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$$S = 2R_{\text{crank}}$$

A displacement for all cylinders

$$V_{\text{displ}} = N_{\text{cyl}} (V_{\text{max}} - V_{\text{min}}) = N_{\text{cyl}} A_{\text{cyl}} S$$

The ratio of the largest to the smallest volume is the compression ratio:

$$r_v = CR = V_{\text{max}} / V_{\text{min}}$$

→ The net specific work in a complete cycle is used to define a mean effective pressure

net work per cylinder per cycle:

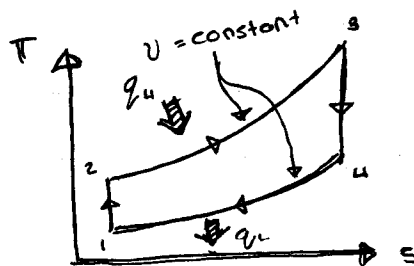
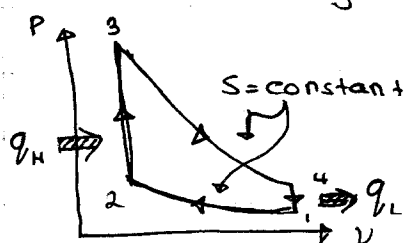
$$W_{\text{net}} = m w_{\text{net}} = P_{\text{meff}} (V_{\text{max}} - V_{\text{min}})$$

the ratio of work for the whole engine

$$\dot{W} = N_{\text{cyl}} m w_{\text{net}} \frac{\text{RPM}}{60} = P_{\text{meff}} V_{\text{displ}} \frac{\text{RPM}}{60}$$

→ result should be corrected with a factor of $(1/2)$ for a 4-stroke engine.

The Otto Cycle



$$q_H = u_3 - u_2$$

(constant volume)

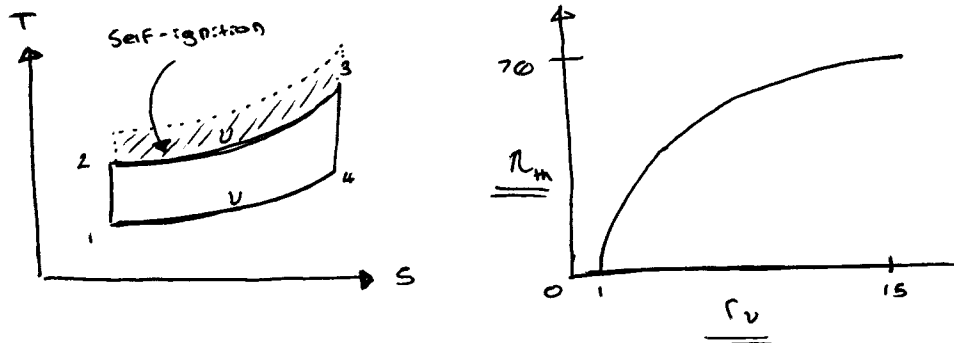
$$q_H = C_v (T_3 - T_2)$$

$$q_L = u_4 - u_1 = C_v (T_4 - T_1)$$

$$\text{then } \eta_{\text{th}} = \frac{q_H - q_L}{q_H} = 1 - q_L / q_H \approx 1 - \frac{C_v (T_4 - T_1)}{C_v (T_3 - T_2)} = 1 - \frac{T_1 (T_4 / T_1 - 1)}{T_2 (T_3 / T_2 - 1)}$$

$$(T_2 / T_1) = (V_1 / V_2)^{\kappa-1} = T_3 / T_4$$

$$\eta_{\text{th}} = 1 - T_1 / T_2 = 1 - (r_v)^{1-\kappa} = 1 - \frac{1}{(r_v^{\kappa-1})}$$



→ Specific heats of actual gases increase with an increase in temp.

→ 10.7

Example

Compression ratio = 10 = $V_r = V_1/V_2$

$P_1 = 0.1 \text{ MPa}$, $T_1 = 15^\circ\text{C}$

$q_H = 1800 \text{ kJ/kg}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\kappa-1} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\kappa}$$

$$T_2 = (10)^{(1.4-1)} \times T_1 = \boxed{723.9 \text{ K}} \rightarrow T_2$$

$$P_2 = (10)^{1.4} \times P_1 = \boxed{2.512 \text{ MPa}} \rightarrow P_2$$

$$q_H = q_{2-3} = u_3 - u_2 = C_v(T_3 - T_2)$$

$$1800 = (0.717)(T_3 - 723.9) \Rightarrow \boxed{3234 \text{ K}} \rightarrow T_3 \text{ (Maximum temp.)}$$

$$PV = RT \Rightarrow \frac{P_3}{T_3} = \frac{R}{V} = \frac{P_2}{T_2}, \text{ thus } P_3/P_2 = T_3/T_2 \text{ for C.V. process for ideal gas}$$

(if constant)

$$P_3 = P_2(T_3/T_2) = 2.512 \left(\frac{3234}{723.9}\right) = \boxed{11.22 \text{ MPa}} \rightarrow P_3$$

$$T_3/T_4 = (V_4/V_3)^{\kappa-1} \Rightarrow (10)^{0.4}$$

$$T_4 = T_3/(10^{0.4}) = \boxed{1289.5 \text{ K}} \rightarrow T_4$$

$$\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^{\kappa} \Rightarrow P_4 = \frac{P_3}{(V_4/V_3)^{\kappa}} = \frac{11.22}{10^{1.4}} = \boxed{0.4467 \text{ MPa}} \rightarrow P_4$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\kappa-1}} = 1 - \frac{1}{10^{0.4}} = 0.602 \text{ or } \boxed{60.2\%}$$

$$m_{ep} = \frac{w_{net}}{(V_1 - V_2)}$$

$$w_{net} = q_H - q_L \rightarrow w_{net} = \boxed{1083.5 \text{ kJ/kg}} \rightarrow w_{net}$$

$$q_L = u_4 - u_1 = C_v(T_4 - T_1) = 0.717(1289.5 - 288.2) = \boxed{716.5 \text{ kJ/kg}} \rightarrow q_L$$

$$Pv = RT \rightarrow v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(288.2)}{(100)} = 0.827 \text{ m}^3/\text{kg}$$

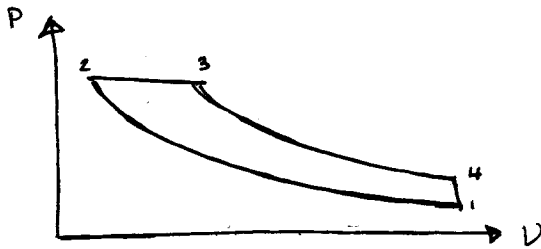
$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287)(725.9)}{(2.512 \times 10^3)} = 0.0827 \text{ m}^3/\text{kg}$$

$$\rightarrow \text{or } v_1/v_2 = v_1/v_2 = 10$$

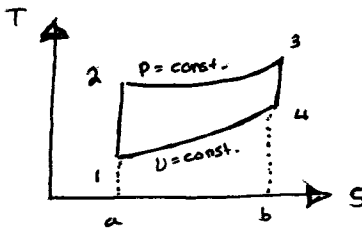
$$\text{then } v_2 = v_1/10 = 0.827 \text{ m}^3/\text{kg}$$

$$P_{\text{mep}} = \frac{1083.5}{(0.827 - 0.0827)} = 1456 \text{ kPa}$$

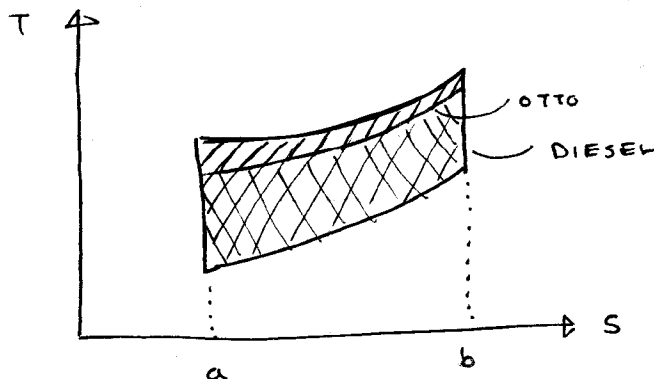
The Diesel Cycle (compression ignition engine)



$$\begin{aligned} q_H &= u_3 - u_2 + u_{2-3} \\ &\Rightarrow u_3 - u_2 + P_2(v_3 - v_2) \\ &\Rightarrow h_3 - h_2 \end{aligned}$$

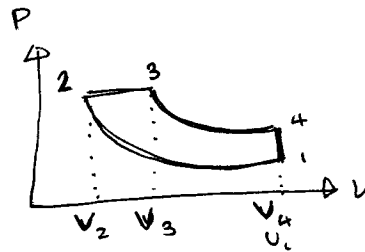


$$\begin{aligned} \eta_{th} &= 1 - q_c/q_H = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \\ &= 1 - \frac{T_1(T_4/T_1 - 1)}{k T_2(T_3/T_2 - 1)} \end{aligned}$$



$$\eta_{di} > \eta_{otto}$$

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Example

$$P_1 = 0.1 \text{ MPa}$$

$$T_1 = 288 \text{ K}$$

$$T_2/T_1 = (V_1/V_2)^{\gamma-1} \Rightarrow T_2 = T_1 (20)^{0.4} = 955.2 \text{ K}$$

$$P_2/P_1 = (V_1/V_2)^{\gamma} \Rightarrow P_2 = P_1 (20)^{1.4} = 6.629 \text{ MPa}$$

$$q_H = 1800 \text{ kJ/kg} = h_3 - h_2 = C_p (T_3 - T_2)$$

$$T_3 = q_H / C_p + T_2 = 1800 / 1.004 + 955.2 = 2748 \text{ K}$$

$$P_3 = P_2 = 6.629 \text{ MPa} \quad (\text{Diesel cycle})$$

$$T_4/T_3 = (V_3/V_4)^{\gamma-1}$$

$$PV = mRT \Rightarrow \frac{V_2}{T_2} = \frac{mR}{P} = \frac{V_3}{T_3}$$

Ideal gas law : for constant pressure process

$$V_3/V_2 = T_3/T_2 \Rightarrow V_3 = V_2 (T_3/T_2)$$

$$V_3/V_2 = 2748/955.2 = 2.8769$$

$$V_3/V_4 = V_3/V_2 \times \frac{V_2}{V_4} =$$

$\leftarrow = V_2/V_1$

$$T_4/T_3 = (V_3/V_4)^{\gamma-1} \Rightarrow \left(\frac{2.8769}{20} \right)^{0.4}$$

$$\Rightarrow T_4 = 1265 \text{ K}$$

$$P_4/P_3 = (V_3/V_4)^{\gamma} \Rightarrow P_4 = (6.629) \left(\frac{2.8769}{20} \right)^{1.4}$$

$$\Rightarrow P_4 = 439 \text{ kPa}$$

$$q_L = C_v (T_1 - T_4) = 0.7157 (288.2 - 1265)$$

$$= -700.4 \text{ kJ/kg}$$

$$w_{\text{net}} = |q_H| - |q_L| = 1800 - 700.4 = 1099.6 \text{ kJ/kg}$$

$$\eta_{\text{th}} = w_{\text{net}} / q_H = 1099.6 / 1800 = 0.611 \text{ or } 61.1 \%$$

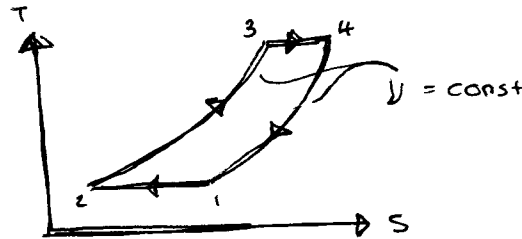
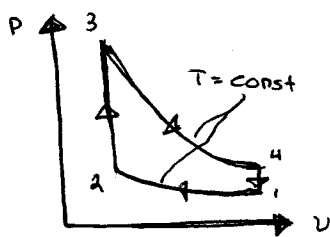
$$m_{\text{ep}} = w_{\text{net}} / (V_1 - V_2)$$

$$V_1 = \frac{RT_1}{P_1} = \frac{(0.287)(288)}{(100)} = 0.827 \text{ m}^3/\text{kg}$$

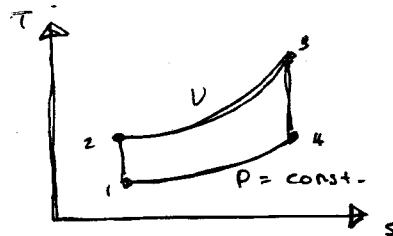
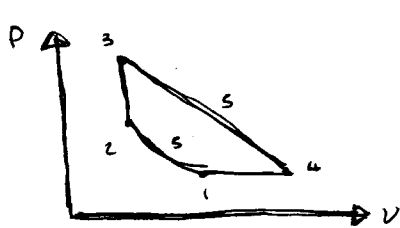
$$V_2 = 0.04135 \text{ m}^3/\text{kg}$$

$$m_{\text{ep}} = \frac{1099.6}{(0.827 - 0.04135)} = 1400 \text{ kPa}$$

The Stirling Cycle



The Atkinson Cycle



For compression and expansion

$$T_2/T_1 = (V_1/V_2)^{\kappa-1} \quad \text{and} \quad T_4/T_3 = (V_3/V_4)^{\kappa-1}$$

$$P = C \quad ; \quad T_4 = (V_4/V_1) T_1 \quad ; \quad \text{and} \quad q_L = h_4 - h_1$$

$$\eta = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

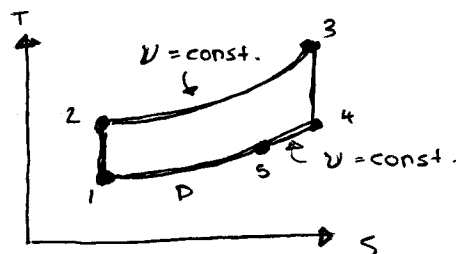
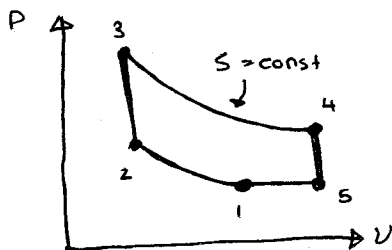
$$\cong 1 - \frac{C_P(T_4 - T_1)}{C_V(T_3 - T_2)} = 1 - \kappa \frac{T_4 - T_1}{T_3 - T_2}$$

$$T_2 = T_1 CR_1^{\kappa-1} \quad ; \quad T_4 = (V_4/V_1) T_1 = \frac{CR}{CR_1} T_1$$

$$\rightarrow T_3 = T_4 CR^{\kappa-1} = (CR/CR_1) T_1 CR^{\kappa-1} = \frac{CR^{\kappa}}{CR_1} T_1$$

$$\eta = 1 - \kappa \left(\frac{CR}{CR_1} - 1 \right) \frac{1}{\left(\frac{CR^{\kappa}}{CR_1} - CR_1^{\kappa-1} \right)} \cong 1 - \kappa \left(\frac{CR - CR_1}{CR^{\kappa} - CR_1^{\kappa}} \right)$$

The Miller Cycle



→ both Atkinson and Miller have higher efficiencies than the Otto Cycle.