEAN, NULL$\rangle$, and
    pointers to new Leaf nodes with keys $\infty_1$ and
    $\infty_2$, respectively, as $left$ and $right$ fields.
```

Figure 4: Type definitions and initialization.

```
RECOVER() {
    Flag $*op = Announce[id]$

    if $op$ of type IInfo then {
        $result := op \rightarrow p \rightarrow update$
        if $result = \langle \text{IFlag}, op \rangle$ then HELPINSERT($op$)          ▷ Finish the insertion
    }
    if $op$ of type DInfo then {
        $result := op \rightarrow gp \rightarrow update$
        if $result = \langle \text{DFlag}, op \rangle$ then HELPDELETE($op$)          ▷ Either finish deletion or unflag
    }
    if $op \rightarrow done$ = TRUE then return TRUE
    else return FAIL
}
```

Figure 5: RECOVER routine

```
SEARCH(Key $k$) : ⟨Internal*, Internal*, Leaf*, Update, Update⟩ {
        ▷ Used by INSERT, DELETE and FIND to traverse a branch of the BST; satisfies following postconditions:
        ▷ (1) $l$ points to a Leaf node and $p$ points to an Internal node
        ▷ (2) Either $p \to left$ has contained $l$ (if $k < p \to key$) or $p \to right$ has contained $l$ (if $k \geq p \to key$)
        ▷ (3) $p \to update$ has contained $pupdate$
        ▷ (4) if $l \to key \neq \infty_1$, then the following three statements hold:
        ▷     (4a) $gp$ points to an Internal node
        ▷     (4b) either $gp \to left$ has contained $p$ (if $k < gp \to key$) or $gp \to right$ has contained $p$ (if $k \geq gp \to key$)
        ▷     (4c) $gp \to update$ has contained $gpupdate$
        Internal *$gp$, *$p$
        Node *$l$ := $Root$
        Update $gpupdate, pupdate$                                  ▷ Each stores a copy of an $update$ field

        while $l$ points to an internal node {
                $gp := p$                                           ▷ Remember parent of $p$
                $p := l$                                            ▷ Remember parent of $l$
                $gpupdate := pupdate$                               ▷ Remember $update$ field of $gp$
                $pupdate := p \to update$                           ▷ Remember $update$ field of $p$
                if $k < l \to key$ then $l := p \to left$ else $l := p \to right$   ▷ Move down to appropriate child
        }
        return $\langle gp, p, l, pupdate, gpupdate \rangle$
}

FIND(Key $k$) : Leaf* {
        Leaf *$l$

        $\langle -, -, l, -, - \rangle$ := SEARCH($k$)
        if $l \to key = k$ then return $l$
        else return NULL
}

INSERT(Key $k$) : boolean {
        Internal *$p$, *$newInternal$
        Leaf *$l$, *$newSibling$
        Leaf *$new$ := pointer to a new Leaf node whose $key$ field is $k$
        Update $pupdate, result$
        IInfo *$op$

        while TRUE {
                $\langle -, p, l, pupdate, - \rangle$ := SEARCH($k$)
                if $l \to key = k$ then return FALSE                ▷ Cannot insert duplicate key
                if $pupdate.state \neq$ CLEAN then HELP($pupdate$)  ▷ Help the other operation
                else {
                        $newSibling$ := pointer to a new Leaf whose key is $l \to key$
                        $newInternal$ := pointer to a new Internal node with $key$ field $\max(k, l \to key)$,
                                $update$ field ⟨CLEAN, NULL⟩, and with two child fields equal to $new$ and $newSibling$
                                (the one with the smaller key is the left child)
                        $op$ := pointer to a new IInfo record containing $\langle p, l, newInternal,$ FALSE$\rangle$
                        $Announce[id] := op$
                        $result$ := CAS($p \to update, pupdate, \langle$IFlag$, op \rangle$)   ▷ iflag CAS
                        if $result = pupdate$ then {                ▷ The iflag CAS was successful
                                HELPINSERT($op$)                    ▷ Finish the insertion
                                return TRUE
                        }
                        else HELP($result$)       ▷ The iflag CAS failed; help the operation that caused failure
                }
        }
}

HELPINSERT(IInfo *$op$) {
        ▷ Precondition: $op$ points to an IInfo record (i.e., it is not NULL)
        CAS-CHILD($op \to p, op \to l, op \to newInternal$)         ▷ ichild CAS
        $op \to done$ := TRUE                                       ▷ announce the operation completed
        CAS($op \to p \to update$, ⟨IFlag, $op$⟩, ⟨CLEAN, $op$⟩)    ▷ iunflag CAS
}
```

Figure 6: Pseudocode for SEARCH, FIND and INSERT.

```
DELETE(Key $k$) : boolean {
    Internal $*gp$, $*p$
    Leaf $*l$
    Update $pupdate, gpupdate, result$
    DInfo $*op$

    while TRUE {
        $\langle gp, p, l, pupdate, gpupdate \rangle$ := SEARCH($k$)
        if $l \rightarrow key \neq k$ then return FALSE                          ▷ Key $k$ is not in the tree
        if $gpupdate.state \neq$ CLEAN then HELP($gpupdate$)
        else if $pupdate.state \neq$ CLEAN then HELP($pupdate$)
        else {                                                                 ▷ Try to flag $gp$
            $op$ := pointer to a new DInfo record containing $\langle gp, p, l, pupdate,$ FALSE$\rangle$
            $Announce[id] := op$
            $result$ := CAS($gp \rightarrow update, gpupdate, \langle$DFlag$, op\rangle$)          ▷ dflag CAS
            if $result = gpupdate$ then {                                          ▷ CAS successful
                if HELPDELETE($op$) then return TRUE                           ▷ Either finish deletion or unflag
            }
            else HELP($result$)                  ▷ The dflag CAS failed; help the operation that caused the failure
        }
    }
}

HELPDELETE(DInfo $*op$) : boolean {
    ▷ Precondition: $op$ points to a DInfo record (i.e., it is not NULL)
    Update $result$                                                            ▷ Stores result of mark CAS

    $result$ := CAS($op \rightarrow p \rightarrow update, op \rightarrow pupdate, \langle$MARK$, op\rangle$)        ▷ mark CAS
    if $result = op \rightarrow pupdate$ or $result = \langle$MARK$, op\rangle$ then {        ▷ $op \rightarrow p$ is successfully marked
        HELPMARKED($op$)                                                       ▷ Complete the deletion
        return TRUE                                                            ▷ Tell DELETE routine it is done
    }
    else {                                                                     ▷ The mark CAS failed
        HELP($result$)                                                         ▷ Help operation that caused failure
        CAS($op \rightarrow gp \rightarrow update, \langle$DFlag$, op\rangle, \langle$CLEAN$, op\rangle$)          ▷ backtrack CAS
        return FALSE                                                           ▷ Tell DELETE routine to try again
    }
}

HELPMARKED(DInfo $*op$) {
    ▷ Precondition: $op$ points to a DInfo record (i.e., it is not NULL)
    Node $*other$

    ▷ Set $other$ to point to the sibling of the node to which $op \rightarrow l$ points
    if $op \rightarrow p \rightarrow right = op \rightarrow l$ then $other := op \rightarrow p \rightarrow left$ else $other := op \rightarrow p \rightarrow right$
    ▷ Splice the node to which $op \rightarrow p$ points out of the tree, replacing it by $other$
    CAS-CHILD($op \rightarrow gp, op \rightarrow p, other$)                                 ▷ dchild CAS
    $op \rightarrow done$ := TRUE                                                  ▷ announce the operation completed
    CAS($op \rightarrow gp \rightarrow update, \langle$DFlag$, op\rangle, \langle$CLEAN$, op\rangle$)              ▷ dunflag CAS
}

HELP(Update $u$) {                                                             ▷ General-purpose helping routine
    ▷ Precondition: $u$ has been stored in the $update$ field of some internal node
    if $u.state$ = IFlag then HELPINSERT($u$.info)
    else if $u.state$ = MARK then HELPMARKED($u$.info)
    else if $u.state$ = DFlag then HELPDELETE($u$.info)
}

CAS-CHILD(Internal $*parent$, Node $*old$, Node $*new$) {
    ▷ Precondition: $parent$ points to an Internal node and $new$ points to a Node (i.e., neither is NULL)
    ▷ This routine tries to change one of the child fields of the node that $parent$ points to from $old$ to $new$.
    if $new \rightarrow key < parent \rightarrow key$ then
        CAS($parent \rightarrow left, old, new$)
    else
        CAS($parent \rightarrow right, old, new$)
}
```

Figure 7: Pseudocode for DELETE and some auxiliary routines.

## 5 Elimination Stack

For simplicity, we assume a value $\perp$, which is different from NULL and any other value the stack can store. Since NULL is used as a legit return value, representing the value of POP operation (when exchanging values using the elimination array), NULL can not be used to represent an initialization value, different then any stack value. The same holds for a Node, since a NULL node represent an empty stack, the value $\perp$ is used to distinguish between initialization value and empty stack.

For simplicity, we split the RECOVER routine into sub-routines, based on which operation (PUSH, POP, EXCHANGE) is pending, or needs to be recover. This can be concluded easily by the type of record stored in *Announce*[*pid*] (ExInfo or OpInfo), thus there is no need to explicitly know where exactly in the code the crash took place. Also, the RECOVER routine returns FAIL in case the last pending operation did not took affect (no linearization point), nor it will take in any future run. In such case, the user has the option to either re-invoke the operation, or to skip it, depends on the needs and circumstances of the specific use of the data structure.

The given implementation ignores the log of failures and successes of the exchange routine when recovering. That is, in case of a crash during an EXCHANGE, a process is able to recover the EXCHANGE routine, however, the log of successes and failures is not update, since it might be the process already updated it. In addition, in case of a FAIL response, we do not know whether the time limit (timeout) was reached, or that the process simply crashed earlier in the routine without completing it. The given implementation can be expanded to also consider the log. Nonetheless, for ease of presentation we do not handle the log in case of a crash. Assuming crash events are rare, the log still gives a roughly good approximation to the number of failures and successes, thus our approach might be useful in practice.

### 5.1 A Lock-Free Exchanger

An exchanger object supports the EXCHANGE procedure, which allows exactly two processes to exchange values. If process A calls the EXCHANGE with argument $a$, and B calls the EXCHANGE of the same object with argument $b$, then A's call will return value $b$ and vice versa.

On the original algorithm [cite the book?!], processes race to win the exchanger using a **CAS** primitive. A process accessing the exchanger first reads its content, and act according to the state of it. The first process observe an EMPTY state, and tries to atomically writes its value and change the state to WAITING. In such case, it spins and wait for the second process to arrive. The second, observing the state is now WAITING, tries to write its value and change the state to BUSY. This way, it informs the first one a successful collision took place. Once the first process notice the collision, it reads the other process value and release the exchanger by setting it back to EMPTY. In order to avoid an unbounded waiting, if a second process does not show up, the call eventually timeout, and the process release the exchanger and return.

Assume a process $p$ successfuly capture the exchanger by setting its status to WAITING, followed by a crash. Now, some other process $q$ complete the exchange by setting the exchanger to BUSY. Upon recovery, $p$ can conclude some exchange was completed, but it can not tell whether its value is part of the exchange, and thus it can not complete the operation. Moreover, $p$ and $q$ must agree, otherwise $q$ will return $p$'s value, and thus the operation of $p$ must be linearized together with $q$ operation.

In order to avoid the above problem, we take an approach resembling the BST implementation. Instead of writing a value to the exchanger, processes will use an info record, containing the relevant information for the exchange. This way, processes use the exchanger in order to exchange info records (more precisely, pointers to such records), and not values. To overcome the problematic scenario described earlier, if a process $q$ observe the exchanger state is WAITING with some record *yourop*, it first update its own record *myop* it is about to try and collide with *yourop*, and only then performs the **CAS**. This way, if the collision is successful, the record *myop* which now stored in the exchanger implies which two records collide. Also, the fact that different processes uses different records guarantee that at most one record can collide with *yourop*.

Using records instead of values, when using wisely, allows us to farther improve the algorithm. First, there is no need to store the exchanger's state in it (by using 2 bits of it to mark the state), but we can rather have this info in the record. Second, if there is a BUSY record in the exchanger, it contains the info of the two colliding records. Therefore, a third process, trying to also use the exchanger, can help the processes to complete the collision, and then can try and set the exchanger back to EMPTY, so it can use it again. In the original implementation, a process observaing a BUSY exchanger, have to wait for the first process to read the value and release the exchanger. Therefore, if the first process crash after the collision, the exchanger will be hold by it forever. The helping mechanism avoids this scenario, making the exchange routine non-blocking.

Notice that no exchange record with EMPTY state is ever created, except for the $default$ record. Therefore, reading EMPTY state is equivalent to the exchanger storing a pointer to $default$. A process $p$ creates a new record *myop* when accessing the exchanger, with a unique address. As long as $p$ fails to perform a successful **CAS**, and thus fails to store *myop* in *slot*, it is allowed to try again. However, once a process performs a successful **CAS** and stores *myop* in *slot*, the only other **CAS** it is allowed to do are in order to try and store $defualt$ in *slot*. Thus, *myop* can be written exactly once to *slot*. It follows that a collision can occur between two processes exactly - once a WAITING record stored in *slot*, only a single **CAS** can replace it with a BUSY record. As the two records can not be written again to *slot*, no other process can collide with any of the records.

The EXCHANGE-RECOVER routine relies on the following argument. If a process $p$ successfully wrote $op_p$ to *slot* using the **CAS** in line 81, the only way to overwrite it by a different process $q$, is by a **CAS** in line 104 with a record $op_q$ such that its state is BUSY, and $op_q.partner = op_p$. In addition, the only way to overwrite $op_q$ is by a **CAS** replacing it with $default$, and this is done only after SWITCHPAIR$(op_p, op_q)$ is completed, and thus both *result* fields are updated.

The correctness of the EXCHANGE-RECOVER routine is based on the above argument. There are few scenarios to consider. If $p$ crash after a successful **CAS** in line 81, then $op_p$ state is WAITING. Therefore, when reading *slot* in the EXCHANGE-RECOVER one of the following must hold. If *slot* contains $op_p$, then no process collide with $p$, and $p$ continue to run as if the time limit has been reached. Otherwise, there was a collision. From the above argument, it must be that either $op_q$ that collide with $op_p$ is stored in *slot*, in this case $op_q.partner = op_p$, and $p$ will try to complete the collision and release *slot*, or that $op_q$ has been overwritten, and in this case the *result* field of $op_p$ is updated. In both cases, $p$ returns $op_p.result$. If $p$ crash after a successful **CAS** in line 104, then $op_p$ state is BUSY. It follows from the argument that the only way to overwrite $op_p$ is only after completing the collision by SWITCHPAIR. Thus, either upon recovery $p$ reads $op_p$ from *slot*, and in this case it tries to complete the the operation, or that $op_p.result$ was already updated. In both cases, $p$ returns it. If non of the above holds, then $op_p$ was not involved in any collision, because either no successful **CAS** was done by $p$, or $p$ reached the time limit while no process show up, and was able to set *slot* back to $defualt$. In any case, after the crash of $p$, $op_p$ will never be written again to *slot*, nor any other $op_q$ such that $op_q.partner = op_p$, as any such $op_q$ tries to perform

**CAS** $(op_p, op_q)$ that will fail. Also, as no process can collide with $op_p$, no SwitchPair with $op_p$ as parameter is ever invoked, and in particular $op_p.result = \perp$ for the rest of the execution. This in turn implies that upon recovery $p$ will return Fail, as required.

## 5.2 Lock-Free Stack

The stack implementation is due to [....]. The TryPush routine tries to atomically have a new node pointing to the old top, and then updating the top to be the new node. The TryPop routine tries to atomically read the top of the stack, and change the top to the next node of it. The two routines uses **CAS** in order to gurantee no change for the top was made between the read and write. Push (resp. Pop) routine is alternating between a TryPush (TryPop) routine, which access the central stack, and the Exchange, trying to collide with an opposite operation.

In order to make the implementation recoverable, we need a way to infer whether a Pop or Push already took affect, in case of a crash. Moreover, in case of a Pop, we also need to infer which process is the one to pop the node. For that, we use an approach similar to the Linked-List implementation. Each node contains a new field *popby* which is used to identify a Push of the node completed, as well as a Pop of the node was completed, and who is the process to pop it. Consider the following scenario. Assume a process $p$ performs a Push operation with node $nd$, and using a **CAS** succeed to update the stack top to point to $nd$, followed by a crash. Now, process $q$ performing a Pop operation performs a **CAS** causing the removal of $nd$ from the stack (by changing top to the next node). In this case, once $p$ recovers, $nd$ is no longer part of the stack, and it is also not marked as deleted. This is indistinguishable from a configuration in which the Push of $nd$ was yet to take affect (a crash before **CAS**), and thus $p$ can not know what the right response is.

One way to solve this issue is by first marking a node for removal, and only then remove it. This way, if a node is no longer part of the stack it must be marked, and thus we can conclude it was in the stack, and the Push routine was successful. However, such an implementation, in addition for the need of to system to support a markable reference, also requires process to help each other. If a node is marked for delete, then a process trying to perform a different operation first needs to complete the deletion, before applying its own operation, otherwise the physical delete of the node may not take place, leaving the node forever in the stack. As the original algorithm avoids any marking, and simply tries to swing the $Top$ pointer, we would like to maintain this property.

A field *popby* is initialised to $\perp$ when a node $nd$ is created. Once the node is successfully insert to the stack by a Push operation, the inserting process tries to mark it by changing *popby* to Null using a **CAS**. Before a process tries to remove the node from the stack during a Pop routine, it first mark it as part of the stack by doing the same thing, helping the inserting process conclude the node is in the stack. This replace the logic delete of the node, as we only need to know the node was part of the stack if it is removed. After a successful **CAS** to remove $nd$ from the stack, another **CAS** is used in order to try and set *popby* to the identifier of the process who performed the **CAS**. The use of **CAS** to change *popby* from $\perp$to Null, and from Null to an identifier guarantee that only the first process to perform each of these **CAS** will succeed. Note that before writing an identifier to *popby* a process must try and set it to Null, and thus it can not store two different identifiers along in any execution.

The correctness proof follows the same guidelines as of the proof for the Linked-List. If a Push operation did not introduce a new node $nd$ into the stack, then no process but $p$ is aware of $nd$. Thus, upon recovery the Search routine will not find $nd$ in the stack, nor its *popby* field has been changed, and the PUSH-ROCEOVER returns FAIL. Otherwise, *nd* was successfully inserted to the stack. As discessed above, the only way to delete *nd* from the stack is by first changing its *popby* field to NULL. Thus, upon recovery *p* will either find *nd* in the stack, using the SEARCH routine, or that *popby* is different then $\perp$ in case it was deleted, and in both cases it returns **true**. For the POP routine, if *p* tries to remove a node *nd* from the top of the stack and crash, then upon recovery it first check if *nd* is still in the stack using the SEARCH routine. If it is so, then clearly *nd* was yet to delete, and it returns FAIL. Otherwise, *nd* was deleted, either by *p* or by some other process. Only the first process of which to performs a **CAS**, writing its identifier to *popby* will return the value stored in *nd*, while the others return either $\perp$ (in the TRYPOP routine) or FAIL (in the POP-RECOVER routine).

Notice that both PUSH-ROCEOVER and POP-RECOVER are wait-free. Due to the structure of stack, no *next* pointer of any node in the stack is ever changed. Therefore, once a process reads $Top$ at the beginning of its RECOVER routine, the chain of pointers from this $Top$ to the last node in the stack is fixed for the rest of the execution, and thus traversing it using the SEARCH routine is wait-free.

```
Type Node {
    T value
    int popby
    Node *next
}

Type PushInfo {                    ▷ subtype of Info
    Node *pushnd
}

Type PopInfo {                     ▷ subtype of Info
    Node *popnd
}

Type ExInfo {                      ▷ subtype of Info
    {EMPTY, WAITING, BUSY} state
    T value, result
    ExInfo *partner, *slot
}
```

Figure 8: Type definition

ExInfo $default$ - global static ExInfo object with state = EMPTY

**Algorithm 6:** T EXCHANGE (ExInfo $*slot$, T $myitem$, long $timeout$)

```
long timeBound := getNanos() + timeout
ExInfo myop := new ExInfo(WAITING, myitem, ⊥, ⊥, slot)
Announce[pid] := myop
while true do
    if getNanos() > timeBound then
        myop.result := TIMEOUT                                  // time limit reached
        return TIMEOUT
    yourop := slot
    switch yourop.state do
        case EMPTY
            myop.state := WAITING                               // attempt to replace default
            myop.partner := ⊥
            if slot.CAS(yourop, myop)  then                     // try to collide
                while getNanos() < timeBound do
                    yourop := slot
                    if yourop ≠ myop then                       // a collision was done
                        if youop.parnter = myop then            // yourop collide with myop
                            SWITCHPAIR(myop, yourop)
                            slot.CAS(yourop, default)           // release slot
                        return myop.result
                end
                // time limit reached and no process collide with me
                if slot.CAS(myop, default) then                 // try to release slot
                    myop.result := TIMEOUT
                    return TIMEOUT
                else                                            // some process show up
                    yourop := slot
                    if yourop.partner = myop then
                        SWITCHPAIR(myop, yourop)                // complete the collision
                        slot.CAS(yourop, default)               // release slot
                    return myop.result
            end
            break
        case WAITING                                            // some process is waiting in slot
            myop.partner := yourop                              // attempt to replace yourop
            myop.state := BUSY
            if slot.CAS(yourop, myop)  then                     // try to collide
                SWITCHPAIR(myop, yourop)                        // complete the collision
                slot.CAS(myop, default)                         // release slot
                return myop.result
            break
        case BUSY                                               // a collision in progress
            SWITCHPAIR(yourop, yourop.parnter)                  // help to complete the collision
            slot.CAS(yourop, default)                           // release slot
            break
    endsw
end
```

**Algorithm 7:** void SwitchPair(ExInfo *first*, ExInfo *second*)

```
    /* exchange the valus of the two operations                                  */
115 $first.result := second.value$
116 $second.result := first.value$
```

**Algorithm 8:** T Visit (T *value*, int *range*, long *duration*)

```
    /* invoke Exchange on a random entery in the collision array                  */
117 int $cell$ := randomNumber($range$)
118 return Exchange($exchanger[cell], value, duration$)
```

**Algorithm 9:** T Exchange-Recover ()

```
119 ExInfo $*myop := Announce[pid]$                      // read your last operation record
120 ExInfo $*slot := myop.slot$                          // and the slot on which it acts
121 if $myop.state$ = Waiting then
        /* crash while trying to exchange $defualt$, or waiting for a collision    */
122     $yourop := slot$
123     if $yourop = myop$ then                          // still waiting for a collision
124         if $slot$.CAS($myop, default$) then               // try to release $slot$
125             return Fail
126         else                                          // some process show up
127             $yourop := slot$
128             if $yourop.partner = myop$ then
129                 SwitchPair($myop, yourop$)            // complete the collision
130                 $slot$.CAS($yourop, default$)                  // release $slot$
131             return $myop.result$
132     else if $yourop.partner = myop$ then             // $yourop$ collide with $myop$
133         SwitchPair($myop, yourop$)                    // complete the collision
134         $slot$.CAS($yourop, default$)                          // release $slot$
135         return $myop.result$
136 if $myop.state$ = Busy then
        /* crash while trying to collide with $myop.partner$                       */
137     $yourop := slot$
138     if $yourop = myop$ then                 // collide was successful and in progress
139         SwitchPair($myop, myop.partner$)              // complete the collision
140         $slot$.CAS($myop, default$)                            // release $slot$
141         return $myop.result$
142 if $myop.result \neq \perp$ then
143     return $myop.result$                         // collide was successfuly completed
144 else
145     return Fail
```

Figure 9: Elimination Array routines

**Algorithm 10:** boolean TryPush (Node $*nd$)

```
/* attempt to perform Push to the central stack                                   */
Node $*oldtop := Top$
$nd.next := oldtop$
if $Top.$CAS$(oldtop, nd)$ then                           // try to declare $nd$ as the new $Head$
    $nd.popby.$CAS$(\perp,$ Null$)$                             // announce $nd$ is in the stack
    return true
return false
```

**Algorithm 11:** boolean Push (T $myitem$)

```
Node $*nd$ = new Node ($myitem$)
$nd.popby := \perp$
PushInfo $data$ := new PushInfo ($nd$)
while true do
    $Announce[pid] := data$                          // declare - trying to push node $nd$
    if TryPush($nd$) then                        // if central stack Push is successful
        return true
    $range$ := CalculateRange()                    // get parameters for collision array
    $duration$ := CalculateDuration()
    $othervalue$ := Visit($myitem, range, duration$)                    // try to collide
    if $othervalue$ = Null then             // successfuly collide with Pop operation
        RecordSuccess ()
        return true
    else if $othervalue$ = Timeout then                                 // failed to collide
        RecordFailure ()
end
```

**Algorithm 12:** boolean Push-Roceover ()

```
Node $*nd := Announce[pid].pushnd$
if $nd.popby \neq \perp$ then                          // $nd$ was announced to be in the stack
    return true
if Search($nd$) || $nd.popby \neq \perp$ then   // $nd$ in the stack, or was announced as such
    $nd.popby.$CAS$(\perp,$ Null$)$                             // announce $nd$ is in the stack
    return true
return Fail
```

**Algorithm 13:** boolean Search (Node $*nd$)

```
/* search for node $nd$ in the stack                                                 */
Node $*iter := Top$
while $iter \neq \perp$ do
    if $iter = nd$ then
        return true
    $iter := iter.next$
end
return false
```

Figure 10: Push routine

**Algorithm 14:** T TryPop()

```
Node *$oldtop$ := $Top$
Node *$newtop$
$Announce[pid].popnd$ := $oldnop$             // declare - trying to pop node oldtop
if $oldtop = \bot$ then                              // stack is empty
    return Empty
$newtop$ := $oldtop.next$
$oldtop.popby$.CAS($\bot$, Null)                // announce oldtop is in the stack
if $Top$.CAS($oldtop, newtop$) then   // try to pop oldtop by changing Top to newtop
    if $newtop.popby$.CAS(Null, $pid$) then     // try to announce yourself as winner
        return $oldtop.value$
else
    return $\bot$
```

**Algorithm 15:** T Pop ()

```
Node *$result$
PopInfo $data$ := new PopInfo ($Top$)
while true do
    $Announce[pid]$ := $data$                  // declare - trying to perform Pop
    $result$ := TryPop()                      // attempt to pop from central stack
    if $result \neq \bot$ then                  // if central stach Pop is successful
        return $result$
    $range$ := CalculateRange()               // get parameters for collision array
    $duration$ := CalculateDuration()
    $othervalue$ := Visit(Null, $range, duration$)                 // try to collide
    if $othervalue$ = Timeout then                              // failed to collide
        RecordFailure ()
    else if $othervalue \neq$ Null then    // successfuly collide with Push operation
        RecordSuccess ()
        return $othervalue$
end
```

**Algorithm 16:** T Pop-Recover()

```
Node *$nd$ := $Announce[pid].popnd$           // crash while trying to pop node nd
if $nd = \bot$ then                                     // pop from an empty stack
    return Empty
if Search($nd$) then                          // nd was not removed from the stack
    return Fail
$nd.popby$.CAS(Null, $pid$)       // nd was removed.  Try to complete the operation
if $nd.popby = pid$ then               // you are the process to win the pop of nd
    return $nd.value$
return Fail
```

Figure 11: Pop routine

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