

INTERRO DE COURS

Exercice 1 – On considère les matrices suivantes :

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \quad \text{et} \quad B = \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix}.$$

Calculer $B - A$, $A + 2B$, $-(A - 2I_3)$ et $I_3 + A + B$.

Solution :

$$\begin{aligned} B - A &= \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 1 & 3 \\ -1 & -4 & -6 \\ 2 & 3 & -4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A + 2B &= \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 2 \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 2 & 2 \\ -2 & -4 & -6 \\ 6 & 4 & -8 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 0 \\ -2 & -2 & -3 \\ 7 & 3 & -8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} -(A - 2I_3) &= - \left(\begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \\ &= - \left(\begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \right) \\ &= - \begin{pmatrix} -1 & 0 & -2 \\ 0 & 0 & 3 \\ 1 & -1 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -3 \\ -1 & 1 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 I_3 + A + B &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} \\
 &= \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \\ 4 & 1 & -3 \end{pmatrix}
 \end{aligned}$$

Exercice 2 – Effectuer les produits suivants :

1. $\begin{pmatrix} 1 & 2 \\ 3 & -4 \\ 0 & 6 \end{pmatrix} \times \begin{pmatrix} 10 & 1 \\ 0 & -3 \end{pmatrix},$

3. $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 7 \\ 8 & -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0 & 3 \\ -3 & 1 & \sqrt{2} \end{pmatrix},$

2. $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ -1 & 5 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix},$

4. $\begin{pmatrix} 1 & 2 & -4 \\ 4 & 6 & -3 \\ 2 & -5 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$

Solution :

$$\begin{aligned}
 \begin{pmatrix} 1 & 2 \\ 3 & -4 \\ 0 & 6 \end{pmatrix} \times \begin{pmatrix} 10 & 1 \\ 0 & -3 \end{pmatrix} &= \begin{pmatrix} 1 \times 10 + 2 \times 0 & 1 \times 1 + 2 \times (-3) \\ 3 \times 10 + (-4) \times 0 & 3 \times 1 + (-4) \times (-3) \\ 0 \times 10 + 6 \times 0 & 0 \times 1 + 6 \times (-3) \end{pmatrix} \\
 &= \begin{pmatrix} 10 & -5 \\ 30 & 15 \\ 0 & -18 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ -1 & 5 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} &= \begin{pmatrix} 1 \times 2 + 2 \times 2 + 1 \times 2 \\ 3 \times 2 + 1 \times 2 + 0 \times 2 \\ -1 \times 2 + 5 \times 2 + 0 \times 2 \end{pmatrix} \\
 &= \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 7 \\ 8 & -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0 & 3 \\ -3 & 1 & \sqrt{2} \end{pmatrix} \\
 &= \begin{pmatrix} 1 \times 2 + 2 \times (-1) + 3 \times (-3) & 1 \times 1 + 2 \times 0 + 3 \times 1 & 1 \times 4 + 2 \times 3 + 3 \times \sqrt{2} \\ (-1) \times 2 + 4 \times (-1) + 7 \times (-3) & (-1) \times 1 + 4 \times 0 + 7 \times 1 & (-1) \times 4 + 4 \times 3 + 7\sqrt{2} \\ 8 \times 2 + (-3) \times (-1) + 2 \times (-3) & 8 \times 1 + (-3) \times 0 + 2 \times 1 & 8 \times 4 + (-3) \times 3 + 2 \times \sqrt{2} \end{pmatrix} \\
 &= \begin{pmatrix} -9 & 4 & 10 + 3\sqrt{2} \\ -27 & 6 & 8 + 7\sqrt{2} \\ 13 & 10 & 23 + 2\sqrt{2} \end{pmatrix}
 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & -4 \\ 4 & 6 & -3 \\ 2 & -5 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 2y - 4z \\ 4x + 6y - 3z \\ 2x - 5y + z \end{pmatrix}$$