INTERRO DE COURS

Exercice 1 – On considère les matrices suivantes :

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \quad \text{et} \quad B = \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix}.$$

Calculer $B-A$, $A+2B$, $-(A-2I_3)$ et I_3+A+B.

Solution :

$$\begin{aligned} B-A &= \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 1 & 3 \\ -1 & -4 & -6 \\ 2 & 3 & -4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A+2B &= \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 2 & 2 \\ -2 & -4 & -6 \\ 6 & 4 & -8 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 0 \\ -2 & -2 & -3 \\ 7 & 3 & -8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} -(A-2I_3) &= -\left(\begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) \\ &= -\left(\begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\right) \\ &= -\begin{pmatrix} -1 & 0 & -2 \\ 0 & 0 & 3 \\ 1 & -1 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -3 \\ -1 & 1 & 2 \end{pmatrix} \end{aligned}$$

$$I_3+A+B=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\begin{pmatrix}1&0&-2\\0&2&3\\1&-1&0\end{pmatrix}+\begin{pmatrix}0&1&1\\-1&-2&-3\\3&2&-4\end{pmatrix}$$
$$=\begin{pmatrix}2&1&-1\\-1&1&0\\4&1&-3\end{pmatrix}$$

Exercice 2 – Effectuer les produits suivants :

1. $\begin{pmatrix}1&2\\3&-4\\0&6\end{pmatrix}\times\begin{pmatrix}10&1\\0&-3\end{pmatrix}$,
2. $\begin{pmatrix}1&2&1\\3&1&0\\-1&5&0\end{pmatrix}\times\begin{pmatrix}2\\2\\2\end{pmatrix}$,
3. $\begin{pmatrix}1&2&3\\-1&4&7\\8&-3&2\end{pmatrix}\times\begin{pmatrix}2&1&4\\-1&0&3\\-3&1&\sqrt{2}\end{pmatrix}$,
4. $\begin{pmatrix}1&2&-4\\4&6&-3\\2&-5&1\end{pmatrix}\times\begin{pmatrix}x\\y\\z\end{pmatrix}$.

Solution :

$$\begin{pmatrix}1&2\\3&-4\\0&6\end{pmatrix}\times\begin{pmatrix}10&1\\0&-3\end{pmatrix}=\begin{pmatrix}1\times10+2\times0&1\times1+2\times(-3)\\3\times10+(-4)\times0&3\times1+(-4)\times(-3)\\0\times10+6\times0&0\times1+6\times(-3)\end{pmatrix}$$
$$=\begin{pmatrix}10&-5\\30&15\\0&-18\end{pmatrix}$$

$$\begin{pmatrix}1&2&1\\3&1&0\\-1&5&0\end{pmatrix}\times\begin{pmatrix}2\\2\\2\end{pmatrix}=\begin{pmatrix}1\times2+2\times2+1\times2\\3\times2+1\times2+0\times2\\-1\times2+5\times2+0\times2\end{pmatrix}$$
$$=\begin{pmatrix}8\\8\\8\end{pmatrix}$$

$$\begin{pmatrix}1&2&3\\-1&4&7\\8&-3&2\end{pmatrix}\times\begin{pmatrix}2&1&4\\-1&0&3\\-3&1&\sqrt{2}\end{pmatrix}$$
$$=\begin{pmatrix}1\times2+2\times(-1)+3\times(-3)&1\times1+2\times0+3\times1&1\times4+2\times3+3\times\sqrt{2}\\(-1)\times2+4\times(-1)+7\times(-3)&(-1)\times1+4\times0+7\times1&(-1)\times4+4\times3+7\sqrt{2}\\8\times2+(-3)\times(-1)+2\times(-3)&8\times1+(-3)\times0+2\times1&8\times4+(-3)\times3+2\times\sqrt{2}\end{pmatrix}$$
$$=\begin{pmatrix}-9&4&10+3\sqrt{2}\\-27&6&8+7\sqrt{2}\\13&10&23+2\sqrt{2}\end{pmatrix}$$

$$\begin{pmatrix}1&2&-4\\4&6&-3\\2&-5&1\end{pmatrix}\times\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x+2y-4z\\4x+6y-3z\\2x-5y+z\end{pmatrix}$$