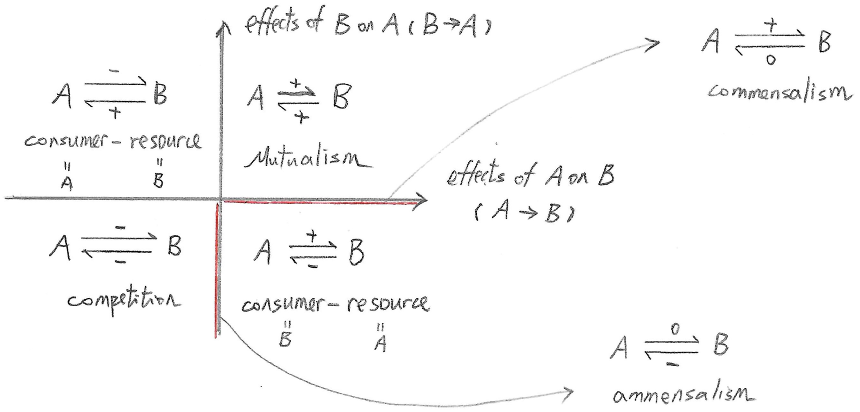
2021.11.9 Competition (Graphical Analysis)

- Model derivation
- Zero net growth isocline & State space diagram
- Graphical analysis

◎ Model derivation for species interaction

A Phenomenological representation of species interactions



N_A & N_B ⇒ Density of species A & B

$$\begin{cases} \dfrac{dN_A}{dt} = f_A(N_A, N_B) \\ \dfrac{dN_B}{dt} = f_B(N_A, N_B) \end{cases}$$

⇒ interaction type can be determined based on per capita rates

⇒ $A \to A : \dfrac{\partial}{\partial N_A}\left(\dfrac{1}{N_A}\dfrac{dN_A}{dt}\right)$

$A \to B : \dfrac{\partial}{\partial N_A}\left(\dfrac{1}{N_B}\dfrac{dN_B}{dt}\right)$

$B \to A : \dfrac{\partial}{\partial N_B}\left(\dfrac{1}{N_A}\dfrac{dN_A}{dt}\right)$

$B \to B : \dfrac{\partial}{\partial N_B}\left(\dfrac{1}{N_B}\dfrac{dN_B}{dt}\right)$

$A \to B$, $B \to A$: Interspecific interaction

$A \to A$, $B \to B$: Intraspecific interaction (usually negative)

Competition: $\frac{1}{dN_A}\left(\frac{1}{N_B}\frac{dN_B}{dt}\right) < 0$ & $\frac{1}{dN_B}\left(\frac{1}{N_A}\frac{dN_A}{dt}\right) < 0$ along w/ negative intras

$$\begin{tikzcd} A \arrow[loop left, "\alpha_{AA}"] \arrow[r, shift left, "\alpha_{BA}"] & B \arrow[l, shift left, "\alpha_{AB}"] \arrow[loop right, "\alpha_{BB}"] \end{tikzcd}$$

α_{ij} = effect of j on i

↑ competitive coefficients

$$\begin{cases} \frac{1}{N_A}\frac{dN_A}{dt} = r_A - \alpha_{AA}N_A - \underline{\alpha_{AB}N_B} \\ \frac{1}{N_B}\frac{dN_B}{dt} = r_B - \alpha_{BA}N_A - \alpha_{BB}N_B \end{cases}$$

→ linear competitive effects

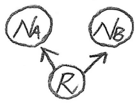
$$\Rightarrow \begin{cases} \frac{dN_A}{dt} = N_A(r_A - \alpha_{AA}N_A - \alpha_{AB}N_B) \\ \frac{dN_B}{dt} = N_B(r_B - \alpha_{BA}N_A - \alpha_{BB}N_B) \end{cases}$$

« Lotka – Volterra competition model »

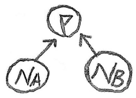
modeling net direct impact
||
« phenomenological model »

↕ vs.

« mechanistic model »
||
explicit modeling of limiting factors



exploitative competition



apparent competition

By setting the above equation to zero, get equilibrium (N_A^*, N_B^*)

$$\Rightarrow \begin{cases} N_A^* = 0 \lor r_A - \alpha_{AA}N_A^* - \alpha_{AB}N_B^* = 0 \\ N_B^* = 0 \lor r_B - \alpha_{BA}N_A^* - \alpha_{BB}N_B^* = 0 \end{cases}$$

⇒ FOUR potential equilibrium:

① E_0 $\begin{cases} N_A^* = 0 \\ N_B^* = 0 \end{cases} \Rightarrow (N_A^* = 0,\ N_B^* = 0)$

carrying capacity of A
(intrinsic rate / self-limitation)

② E_A $\begin{cases} r_A - \alpha_{AA}N_A^* - \alpha_{AB}N_B^* = 0 \\ N_B^* = 0 \end{cases} \Rightarrow \left(N_A^* = \frac{r_A}{\alpha_{AA}},\ N_B^* = 0\right)$

③ E_B $\begin{cases} N_A^* = 0 \\ r_B - \alpha_{BA}N_A^* - \alpha_{BB}N_B^* = 0 \end{cases} \Rightarrow \left(N_A^* = 0,\ N_B^* = \frac{r_B}{\alpha_{BB}}\right)$

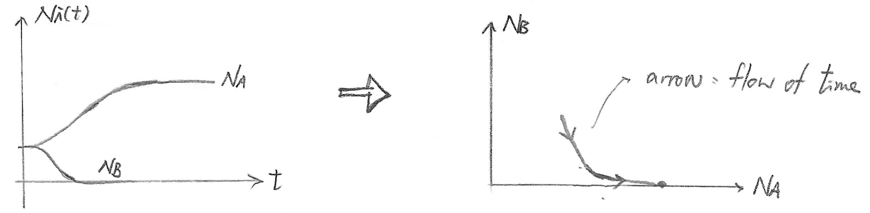
④ E_{AB} $\begin{cases} r_A - \alpha_{AA} N_A^* - \alpha_{AB} N_B^* = 0 \\ r_B - \alpha_{BA} N_A^* - \alpha_{BB} N_B^* = 0 \end{cases}$ $\Rightarrow$ ($N_A^* > 0$, $N_B^* > 0$)

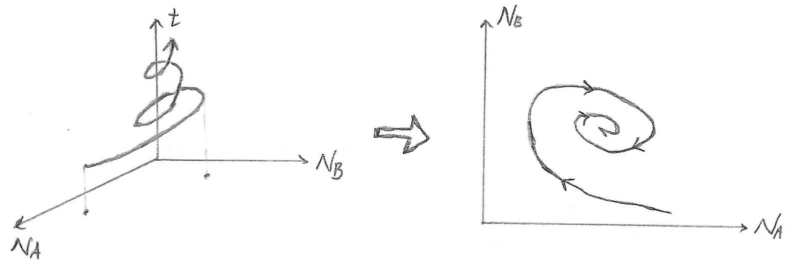
$$N_A^* = \frac{r_A r_B \left(\frac{\alpha_{BB}}{r_B} - \frac{\alpha_{AB}}{r_A} \right)}{\alpha_{AA}\alpha_{BB} - \alpha_{AB}\alpha_{BA}} \qquad N_B^* = \frac{r_A r_B \left(\frac{\alpha_{AA}}{r_A} - \frac{\alpha_{BA}}{r_B} \right)}{\alpha_{AA}\alpha_{BB} - \alpha_{AB}\alpha_{BA}}$$

⑥ Isocline & State space

State space $\Rightarrow$ A plot with the state variables on the axis

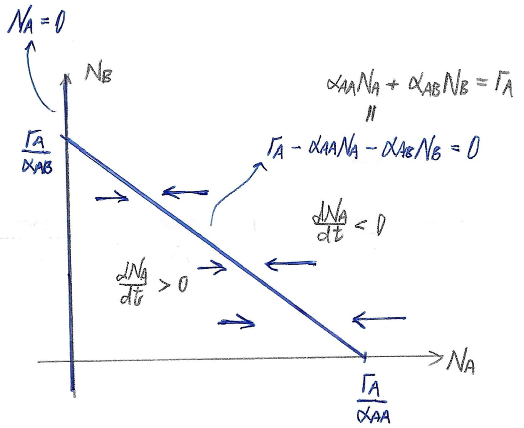
Each point within the state space is a combination of state variables





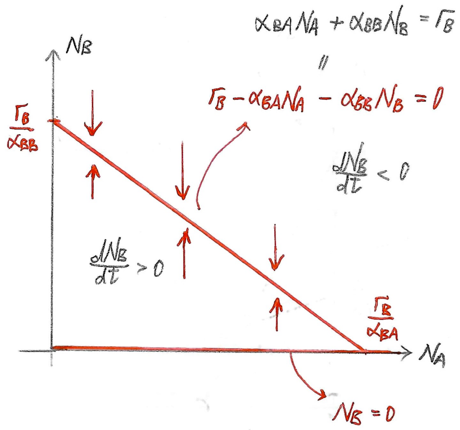
Isocline (Zero net growth isocline , ZNGI)

$\Rightarrow$ combinations of state variable that result in zero growth



« For N_A »

ZNGI that cause $\frac{dN_A}{dt} = 0$



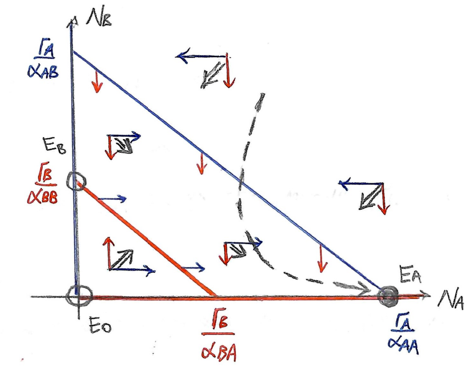
« For N_B »

ZNGI that cause $\frac{dN_B}{dt} = 0$

◎ Graphical Analysis 3

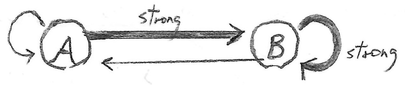
put ZNGIs together on the state space and check movements

« Case 1 » E_A is stable, N_A wins competition

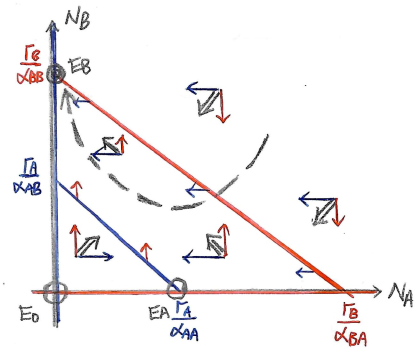


$$\begin{cases} \frac{r_A}{\alpha_{AA}} > \frac{r_B}{\alpha_{BA}} \\ \frac{r_A}{\alpha_{AB}} > \frac{r_B}{\alpha_{BB}} \end{cases} \Rightarrow \begin{cases} \frac{\alpha_{AA}}{r_A} < \frac{\alpha_{BA}}{r_B} \\ \frac{\alpha_{BB}}{r_B} > \frac{\alpha_{AB}}{r_A} \end{cases}$$

when special case $r_A = r_B$: $\begin{cases} \alpha_{AA} < \alpha_{BA} \\ \alpha_{BB} > \alpha_{AB} \end{cases}$

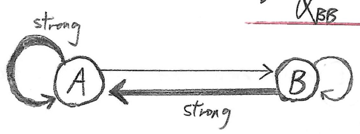


« Case 2 » E_B is stable, N_B wins competition



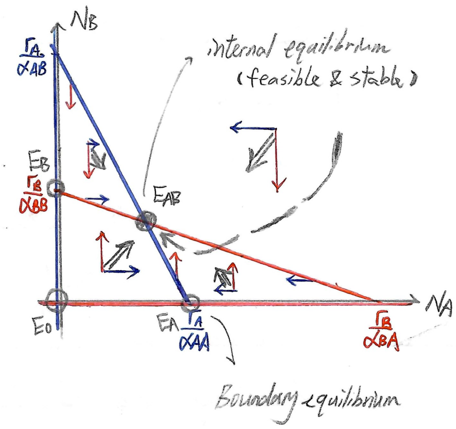
$$\begin{cases} \frac{r_B}{\alpha_{BA}} > \frac{r_A}{\alpha_{AA}} \\ \frac{r_B}{\alpha_{BB}} > \frac{r_A}{\alpha_{AB}} \end{cases} \Rightarrow \begin{cases} \frac{\alpha_{AA}}{r_A} > \frac{\alpha_{BA}}{r_B} \\ \frac{\alpha_{BB}}{r_B} < \frac{\alpha_{AB}}{r_A} \end{cases}$$

when special case $r_A = r_B$: $\begin{cases} \alpha_{AA} > \alpha_{BA} \\ \alpha_{BB} < \alpha_{AB} \end{cases}$



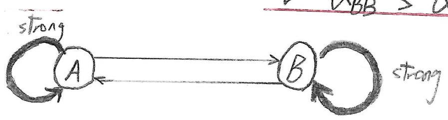
↗ the internal equilibrium is feasible in this case (i.e., $N_i^* > 0$)

« Case 3 » E_{AB} is stable, species coexist

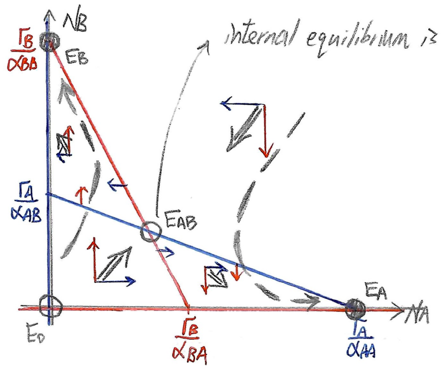


$$\begin{cases} \frac{r_B}{\alpha_{BA}} > \frac{r_A}{\alpha_{AA}} \\ \frac{r_A}{\alpha_{AB}} > \frac{r_B}{\alpha_{BB}} \end{cases} \Rightarrow \begin{cases} \frac{\alpha_{AA}}{r_A} > \frac{\alpha_{BA}}{r_B} \\ \frac{\alpha_{BB}}{r_B} > \frac{\alpha_{AB}}{r_A} \end{cases}$$

when special case $r_A = r_B$: $\begin{cases} \alpha_{AA} > \alpha_{BA} \\ \alpha_{BB} > \alpha_{AB} \end{cases}$

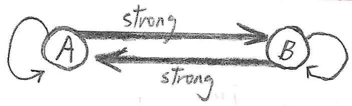


« Case 4 » E_A & E_B are both stable, Alternative stable states



$$\begin{cases} \dfrac{r_A}{\alpha_{AA}} > \dfrac{r_B}{\alpha_{BA}} \\ \dfrac{r_B}{\alpha_{BB}} > \dfrac{r_A}{\alpha_{AB}} \end{cases} \Rightarrow \begin{cases} \dfrac{\alpha_{AA}}{r_A} < \dfrac{\alpha_{BA}}{r_B} \\ \dfrac{\alpha_{BB}}{r_B} < \dfrac{\alpha_{AB}}{r_A} \end{cases}$$

when special case $r_A = r_B$: $\begin{cases} \alpha_{AA} < \alpha_{BA} \\ \alpha_{BB} < \alpha_{AB} \end{cases}$



Summary: competitive outcome depends on two sets of comparison

$\begin{cases} \alpha_{AA} \text{ vs } \alpha_{BA} \Rightarrow \text{A effect on A vs. A effect on B} \\ \alpha_{BB} \text{ vs } \alpha_{AB} \Rightarrow \text{B effect on B vs. B effect on A} \end{cases}$

⇒ Depends on species effect on conspecifics vs. species effect on heterospecifics

⇒ Coexist when each species effect on conspecifics > effect on heterospecifics

Often hear that coexistence requires Intra > Inter-specific competition but this statement is imprecise, the correct comparison for coexistence is $\alpha_{ii} > \alpha_{ji}$, not $\alpha_{ii} > \alpha_{ij}$

(V) each species IMPOSES stronger impact ON conspecifics than on heterospecifics

(X) each species is AFFECTED BY conspecifics more than by the heterospecifics