Sets and Indices

T, t	Set and index for 15-minute time intervals $t \in \{1, ..., 35040\}$
B, b	Set and index for 4-hour ancillary service blocks $b \in \{1, ..., 2190\}$
J, j	Set and index for cycle degradation cost segments $j \in \{1, ..., J\}$
I, i	Set and index for calendar degradation cost breakpoints $i \in \{1, ..., I\}$

Parameters

Symbol	Description	Unit
$P_{DA}(t)$	Day-ahead electricity price	EUR/MWh
$P_{FCR}(b)$	FCR capacity price	EUR/MW/h
$P^{\text{pos}}_{aFRR}(b)$	Positive aFRR capacity price	EUR/MW/h
$P^{\text{neg}}_{aFRR}(b)$	Negative aFRR capacity price	EUR/MW/h
$P^{\text{pos}}_{aFRR,E}(t)$	**(New)** Positive aFRR energy price	EUR/MWh
$P^{\text{neg}}_{aFRR,E}(t)$	**(New)** Negative aFRR energy price	EUR/MWh
E_{nom}	Nominal energy capacity	kWh
$P^{\text{config}}_{\text{max}}$	Max charge/discharge power	kW
$\eta_{\text{ch}}, \eta_{\text{dis}}$	Charging/discharging efficiencies	-
$SOC_{\text{min}}, SOC_{\text{max}}$	Min/max SOC limits (fraction)	-
E^{seg}_j	**(New)** Max energy capacity of segment j	kWh
c^{cost}_j	**(New)** Marginal cyclic degradation cost for segment j	EUR/kWh
SOC^{point}_i	**(New)** SOC breakpoint i for calendar cost	kWh
$Cost^{\text{point}}_i$	**(New)** Calendar degradation cost at breakpoint i	EUR/hr
α	**(New)** Degradation price (meta-parameter)	-

Decision Variables

Symbol	Description	Type
$p_{\text{ch}}(t), p_{\text{dis}}(t)$	DA charge/discharge power	Cont. ≥ 0
$p^{\text{pos}}_{aFRR,E}(t)$	**(New)** aFRR-E positive (discharge) power	Cont. ≥ 0
$p^{\text{neg}}_{aFRR,E}(t)$	**(New)** aFRR-E negative (charge) power	Cont. ≥ 0
$c_{fcr}(b)$	FCR capacity bid	Cont. ≥ 0
$c^{\text{pos}}_{aFRR}(b)$	Positive aFRR capacity bid	Cont. ≥ 0
$c^{\text{neg}}_{aFRR}(b)$	Negative aFRR capacity bid	Cont. ≥ 0
$p^{\text{total}}_{\text{ch}}(t), p^{\text{total}}_{\text{dis}}(t)$	**(New)** Total charge/discharge power	Cont. ≥ 0
$p^{\text{ch}}_j(t), p^{\text{dis}}_j(t)$	**(New)** Charge/discharge power for segment j	Cont. ≥ 0
$e_{\text{soc},j}(t)$	**(New)** Energy stored in segment j	Cont. ≥ 0
$e_{\text{soc}}(t)$	**(New)** Total energy stored in BESS	Cont. ≥ 0
$c^{\text{cal}}_{\text{cost}}(t)$	**(New)** Calendar cost at time t	Cont. ≥ 0
$\lambda_{t,i}$	**(New)** SOS2 variable for calendar cost	Cont. ≥ 0
$y_{\text{ch}}(t), y_{\text{dis}}(t)$	DA bid binaries	Binary
$y_{fcr}(b), y^{\text{pos}}_{aFRR}(b), ...$	Ancillary service bid binaries	Binary
$y^{\text{pos}}_{aFRR,E}(t), ...$	**(New)** aFRR-E bid binaries	Binary
$y^{\text{total}}_{\text{ch}}(t), y^{\text{total}}_{\text{dis}}(t)$	**(New)** Total operation binaries	Binary

1 Model (i): Base Model + aFRR Energy Market

1.1 Objective Function

The objective is to maximize the total profit from market participation.

$$\max\ Z = \mathbb{P}^{DA} + \mathbb{P}^{ANCI} + \mathbb{P}^{aFRR\_E} \tag{1}$$

Where:

$$\mathbb{P}^{DA} = \sum_{t\in T}\left(\frac{P_{DA}(t)}{1000}\,p_{\text{dis}}(t) - \frac{P_{DA}(t)}{1000}\,p_{\text{ch}}(t)\right)\Delta t \tag{2}$$

$$\mathbb{P}^{ANCI} = \sum_{b\in B}\left(P_{FCR}(b)\,c_{fcr}(b) + P^{\text{pos}}_{aFRR}(b)\,c^{\text{pos}}_{aFRR}(b) + P^{\text{neg}}_{aFRR}(b)\,c^{\text{neg}}_{aFRR}(b)\right)\Delta b \tag{3}$$

$$\mathbb{P}^{aFRR\_E} = \sum_{t\in T}\left(\frac{P^{\text{pos}}_{aFRR,E}(t)}{1000}\,p^{\text{pos}}_{aFRR,E}(t) - \frac{P^{\text{neg}}_{aFRR,E}(t)}{1000}\,p^{\text{neg}}_{aFRR,E}(t)\right)\Delta t \tag{4}$$

1.2 Constraints

1.2.1 Standard SOC Dynamics

$$e_{\text{soc}}(t) = e_{\text{soc}}(t-1) + \left(p^{\text{total}}_{\text{ch}}(t)\eta_{\text{ch}} - \frac{p^{\text{total}}_{\text{dis}}(t)}{\eta_{\text{dis}}}\right)\Delta t \qquad \forall t \tag{5}$$

$$p^{\text{total}}_{\text{ch}}(t) = p_{\text{ch}}(t) + p^{\text{neg}}_{aFRR,E}(t) \qquad \forall t \tag{6}$$

$$p^{\text{total}}_{\text{dis}}(t) = p_{\text{dis}}(t) + p^{\text{pos}}_{aFRR,E}(t) \qquad \forall t \tag{7}$$

1.2.2 SOC Limits

$$SOC_{\min}\,E_{\text{nom}} \le e_{\text{soc}}(t) \le SOC_{\max}\,E_{\text{nom}} \qquad \forall t \tag{8}$$

1.2.3 Simultaneous Operation Prevention

$$p^{\text{total}}_{\text{ch}}(t) \le y^{\text{total}}_{\text{ch}}(t)\cdot P^{\text{config}}_{\max} \qquad \forall t \tag{9}$$

$$p^{\text{total}}_{\text{dis}}(t) \le y^{\text{total}}_{\text{dis}}(t)\cdot P^{\text{config}}_{\max} \qquad \forall t \tag{10}$$

$$y^{\text{total}}_{\text{ch}}(t) + y^{\text{total}}_{\text{dis}}(t) \le 1 \qquad \forall t \tag{11}$$

1.2.4 Market Co-optimization Power Limits

$$p^{\text{total}}_{\text{dis}}(t) + 1000\,c_{fcr}(b) + 1000\,c^{\text{pos}}_{aFRR}(b) \le P^{\text{config}}_{\max} \qquad \forall b, t\in b \tag{12}$$

$$p^{\text{total}}_{\text{ch}}(t) + 1000\,c_{fcr}(b) + 1000\,c^{\text{neg}}_{aFRR}(b) \le P^{\text{config}}_{\max} \qquad \forall b, t\in b \tag{13}$$

1.2.5 Ancillary Service Energy Reserve

$$\frac{\left(1000\,c_{fcr}(b) + 1000\,c^{\text{pos}}_{aFRR}(b)\right)\tau}{\eta_{\text{dis}}} \le e_{\text{soc}}(t) - SOC_{\min}\,E_{\text{nom}} \qquad \forall b, t\in b \tag{14}$$

$$\left(1000\,c_{fcr}(b) + 1000\,c^{\text{neg}}_{aFRR}(b)\right)\tau\,\eta_{\text{ch}} \le SOC_{\max}\,E_{\text{nom}} - e_{\text{soc}}(t) \qquad \forall b, t\in b \tag{15}$$

(Where τ is the reserve duration, e.g., 0.25h)

1.2.6 Ancillary Capacity Market Exclusivity

$$y_{fcr}(b) + y^{\text{pos}}_{aFRR}(b) + y^{\text{neg}}_{aFRR}(b) \leq 1 \qquad \forall b \tag{16}$$

1.2.7 Cross-Market Mutual Exclusivity

$$y^{\text{total}}_{\text{ch}}(t) + y_{fcr}(b) + y^{\text{pos}}_{aFRR}(b) \leq 1 \qquad \forall b, t \in b \tag{17}$$

$$y^{\text{total}}_{\text{dis}}(t) + y_{fcr}(b) + y^{\text{neg}}_{aFRR}(b) \leq 1 \qquad \forall b, t \in b \tag{18}$$

1.2.8 Minimum Bids & Binary Logic

$$y_{\text{ch}}(t)\,\text{MinBid}_{da} \cdot 1000 \leq p_{\text{ch}}(t) \leq y_{\text{ch}}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{19}$$

$$y_{\text{dis}}(t)\,\text{MinBid}_{da} \cdot 1000 \leq p_{\text{dis}}(t) \leq y_{\text{dis}}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{20}$$

$$y^{\text{pos}}_{aFRR,E}(t)\,\text{MinBid}_{afrr\_e} \cdot 1000 \leq p^{\text{pos}}_{aFRR,E}(t) \leq y^{\text{pos}}_{aFRR,E}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{21}$$

$$y^{\text{neg}}_{aFRR,E}(t)\,\text{MinBid}_{afrr\_e} \cdot 1000 \leq p^{\text{neg}}_{aFRR,E}(t) \leq y^{\text{neg}}_{aFRR,E}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{22}$$

$$y^{\text{total}}_{\text{ch}}(t) \geq y_{\text{ch}}(t); \quad y^{\text{total}}_{\text{ch}}(t) \geq y^{\text{neg}}_{aFRR,E}(t) \qquad \forall t \tag{23}$$

$$y^{\text{total}}_{\text{dis}}(t) \geq y_{\text{dis}}(t); \quad y^{\text{total}}_{\text{dis}}(t) \geq y^{\text{pos}}_{aFRR,E}(t) \qquad \forall t \tag{24}$$

(Similar min/max bid constraints for c_{fcr}, c^{pos}_{aFRR}, c^{neg}_{aFRR} remain from Phase I)

2 Model (ii): Model (i) + Cyclic Aging Cost

2.1 Objective Function

The objective is to maximize the total profit, defined as market revenues minus cyclic degradation costs.

$$\max\ Z = \mathbb{P}^{DA} + \mathbb{P}^{ANCI} + \mathbb{P}^{aFRR\_E} - \alpha \cdot C^{\text{cyc}} \tag{25}$$

Where:

$$C^{\text{cyc}} = \sum_{t \in T} \sum_{j \in J} \left(c^{\text{cost}}_j \cdot \frac{p^{\text{dis}}_j(t)}{\eta_{\text{dis}}} \cdot \Delta t \right) \tag{26}$$

2.2 Constraints

2.2.1 Degradation-Aware SOC Dynamics

$$e_{\text{soc},j}(t) = e_{\text{soc},j}(t-1) + \left(p^{\text{ch}}_j(t)\eta_{\text{ch}} - \frac{p^{\text{dis}}_j(t)}{\eta_{\text{dis}}} \right) \Delta t \qquad \forall t, \forall j \tag{27}$$

$$e_{\text{soc}}(t) = \sum_{j \in J} e_{\text{soc},j}(t) \qquad \forall t \tag{28}$$

$$p^{\text{total}}_{\text{ch}}(t) = \sum_{j \in J} p^{\text{ch}}_j(t) \qquad \forall t \tag{29}$$

$$p^{\text{total}}_{\text{dis}}(t) = \sum_{j \in J} p^{\text{dis}}_j(t) \qquad \forall t \tag{30}$$

$$p^{\text{total}}_{\text{ch}}(t) = p_{\text{ch}}(t) + p^{\text{neg}}_{aFRR,E}(t) \qquad \forall t \tag{31}$$

$$p^{\text{total}}_{\text{dis}}(t) = p_{\text{dis}}(t) + p^{\text{pos}}_{aFRR,E}(t) \qquad \forall t \tag{32}$$

2.2.2 SOC & Segment Limits

$$SOC_{\min} E_{\text{nom}} \le e_{\text{soc}}(t) \le SOC_{\max} E_{\text{nom}} \qquad \forall t \tag{33}$$
$$0 \le e_{\text{soc},j}(t) \le E_j^{\text{seg}} \qquad \forall t, \forall j \tag{34}$$

2.2.3 Simultaneous Operation Prevention

$$p_{\text{ch}}^{\text{total}}(t) \le y_{\text{ch}}^{\text{total}}(t) \cdot P_{\max}^{\text{config}} \qquad \forall t \tag{35}$$
$$p_{\text{dis}}^{\text{total}}(t) \le y_{\text{dis}}^{\text{total}}(t) \cdot P_{\max}^{\text{config}} \qquad \forall t \tag{36}$$
$$y_{\text{ch}}^{\text{total}}(t) + y_{\text{dis}}^{\text{total}}(t) \le 1 \qquad \forall t \tag{37}$$

2.2.4 Market Co-optimization Power Limits

$$p_{\text{dis}}^{\text{total}}(t) + 1000\, c_{fcr}(b) + 1000\, c_{aFRR}^{\text{pos}}(b) \le P_{\max}^{\text{config}} \qquad \forall b, t \in b \tag{38}$$
$$p_{\text{ch}}^{\text{total}}(t) + 1000\, c_{fcr}(b) + 1000\, c_{aFRR}^{\text{neg}}(b) \le P_{\max}^{\text{config}} \qquad \forall b, t \in b \tag{39}$$

2.2.5 Ancillary Service Energy Reserve

$$\frac{\left(1000\, c_{fcr}(b) + 1000\, c_{aFRR}^{\text{pos}}(b)\right)\tau}{\eta_{\text{dis}}} \le e_{\text{soc}}(t) - SOC_{\min} E_{\text{nom}} \qquad \forall b, t \in b \tag{40}$$
$$\left(1000\, c_{fcr}(b) + 1000\, c_{aFRR}^{\text{neg}}(b)\right)\tau\, \eta_{\text{ch}} \le SOC_{\max} E_{\text{nom}} - e_{\text{soc}}(t) \qquad \forall b, t \in b \tag{41}$$

(Where τ is the reserve duration, e.g., 0.25h)

2.2.6 Ancillary Capacity Market Exclusivity

$$y_{fcr}(b) + y_{aFRR}^{\text{pos}}(b) + y_{aFRR}^{\text{neg}}(b) \le 1 \qquad \forall b \tag{42}$$

2.2.7 Cross-Market Mutual Exclusivity

$$y_{\text{ch}}^{\text{total}}(t) + y_{fcr}(b) + y_{aFRR}^{\text{pos}}(b) \le 1 \qquad \forall b, t \in b \tag{43}$$
$$y_{\text{dis}}^{\text{total}}(t) + y_{fcr}(b) + y_{aFRR}^{\text{neg}}(b) \le 1 \qquad \forall b, t \in b \tag{44}$$

2.2.8 Minimum Bids & Binary Logic

$$y_{\text{ch}}(t)\, \text{MinBid}_{da} \cdot 1000 \le p_{\text{ch}}(t) \le y_{\text{ch}}(t)\, P_{\max}^{\text{config}} \qquad \forall t \tag{45}$$
$$y_{\text{dis}}(t)\, \text{MinBid}_{da} \cdot 1000 \le p_{\text{dis}}(t) \le y_{\text{dis}}(t)\, P_{\max}^{\text{config}} \qquad \forall t \tag{46}$$
$$y_{aFRR,E}^{\text{pos}}(t)\, \text{MinBid}_{afrr\_e} \cdot 1000 \le p_{aFRR,E}^{\text{pos}}(t) \le y_{aFRR,E}^{\text{pos}}(t)\, P_{\max}^{\text{config}} \qquad \forall t \tag{47}$$
$$y_{aFRR,E}^{\text{neg}}(t)\, \text{MinBid}_{afrr\_e} \cdot 1000 \le p_{aFRR,E}^{\text{neg}}(t) \le y_{aFRR,E}^{\text{neg}}(t)\, P_{\max}^{\text{config}} \qquad \forall t \tag{48}$$
$$y_{\text{ch}}^{\text{total}}(t) \ge y_{\text{ch}}(t); \quad y_{\text{ch}}^{\text{total}}(t) \ge y_{aFRR,E}^{\text{neg}}(t) \qquad \forall t \tag{49}$$
$$y_{\text{dis}}^{\text{total}}(t) \ge y_{\text{dis}}(t); \quad y_{\text{dis}}^{\text{total}}(t) \ge y_{aFRR,E}^{\text{pos}}(t) \qquad \forall t \tag{50}$$

(Similar min/max bid constraints for c_{fcr}, c_{aFRR}^{pos}, c_{aFRR}^{neg} remain from Phase I)

3 Model (iii): Model (ii) + Calendar Aging Cost

3.1 Objective Function

The objective is to maximize the total profit, defined as market revenues minus degradation costs.

$$\max\ Z = \mathbb{P}^{DA} + \mathbb{P}^{ANCI} + \mathbb{P}^{aFRR\_E} - \alpha \cdot (C^{\text{cyc}} + C^{\text{cal}}) \tag{51}$$

Where:

$$C^{\text{cal}} = \sum_{t \in T} c^{\text{cal}}_{\text{cost}}(t) \cdot \Delta t \tag{52}$$

3.2 Constraints

3.2.1 Degradation-Aware SOC Dynamics (Replaces Cst-1)

$$e_{\text{soc},j}(t) = e_{\text{soc},j}(t-1) + \left(p^{\text{ch}}_j(t)\eta_{\text{ch}} - \frac{p^{\text{dis}}_j(t)}{\eta_{\text{dis}}} \right) \Delta t \qquad \forall t, \forall j \tag{53}$$

$$e_{\text{soc}}(t) = \sum_{j \in J} e_{\text{soc},j}(t) \qquad \forall t \tag{54}$$

$$p^{\text{total}}_{\text{ch}}(t) = \sum_{j \in J} p^{\text{ch}}_j(t) \qquad \forall t \tag{55}$$

$$p^{\text{total}}_{\text{dis}}(t) = \sum_{j \in J} p^{\text{dis}}_j(t) \qquad \forall t \tag{56}$$

$$p^{\text{total}}_{\text{ch}}(t) = p_{\text{ch}}(t) + p^{\text{neg}}_{aFRR,E}(t) \qquad \forall t \tag{57}$$

$$p^{\text{total}}_{\text{dis}}(t) = p_{\text{dis}}(t) + p^{\text{pos}}_{aFRR,E}(t) \qquad \forall t \tag{58}$$

3.2.2 SOC & Segment Limits (Replaces Cst-2)

$$SOC_{\min} E_{\text{nom}} \le e_{\text{soc}}(t) \le SOC_{\max} E_{\text{nom}} \qquad \forall t \tag{59}$$

$$0 \le e_{\text{soc},j}(t) \le E^{\text{seg}}_j \qquad \forall t, \forall j \tag{60}$$

3.2.3 Simultaneous Operation Prevention (Replaces Cst-3)

$$p^{\text{total}}_{\text{ch}}(t) \le y^{\text{total}}_{\text{ch}}(t) \cdot P^{\text{config}}_{\max} \qquad \forall t \tag{61}$$

$$p^{\text{total}}_{\text{dis}}(t) \le y^{\text{total}}_{\text{dis}}(t) \cdot P^{\text{config}}_{\max} \qquad \forall t \tag{62}$$

$$y^{\text{total}}_{\text{ch}}(t) + y^{\text{total}}_{\text{dis}}(t) \le 1 \qquad \forall t \tag{63}$$

3.2.4 Market Co-optimization Power Limits (Replaces Cst-4)

$$p^{\text{total}}_{\text{dis}}(t) + 1000\, c_{fcr}(b) + 1000\, c^{\text{pos}}_{aFRR}(b) \le P^{\text{config}}_{\max} \qquad \forall b, t \in b \tag{64}$$

$$p^{\text{total}}_{\text{ch}}(t) + 1000\, c_{fcr}(b) + 1000\, c^{\text{neg}}_{aFRR}(b) \le P^{\text{config}}_{\max} \qquad \forall b, t \in b \tag{65}$$

3.2.5 Ancillary Service Energy Reserve (Keeps Cst-6 Logic)

$$\frac{\left(1000\, c_{fcr}(b) + 1000\, c^{\text{pos}}_{aFRR}(b)\right)\tau}{\eta_{\text{dis}}} \le e_{\text{soc}}(t) - SOC_{\min} E_{\text{nom}} \qquad \forall b, t \in b \tag{66}$$

$$\left(1000\, c_{fcr}(b) + 1000\, c^{\text{neg}}_{aFRR}(b)\right)\tau\, \eta_{\text{ch}} \le SOC_{\max} E_{\text{nom}} - e_{\text{soc}}(t) \qquad \forall b, t \in b \tag{67}$$

(Where τ is the reserve duration, e.g., 0.25h)

3.2.6 Ancillary Capacity Market Exclusivity (Keeps Cst-7)

$$y_{fcr}(b) + y^{\text{pos}}_{aFRR}(b) + y^{\text{neg}}_{aFRR}(b) \leq 1 \qquad \forall b \tag{68}$$

3.2.7 Cross-Market Mutual Exclusivity (Replaces Cst-8)

$$y^{\text{total}}_{\text{ch}}(t) + y_{fcr}(b) + y^{\text{pos}}_{aFRR}(b) \leq 1 \qquad \forall b, t \in b \tag{69}$$

$$y^{\text{total}}_{\text{dis}}(t) + y_{fcr}(b) + y^{\text{neg}}_{aFRR}(b) \leq 1 \qquad \forall b, t \in b \tag{70}$$

3.2.8 Minimum Bids & Binary Logic (Updates Cst-9)

$$y_{\text{ch}}(t)\,\text{MinBid}_{da} \cdot 1000 \leq p_{\text{ch}}(t) \leq y_{\text{ch}}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{71}$$

$$y_{\text{dis}}(t)\,\text{MinBid}_{da} \cdot 1000 \leq p_{\text{dis}}(t) \leq y_{\text{dis}}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{72}$$

$$y^{\text{pos}}_{aFRR,E}(t)\,\text{MinBid}_{afrr\_e} \cdot 1000 \leq p^{\text{pos}}_{aFRR,E}(t) \leq y^{\text{pos}}_{aFRR,E}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{73}$$

$$y^{\text{neg}}_{aFRR,E}(t)\,\text{MinBid}_{afrr\_e} \cdot 1000 \leq p^{\text{neg}}_{aFRR,E}(t) \leq y^{\text{neg}}_{aFRR,E}(t)\,P^{\text{config}}_{\max} \qquad \forall t \tag{74}$$

$$y^{\text{total}}_{\text{ch}}(t) \geq y_{\text{ch}}(t); \quad y^{\text{total}}_{\text{ch}}(t) \geq y^{\text{neg}}_{aFRR,E}(t) \qquad \forall t \tag{75}$$

$$y^{\text{total}}_{\text{dis}}(t) \geq y_{\text{dis}}(t); \quad y^{\text{total}}_{\text{dis}}(t) \geq y^{\text{pos}}_{aFRR,E}(t) \qquad \forall t \tag{76}$$

(Similar min/max bid constraints for c_{fcr}, c^{pos}_{aFRR}, c^{neg}_{aFRR} remain from Phase I)

3.2.9 Calendar Aging PWL Constraints (New)

$$e_{\text{soc}}(t) = \sum_{i \in I} \lambda_{t,i} \cdot SOC^{\text{point}}_i \qquad \forall t \tag{77}$$

$$c^{\text{cal}}_{\text{cost}}(t) = \sum_{i \in I} \lambda_{t,i} \cdot Cost^{\text{point}}_i \qquad \forall t \tag{78}$$

$$\sum_{i \in I} \lambda_{t,i} = 1 \qquad \forall t \tag{79}$$

$$\{\lambda_{t,i}\}_{i \in I} \text{ are SOS2 variables} \qquad \forall t \tag{80}$$