2021. 11. 2 Metapopulation models & Patch occupancy models

$\frac{dP}{dt} = I - E$

fraction of cites occupied

Model formulation

Model variation

How to build a model

◎ Model formulation

Metapopulation : a group of local populations connected by immigration & emigration

use multiple populations for modeling ($N_1, N_2 \dots N_i$), $\frac{dN_i}{dt}$

use probability [Levins : 1970]

each local patch can be empty (locally extinction) OR occupied (local persistant)

$\Rightarrow P_{(t)}$ = the fraction (frequency) of occupied patches in the metapopulation

$$\Rightarrow \frac{dP}{dt} = [\text{local colonization}] - [\text{local extinction}] = I - E \quad \dots ①$$

◎ Assumptions

1. Homogeneous patches
2. No explicit spatial structure (spatially implicit)
   movement isn't distance - dependent , Global dispersal
3. No time lag
4. No time - dependent colonization / extinction rates , but can depend on current occupancy
5. Large (infinit) number of patches
   only local stochastizity (local patches can go extinct) , no global stochastizity

empty patches available for colonization

occupied patch that may go extinct

$$\Rightarrow \frac{dP}{dt} = C(P) \times (1 - P) - e(P) \times P \quad \dots ②$$

local colonization rate

local extinction rate

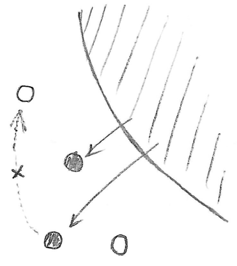
2 Model variation

Model 1 : Mainland - island model

permanent external source

continuous source for migrants

constant immigration rate from large mainland, no dispersal between islands (propagule rain)

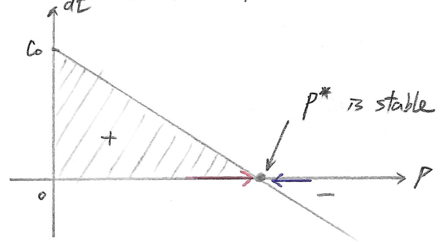


$\Rightarrow$ $C(P) = C_0$ } $\rightarrow$ independent to number of patches
$e(P) = e_0$

$$\Rightarrow \frac{dP}{dt} = C_0(1-P) - e_0 P \quad \ldots (3)$$

- Solve for equilibrium: $C_0(1-P^*) = e_0 P^* \rightsquigarrow P^* = \frac{C_0}{C_0 + e_0} > 0$ ...(4)
- Stability via graphical method:

$\frac{dP}{dt} = C_0 - (C_0 + e_0)P$



- Stability via local stability analysis: $\frac{d}{dP}\left(\frac{dP}{dt}\right)\Big|_{P^*} = -(C_0 + e_0) < 0$ — stable as long as $C_0, e_0 > 0$

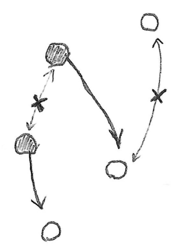
$\Rightarrow P^* = \frac{C_0}{C_0 + e_0}$ is locally stable, a balance between colonization / extinction

metapopulation ALWAYS persist in mainland-island models even if $e_0 \gg C_0$

Model 2 : Internal colonization

migrant source is from other colonized patches

Populations on occupied patches contribute to propagule pool, no dispersal to already colonized patches



$\rightarrow$ $C(P) = C_1 P$; $\frac{dC(P)}{dP} > 0$

$e(P) = e_0$ $\quad\searrow$ larger P, more colonization

C_1 measures how colonization rate increases w/ each additional occupied patch

$$\Rightarrow \frac{dP}{dt} = c_1 P(1-P) - e_0 P \quad \cdots ⑤$$

- Solve for equilibrium : $c_1 P^*(1-P^*) - e_0 P^* = 0$

$$\Rightarrow P^*\left[c_1(1-P^*) - e_0\right] = 0 \rightsquigarrow P^* = 0 \ \vee \ 1 - \frac{e_0}{c_1} \quad \cdots ⑥$$

- Solve for stability : $\frac{d}{dP}\left(\frac{dP}{dt}\right) = \left[c_1(1-P) - e_0\right] + P(-c_1)$

with $P^* = 0 \Rightarrow \frac{d}{dP}\left(\frac{dP}{dt}\right)\Big|_{P^*=0} = c_1 - e_0$

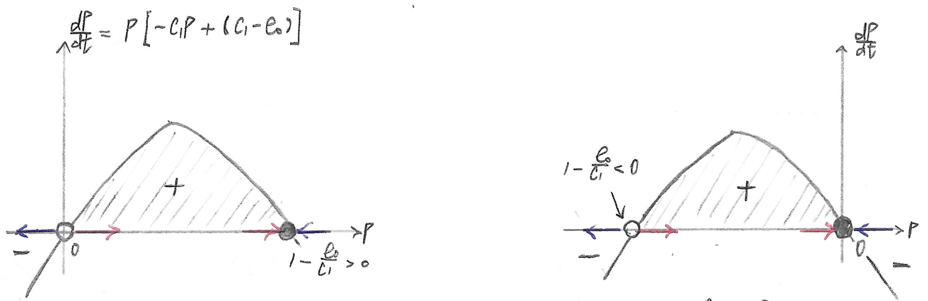
with $P^* = 1 - \frac{e_0}{c_1} \Rightarrow \frac{d}{dP}\left(\frac{dP}{dt}\right)_{P^*} = -c_1 + e_0$

$\Rightarrow$ if $c_1 > e_0$, $-c_1 + e_0 < 0 \Rightarrow P^* = 1 - \frac{e_0}{c_1}$ is stable ($P^* = 0$ is unstable)

if $c_1 < e_0$, $c_1 - e_0 < 0 \Rightarrow P^* = 0$ is stable between $0 \sim 1$

$\Rightarrow P^* = 1 - \frac{e_0}{c_1}$ is locally stable if $c_1 > e_0$

metapopulation is not always persistant , depends on the relative magnitude of c_1 & e_0

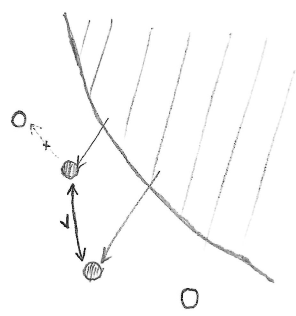


« $c_1 > e_0$ » « $c_1 < e_0$ »

Model 3 : Rescue effect

immigrants arrive at colonized sites , increases population size and decrease extinction rate

Reduction in extinction probability as more sites are occupied



$\Rightarrow C(P) = C_0$

$e(P) = e_1(1-P)$; $\frac{\partial e(P)}{\partial P} < 0$

↘ large P, less local extinction

e_1 measures how extinction rate decreases w/ P

$$\Rightarrow \frac{dP}{dt} = C_0(1-P) - e_1(1-P)P \quad \text{...①}$$

- Solve for equilibrium: $(1-P^*)[C_0 - e_1 P^*] \rightsquigarrow \underline{P^* = 1 \vee \frac{C_0}{e_1}}$ ...②

- Solve for stability: $\frac{d}{dP}\left(\frac{dP}{dt}\right) = -(C_0 - e_1 P) + (1-P)(-e_1)$

  with $P^* = 1 \Rightarrow \frac{d}{dP}\left(\frac{dP}{dt}\right)\Big|_{P^*=1} = -(C_0 - e_1)$

  with $P^* = \frac{C_0}{e_1} \Rightarrow \frac{d}{dP}\left(\frac{dP}{dt}\right)\Big|_{P^* = \frac{C_0}{e_1}} = -e_1 + C_0$

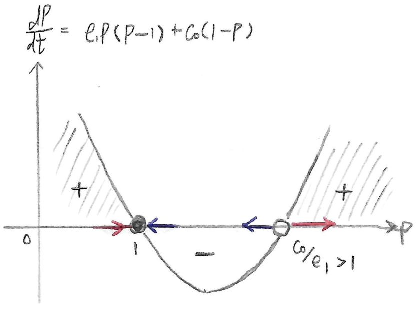
$\Rightarrow$ if $C_0 > e_1$, $-(C_0 - e_1) < 0 \Rightarrow P^* = 1$ is stable ($\frac{C_0}{e_1}$ is unstable)

if $C_0 < e_1$, $-e_1 + C_0 < 0 \Rightarrow P^* = \frac{C_0}{e_1}$ is stable (1 is unstable)

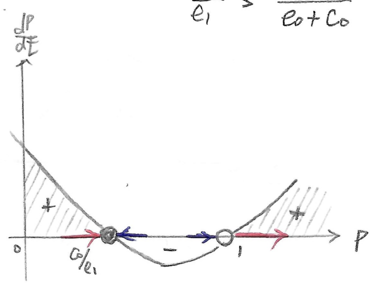
$\Rightarrow$ with propagule rain & rescue effect, the metapopulation ALWAYS persist

it can reach full occupancy if $C_0 > e_1$, but can still reach $\frac{C_0}{e_1}$ even if $e_1 > C_0$

↘ $\frac{C_0}{e_1} > \frac{C_0}{e_0 + C_0}$



« $C_0 > e_1$ »



« $C_0 < e_1$ »

ⓐ Extra: Metapopulation model with habitat destruction

$$\frac{dP}{dt} = c_1 P(1-\lambda-P) - e_0 P \quad \cdots ⑨$$

unsuitable habitats

- Solve for equilibrium: $P^* = 0 \vee 1 - \frac{e_0}{c_1} - \lambda$ ⋯ ⑩

- Stability: $\frac{d}{dP}\left(\frac{dP}{dt}\right) = \left[c_1(1-\lambda-P) - e_0\right] + P(-c_1) = f(P)$

with $P^*=0$, $f(P^*) = c_1 - c_1\lambda - e_0$

extinction is stable if $c_1 < \frac{e_0}{1-\lambda}$ ⋯ ⑪

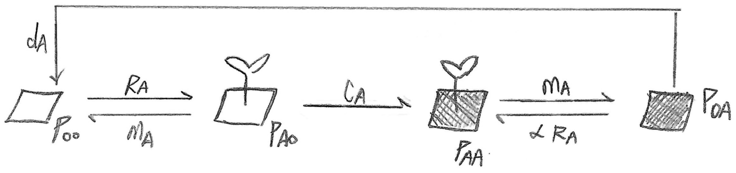
with unsuitable habitats, c_1 needs to be larger in order to persist

with $P^*>0$, $f(P^*) = -c_1 + c_1\lambda + e_0$

persistance is stable if $c_1 > \frac{e_0}{1-\lambda}$

λ does not need to be as large as 1 to cause extinction

⑩ Extra: PSF patch occupancy model exercise



$$\frac{dP_{00}}{dt} = -R_A(P_{A0}+P_{AA})P_{00} + m_A P_{A0} + d_A P_{0A}$$

$$\frac{dP_{A0}}{dt} = +R_A(P_{A0}+P_{AA})P_{00} - C_A P_{A0}$$

$$\frac{dP_{AA}}{dt} = +C_A P_{A0} + \alpha R_A(P_{A0}+P_{AA})P_{0A} - m_A P_{AA}$$

$$\frac{dP_{0A}}{dt} = +m_A P_{AA} - \alpha R_A(P_{A0}+P_{AA})P_{0A} - d_A P_{0A}$$

$R_A = 0.5$, $m_A = 0.1$, $C_A = 0.5$, $d_A = 0.4$, $\alpha = 0.7$