AS | Physics - 9702 - 1.3 Errors and Uncertainties

INTRODUCTION

As we have been learning, a physical quantity is a measurable quantity. Scientists often make measurements. These need to be stated with the units of measurement, and the associated degree of accuracy. In this unit we will learn about the degrees of accuracy, errors and uncertainties of physical quantities.

LEARNING OBJECTIVES

By the end of this unit you should be able to:

1. Understand and explain the effects of systematic errors (including zero errors) and random errors in measurements.
2. Understand the distinction between precision and accuracy.
3. Assess the uncertainty in a derived quantity by the simple addition of absolute or percentage uncertainties.

Errors and Uncertainties

Errors and Uncertainties

Introduction

Measurements of quantities are made to find the true value of that quantity. The true value refers to a perfect measurement value that reflects the quantity being measured with no errors. However, this is, in reality, quite impossible. There will always be a degree of ***uncertainty***.

The uncertainty is an estimate of the difference between a measurement reading and the true value.

LEARNING OBJECTIVES

By the end of this section you should be able to:

1. Understand and explain the effects of systematic errors (including zero errors) and random errors in measurements.
2. Understand the distinction between precision and accuracy.
3. Assess the uncertainty in a derived quantity by the simple addition of absolute or percentage uncertainties.

TABLE OF CONTENTS

TERMS AND DEFINITIONS

Anomalous - An outlier which noticeably stands out from other data entries in a set.
Significant figure - A method of rounding off. For example, in the number 7 483, the most significant figure is 7, as its value is 7 000. To give 7 483 correct to one significant figure (1 s.f.) would be 7 000. To 2 s.f., it would be 7 500.
True value - A perfect measurement value that reflects the quantity being measured with no errors.
Uncertainty - An estimate of the difference between a measurement reading and the true value.

1. Accuracy vs Precision

In everyday conversation, accuracy and precision might sound like the same thing, but they actually mean different things.

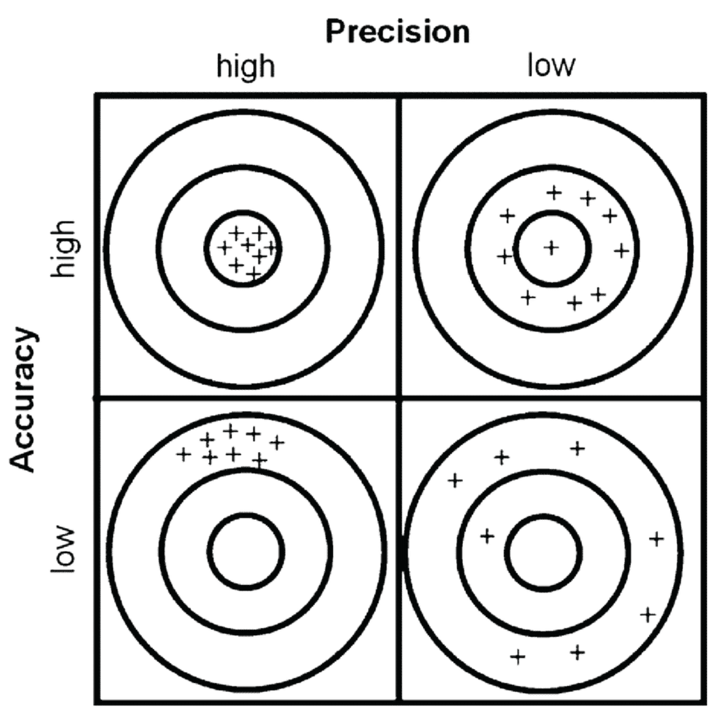
- **Accuracy** is how close a measured value is to the actual (or true) value. The accuracy can be increased by repeating measurements and calculating the mean of the measurements. For example, $9.75ms^{-2}$ is a more accurate value of the acceleration due to gravity ($g = 9.81ms^{-2}$) than $11.6ms^{-2}$.

- **Precision** is how close the measured values are to each other. Precise measurements are spread over a range. The measurements to a greater number of decimal places are said to be more precise than those to a whole number.

An ***anomaly*** is a data entry that falls outside the range of precise values. It is sometimes called an ***outlier***.

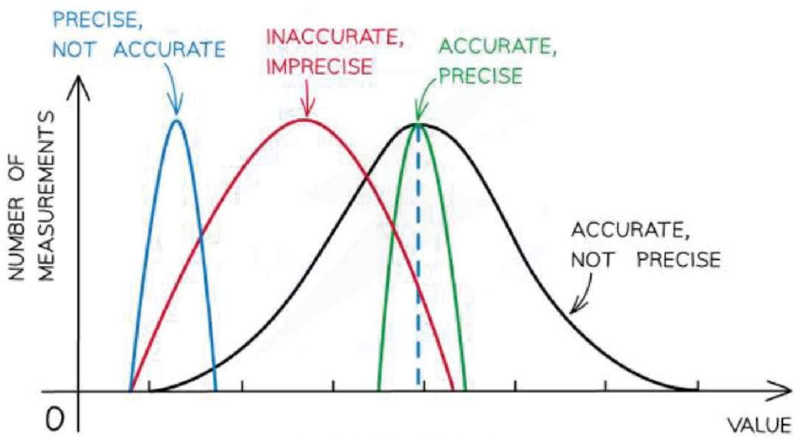
NOTE : A measurement can be precise but inaccurate however a measurement can not be accurate but imprecise.

To illustrate the difference, we can use the example of a target:



Illustrating precision and accuracy

It can also be shown on a graph of data as shown below. Each graph colour has a caption in its colour.



Demonstrating precision and accuracy

When you make a measurement, you have to be aware of the **uncertainty** in the measurement. Often (but not always), it is determined by the **smallest division on the measurement instrument**.

When carrying out practical work, you must consider two things:

- how the equipment or technique can be **improved** to reduce the uncertainty associated with measurements.
- how to **present** the uncertainty in your findings.

Good equipment and good technique will reduce uncertainties, but difficulties and judgements in making observations will limit the precision of your measurements.

How to remember the difference:

- a**C**curacy is **C**orrect (a bullseye)
- p**R**ecise is **R**epeating (hitting the same spot, but maybe not the correct spot)

The following section will introduce the types and examples of errors.

2. Random and Systematic Errors

2.1 Random error

When a judgement has to be made by the **observer**, the measurement will sometimes be above and/or below the ***true value***.

Random errors cause unpredictable fluctuations in an instrument's reading as a result of uncontrollable factors, such as environmental conditions. This affects the precision of the measurements taken, causing a wider spread of results about the mean value.

Repeating measurements more than once and then **calculating the mean** will reduce random error.

2.2 Systematic error

This is an error that arises as a result of faulty equipment. A systematic error causes a set of measurements to all be wrong by the same amount. Taking multiple readings does not eliminate a systematic error. Either the faulty equipment may be **adjusted** or **recalibrated** or the measurements are **corrected** to account for the systematic error.

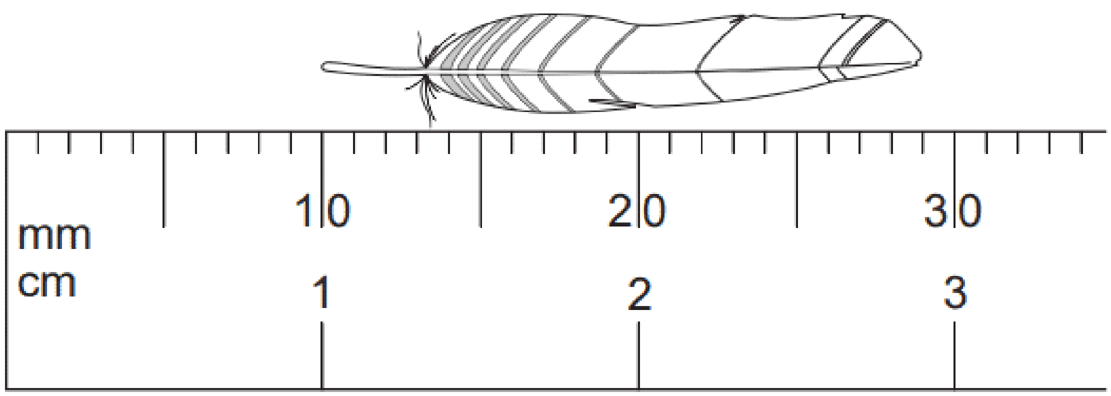
For example, the spring on a force meter might have become weaker over time, resulting in the force meter showing results that read consistently higher than the actual amount by say 3N. This means that all the readings from this force meter will be 3N more than the actual value.

To reduce systematic errors, instruments should be **recalibrated**, or the **technique** being used should be corrected or adjusted.

2.2a Zero error

Zero error is an example of a systematic error which occurs when an instrument starts off its readings from a point that is not 0. The instrument whether digital or analogue, may start at a value greater or lower than 0. All the readings become either greater or lesser than the true value by the same amount.

The diagram below shows how a zero error may come about. A student may read-off the length of the feather as 2.9cm but this is inaccurate as the feather's length starts off at 1.0cm and not 0cm. The true value in this case is 2.9cm - 1.0cm = 1.9cm.



An example of a zero error

3. Estimating Uncertainties

An uncertainty is a range of values by which a measured value is expected to deviate from the true value. The uncertainty is usually expressed with the measured value as ±.

Using the feather example above, the length of the feather can be expressed with its uncertainty as: (2.9 ± 0.1) cm. This means that the length of the feather is between 2.8 cm and 3.0 cm. ± 0.1 is the uncertainty in the length of the feather.

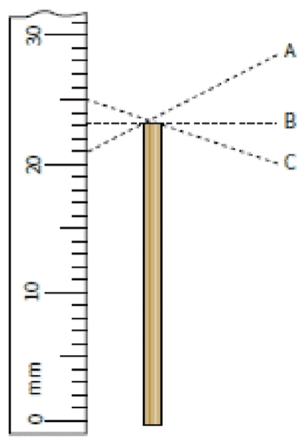
Uncertainties are usually given where one has to express it. However, in experimental work, the uncertainty is either the smallest division or half the smallest division of the measuring instrument used. Below is an explanation of how you would know which is which.

3.1 Using the division on the scale

Look at the smallest division on the scale used to take the reading. Then, decide whether you can **read the scale better** than the smallest division. This will depend on the **instrument** you are using and whether the instrument's scale is accurate.

Example:

Look at the example below. The division of the ruler is 1 mm. You have to decide whether it is possible to read the scale to better than 1 mm. When reading the scale, a parallax error might occur, which will result in you thinking that an uncertainty of 0.5 mm or 1 mm is reasonable. In general, the position of a mark on a ruler is generally measured to an uncertainty of ± 0.5mm.



When using a protractor, you can measure to the nearest 1°. However, it is unlikely that you can measure an angle better than ± 0.5° with your eye.

3.2 Repeat the readings

You should **repeat the readings** several times. You can then determine the uncertainty by taking half of the range of the values obtained. In other words, subtract the smallest reading from the largest reading. Then, halve the result.

This method should always be tried because it may reveal random errors (but not systematic errors), and it gives an easy way to estimate the uncertainty.

Although you should **always try this method**, you have to try both methods when the readings are all the same.

4. Calculating Uncertainties

Uncertainties can be represented in several ways:

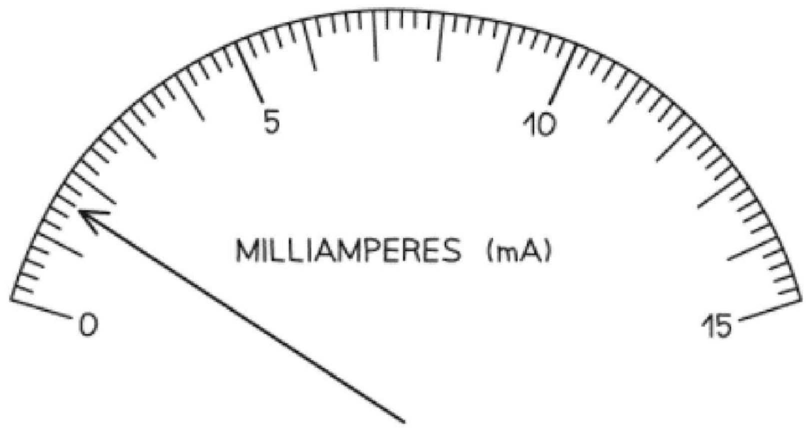
Considering a mass measurement m,

- **Absolute uncertainty:** the uncertainty is given as a fixed quantity Δm
- **Fractional uncertainty:** the uncertainty given as a fraction Δm/m
- **Percentage uncertainty:** the uncertainty given as a percentage Δm/m x 100%

The symbol Δ means "uncertainty in," therefore ΔI means "uncertainty in I"

Example:

Let's calculate the absolute, fractional and percentage uncertainty of the example below. The smallest division is 0.2 mA and the reading is 1.6 mA.



- **Absolute uncertainty** = ΔI = ½ x 0.2 mA = 0.1 mA *therefore* I = (1.6 ± 0.1) mA
- **Fractional uncertainty** = ΔI / I = 0.1 / 1.6 = 1/16 *therefore* I = (1.6 ± 1/16) mA
- **Percentage uncertainty** = ΔI / I x 100% = 0.1 / 1.6 × 100 = 6.2 % *therefore* I = (1.6 ± 6.2%) mA

4.1 Finding uncertainties in different situations

To find uncertainties in different situations:

- **Uncertainty in a reading:** ± half the smallest division.
- **Uncertainty in a measurement:** at least ±1 the smallest division.
- **Uncertainty in repeated data:** half the range, in other words, ± ½ (largest - smallest value)
- **Uncertainty in digital readings:** ± the last significant digit unless otherwise quoted.

EXAM - STYLE QUESTIONS

Consider the EXAM-STYLE QUESTION below. After reflecting on this question, click the FEEDBACK and EXPLANATION tabs to reveal the suggested answer.

QUESTION	FEEDBACK	EXPLANATION

The acceleration of free fall g may be determined from an oscillating pendulum using the equation

$$g = 4\pi^2 l / T^2$$

where l is the length of the pendulum and T is the period of oscillation. In an experiment, the measured values for an oscillating pendulum are

l = 1.50m ± 2% and

T = 2.48s ± 3%.

1. Calculate the acceleration of free fall g. [1]

2. Determine the percentage uncertainty in g. [2]

3. Use your answers in (1) and (2) to determine the absolute uncertainty of the calculated value of g. [1]

QUESTION	FEEDBACK	EXPLANATION

1. $g = 9.63\ \text{m s}^{-2}$

2. percentage uncertainty = 8%

3. absolute uncertainty = = $0.8\ \text{m s}^{-2}$

QUESTION	FEEDBACK	EXPLANATION

1. $g = (4\ \pi^2 \times 1.50) / (2.482) = 9.63\ \text{m s}^{-2}$

2. percentage uncertainty = 2 + (3 × 2) or
   fraction uncertainty = 0.02 + (0.03 × 2)
   percentage uncertainty = 8%

3. absolute uncertainty = 0.08 × 9.6
   = $0.8\ \text{m s}^{-2}$

5. Combining Uncertainties

The basics of determining uncertainty are quite simple, but combining two uncertain numbers gets more complicated. However, there are simple rules to follow.

5.1 Adding or subtracting data

The rule is to **add the absolute value**:

A student is required to find the difference between the inner and outer diameters of a Goodwill tyre.

Outer diameter of tyre (d_1) = 55.0 ± 0.5cm
Inner diameter of tyre (d_2) = 21.0 ± 0.7cm

Difference in diameters (d_1 - d_2)
= 55.0 - 21.0 = 34.0cm
Uncertainty in difference = ± (0.5 + 0.7) = ±1.2cm

D = (34.0 ± 1.2) cm

5.2 Multiplying or dividing data

The rule is to **add the percentage uncertainties**:

- Distance = 50.0 ± 0.1m
- Time = 5.00 ± 0.05s

Speed (v) = Distance (s) / Time (t)
V = 50.0/5.0 = 10ms^{-1}

$\triangle v/v = \triangle s/s + \triangle t/t$
0.1/50.0 + 0.05/5.00
= 0.002 + 0.01
= 0.012

Absolute uncertainty ($\triangle v$)

10.0 * 0.012
$= \pm 0.12 ms^{-1}$
$v = (10.00 \pm 0.12)\ m\ s^{-1}$

You can read a bit more about how to calculate uncertainty ▶here◀.

KEY TAKEAWAYS

- Accuracy is how close a measured value is to the actual (or true) value. The accuracy can be increased by repeating measurements and finding the mean.
- Precision is how close the measured values are to each other.
- Uncertainty can be thought of as the difference between the actual reading taken (caused by the equipment or techniques used) and the true value. Uncertainties are not the same as errors.

- Uncertainty can be estimated in two ways: using the division on the scale and repeating the readings.
- The measurement and uncertainty must be given to the same decimal places.

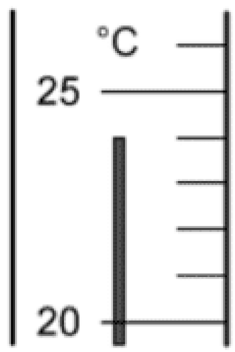
TEST YOUR KNOWLEDGE

1. The density of the material of a coil of thin wire is to be found. Which set of instruments could be used to do this the most accurately?

- ○ metre rule, protractor, spring balance
- ○ micrometer, metre rule, top-pan balance
- ○ stopwatch, newton-metre, vernier calipers
- ○ tape measure, vernier calipers, lever balance

SUBMIT

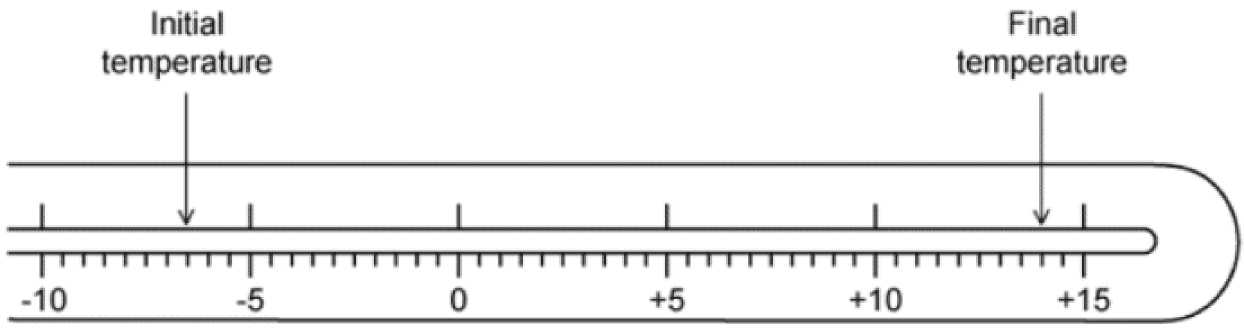
2. The diagram shows part of a thermometer. What is the correct reading on the thermometer and the uncertainty in this reading?



- 24 °C ± 1 °C
- 24 °C ± 0.1 °C
- 24.0 °C ± 0.2 °C
- 24.0 °C ± 0.5 °C

SUBMIT

3. The diagram shows the stem of a Celsius thermometer marked to show initial and final temperature values. What is the temperature change expressed to an appropriate number of significant figures?



- 14 °C
- 20.5 °C
- 21 °C
- 22.0 °C

SUBMIT

4. A micrometre is used to measure the diameters of two cylinders.

1. Diameter of first cylinder = 12.78 ± 0.02 mm
2. Diameter of second cylinder = 16.24 ± 0.03 mm

The difference in the diameters is calculated. What is the uncertainty in this difference?

- ± 0.01 mm
- ± 0.02 mm
- ± 0.03 mm
- ± 0.05 mm

SUBMIT

5. A student carries out a series of determinations of the acceleration of free fall *g*. The table shows the results. What can be said about this experiment?

g/ms^{-2}
4.91
4.81
4.88
4.90
4.92

- ○ It is accurate and precise.
- ○ It is accurate but not precise.
- ○ It is not accurate and not precise.
- ○ It is not accurate but precise.

SUBMIT

REFERENCES

- Crundell M, Goodwin G (2020), Cambridge International AS & A Level Physics Student's Book, https://www.hoddereducation.co.uk/
- Sang D, Jones G, Woodside R and Chadha G (2020), Cambridge International AS & A Level Physics: Coursebook, Third Edition, http://www.cambridge.org/education

END OF UNIT