

Introduction to Theoretical Ecology Assignment 6

Graphical Analysis of Lotka-Volterra Competition Model

The Lotka-Volterra competition model can be written in terms of the carrying capacities of the two competing species N_1 and N_2 :

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \beta N_1}{K_2}\right)$$

, where r_1 and r_2 are the intrinsic population growth rates; K_1 and K_2 are the carrying capacities; α is the effect of N_2 on the population growth of N_1 ; β is the effect of N_1 on the population growth of N_2 .

1. Find all possible equilibrium population sizes of the two species. (5 pts)

Solution:

$$\left\{ \begin{array}{l} 0 = r_1 N_1^* \left(1 - \frac{N_1^* + \alpha N_2^*}{K_1}\right) \\ 0 = r_2 N_2^* \left(1 - \frac{N_2^* + \beta N_1^*}{K_2}\right) \end{array} \right.$$

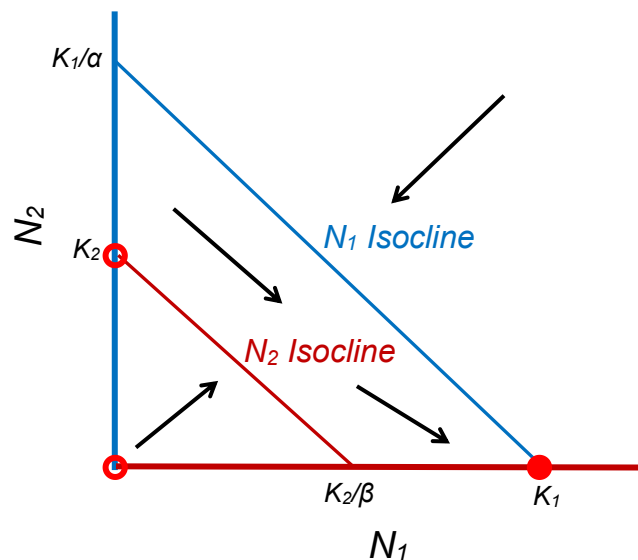
$$(N_1^*, N_2^*) = (0, 0), (K_1, 0), (0, K_2), \text{ and } \left(\frac{K_1 - \alpha K_2}{1 - \alpha\beta}, \frac{K_2 - \beta K_1}{1 - \alpha\beta}\right)$$

2. Use graphical analysis to determine the stability of the system for all possible scenarios. Please (1) show the stability criteria in each scenario; (2) mark the equilibrium points (both stable and unstable) in the phase plane; and (3) denote all the intercepts between the isoclines and axes. (10 pts)

Solution:

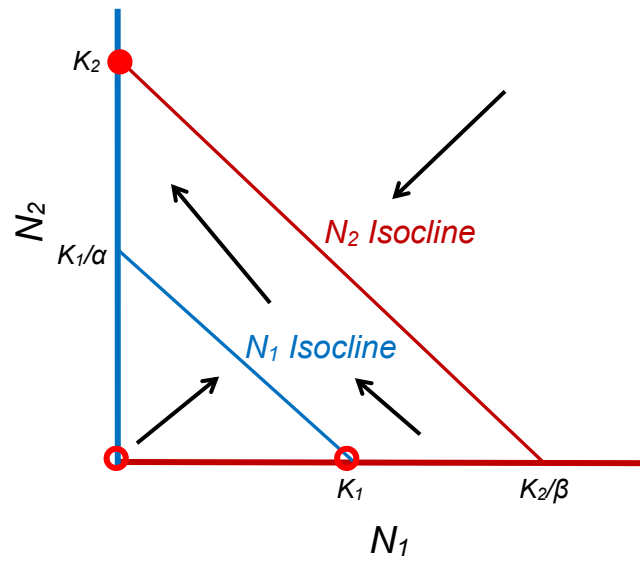
Scenario 1: N_1 wins

- Stable equilibrium point: $(K_1, 0)$
- Stability criteria: $K_1 > K_2/\beta$ and $K_1/\alpha > K_2$



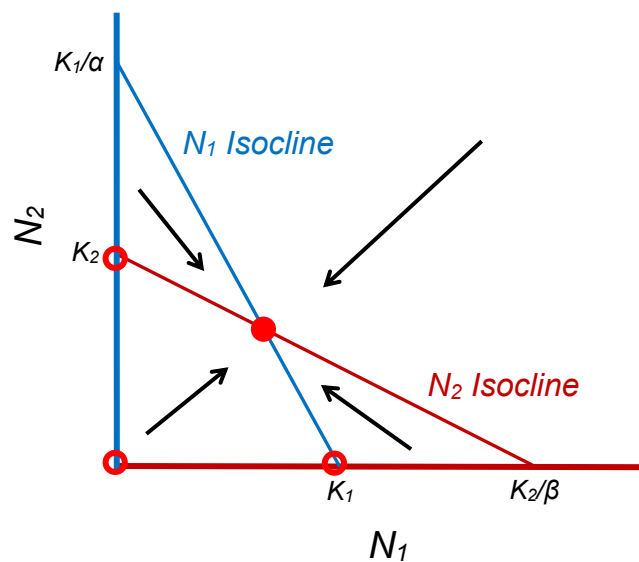
Scenario 2: N_2 wins

- Stable equilibrium point: $(0, K_2)$
- Stability criteria: $K_2 > K_1/\alpha$ and $K_2/\beta > K_1$



Scenario 3: Stable coexistence

- Stable equilibrium point: $\left(\frac{K_1 - \alpha K_2}{1 - \alpha\beta}, \frac{K_2 - \beta K_1}{1 - \alpha\beta} \right)$
- Stability criteria: $K_1/\alpha > K_2$ and $K_2/\beta > K_1$



Scenario 4: Unstable coexistence (saddle)

- Stable equilibrium point: $(K_1, 0)$ or $(0, K_2)$ (depending on the initial conditions)
- Stability criteria: $K_1 > K_2/\beta$ and $K_2 > K_1/\alpha$

