INTERRO DE COURS – SEMAINE 1

Exercice 1 – Donner l'écriture des nombres suivants sous la forme d'un entier ou d'une fraction irréductible.

1. $A = \frac{2}{3} + \frac{1}{4} - \frac{1}{2}$

Solution :

$$\begin{aligned} A &= \frac{2}{3} + \frac{1}{4} - \frac{1}{2} \\ &= \frac{8}{12} + \frac{3}{12} - \frac{6}{12} \\ &= \frac{5}{12} \end{aligned}$$

2. $B = 2\left(\frac{1}{2} - \frac{1}{3}\right) - \left(1 + \frac{1}{5}\right)$

Solution :

$$\begin{aligned} B &= 2\left(\frac{1}{2} - \frac{1}{3}\right) - \left(1 + \frac{1}{5}\right) \\ &= 2\left(\frac{3}{6} - \frac{2}{6}\right) - \left(\frac{5}{5} + \frac{1}{5}\right) \\ &= 2 \times \frac{1}{6} - \frac{6}{5} \\ &= \frac{1}{3} - \frac{6}{5} \\ &= \frac{5}{15} - \frac{18}{15} \\ &= -\frac{13}{15} \end{aligned}$$

3. $C = \left(1 - \frac{1}{3}\right) \times \left(-1 + \frac{5}{6}\right) \div \left(\frac{4}{5} - \frac{2}{3}\right)$

Solution :

$$\begin{aligned} C &= \left(1 - \frac{1}{3}\right) \times \left(-1 + \frac{5}{6}\right) \div \left(\frac{4}{5} - \frac{2}{3}\right) \\ &= \left(\frac{3}{3} - \frac{1}{3}\right) \times \left(\frac{-6}{6} + \frac{5}{6}\right) \div \left(\frac{12}{15} - \frac{10}{15}\right) \\ &= \frac{2}{3} \times \frac{-1}{6} \div \frac{2}{15} \\ &= \frac{2}{3} \times \frac{-1}{6} \times \frac{15}{2} \\ &= \frac{-5}{6} \end{aligned}$$

4. $D = \dfrac{1+\frac{1}{2}\times\frac{2}{3}}{1-\frac{1}{3}\times\frac{3}{2}}$

Solution :

$$\begin{aligned} D &= \frac{1+\frac{1}{2}\times\frac{2}{3}}{1-\frac{1}{3}\times\frac{3}{2}} \\ &= \frac{1+\frac{1}{3}}{1-\frac{1}{2}} \\ &= \frac{\frac{4}{3}}{\frac{1}{2}} \\ &= \frac{4}{3}\times\frac{2}{1} \\ &= \frac{8}{3} \end{aligned}$$

Exercice 2 – Simplifier les nombres suivants.

1. $A = 2^3\times 2^{-1}\times 2^4\times 2$

Solution : $A = 2^3\times 2^{-1}\times 2^4\times 2 = 2^{3-1+4+1} = 2^7$

2. $B = \dfrac{3^2\times 3^{-4}}{3\times 3^{-3}}$

Solution : $B = \dfrac{3^2\times 3^{-4}}{3\times 3^{-3}} = \dfrac{3^{2-4}}{3^{1-3}} = \dfrac{3^{-2}}{3^{-2}} = 3^{-2+2} = 3^0 = 1$

3. $C = \dfrac{81^{-2}\times 3^4}{27^2\times 9^{-1}}$

Solution : $C = \dfrac{(3^4)^{-2}\times 3^4}{(3^3)^2\times(3^2)^{-1}} = \dfrac{3^{-8}\times 3^4}{3^6\times 3^{-2}} = \dfrac{3^{-4}}{3^4} = 3^{-4-4} = 3^{-8}$

4. $D = \dfrac{(2\times 3^4)^{-2}}{4\times 9}$

Solution : $D = \dfrac{(2\times 3^4)^{-2}}{4\times 9} = \dfrac{2^{-2}\times 3^{-8}}{2^2\times 3^2} = 2^{-2-2}\times 3^{-8-2} = 2^{-4}\times 3^{-10}$

Exercice 3 – Développer, réduire et ordonner les expressions suivantes.

1. $A(x) = 4(-2x+1)$

Solution : $A(x) = 4(-2x+1) = -8x+4$

2. $B(x) = (-2x+1)(x+5)$

Solution : $B(x) = (-2x+1)(x+5) = -2x^2 - 10x + x + 5 = -2x^2 - 9x + 5$

3. $C(x) = (3-8x)(11x+3)$

Solution : $C(x) = 33x + 9 - 88x^2 - 24x = -88x^2 + 9x + 9$

4. $D(x) = (4-5x)^2$

Solution : $D(x) = 4^2 - 2 \times 4 \times 5x + (5x)^2 = 16 - 40x + 25x^2 = 25x^2 - 40x + 16$

5. $E(x) = (3-2x)(3+2x) + (1-2x)^2$

Solution :

$$\begin{aligned} E(x) &= (3-2x)(3+2x) + (1-2x)^2 \\ &= 3^2 - (2x)^2 + 1^2 - 2 \times 1 \times 2x + (2x)^2 \\ &= 9 - 4x^2 + 1 - 4x + 4x^2 \\ &= -4x + 10 \end{aligned}$$

Exercice 4 – Factoriser **au maximum** les expressions suivantes.

1. $A(x) = 4x - 8$

Solution : $A(x) = 4x - 8 = 4 \times x - 4 \times 2 = 4(x-2)$

2. $B(x) = (5-4x)(x-3) + (6+2x)(5-4x)$

Solution : $B(x) = (5-4x)\,(x - 3 + 6 + 2x) = (5-4x)(3x+3) = 3(5-4x)(x+1)$

3. $C(x) = (2x+1)^2 - (2x+1)(x-3)$

Solution :

$$\begin{aligned} C(x) &= (2x+1)(2x+1) - (2x+1)(x-3) \\ &= (2x+1)(2x+1-(x-3)) \\ &= (2x+1)(2x+1-x+3) \\ &= (2x+1)(x+4) \end{aligned}$$

4. $D(x) = 4x^2 - 40x + 100$

Solution : $D(x) = (2x)^2 - 2 \times 2x \times 10 + 10^2 = (2x-10)^2$

5. $E(x) = (x-1)(2x-3) - (4x^2 - 12x + 9)$

Solution :

$$
\begin{aligned}
E(x) &= (x-1)(2x-3) - ((2x)^2 - 2\times 2x\times 3 + 3^2)\\
&= (x-1)(2x-3) - (2x-3)^2\\
&= (x-1)(2x-3) - (2x-3)(2x-3)\\
&= (2x-3)(x-1-(2x-3))\\
&= (2x-3)(x-1-2x+3)\\
&= (2x-3)(-x+2)
\end{aligned}
$$