INTERRO DE COURS 15

Exercice 1 – Soient x et y deux réels. Simplifier le plus possible les expressions suivantes.

1. $A = \dfrac{e^x}{e^{2x}}$

> **Solution :**
> $$A = \frac{e^x}{e^{2x}} = e^{x-2x} = e^{-x}$$

2. $B = e^x \times \left(e^x\right)^3$

> **Solution :**
> $$B = e^x \times \left(e^x\right)^3 = e^x \times e^{3x} = e^{x+3x} = e^{4x}$$

3. $C = \dfrac{e^x}{(e^y)^2} \times e^{2y}$

> **Solution :**
> $$C = \frac{e^x}{(e^y)^2} \times e^{2y} = \frac{e^x}{e^{2y}} \times e^{2y} = e^{x-2y} \times e^{2y} = e^{x-2y+2y} = e^x$$

4. $D = e^{-x} \times \dfrac{1}{(e^x)^2}$

> **Solution :**
> $$D = e^{-x} \times \frac{1}{(e^x)^2} = e^{-x} \times \frac{1}{e^{2x}} = e^{-x} \times e^{-2x} = e^{-3x}$$

5. $E = e^x \times \left(e^y\right)^2 \times \dfrac{e^{-y}}{(e^x)^2}$

> **Solution :**
> $$E = e^x \times \left(e^y\right)^2 \times \frac{e^{-y}}{(e^x)^2} = e^x \times e^{2y} \times \frac{e^{-y}}{e^{2x}} = e^x \times e^{2y} \times e^{-y-2x} = e^{x+2y-y-2x} = e^{y-x}$$

Exercice 2 – Résoudre dans **R** les équations suivantes.

1. $e^x = 2$

> **Solution :**
> $$e^x = 2 \iff x = \ln(2)$$
> Donc $\mathcal{S} = \left\{\ln(2)\right\}$.

2. $e^{x^2-5x+6} = 1$

> **Solution :**
> $$e^{x^2-5x+6} = 1 = e^0 \iff x^2 - 5x + 6 = 0$$
>
> Je calcule le discriminant $\Delta = 25 - 24 = 1$. Il y a donc deux racines
> $$x_1 = \frac{5-1}{2} = 2 \quad \text{et} \quad x_2 = \frac{5+1}{2} = 3.$$
>
> Donc $\mathcal{S} = \{2, 3\}$.

3. $e^{2x+1} = 1$

> **Solution :**
> $$e^{2x+1} = 1 = e^0 \iff 2x + 1 = 0 \iff 2x = -1 \iff x = -\frac{1}{2}$$
>
> Donc $\mathcal{S} = \left\{-\frac{1}{2}\right\}$.

4. $\ln(2x-3) = 0$

> **Solution :** On a :
> $$\ln(2x-3) = 0 = \ln(1) \iff 2x - 3 = 1 \iff 2x = 4 \iff x = \frac{4}{2} = 2$$
>
> Donc $\mathcal{S} = \{2\}$.

5. $\dfrac{e^{2x+3}}{e^{4x-1}} = e^x$

> **Solution :**
> $$\frac{e^{2x+3}}{e^{4x-1}} = e^x \iff e^{2x+3-4x+1} = e^x \iff e^{-2x+4} = e^x \iff -2x + 4 = x \iff -3x = -4$$
> $$\iff x = \frac{-4}{-3} = \frac{4}{3}$$
>
> Donc $\mathcal{S} = \left\{\frac{4}{3}\right\}$.