Introduction to Theoretical Ecology Assignment 10

Graphical and Local Stability Analysis of the SIR Model

In the lecture, we have learned the SIR disease model with demography. Here, we assume that the mortality rate of the infected individuals is the same as that of the susceptible and the recovered individuals (i.e., all δ):

$$\frac{dS}{dt} = \theta - \beta SI - \delta S$$

$$\frac{dI}{dt} = \beta SI - \rho I - \delta I$$

$$\frac{dR}{dt} = \rho I - \delta R$$

then

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = \theta - \delta N$$

The dynamics of the original SIR system can be understood by analyzing the more tractable two-dimensional SI system:

$$\frac{dS}{dt} = \theta - \beta SI - \delta S$$

$$\frac{dI}{dt} = \beta SI - \rho I - \delta I$$

with $R_{(t)}$ simply being:

$$R_{(t)} = N_{(t)} - S_{(t)} - I_{(t)}$$

1. Use graphical analysis to determine the stability of the SI system. Please mark the equilibrium points in the phase plane and show the stability criteria. (Hint: There are 2 scenarios; 2.5 pts each)

2. Perform local stability analysis on the two equilibrium points and discuss the stability criteria. (2.5 pts for each equilibrium point)