

**Operations Research: theory and  
applications to networking**  
*Lesson 1 – Warm-Up*

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**CCNE+ICT**

# Mathematical Optimization

= Select the “best” element from a set of available alternatives

Simplest case: minimizing a real function

- choosing input values from an allowed input set
- computing the value of the function

Area of Applied Mathematics

# Structure of an Optimization Problem

**Minimize or Maximize**

**Objective Function**

**Subject to**

**Constraints**

**With Variables**

**Variables**

# Parameters vs Variables

## Parameters

**Constant terms**  
**The input of the program**

## Variables

**What we want to optimize**  
**The output of the program**

# Taxonomy of Mathematical Optimization

## Convex Programming

- Objective function and constraints set are convex
- The variables are continuous
- Includes linear programming
  - Objective function and constraints are linear

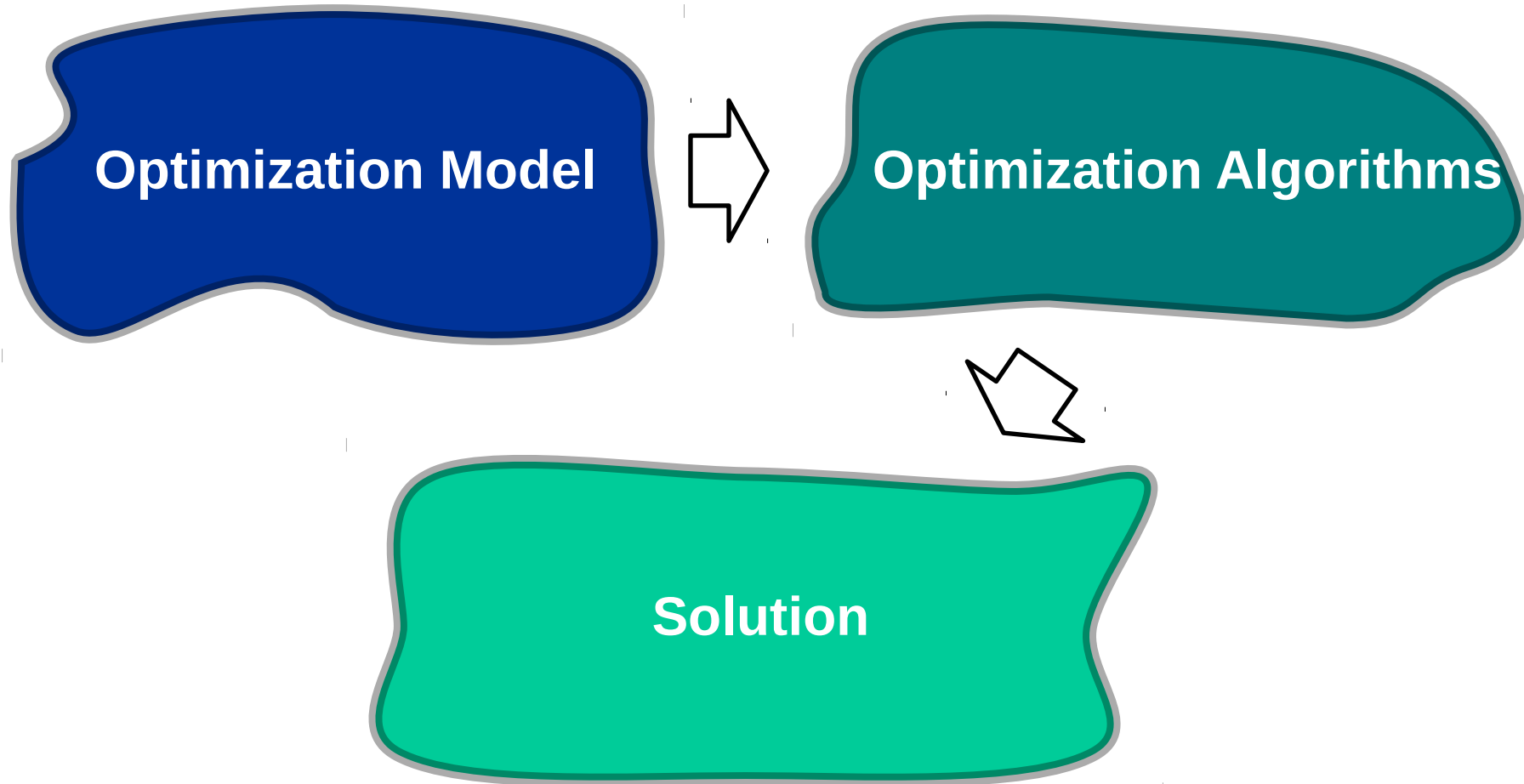
## Integer Programming

- Some variables take INTEGER values  
example  $c = \{0,1,2,3,4\}$

## Non-Linear Programming

- Non-linear objective functions and constraints

# Building Blocks of Optimization



# Optimization & Networks

Telecommunication Networks are one of the most important application fields for optimization

Internet: moves information from one point to another in an efficient way

Choosing how to move traffic in the network (routing)

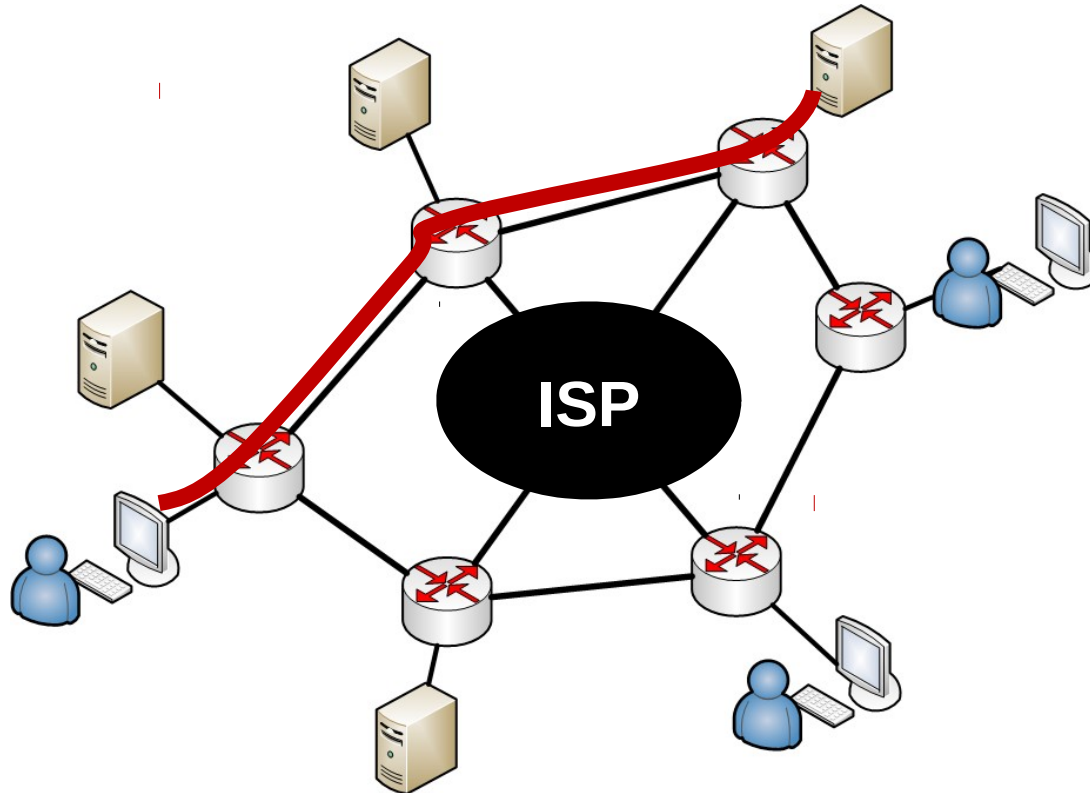
Dimensioning the network resources (design)

Guaranteeing user performance (Quality of Service)

## Example I: routing

Route traffic between servers and users over an Internet Service Provider Network.

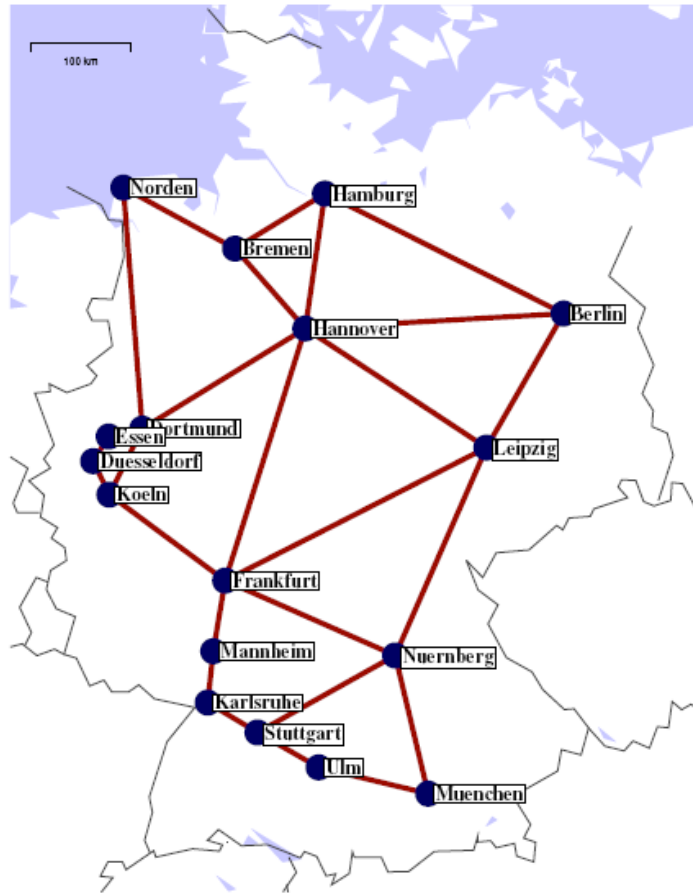
Choose the best route that minimizes the network delay.





# Example II: design

Design the Internet Service Provider Core Network



IP-over-WDM architecture

Minimize the Capital Expenditures (CAPEX)

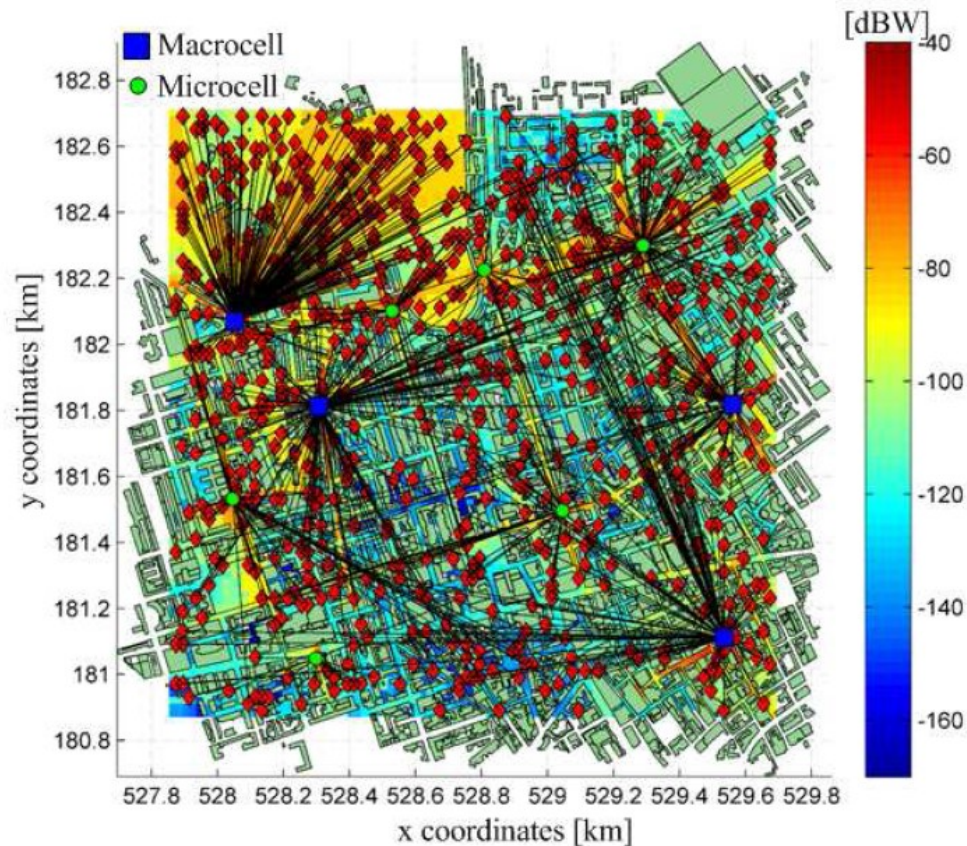
How many fibers on physical links?

How many routers?

## Example III (Quality of Service)

Associate mobile phones to Base Stations in a cellular network

Best association so that the Quality of Service experienced by users is maximized.



# The decision process

## Problem Definition

“I want to minimize the total delay in my network”

## Mathematical model

$$\begin{array}{l} \text{Minimize} \\ D_T = \sum_{(i,k) \in L} D_{ik}(f_{ik}) \end{array}$$

## Computer Program

```
delay=minimize_delay(flows);
```

## Optimization Solver



## Solution

```
Minimized Delay = 100 ms
```

# Optimization Solvers

IBM ILOG CPLEX Optimizer

**Solution of IBM**



**Available on-line**



**Very Powerful**



**Optimization Toolbox**



**Used in this Lab**

**You can download the student version**

# Interaction with solvers

## System Calls

- 0 Use a standard language, like “C”, “Java”, or scripting language like “python”
- 0 Functions to send/receive information from the solver
- 0 Less easy to use
  - Many functions to be invoked
  - Interaction with the solver need to be managed

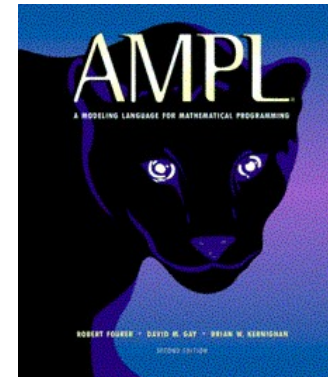
## Modeling Language

- 0 Use a proprietary language
- 0 Compact structures to define the problem
- 0 Easy to formalize complex problems
- 0 Syntax is different from standard languages

# Modeling Languages

## AMPL

- A Modeling Language for Mathematical Programming
- Developed at Bell Laboratories



## GAMS

- General Algebraic Modeling System



## MOSEL

- Used in this Lab together with XPress<sup>MP</sup>

**Mosel**

# What you will learn with the lab activities

Formulate an optimization problem with a mathematical formulation

Translate the mathematical formulation to a problem written in MOSEL language

Learn how to configure the XPress<sup>MP</sup> solver to run the problem

Solve the MOSEL problem with XPress<sup>MP</sup>

Analyze the results

Compare the optimal results with custom heuristics



Questions?