MR Bugiardi

Lorenzo Livio Vaccarecci (matr. 5462843)

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1 Codice

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1 import random
3 dict = {
      "P": "Probabilmente primo",
      "C": "Probabilmente composto"
5
6 }
8 def MCPrimalityTest(n, a=None):
9
    s = 0
    q = n - 1
10
11
    while q % 2 == 0:
12
     s += 1
      q //= 2
13
    if a is None:
14
15
       a = random.randint(2,n-2)
16
    x = pow(a, q, n)
17
    if x == 1 or x == n-1:
18
      return dict["P"], s, q
19
    i = s
20
    while i-1 >= 0:
      x = pow(x, 2, n)
21
22
      if x == n-1:
23
       return dict["P"], s, q
      i -= 1
24
25
    return dict["C"], s, q
26
27 \text{ def MCD(a, b)}:
28
     while b:
29
           a, b = b, a \% b
30
       return a
31
32 def Z(n):
33
    return [a for a in range(1,n) if MCD(a,n) == 1]
34
35 def H(n):
    return [a for a in Z(n) if pow(a,n-1,n) == 1]
36
37
38
39
41 def MRLiars(witnesses, n):
   L = []
42
43
    for a in witnesses:
44
      if MCPrimalityTest(n,a)[0] == dict["P"]:
45
        L.append(a)
46
    return L
48 carmichael = [561,1105,1729,2465,2821,6601,8911]
49 for n in carmichael:
50 primality = MCPrimalityTest(n)
51 if primality[0] == dict["C"]:
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Hn = H(n)
liars = MRLiars(Hn, n)
print(f"I bugiardi per {n}({len(liars)}) sono: {liars}")
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Numero	Bugiardi
561	1, 50, 101, 103, 256, 305, 458, 460, 511, 560
1105	1, 47, 72, 98, 123, 132, 242, 256, 268, 293, 327, 341, 353, 463, 548, 557, 642, 752, 764, 778, 812, 837, 849, 863, 973, 982, 1007, 1033, 1058, 1104
1729	$\begin{array}{c} 1, 9, 10, 12, 16, 69, 74, 75, 81, 90, 92, 100, 103, 108, 120, 129, 144, 160, 166, 172, 173, 181, 191, 192, 235, 256, 257, 263, 282, 289, 302, 335, 347, 355, 363, 365, 374, 376, 386, 402, 426, 433, 438, 439, 443, 484, 493, 529, 536, 545, 555, 562, 563, 568, 575, 584, 621, 625, 638, 649, 653, 654, 666, 675, 690, 699, 706, 729, 740, 750, 757, 797, 802, 807, 809, 810, 828, 829, 831, 841, 857, 872, 888, 898, 900, 901, 919, 920, 922, 927, 932, 972, 979, 989, 1000, 1023, 1030, 1039, 1054, 1063, 1075, 1076, 1080, 1091, 1104, 1108, 1145, 1154, 1161, 1166, 1167, 1174, 1184, 1193, 1200, 1236, 1245, 1286, 1290, 1291, 1296, 1303, 1327, 1343, 1353, 1355, 1364, 1366, 1374, 1382, 1394, 1427, 1440, 1447, 1466, 1472, 1473, 1494, 1537, 1538, 1548, 1556, 1557, 1563, 1569, 1585, 1600, 1609, 1621, 1626, 1629, 1637, 1639, 1648, 1654, 1655, 1660, 1713, 1717, 1719, 1720, 1728 \\ \end{array}$
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2821	1, 9, 12, 16, 17, 75, 81, 100, 107, 108, 113, 144, 153, 165, 166, 181, 183, 191, 192, 199, 204, 211, 220, 235, 244, 251, 256, 272, 285, 289, 290, 295, 326, 363, 373, 380, 386, 402, 417, 426, 433, 438, 443, 446, 484, 517, 529, 530, 534, 536, 547, 555, 556, 562, 568, 584, 599, 615, 625, 627, 638, 647, 649, 653, 654, 659, 675, 690, 699, 706, 712, 718, 719, 729, 740, 766, 790, 797, 807, 818, 820, 829, 836, 841, 872, 881, 893, 894, 900, 919, 920, 922, 932, 939, 963, 972, 985, 989, 1002, 1010, 1013, 1017, 1018, 1030, 1076, 1082, 1083, 1091, 1093, 1101, 1102, 1109, 1121, 1166, 1174, 1192, 1193, 1195, 1200, 1221, 1252, 1275, 1284, 1286, 1290, 1296, 1303, 1327, 1336, 1353, 1356, 1366, 1374, 1375, 1377, 1444, 1446, 1447, 1455, 1465, 1468, 1485, 1494, 1518, 1525, 1531, 1535, 1537, 1546, 1569, 1600, 1621, 1626, 1628, 1629, 1647, 1655, 1700, 1712, 1719, 1720, 1728, 1730, 1738, 1739, 1745, 1791, 1803, 1804, 1808, 1811, 1819, 1832, 1836, 1849, 1858, 1882, 1889, 1899, 1901, 1902, 1921, 1927, 1928, 1940, 1949, 1980, 1985, 1992, 2001, 2003, 2014, 2024, 2031, 2055, 2081, 2092, 2102, 2103, 2109, 2115, 2122, 2131, 2146, 2162, 2167, 2168, 2172, 2174, 2183, 2194, 2196, 2206, 2222, 2237, 2253, 2259, 2265, 2266, 2274, 2285, 2287, 2291, 2292, 2304, 2337, 2375, 2378, 2383, 2388, 2395, 2404, 2419, 2435, 2441, 2448, 2458, 2495, 2526, 2531, 2532, 2536, 2549, 2565, 2570, 2577, 2586, 2601, 2610, 2617, 2622, 2629, 2630, 2638, 2640, 2655, 2656, 2668, 2677, 2708, 2713, 2714, 2721, 2740, 2746, 2804, 2805, 2809, 2812, 2820
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