Problems*

CHAPTER 1

1. Show that the set of points on a semicircle has the cardinality of the continuum.

2. Show that the union of two countable sets is countable.

3. Show that the set of rational numbers is countable.

4. We know that an infinite subset of a countable set is countable. Use this fact to show that the set of primes is countable.

5. Use equivalent transformations to convert the following six functions to a form containing only disjunction, conjunction, and negation:

   1. $f(x_1, x_2) = (x_1 \bigtriangledown x_2) \bigtriangledown (1/x_2)$
   2. $f(x_1, x_2) = x_2/(x_1 \leftarrow x_2)$
   3. $f(x_1, x_2) = x_1 \sim (x_1 \downarrow x_2)$
   4. $f(x_1, x_2) = x_1 \bigtriangledown (x_1/x_2)$
   5. $f(x_1, x_2, x_3) = (x_1 \downarrow x_2) \leftarrow (x_3/x_1)$
   6. $f(x_1, x_2, x_3) = (x_1/x_2) \bigtriangledown (\bar{x}_3 \rightarrow x_1)$

6. Find the complete disjunctive normal form of the function

$$f(x_1, x_2) = x_1 \bigtriangledown x_2 \,\&\, (x_1 \,\&\, x_2) .$$

7. Find the complete conjunctive normal form of the function

$$f(x_1, x_2, x_3) = (x_1 \bigtriangledown x_2) \,\&\, (\bar{x}_2 \bigtriangledown x_3) \,\&\, (x_1 \bigtriangledown \bar{x}_2) \,\&\, (x_2 \bigtriangledown x_3) .$$

8. For each of the following two functions, find the complete disjunctive and conjunctive normal forms, constructing as a preliminary the characteristic table:

$$f_1(x_1, x_2) = x_1 \rightarrow \bar{x}_2$$
$$f_2(x_1, x_2) = x_1 \sim x_2$$

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*In all problems the symbol x corresponds to the symbol $\mathscr{x}$ originally used in the Russian edition and throughout the text. Reader is advised to note the difference in solving the problem.

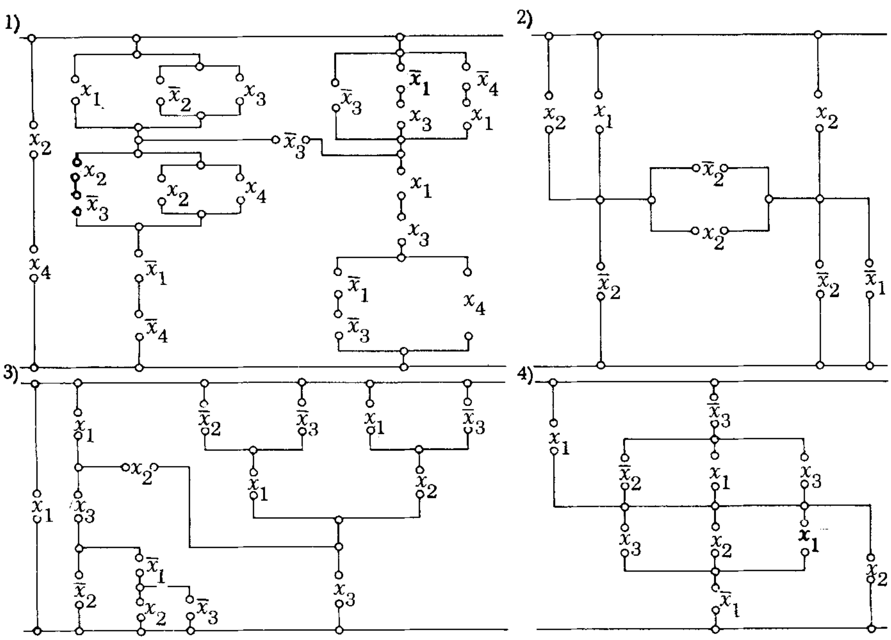
9. For the function $f(x_1, x_2) = x_1 \vee \bar{x}_2$ construct the complete disjunctive normal form, simplifying this function as a preliminary.

10. Given the predicate $P(x, y, z) = [x \to (y \to z)]$ find the predicate $Q(y, z) = (\forall x) P(x, y, z)$

11. Given the predicate $P(x, y, z) = [\overline{x \vee (y \sim z)}]$ find the predicate $Q(y, z) = (\exists x) P(x, y, z)$.

CHAPTER 2

1. For the following four contact diagrams, find the simplest equivalent circuits:



2. Each of the following two tables gives the values of two logical functions y_1 and y_2 (all told, four functions). Construct the contact diagrams corresponding to these functions

   a) by the canonical method,

   b) by the block method.

Table 1

x_1	0	I	0	I
x_2	0	0	I	I
y_1	I	I	I	0
y_2	0	I	I	0

Table 2

x_1	0	I	0	I	0	I	0	I
x_2	0	0	I	I	0	0	I	I
x_3	0	0	0	0	I	I	I	I
y_1	I	0	I	0	I	0	0	I
y_2	0	I	0	0	0	I	I	0

3. From a given logical function $y = (x_1 \,\&\, x_2) \nabla (\bar{x}_1 \,\&\, x_2)$ construct the diagram at the diodes that realize this function.

4. From a given logical function $y = (x_2 \,\&\, x_3) \,\&\, \bar{x}_1$ construct the scheme on the triodes that realize this function.

5. With the aid of Quine's algorithm, find the minimum disjunctive normal form of the following functions:

$$y(x_1, x_2, x_3) = (x_1 \,\&\, \bar{x}_2 \,\&\, x_3) \nabla (\bar{x}_1 \,\&\, \bar{x}_2 \,\&\, x_3) \nabla (x_1 \,\&\, x_2 \,\&\, x_3) \nabla (x_1 \,\&\, x_2 \,\&\, \bar{x}_3)$$

$$y(x_1, x_2, x_3) = (\bar{x}_1 \,\&\, \bar{x}_2 \,\&\, \bar{x}_3) \nabla (x_1 \,\&\, x_2 \,\&\, x_3) \nabla (x_1 \,\&\, \bar{x}_2 \,\&\, x_3) \nabla (x_1 \,\&\, \bar{x}_2 \,\&\, \bar{x}_3).$$

CHAPTER 3

1. Give an example of a dynamic system that can be treated as a finite automaton.

2. The table below is a combined table of an automaton and a transformer. Construct the graph and the interconnection matrix of unions of this sequential machine.

$x \backslash \rho$	ρ_1	ρ_2
x_1	$x_4\lambda_0$	$x_1\lambda_1$
x_2	$x_1\lambda_0$	$x_3\lambda_1$
x_3	$x_2\lambda_1$	$x_1\lambda_0$
x_4	$x_2\lambda_0$	$x_2\lambda_1$

3. Suppose that you are given the interconnection matrix C shown. Construct a table of an automaton of the P-P type and the table of transformations.

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 0 & \rho_1\lambda_1 & \rho_3\lambda_0 & 0 & \rho_2\lambda_1 \\ \rho_3\lambda_0 & \rho_1\lambda_0 & 0 & \rho_2\lambda_0 & 0 \\ 0 & \rho_2\lambda_1 & \rho_1\lambda_1 & \rho_3\lambda_1 & 0 \\ 0 & 0 & \rho_2\lambda_0 \vee \rho_3\lambda_1 & 0 & \rho_1\lambda_1 \\ \rho_1\lambda_0 \vee \rho_3\lambda_0 & 0 & 0 & \rho_2\lambda_0 & 0 \end{bmatrix} \end{array}$$

4. Suppose that we have an s-machine of the P-Pr type defined by two tables, namely, the basic table of the automaton and the table of the output transformer. Construct its diagram of states and interconnection matrix.

Automaton

$x \backslash \rho$	ρ_1	ρ_2
x_1	x_2	x_5
x_2	x_3	x_4
x_3	x_4	x_3
x_4	x_2	x_5
x_5	x_1	x_5

Transformer

$x \backslash \rho$	ρ_1	ρ_2
x_1	λ_1	—
x_2	λ_1	—
x_3	λ_0	λ_1
x_4	λ_0	λ_0
x_5	—	λ_0

5. Do the same thing for the following machine of the P-Pr type:

Automaton

$x \backslash \rho$	ρ_1	ρ_2
x_1	x_2	x_3
x_2	x_1	x_3
x_3	x_6	x_6
x_4	x_3	x_4
x_5	x_6	x_3
x_6	x_2	x_6

Transformer

$x \backslash \rho$	ρ_1	ρ_2
x_1	λ_0	—
x_2	λ_1	—
x_3	λ_0	λ_1
x_4	—	λ_0
x_5	—	—
x_6	λ_1	λ_1

6. On the basis of the examples given, state the general properties of the interconnection matrix of an s-machine of the P-Pr type.

7. Suppose that we are given a finite automaton of the P-Pr type with output transformer as shown:

Automaton

$x \backslash \rho$	ρ_1	ρ_2	ρ_3
x_1	x_4	x_2	x_5
x_2	x_3	x_3	x_5
x_3	x_5	x_1	x_2
x_4	x_1	x_2	x_2
x_5	x_3	x_3	x_2

Transformer

x	x_1	x_2	x_3	x_4	x_5
λ	λ_2	λ_3	λ_1	λ_2	λ_3

Is the entire system an automaton, that is, does there exist a single-valued function F^* such that $\lambda^p = F^*(\lambda^{p-1}, \rho^p)$.

8. Suppose that we are given an s-machine of the P-P type together with a table of the automaton and the output transformer. Let us assign to each pair $\rho_i\lambda_j$ the symbol θ_k from the alphabet $\{\theta_1, \theta_2, \theta_3, \ldots, \theta_{16}\}$. Is there a single-valued function F^* such that $\theta^p = F^*(\theta^{p-1}, \rho^p)$?

$x \backslash \rho$	ρ_1	ρ_2	ρ_3	ρ_4
x_1	$x_1\lambda_3$	$x_3\lambda_2$	$x_2\lambda_1$	$x_4\lambda_1$
x_2	$x_3\lambda_1$	$x_1\lambda_4$	$x_5\lambda_3$	$x_2\lambda_2$
x_3	$x_3\lambda_2$	$x_6\lambda_3$	$x_4\lambda_2$	$x_3\lambda_4$
x_4	$x_2\lambda_3$	$x_4\lambda_2$	$x_2\lambda_1$	$x_1\lambda_1$
x_5	$x_1\lambda_4$	$x_3\lambda_1$	$x_5\lambda_4$	$x_4\lambda_3$
x_6	$x_3\lambda_1$	$x_4\lambda_4$	$x_5\lambda_3$	$x_6\lambda_2$

CHAPTER 4

1. Construct a block diagram of the automaton described by table 4.10 in the following cases:

   (a) for every $i, k_i = 2$ and $r_i = 3$;

   (b) for every $i, k_i = 3$ and $r_i = 2$;

   (c) for every $i, k_i = r_i = 3$.

2. From the neurons of McCulloch and Pitts all logical functions of two variables.

3. Construct a trigger from the neurons of McCulloch and Pitts.

CHAPTER 5

1. From the following four tables, determine the types of automata or sequential machines.

Table 1

$x \backslash \rho$	ρ_1	ρ_2	ρ_3	ρ_4
x_1	$x_2\lambda_1$	$x_1\lambda_1$	$x_1\lambda_1$	$x_3\lambda_1$
x_2	$x_1\lambda_3$	$x_3\lambda_3$	$x_2\lambda_3$	$x_2\lambda_3$
x_3	$x_1\lambda_2$	$x_2\lambda_2$	$x_2\lambda_2$	$x_3\lambda_2$

Table 2

$x \backslash \rho$	ρ_1	ρ_2	ρ_3	ρ_4
x_1	$x_2\lambda_1$	$x_1\lambda_1$	$x_1\lambda_1$	$x_3\lambda_2$
x_2	$x_1\lambda_3$	$x_2\lambda_2$	$x_2\lambda_3$	$x_2\lambda_3$
x_3	$x_1\lambda_2$	$x_2\lambda_2$	$x_1\lambda_1$	$x_3\lambda_2$

Table 3

$x \backslash \rho$	ρ_1	ρ_2	ρ_3	ρ_4
x_1	$x_2\lambda_1$	$x_1\lambda_1$	$x_1\lambda_3$	$x_3\lambda_3$
x_2	$x_1\lambda_3$	$x_3\lambda_2$	$x_2\lambda_2$	$x_2\lambda_1$
x_3	$x_1\lambda_3$	$x_2\lambda_3$	$x_2\lambda_2$	$x_3\lambda_3$

Table 4

$x \backslash \rho$	ρ_1	ρ_2	ρ_3	ρ_4
x_1	$x_2\lambda_1$	$x_1\lambda_2$	$x_1\lambda_2$	$x_3\lambda_3$
x_2	$x_1\lambda_2$	$x_3\lambda_3$	$x_2\lambda_1$	$x_2\lambda_1$
x_3	$x_1\lambda_2$	$x_2\lambda_1$	$x_2\lambda_1$	$x_3\lambda_3$

2. From the tables of the preceding exercise, construct the tables of transitions and minimize them.

3. From the same tables, construct $(2S_0 + 1)$ realizations of Huffman.

CHAPTER 6

1. Synthesize an s-machine with input alphabet $\{\rho_1, \rho_2, \rho_3\}$ and output $\{\lambda_1\lambda_2\}$ such that, for an initial state x_0 and fixed ρ^*, if $\rho^* = \rho_1$, the periodic sequence $\lambda_1\lambda_1\lambda_2\lambda_1$ will be generated; if $\rho^* = \rho_2$, the periodic sequence $\lambda_1\lambda_2\lambda_2$ will be generated; for $\rho^* = \rho_3$, the periodic sequence $\lambda_2\lambda_1\lambda_2$ will be generated.

2. Do the same thing as in problem 1 but with the alphabet $\rho = \{\rho_1, \rho_2\}$ and the alphabet $\lambda = \{\lambda_1, \lambda_2, \lambda_3\}$
If $\rho^* = \rho_1$, the sequence $\lambda_3\lambda_1\lambda_1\lambda_2\lambda_2\lambda_1\lambda_2\lambda_2$ will be generated with period $\lambda_1\lambda_2\lambda_2$;
if $\rho^* = \rho_2$, the sequence $\lambda_2\lambda_3\lambda_2\lambda_3\lambda_1\lambda_1\lambda_1\lambda_1$ will be generated with period λ_1.

3. A periodic input sequence is applied at the input of an arbitrary s-machine. Show that the periodic sequence of output symbols is determined by a finite number of moments at the output of that machine.

CHAPTER 7

1. Show that the events mentioned in examples 1-14 of Section 7.2 are regular and, by using the concept of chains of triads, construct automata representing these events.

2. Suppose that we are given the alphabet $\{\rho_1, \rho_2\}$. The set L contains all words consisting of letters of that alphabet with the exception of words in which the same letter occurs twice in a row. Show that the set L is regular. Write the regular expression for it.

3. Do the same thing as in problem 2 for the alphabet $\{\rho_1, \rho_2, \rho_3\}$. Is the assertion of regularity of the set L so constructed true for an arbitrary finite alphabet?

4. What event is represented by the automaton shown in Fig. 3 of Chapter 2 by means of the set of events $\{x_2, x_3\}$ if it begins from the initial state x_1? Write the regular expression for this event.

5. Show that the intersection of two regular sets is regular.

6. An s-machine is said to be strongly connected if, for every pair x_i and x_j of states of that machine there exists an input sequence that takes the machine from the state x_i into the state x_j.
Let S denote the subset of the states of a strongly connected machine. Let x_k denote the initial state. We denote by R_k the event that the subset S is in the initial state x_k.
Show that $\bigcup_K R_K = ER$, where E is the universal event and R is some regular event.
7. Let $f(t)$ denote an integer-valued function such that $0 \leq f(t) \leq t$ and

$$\overline{\lim_{t \to \infty}}\, f(t) = \overline{\lim_{t \to \infty}}\, [t - f(t)] = \infty$$

Show that the event "the number of symbols ρ_1 from the zeroth to the tth moment is equal to $f(t)$" is not regular.

8. Suppose that we are given a finite automaton A with initial state x^0. Let R denote a set of sequences at the input. Suppose that to each of these sequences is assigned a sequence in a set K of sequences of states. Show that, if R is regular, so is K. Does nonregularity of R imply nonregularity of K?

9. Suppose that we are given an s-machine with initial state x^0. At the input of this machine, sequences from the universal set E are applied. Show that the set of output sequences of the machine is regular.

10. Under the conditions of exercise 9, suppose that only sequences belonging to some regular set L are applied at the input. Is the assertion that the output sequences constitute a regular set valid in this case?

CHAPTER 8

1. Synthesize a finite automaton that represents by the appearance of the symbol λ_1 at the output the regular event

$$R = \{[\rho_1(\rho_1)^* \vee \rho_3(\rho_3)^* \cdot \rho_1(\rho_1)^*] \cdot \rho_2(\rho_2)^*\}^*.$$

2. Do the same thing for

$$R = [(\rho_1 \triangledown \rho_3\rho_1)\rho_2]^* .$$

3. Synthesize a finite automaton representing the following definite event: the input sequence terminates with the symbols $\rho_1\rho_2$ or $\rho_3\rho_4\rho_1$. Write the regular expression for this event.

4. Synthesize the indicator of evenness of a discrete time moment. The regular expression corresponding to it has the form

$$R = [(\rho_1 \triangledown \rho_2 \triangledown \rho_3 \triangledown \rho_4)(\rho_1 \triangledown \rho_2 \triangledown \rho_3 \triangledown \rho_4)]^*.$$

CHAPTER 9

1. By using the algorithm of Aufenkamp and Hohn, show that the machine of Fig. 1 is minimal. The alphabets: $\rho = \{\rho_1, \rho_2\}$

$$x = \{x_1, x_2, x_3, x_4\} ; \lambda = \{\lambda_0, \lambda_1\}$$

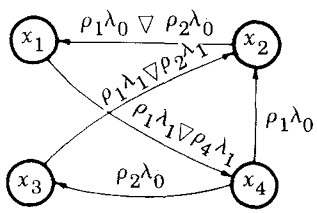


Fig. 1

2. Minimize the machine with interconnection matrix C by using the algorithm of Aufenkamp and Hohn:

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 0 & \rho_1\lambda_0 & 0 & \rho_2\lambda_1 & 0 & \rho_3\lambda_0 \\ \rho_2\lambda_0 & 0 & \rho_3\lambda_0 & 0 & \rho_1\lambda_1 & 0 \\ 0 & \rho_1\lambda_1 & 0 & \rho_3\lambda_0 & 0 & \rho_2\lambda_1 \\ \rho_2\lambda_0 & 0 & \rho_3\lambda_0 & 0 & \rho_1\lambda_1 & 0 \\ 0 & \rho_1\lambda_0 & \rho_2\lambda_0 & \rho_3\lambda_1 & 0 & 0 \\ \rho_3\lambda_1 & 0 & \rho_1\lambda_0 & 0 & \rho_2\lambda_0 & 0 \end{bmatrix} \end{array}$$

3. Show whether the following machine does or does not have equivalent states:

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & \rho_2\lambda_0 & \rho_1\lambda_1 \\ 0 & 0 & \rho_1\lambda_1 & 0 & \rho_2\lambda_0 \\ 0 & \rho_2\lambda_0 & \rho_1\lambda_1 & 0 & 0 \\ 0 & 0 & \rho_2\lambda_1 & \rho_1\lambda_1 & 0 \\ \rho_2\lambda_0 & 0 & 0 & \rho_1\lambda_1 & 0 \end{bmatrix} \end{array}$$

4. Minimize the following machine (with respect to strong equivalence):

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 0 & \rho_1\lambda_0 & 0 & \rho_2\lambda_0 & 0 & 0 \\ 0 & 0 & \rho_1\lambda_1 & 0 & 0 & \rho_2\lambda_0 \\ \rho_2\lambda_0 & 0 & 0 & \rho_1\lambda_0 & 0 & 0 \\ 0 & \rho_2\lambda_0 & 0 & 0 & 0 & \rho_1\lambda_0 \\ \rho_1\lambda_0 & 0 & \rho_2\lambda_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_1\lambda_1 \nabla \rho_2\lambda_0 & 0 \end{bmatrix} \end{array}$$

5. From the interconnection matrix C draw a diagram of the states of the machine. Minimize it by using the algorithm of Aufenkamp and Hohn (strong equivalence).

$$C = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} \rho_2\lambda_0 & \rho_1\lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \rho_2\lambda_0 & \rho_1\lambda_0 & 0 & 0 & 0 \\ 0 & 0 & \rho_2\lambda_0 & \rho_1\lambda_0 & 0 & 0 \\ 0 & 0 & 0 & \rho_2\lambda_0 & \rho_1\lambda_1 & 0 \\ 0 & 0 & 0 & 0 & \rho_2\lambda_0 & \rho_1\lambda_0 \\ \rho_1\lambda_0 & 0 & 0 & 0 & 0 & \rho_2\lambda_0 \end{bmatrix} \end{array}$$

6. Show that the machine of Fig. 2 is minimal (strong equivalence). Find the groups of equivalent states in the case of the set of admissible input sequences L that contain all sequences of length 2.

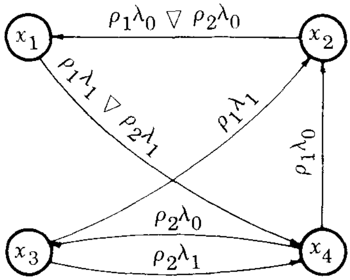


Fig. 2

7. Show whether the machine of Fig. 3 does or does not have equivalent states. For this machine, construct partitions of all states into equivalence classes with respect to input sequences of length 1, 2, 3, and 4.

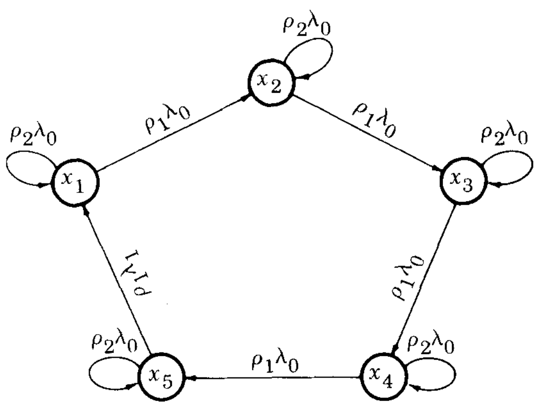


Fig. 3

8. Show whether the machine of Fig. 4 has equivalent states or not. Construct partitions into groups of equivalent states with respect to input sequences of length 1, 2, and 3.

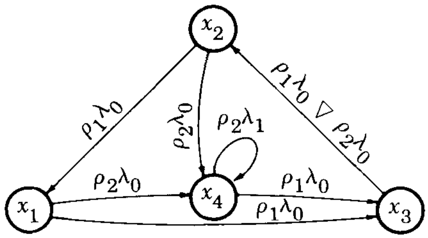


Fig. 4

9. The machine of Fig. 5 does not have equivalence states. However, if we take the definition of weak equivalence, corresponding to this machine is a minimal machine with three states. Construct it.

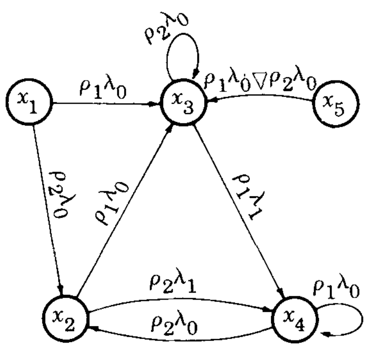


Fig. 5

10. Minimize the machine of Fig. 6 with respect to weak equivalence.

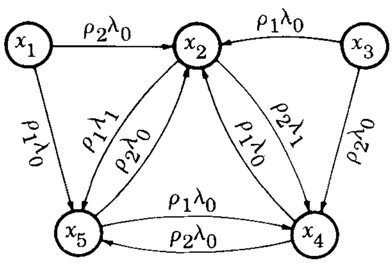
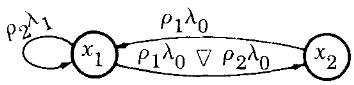


Fig. 6

11. Denote by E_l the set of all input sequences of length l. For arbitrary fixed l^*, construct for a given machine A (see Fig. 7) a machine B equivalent to A in the sense of weak equivalence with respect to E_{l^*} but not equivalent with respect to $E_{l^{**}}$ for $l^{**} > l^*$.



Machine A

Fig. 7

12. An s-machine is said to be strongly connected if, for every pair x_i and x_j of states of that machine there exists an input sequence that converts the machine from the state x_i to the state x_j. Show that, for strongly connected machines, weak equivalence of two machines implies their strong equivalence (so that in this case the concepts of strong and weak equivalence coincide.)

13. Show that, for completely determined automata (that is, automata without restrictions of the Aufenkamp type) for an arbitrary set of input sequences L and arbitrary x_i, x_j, x_k the equivalences with respect to L

$$x_i \sim x_j \quad ; \quad x_j \sim x_k$$

imply $x_i \sim x_k$ (transitivity of equivalence). Show that the number of groups of equivalent states with respect to L is the same in all machines that are pairwise equivalent with respect to L.

14. The set L contains a single sequence $\rho_1\rho_2$. Minimize the machine of Fig. 8 up to the 4th states with respect to L (strong equivalence)

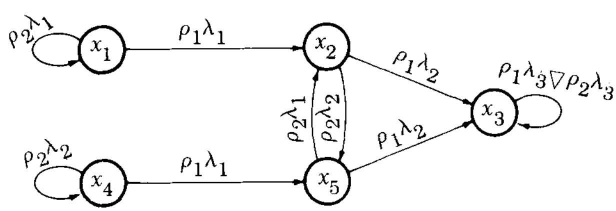


Fig. 8

(Hint: direct the arrow λ_1 from the state x_4x_5 to the state x_1 and remove the arrow $\rho_2\lambda_1$ from x_5 to x_2.)

15. The set L contains a single sequence $\rho_1\rho_2\rho_2$. Minimize up to the 4th states the machine of Fig. 9 with respect to L (strong equivalence).

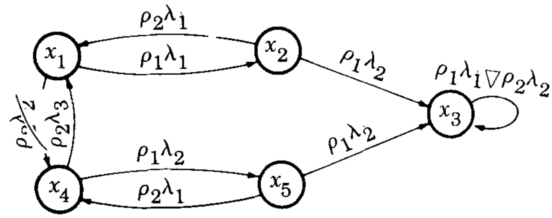


Fig. 9

16. Suppose that we are given the machine of Fig. 10 and the set L containing a single sequence $\rho_1\rho_2\rho_2$. In the machine the states x_2 and x_5 are equivalent with respect to L. However, it is imposible to minimize the machine of Fig. 10. Prove this. (Here, it is a question of minimization with respect to L in the sense of either strong or weak equivalence.)

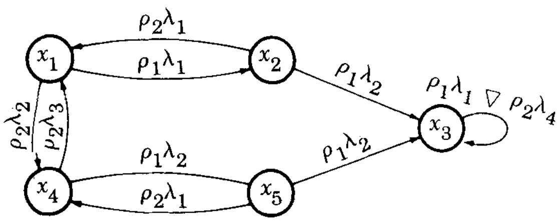


Fig. 10

17. The set L contains all sequences of input symbols such that a single symbol does not appear twice in a row. Minimize the machine of Fig. 11 with respect to L up to the 3rd states with respect to strong equivalence and up to the second states with respect to weak equivalence.

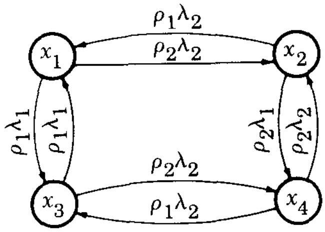


Fig. 11

18. Suppose that we are given the automaton of Fig. 12 with input alphabet $\{\rho_1, \rho_2, \rho_3\}$ and restrictions of the Aufenkamp type. Write the regular expressions of the set of admissible input sequences for the states $x_1, x_2, x_3, x_4,$ and x_5

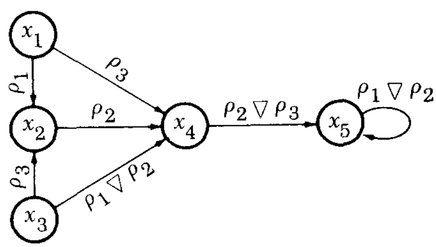


Fig. 12

19. Do the same thing as in exercise 18 for the automaton of Fig. 13.

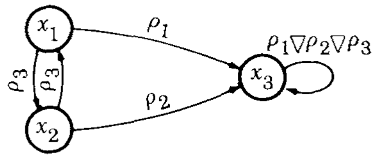


Fig. 13

Is the set of admissible input sequences for an arbitrary state x_i of the automaton with restrictions of the Aufenkamp type always regular?

20. Suppose that we are given an arbitrary s-machine with restrictions of the Aufenkamp camp that is in the initial state x_0. All sequences in the set L_{x_0} of sequences admissible for the state x_0 are applied at the input. Is the set of output sequences of this machine regular?

21. Is the theorem of regularity of representable events for an automaton with restrictions of the Aufenkamp type valid?

22. Minimize the s-machine of Fig. 14 with restrictions of the Aufenkamp type.

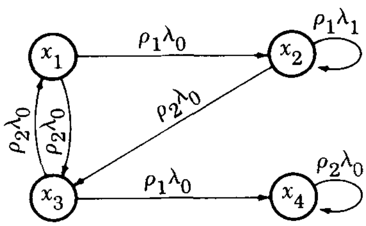


Fig. 14

23. To simplify the work, suppose that the following procedure is chosen for minimizing an s-machine with restrictions of the Aufenkamp type: First, we minimize the given machine in accordance with Aufenkamp's algorithm (symmetric partitioning into generalized i-matrices) and then we minimize the resulting machine by Hill's method. Does this approach guarantee construction of a minimal machine?

24. Figure 15 shows a diagram of the states of the s-machine. In that machine there are no restrictions of the Aufenkamp type (all transfers are determined) but the output transformer is undetermined: we do not know the value of λ for x_2 and ρ_1 (the loop in the diagram of the states). One can easily show that no extension of the definition of the output transformer will make it possible to minimize the machine of Fig. 15; that is, equivalence states do not arise.

However, it is possible to minimize this machine if we understand the word minimize in the following way: for a given s-machine M, it is required to construct an s-machine N such that the following two conditions are satisfied: (1) To every state x_i of the machine M there corresponds at least one state x_j of the machine N such that, for an arbitrary input sequence with the initial states x_i and x_j, the output sequence of the machine M coincides, wherever it is designed, with the output sequence of the machine N.

(2) No s-machine N' exists satisfying condition (1) with fewer states than the machine N.

Minimize the machine of Fig. 15 in accordance with this definition up to the 2nd states.

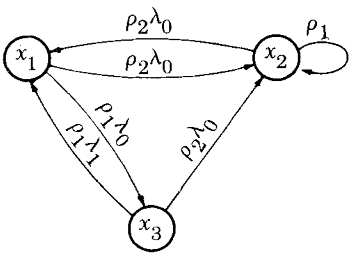


Fig. 15

CHAPTER 10

1. Suppose that we are given a slow machine G (see Fig. 1). Construct a minimal fast machine reproducing it under a clock-rate transformation with the clock rate determined by a change in the state at the input of the machine G.

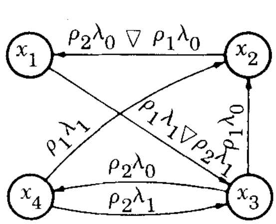


Fig. 1

2. Do the same thing as in problem 1 for the slow machine G of Fig. 2.

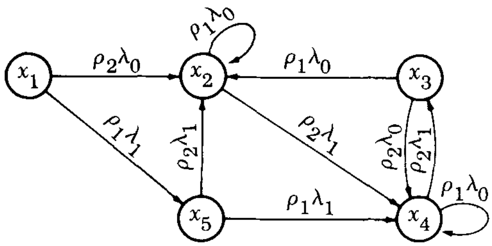


Fig. 2

In the following problems, the law of transformation of the clock rate is as follows: a slow moment occurs at the instant $\rho + 1$ if a regular event R occurs at the instant ρ, that is, the sequence $\rho_0\rho_1 \cdots \rho_p$ belongs to a regular set R.

3. Show that the machine S the diagram of the states of which is shown in Fig. 3 does not represent any slow machine if the law of transformation of clock rate is given by the regular set $R = (\rho_1^*\rho_2)^*$.

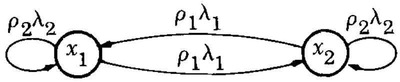


Fig. 3

4. A fast machine S is shown in Fig. 4. The law of transformation of clock rate is determined by a regular set R that can be represented in the automaton A (see Fig. 5) from the initial state a_0 by a set of states $\{a_0, a_1, a_2, a_3\}$. Construct a machine G that the machine S represents under such a transformation of clock rate.

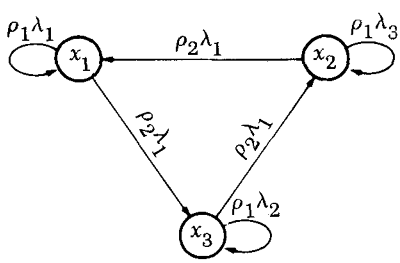


Fig. 4

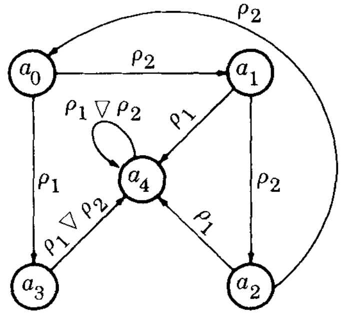


Fig. 5

5. Let S denote an arbitrary machine with initial state x_0 and let R denote a regular set defining a transformation of clock rate. Consider the set of all slow tapes obtained from fast ones as a result of the transformation of clock-rate. In this set, let us denote by λ_1 some event G_i (corresponding to the set of slow input sequences that lead to the occurence λ_i). Show that the set G_i is regular.

6. For the machine S shown in Fig. 4 and the machine G shown in Fig. 6, construct a regular set R such that the machine S reproduces the machine G under a transformation of clock rate determined by the set R.

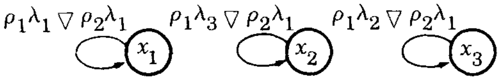


Fig. 6

(Hint: consider the following correspondence of events:

$$x_1^S \leftrightarrow x_1^G \,;\; x_2^S \leftrightarrow x_2^G \,;\; x_3^S \leftrightarrow x_3^G \;.)$$

7. Let S and G denote machines. Let x^G denote a state of the machine G. What conditions must the machine S satisfy for it to be possible to construct a regular set R such that corresponding to the state x^G there will be a state x^S of the machine S in the sense of reproduction.

CHAPTER 11

1. Suppose that Fig. 1 is the diagram of states of a strongly connected s-machine S.

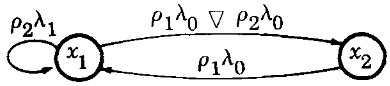


Fig. 1

Construct a diagram of states of the s-machine N for which the result of an experiment of length q coincides with the result of the experiment with the given machine S for arbitrary initial states of the s-machine N. (this last condition differs from the condition of the example given on page 399 of original Russian, Figs. 11.1 and 11.2).

2. Suppose that Fig. 2 is the diagram of states of an s-machine. Find the shortest experiment determining the last state of this machine under the condition that all the states can be initial.

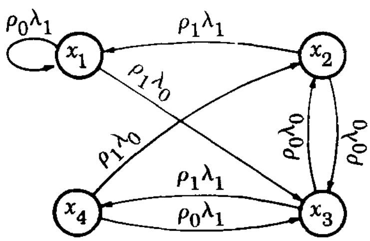


Fig. 2

3. Show that the estimates (11.17) and (11.18) are exact for arbitrary N. To do this, construct diagrams of the states of an s-machine and a finite automaton with N states. To determine the last states of these, it is necessary to make experiments the lengths of which are determined by these estimates.

4. Figure 3 shows the diagrams of the states of three finite automata. Show that it is possible to single out any one of them by an experiment of length not exceeding 4.

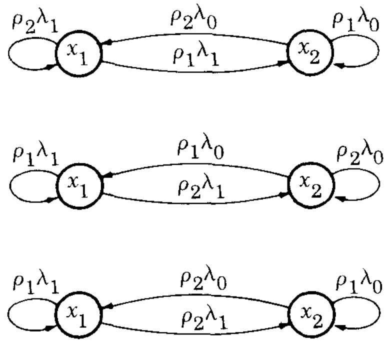


Fig. 3

5. Find an experiment with the aid of which we can ascertain the structure of any one of four finite automata shown in Fig. 4, that is, we can run through all states and all arrows of any one of these automata if the state κ_2 is the initial state.

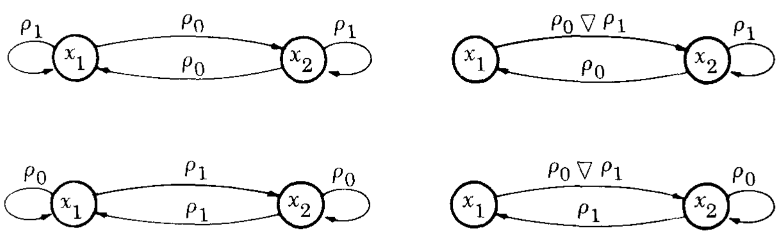


Fig. 4

6. Construct the diagram of the states of a strongly connected s-machine with N states to which it necessary to apply an input sequence of length $N^2/4$ in order to go through all its states. Is the quantity $N^2/4$ maximum for the length of the experiment as a result of which an arbitrary strongly connected machine goes through all states or is it possible to find a strongly connected machine for which an experiment of greater length is required if the machine is to go through all states?

7. Suppose that we are given the set of all strongly connected machines with N states. Show that there exists an experiment making it possible to go through all states of any one of these

machines independently of its initial state and estimate the length of the experiment.

8. Figure 5 shows the diagram of the states of a strongly connected s-machine with four nonequivalent states. One can easily show that it is possible to set up the following one-to-one correspondence between the results of experiments and states into which the s-machine goes at the end of these experiments.

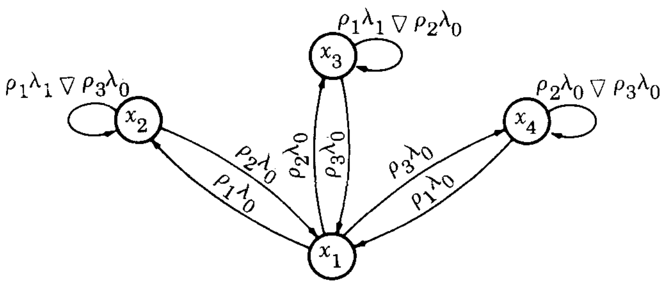


Fig. 5

The result corresponds to a transfer of the s-machine into the state

ρ_1	ρ_1	ρ_2		κ_1
λ_0	λ_1	λ_0		

ρ_1	ρ_1	κ_2
λ_0	λ_1	

ρ_1	ρ_1	ρ_2	ρ_2	κ_3
λ_0	λ_1	λ_0	λ_0	

ρ_1	ρ_1	ρ_2	ρ_3	κ_4
λ_0	λ_1	λ_0	λ_0	

Try to show that for a given strongly connected s-machine with N nonequivalent states, then to each of its states x_i it is possible to assign an experiment of length not exceeding N the result of which indicates unambiguously that the machine has gone into just that state x_i. (In this example, this estimate is attained.)

9. Suppose that we are given a set of 2^n strongly connected finite automata of the form shown in Fig. 6 (locks).

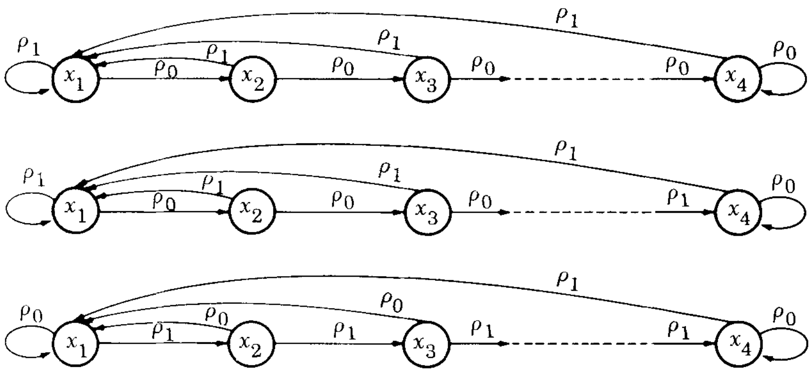


Fig. 6

At the beginning of the experiment, the finite automata are in the state κ_1. Show that an upper bound q of the length of the experiment necessary for attaining the state κ_n of any one of these automata is determined by the equation $q = 2^n \ln n$.