## 1 Elimination Stack

For simplicity, we assume a value  $\perp$ , which is different from Null and any other value the stack can store. Since Null is used as a legit return value, representing the value of Pop operation (when exchanging values using the elimination array), Null can not be used to represent an initialization value, different then any stack value. The same holds for a Node, since a Null node represent an empty stack, the value  $\perp$  is used to distinguish between initialization value and empty stack.

For simplicity, we split the RECOVER routine into sub-routines, based on which operation (PUSH, POP, EXCHANGE) is pending, or needs to be recover. This can be concluded easily by the type of record stored in Announce[pid] (ExInfo or OpInfo), thus there is no need to explicitly know where exactly in the code the crash took place. Also, the RECOVER routine returns FAIL in case the last pending operation did not took affect (no linearization point), nor it will take in any future run. In such case, the user has the option to either re-invoke the operation, or to skip it, depends on the needs and circumstances of the specific use of the data structure.

The given implementation ignores the log of failures and successes of the exchange routine when recovering. That is, in case of a crash during an EXCHANGE, a process is able to recover the EXCHANGE routine, however, the log of successes and failures is not update, since it might be the process already updated it. In addition, in case of a FAIL response, we do not know whether the time limit (timeout) was reached, or that the process simply crashed earlier in the routine without completing it. The given implementation can be expanded to also consider the log. Nonetheless, for ease of presentation we do not handle the log in case of a crash. Assuming crash events are rare, the log still gives a roughly good approximation to the number of failures and successes, thus our approach might be useful in practice.

## 1.1 A Lock-Free Exchanger

An exchanger object supports the Exchange procedure, which allows

```
Type Node {
     T value
     int popby
     Node *next
}
                                 \rhd subtype of Info
Type Push
Info {
     Node *pushNd
Type PopInfo {
                                 \triangleright subtype of Info
     Node *popNd
}
Type ExInfo \{
                                  \triangleright subtype of Info
     {Empty, Waiting, Busy} state
     \Tau\ value, result
     {\bf ExInfo} *partner, *slot
}
```

Figure 1: Type definition

## Algorithm 1: T Exchange (ExInfo \*slot, T myitem, long timeout)

```
1 long timeBound := getNanos() + timeout
2 ExInfo myop := \text{new ExInfo}(\text{Waiting}, myitem, \bot, \bot, slot)
3 Announce[pid] := myop
4 while true do
      if getNanos() > timeBound then
          myop.result := Timeout
                                                                           // time limit reached
6
          return Timeout
      yourop := slot
8
      \mathbf{switch}\ your op. state\ \mathbf{do}
          case Empty
10
             myop.state := Waiting
                                                                       // try to replace default
11
              myop.partner := \bot
12
             if slot.CAS(yourop, myop) then
13
                 while getNanos() < timeBound do
14
                     yourop := slot
15
                     if yourop \neq myop then
                                                                         // a collision was done
16
                         if youop.parnter = myop then
                                                                    // yourop collide with myop
                            SWITCHPAIR(myop, yourop)
18
                            slot. \mathbf{CAS}(yourop, default)
                                                                                   // release slot
19
                         return myop.result
20
21
                 end
                 // time limit reached and no process collide with me
                 if slot.\mathbf{CAS}(myop, default) then
                                                                           // try to release slot
22
                     myop.result := Timeout
23
                     return Timeout
24
                 else
                                                                         // some process show up
25
                     yourop := slot
26
                     if yourop.partner = myop then
27
                         SWITCHPAIR(myop, yourop)
                                                                      // complete the collision
28
                         slot. CAS(yourop, default)
                                                                                   // release slot
                     return myop.result
30
              end
31
             break
32
          case Waiting
                                                              // some process is waiting in slot
33
             myop.partner := yourop
                                                                   // attempt to replace yourop
34
              myop.state := Busy
35
             if slot.CAS(yourop, myop) then
                                                                                // try to collide
36
                 SWITCHPAIR(myop, yourop)
                                                                       // complete the collision
37
                 slot. \mathbf{CAS}(myop, default)
                                                                                   // release slot
                 return myop.result
39
             break
40
          case Busy
                                                                      // a collision in progress
41
             SWITCHPAIR(yourop, yourop.parnter)
                                                             // help to complete the collision
42
              slot. CAS(yourop, default)
                                                                                   // release slot
43
              break
44
      endsw
45
46 end
```

```
Algorithm 2: void SWITCHPAIR(ExInfo first, ExInfo second)
  // exchange the valus of the two operations
47 first.result := second.value
48 second.result := first.value
 Algorithm 3: T VISIT (T value, int range, long duration)
  // invoke EXCHANGE on a random selected entery in the collision array
49 int cell := randomNumber(range)
50 return EXCHANGE(exchanger[cell], value, duration)
 Algorithm 4: T Exchange-Recover ()
\texttt{51} ExInfo *myop := Announce[pid]
                                                        // read your last operation record,
52 ExInfo *slot := myop.slot
                                                             // and the slot on which it act
if myop.state = Waiting then
      // crash while trying to exchange defualt, or waiting for a process to collide
         with me
      yourop := slot
54
      if yourop = myop then
                                                              // still waiting for a collide
55
         if slot.CAS(myop, default) then
                                                                       // try to release slot
56
            return Fail
57
         else
                                                                      // some process show up
58
             yourop := slot
59
             if yourop.partner = myop then
60
                SWITCHPAIR(myop, yourop)
                                                                   // complete the collision
61
                slot. CAS(yourop, default)
                                                                               // release slot
62
             return myop.result
63
      if yourop.partner = myop then
                                                                 // yourop collide with myop
64
         SWITCHPAIR(myop, yourop)
                                                                   // complete the collision
65
         slot. CAS(yourop, default)
                                                                               // release slot
66
         return myop.result
67
68 if myop.state = Busy then
      // crash while trying to collide with myop.partner
      yourop := slot
69
      if yourop = myop then
                                                                   // collide was successful
70
         SWITCHPAIR(myop, myop.partner)
                                                                   // complete the collision
71
         slot. \mathbf{CAS}(myop, default)
                                                                               // release slot
72
         return myop.result
  if myop.result \neq \bot then
      return myop.result
                                                        // collide was successfuly completed
75
76 else
      return Fail
77
```

Figure 2: Elimination Array routines

```
Algorithm 5: boolean TryPush (Node *nd)
78 Node *oldTop := Top
 79 nd.next := oldTop
 so if Top.CAS(oldTop, nd) then
       nd.popby.\mathbf{CAS}(\bot, \text{NULL})
 82
       return true
 83 return false
  Algorithm 6: boolean Push (T myitem)
84 Node *nd = \text{new Node } (myitem)
 85 nd.popby := \bot
 86 PushInfo data := \text{new PushInfo } (nd)
 87 while true do
       Announce[pid] := data
 88
       if TryPush(nd) then
89
          return true
90
       range := CalculateRange()
91
       duration := CalculateDuration()
 92
       othervalue := Visit(myitem, range, duration)
 93
       if othervalue = Null then
94
95
           RecordSuccess ()
           return true
 96
       else if othervalue = Timeout then
 97
           RecordFailure ()
 98
99 end
  Algorithm 7: boolean Push-Roceover ()
100 Node *nd := Announce[pid].pushNd
101 if nd.popby \neq \bot then
       return true
103 if Search(nd) || nd.popby \neq \bot then
       nd.popby.\mathbf{CAS}(\bot, \mathrm{Null})
104
       return true
105
106 return Fail
  Algorithm 8: boolean Search (Node *nd)
107 Node *iter := Top
108 while iter \neq \bot do
       if iter = nd then
109
          return true
110
       iter := iter.next
111
112 end
113 return false
```

Figure 3: Push routine

```
Algorithm 9: T TryPop()
114 Node *oldTop := Top
115 Node *newTop
116 Announce[pid].popNd := oldTop
117 if oldTop = \bot then
       return Empty
119 newTop := oldTop.next
120 oldTop.popby.\mathbf{CAS}(\bot, Null)
121 if Top.\mathbf{CAS}(oldTop, newTop) then
       if newTop.popby.CAS(Null, pid) then
          return oldTop.value
123
124 else
       return \perp
125
  Algorithm 10: T POP ()
126 Node *result
127 PopInfo data := new PopInfo (Top)
128 while true do
       Announce[pid] := data
129
       result := TryPop()
130
       if result \neq \bot then
131
132
          return result
       range := CalculateRange()
133
       duration := CalculateDuration()
134
       othervalue := Visit(Null, range, duration)
135
       if othervalue = Timeout then
136
          RecordFailure ()
137
       else if othervalue \neq Null then
138
          RecordSuccess ()
139
          return othervalue
140
141 end
  Algorithm 11: T POP-RECOVER()
142 Node *nd := Announce[pid].popNd
143 if nd = \bot then
       return Empty
145 if SEARCH(nd) then
       return Fail
147 nd.popby.CAS(Null, pid)
148 if nd.popby = pid then
       return nd.value
149
150 return Fail
```

Figure 4: Pop routine