

NOM :

INTERRO DE COURS – SEMAINE 18

Exercice 1 – Calculer les limites suivantes.

1. $\lim_{x \rightarrow -1} x^2 - 5x + 6$

Solution :

$$\lim_{x \rightarrow -1} x^2 - 5x + 6 = (-1)^2 - 5 \times (-1) + 6 = 1 + 5 + 6 = 12$$

2. $\lim_{x \rightarrow 3} \frac{2x-1}{x+6}$

Solution :

$$\lim_{x \rightarrow 3} \frac{2x-1}{x+6} = \frac{2 \times 3 - 1}{3 + 6} = \frac{5}{9}$$

3. $\lim_{x \rightarrow 2^+} \frac{3x-1}{2x-4}$

Solution :

$$\left. \begin{array}{l} \lim_{x \rightarrow 2^+} 3x - 1 = 5 \\ \lim_{x \rightarrow 2^+} 2x - 4 = 0^+ \end{array} \right\} \text{Par quotient, } \lim_{x \rightarrow 2^+} \frac{3x-1}{2x-4} = +\infty.$$

4. $\lim_{x \rightarrow 1^-} \frac{2x+1}{-x+1}$

Solution :

$$\left. \begin{array}{l} \lim_{x \rightarrow 1^-} 2x + 1 = 3 \\ \lim_{x \rightarrow 1^-} -x + 1 = 0^+ \end{array} \right\} \text{Par quotient, } \lim_{x \rightarrow 1^-} \frac{2x+1}{-x+1} = +\infty.$$

5. $\lim_{x \rightarrow -\infty} 1 + \frac{1}{x} + \frac{3}{x^3}$

Solution :

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} 1 = 1 \\ \lim_{x \rightarrow -\infty} \frac{1}{x} = 0^- \\ \lim_{x \rightarrow -\infty} \frac{3}{x^3} = 0^- \end{array} \right\} \text{Par somme, } \lim_{x \rightarrow -\infty} 1 + \frac{1}{x} + \frac{3}{x^3} = 1.$$

6. $\lim_{x \rightarrow -\infty} x^3 - 5x^2 + 4x - 7$

Solution :

$$\lim_{x \rightarrow -\infty} x^3 - 5x^2 + 4x - 7 = \lim_{x \rightarrow -\infty} x^3 = -\infty$$

7. $\lim_{x \rightarrow +\infty} \frac{3x^2 - 2x + 1}{-4x^3 + 2x - 5}$

Solution :

$$\lim_{x \rightarrow +\infty} \frac{3x^2 - 2x + 1}{-4x^3 + 2x - 5} = \lim_{x \rightarrow +\infty} \frac{3x^2}{-4x^3} = \lim_{x \rightarrow +\infty} \frac{-3}{4x} = 0^-$$

8. $\lim_{x \rightarrow -\infty} (2x^2 + 1) \times \frac{2x^2 + 3x - 1}{x^2 + 5}$

Solution :

$$\lim_{x \rightarrow -\infty} 2x^2 + 1 = \lim_{x \rightarrow -\infty} 2x^2 = +\infty \quad \text{et} \quad \lim_{x \rightarrow -\infty} \frac{2x^2 + 3x - 1}{x^2 + 5} = \lim_{x \rightarrow -\infty} \frac{2x^2}{x^2} = \lim_{x \rightarrow -\infty} 2 = 2$$

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} 2x^2 + 1 = +\infty \\ \lim_{x \rightarrow -\infty} \frac{2x^2 + 3x - 1}{x^2 + 5} = 2 \end{array} \right\} \text{Par produit, } \lim_{x \rightarrow -\infty} (2x^2 + 1) \times \frac{2x^2 + 3x - 1}{x^2 + 5} = +\infty.$$

9. $\lim_{x \rightarrow +\infty} \sqrt{\frac{2}{x^2}} + 4$

Solution :

$$\lim_{x \rightarrow +\infty} \frac{2}{x^2} = 0^+ \Rightarrow \lim_{x \rightarrow +\infty} \frac{2}{x^2} + 4 = 4 \Rightarrow \lim_{x \rightarrow +\infty} \sqrt{\frac{2}{x^2}} + 4 = \sqrt{4} = 2$$

10. $\lim_{x \rightarrow 2^+} \left(\sqrt{\frac{1}{x-2}} + 3 \right)^2$

Solution :

$$\lim_{x \rightarrow 2^+} \frac{1}{x-2} = +\infty \Rightarrow \lim_{x \rightarrow 2^+} \sqrt{\frac{1}{x-2}} = +\infty \Rightarrow \lim_{x \rightarrow 2^+} \sqrt{\frac{1}{x-2}} + 3 = +\infty$$

$$\Rightarrow \lim_{x \rightarrow 2^+} \left(\sqrt{\frac{1}{x-2}} + 3 \right)^2 = +\infty$$