1.33:

Least Square Estimate of β_0

$$Q = \sum_{i=1}^{n} (y_i - \hat{y})^2$$

$$= \sum_{i=1}^{n} (y_i - (\hat{\beta_0}))^2$$

$$\frac{\partial Q}{\partial \hat{\beta_0}} = \sum_{i=1}^{n} 2 \cdot (y_i - \hat{\beta_0}) \cdot (-1)$$

$$= -2 \cdot \sum_{i=1}^{n} (y_i - \hat{\beta_0}) = 0$$

$$\therefore \sum_{i=1}^{n} (y_i - \hat{\beta_0}) = 0$$

$$\therefore n \cdot \hat{\beta_0} = \sum_{i=1}^{n} y_i$$

$$\hat{\beta_0} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\beta_0} = \overline{y_i}$$

1.34:

Show $\hat{\beta}_0$ is unbiase

$$E(\hat{\beta}_0) = E(\bar{y}_i)$$

$$= E\left(\frac{y_1 + y_2 + \cdots y_n}{n}\right)$$

$$= E\left(\frac{\sum_{i=1}^{n} y_i}{n}\right)$$

$$= E\left(\frac{\sum_{i=1}^{n} (\beta_0 + \varepsilon_i)}{n}\right)$$

$$= E(\beta_0 + \varepsilon_i)$$

$$= E(\beta_0) + E(\varepsilon_i)$$

where $E(\varepsilon_i) = 0$, we have:

$$E(\hat{\beta}_0) = E(\beta_0) = \beta_0$$

Thus $\hat{\beta}_0$ is unbiased

1.39:

To prove they're Indentical, First we calculate the former one's parameters:

① Three points: $(5, \bar{Y}_1), (10, \bar{Y}_2), (15, \bar{Y}_3)$

$$\bar{X} = \frac{5+10+15}{3} = 10, \quad \bar{Y} = \frac{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3}{3}$$

$$b_1 = \frac{(5-10)(\bar{Y}_1 - \bar{Y}) + (10-10)(\bar{Y}_2 - \bar{Y}) + (15-10)(\bar{Y}_3 - \bar{Y})}{(5-10)^2 + (10-10)^2 + (15-10)^2}$$

$$= \frac{-5\bar{Y}_1 + 5\bar{Y}_3}{50} = \frac{-\bar{Y}_1 + \bar{Y}_3}{10}$$

$$b_0 = \bar{Y} - b_1 \cdot \bar{X} = \bar{Y} - 10\bar{X}$$

Then we calculate the later one's parameters:

② Six points: $(5, Y_1), (5, Y_1'), (10, Y_2), (10, Y_2'), (15, Y_3), (15, Y_3')$

where: $\bar{Y}_1 = (Y_1 + Y_1')/2$, $\bar{Y}_2 = (Y_2 + Y_2')/2$,

$\bar{Y}_3 = (Y_3 + Y_3')/2$,

$$\bar{X} = 10, \quad \bar{Y} = \frac{Y_1 + Y_1' + Y_2 + Y_2' + Y_3 + Y_3'}{6}$$

$$b_1' = \frac{(5-10)(Y_1 - \bar{Y}) + (5-10)(Y_1' - \bar{Y}) + (10-10)(Y_2 - \bar{Y}) + (10-10)(Y_2' - \bar{Y}) + (15-10)(Y_3 - \bar{Y}) + (15-10)(Y_3' - \bar{Y})}{(5-10)^2 + (5-10)^2 + (10-10)^2 + (10-10)^2 + (15-10)^2 + (15-10)^2}$$

$$b_1' = \frac{-5Y_1 + 5\bar{Y} - 5Y' + 5\bar{Y} + 5Y_3 - 5\bar{Y} + 5Y_3' - 5\bar{Y}}{100}$$

$$b_1' = \frac{-Y_1 - Y_1' + Y_3 + Y_3'}{20} = \frac{-\frac{(Y_1 + Y_1')}{2} + \frac{(Y_3 + Y_3')}{2}}{10}$$

$$b_1' = \frac{-\bar{Y_1} + \bar{Y_3}}{10} = b_1$$

$$b_0' = \bar{Y} - b_1' \cdot \bar{X}$$

$\because b_1' = b_1$,

$\therefore b_0' = b_0$

Thus $b_1' = b_1$, $b_0' = b_0$, Those two lines are Indentical. $\blacksquare$