# *Introduction to Theoretical Ecology Assignment 6*

Graphical Analysis of Lotka-Volterra Competition Model

The Lotka-Volterra competition model can be written in terms of the carrying capacities of the two competing species $N_1$ and $N_2$:

$$\frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1 + \alpha N_2}{K_1})$$

$$\frac{dN_2}{dt} = r_2 N_2 (1 - \frac{N_2 + \beta N_1}{K_2})$$

, where $r_1$ and $r_2$ are the intrinsic population growth rates; $K_1$ and $K_2$ are the carrying capacities; $\alpha$ is the effect of $N_1$ on the population growth of $N_2$; $\beta$ is the effect of $N_2$ on the population growth of $N_1$.

1. Find all possible equilibrium population sizes of the two species. (5 pts)

***Solution:***

$$\begin{cases} 0 = r_1 N_1^* (1 - \frac{N_1^* + \alpha N_2^*}{K_1}) \\ 0 = r_2 N_2^* (1 - \frac{N_2^* + \beta N_1^*}{K_2}) \end{cases}$$

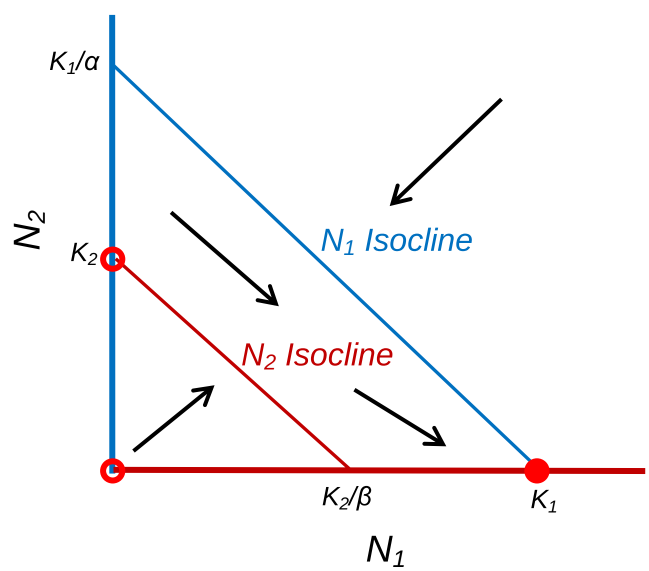
$$(N_1^*, N_2^*) = (0,0), (K_1, 0), (0, K_2), and\ (\frac{K_1 - \alpha K_2}{1 - \alpha\beta}, \frac{K_2 - \beta K_1}{1 - \alpha\beta})$$

2. Use graphical analysis to determine the stability of the system for all possible scenarios. Please (1) show the stability criteria in each scenario; (2) mark the equilibrium points (both stable and unstable) in the phase plane; and (3) denote all the intercepts between the isoclines and axes. (10 pts)

***Solution:***

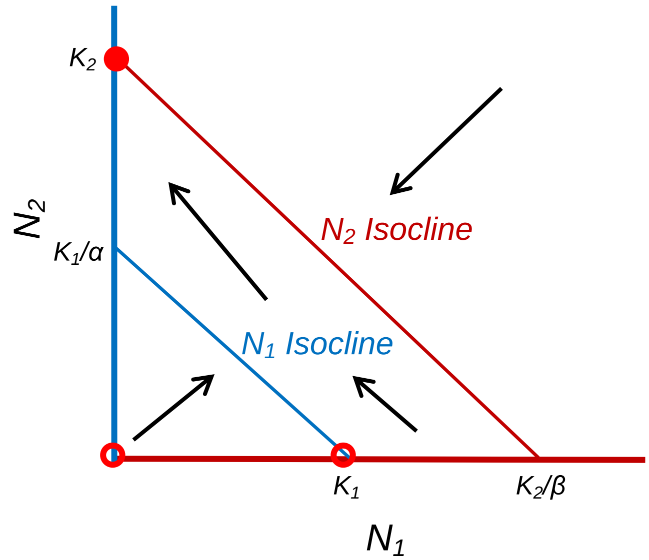
Scenario 1: $N_1$ wins

- Stable equilibrium point: ($K_1$, 0)
- Stability criteria: $K_1 > K_2/\beta$ and $K_1/\alpha > K_2$



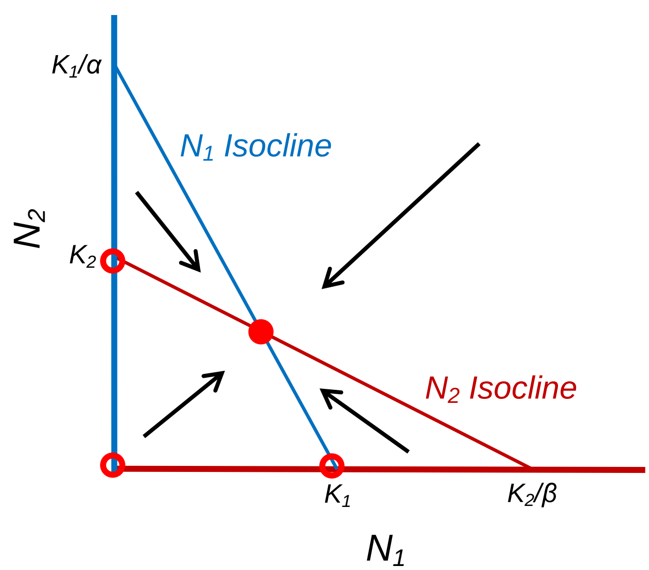

Scenario 2: $N_2$ wins

- Stable equilibrium point: (0, $K_2$)
- Stability criteria: $K_2 > K_1/\alpha$ and $K_2/\beta > K_1$




Scenario 3: Stable coexistence

- Stable equilibrium point: $\left(\frac{K_1 - \alpha K_2}{1 - \alpha\beta}, \frac{K_2 - \beta K_1}{1 - \alpha\beta}\right)$
- Stability criteria: $K_1/\alpha > K_2$ and $K_2/\beta > K_1$



Scenario 4: Unstable coexistence (saddle)

- Stable equilibrium point: ($K_1$, 0) or (0, $K_2$) (depending on the initial conditions)
- Stability criteria: $K_1 > K_2/\beta$ and $K_2 > K_1/\alpha$