From previous example:

$\begin{cases} \text{Table A} - 9 \\ \text{Case 10} \end{cases}$

$$y_{12} = y_{AB}|_{x=0.45} = \frac{(-700)(0.225)(0.45)(0.9^2 - 045^2)}{(6 * 0.9)EI}$$
$$= -\frac{7.9738}{EI}$$

$$y_{22} = y_t = -\frac{(-700)(0.225)^2(0.9 + 0.225)}{3EI} = \frac{13.289}{EI}$$

$\begin{cases} \text{Table A} - 9 \\ \text{Case 5} \end{cases}$

$$y_{11} = y_{max} = -\frac{(900)(0.9)^3}{48EI} = -\frac{(13.669)}{EI}$$
$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$\theta_{AB} = \frac{dY_{AB}}{dx} = \frac{F}{16EI}(4x^2 - l^2) \quad ; \quad 0 \le x \le \frac{l}{2}$$
$$\theta_A = \frac{(900)(-0.9^2)}{16EI} = -\frac{45.563}{EI}$$
$$y_{21} = (-\theta_A)(0.225) = \frac{10.252}{EI}$$

Or:

$$y_{21} = (\theta_C)(0.225) = \frac{10.252}{EI}$$

$$\therefore y_1 = y_{11} + y_{12} = -\frac{21.643}{EI}$$
$$y_2 = y_{21} + y_{22} = \frac{23.541}{EI}$$

Rayleigh’s Method:

$$\therefore \omega_1 = \sqrt{\frac{g(\sum w_i |y_i|)}{(\sum w_i {y_i}^2)}}$$
$$= \sqrt{(9.81)\frac{(900)\left(\frac{21.643}{EI}\right) + (700)\left(\frac{23.541}{EI}\right)}{(900)\left(\frac{21.643}{EI}\right)^2 + (700)\left(\frac{23.541}{EI}\right)^2}}$$
$$= 0.66011\sqrt{EI}$$

Dunkerley’s Method:

$$y_{11} = -\frac{13.669}{EI}$$
$$y_{22} = \frac{13.289}{EI}$$
$$\omega_{11}^2 = \frac{g}{|y_{11}|} = 0.71768\, EI$$

$$\omega_{22}^2 = \frac{g}{|y_{22}|} = 0.73820\ EI$$

$$\therefore \frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{2.7480}{EI}$$

$$\omega_1 = \sqrt{\frac{EI}{2.7480}} = 0.60324\sqrt{EI}$$

Take $E = 200\ GPa, d = 25\ mm$
$\sqrt{EI} = 1261.6\ (N \cdot m^2)$

$\therefore\ \omega_{1(Dunkerley)} = (0.60324)(1261.6)$
$= 761.0\ rad/s$
$Or\ n_{1(Dunkerley)} = 7267\ rpm$
$Also\ \omega_{1(Rayleigh)} = 832.8\ rad/s$
$Or\ n_{1(Rayleigh)} = 7952\ rpm$

$\therefore Operating\ Speed \leq \frac{1}{3}\omega_1$
$Or\ n \leq 2422\ rpm$

7-3 Shaft Layout

- Between a shaft and its components (e.g., gears, bearings, pulleys, etc.), the latter must be located axially and circumferentially.
- Means to provide for torque transmission
  Keys
  Splines
  Setscrews
  Pins
  Press/shrink fits
  Tapered fits
  etc.
- Means to provide for axial location – large axial load shoulders
  Shoulders
  Retaining rings
  sleeves
  Collars
  etc.
- Means to provide for axial location – small axial load
  Press/shrink fits
  Setscrews
  etc.
- Locating rolling element bearings
  See Ch. 11

7-7 Miscellaneous Shaft Components

Includes:

- Setscrews
- Keys and pins
- Retaining rings

Focus:

Keys

Example 7-6

7-8 Limits and Fits

- Fits (clearance, transition, and interference) are to ensure that a shaft and its components/attachments will function as intended.
- Preferred fits are listed in Table 7-9
- Medium drive fit and force fit will give rise to the press/shrink fits.
- Press-fit is typically for small hubs; Shrink fit (or expansion fit) it used with larger hubs
- How much diameter interference to have?
  0.001" for up to 1" of diameter
  0.002" for diameter 1" to 4"
- Press/shrink fits can be designed to transfer torque and axial load.
- Press/shrink fits are known to be associated with fretting corrosion (loss of material from the interface)
- (Section 3-14) for stress distributions in thick-walled cylinder under pressures.
- (Section 3-16) for stresses developed in the shaft and hub due to pressure induced by a press/shrink fit; or (Eq 7-39) to (Eq. 7-47)
- Axial load and torque capacities: (Eq. 7-48) and (Eq. 7-49)
- Radial interference versus diametral interference (not necessarily the same thing).

Chapter 11

Rolling-Contact Bearings

Part 1: (Introduction)

11-1 Bearing Types

Part 2: (The Basics)

11-2 Bearing Life

11-3 Bearing Life at Rated Reliability

11- 4Reliability versus Life – The Weibull Distribution

11-5 Relating Bearing Load, Life and Reliability

Part 3: (Selection of Bearing)

11-6 Combined Radial and Thrust Bearing

11-8 Selection of Ball and Roller Bearings

11-7 Variable Loading

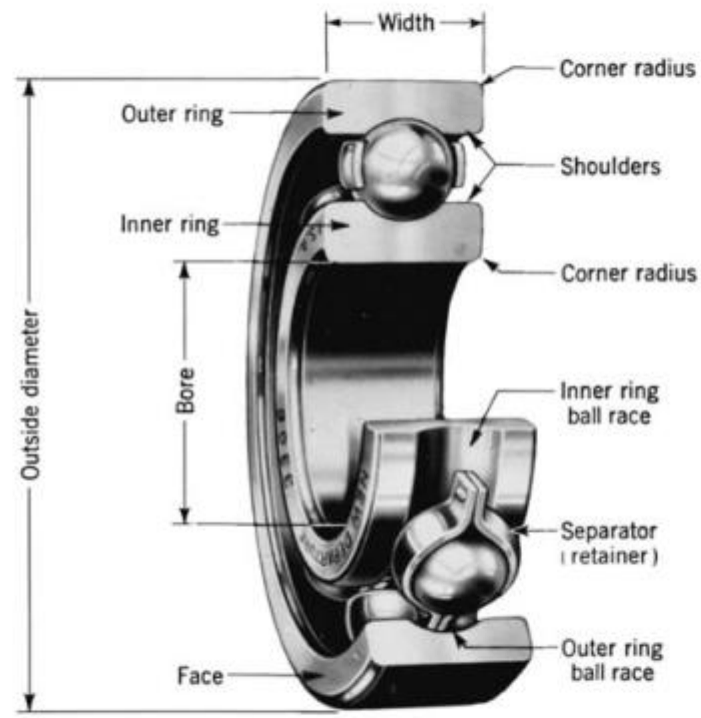
11-10 Design Assessment

Part 4: Others
11-12 Mounting and Enclosure
11-11 Lubrication
11-9 Selection of Tapered Roller Bearings

11-1 Bearing Types
Nomenclature
See Figure 11-1

Figure 11–1

Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*



Classifications

- By shape of rolling elements (sphere, cylinder, tapered, etc.)
- By type of loads taken (radial only, axial only, combination)
- By permissible slope (Self-aligning, non-self-aligning)
- Sealed? Shielded?
- Figures 11-2, 11-3

Figure 11–2

Various types of ball bearings.

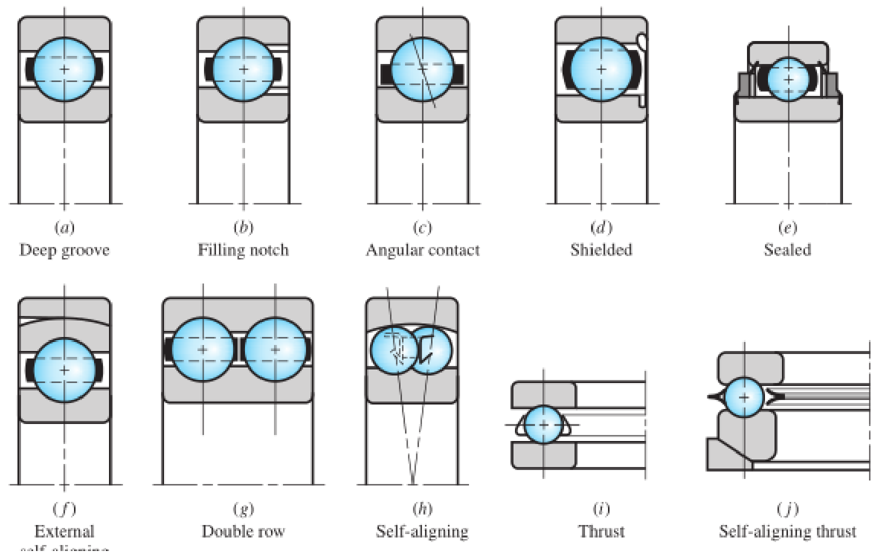
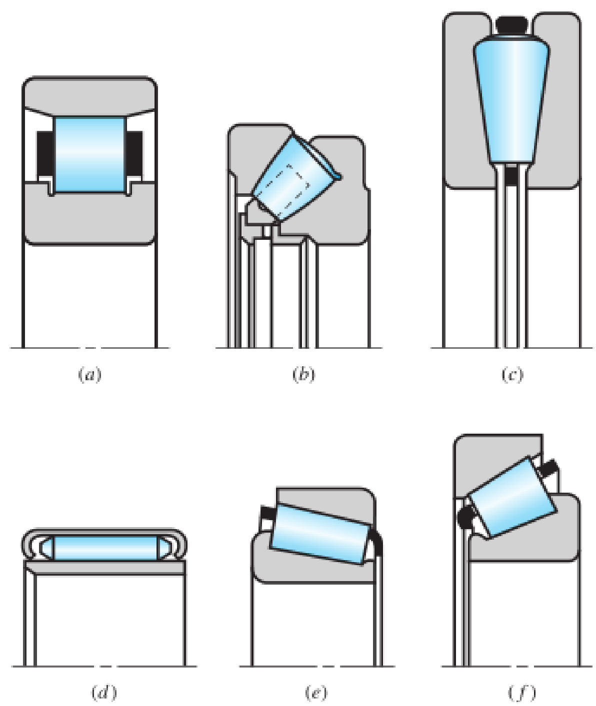


Figure 11–3

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*



11-2 Bearing Life

Why Bearing Life?

- Bearings are under cyclic contact stresses (compressive as well as shear). As a result, they may experience crack, putting, spalling, fretting, excessive noise, and vibration.
- Common life measures are:
  - Number of revolutions of the inner ring (outer ring stationary) until the first evidence of failure; and
  - Number of hours of use at standard angular speed until the first evidence of fatigue
- Number of revolutions is more common

Rating Life (or **Rated Life)**

- This is the terminology used by ABMA (American Bearing Manufacturers Association) and most bearing manufacturers.
- It is defined as, of a group of nominally identical bearings, the number of revolutions that 90% of the bearings in the group will achieve or exceed, before failure occurs.
- It is denoted as L_{10} or B_{10} life.
- The typical value for L_{10} or B_{10} is 1 million.
- However, a manufacturer can choose its own specific rating life.
- For example, Timken uses 90 million for tapered roller bearings, but 1 million for its other bearings.
- Refer to bearings catalog for value(s) of L_{10}.

11-3 Bearing Life at Rated Reliability

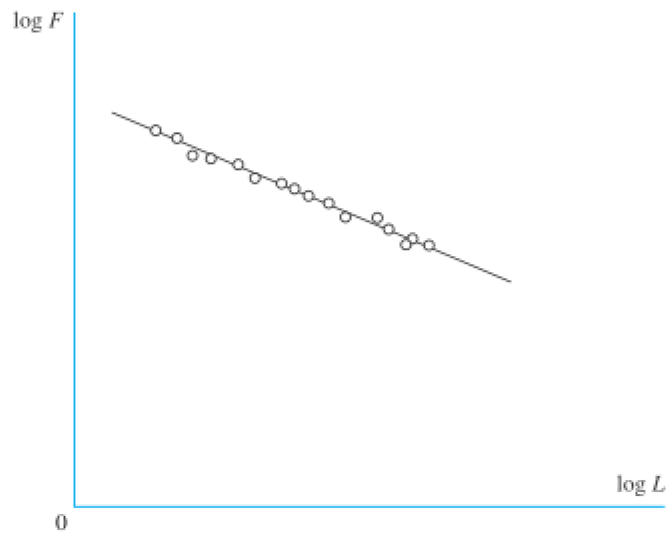
- At rated reliability of 90%, bearing life L relates to bearing's radial load F by:

$$FL^{1/a} = constant$$

  Where $a = 3$ for ball bearings and $a = 10/3$ or roller bearings.
  Figure 11-4 shows the meaning of (Eq. 11-1)

Figure 11–4

Typical bearing load-life log-log curve.



  - It's a straight line on log-log scales;
  - Points on the line will have the same reliability;
  - The line corresponding to 90% reliability is called the rated line.
- To determine the constant on the RHS of (Eq. 11-1), L is set to L_{10}. The corresponding F is designated as C_{10}.
  C_{10} is called the **Basic Dynamic Load Rating**, or the **Basic Dynamic Rated Load** It is defined as the radial load that causes 10% of the group of nominally identical bearings to fail at or before L_{10} (1 million, or 90 million, or revs as chosen by a manufacturer).
- (Equation 11-1) becomes:

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a} \qquad (11-3)^*$$

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R}\right)^{1/a} = F_D \left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \qquad (11\text{–}3)$$

Where the subscript D means design. (Equation 11-3)* is essentially (Eq. 11-3) of the text. In (Eq. 11-3), the subscript R means rated.

- Example 11-1: find C_{10} from known L_D, F_D and L_{10}

11-4 Reliability versus Life – The Weibull Distribution

11-5 Relating Load, Life and Reliability

- At 90% reliability, (Eq. 11-3*) forms the basis for selecting a bearing.
- What if the reliability is not 90%?
  Figure 11-5 shows the process of going from the rated line to a different line.
  $A \to D$, with B being the intermediary.

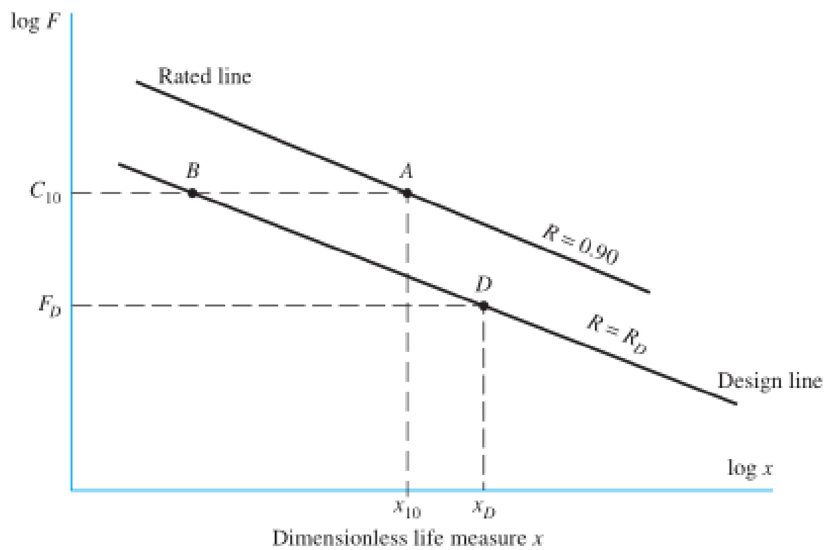


Figure 11–5

Constant reliability contours. Point A represents the catalog rating C_{10} at $x = L/L_{10} = 1$. Point B is on the target reliability design line R_D, with a load of C_{10}. Point D is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load F_D.

- $A \to B$: Load is constant, and life measure (which is a random variable) follows a three-parameter Weibull distribution
- $B \to D$: Reliability is constant, and (Eq. 11-3*) is valid; The mathematics is given in (Sec. 11-4) and (Sec. 11-5).
- Another approach is to use a reliability factor a_1, the value of which depends on R, the reliability. Values of a_1 are available from a number of references.

R, %	**Reliability Factor, a_1**
90	1.00
95	0.62
96	0.53
97	0.44
98	0.33
99	0.21

- The factor a_1 can be determined by (courtesy of, for example SKF catalog)

$$a_1 = 4.48\left(\ln\frac{100}{R}\right)^{2/3}$$

Where R is in %, e.g., $R = 92.5$. Note: $R \le 99$.

- Timken recommends the following formula for a_1

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Where $R \le 99.9$

The Basic Bearing Equation

The basic bearing equation can now be obtained by, in (Eq. 11-3), introducing the reliability factor a_1 and a load-application factor k_a. That is,

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a}$$

Becomes,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

k_a is given in (Table 11-5)

Table 11-5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

This basic equation can be used for bearing selection and for assessment after selection.

Example 1

A SKF deep-groove ball bearing is subjected to a radial load of 495 lb. The shaft rotates at 300 rpm. The bearing is expected to last 30,000 hours (continuous operation). Catalog shows a $C_{10} = 19.5\ kN$ on the basis of 10^6 revs. (1) is the bearing suitable for 90% reliability? (2) Also assess the bearing's reliability. Set $k_a = 1$.

Solution:

The basic bearing equation is:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

(Where a and L_{10} are generally set by the manufacturer, k is a variable we can change.)

Where:

$C_{10} = 19.5\ kN = 4387.5\ lb$; $L_{10} = 10^6\ revs$; $k_a = 1$; $a = 3$

Also $F_D = 495\ lb$; and $L_D = (30{,}000)(60)(300) = (540)(10^6)\ revs$.

Assume 90% reliability, then $a_1 = 1$. From the basic bearing equation,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting values, $LHS = 8.92$, $RHS = 8.14$. Therefore, (L_D, F_D) is <u>not</u> on the rated line.

There are a number of ways to seek the answer.
90% reliability, $a_1 = 1$;

<u>The first</u>: *similar to the typical calculations done for selecting a bearing*
Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$; and $L_{10} = 10^6\ revs$; find C_{10} and check if it is less than the $4387.5\ lb$ that the bearing is capable of providing.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{C_{10}}{(1)(495)} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Resulting in $C_{10} = 4031\ lb$
Since it's less than the catalog's C_{10}, or $4387.5\ lb$, the selected bearing is suitable for 90% reliability.

<u>The second:</u> *can be used to select a bearing (pre-selecting a bearing, then checking to make sure it is suitable)*
Set $L_D = (540)(10^6)\ revs$, find F_D, and check is $F_D \geq 495\ lb$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)F_D} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Solving gives $F_D = 538.8\ lb$
$\therefore$ with 90% reliability and a life of $(540)(10^6)\ revs$, the bearing can take on a maximum radial load of $538.8\ lb$. Since the applied radial load is only $495\ lb$, the bearing will have better than 90% reliability.

<u>The third:</u> *similar to post-selection calculation to evaluate the life of the bearing.*
Set $F_D = 495\ lb$, find L_D, and check if $L_D \geq (540)(10^6)\ revs$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{L_D}{(1)(10^6)}\right)^{1/3}$$

Solving gives $L_D = (696)(10^6)\ revs$
With a radial load at $495\ lb$, the bearing has 90% chance proabability to survive at least $(696)(10^6)\ revs$. The chance of surviving only $(540)(10^6)\ revs$ is better than 90%.

(2) Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$
To assess reliability means to evaluate a_1. This is typically done after selection.

From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{(540)(10^6)}{a_1(10^6)}\right)^{1/3}$$

Solving gives $a_1 = 0.775$

Finally, from:

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Solving for R results in $R = 93\%$.
With the radial load at $495\ lb$, there is a 7% of chance that the bearing would fail at or before 540 millions of revs.
It shows that the bearing is more than suitable for 90% reliability.

Example 2
Select bearings A and B for the shaft of Example 7-2. They are to be used for a minimum of 1,000 hours of continuous operation. Shaft rpm is 450. Radial loads are, $F_A = 375\ lb$ and $F_B = 1918\ lb$. Shaft diameter at both locations is $1"$ (D_1 and D_7 in Figure 7-10). Assume 90% reliability.

Solution:
Table 11-2
Table 11-3
Since there is no thrust load, deep-groove ball bearings may