A: Theory

We consider a unigram as a single word and a bigram as an ordered pair of words in the form $\{w_1, w_2\}$ separated by space. We have two dictionaries Pw and Pw2 that contains unigrams and bigrams respectively along with their frequencies. We use the following notations.

Notations	Meaning
Pw[w]	frequency of w in Pw dictionary
$Pw2[w_1, w_2]$	frequency of $\{w_1, w_2\}$ bigram in Pw2 dictionary
V	Total number of different unigrams or bigrams
N	Sum of frequencies
voc[w]	Number of different bigrams started by w

There can be different cases that might happen when we analyze any bigram. However, when we analyze the probability of the very first word of the document or when we can not find the first word w_1 of an ordered pair $\{w_1, w_2\}$ in the dictionary Pw, we only use the unigram model to assign the probability of the word.

According to the unigram model, The probability of a word w is given by:

$$P(w) = \frac{Pw[w] + 1}{Pw.N + Pw.V + 1} \tag{1}$$

if it is found in Pw or

$$P(w) = \frac{1}{Pw.N + Pw.V + 1} \tag{2}$$

if it is not found in Pw

However, we use bigram model for every ordered pair $\{w_1, w_2\}$ when w_1 is found in Pw. According to the bigram model, the conditional probability of w_2 given that we already observed w_1 is calculated. There can be any of the following three cases.

(i) Bigram $\{w_1, w_2\}$ in Pw2

In this case there is a connection between two known words. We consider conditional probability for w_2 as:

$$P(w_2|w_1) = \frac{Pw2[w_1, w_2] + 1}{Pw[w_1] + Pw2.voc[w_1] + 1}$$
(3)

(ii) Bigram not in Pw2, w_2 in Pw

Although there is no observed connection, both are known words. In this case, we assign the conditional probability based on the frequency of w_2 in Pw.

$$P(w_2|w_1) = \frac{Pw[w_2] + 1}{(Pw[w_1] + Pw2.voc[w_1] + 1)(Pw.N + Pw.V + 1)}$$
(4)

(iii) Bigram not in Pw2, w_2 is not in Pw

In this case, w_2 is completely unknown. Instead of assigning a zero probability, we assign probability of w_2 as

$$P(w_2|w_1) = \frac{1}{(Pw[w_1] + Pw2.voc[w_1] + 1)(Pw.N + Pw.V + 1)}$$
(5)

B: Data Structure

Data Structure	Description
Pw,Pw2	Dictionaries with frequencies of unigrams and bigrams respectively
chart	Dynamic programming Table to store the words
entry	A structure {word, start position, log probability, back pointer}
pq	priority queue of entries, priority= lowest starting

C: Algorithm

The algorithm is given below:

Algorithm 1 Word Segmentation

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Input:
           File with non segmented words Input,
    unigram dictionary Pw,
    bigram dictionary Pw_2
Output: File with segmented words
 1: Initialize chart = \{\} and maxlen \leftarrow longest bigram length
 2: Initialize priority queue with starting position as priority
 3: for l in range(1,...,maxlen) do
      w_1 \leftarrow \text{substring input}[0,...,l-1]
      if w_1 is found in Pw then
 5:
 6:
         calculate P(w_1) according to unigram model
         add \{w_1, 0, P[w_1], None\} into pq
 7:
 8: if nothing inserted in last for loop then
      w_1 \leftarrow \text{input}[0]
 9:
      calculate P(w_1) according to unigram model
10:
      add \{w_1, 0, Pw[w_1], None\} into pq
11:
12: while pq not empty do
      item \leftarrow highest priority item from pq
13:
      end \leftarrow item.length + item.start
14:
      if item.probability > chart[end].probability then
15:
16:
         chart[end] \leftarrow item
      nextstart \leftarrow end + 1
17:
      if w_1 is found in Pw then
18:
         for l in range(1,...,maxlen) do
19:
            w_2 \leftarrow \text{substring input[nextstart,...,nextstart+l-1]}
20:
21:
            calculate P(w_2|w_1) according to bigram model
            add \{w_2, nextstart, P(w_1) * P(w_2|w_1), w_1\} into pq
22:
         if nothing inserted in last for loop then
23:
            w_1 \leftarrow \text{input}[0]
24:
            calculate P(w_1) according to unigram model
25:
            add \{w_2, nextstart, P(w_1) * P(w_2|w_1), w_1\} into pq
26:
27:
      else
         for l in range(1,...,maxlen) do
28:
            w_2 \leftarrow \text{substring input[nextstart,...,nextstart+l-1]}
29:
            calculate P(w_2) according to unigram model
30:
            add \{w_2, nextstart, P(w_1) * P(w_2), w_1\} into pq
31:
         if nothing inserted in last for loop then
32:
            w_2 \leftarrow \text{input}[\text{nextstart}]
33:
34:
            calculate P(w_2) according to unigram model
            add \{w_2, nextstart, P(w_1) * P(w_2), w_1\} into pq
35:
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