Department of Decision Sciences

HONPR2C
Project II

A statistical analysis of heuristics for the p-median problem

Assignment 2: Preliminary Report

Assignment Unique Number: 650898

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30 August 2017

Contents

Signed Declaration

TO DO:

1. extend literature study to include more details on algorithms used
2. STINGY algorithm - code (Classic - CH)
3. ALTERNATE algorithm - code (Classic - LS)
4. check the rules of citations and full stops
5. add a section discussing the diff between classical and meta heuristics
6. add explanation around how choices between identical improvements are made (i.e. randomly)

Introduction

The network facility location problem of finding a location for facilities or public services within a given space by optimising some predefined objective, is not a simple one. Within the single-objective branch of network location problems, there is a plethora of papers exploring minisum, minimax, and covering problems.

If the number of facilities needed is known and these facilities need to be located in the manner of minimising the total distance between demand nodes and their assigned facility, then the decision-maker is facing a p-median problem.

The p-median problem is well studied and many different approaches to finding both exact and approximate solutions have been proposed, but determining which method would be best for a particular problem is not clear.

The study proposed will analyse and compare several heuristics applied to the p-median problem in the hope that this decision may be made more clear.

Description of the Problem

The uncapacitated p-median problem has been widely studied and many different approaches to finding an acceptable, if not optimal, solution have been presented over the past fifty years.

The focus of this study is to statistically analyse and compare the results obtained from various heuristic and metaheuristic algorithms applied to the p-median problem.

Different heuristics applied to p-median problems can produce varying quality of results dependant on the size of the original problem. As algorithms take a considerable amount of time and research to be suitably set-up and run, it may be of benefit to have some prior knowledge of which heuristics may be suitable for the problem on hand.

As such, the main objective of this proposed study would be to provide an indication of which method would be best suited to a given problem.

This study will also investigate the computational time each algorithm requires to generate a solution, taking into account the size of the problem being solved.

Since parametrisation of heuristics play a crucial part of the efficiency of the algorithm and the quality of the results it produces, this will be explained in detail in the study. Given time, different approaches to the same heuristic may be explored as well.

Methodology

All algorithms will be coded in R [1], an open source programming language, due to its ease of use, extensive availability of online resources as well as the functionality provided by downloadable packages that provide useful data handling and visualisation.

As the results of the algorithms under investigation are to be compared, identical problems need to be presented to each algorithm to solve. Beasley [2] offers a forty-instance dataset which comprises of problems ranging from 100 to 900 demand nodes that require the locations of between five and 200 medians to be determined. These datasets ae given in a text format presenting the pairs of nodes along with their associated cost. These text files were converted to cost matrices using the R package igraph [3]. These matrices provide the cost of traversing from one node to any other node in each network of the forty instances of the dataset.

Beasley's library also provides the optimal solution's value of the weighted total distance objective function for each of the forty instances in the dataset. This will allow the results obtained from the algorithms to be objectively assessed against the optimal solution. The deviation of each algorithm's solution from the optimal solution will be analysed. In addition, the computational time required for each algorithm will be logged and compared.

Literature Study

While the p-median problem may have been tackled by academics before 1964, it was first formally defined and formulated by Hakimi [4]. It was also in this paper that Hakimi proved that the optimal location of a facility in a connected discrete network would always be found to correspond to the location of a demand node. This considerably reduces the solution search space.

A few years later, ReVelle and Swain [5] formulated the p-median problem as an integer programme which provided the answers to both the question of where the optimal medians were located as well as which demand nodes should be allocated to each median. In this paper, the solution properties of solving this problem with linear programming using the branch-and-bound technique was also discussed.

Later that decade, it was shown that the p-median problem is NP-hard and as such there exists no algorithm that can find the optimal solution within polynomial time. Since most real-world problems are very large in nature, this poses a great problem. Well developed heuristics can provide satisfactory solutions and many such approaches have been proposed. Mladenović et al. [6] produced a comprehensive survey of the heuristics that have been used to solve the p-median problem. This survey divides heuristics into two groups, classical heuristics and metaheuristics. The classical heuristics group consists of constructive heuristics, such as the greedy algorithm, local search, and mathematical programming. The metaheuristics group lists tabu search, variable neighbourhood search, and simulated annealing among others.

There do exist studies that analysis the efficiency of heuristics in solving the p-median. Simulated annealing was analysed by Chiyoshi and Galvão [7] while Alp et al. [8] investigated their proposed genetic algorithm. Hansen and Mladenović [9] applied the variable neighbourhood search algorithm to the p-median problem as well as heuristic concentration, tabu search, and the greedy interchange algorithm. These studies used Beasley's test problem dataset although Hansen and Mladenović's review only used a portion of the available test problems.

Rolland et al. [10] explored the efficiency of tabu search using randomly generated datasets as test problems, and this was compared to results obtained from a two-exchange heuristic and an integer programming algorithm. A heuristic concentration was compared to tabu search by Rosing et al. [11]. Since datasets were independently and randomly generated for each of these studies, it makes it very difficult to compare results.

ADD MORE ABOUT THE ALGORITHMS USED IN THIS REPORT

Heuristics and Metaheuristics

explain DIFFERENCES

Pseudocode Notation

For each algorithm examined in this study, a pseudocode will be presented using notation given below. This notation is the same as to that used by Whitaker [12].

G : the problem network
n : the number of nodes within the network
m : the number of nodes nodes that are available to be assigned as a median
p : the number of medians by required by the solution
M : the set of nodes that are available to be assigned as a median
P^* : the set of medians where $P^* \subset M$
P : the set of nodes not in the solution median set where $P \subset M$ and $P \cup P^* = M$
S^* : objective function value for supplying the network G
k : an iteration parameter
∞ : an arbitrarily large number
d_{ij} : the cost of travelling from node i to potential median j
u_i^k : the cost of travelling from node i to the closest median j where $j \in P^*$ during iteration k
w_i^k : the cost of travelling from node i to the second closest median j where $j \in P*$ during iteration
c_j^k : the potential value of S^* if node j is added to the median set P^* during iteration k
S_r : the value of S^* after adding node r to the median set P^*
S_{rt} : the potential increase (or decrease) to S* resulting from the interchange of node r with median t
I, J, K_j : subsets of the n nodes as defined XXXXXDEFINE THESE!XXXXX
a, b, z, q : indices and parameters as defined XXXXDEFINE!XXXX
$\emptyset$: an empty set

Classical Heuristics

Constructive Heuristics

A constructive heuristic is a technique that starts with an empty solution and each iteration extends this solution until a complete solution has been found. Within the context of the unconstrained p-median problem, for each iteration of the algorithm, a median is identified by some deterministic method and added to the current solution. This process continues until the solution contains p entries.

The Greedy Algorithm

The greedy algorithm is a simple recursive algorithm that starts with an empty median set and adds a node to the median set with each iteration of the algorithm. The node that that is added to the median set is the one that decreases the objective function value the most when compared to the other nodes that are not already designated as a median XXXXXXXXXXREPHRASEXXXXXXXXXXXXXXXXX.

The pseudocode for this algorithm is given below:

Algorithm 1: The Greedy Algorithm

Data: Cost Matrix

Result: P^* populated with the set of nodes designated as the solution medians

Step 0: Initialization

Set $P^* = \emptyset$; $k = 1$; and $u_i^1 = \infty$

Step 1: For all nodes that are not already designated as a median, find the sum of the minimum between the cost of travelling to the closet node that is not a median and the cost of travelling to the closest median.

$$c_j^k = \sum_{i=1}^{n} \min(d_{ij},\, u_i^k) \quad \forall j \in M,\, j \notin P^*$$

Step 2: Determine which free node has the smallest objective function value, XXXXXXXXXXXXXXXXXXXX

$$S_r = \min_{j \in P*}(c_j^k) \quad \text{for } r \in M,\, r \notin P^*$$

Step 3: Add node r to the set of medians and then determine if the algorithm has completed the number of required iterations.

$$P^* = P^* \cup r$$

if $k = p$ **then**
  go to Step 5;
else
  go to Step 4;
end

Step 4: Increase the value of iteration parameter and update the cost of travelling from any node to its closet median given that node r has been added to the median set.

$$k = k + 1$$
$$u_i^k = \min(d_{ir},\, u_i^{k-1}) \quad \text{for } i = 1,2,3\ldots$$

Step 5: Once p iterations are complete, set the final objective function value equal to the current iteration's objective function value and STOP.

$$S^* = S_r$$

The Fast Greedy Algorithm

The Stingy Algorithm

Computational Results - Constructive Heuristics

Local Search

A local search heuristic differs from a constructive heuristic in that it starts with a feasible solution and each iteration attempts to improve this solution by some deterministic method. This process continues until there is no better solution within reach of the current solution or some predetermined time-out is reached.

The Interchange Algorithm

The Fast Interchange Algorithm

The Alternate Algorithm

Computational Results - Local Search

Conclusion

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