DBS

Microsoft

IONQ

8 DECENT WORK AND ECONOMIC GROWTH

Counterparty Credit Risk

1. Context and Relevance

Risk management is at the core of modern finance. Institutions such as banks, asset managers, and hedge funds rely on precise risk calculations to ensure regulatory compliance, safeguard investor assets, and optimize returns. In the case of counterparty credit risk one investigates the risks of a counterparty to a transaction to default before the settlement of the transaction's cash flows. One of the key tools in this process is potential future exposure (PFE), which quantifies the potential loss of a portfolio under extreme market conditions. Traditional risk models rely on Monte Carlo simulations to approximate probability distributions of losses by sampling thousands or millions of possible market scenarios.

While classical Monte Carlo simulations are robust and widely used, they become computationally intensive as the size of the portfolio and the number of market variables increases. High-dimensional correlations, nonlinear payoffs, and fat-tailed distributions exacerbate these computational costs, making accurate risk assessments difficult and expensive. This creates a potential entry point for quantum computing methods like quantum amplitude estimation (QAE), which promise potential speed-ups in sampling and estimation under certain conditions.

2. Challenge Overview

In this hackathon challenge, participants are tasked with developing quantum-enhanced Monte Carlo methods to compute the potential future exposure (PFE) as a risk measure for a financial portfolio of derivatives. Participants will explore how quantum algorithms can be used as an alternative to classical Monte Carlo methods.

Participants are encouraged to:

- Create a model of the derivatives' underlying assets in a quantum computing circuit. Start with a small model of one or two assets before moving to larger models

- Append the quantum circuit to compute the options payoff. In the beginning it is easier to deal with European options (see below)
- Apply techniques like quantum amplitude estimation to calculate the PFE
- Clearly document their workflow to ensure reproducibility and extensibility
- Demonstrate a compelling path toward a potential quantum advantage, with a clear articulation of the challenges that must be overcome.
- Further improve and/or extend the model e.g. for different types of derivatives (like path-dependent ones) or by incorporating classical pre- or post-processing steps

3. Detailed Problem Description

a. Mathematical Problem / Model Description

The goal of the challenge is to compute PFE of a portfolio of derivative options. The portfolio consists of a set of derivatives which deliver a certain payoff $f_1(t), f_2(t), \ldots, f_n(t)$ at a time t whose value is based on underlying assets with a stochastic value $S_1, S_2, \ldots, S_n$ and according weights w_1, $w_2, \ldots w_n$. So the portfolio value V(t) is given as their weighted sum:

$$V(\tau) = \sum_{i=0}^{n} w_i f_i\left(S_i(\tau)\right)$$

PFE is a forward-looking measure of counterparty credit risk that estimates the maximum potential loss a financial institution could incur if a counterparty defaults on a derivative or other contract at a specific future time. PFE is traditionally calculated by simulating gains or losses on a portfolio of trades over time. The resulting figure is expressed at a given confidence level α (typically 95%) and a time horizon τ:

$$\mathrm{PFE}_\alpha(\tau) \;=\; \inf\{y \;\mid\; P(E(\tau) \leq y) \geq \alpha\}$$
$$E(\tau) = \max(V(\tau), 0)$$

where the exposure E(t) is defined as the positive maximum of the portfolio value V(t) at the time horizon τ and P is the cumulative distribution function that E(t) takes a value below y. For simplicity, in this challenge it is assumed that the value of each asset S is an independent stochastic variable following a normal distribution:

$$p(S(t))) = \frac{1}{\sigma\sqrt{2\pi t}} e^{-\frac{1}{2\sigma^2 t}(S-\mu t)^2}$$

Where μ is the drift, σ the volatility and t the time. In the literature this is also known as a Wiener process. In this challenge the focus is on European call and put options as derivatives as they are the easiest option class[1]. A European call and put option on an asset gives the owner the right to buy or sell it for a strike price K at their expiration date of the option τ. The payoff f(τ) is given by:

$$f(t) = \max\{(K - S(t)), 0\}, \text{ for a put option}$$

$$f(t) = \max\{(S(t) - K), 0\}, \text{ for a call option}$$

b. Classical Monte Carlo approach

Nowadays many financial problems like computing PFE are studied via Monte Carlo methods. The basic idea behind all of them is to simulate random iterations of m paths of the underlying stochastic assets of the portfolio of interest and to compute the overall portfolio value V(t) for each of them. Quantities of interest like the average payoff can then be readily obtained:

$$E\left[V\right] = \frac{1}{m}\sum_{j=0}^{m} V_m$$

The higher the chosen number of iterations m the more accurate the predictions of the Monte Carlo techniques. For standard Monte Carlo techniques the estimation error scales as:

$$\epsilon = O(1/\sqrt{m})$$

which means that e.g. one needs a 100 times more samples to improve the estimate by one decimal place. This is the main bottleneck in many applications.

c. Quantum Monte Carlo/Quantum Amplitude Estimation

To illustrate how quantum computing can provide an advantage we take the simple example of estimating the distribution of a coin toss with probability p for heads and 1-p for tails. In classical Monte Carlo we would flip the coin m times and count the number of times we see head k to estimate p via p=k/m.

In order to use a quantum approach first encode the probabilities in the amplitudes of a set of qubits. Then construct an operator A which maps the probability p on the amplitudes of a qubit:

$$A\left|0\right\rangle = \sqrt{p}\left|1\right\rangle + \sqrt{1-p}\left|0\right\rangle$$

[1] As compared to e.g. American options which can be exercised at any time before their expiration date.

Then apply a so-called Grover iterate Q which consists of making a phase-flip around the I1> state S_x and around the initial state I0>, S_0. That means only the sign of I1> is changed and the I0> is left unchanged respectively:

$$Q = -AS_0A^\dagger S_x, \text{ where: } S_0 = 1 - |0\rangle\langle 0|\,, \quad S_x = 1 - |1\rangle\langle 1|$$

By rewriting p in terms of an angle:

$$p = \sin^2(\theta)$$

$$A = \sin(\theta)\,|1\rangle + \cos(\theta)\,|0\rangle$$

One can verify that Q corresponds to a rotation:

$$Q^m\,|0\rangle = \cos((2m+1))\theta)\,|1\rangle + \sin((2m+1)\theta)\,|0\rangle$$

One can then use quantum phase estimation to extract the angle of the rotation Q from the amplitudes and thus obtain the probability p. Contrary to classical Monte Carlo methods, the error scales as:

$$\epsilon = O(\frac{1}{n})$$

Which corresponds to a quadratic advantage compared to the classical method. For a more thorough derivation see the literature below.

d. Proposed Quantum Approach

Quantum approaches should start by encoding the portfolio's underlying assets' probability distribution in a qubit register. It is easier to begin with a circuit for a single asset and increase the portfolio size once a first working quantum circuit has been set up. Next the payoff of the European call or put option has to be computed with the circuit. By encoding the exposure and a threshold oracle for the corresponding of the cumulative distribution function P for a given value one can use Quantum Amplitude Estimation (QAE) to obtain an estimate of P, see literature under resources below. This is only a proposal. Participants are encouraged to innovate and test alternative approaches.

e. (Optional) path dependent options

Participants who want to challenge themselves more can alternatively investigate path dependent options like Asian options, defined as:

$$f(\bar{S}) = \max(0, \bar{S} - K)$$

Where S is the arithmetic average over the underlying assets value at d points until the option maturity:

$$\bar{S} = \frac{1}{d}\sum_{t=1}^{d} S_t.$$

Or Barrier options. Those are similar to the above mentioned European options but are only activated or extinguished if the asset crosses a predetermined level, called a barrier. There are in general two different types of barrier options:

- **Knock-Out:** The option expires worthless if the underlying asset crosses a certain price level before the option's maturity.
- **Knock-In:** The option has no value unless the underlying asset crosses a certain price level before maturity.

A knock-in barrier e.g. would be fined as:

$$f(S) = \begin{cases} \max(0, S_T - K) & \text{if } \exists t \text{ s.t. } S_t \geq B \\ 0 & \text{otherwise} \end{cases}$$

Where T is the time to maturity and S_t is the asset price at time t $0 < t \leq T$ over a given barrier level B. In order to implement these types of option in a quantum circuit one has to discretize the total available time and encode the value of each underlying asset at time t in a respective quantum register. More information can be found in the literature provided below.

4. Expected Deliverables

- **Working Prototype:** A functional model that takes data as input and produces a solution to the posed challenge. We recommend a step-by-step approach, beginning with a proof-of-concept for a very simple toy model before expanding to a full model, e.g. tackling larger data sets or problem instances.
- **Technical Explanation:** Participants must be able to explain the quantum algorithms used, the data model, and the rationale for choosing the specific approach. The focus should be on the clear and deliberate use of methodology and the benchmarking process, rather than solely on high-performing results. Participants should benchmark their quantum model against standard classical solutions. If more than one model was

developed during the hackathon the team may also benchmark against those.

- **Classical Bottleneck Analysis and Justification for Quantum Computing Approach:** An analysis of standard approaches and their bottlenecks. This is the basis for arguing why a quantum computing approach may be beneficial for the problem at hand.
- **SDG Impact Assessment:** An assessment of the impact of your solution on the main SDG tackled by this challenge. Your arguments should be backed by references to publications and publicly available data. Also: What other SDGs may be impacted by your solution (both to the positive or to the negative)? List them and argue for why your solution is relevant also to those.
- **Business Case:** A concise plan outlining the commercial viability, target users, and market strategy. Think about how your solution could be commercialized, how you would approach commercialization, what would the product be, who would be the customers, how you would make money with it.
  Also: In what other fields / problems could your solution be useful?
- **Pitches & Technical Deep Dive:** At the end of the hackathon you will have the chance to present your solution in a final presentation as a 5 min pitch with slides. Please explain the solution's potential and its future roadmap, including an honest discussion of the challenges that need to be addressed before it can be a fully realized product, all based on the above deliverables. The pitch will be followed by a 3 min Q&A with the jury for the respective challenge, which consists of 50% technical experts and 50% business experts.
  The overall 10 best teams across all challenges get to show the same 5 min pitch in front of all participants and all jury members again, this time without Q&A.

Importantly: A deeper investigation into one quantum computing approach is preferred over a superficial overview of many. The goal is not to find a single "best" model, but to demonstrate a clear understanding of the chosen approach. The solution should focus on demonstrating how the quantum approach can be applied to the problem, specifically addressing the challenge specified above. The goal is to articulate a compelling case for a future computational speed-up or greater accuracy, rather than proving an immediate advantage.

Target audience: The project should be compelling to technical professionals who are interested in exploring how quantum computing can be applied to their domain.

5. Resources

[Context] SDG 8: https://sdgs.un.org/goals/goal8

[Tutorial] Quantum amplitude estimation
https://qiskit-community.github.io/qiskit-finance/tutorials/00_amplitude_estimation.html

[Literature] Option pricing using quantum computers
https://arxiv.org/pdf/1905.02666

[Literature] Risk analysis using quantum computers
https://www.nature.com/articles/s41534-019-0130-6

[Literature] Quantum risk models:
https://arxiv.org/pdf/2103.05475