## MR Bugiardi

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A.A. 2023/2024

## 1 Codice

```
1 import random
3 dict = {
      "P": "Probabilmente primo",
      "C": "Probabilmente composto"
5
6 }
8 def MCPrimalityTest(n, a=None):
9
    s = 0
    q = n - 1
10
11
    while q % 2 == 0:
12
     s += 1
      q //= 2
13
    if a is None:
14
15
       a = random.randint(2,n-2)
16
    x = pow(a, q, n)
17
    if x == 1 or x == n-1:
18
      return dict["P"], s, q
19
    i = s
20
    while i-1 >= 0:
      x = pow(x, 2, n)
21
22
      if x == n-1:
23
       return dict["P"], s, q
      i -= 1
24
25
    return dict["C"], s, q
26
27 \text{ def MCD(a, b)}:
28
     while b:
29
           a, b = b, a \% b
30
       return a
31
32 def Z(n):
33
    return [a for a in range(1,n) if MCD(a,n) == 1]
34
35 def H(n):
    return [a for a in Z(n) if pow(a,n-1,n) == 1]
36
37
38
39
41 def MRLiars(witnesses, n):
   L = []
42
43
    for a in witnesses:
44
      if MCPrimalityTest(n,a)[0] == dict["P"]:
45
        L.append(a)
46
    return L
48 carmichael = [561,1105,1729,2465,2821,6601,8911]
49 for n in carmichael:
50 primality = MCPrimalityTest(n)
51 if primality[0] == dict["C"]:
```

```
Hn = H(n)
liars = MRLiars(Hn, n)
print(f"I bugiardi per {n}({len(liars)}) sono: {liars}")
```

Numero	Bugiardi
561	1, 50, 101, 103, 256, 305, 458, 460, 511, 560
1105	1, 47, 72, 98, 123, 132, 242, 256, 268, 293, 327, 341, 353, 463, 548, 557, 642, 752, 764, 778, 812, 837, 849, 863, 973, 982, 1007, 1033, 1058, 1104
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2465	1, 47, 72, 98, 157, 217, 242, 254, 256, 293, 302, 327, 387, 426, 438, 472, 548, 582, 596, 693, 727, 752, 888, 897, 934, 1007, 1019, 1033, 1058, 1092, 1143, 1152, 1177, 1203, 1228, 1237, 1262, 1288, 1313, 1322, 1373, 1407, 1432, 1446, 1458, 1531, 1568, 1577, 1713, 1738, 1772, 1869, 1883, 1917, 1993, 2027, 2039, 2078, 2138, 2163, 2172, 2209, 2211, 2223, 2248, 2308, 2367, 2393, 2418, 2464
2821	1, 9, 12, 16, 17, 75, 81, 100, 107, 108, 113, 144, 153, 165, 166, 181, 183, 191, 192, 199, 204, 211, 220, 235, 244, 251, 256, 272, 285, 289, 290, 295, 326, 363, 373, 380, 386, 402, 417, 426, 433, 438, 443, 446, 484, 517, 529, 530, 534, 536, 547, 555, 556, 562, 568, 584, 599, 615, 625, 627, 638, 647, 649, 653, 654, 659, 675, 690, 699, 706, 712, 718, 719, 729, 740, 766, 790, 797, 807, 818, 820, 829, 836, 841, 872, 881, 893, 894, 900, 919, 920, 922, 932, 939, 963, 972, 985, 989, 1002, 1010, 1013, 1017, 1018, 1030, 1076, 1082, 1083, 1091, 1093, 1101, 1102, 1109, 1121, 1166, 1174, 1192, 1193, 1195, 1200, 1221, 1252, 1275, 1284, 1286, 1290, 1296, 1303, 1327, 1336, 1353, 1356, 1366, 1374, 1375, 1377, 1444, 1446, 1447, 1455, 1465, 1468, 1485, 1494, 1518, 1525, 1531, 1535, 1537, 1546, 1569, 1600, 1621, 1626, 1628, 1629, 1647, 1655, 1700, 1712, 1719, 1720, 1728, 1730, 1738, 1739, 1745, 1791, 1803, 1804, 1808, 1811, 1819, 1832, 1836, 1849, 1858, 1882, 1889, 1899, 1901, 1902, 1921, 1927, 1928, 1940, 1949, 1980, 1985, 1992, 2001, 2003, 2014, 2024, 2031, 2055, 2081, 2092, 2102, 2103, 2109, 2115, 2122, 2131, 2146, 2162, 2167, 2168, 2172, 2174, 2183, 2194, 2196, 2206, 2222, 2237, 2253, 2259, 2265, 2266, 2274, 2285, 2287, 2291, 2292, 2304, 2337, 2375, 2378, 2383, 2388, 2395, 2404, 2419, 2435, 2441, 2448, 2458, 2495, 2526, 2531, 2532, 2536, 2549, 2565, 2570, 2577, 2586, 2601, 2610, 2617, 2622, 2629, 2630, 2638, 2640, 2655, 2656, 2668, 2677, 2708, 2713, 2714, 2721, 2740, 2746, 2804, 2805, 2809, 2812, 2820
6601	$\begin{array}{c} 1, 16, 18, 40, 45, 66, 78, 100, 122, 141, 165, 195, 242, 250, 256, 286, 288, 303, 305, 318, 324, 327, 332, 338, 433, 474, 482, 508, 517, 523, 573, 605, 611, 619, 625, 640, 652, 715, 720, 739, 769, 795, 804, 810, 821, 824, 830, 843, 845, 877, 898, 927, 961, 983, 1002, 1056, 1082, 1091, 1108, 1111, 1117, 1132, 1147, 1166, 1185, 1188, 1193, 1199, 1205, 1248, 1270, 1289, 1313, 1369, 1378, 1404, 1417, 1451, 1453, 1466, 1513, 1527, 1576, 1581, 1600, 1630, 1644, 1671, 1682, 1691, 1721, 1738, 1753, 1762, 1767, 1773, 1779, 1788, 1800, 1868, 1887, 1931, 1952, 1964, 1972, 1993, 2008, 2010, 2025, 2027, 2054, 2060, 2075, 2101, 2109, 2131, 2174, 2196, 2204, 2218, 2245, 2256, 2259, 2312, 2347, 2396, 2483, 2491, 2505, 2526, 2538, 2543, 2567, 2584, 2601, 2614, 2634, 2640, 2661, 2705, 2729, 2770, 2813, 2830, 2869, 2888, 2907, 2915, 2927, 2936, 2948, 2962, 2970, 3057, 3079, 3091, 3117, 3120, 3139, 3156, 3175, 3188, 3202, 3249, 3298, 3303, 3352, 3399, 3413, 3426, 3445, 3462, 3481, 3484, 3510, 3522, 3544, 3631, 3639, 3653, 3665, 3674, 3686, 3694, 3713, 3732, 3771, 3788, 3831, 3872, 3896, 3940, 3961, 3967, 3987, 4000, 4017, 4034, 4058, 4063, 4075, 4096, 4110, 4118, 4205, 4254, 4289, 4342, 4345, 4356, 4383, 4397, 4405, 4427, 4470, 4492, 4500, 4526, 4541, 4547, 4574, 4576, 4591, 4593, 4608, 4629, 4637, 4649, 4670, 4714, 4733, 4801, 4813, 4822, 4828, 4834, 4839, 4848, 4863, 4880, 4910, 4919, 4930, 4957, 4971, 5001, 5020, 5025, 5074, 5088, 5135, 5148, 5150, 5184, 5197, 5223, 5232, 5288, 5312, 5331, 5353, 5396, 5402, 5408, 5413, 5416, 5435, 5454, 5469, 5484, 5490, 5493, 5510, 5519, 5545, 5599, 5618, 5640, 5674, 5703, 5724, 5756, 5758, 5771, 5777, 5780, 5791, 5797, 5806, 5832, 5862, 5881, 5886, 5949, 5996, 5996, 6028, 6078, 6084, 6093, 6119, 6127, 6168, 6263, 6269, 6274, 6277, 6283, 6296, 6298, 6313, 6315, 6345, 6351, 6359, 6406, 6436, 6460, 6479, 6501, 6523, 6535, 6556, 6561, 6583, 6585, 6600.$

## 8911

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