

Draw a straight line between the two points of (T_{av}, μ) . The intersecting point with the SAE20 line is (117, 5.2).

So, $T_{av} = 117^\circ F$ or $\Delta T_F = 34^\circ F$ and $5.2 \mu\text{reyn}$

(Keep in mind every time we have a new viscosity, we have to find a new Sommerfeld number)

The last part is as follows:

$$T_{max} = 134^\circ F < 250^\circ F$$

Figure 12-15: $\frac{h_0}{c} = 0.49$

$$h_0 = (0.49)(0.0015) = 0.000735"$$

$$h_{min} = 0.0002 + 0.00004d = 0.00026"$$

So, $h_0 > h_{min}$ and $(S, h_0/c)$ is inside the optimal zone.

(Where $S = 0.176$ and $h_0/c = 0.49$)

Figure 12-17: $(r/c)f = 4.2$

$$f = \frac{4.2}{500} = 0.0084 < 0.01$$

Figure 12-20: $\frac{P}{p_{max}} = 0.45$

$$p_{max} = \frac{222.2}{0.45} = 494 \text{ psi}$$

Power loss:

$$H_{loss} = \frac{fWrN}{1050} = \frac{(0.0084)(500)(0.75)(30)}{1050} = 0.0897 \text{ hp}$$

Example 3

A journal bearing has $r = l = 1.5"$, $c = 0.0015"$, $W = 1000 \text{ lb}$, and $N = 30 \text{ rps}$. Lubricant's inlet temperature is assumed to be $120^\circ F$. The bearing is to be designed with high load capacity. Select a lubricant, and design and evaluate the bearing.

Solution:

The first part of the process is as follows:

$$P = \frac{W}{ld} = \frac{1000}{1.5 \times 3} = 222.2 \text{ psi}, N = 30 \text{ rps}, \frac{l}{d} = \frac{1}{2}, \text{ and } \frac{r}{c} = \frac{1.5}{0.0015} = 1000$$

$$\text{Also, } T_1 = 120^\circ F$$

For the second part,

Figure 12-16: on the plot of $\frac{l}{d} = \frac{1}{2}$, select a point located close to the "Max W" edge, say $S = 0.3$

$$S = 0.3 = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = (1000)^2 \frac{\mu(30)}{222.2}$$

$$\text{So, } \mu = 2.22 \mu\text{reyn}$$

Figure 12-24 with $S = 0.3$:

$$\frac{9.70 \Delta T_F}{p_{psi}} = 0.394552 + 6.392527S - 0.036013S^2 = 2.309$$

$$\Delta T_F = (2.309)(222.2)/(9.70) = 52.9^\circ F$$

$$\text{So, } T_{av} = T_1 + \frac{\Delta T_2}{2} = 146^\circ F$$

Figure 12-12: locate the point (146, 2.22). It is below the SAE20 line. On the line, $\mu = 2.5 \mu\text{reyn}$, which gives a Sommerfeld number $S = 0.34$. The design remains inside the optimal zone.

Or, the average temperature needs to be $153^\circ F$ for the lubricant to have a viscosity of $2.22 \mu\text{reyn}$. That means inlet temperature needs to be at $126.5^\circ F$.

The last part gives the following results: (based on $S = 0.34$)

$$h_0 = (0.43)c = 0.000625''$$

$$h_{min} = 0.0002 + 0.00004d = 0.00032'' < h_0$$

$$f = (8.5)/(r/c) = 0.0085 < 0.01$$

$$Q = (4.8)/(rcNl) = 1394 \text{ in}^3/\text{s}$$

$$Q_s = 0.72 \cdot Q = 1004 \text{ in}^3/\text{s}$$

$$p_{max} = P/0.375 = 593 \text{ psi}$$

12-10 Clearance

Why this section?

- Clearance c has a range due to manufacture and assembly
- It tends to increase due to wear

How to take into consideration change in clearance?

A suitable fit is assigned between journal and bushing. For example, H8/f7 (close to running fit) or H9/d9 (free running);

Then the range of clearance is determined (see Table 12-3, for example);

Performance of the bearing (As indicated by h_0 , T_2 the outlet temperature, Q , and H the power loss, for example) is calculated and plotted against clearance c . See Figure 12-25.

The initial clearance band (i.e., the tolerance specified for manufacturing) should be located to the left of the peak of the $h_0 - c$ plot.

Chapter 13

Gears - General

Part 1: Geometry and Tooth System

13-1 ... 13.8:

Types of Gears, ..., The Forming of Gear Teeth

13-12: Tooth Systems

Part 2: Kinematics

13-13: Gear Trains

Part 3: (to be discussed with Chapters 14 and 15)

13-19 ... 13-11:

Bevel Gears, Parallel Helical Gears, Worm Gears

13-14 ... 13-17:

Force Analysis – Spur, Bevel, Helical and Worm Gears

13-1 Types of Gears

Why Gears?

Of constant-speed mechanical transmission elements, the frequencies of usage are:

- Gears: 50%
- Couplings: ~20%
- Chain Drives: 10–20%
- Belt Drives: 10-12%
- Power screws, wire ropes, friction wheels, etc.: 5-10%

Types of Gears:

- Spur gears: most common; transmit power between two parallel shafts
- Helical gears: between two intersecting shafts
- Bevel gears: between intersecting shafts
- Worm gear sets: between non-parallel and non-intersecting shafts
- And many other types

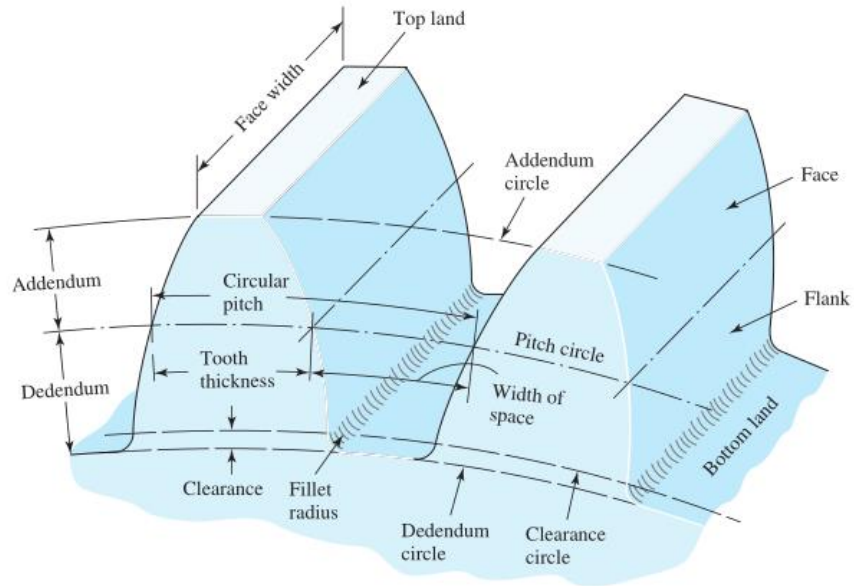
13-2 Nomenclature

13-12 Tooth Systems

Nomenclature: Figure 13-5, for spur gears only.

Figure 13-5

Nomenclature of spur-gear teeth.



Tooth System: refers to the standard that specifies the tooth geometry and so on. There are the metric system and the US customary system.

- Pitch circle
- Circular pitch p , pitch diameter d , in *in* or *mm*
- Number of teeth N
- Diametral pitch $P = N/d$, in teeth/in; or
- Module $m = d/N$, in *mm*
- P and m are standardized, see Table 13-2
- Metric gears and US customary gears are NOT interchangeable

Tables 13-1, 13-3, 13-4: formulas for spur gears, 20° straight bevel gears, and helical gears

Table 13-5: information for worm gearing

Typical values for face width

$$\frac{3\pi}{P} \leq F \leq \frac{5\pi}{P}$$

Or

$$3\pi m \leq F \leq 5\pi m$$

13-3 Conjugate Action

13-4 Involute Properties

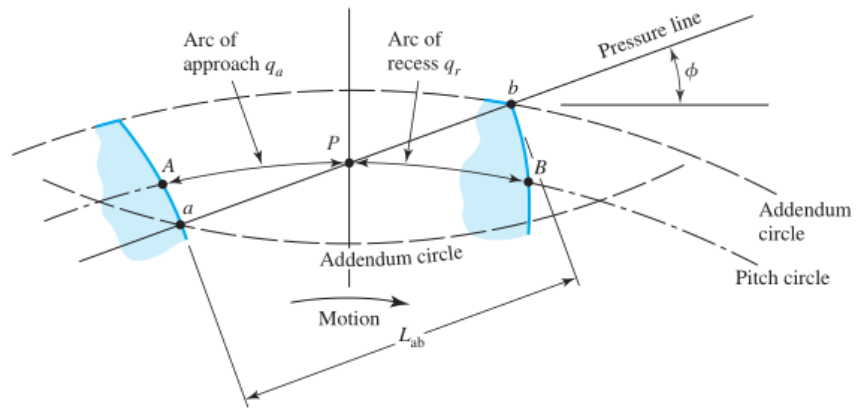
13-5 Fundamentals

- The fundamental Law of Gearing:
Angular velocity ratio between the gears of a gearset must remain constant throughout the mesh
- Involute tooth form meets the fundamental law, and has the advantage that error in center-to-center distance will not affect the angular velocity ratio.

13-6 Contact Ratio

Figure 13-15

Definition of contact ratio.



- Addendum circles
- Pressure line (passing through pitch circles)
- A part of gear teeth enters into contact at point a and exits from contact at point b on the same line
- The distance between these points in the length of action L_{ab} also labelled as Z .

Gear contact ratio m_c defines the average number of teeth that are in contact at any time,

$$m_c = \frac{L_{ab}}{p_b} = \frac{L_{ab}}{p \cos \phi}$$

Where $p_b = p \cdot \cos \phi$ is the base pitch and ϕ is the pressure angle.

Significance:

For a pair of gears the mesh properly, their diametral pitch or module, and pressure angle must be the same. In addition, contact ratio must meet certain requirements.

$$m_c = \frac{L_{ab}}{p \cos \phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin \phi}{p \cos \phi}$$

Or

$$m_c = \frac{L_{ab}}{\pi m \cos \phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin \phi}{\pi m \cos \phi}$$

Where:

r_{ap}, r_{bp} : radii of addendum circle and base circle of the pinion;

r_{ag}, r_{bg} : radii of addendum circle and base circle of the gear;

c : center-to-center distance;

The radius of base circle of a gear is $r_b = \frac{d}{2} \cos \phi$, with d being the pitch diameter.

Example 1

A gear set has diametral pitch of $P = 10 \text{ teeth/in}$ and pressure angle of 20° . Teeth numbers are $N_p = 30$ and $N_g = 75$. Determine the contact ratio of the set. Assume full depth tooth profile.

Solution:

	Pinion	Gear
Pitch radius, <i>in</i>	1.5	3.75
Addendum radius, <i>in</i>	1.6	3.85
Base radius, <i>in</i>	1.4095	3.5238
Center-to-centre distance, <i>in</i>	5.25	

$$L_{ab} = 0.7572 + 1.5509 - 1.7956 = 0.5125''$$

$$p = \frac{\pi}{P} = 0.3142''$$

$$m_c = \frac{0.5125}{0.3142 \cdot \cos(20^\circ)} = 1.74$$

Example 2

A set of stub-profiled gear has, $N_p = 18$ and $N_g = 72$. Module is $m = 5 \text{ mm}$. Determine the set's contact ratio. Pressure angle is 22.5° .

Solution:

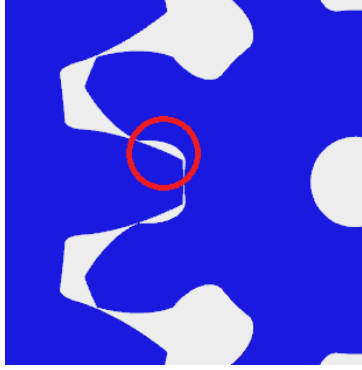
	Pinion	Gear
Pitch radius, <i>mm</i>	45	180
Addendum radius, <i>mm</i>	49	184
Base radius, <i>mm</i>	41.575	166.298
Center-to-centre distance, <i>mm</i>	225	

$$L_{ab} = 25.933 + 78.746 - 86.104 = 18.575 \text{ mm}$$

$$m_c = \frac{18.575}{n \cdot \pi \cdot \cos(22.5^\circ)} = 1.28$$

13-8 The Forming of Gear Teeth**13-7 Interference**

- Mainly, there are form cutting and generating cutting
 - Form cutting: the cutter is the exact shape of the tooth space; expensive
 - Generating cutting: the cutter has a shape different from the tooth space; more common
- Of interest to discussing interference is generating cutting which includes,
 - Shaping: pinion cutting (Figure 13-17) and rack cutting (Figure 13-18)
 - Hobbing: using hob, a worm-like cutting tool, to cut a blank (Figure 13-19)
- Interference refers to contact taking place on the non-involute portion of the tooth profile (inside base circle)
- Undercut refers to the removal of interfering material during generating cutting.



- To avoid interference, the pinion requires a minimum number of teeth, while the gear has a restriction on maximum number of teeth.
- The text has three equations, (Eq. 13-10), (Eq. 13-11) and (Eq. 13-13) for determining the minimum number of teeth to avoid interference.
- (Eq. 13-11) is for general cases, and recommended. For a pinion-gear set, the minimum number of teeth on pinion without interference is, (Eq. 13 – 11):

$$N_P = \frac{2k}{(1 + 2m) \sin^2 \phi} (m + \sqrt{m^2 + (1 + 2m) \sin^2 \phi}) \quad (13-11)$$

Where:

m is the gear ratio $m = N_G/N_P$. $m > 1$.

$k = 1$ for full-depth teeth, and 0.8 for stub teeth

ϕ is the pressure angle

- (Eq. 13-10) is for cases of one-to-one gear ratio.

$$N_P = \frac{2k}{3 \sin^2 \phi} (1 + \sqrt{1 + 3 \sin^2 \phi}) \quad (13-10)$$

- (Eq. 13-13) is for cases of pinion meshing with a rack.

$$N_P = \frac{2(k)}{\sin^2 \phi} \quad (13-13)$$

- Maximum number of teeth on a gear mating with a specific pinion is determined by (Eq. 13-12)

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \quad (13-12)$$

- A number of examples are shown within the section.

1st example: a set of gears, shapes by pinion cutter, 20° pressure angle, full-depth teeth; then smallest N_p is 13 and largest N_G is 16.

2nd example: a set of gears, 20° pressure angle, full-depth teeth, cut by hobbing; then smallest N_p is 17 and largest N_G is 1309.

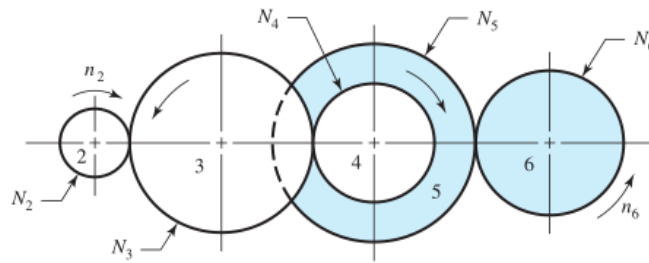
13-13 Gear Trains

Types of Gear Trains

- Simple of series trains, See Figure 13-27

Figure 13-27

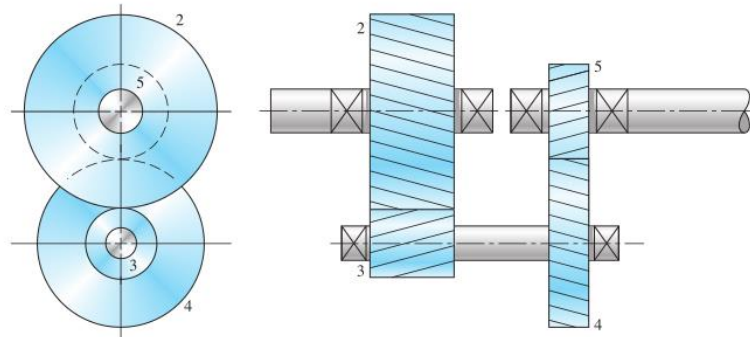
A gear train.



- Compound trains
 - Reverted (Figure 13-29)

Figure 13-29

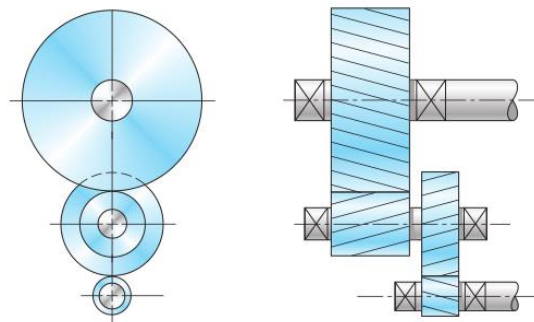
A compound reverted gear train.



- Non-reverted (Figure 13-28)

Figure 13-28

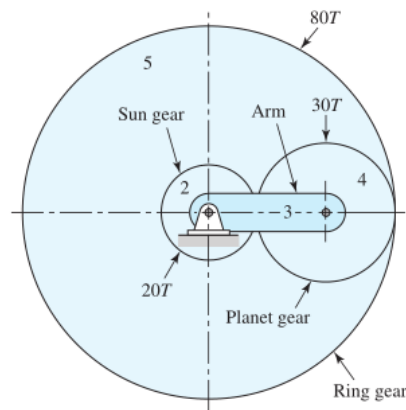
A two-stage compound gear train.



- Planetary or epicyclic trains (Figure 13-30)

Figure 13-30

A planetary gear train.



Train Value, Speed Ratio, Gear Ratio, and so on

- In the text, train value e is used. It is defined as

$$e = \pm \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

e is positive if the last gear rotates in the same sense as the first, and negative if the last gear rotates in the opposite sense.

e is also the ratio of n_L , the speed of the last gear, over the speed of the first gear n_F .

$$e = \frac{n_L}{n_F}$$

- Speed ratio = velocity ratio = transmission ratio – train value.
- Gear ratio is commonly used in daily conversions. Gear ratio = $1/e$

Problem-Solving

- Given a train, find velocity ratio;
- Given required gear ratio, determine the type of train and teeth numbers.
See Examples 13 – 3 ~ 13 – 5.

Velocity Ratio – Planetary Gear Trains

- Tabular method see “Dynamics of Machinery”, R. L. Norton.
 - Follow the power flow
 - Velocity difference equation

$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

(NOTE: The last term represents the velocity of gear relative to the arm.)

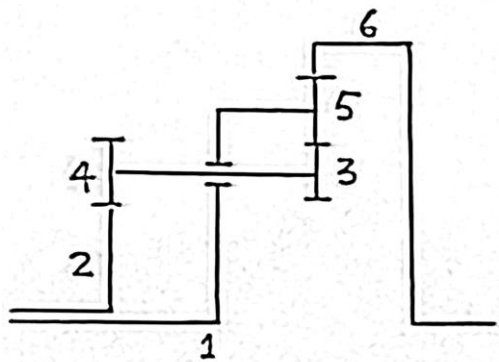
- Relative velocity obeys

$$VR = \frac{[\omega_{gear/arm}]_{driven}}{[\omega_{gear/arm}]_{driver}} = \pm \frac{N_{driver}}{N_{driven}}$$

(NOTE: Where the “+” is used with internal set and “–” is used with external set.)

Example 3:

The schematic of a planetary gear train is shown below, with “1” being the arm. Gears 2 and 6 rotate about the same axis as the arm. Input is to Gear 2.



- (1) Given $N_2 = 30$, $N_3 = 25$, $N_4 = 45$, $N_5 = 30$, $N_6 = 160$, $\omega_2 = 50 \text{ rad/s}$, $\omega_{\text{arm}} = -75 \text{ rad/s}$
Find ω_6 .
- (2) Given $N_2 = 30$, $N_3 = 25$, $N_4 = 45$, $N_5 = 50$, $N_6 = 200$, $\omega_2 = 50 \text{ rad/s}$, $\omega_6 = 0$
Find ω_{arm} , ω_3 , ω_4 , ω_5

Solution:

(1): Power flow: $2 \rightarrow 4 \text{ \& } 3 \rightarrow 5 \rightarrow 6$

Gear	$\omega_{\text{gear}} =$	ω_{arm}	$+$	$\omega_{\text{gear/arm}}$	VR
2	50	-75		125	$-N_2/N_4$
4		-75		$(125) \left(-\frac{N_2}{N_4} \right)$	
3		-75			$-N_3/N_5$
5		-75		$(125) \left(-\frac{N_2}{N_4} \right) \left(-\frac{N_3}{N_5} \right)$	
6	ω_6	-75		$(125) \left(-\frac{N_2}{N_4} \right) \left(-\frac{N_3}{N_5} \right) \left(\frac{N_5}{N_6} \right)$	N_5/N_6

$$\therefore \omega_6 = -61.98 \text{ rad/s}$$

(2): Power flow: $2 \rightarrow 4 \text{ \& } 3 \rightarrow 5 \rightarrow 6$

Gear	$\omega_{\text{gear}} =$	ω_{arm}	$+$	$\omega_{\text{gear/arm}}$	VR
2	50	x			
4		x			
3		x			
5		x			
6	0	x			

$$\omega_{\text{arm}} = x = -4.545 \text{ rad/s}$$

$$\omega_3 = \omega_4 = -40.91 \text{ rad/s}$$

$$\omega_5 = 13.64 \text{ rad/s}$$